This presentation reports results of an investigation of the role of abstract thinking in human learning. Involved were 37 mentally retarded subjects, 62 normal subjects with equivalent chronological ages, and 50 normal subjects with chronological ages equivalent to the mean mental age of the mentally retarded group. Each subject was taught a six level order of classification with an increasing degree of difficulty and abstraction. At each level, the subject was presented with two concrete examples, then asked to select two others from a choice of six. The mentally retarded subjects performed significantly below their mental age and chronological age equivalents. The analysis indicated a strong dependence upon concrete cues and an incapacity of organizing information into conceptual hierarchical arrays by mentally retarded children. (SL)
INTRODUCTION

Educators and psychologists have long been aware of abstraction as a significant dimension of human intelligence. Few definitions of intelligence are ever presented without acknowledging the importance of the role of abstraction [1, 2, 3, 4]. However, while educators and psychologists find agreement in the significance of abstraction in human intelligence, few are in agreement on a definition of abstraction as it relates to human learning. These disparities in the uses of the term abstraction have been extensively explored and reported in the literature elsewhere [1]. Yet clearly, until researchers are capable of agreement on definition and terminology, little progress can be made in their field of inquiry. Such is the case with abstraction; the definition of abstraction as found in the literature is varied and lacks specificity. In addition, many studies subsume abstraction under a different topic, such as discrimination learning, concept formation, or classification/categorization processes, therefore, any definition to be set forth must consider these basic processes as instrumental.

This study was undertaken to construct a theoretical model of abstraction accountable to the variety and scope of research in this field and, further, to explore the role of abstraction in human thinking. To this end, a hierarchical model of abstraction and a test based on this model were constructed. One dimension on the model, the order of classification, was evaluated using mentally retarded and normal ability
subjects. The evaluation consisted of a hierarchical test constructed to ascertain whether human subjects order their world in hierarchical arrays and the relationship of mental ability to this process.

Since abstraction in general and hierarchical structures in particular have been the focus of much research and curricula emphasis in science education in the last decade, [5, 6, 7, 8] it is hoped that such a model will provide a systematic theoretical framework for further studies in this area.

Theory

Abstraction is considered here as a cognitive process of discriminating specific attributes of the environment that can be combined to form generalized representations of experience. These representations can be categories, sets of relations, or operations. The ability to form categories or to classify experience is considered to be the most basic of the three modes of representing experience and fundamental to representations which involve sets of relations and operations. Representations which involve the formation of sets of relations (establishing logical or causal connection between attributes, objects or classes) are considered more complex than categorization, since relational statements can only be made subsequent to categorization. The ability to perform operations, changing one thing into something else, will be defined as the most complex of the three modes of representing experience since before operations can be performed categories and/or relations must be set forth. The organization of these three modes of representing experience as set forth here is not unique and is well supported in the literature [7,9].
These three modes of abstraction, (1) Classification, (2) Relations, and (3) Operations are referred to as Orders of abstractions. An Order of abstraction is a cognitive mode of representing experience. The three Orders to be dealt with in this study, the Order of Classification, the Order of Relations, and the Order of Operations, are hierarchically arranged as shown in Figure 1.

The first Order in the hierarchical model is the Order of Classification, followed by the Order of Relations, with the Order of Operations located at the top. Within each Order of abstraction there exist Levels of abstractions. These levels of abstraction exist in each Order and are hierarchically arranged in increasing generality and inclusiveness. In Figure 1, Levels of Abstraction are depicted by horizontal lines of increasing length transitig each Order. As one progresses up the hierarchy from level to level within each Order, the power of abstraction or generality and inclusiveness of the instances increases commensurately. An example of this progression would be the concept "mouse" subsumed by the concept "rodent" subsumed by the concept "mammal." The progression from a lower level abstraction to a higher level abstraction, in this case mouse-rodent-mammal, is indicative of an increase in generality and inclusiveness (of attributes).

1 Levels of abstraction will be defined as follows: A level is an assembly of things of a definite kind, e.g., a collection of systems characterized by a definite set of properties and laws, and such that it belongs to an evolutionary line, though not necessarily to a line of biological descent. Some of the emergent characteristics or 'nova' are the exclusive property of the given level. For every 'novum' has presumably emerged, in the course of a process, from preexisting levels [9].
The three cone-shaped structures represent the Orders of Classification, Relations, and Operations, respectively. The horizontal lines designate the various levels of abstraction in each Order. The levels are hierarchically arranged in order of increasing abstraction. Six levels are designated for the Order of Classification; however, it is conceivable more exist beyond six. These six levels are: Level I, Attribute Identification; Level II, Attribute Recognition; Level III, Object Recognition; Level IV, Class Recognition; Level V, Class of Classes Recognition, one class; Level VI, Class of Classes Recognition, two classes.
It is usually the case for one to infer identity or some other categorical significance not from a single attribute exhibited by an instance but from several attributes taken together [8]. Since at high levels of abstraction, the subsumptive power of an instance can be extremely large, this implies an overlap, or sharing of attributes. To account for the overlap, or sharing of attributes at any level of abstraction, the term Degrees of Complexity is used. The Degree of Complexity is defined as the number of shared attributes among instances within a level, as the level of abstraction increases, the possibility for instances to share attributes in common also increases. This increase in complexity derives in part from the increase in generality and number of instances contained in superordinate categories. This is shown in Figure 1 as a broadening of the cone upward and hence a commensurate increase in the length of a line at each level. For example: an alligator and a hippopotamus would be an example of an instance with low complexity. Although both share attributes, live in or near water, walk with four legs, are found in tropical areas, etc., they are clearly distinct creatures physiologically and morphologically; one is a reptile, the other a mammal. Another example: two primitive members of the horse family, Eohippus and Mesohippus, share almost all of the same attributes with the exception of size -- Mesohippus was larger and with different foot and tooth structure -- yet they are clearly distinct species since one, Eohippus, lived 10,000 years before the other. The Degree of Complexity in this example is considered high due to the sharing of many attributes for each instance. There is a greater opportunity for shared attributes among instances at higher levels of abstraction since more attributes are subsumed by each instance within these higher levels.
In Figure 1, as one progresses from a lower level to a higher level, the lines transiting each order increase in length. The increase in length is representative of greater and greater Degrees of Complexity, or greater and greater overlap or sharing of attributes. In Figure 1, the Degree of Complexity increases at all levels, from left to right. As the number of shared attributes increases (increased complexity), the degree of discrimination required to detect the individual attributes within a level becomes more acute, hence explaining the increased complexity.

Although each Order of Abstraction demands different cognitive strategies in constructing representations, these Orders are not mutually exclusive. The three Orders can be thought of as existing in a hierarchy as exemplified in Figure 1. The Order of Operations, as represented by Figure 1, can be thought of as dominating, while at the same time, depending upon its two lower subsidiaries, i.e., (1) the Order of Relations and (2) the Order of Classifications, for its existence. This means operations cannot be performed until relational statements have been set forth, and relational statements cannot be set forth until units have been identified or classified. Thus, the Order of Classification is primary to the two higher Orders of Relations and Operations. This dependency of Orders is indicated in Figure 1, by lines connecting the base of each Order to its immediate predecessor. The spatial position of each Order, namely, Relations higher than Classifications, Operations higher than Relations, is set forth in this manner to indicate greater cognitive complexity as one progresses from Classifications to Operations. Each Order is represented by a cone, containing a series of levels. The shape of a cone is such that at the top, broad generalized concepts are represented which subsume less inclusive concepts as one progresses toward the base. Taken together,
these three Orders are fundamental to a body of knowledge - since they represent various cognitive strategies used to represent particular experiences with either immediate sense data or the symbolic representations, which make up a body of knowledge.

The Order of Classification

The hierarchy begins with the Order of Classification. This Order begins, as presented in Figure 1, with immediate sense data. The Order of Classification concerns itself primarily with object to class identification. (A detailed elaboration of this Order will be presented later in the discussion since this Order has been selected as the main focus of the study.) Once objects or classes have been identified, relational statements about them can be made, hence the next Order, Relations.

The Order of Relations

The Order of Relations, as a hierarchy, begins with simple relational statements concerning attributes and progresses upward to complex statements concerning relations of relations. The Order of Relations, as indicated by Figure 1, is dependent upon Order of Classification, and is connected to it by a line. This dependency illuminates several important criteria about the Order of Relations. The dependency indicates that relations are not concrete, they are not found directly in sense data; they arise out of it. Immediate sensory experience does not contain ready-formed entities called "less than," "same as," "equivalent to," "brighter than," etc. For any of these relational situations to exist there must be units (i.e., attributes, objects, classes), identified prior to being related. Before a predator-prey relationship can be set forth, one must be capable of identifying predator and prey.
The Order of Relations, as indicated by Figure 1, is a hierarchy of various levels of abstraction. The lower levels indicate simple relational statements, such as \( A = A \), a relational statement involving only one element. This statement would be equivalent to saying, "I am as tall as myself," an obvious and simple tautological relation. However, as one progresses to higher levels of abstraction in the hierarchy, relational statements become more complex. For example: \( A = B \) or \( B > C \), here more than one element is being related. At still higher levels, \( A = B = C \), therefore \( A = C \), the complexity increases with an increase in elements to be related. In still higher levels of abstractions, complex statements of relations can be set forth by increasing the number of elements to be related, \( A = B = C = D \ldots \), while at the same time using different varieties of relational statements (such as symmetry-asymmetry, transitive-intransitive, reflexive-irreflexive) in conjunction with a number of elements. Relational statements at this level would be exceedingly complex. Once units have been classified and/or relational statements have been set forth, it is then possible to perform operations on these units and/or relations.

The Order of Operations

The Order of Operations, by definition, includes transformations, which is a cognitive process of changing one thing into another thing. As shown in Figure 1, it is the highest Order in the hierarchy and thus dominates it, since it is possible to perform operations by using instances from the Order of Relations and/or the Order of Classification. While dominating the hierarchy, its existence is totally dependent upon the subordinate Orders. To perform an operation, one must have something, a unit or relation, to operate on. As with the other Orders, the Order
of Operations is a hierarchy which begins with simple one-step operations and progresses upward to ever more inclusive operations of subsuming operations. The recognition of going from a one-dimensional measure to a two-dimensional measure to a three-dimensional measure would be an instance of simple one-step operations to more complex, subsuming operations.

The Order of Classification

A detailed description of the Order of Classification is presented here since it was the object of investigation in this study. As stated previously, the Order of Classification is a hierarchy consisting of broad inclusive concepts at the top, subsuming less inclusive concepts at lower levels. The lowest levels of the Order of Classification are concerned with representations of immediate sensory data at the attribute level, while higher levels contain classes of categories or concepts built upon attributes assimilated in lower levels. At still higher levels are concepts subsuming lower-level concepts. The lowest levels of the Order of Classification contain attribute identification and recognition, while at higher levels are classes, or classes of classes. Figure 2 presents a schematic view of the various structural features in the Order of Classification.

The cone has been selected to represent the hierarchy because the notion of greater inclusiveness can be conveniently shown by such a structure. At the top of the cone are the most abstract concepts, subsuming less abstract and less inclusive concepts and becoming even less inclusive as one progresses toward the base. In Figure 2, the Order of Classification displays six levels of abstraction. They are: Attribute Identification,
Levels of Complexity:

- Level VI
- Level V
- Level IV
- Level III
- Level II
- Level I

Figure 2

Order of Classification
Attribute Recognition, Object Recognition, Class Recognition, Class of Classes Recognition (one class), and Class of Classes Recognition (two classes). In this study, the first six levels have been selected since they can be assessed in a study of this size and, moreover, they are essential to the acquisition of higher levels of abstraction. An individual incapable of performing successfully at the first three levels in this order would not be expected to perform successfully at higher levels. The six levels will be defined as follows:

**Level I - Attribute Identification**

In the Order of Classification, Level I provides loci for attribute identification. Discrimination of sense data begins at Level I. At this level, the environment is perceived by the individual to be consisting of discrete attributes (different shapes, sizes, colors, tastes, etc.). While attributes are discriminated at this level they are not verbally labeled. As an example: a child may be shown a hammer and a nail, then he may successfully find these objects in a chest of tools without verbally labeling either object's name or attributes. The child need not verbalize the attributes to successfully make the selection and thus this example demonstrates that object and attribute identification can proceed independently of labeling. Since attribute identification is so fundamental to human abstract thinking, Level I is the most basic and lowest level of abstraction in the Order of Classification.
Level II - Attribute Recognition

This is the level where an individual is not only aware of discernible features of an event, but is capable of verbally recognizing a discrete attribute. A child is displaying a Level II abstraction when he is capable of verbally identifying all red marbles from a jar of different colored marbles. Such a discrimination indicates that he is aware of differences, at Level I, as discrete units in his environment and, moreover, can correctly assign verbal labels to these perceived characteristics. The distinguishing feature at this level is the cognitive capability of correctly matching symbolic representations to the appropriate attribute(s). As indicated in Figure 2, Level II subsumes Level I.

Level III - Object Recognition

This refers to that level where an individual groups several specific attributes together and constructs an object name to stand in place of these attributes. The name of the object subsumes the specific attributes which refer to the object. It is at this level where clear economy of thought begins. An object that is classified as an apple must be round, derived from a particular tree, of a particular color, cellular in structure, and so forth. A chemist need only see a symbol Cu (copper) to be able to call forth a number of highly specific attributes which are represented by it. Objects and events are categorized on the basis of their attributes or characteristics [10]. Clearly, only until the lower levels are mastered can an individual manipulate abstractions at this higher level.
Level IV - Class Recognition

Classes can be somewhat operationally defined as recognized sets of items of information grouped by virtue of their common properties [11]. Class recognition, even more than object recognition, "reduces the complexity of the environment and thus represents even greater economy of thought [12]." At this level, an individual can dispense with specific names and deal with generalized representations of specific names. The class: "rodents," which not only refers to rats but also to squirrels, chipmunks, rabbits, and beavers, provides a case in point. Abstractions at this level are more difficult to attain than lower-level categories because the amount of generality is much greater than in these lower levels; thus, more information must be synthesized to recognize these higher level categories. In Figure 2, this increasing generality is portrayed by an expanding cone.

Level V - Class of Classes Recognition

This level consists of classes with similar attributes which can be grouped together to form a class of classes. The term Hominoids subsumes the lower groups of Hominids (man-like) and Pongids (ape-like). The term Hominoids is more inclusive than either Hominids or Pongids since it includes all attributes from these two categories. Again, there is a reduction of complexity of the environment by such abstractions, but the amount of intellectual capacity needed to attain this level of abstraction is considerable due to the ever-increasing generality of classes.
Level VI - Classes of Classes Recognition

This level constitutes the highest level of abstraction to be dealt with in this study. Level VI refers to extremely broad generalized categories which demonstrate powerful subsumptivity. An example of subsumption in Level VI is presented using the same concepts as in Level V. The class mammal subsumes all living animals with mammary glands and that among other things have a backbone and a four-chambered heart. Mammals therefore subsume Anthropoids of Level V. The inclusivity of attributes of objects at this level is immense. Hence, a commensurate increase in intellectual capacity to achieve abstractions at this level. In Figure 2, Level VI subsumes all previous levels of abstractions in the Order of Classification.

Degrees of Complexity

The existence of levels of abstraction does not take into account the overlap of attributes from instance to instance in Level III and above. (Level III is the level where attributes begin to be grouped.) At these upper levels of abstraction, there exists various degrees of perceptual differentiation that an individual must bring to bear in order to discriminate among instances. This differentiation acuity has been referred to as Degrees of Complexity. The degree of complexity is the amount of perceptual differentiation that one must use in order to correctly discriminate instances of an object or class from another instance of the same object class. The degree of complexity is related to the amount of attributes that the particular objects/classes hold in
The more shared attributes, the greater the complexity.

In Figure 2, the Degree of Complexity is indicated by the length of the solid lines transiting the cone. As the level of abstraction increases, the Degree of Complexity increases also. This is due simply to an increase in the amount of attributes available to be shared at these high levels of abstraction. However, from Level III upward, any particular level may have instances which either share one or many attributes. To account for the sharing of attributes at any particular level, the Degree of Complexity will be defined as increasing from the left to the right at any level. In Figure 2, the left side of the cone, in all levels, represents disjunctivity, or instances (objects/classes) which share only one attribute. For example, discriminating the object, orange, from two objects, an orange and a black-colored ballp... Here the only clearly appre...ensible attribute shared is solidness; therefore, the amount of perceptual differentiation needed to make the discrimination is slight due to the sharing of only one attribute. Such an instance would be located on the extreme left side of the cone.

As one progresses across a level, from the extreme left (disjunctive side) to the extreme right (conjunctive side), the amount of shared attributes increases. Instances which share two or more attributes are referred to as conjunctive; thus, in Figure 2, conjunctivity increases to the right. Objects/classes which share large numbers of attributes, such as Hominids and Pongids (men and apes), would be located on the far right side of the cone. These instances would require significantly more discrimination acuity than those instances located on the far left side of the cone.
METHODOLOGY

Subjects

The study was based on a sample of 149 subjects divided into five groups. The sample parameters used for the study are displayed in Table 1. The mentally retarded group, consisting of 37 subjects, was randomly selected from special education classes in an urban New Jersey school district. The four remaining groups consist of normal subjects selected from New York City and New Jersey schools. Two of these four groups were selected from private school populations and the remaining two groups were selected from public schools. The two groups selected from the New York City and New Jersey public schools consist of 32 normal subjects whose chronological age was equivalent to the mean mental age of the mentally retarded group and 33 normal subjects whose chronological age was equivalent to the mean chronological age of the mentally retarded group. The two private school groups consist of approximate mental age and chronological age equivalents. Since I.Q. information was inaccessible in the New York City schools, normal subjects were considered to be any students reading on or near grade level. In one private school where the normal mental age equivalent subjects were drawn, no reading scores or I.Q. information was available; subjects were therefore selected as average by teacher evaluation of their classroom performance. Any subjects with language problems or vision problems were eliminated from the study.
TABLE 1

Population Parameters

<table>
<thead>
<tr>
<th></th>
<th>Group 1</th>
<th>Group 2</th>
<th>Group 3</th>
<th>Group 4</th>
<th>Group 5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Educable.</td>
<td>Normal Chronological</td>
<td>Normal</td>
<td>Normal Chronological</td>
<td>Normal Chronological</td>
</tr>
<tr>
<td></td>
<td>Mentally Retarded</td>
<td>Mental Age Equivalent</td>
<td>Mental Age</td>
<td>Equivalent</td>
<td>Equivalent</td>
</tr>
<tr>
<td><strong>Age (in years)</strong></td>
<td>Range 13-2-16-0</td>
<td>9-0-10-4</td>
<td>12-10-15-6</td>
<td>8-0-10-10</td>
<td>14-11-18-5</td>
</tr>
<tr>
<td><strong>Mean</strong></td>
<td>14.2</td>
<td>9.4</td>
<td>14.0</td>
<td>9.42</td>
<td>16.5</td>
</tr>
<tr>
<td><strong>SD</strong></td>
<td>1.13</td>
<td>.42</td>
<td>.77</td>
<td>.92</td>
<td>.98</td>
</tr>
<tr>
<td><strong>I.Q.</strong></td>
<td>Range 60-75</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>a</td>
</tr>
<tr>
<td><strong>Mean</strong></td>
<td>66.5</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>92-122b</td>
</tr>
<tr>
<td><strong>SD</strong></td>
<td>4.8</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>104.1</td>
</tr>
<tr>
<td><strong>N</strong></td>
<td>37</td>
<td>32</td>
<td>33</td>
<td>18</td>
<td>29</td>
</tr>
</tbody>
</table>

*I.Q. scores unavailable, Ss' MAT reading score, on or near grade level, used as criteria for group placement.

*I.Q. and reading scores unavailable.

*I.Q. statistic available for only 17 Ss.
Testing Procedure

All subjects were individually taught a first order hierarchy. This hierarchy, based on the theoretical model presented above, consisted of a series of tests representing each level of abstraction in the Order of Classification. As the level of abstraction increases, from Level I to Level VI, there is a commensurate increase in generality and inclusiveness and thus an increase in the degree of difficulty for each test.

Each test consists of two boxes. The contents of the first box presented to the Ss contained two examples of the instance to be taught. Once the examples have been presented, the box and its contents were removed and the second box was presented. The second box contained two examples of the instance taught plus four distractors. The subjects were asked to select examples of the instance taught to them. Since the contents of the second box were used to assess the S's ability to acquire the abstraction taught to him, no verbal cues were given. All tests for the six levels of abstraction in the Order of Classification were constructed and presented in this manner.

The criterion level of achievement was successful completion of the task within two trials. Upon successful completion of Level I, the subject was taught Level II, in the same manner and with the same criterion for success. Successful completion for all tests occurred when the subject identified both instances correctly. Unsuccessful completion of the test occurred when the subject (a) failed to identify either instance

1Probability of a chance combination of two correct instances on each trial is $\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$. 

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correctly, (b) only identified one instance correctly, or (c) identified both instances correctly but placed one or more incorrect instances in the correct group.

The presentation sequence and the success criterion continued until the Level VI test terminated the testing sequence.

If a subject failed to achieve a particular level within two trials, the testing sequence was still continued to the highest level. By testing to the highest level, it was possible to detect whether subjects can, by some cognitive process that was not predicted in the rationale of this study, go to a higher level after failing a lower one. It is anticipated that, if the hierarchy is properly organized, most subjects will not be able to achieve a higher level after failing a lower one.

The six levels tested in the Order of Classification were: Level I, Attribute Identification; Level II, Attribute Recognition; Level III, Object Recognition; Level IV, Class Recognition; Level V, Class of Classes Recognition, subsuming one class; Level VI, Class of Classes Recognition, subsuming two classes. In Level I, two objects, galena and pyrite, were presented to the subjects. The Ss were asked to identify these two instances from a cluster of six similar but non-identical objects. The identification was based on the attributes of the objects and the selection required no verbalization. At Level II, two attributes of each of the two objects were described for the subject (i.e., color and luster). When subjects received the Level II test box they were expected to select from its contents the appropriate instances and verbally identify the attributes taught to them. In Level III, the specific name of the objects was stated, for example, galena and pyrite. Again

1 The terms iron and lead were used in the study.
the subjects were expected to select and state the appropriate instance.

In Level IV, two classes of objects were presented, metallic and non-metallic. A separate test was given for each class. In Level V, two more-inclusive classes were presented, minerals which subsumed metallic and non-metallic and life products. In Level VI, minerals and life products were subsumed by the concept Natural phenomena. Figure 3 displays how the six test categories are hierarchically arranged according to levels of abstraction.

Data Analysis

A record for each student was kept showing the maximum level of abstraction achieved and the number of trials taken to reach criterion at each level. A summary graph for each of the five groups was constructed. The ordinate of this graph represents the cumulative number of trials to criterion for all subjects. The abscissa represents the levels of abstraction achieved; thus, a cumulative record of trials to criterion was obtained. The mean level achieved for each group was statistically analyzed. A one-way analysis of variance was used to determine if there was a statistically significant difference among the means of the groups. A confidence level of p<.05 was used. The average levels achieved for each group were compared by using the Scheffé contrast formula (p<.05) and their relationships set forth as presented in the results section.
Level VI

Natural Phenomena

Level V

Mineral

Non-metallic

Living Product

Level IV

Metallic

Level III

Objects

Objects

Level II

Attributes

Attributes

Level I

Identification

Identification

Figure 3

Hierarchical arrangement of test situation for the Order of Classification
Results

Average Level Achieved

The maximum level that a subject could achieve was 6, the highest level in the abstraction hierarchy. To reach Level 6, the subject had to pass eight subtests which comprise the Order of Classification Natural Things Test. The average test score and the average level achieved for each group was computed and is set forth in Table 2. As observed in this table, Group 1, mentally retarded Ss, achieved the lowest level 2.96, of all groups. Group 2, the mental age equivalents of Group 1, achieved the next to the lowest level, 3.75, among all groups. The remaining Groups 3, 4, and 5 achieved consistently higher levels than either Group 1 or Group 2. Group 3, the chronological age equivalents to Group 1, while achieving substantially higher than Groups 1 and 2, achieved slightly less than Group 4 and Group 5. However, the difference is not significant and could be attributed to the fact that Groups 4 and 5 come from a New York City private school population and thus constitute a socially and economically advantaged group compared to subjects from the public school.

For the average highest test score per group (i.e., the highest test passed), Group 1 again scored lowest of all groups, with Group 2 again being next to the lowest. Groups 4 and 5 once again scored very high, in both cases almost reaching the highest possible criterion score.

Analysis of Variance of Average Level Achieved

A one-way analysis of variance of the average level achieved among all groups was computed (p < .05). The results of the ANOVA, given in Table 3, show there is a significant difference among the groups at p < .05.
Table 2

Average Level Achieved Per Group

<table>
<thead>
<tr>
<th>Group</th>
<th>Average Level Achieved</th>
<th>SD</th>
<th>Average Test Score Achieved</th>
<th>SD</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group 1</td>
<td>2.96</td>
<td>1.05</td>
<td>3.78</td>
<td>1.51</td>
<td>37</td>
</tr>
<tr>
<td>Group 2</td>
<td>3.75</td>
<td>1.01</td>
<td>4.96</td>
<td>1.51</td>
<td>32</td>
</tr>
<tr>
<td>Group 3</td>
<td>5.06</td>
<td>1.24</td>
<td>7.03</td>
<td>1.48</td>
<td>33</td>
</tr>
<tr>
<td>Group 4</td>
<td>5.67</td>
<td>.84</td>
<td>7.50</td>
<td>1.04</td>
<td>18</td>
</tr>
<tr>
<td>Group 5</td>
<td>5.62</td>
<td>.77</td>
<td>7.65</td>
<td>.72</td>
<td>29</td>
</tr>
</tbody>
</table>

Group 1 = Mentally retarded  
Group 2 = Mental age equivalent of average scholastic ability (Public school)  
Group 3 = Chronological age equivalent (Public school)  
Group 4 = Mental age equivalent of average scholastic ability (Private school)  
Group 5 = Chronological age equivalent (Private school)
Table 3
Analysis of Variance of the Average Level Achieved for All Groups

<table>
<thead>
<tr>
<th>Sum of Squares</th>
<th>Degrees of Freedom</th>
<th>Mean Square</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between</td>
<td>178.0461</td>
<td>4</td>
</tr>
<tr>
<td>Within</td>
<td>150.5983</td>
<td>144</td>
</tr>
<tr>
<td>Total</td>
<td>328.6444</td>
<td>148</td>
</tr>
</tbody>
</table>

\[ F = 42.5613 \]
\[ F(4,144), \alpha=.05=2.44 \]

Table 4
Results of Scheffé Contrast Among Sample Means
\( P \leq .05 \)

<table>
<thead>
<tr>
<th>Contrast Groups</th>
<th>Upper Limit</th>
<th>Lower Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 - 2</td>
<td>-.03284</td>
<td>-1.57535</td>
</tr>
<tr>
<td>1 - 3</td>
<td>-1.3497</td>
<td>-2.8796</td>
</tr>
<tr>
<td>1 - 4</td>
<td>-1.8026</td>
<td>-3.6389</td>
</tr>
<tr>
<td>1 - 5</td>
<td>-1.8824</td>
<td>-3.4671</td>
</tr>
<tr>
<td>2 - 4</td>
<td>- .9754</td>
<td>-2.8579</td>
</tr>
</tbody>
</table>
Groups 2, 3, 4, and 5 were contrasted with Group 1 and Group 2 with Group 4, using the Scheffé formula \((p=.05)\). The results are given in Table 4. Group 1's performance was significantly different \((p<.05)\) for all contrasts. The significant difference between Group 1 and Group 2 is particularly interesting in light of the fact that both samples are drawn from urban public schools and are equivalent in mental age. The significant difference between Group 2 and Group 4 is also noteworthy in that both groups are ostensibly mental age equivalents coming from different school population, the former urban and public and the latter urban and private.¹

**Average Cumulative Trials to Criterion**

The six level Natural Things Test is composed of eight subtests. The minimum number of cumulative trials to reach Level 6 is eight (when criterion is achieved in one trial for each of the eight subtests) and the maximum number is sixteen (two trials required to reach criterion on each subtest). Since all eight tests were administered to each subject, the average cumulative number of trials taken by each group for all eight tests provides evidence as to the amount of difficulty each group was experiencing with the Natural Things Test. The average cumulative trials for all eight tests was tabulated and is set forth in Table 5. It was expected that Group 1, the mentally retarded group, would experience the greatest cognitive strain, especially on the upper levels (4, 5, and 6) on the Natural Things Test. Evidence of cognitive strain for this group

¹Since no I.Q. data were available for Group 4, it need not be assumed these Ss were of equal M.A. These Ss were reported by their teacher to be achieving at grade level or slightly below it as compared to their peers in the same class. This is the extent of our knowledge about scholastic ability for this group.
Table 5
Average Cumulative Trials to Criterion Per Group

<table>
<thead>
<tr>
<th>Level</th>
<th>Test</th>
<th>Group 1</th>
<th>Group 2</th>
<th>Group 3</th>
<th>Group 4</th>
<th>Group 5</th>
<th>Group 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>1</td>
<td>1.30</td>
<td>1.13</td>
<td>1.09</td>
<td>1.22</td>
<td>1.03</td>
<td></td>
</tr>
<tr>
<td>II</td>
<td>2</td>
<td>2.60</td>
<td>2.25</td>
<td>2.24</td>
<td>2.27</td>
<td>2.14</td>
<td></td>
</tr>
<tr>
<td>III</td>
<td>3</td>
<td>4.00</td>
<td>3.35</td>
<td>3.33</td>
<td>3.33</td>
<td>3.24</td>
<td></td>
</tr>
<tr>
<td>IV</td>
<td>4</td>
<td>5.39</td>
<td>4.81</td>
<td>4.54</td>
<td>4.55</td>
<td>4.45</td>
<td></td>
</tr>
<tr>
<td>V</td>
<td>5</td>
<td>7.35</td>
<td>6.56</td>
<td>6.24</td>
<td>5.94</td>
<td>5.97</td>
<td></td>
</tr>
<tr>
<td>VI</td>
<td>6</td>
<td>9.24</td>
<td>8.38</td>
<td>7.81</td>
<td>7.16</td>
<td>7.31</td>
<td></td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>11.14</td>
<td>10.21</td>
<td>9.18</td>
<td>8.77</td>
<td>8.41</td>
<td></td>
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<tr>
<td></td>
<td>8</td>
<td>13.03</td>
<td>12.06</td>
<td>10.72</td>
<td>10.00</td>
<td>10.03</td>
<td></td>
</tr>
</tbody>
</table>

N=37  N=32  N=33  N=18  N=29

Group 1 = Mentally retarded
Group 2 = Mental age equivalent (Public School)
Group 3 = Chronological age equivalent (Public School)
Group 4 = Mental age equivalent (Private School)
Group 5 = Chronological age equivalent (Private School)
is a greater number of trials at all levels when compared to the remaining
groups. Group 2, the mental age equivalents of Group 1, also appear to be
experiencing difficulty with the test, especially Levels 4, 5, and 6.

Groups 3, 4, and 5 progress through the lower levels without much difficulty, but as can be seen in Table 5, Levels 5 and 6 apparently were difficult for some subjects in these groups. The data displayed in Table 5, while providing evidence of the amount of difficulty Ss experience with the Natural Things Test, do not directly support the validity of the hierarchy. If the hierarchy in the Natural Things Test is valid, Ss failing low-level tests should continue to fail upper-level tests. In Figure 4 a graph has been constructed showing the average cumulative trials to criterion for each group and the number of subjects who successfully completed each level but failed higher levels.

Group 1 is taken as an example, at Level 3, 30 Ss (N=37) achieved that level but failed the higher levels. However, progressing to Level 4, the next highest level, all but three Ss have failed to achieve this level and the subsequent two higher levels. Of the 37 Ss in Group 1 taking the Natural Things Test, only two succeeded in achieving all six levels. Failure at a low level and then subsequent failure on higher levels, as demonstrated by Group 1, provides further evidence for the validity of the subsumptive organization of the Order of Classification as presented in Figure 2 - and the hierarchical test of Natural Things.
Figure 6

Average cumulative trials to criterion vs Levels of Abstraction

GROUPS = Total number of Ss achieving that particular level.

<table>
<thead>
<tr>
<th>Levels</th>
<th>GROUPS</th>
<th>GROUPS</th>
<th>GROUPS</th>
<th>GROUPS</th>
<th>GROUPS</th>
<th>GROUPS</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>1 = 37</td>
<td>1 = 35</td>
<td>1 = 30</td>
<td>1 = 3</td>
<td>1 = 3</td>
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<td>2 = 17</td>
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<tr>
<td>VI</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Discussion

When the average level of abstraction achieved by Group 1 is compared to Group 2 (equivalent mental age normals), a significant difference, \( p < .05 \), was observed. Group 2 attained an average level of 3.75 which approximates the first class level, Level 4, in the Order of Classification. The average test score for Group 2 is also higher (4.96). Although one might expect to find similar if not identical performance by mentally retarded subjects and their equal mental age normal counterparts, this study showed that the equal mental age normal subjects performed better on the test of natural things \( (p < .05) \). Such a significant difference in level achieved between these two groups suggests that the mentally retarded individuals are not merely retarded in mental development, but exhibit different cognitive processes than the normal mental age equivalents. When the level achieved by Group 1 is compared to that of the remaining Groups 3, 4, and 5, further evidence of the low ability of Group 1 is observed. Of special interest in Groups 3, 4, and 5 is the performance of Group 4. This group achieved an almost perfect score, 5.67.

What is of interest here is that this group, while purportedly a normal mental age equivalent group drawn from an advantaged sociocultural-economic population, achieved the highest score. The school from which this sample was drawn did not provide I.Q. data and therefore it cannot be definitely stated that these subjects are precise mental age equivalents to the mentally retarded subjects. Therefore, the difference in performance

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1 At this level and all subsequent levels the number of attributes to be dealt with for each instance increases greatly; therefore, to simplify the task, verbal cues along with concrete exemplars of the instances were presented. This method of assigning class names considerably simplifies the task for the Ss as reported in previous concept acquisition studies.\[13,14]\.
between the mentally retarded subjects and these "equal" mental age normals may be attributed to cultural advantages. If this is the case, a significant observation is raised about the role of early, pre-school experience in determining performance on abstract learning tasks. Indeed, many students who are determined to be mentally retarded may be merely socially disadvantaged due to lack of educational opportunities during early childhood. Such an observation lends further credence to enriched learning experiences in the early preschool years for otherwise culturally deprived urban children.

It is worth noting that while Groups 3, 4, and 5 all were capable of achieving high levels, no group had a perfect score. This indicates that at Level 6, the highest level, all groups were exhibiting some difficulty. Since the test was carefully constructed so that each subtest represents the prescribed characteristics as presented in the Order of Classification hierarchy, validity of the test would be supported if Ss failing lower levels also failed higher levels and Ss passing lower levels exhibited increasing difficulty in reaching criterion on upper levels. Clearly, the validity of the test is supported in both these areas by (1) the low achievement in Group 1 and their successive failure on upper levels and (2) the difficulty exhibited by Groups 3, 4, and 5 in achieving criterion at the upper levels of the test.

By observing the average cumulative trials to criterion for each group it is possible to observe the amount of cognitive strain each group experienced with the Natural Things Test. Since the Natural Things Test is organized in increasing difficulty, it was expected that as Ss reach the upper levels they would require more trials to criterion than experienced at lower levels. This difficulty can be detected to a large degree by the average cumulative number of trials each group exhibited for each level.
(and hence its corresponding test), and the total cumulative trials to criterion for the total test. Even though most subjects in Group 1 failed at or near the third level in the hierarchy, they were nonetheless presented the higher level tests to determine their performance capacity and their cumulative trials to criterion. Since they consistently failed these upper-level tests each subject accumulated two trials at each level. Therefore, their cumulative trials to criterion score were quite high compared to the remaining groups where performance was not so debilitated. These data once again exhibit the cognitive strain endured by the mentally retarded subjects in performing higher abstraction tasks. Particular attention is once again directed to a comparison between Groups 1 and 2. Clearly, Group 2 performed better than Group 1 with respect to trials to criterion, thus further indicating greater facility in performing the task by the equal mental age normal subjects and hence suggesting they are operating in a different cognitive mode than the mentally retarded subjects.

Retest

Five weeks after the initial testing, ten subjects from Group 1 were selected for retesting based on prior test performance to yield a representative group of individuals who varied in level achieved. The retesting accomplished two objectives: (1) it provided evidence of reliability and (2) served as an attempt to determine the cognitive strategy the Ss used by having the Ss verbalize their reasons for selecting items in the test. The criterion for test reliability used here was that retest performance be identical to or slightly below original test performance and that Ss failing at low levels of the test exhibit failure
on all successively higher level tests in the hierarchy. With respect to retest reliability for these ten Ss, six Ss achieved the exact same level; two Ss actually dropped from Level 3 to Level 2; two Ss achieved higher levels, one going from Level 5 to Level 6, the other going from Level 1 to Level 2. With one exception -- a S going from Level 5 to Level 6 -- all retested Ss who failed at low levels continued to fail at the upper levels. The test is considered reliable since all but two Ss scored at or below their original performance and all but one exhibited consistently low-level performance.

With respect to the retest group's ability to verbalize, only one subject was capable of setting forth reasons for selecting items in the test. (This subject also achieved the highest level on the test and retest.) Since the mentally retarded are widely recognized as being unable to verbalize about their cognitive behavior, such results are not unusual.

In conclusion, if the assumption of the hierarchical organization of abstraction was correct, then subjects who fail to achieve a lower level in the test of natural things should also fail successive higher levels. Among all groups tested, 82.6 percent of the subjects met this criterion. The remaining subjects exhibited varying degrees of success on higher-level categories after failing a lower one, but there was no clear pattern of outstanding performance on the very top levels of the hierarchy.

1 Among all groups tested, only 17.4 percent did not meet this criterion. Some of this anomalous behavior could be due to differences in perceptual set for subjects vis-à-vis the instructional presentation and therefore they did not categorize as predicted in some tests. In particular, since the higher levels are more complex, these anomalous Ss may have been attending to other than the specific attributes according to the particular mental set brought to the task.
test sequence after failing a lower level. Therefore, substantial evidence has been obtained to support the assumption that the hierarchical model of abstraction is valid as a predictor of human performance. One must recognize, however, that the test used is a mediating mechanism in arriving at this conclusion; a different form of the test (insofar as it has face validity) might yield different results. Since the test was carefully constructed to represent each of the levels of the hierarchy, as prescribed in the general model, it can be concluded with fair confidence that this admittedly limited test of the hierarchy has yielded strong support for the model.

These results support previous studies which allude to the strong dependence of the mentally retarded child on concrete cues [15, 16, 17] and suggest that the mentally retarded may be incapable of organizing information into conceptual hierarchical arrays. In addition, the results support the hierarchical model of abstraction and, in particular, the validity of the classification test.

Acknowledgement

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References


