This unit of the Flexible Learning System (FLS), the second of a 3-volume series on children’s thinking, focuses on the development of order relations (seriation) in children between 3 and 8 years of age. The series is based on the application of Jean Piaget’s work to early childhood education. Seriation concerns the way children reason about relationships between elements of a series or sequence. The unit is designed for use with a group of adults to help them understand order and its organization in the preconceptual, intuitive and concrete-operational stages of development. Practice in exploring children’s thinking is provided. Activities include thought problems, role playing, child interviews, discussions, reading and viewing a color videotape or seriation from a series entitled The Growing Mind: A Piagetian View of Young Children. Rip-out guided interview forms are provided for all interviews. Also included are an introduction to Piaget and his general theory, educational implications of the theory, seriation materials, a transcript of the videotapes, and an annotated bibliography. Related FLS units: "Exploring Children's Thinking: Classification"; "Exploring Children's Thinking: Conservation"; "Working with Children's Concepts"; "Using Toys and Games with Children"; "Developing Children's Sense Perception." (Author/SS)
exploring children's thinking
Part 2, series A: Learning Units for Adults in Early
EXPLORING CHILDREN'S THINKING:
Part 2
The Development of Order Relations:
Seriation

Preschool - Third-Grade

by
Keith R. Alward

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Far West Laboratory for Educational Research and Development
San Francisco
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ACTIVITY II: Sharing and Assessing Your Interviews of Children on Seriation Problems

SIMPLE SERIATION WITH INSERTION: CHILD RECORD

DOUBLE SERIATION: CHILD RECORD

TESTS OF ORDINAL CORRESPONDENCES: CHILD RECORD
OVERVIEW

This unit is about how children between four and eight years of age think about order relations (sequencing and seriation). It is accompanied by a 30 minute color videotape showing the development of order relations. Though this book is designed to be used with a trainer and a group of learners, it will also prove useful to the individual student.

There are 12 activities that serve as the backbone of the unit. You will do some by yourself, some with other adults, and some with children. The main learning will come from pursuing the activities and sharing your questions and insights with fellow learners. Chapter 4 provides additional activities that can be carried out with children to extend your exploration of their understanding of order relations. Chapter 5 discusses the educational implications of what you will have learned.

When you have finished this unit, you will:

- be familiar with the general nature of Piaget's work on child development;
- be able to demonstrate the differences between adult and child thought as revealed in a variety of seriation problems;
- be able to interview children between four and eight years of age to determine their understanding of order relations;
- be able to demonstrate the differences in the way children seriate within the pre-conceptual, intuitive and concrete-operational stages of development;
- be able to develop a system for interviewing children and keeping records of your findings;
- recognize how order relations are involved in a wide variety of activities.
PREFACE

If you plan to or are presently affecting the experiences of children, then you should learn about children's thinking. This book is part of a series called Exploring Children's Thinking (ECT). This series covers four areas of mental development between four and eight years of age: classification, seriation (order relations), number and measurement (quantitative relations), and spatial relations. The first three topics are covered by individual books (Parts 1 through 3) and by corresponding 30 minute color videotapes illustrating children's reasoning. The fourth topic (spatial relations) is covered by a fourth videotape.

In reading through this book, you will explore how children develop in their understanding of order relations. Order relations are involved whenever objects, events, or experiences are sequenced or thought about in terms of a series. "First, second...last," "less-than and greater-than," "to the right of," "north of," "in the middle of," "older-than and younger-than," are all concepts that depend upon an understanding and use of order relations. Concepts of time and measurement are likewise dependent upon a use of order relations.

If this is your first exposure to this topic, you will be surprised by what you learn. Below a certain stage in development children do not reason about order or sequences in the same way as adults, no matter how they are taught or raised. Their ordering skills, like their thinking in

general, change in fundamental ways as they develop. You will see that children at the same stage of development reason about order relations in similar ways, no matter what their educational or cultural experiences.

In addition to the topic of order relations, this book introduces the work of the Swiss psychologist, Jean Piaget. Piaget is internationally recognized as a leading authority on the character and development of children's thinking. He has shown with astonishing breadth and detail the nature of what children know and understand and how their understanding changes as they develop mentally.

Piaget's contribution lies not only in the numerous areas he and his co-workers have investigated, but equally important, in the method of investigation called "structuralism." Structuralism embodies not only a means of studying thinking and its development, but also a theory that guides the exploration and provides a framework for understanding what is found. As you work through this book you will learn how to investigate children's understanding of order relations and analyse your findings according to the methods of structuralism. This way of looking at thinking takes the observer beyond exploring facts that children may or may not know and focuses instead on the underlying pattern or organization of children's thinking. As you learn to detect this pattern, you will find that what children know is not simply less than what adults know, it is fundamentally different; and what children of the same stage know is fundamentally similar.

2. Piaget, Jean, Structuralism.
Piaget has argued that children's understanding of physical causality, numbers, time, movement-speed-distance, probability, spatial relations, and geometry is also tied to a developing understanding of order relations. The organization and understanding of a series (which is what Piaget means by the expression "order relations"), partially describes the organization of all forms of systematic knowledge.3

The Introduction to this book provides a brief description of Piaget's theory and some information about Piaget himself. In addition, it gives an overview of the ECT series and its relationship to Piaget's theory. Chapter 1 introduces the topic of order relations and the characteristics of its development. Chapter 2 involves working with children to explore their understanding of order relations. Chapter 3 provides a more detailed and theoretical description of the development of order relations and the means of keeping records on the skills and stages of the children you work with. Chapter 4 presents a number of additional activities to further explore children's understanding of order relations. Chapter 5 provides a brief discussion of the educational significance of what you have learned about children's thinking and Piaget's theory. The Appendices contain some materials that can be used in seriation activities, a transcript of the videotape on order relations, and a list of resources for further

Piaget, Jean, The Psychology of Intelligence.
Inhelder and Piaget, The Early Growth of Logic in the Child.

These works describe the various forms of classification structure as well as the structures of relations (order relations).
exploring not only the topic of order relations but much of Piaget's other work, and the relationship of this work to educational process.

Now that you have a better sense of what this book is about, we can suggest ways it might help you as a teacher. For one, you will learn to look at children's thinking in a way that reveals its underlying organization. You will come to appreciate the differences between how you and the child view the world. You will learn how to engage children in enjoyable activities that allow you to assess their developmental level. You will come to see similarities and differences between children of the same age, and to find the underlying causes of some of the difficulties children may have with their schoolwork. Ideally, you will be better able to help children reveal their inner thoughts, and be better able to understand the nature of these thoughts. It is my hope that you find this exploration rewarding.

INTRODUCTION

PIAGET -- THE PERSON

This book is one of a series of three dealing with Piaget's study of children's thinking. We selected Piaget's work as our focus because to date he has provided the most complete description and theoretical account of mental development in children.

Jean Piaget was born in Switzerland in 1896 where he has spend most of his life and continues to work. At twenty-two he received his Ph.D. in biology (a field in which he first published at the age of ten) and soon began work in the laboratory of Simon Binet, one of the founders of intelligence testing. While pursuing studies as a biologist, Piaget was developing a dominant interest in knowledge. He began to view its acquisition not as a set of facts and experiences, but rather as an evolutionary process in which knowledge was an outcome of how the mind organizes mental and physical activities. He proposed that the manner in which activities and experiences are organized goes through a series of regular steps or stages.

His early work in Binet's laboratory provided him with much information on the thoughts produced by children. He noticed regular inaccuracies in their thinking that were gradually eliminated with age. On the basis of three papers describing these common inaccuracies, Piaget at the age of twenty-five was made "director of studies" at the Institute J.J. Rousseau in Geneva. He continued his work at the Institute until 1940, at which time he was named Director of the Psychology Laboratory at the University.

4. This Introduction appears in each book in the ECT series.
of Geneva. Along with numerous other posts and duties, Piaget is presently the Director of the International Center for Genetic Epistemology (Geneva), which he founded in 1956.

Throughout this more than fifty years, Piaget has been incredibly productive. He has produced well over two hundred works investigating numerous areas of human knowledge. He has virtually mapped the domain of intelligence from birth to late adolescence and has brought his nearly endless observations into a theoretical perspective drawing from logic, mathematics, physics, biology, psychology, and computer theory.

**PIAGET'S VIEW OF KNOWLEDGE**

Piaget's theory has evolved in response to questions asked throughout history: "What is intelligence?" "How are universally true ideas derived?" "Is knowledge really no more than memory?" As the science of psychology developed, it addressed these issues, yielding two views. The first holds that we are born with particular ways of organizing experience, and that knowledge reflects these inborn patterns of organization. The second view is the behavioristic one that has dominated American psychology. It holds that knowledge is a copy of reality and/or learning from others. Piaget has brought a third view to bear, one that strongly suggests the inadequacy of the "inborn" and the behavioristic "copy" theories of knowledge.

As a biologist, Piaget formulated his view around three elements: the organism, the environment, and the interaction between the two. The term "environment" refers to those things that are outside the organism but which affect how the organism functions.
beginning elements come two biological processes that result in change. One is the process of acting on the environment, which is the same as incorporating the environment into actions. Piaget calls this "assimilation" of the environment to the organism. Grasping objects, recognizing a familiar object, and cooking dinner are ways we act upon our environment. The other process is an alteration in the organization of actions as a result of their use. Piaget calls this "accommodation," or the adaptation of actions to the environment. Learning to grasp differently, finding out that something is different than expected, modifying recipes for a meal are examples of how actions are modified through use. "Assimilation" and "accommodation" make up the dynamics of life—all life being a process of acting on or taking in the environment with resulting changes in the actions themselves and their organization. Changes in the organization or structures underlying actions can be viewed as evolution or development.

Piaget sees knowledge as based in biology. He suggests that the underlying process by which an organism comes to survive is the same as that by which man can arrive at objective knowledge. In both instances the process is composed of the assimilation of reality by the organism and a resulting change in the structures that assimilate. Mathematical thought and primitive biological processes are both based in action systems. The difference is one of the degree of development of those systems.

STAGES OF DEVELOPMENT

Piaget is probably most widely recognized for his theory that children's

6. Knowledge that is universally accepted as provable and true.
Thinking advances through a series of distinct stages. The essential aspects of a stage theory are that each new stage follows from and depends upon the completion of earlier stages, and that the sequence of development is the same for everyone. Piaget describes a stage in terms of how a child's thinking is organized. The thinking in earlier stages is less well organized than in later stages.

Piaget and his co-workers in Geneva, and a large number of researchers in other countries, have shown that children's thinking in a wide range of knowledge areas goes through a similar developmental pattern. This pattern is described by four major periods. During the first two years, the sensory-motor period, children progress through six distinct stages of intelligence. A second broad period, the pre-operational period, generally lasts between two and eight years of age. During this period, children develop their ability to represent reality with language, imagery, play, drawing, etc., and develop in their understanding of reality. The next period is the concrete-operational stage, during which children develop logical structures (from the adult's view) and apply them to a systematic understanding of a wide range of problems. By early adolescence children enter the formal operational period, considered to be the highest level of mental organization.

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It is important to keep in mind that the age at which a child enters or leaves a stage is not specified by the theory. Children of the same age may be in different stages of development. What is so far shown to be true of all children is that all children go through the same series of stages, although not all children progress beyond the concrete-operational stage.

AN EXAMPLE OF STAGES IN INTELLECTUAL DEVELOPMENT: CONSERVATION

One of the more familiar aspects of Piaget's work is the study of conservation. As adults, we recognize that a given amount of something does not change when only its shape has changed. For example, if you pour a tall glass of water into a short fat one, you know that the shape of the water may change, but its amount remains the same. This is called conservation. Conservation is assumed by adults for anything that can be thought of in quantitative terms: a quantity of clay, a measure of distance, a unit of weight, a number of objects, a unit of volume, and so forth.

Piaget and numerous researchers throughout the world have shown that all children progress through the same sequence of stages in their understanding of conservation. Children in the pre-conceptual stage always think that changing the shape or arrangement of objects changes their amount. Children in the next stage believe that quantity is conserved under some circumstances, but not others. By the concrete-operational stage, children firmly believe that changes in shape, arrangement,

7. See The Development of Quantitative Relations, Part 3 of the ECT unit—a unit of the FLS

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and appearance do not change amount. Furthermore, all children conserve substance (amounts of clay, rice, water, etc.) before they conserve weight (understand that the weight of something does not change when its shape changes); and all children conserve weight before they conserve volume (understand, for example, that a quantity of clay will displace the same amount of water even if the shape of the clay is changed).

THE SOURCES AND DIRECTION OF INTELLECTUAL DEVELOPMENT

Although conservation is only one of many areas of knowledge examined by Piaget, it provides a focus for discussing his theoretical views on how knowledge is acquired.

It has been widely understood that memory, associations between one experience and another, sensory impressions, trial and error learning, and imitations of others, all play a role in thought and affect what we learn. However, each of these, singly or in concert, cannot account for what Piaget and others have found to be true of children's intellectual development. For example, the fact that children think an amount of liquid changes when it is poured, cannot be attributed to a prior memory, to experience, or to the teaching of others. While it is surprising to find children making such judgments, all children think this way at some point in their development.

On the other hand, all children eventually come to know that amounts are conserved, and they do so after passing through the same sequence of earlier stages. When asked why an amount of liquid stays the same when it is poured into a wider container, the conserving child almost universally gives one of the following arguments: "Nothing was added or taken away, so
it's still the same." "The water is now wider than before, but it is also not as tall." "You could pour the water back into the glass and it would be the same as before." These are logically precise arguments for why the amount has not changed even though it looks different. The question naturally arises as to how children come to reason in such systematic terms.

The arguments given by children for why amounts are conserved provide the basis for suggesting some of the likely and unlikely sources of objective knowledge. For example, consider the argument that if nothing is added or subtracted, amounts stay the same. It's easy to imagine that such a principle might be taught, or that it might be experienced through counting activities. However, it is well known that it is virtually impossible to teach this principle to pre-conceptual children; also, children in the intuitive stage either already know or can be taught this with respect to counting, but they do not necessarily apply the principle to other areas of conservation, such as substance and length. Furthermore, all children arrive at an understanding of this principle irrespective of whether it is taught. The intuitive child must be repeatedly convinced of its truth, whereas children a few months older regard it as an obvious fact of nature.

Children who spontaneously understand that amounts do not change when nothing is added or subtracted may just as easily express the argument that liquid poured, for example, from a tall narrow glass into a wide one is conserved, because the water level is now lower, but further around, or wider. This expresses an understanding that changes in one dimension (height) can be compensated by those in another (width). It is unlikely that this principle of compensation was ever taught to most children who express it.

8. Siegel, Irving and Hooper, Frank (Eds.), Logical Thinking in Children: Research Based on Piaget's Theory.
Without measurement there is no way to tell that changes in one dimension compensate those in another, and the ability to measure, itself, follows rather than precedes an understanding of conservation. When children express an argument of compensation as their basis for conserving, they are simply expressing what they know must be true. As you will see, the argument of compensation is an important clue to their basis for conserving.

A third argument typically given is that if a quantity changes in appearance, there must still be the same amount because it can be changed back to its original appearance. This can be experienced directly. You can pour a glass of water into a bucket and pour from the bucket back into the glass and witness that there is as much water as when you started. But here is an interesting fact. While children may experience this from their first water play on, and while it may even be pointed out to them, this observation does not lead them to conserve. It is not until very close to the concrete-operational stage that such repeated demonstrations lead to an idea of conservation. And, again, children a few months older come to invent this principle for themselves.

The above arguments suggest that an understanding of conservation does not result from experience alone, whether that experience is manipulative and/or social. Piaget has argued that social experiences, physical experiences and maturation (physical growth) are necessary to intellectual development. But they alone are not sufficient to account for something as simple and obvious to adults as conservation.

Piaget has suggested two additional factors that underlie the source and direction of intellectual development. One of these is the coordin...
tion of actions and the other is the tendency of this coordination to be-

come reversible.

The concept of a reversible coordination of mental actions is abstract
and foreign to most of us. We can give some sense of its meaning by return-
ing to the arguments given by children for conservation. One of the argu-
ments is that a quantity is conserved if nothing is added or taken away.
The concept of addition is a mental activity of joining things together.

Subtraction is a mental activity of separating. When it is understood that
subtracting amounts can exactly compensate adding amounts, then these two
mental activities are in a reversible relationship to one another. Such a
relationship makes it possible to reason that adding and/or subtracting
lead to changes in amounts, and that doing neither leaves amounts the same
or conserved. A similar expression of reversible reasoning is demonstrated
in the understanding that changes in one dimension can compensate those in
another. Changes in height, for example, are reversibly related to changes
in width. It is therefore possible for an amount to change in one way and
still be the same, because the first change is compensated by a second
change. Reversible reasoning is likewise expressed in an understanding
that a quantity can be changed in appearance and then changed back to its
original form.

Children's understanding of conservation and their supporting argu-
ments do not reflect things that have been taught or recorded from exper-
ience. Unlike facts, experiences, or things taught, the principle of con-
servation cannot be forgotten any more than one can forget that one's bro-
ther (sister) also has a brother (sister). Conservation is a product of
reversible reasoning applied not just to objects, but to actions upon ob-
jects and, more importantly, to internal or mental representations of
An understanding that actions (adding, pouring, narrowing, lengthening, etc.) can be reversed, inevitably results in a conception of conservation which itself is central to all measurement and all conceptions of units. Numbers themselves are simply abstract representations of units that can be counted and separated. And at its core, a unit is no more than a conception of an amount that is conserved in spite of spatial displacements -- changes in arrangements, appearances, and so forth. An ability to conserve is evidence that a child has achieved reversible thought and is capable of thinking of quantity in terms of units that can be measured.

YOUR AND PIaget's EXPLORATION OF CHILDREN'S THINKING

By eight years of age most children begin to evidence dramatic changes in much of their thinking. Piaget and his co-workers have provided a description of these changes in a wide range of areas and have proposed that the emergence of reversible thought is a primary factor in the nature of these changes.

The majority of Piaget's work has concerned "objective knowledge:" knowledge that is subject to proof through agreed-upon arguments. For example, one can prove by the agreed-upon argument of counting, that eight blocks will remain eight blocks, even if their arrangement is changed. There is a range of similar problems that concern areas of quantitative reasoning other than conservation. Some of these are: time, speed-distance-movement, probability, proportionality, geometry, density, force, pressure, and velocity.

In Part 3 of this volume on Exploring Children's Thinking (ECT) we
focus on children's developing understanding of the conservation of number, substance, and length; measurement of distance; and the relation of time, speed and movement. Part 3 consists of a learning unit containing guidelines for exploring children's quantitative thinking and a 30 minute video-tape demonstrating the methods of investigation and the developing character of quantitative thought between four and eight years of age.

Other subjects investigated by Piaget are not concerned with quantification. For example, Piaget has studied the understanding of space from early infancy to late adolescence. At some points an understanding of space uses quantitative concepts and at other points it does not. For example, geometry uses units to describe space. A square contains four equal straight lines connected at end points to form an enclosed space. The concepts "four" and "equal" are statements about units and are thus quantitative. However, space can be described without units. For example, the notion of an "enclosed space" does not use any quantitative units.

Part 4 of ECI concerns Piaget's investigation of developing spatial concepts in children between four and eight years of age. Again, we see the same stages as revealed in quantitative thought. Part 4 consists of a videotape demonstrating the methods and results of interviewing children between four and eight years of age on their concepts of straight lines, left-right and foreground-background orientations, and horizontal spatial orientations as demonstrated by the surface of a liquid.

Piaget argues that the similar pattern of stages in quantitative and spatial reasoning results from the general underlying tendency of mental activity to become increasingly organized and reversible. He has attempted to analyze, in terms of reversible classifying and sequencing activities, all that he has demonstrated in spatial and quantitative reasoning, as well
as other areas such as causality and genealogical relations.

In Parts 1 and 2 we explore the development of classifying and sequencing in children between four and eight years of age. Because of the importance attributed by Piaget to these two topics, we have provided a book for each. Part 1 presents a detailed description of the developing understanding of classification and how to explore this development with children. In Part 2 we likewise treat ordinal relations, or the logic of sequences. Each book is accompanied by a 30 minute color videotape.

The topic of classification concerns the coordination of judgments about how objects and events are similar and/or different, and the logic of some and all. For example, all cats and dogs are animals. Because all of the cats are only some of the animals, there are logically more animals than cats. Piaget argues that the logic of classification is based upon a reversible coordination between combining and separating activities; and that prior to the concrete-operational stage, this reversibility is absent, giving the young child's concepts an illogical appearance. However, it is wrong to call the classification of pre-operational children illogical. It is different from adult thought, yet systematically organized and consistent in its application.

The second topic, ordinal relations, concerns how children coordinate judgments about such things as before-after, first-next, less than-greater than, shorter than-taller than, and so on. Here, as well, there is a logic as expressed in the following: If Steve is older than Leon, and Steve is younger than Pete, then Leon is the youngest. Piaget describes this logic as the reversal of relations such as: If Steve is older than Leon, then Leon is younger than Steve.

Piaget's analysis of knowledge is complex. Some parts are more under-
standable than others and some are more worthwhile to the practicing educator. In this volume on children's thinking we attempt to guide you in a first-hand exploration of part of what Piaget has observed in children's thinking. In pursuing the reading, interviewing, and discussion activities, and by viewing the videotapes, we hope to stimulate not only an appreciation for the character of children's thinking, but a way of looking at thinking itself.

As you work through this volume, you will gradually gain new lenses through which to look not only at children's thinking, but at your own as well. If this topic sparks an interest, you will come to sense the broad patterns of commonality that touch upon a wide range of understandings. In another unit of the Flexible Learning System we help you prepare for exploring children's concepts in general.

Educators commonly ask about the implications of Piaget's theory for education. We address this issue in the concluding chapters of Parts 1 and 2. However, a general view can be expressed quite briefly. Any significant educational implications from Piaget's theory are to ultimately be decided by educators who have come to experience the character of reasoning revealed by Piaget's methods. To translate Piaget's theory to educational prescription must be preceded by an appreciation of what he has discovered. This is the function of the ECT volume.

10. Alward, Keith R., Working With Children's Concepts, a unit of the FLS.

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AN INTRODUCTION TO ORDER RELATIONS

WHAT ARE ORDER RELATIONS?

When we speak of order relations we are essentially talking about the relationships that hold between the elements of any series or sequence. A series is any conception in which the elements are organized so that they precede and follow one another in the sense of, "first, second, third, next, and last."

Order relations are involved in the mental organization both of objects and events. For example, objects can be organized in terms of size (shortest, next shortest...tallest) or color (dark red, medium red, light red), weight (heaviest, next heaviest...lightest), age (oldest, next oldest...youngest), location (closest, next closest...furthest), or even in terms of how much one likes something (favorite, next favorite, least favorite). The nature of objects has no direct bearing on order relations. A group of sticks of various lengths might lend themselves to being arranged in a series from shortest to tallest, but they could just as easily be placed in a series of physical location, such as "furthest from my right shoulder, next furthest from my right shoulder...furthest from my left shoulder." Order relations are simply relationships of order imposed on objects or events by the organizing mind.

All events and experiences can be ordered in time. For example,
sensations, ("I felt terrible, then I felt a little better, and in a few days I was back to my old self."), or actions, ("The man opened the door, entered the room, and started reading the menu."), or natural events, ("The caterpillar emerged from the egg, ate some leaves, made a cocoon, turned into a butterfly, and laid some eggs."), all involve a sequencing in time. Our understanding of time and other order relations is the result of an active mental construction. Piaget's work shows that the child's understanding of time goes through a course of mental development parallel to that of order relations, classification, and all other areas that have been similarly studied.

It is important to distinguish between units of measure and order relations, per se. All conceptions of measurement involve order relations, but order relations do not require the use of units. "I like this painting the best of the three," is just as much an order relation as the statement "This box weighs 10 lbs., this one 30 lbs., and the third one weighs 35 lbs." Order relations do not concern measurement, but rather, the order of elements in a sequence. Of course, the act of ordering may draw from measurement. For example, by measuring their exact weight, I might place objects in a series, but I could just as easily create the series simply by comparing appropriate pairs to see which feels the heaviest. Of course, measurement is impossible without an appreciation and use of order relations.

In our discussion of class relations (Part I of ECT), we showed that classes do not exist in isolation. To say that something belongs to a

class requires specifying at least one property or dimension by which objects can be compared, and reasoning about how, on the basis of this property, some objects are similar and others are different. For example, focusing on the dimension of color, we can put red things together to form a class because they are similar to each other and, at the same time, different from things that are not red. A parallel situation exists for order relations. An isolated object or event does not have an order. The construction of order relations requires comparisons among objects or events in terms of a specific dimension such as length, weight, location time etc.

In the Introduction, we emphasized that children's thinking progresses through a number of stages between birth and adolescence. The character of these stages is clearly revealed in children's understanding and use of order relations. In fact, Piaget has argued that in conjunction with classification, order relations underlie the development of thinking in general.\textsuperscript{12} The ability to understand order relations is one of the differences between an older child and a preschooler. This difference is revealed in everyday situations. For example, an early pre-conceptual child (two to three years of age) may try to drive a toy train under a bridge and, finding the train too large, go on to try an obviously lower bridge. The child does not reason: if the train was too big for the first bridge, it'll certainly be too big for a smaller bridge.

Many a parent has found his/her three year old expressing concern

\textsuperscript{12} Inhelder and Piaget, \textit{The Early Growth of Logical Thinking in the Child}. 

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about going down the bathtub drain. The toilet looms still more ominously.
To the distress of plumbers and parents alike, children experiment to see
what will and won't make it through. And observations that objects
obviously smaller than the child will not make it, do not convince the
child that they too are necessarily too big. An inquisitive child will
step in to find out.

Another expression of young children's inability to reason about
order relations is revealed in their inability to conceive of continuity.
For instance, the pre-conceptual child cannot understand that pictures of
a person from infancy to adulthood can be pictures of the same person.
They look different and without a means of thinking about changes in
appearance over time, there is no reason to suspect they are pictures of
the same person. Children hold this view while at the same time telling
us that someday they'll be grown up and become like daddy or mommy. Yet if
you show a three year old a picture of him/herself when they were a baby,
they'll believe you only with suspicion. The child reasons: "How can I be
me and at the same time be this baby?" The child does not reason that the
picture was taken a long time ago, that later the child will look more
like older children and later still, like an adult. Instead, the young
child thinks they become an adult without a necessary series of changes
in-between.
ACTIVITY 1: A Variety of Seriation Activities

The purpose of this activity is to have you actually experience some of your own reasoning about order and sequences. There are a number of activities, each with its own follow-up discussion to be read after you've done the activity. You can do all or only some of them and you can do them on your own, although they'll be more informative if you share with others.

INSTRUCTIONS FOR ACTIVITY 1A:
Take at least 10 narrow objects of various length (at least an inch in length). You can use spaghetti noodles, strips of paper, used pencils, kitchen utensils, etc. Arrange them in a series from shortest to longest.

FOLLOW-UP TO 1A:
Your series was probably not composed of equal units of distance between each successive object in the sequence. That's unless you managed to get a very specially constructed set of 10 objects (such as Cuisenaire-Rods or unit sticks), your series was probably uneven.

13. Cuisenaire Rods are color coded sticks increasing in length by equal amounts.
Nonetheless, if it was arranged so that each successive member of the series was taller than all those before it, then it was a true series and you most likely used a logical understanding of order relations to construct it. As we said earlier, order relations are not dependent upon measurement or conceptions of equal units. But when the objects are uneven, it is more difficult to seriate them because you cannot use the appearance of the series to judge whether a stick just added to the series is too tall or just right.

One last point. You might have found that some of your objects were very close to being the same length. In order to find out, you were probably careful to match their ends. As you will see in Part 3, young children do not do this when comparing lengths and do not develop consistent notions of length until the concrete-operational stage in middle childhood.\(^4\)

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\(^4\) See Part 3 of ECT: *The Development of Quantitative Relations: Conservation.*
INSTRUCTIONS FOR ACTIVITY 1B:

Suppose you started on page 48 of a book and you finished reading page 58. How many pages did you read? _________

Do this in your head. Now turn to page 48 and count how many pages you would have read when you finish reading page 58. _________

FOLLOW-UP TO 1B: (read after completing 1B)

If you are like most people, you probably used arithmetic to come up with 10 pages when you did it in your head and 11 pages when you actually counted the pages. This problem requires a coordination between order relations and counting. You count page 48 as one page, then continue to count until you get to page 58. At this point you have read the ten preceding pages. Counting page 58 gives you eleven pages altogether. The decision you have to make is whether a page number stands for having read up to that page or having read the page.

48 49 50 51 52 53 54 55 56 57 58

While this problem is difficult to understand when expressed in words, children in the concrete-operational stage show that they can coordinate counting and order relations. The reason adults typically fail this problem is because of its abstract presentation, and because adults are

quick to use an arithmetic solution without considering whether they are counting the last page as read or not read.

INSTRUCTIONS FOR ACTIVITY 1C:

Cut up 20 different length strips of paper. Select half of them by chance and put X's on them. Now arrange the strips so that the longest strip with an X on it is paired with the longest strip without an X, and so on, until the shortest strip with an X is paired with the shortest strip without an X.
FOLLOW-UP TO 1C: (read after completing 1C)

You probably solved this problem by using an understanding of order relations. You might have begun with the longest strip with an X, paired it with the longest strip without an X, then selected the longest remaining strip with an X and paired it with the longest remaining strip without an X, and so on. This seems like an obvious way to proceed and you know that it will work. While you might not have proceeded in exactly this fashion, if you solved the problem you did something that is structurally the same, that is, using the same logic of order relations as those involved in our example. Prior to the concrete-operational stage, children do not yet coordinate order relations, and cannot solve a problem like this.

INSTRUCTIONS FOR ACTIVITY 1D:

This activity is like playing solitaire with a "fixed" deck. You have to think about three piles of cards (A, B, C). The goal is to get all the cards from pile A onto pile C in a serial order.
SETTING UP THE PROBLEM:

You use 8 cards (2-9). At the start of the activity, pile C has only the 9 card in it (face up); pile A has cards 2-8 face down; and pile B does not yet have any cards.

The next step is to turn over the top card in A and put it in pile B (face up).

If the card you turn over is the next smallest number of all the cards in pile C, then you can put it on top of the C pile, e.g., an 8 on the 9, a 7 on the 8, a 6 on the 7, and so on. If a card from A will not go onto pile C, then take another card from pile A and repeat this process until you have gone through all the cards in A. Any card put face up in pile B can go to C if it is the next smallest number.

FIRST GAME: AN EXAMPLE

Start with the following "fixed deck" for the A pile. Put the cards in the order 2, 3, 4, 5, 6, 7, 8, so that the top card is the 8. Now follow the procedure outlined above.
SECOND GAME

Think of another way you can order the cards in the A pile so that you can get them all on the C pile in serial order by following the rules. If you can think of two that's good, three is great, and more is super.

THIRD GAME

Think of a way you can order the cards in the A pile so that it is impossible to get all the cards onto pile C if the rules are followed. Explain to a partner why it couldn't work.

FOLLOW-UP TO 1D: (read after completing 1D)

Here are two other possible orders for the A pile so that the game will work: 8, 7, 6, 5, 4, 3, 2 with the 2 card being the first card placed on the B pile. Try the game with this order. If you compare it to the A deck used in the first game, you'll see that it is exactly the opposite order. In the first game, all the cards go directly from A to pile C, whereas with this order, they stack up in B and then go to C.

A second possible order for the A deck is: 4, 7, 8, 6, 5, 3, 2 with the 2 being the first card to turn over. Try it out. Try the cards in the same order, but this time make the 4 card the first card to be taken off. Does it work? Can you invent any other possible orders for the A deck so that it will work? Here is an order that won't work: 7, 6, 8, 5, 4, 3, 2, with the 7 being the top card. Notice that it works if the 2 is on top. Can you invent any other possible orders for the A deck that won't work?

INSTRUCTIONS FOR ACTIVITY 1E:

Tom is younger than John, Tom is older than Lynn. Who is the oldest?
Because we understand order relations, we know that the answer is John. Because Tom is younger than John and older than Lynn, then Tom is in the middle. And since John is older than Tom, John must be the oldest.

The following is a mathematical expression of these relations, using the symbol (\( < \)) for "less than" and (\( > \)) for "greater than."

- Tom is younger than John: \( T < J \)
- Tom is older than Lynn: \( T > L \)
- (which is the same as saying that Lynn is younger than Tom: \( L < T \))

The fact that Tom is younger than John (\( T < J \)) and Tom is older than Lynn (\( T > L \)) means that John is also older than Lynn (\( J > L \)); or, \( (T < J) + (T > L) = (J > L) \)

DISCUSSION OF ACTIVITY 1:

All of the above problems and tasks require a coordination of order relations. In some, the order is based on the relative length of the elements in the series; in others, it concerns numbers that represent a series; and in the last problem it involves order based upon age. What is common to the problems is that all involve order in the sense of first, second, third, and so on. An equally valid way of talking about order relations uses the expressions "less-than" and "greater-than."

The importance of order relations does not lie in the child's ability to organize objects into a series, but in the very nature of reasoning itself. Order relations are involved in conceptual thinking. Some form of sequencing is involved in all reasoning or thinking. And, of course, many concepts are directly concerned with order. Think of all the forms of
knowledge that draw upon notions of more-less, before-after, greater than-
less than, etc. All conceptions of time, distance, amounts, measurement,
causality and space, for example, depend upon the use of order relations.
And even at the sensory-motor level, all actions are carried out in a se-
quence. As the child develops, the capacity to understand the relation-
ships implied by a sequence, s/he, at the same time, develops the capacity
to regulate activity, to anticipate outcomes, coordinate activity to reach
goals, and in essence, conduct the business we call thinking.

ORDER RELATIONS (SERIATION): STAGES OF DEVELOPMENT

For the remainder of this book, we'll use seriation tasks to inves-
tigate children's understanding of order relations. In seriation problems,
children are asked to arrange objects in a sequence or series and are
questioned or provided with additional problems to explore their under-
standing of the relationships among elements of the series. The following
is a brief introduction to the characteristics of three stages of reasoning
in children between four and eight years of age.

Pre-conceptual Seriation (generally four-five years of age)

Most children between four and five years of age are still in the
pre-conceptual stage of intellectual development. The pre-conceptual
stage is a long stage, emerging somewhere around two years of age and
generally lasting to about five years of age. Children must first pass
through earlier stages or ways of thinking before arriving at the pre-
conceptual stage, and when they leave this stage, they always enter the
next stage called the intuitive stage. There is, however, no absolute
relationship between the age of a child and the stage s/he may be in.

The pre-conceptual child does not have a stable understanding of order relations. For the most part, a four-year old can compare opposites like above/below, up/down, older than/younger than. Given two objects of different sizes, s/he can say which is bigger or which is smaller. What the pre-conceptual child cannot do, however, is coordinate relations among more than two elements.

Let's imagine a task in which a child is given 16 different length sticks to be arranged in a series. All the sticks are of the same color and large enough to be comfortably handled. Furthermore, the difference between each successive stick is large enough to be easily detected.

After becoming familiar with the materials, the child is asked to arrange the sticks in a series from shortest to longest. To help the child

16. Piaget criticizes the use of Cuisenaire Rods (color coded sticks increasing in length by equal amounts) because of their small size and the color coding which leads children to order on the basis of memory rather than reason.
understand what is expected, the interviewer may give the child examples or help him/her arrange a short series.

Children in the pre-conceptual period are not able to construct an ordered series and instead, produce approximate imitations.

Here are some examples of the types of series constructed by children in the pre-conceptual stage:
There are a number of significant points regarding these three examples:

1. Each shows sticks arranged in an approximate series. This indicates that the child has some understanding of the concepts "shorter than" and "longer than", since the sticks at one end generally appear shorter than those at the other.

2. In examples A and C, the child disregards the actual length of the sticks as illustrated by the uneven bottoms.

3. In none of the examples does the child achieve an ordered series from shortest to longest.

4. In all three examples, it's as if the child were constructing an approximate picture of one or more series. In this respect, the series have a graphic character similar to the "graphic collections" formed by pre-conceptual children in response to classification tasks.17

The results of a child's efforts are only partially revealing. The real character of pre-conceptual seriation is shown in the process of construction. The following is an example of how a pre-conceptual child might go about constructing a series. After determining whether the child knows which of any two sticks is the "shortest", for example, s/he may be asked to "put the sticks in order so that the littlest one is here, then the next littlest, all the way to the tallest one: like this (pointing to an example of a series)." The child may proceed by selecting a short-looking stick and continue by selecting another short-looking one, as if s/he were thinking, "I'm going to line up the short sticks here."

17. See Part 1 of ECT: The Development of Classification.
The child adds small-looking sticks to the series being constructed.
Now, if a child were to systematically pursue this approach, s/he would, in fact, construct an order series. But the pre-conceptual child's approach is anything but systematic. For rather than being sure that each successive stick added to the developing series is the shortest stick remaining in the pile, the child simply adds one that looks small compared to the remaining sticks.

As a result, the series is either not composed of progressively taller (shorter) sticks (example B, p. 15); or forms a progressive series, but at the expense of disregarding the actual length of the sticks (examples C and A); or is composed of a number of small series (example C). In some cases, the child leaves a remainder of sticks that do not visually fit at the end(s) of the developing series (example A). In short, the child proceeds as if all the sticks are either big or small, but not necessarily with respect to comparisons among all the sticks already in the developing series or those remaining to be considered.

The pre-conceptual child has no way of considering a relationship among only one stick and all the others. The concepts "biggest-smallest" or "shortest-longest", are reduced in the pre-conceptual child's mind to "big-small" or "short-long". It is not that children in this stage have not been taught the meaning of the words "shortest and tallest". It's simply that a consistent conception of "shortest-tallest" requires an ability to consider at least one stick with all the others, and this is what the pre-conceptual child cannot do. For the pre-conceptual child, a series is not guided by a systematic comparison of sticks, but rather, by a "mental picture" or "graphic representation" of what the child sees.
What s/he sees are sticks, some of which are big and some of which are not.

Intuitive Stage -- Trial and Error Seriation (generally six-seven years of age)

Most children between six and seven years of age are still in the intuitive stage of mental development. During this stage children can construct a series by trial and error, and have developed the ability to compare one stick to all the others. As a result, the concepts "shortest" or "longest" take on a more precise meaning than during the earlier stage. However, the concepts are considered separately and not as opposite and reciprocal terms. That is, the child does not coordinate judgments of "shortest and tallest" with an understanding for example, that if the shortest is taken from pile A and put into pile B, it becomes the tallest stick in B.

The shortest stick becomes the tallest
The intuitive child constructs the series with a concern for either the concept "shortest" or "tallest", but not both together. In other words, the intuitive stage child still does not think in terms of the tallest stick in the developing series (B) as the shortest stick if compared to those remaining to be put into the series (A). As a result, children in this stage proceed in a trial and error fashion.

For example, if the child starts out building from the shortest to the tallest, s/he will probably start with the shortest stick and then add a stick that is taller and so on, being sure that each added stick is taller than all those already in the series. This approach generally leads to errors, because there is no immediate way to know how much taller the next stick should be. The child is focusing on the fact that the series should go up, and thus each added stick must be taller than all those before it. But as a result, the developing series is likely to run into problems, such as none of the remaining sticks being taller than all those already in the series.

The sticks remaining to be put into the series (B) are too short.
The intuitive child generally corrects this situation by removing all the sticks in the series (B) that are taller than those remaining (A) and then starts again with a trial and error approach.

The child often avoids this dilemma by basing judgments upon the appearance of "eveness" in the series, and rejecting a stick that appears too tall.

Stick #4 is simply "too tall" and is rejected by the child, who looks for another stick that is taller than #3, but not "too much taller". Thus, by trial and error guided by appearances, the child eventually forms an ordered series of progressively taller or shorter sticks. But the understanding of order relations has not achieved the level of organization characterizing the concrete-operational child's performance.

When given a new stick and asked to find where it goes in an already constructed series, the intuitive child has difficulty. The problem requires finding a place in the series where the new stick is both taller than those before it and shorter than those that follow.
Stick B is both taller and shorter than its neighbors.

Because children in this stage cannot coordinate shortest and tallest with each other, they find it difficult to insert new sticks into the series.

Concrete-Operational Seriation — "True" Seriation (generally starting between seven-eight years of age).

The last stage to concern us, although the child's ability to reason about order relations continues to develop, is the concrete-operational stage. Most children between seven and eight years of age show some characteristics of this stage, however, concrete-operational reasoning is not fully developed until late middle-childhood, around 11 or 12 years of age. It is important to keep in mind that the concrete-operational child does not work with order relations at the same level when they are verbal and abstract as when they are concrete and manipulative.

Children in the concrete-operational stage seriate (order) objects in
a way that is essentially indistinguishable from the way adults arrange things in series. Seriation at this stage is characterized by the child's ability to reason about how something is shorter than one thing and, at the same time, longer than something else. This coordination is an example of reversible thought. The concrete-operational child's use of reversible thought is clearly seen in how the child forms a series.

Concrete-operational children generally proceed very systematically. They start with the longest stick in the pile, and then add the next longest stick, and then the third longest stick, and so on until they have put all the sticks into a series. Younger children could be led to this, particularly if they were given a great deal of training. But what distinguishes the reasoning of the concrete-operational child, is that s/he knows with logical certainty that the longest stick in the pile of remaining sticks, becomes the shortest stick in the developing series.

Sometimes concrete-operational children, like adults, do not approach the problem this way. They might start the series and if they run into a problem, like finding that a remaining stick will not fit on either end of the series, they simply find the right place inside the series, and put the stick there. The intuitive child has a great deal of difficulty with inserting a stick in the series, and generally takes the series apart or succeeds, more by accident than by a coordination of less-than and greater-than.

The general pattern or sequence of development as we have outlined it has been shown to characterize children throughout the world.18 These

findings tend to support Piaget's claim that experience, in the specific sense of being taught how to seriate, is not the underlying cause of this developmental pattern. Seriation ultimately concerns the child's ability to reason about order relations such as bigger-than/smaller-than, before-after, more-favorable/less-favorable, first-second-third-etc. It requires a coordination of relations, an understanding that an element in the series comes after some elements and, at the same time, before others. The fact that these coordinations are not seen in children before the concrete-operational stage indicates that children's thinking about all things involving sequences is very different from that of adults. As we will see in Part 3, an inability to reason logically about order relations is accompanied by an inability to reason logically about quantitative relations such as number, amount, length, distance, etc. 19

AN INTRODUCTION TO INVESTIGATING SERIATION

ACTIVITIES 2 AND 3: Simple Seriation With Insertions

The purpose of Activities 2 and 3 is to get you started in doing a seriation task with a young child. You will first role-play the activity with an adult and then interview a child.

MATERIALS:
You will need 10 sticks (or other objects) of 10 different lengths (Set 1). The sticks should form an even series such as 2, 3, 4, ..., 11 inches. The sticks should be of the same color and material.

You will also need three or four additional sticks (Set 2) that fit between successive sticks in Set 1. These can be of a different color than those in Set 1. The following is an example:
Set 1: 10 sticks (2, 3, 4, 5, 6, 7, 8, 9, 10, 11 inches in length)
Set 2: 4 sticks (4 1/4, 6 1/2, 7 1/4, 9 1/2 inches in length).

ACTIVITY 2: Simple Seriation with Insertion (Interviewing an Adult)

In Activity 2, you should carry out the task with an adult. This is practice for doing the same activity with a child (Activity 3). The instructions are the same as those used for interviewing the child.

INSTRUCTIONS FOR ACTIVITY 2:

1. Use Form A to complete the activity. You will use the same form with the children in Activity 3.

2. Put out the 10 sticks in Set 1 and mix them all up. Ask your partner to point to the shortest stick, and the longest stick. Then ask him/her to arrange the sticks in a series. You might say "I want you to make a staircase going from the smallest stick up, until you've used all the sticks." If this is not understood,

20. See Appendix A for directions on constructing Sets 1 and 2.
take three or four sticks and put them in order as an example and then mix them up again and ask your partner to do the task.

3. After your partner is finished, take out the four sticks in Set 2. "Here are some more sticks. I want you to find where they go in the staircase." Give your partner one of the sticks and ask him/her to find where it goes in the series. If s/he succeeds, give him/her another stick to insert in the series.

4. Record the information asked for on Form A.

5. Switch roles with your partner and have your partner interview you. If you wish, you can role-play how you think a child might proceed.
ACTIVITY FORM A: Simple Seriation and Insertion

Your Name: ___________________ Child's Name: ___________________

Date: ___________________ Child's Age: ___________________

Name of Child's Teacher: ___________________

This form is to be used to complete Activities 2, 3, and 8 (pages 25, 29, 61).

**Do (Conditions)**

Put out the 10 sticks (set 1) - messed up.

**Say**

HERE ARE A BUNCH OF STICKS. SOME ARE SHORT AND SOME ARE TALL. FIND ME THE SHORTEST ONE. FIND ME THE TALLEST ONE.

Record child's responses.

Describe what the child does and draw a picture of the child's series.

I WANT YOU TO PUT THE STICKS IN ORDER SO THAT THE SHORTEST IS HERE. MAKE THEM GO UP FROM THE SHORTEST TO THE TALLEST (give the child time to understand you and give examples if necessary).
If child did not form a series, arrange the sticks for him/her.

Bring out the 4 extra sticks (set 2).

Record child's responses.

If child didn't get it right, put the stick in the right location in the series, and ask him/her to try another one.

I'M GOING TO CHANGE THE STICKS LIKE THIS...

HERE ARE SOME MORE STICKS THAT WE DIDN'T USE. I WANT YOU TO PUT THIS ONE (one of the sticks) WHERE IT BELONGS IN THIS ROW OF STICKS.

Record child's responses.

LOOK, IF WE PUT THE STICK HERE, THEN ALL THE STEPS KEEP GOING UP. SEE? (Show by pointing.) HERE'S ANOTHER ONE. PUT IT IN THE STAIRCASE SO THAT ALL THE STEPS ARE GOING UP.

Record child's responses.
ACTIVITY 3: Simple Seriation with Insertion (Interviewing a Child)

MATERIALS:

Same as those used for Activity 2.

INSTRUCTIONS FOR ACTIVITY 3:

1. Find a child between four and eight years of age, ideally, someone that is familiar with you and used to working with you. If not, pay particular attention to putting the child at ease.

2. Review notes on interviewing children found on pages 51 - 56.

3. Follow the same basic instructions used in Activity 2.

4. Use Form A to complete the activity with the child.

5. Ask the child to arrange the 10 sticks of Set 1 in a series from the shortest to tallest. Use simple language and ideas such as: "Let's pretend these sticks are people. The smallest person goes here (pointing), and then the next smallest, so that they are all lined up." Or, "Let's make a staircase (etc.)." Give examples if possible. Once a child gets going and, you feel, is trying to do what you've asked, give assurances that s/he is following your directions correctly. Let the child work at his/her own pace.

6. If the child is able to form the series, ask him/her to insert a stick from Set 2 into the series. If s/he does not complete an ordered series, correct it saying, "I'm going to do this with your sticks." Then ask the child to put the new stick into the series. Say something like, "Here is another stick. It goes in here somewhere (pointing to the series). See if you can find the right place for it." If the child does not succeed, help him/her put...
it in the right place and explain why it is right. "See, even with these new sticks the staircase keeps going up." Then have the child try one of the other sticks in Set 2.
ACTIVITY FORM A: Simple Seriation and Insertion

Your Name: ____________________ Child's Name: ____________________

Date: ____________________ Child's Age: ____________________

Name of Child's Teacher: ____________________________________________

This form is to be used to complete Activities 2, 3, and 8 (pages 25, 29, 61).

Do (Conditions) Say

Put out the 10 sticks (set 1) - messed up. HERE ARE A BUNCH OF STICKS.
SOME ARE SHORT AND SOME ARE TALL. FIND ME THE SHORTEST ONE. FIND ME THE TALLEST ONE.

Record child's responses.

Describe what the child does and draw a picture of the child's series. I WANT YOU TO PUT THE STICKS IN ORDER SO THAT THE SHORTEST IS HERE. MAKE THEM GO UP FROM THE SHORTEST TO THE TALLEST (give the child time to understand you and give examples if necessary).
If child did not form a series, arrange the sticks for him/her.

I'M GOING TO CHANGE THE STICKS LIKE THIS...

Bring out the 4 extra sticks (set 2).

HERE ARE SOME MORE STICKS THAT WE DIDN'T USE. I WANT YOU TO PUT THIS ONE (one of the sticks) WHERE IT BELONGS IN THIS ROW OF STICKS.

Record child's responses.

If child didn't get it right, put the stick in the right location in the series and ask him/her to try another one.

LOOK, IF WE PUT THE STICK HERE, THEN ALL THE STEPS KEEP GOING UP. SEE? (Show by pointing.) HERE'S ANOTHER ONE. PUT IT IN THE STAIRCASE SO THAT ALL THE STEPS ARE GOING UP.

Record child's responses.
FOLLOW-UP TO ACTIVITY 3:

The best follow-up to this activity with children is for you to discuss with other adults what you did and observed.

The following provides some focal points for your discussion:

1. Recall what you did, asked the child to do, and what the child did.
2. In what ways did the interview go well?
3. In what ways did it not go well?
4. If there were problems or misunderstandings, how could you make the interview better if you did it again?
5. How was the child's performance similar to or different from how you would solve the problem?
6. How do you think a younger or older child might deal with the task?
CHAPTER 2:

WORKING WITH CHILDREN

This chapter is an extension of the last activity in which you interviewed the child on a simple seriation task. Here, you will learn to administer a double seriation problem and to investigate order correspondences between two series. As before you will role-play the activity with other adults before you interview children. To learn more about seriation, Chapter 3 includes the use of a 30-minute videotape called The Development of Order Relations: Seriation.

INTERVIEWING A PARTNER

ACTIVITY 4: Double Seriation Problems (Interviewing an Adult)

You will start by administering a double seriation problem to your partner. Then you will take the role of a child and complete the tasks administered to you. In a double seriation task, two series are arranged so that they have an ordinal correspondence. This means that the first member of one series is matched to the first member of the second series, and so on.
MATERIALS:

You will need two sets of objects that can be seriated. Ideally, they should be different in appearance and size. There should be the same number of elements in each series (approximately 10). All the materials should be the same color or those of one series, one color, and of the other series, a different color. (See Appendix A for a description of materials.)
INSTRUCTIONS FOR ACTIVITY 4:

1. Use Activity-Form B to determine what you should do and what questions you should ask. Make sure you read it thoroughly and understand the procedures before you begin. (Read pages 56 to 57 for more information on this task.)

2. Record the response of your partner in the appropriate spaces.

3. Switch roles and have your partner interview you. You can play the role of a child if you choose.
ACTIVITY FORM B: Double Seriation

Your Name: ____________________  Child’s Name: ____________________
Date: ________________________  Child’s Age: ________________________
Name of Child’s Teacher: ____________________________________________

This form is to be used for Activities 4 and 9 (pages 35 and 65).

Do (Conditions)                                      Say

Put out dolls and bats in a mixed up order.*         HERE I HAVE SOME DOLLS AND STICKS (BATS). HOW ARE THE DOLLS DIFFERENT FROM EACH OTHER?

Move child to an understanding of your desire to have him/her pair the smallest bat with the smallest doll and so on until the largest doll has the largest bat.

I WANT YOU TO GIVE EACH DOLL THE RIGHT BAT. WHICH BAT SHOULD THE SMALLEST DOLL HAVE?

Once the child indicates an intention to give the small bat to the small doll...

O.K., GIVE ALL THE DOLLS THE RIGHT BATS.

* Dolls and bats will be used to illustrate the procedure.
Record what the child does.

When the child is finished...

Record what child does and says.

When child is finished...

ARE THEY ALL RIGHT NOW?

WHY DOES THIS DOLL GET THIS BAT? (pointing to one of the errors) IS IT RIGHT?
ACTIVITY 5: Ordinal Correspondences (Interviewing an Adult)

This activity is an extension of the double seriation task. Once a double series has been established, you can investigate whether it was constructed with an understanding of the order relations within and between both series.

MATERIALS:
Same as those used in Activity 4.

INSTRUCTIONS FOR ACTIVITY 5:
1. Use Activity Form C to determine what you should do and the questions to ask. (Read pages 57 to 58 for more information on this task).
2. Record the responses of your partner in the appropriate spaces.
3. Switch roles and have your partner interview you.
ACTIVITY FORM C: Ordinal Correspondences

Your Name: ___________________  Child's Name: ___________________
Date: _______________________  Child's Age: ________________
Name of Child's Teacher: ______________________________________

This form is to be used to complete Activities 5 and 9 (pages 41 and 65).

Do (Conditions)                          Say

After the child has formed a double series with or without your help (Form B), put the dolls closer together and the bats further apart, (or vice-a-versa).

After the dolls are bunched together.

O.K. I'M GOING TO TAKE THE DOLLS AND PUT THEM CLOSER TOGETHER.

NOW WHICH BAT SHOULD THIS DOLL HAVE (pointing to the tallest doll)?

WHICH BAT SHOULD THIS DOLL HAVE (pointing to the shortest doll)?

Now pointing to a bat that is opposite to the 4th or 5th doll from the end.

WHICH BAT SHOULD THIS DOLL HAVE (pointing to a middle one)?

WHY?
If the child has been successful with the last two questions, switch the dolls around so that one series goes from short to tall and the other series goes in the same direction from tall to short.

WHAT IF A FRIEND SAID YOU WERE WRONG, HOW WOULD YOU SHOW THEM YOU WERE RIGHT?

NOW THE DOLLS ARE GOING TO SWITCH AND LINE UP IN THE OPPOSITE WAY.

WHICH BAT SHOULD THIS DOLL HAVE (pointing to the smallest)?

WHICH BAT SHOULD THIS DOLL HAVE (pointing to the tallest)?

WHICH BAT SHOULD THIS DOLL HAVE (pointing to the fourth or fifth doll from the end)?

WHY? EXPLAIN WHY THAT BAT GOES WITH THAT DOLL.
ACTIVITY 6: Ordinal Correspondences — Destroyed Series (Interviewing an Adult)

This activity is also an extension of the double seriation task (Activity 4). It can be done after Activity 4 or as a continuation of Activity 5. Once a child knows that certain bats belong with certain dolls, the two series can be destroyed and you can pick out a doll and ask the child to find which bat goes with that doll.

MATERIALS:

Same as for Activity 4.

INSTRUCTIONS FOR ACTIVITY 6:

1. Use Activity Form D to determine what to do and say. (Read page 59 for more information on this task.)
2. Recall the responses of your partner.
3. Switch roles and have your partner interview you.
ACTIVITY FORM D: Ordinal Correspondences (Destroyed Series)

Your Name: ___________________________  Child's Name: ___________________________
Date: ___________________________  Child's Age: ___________________________
Name of Child's Teacher: ___________________________

To be used for Activities 6 and 9 (pages 45 and 65).

Do (Conditions):

After child has formed a double series with or without your help, mess up the two series.

Say

NOW I'M GOING TO MAKE IT DIFFICULT. I'M GOING TO MESS UP YOUR NICE WORK AND SEE IF YOU CAN FIND THE RIGHT BAT FOR THIS DOLL (pick a doll that belongs somewhere in the middle of the series). FIND THE BAT THAT BELONGS TO THIS DOLL.

Record what the child does.

Does child reconstruct one series

_________ both series _______

part of one series: ____________

part of both series: ____________?

Does the child find the right bat?
When child selects a bat... HOW DO YOU KNOW THAT'S THE RIGHT BAT?

IF YOUR FRIEND THOUGHT THAT ANOTHER BAT WAS RIGHT, HOW WOULD YOU SHOW YOUR FRIEND THAT YOU ARE RIGHT?

Record what child does or says.
WATCHING A VIDEOTAPE

In this section of Chapter 2, you will view a videotape showing children four to eight years of age being interviewed on all the tasks you did in Activities 2 through 6.

This tape was made at the Hillside Primary School in Berkeley, California, with the cooperation of the school, teachers, children and their parents. The children were selected by their teacher, primarily on the basis of their ability to cope with the confusion of a video-production and the stress of being filmed.

The youngest children were interviewed by their teacher, Daniel Peletz. The six, seven and eight year olds were interviewed by someone they had just met for the first time. There was no rehearsing and there were no retakes. Despite the presence and observation of strangers, the physical discomfort and chaos, and the general unfamiliarity of the situation, the children were able to concentrate on the tasks, were seriously involved, interested and, on the whole, enjoyed themselves. This illustrates the mental power that children of any age bring to bear on activities that tap their intellect and reasoning. While the raw footage was edited for clarity and the restrictions of a 30-minute show, what you see is an honest and accurate portrayal of children actively reasoning about order relations.
INSTRUCTIONS FOR ACTIVITY 7:

1. Review the following outline of the tape:

<table>
<thead>
<tr>
<th>Child</th>
<th>Age</th>
<th>Stage</th>
<th>Problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>Darrilyn</td>
<td>4</td>
<td>Pre-conceptual</td>
<td>Simple seriation with insertion</td>
</tr>
<tr>
<td>Brock</td>
<td>6</td>
<td>Intuitive</td>
<td>Simple seriation with insertion</td>
</tr>
<tr>
<td>Bebe</td>
<td>7</td>
<td>Intuitive</td>
<td>Double seriation; ordinal correspondences</td>
</tr>
<tr>
<td>Ryan</td>
<td>8</td>
<td>Concrete-operational</td>
<td>Simple seriation with insertion</td>
</tr>
<tr>
<td>Tanya</td>
<td>7</td>
<td>Concrete-operational</td>
<td>Double seriation, ordinal correspondences with and without destruction of the two series.</td>
</tr>
</tbody>
</table>

2. Look for how Darrilyn, Brock, and Bebe handled the task differently than adults. Look for similarity as well.

3. Look for how Ryan's and Tanya's performances are similar to adults. (You can use the transcript of the tape, Appendix B, to make notes.)

INTERVIEWING CHILDREN

In this section of Chapter 2, we'll discuss a number of issues that will help you prepare to interview children on the tasks you saw in the videotape.

Do's and Don'ts of Interviewing:

1. It is important that you enter the interview with the right attitude. This should be a curiosity about how children serjate and a willingness to explore ways of finding out. It is not a question of whether children have the "right" answers. There are no "right" answers to these types of tasks. Given the opportunity, children will do as well as they can, and the object is to see what they do so that you can gain more insight into your children and the concerns of this unit. If you are concerned with the child "doing well", you will prejudice the results and most likely make the child uncomfortable.

A typical mistake of beginning interviewers is to try to get the best/most advanced response from the child. This is understandable, but it inevitably results in hounding the child, making instructions too complex, and generally communicating to the child that you want something s/he is not giving you; that there is something wrong with his/her thinking. It's better to go slow and let the child take the lead with as little interference from you as possible. The child's reasoning is a sensitive and
personal issue and you should maintain a respect for it throughout the interview.

2. Present the seriation materials to the child in a natural manner, saying something like, "let's work with these for a while," or "I have some things I want you to do with these." The tasks will strike children as natural and reasonable. In one form or another, children naturally engage in seriation from birth on. Rather than providing a lot of explanations about what it's all about, just get going.

3. Remain as flexible as possible throughout the interview. If the child does not immediately respond to your questions, wait and ask again. If the child does not do what you ask, let him/her fool around a bit. The child may need some time to feel comfortable with the materials. If the child commences on things, even if they do not relate to the task, listen and respond to what is said. When you feel that the child is drifting from the focus, remind him/her of what you want. The experience itself should be pleasurable for you and the child. If you feel uncomfortable, or the child is feeling anxious and uncomfortable, then it is best to discontinue the interview. If the child gives any indication of wanting to quit, you might see if s/he just needs a little encouragement. If s/he still asks to quit (either verbally or nonverbally), then stop the interview.

4. Try not to be anxious with yourself or the child. You can and will make "mistakes," fail to ask the "right" questions, misunderstand the child and so on. It takes experience to become a good
interviewer. You will have to make mistakes to learn. So don't worry about it. If you worry, you'll probably worry the child as well. **Trust the child.** Children like to share their thoughts with adults, if adults seem interested and respectful.

5. Remember, you are the adult, it's your interview and you're in control. Be clear to the child about what you want. If you let the child "take over," s/he will do whatever strikes his/her fancy and you will have learned little. For example, if the child starts using the sticks to build a house, say, "You can play with the sticks in any way you like after we've finished, but now I want you to..." If the child persists, end the interview. You can always return when conditions are better.

**The Form of the Interview**

A good interview is not the result of following a prescribed procedure word for word or being good at following a set of instructions. A good interview results from being attentive to what the child is doing, how s/he is understanding your instructions, and forming "educated guesses" about why the child is doing one thing or another. Your development as an interviewer depends upon your concern for exploring the child's thinking, your clarity about what you are looking for, and your ability to invent questions "on the *w*" that help you get a clear picture of the child's thinking. This skill takes time and practice to develop.
1. **Simple Seriation Tasks with Insertion**

   The procedures for this task are outlined on Form A (page 27). Use this form for your first few interviews. After some practice you can work without it.

   Some general procedures are outlined below:

   **Constructing the series**

   a. Make sure the child is aware of the nature of the material. The interview might go like this "What are these? That's right, sticks. Tell me about them. That's right, they're different lengths."

   b. After the child is aware of the various lengths, ask him/her to arrange them in a series. You might have to try various ways of expressing this, particularly to younger children. You might say something like, "Do you know how you walk up steps? Well, I want you to arrange these sticks like steps so that each step goes up." If the child does not get the idea, you might arrange the sticks in a series and point out how each one is taller. Then mix them up and ask the child to arrange them in the same way.

   c. If the child creates an error, you might point to one of the sticks and ask the child if it is right. If the answer is "yes," accept it. Do not point out all the errors.

   d. If the child does not line the sticks up so that their bottoms are even you might question the child, by pointing...
to the bottom ends and saying, "How about the bottom of the sticks, are they right too?" If the child doesn't consider them, it is most likely because at his/her stage, it is difficult to consider what's happening to the tops and bottoms of the sticks at the same time.

e. If the child does not arrange all the sticks in a series, ask him/her to use all the sticks. "How about these (remaining sticks); can you use them too?"

f. If the child creates a number of separate series, suggest that s/he construct one series, "That's good, you made a lot of staircases. I'm wondering if you could use all the sticks to make just one staircase."

Inserting new sticks in the series:

g. If the child is able to form a series with the 10 sticks, go on to the insertion problem. If not, correct the series. But do so in a way that does not put the child down. For example, "Now I'm going to take your staircase and make some changes, O.K.?

h. Bring out the second set of sticks. Introduce them with something like, "Here are some more sticks that can go with the sticks in the staircase. I want you to find where they go. Here's one, see if you can find where it goes in the staircase." If the child does not understand, take a stick and demonstrate, leaving it in the series.

i. When a child places a new stick in the series, ask why that's the right place for it.
j. If new sticks are put in an obviously incorrect place, you might point it out saying something like, "How about this; is this right? If you were to walk up the steps, what would happen here?"

k. Check to see if the child is still willing to go on by asking something like, "Do you want to find where this stick (a new one) goes?"

2. Double Seriation Tasks
   a. Only do a double seriation problem if the child is able to create a single series. If the child can, s/he will be in either the intuitive or concrete-operational stage.
   b. Follow the procedures outlined on Form B (page 39).
   c. The object of a double seriation task is to see how the child goes about solving a task that requires thinking of the materials as two series, in which the first member of one series is matched to the first member of the second series, and so on.
   d. The problem should be introduced with simple language and, if necessary, use of imagery that describes one series as "belonging" with another. For example, different size people with "walking sticks", "bats", "batons," "umbrellas," etc. Once there is an understanding that the bats, for example, are of different sizes, and so are the dolls, then lead the child to pair the shortest bat with the shortest doll and likewise, for the longest bat.
"Which bat should the shortest doll have? How about the tallest doll?" "Give the rest of the dolls their bats."
e. If there are some obvious errors in the child's construction, such as a big doll with a short bat next to a shorter doll with a longer bat, ask the child, "Is it right for the small doll to have such a big bat?" If the child says "yes," accept it, but if the child says "no," ask him/her to make them right.

3. Ordinal Correspondence Tasks
   a. This task should not be done with children who have difficulty forming a series.
   b. Use Activity Form C (page 43) for interviewing children. After you are familiar with doing this task, you can proceed without Form C.
   c. "Ordinal correspondence" means a double series in which the first element of one series corresponds (is matched) to the first element in the second, and so on. In the ordinal correspondence task, the question is whether these order relations are still understood when the elements of the series are spread out or put closer together.
   d. When the materials are spread out, one series should extend beyond the ends of the other.
Start by asking about the ends of the short series (dolls).

"Which bat goes with this doll (tallest); which bat goes with this doll" (shortest)?" Next, ask about a doll that is fourth or fifth in from the ends.

e. Be sure to ask the child why his/her answer is correct. Possible approaches are: "How would you show someone you're right?" "Why is that the right bat for this doll?" Accept the child's answer or probe sensitively and briefly for more understanding on your part.

f. If the child has been successful in the above task, reverse the series so that one series (bats) goes from shortest to longest and the other series (dolls) goes from tallest to shortest. Follow the same general procedures outlined above in step 3 (a-e).
4. **Ordinal Correspondence Tasks -- with destruction of the two series**
   
a. This task should not be done unless the child can construct a double series.
   
b. Follow the procedures on Form D (page 47). After some practice, you'll be able to work without Form D.
   
c. The purpose of this task is to see how a child thinks of two series when the elements are all mixed up. A doll is picked out (one belonging to the middle of the series), and the child is asked to find its bat. This involves doing the double seriation over again. The two things to look for are whether the child sees the necessity for reconstructing the two series; and if the child does, how much of the two series s/he reconstructs. The child may rebuild the entire double series, or only up to the doll in question, which is an adequate solution if done systematically.
   
d. When the child finds an answer, you should probe to find out the child's reasoning. "How do you know that's the right bat? What if a friend said you were wrong. How would you show them that you were right?"

---

**Increasing and Decreasing the Task Difficulty**

The purpose is not to make the task "easy" so that the child gets the "right" answers (except where you want the child to get a right answer in order to understand your instructions) but rather, to keep the task at a level where the child feels comfortable dealing with it.
Experience will be your best guide to what children of various ages can handle.

There are two issues to consider when selecting a task for a child. One is the nature of the task, and the other concerns the materials. The tasks presented on Activity F make up a series of increasingly difficult tasks. Most children beyond four years of age can deal with the simple seriation and insertion task. While the younger children will not create "true" series, they are still fairly comfortable dealing with the task. One way of making simple seriation problems easier is to form parts of the series for the child. The double seriation tasks should probably not be given to children who cannot construct a series at least by trial and error. To simplify a double seriation problem, you can complete one of the series for the child.

The materials can be simplified in a number of ways, but if they are made too easy you will not be able to explore the child's reasoning about order relations because you will make it possible to solve the task by simpler means.

Here are some ways of simplifying the materials:

- reduce the number of things to be seriated
- increase the size intervals between successive members of the series.

Increasing the number of objects and/or decreasing the distance between successive elements in the series are ways of increasing the task difficulty.
ACTIVITY 8: Simple Seriation with Inception (Interviewing a Child)

In this activity, you will interview another child between the ages of four and eight on the simple seriation task. You should use Form A as a guide. This is a repeat of the same task you did for Activity 3 (page 29). You should interview a different child than the one you did in Activity 3.
ACTIVITY FORM A: Simple Seriation and Insertion

Your Name: ________________  Child's Name: ________________
Date: ________________  Child's Age: ________________
Name of Child's Teacher: ______________________________________________________________________

This form is to be used to complete Activities 2, 3, and 8 (pages 25, 29, 61).

Do (Conditions)                    Say

Put out the 10 sticks (set 1) - messed up.                            HERE ARE A BUNCH OF STICKS.

HERE ARE A BUNCH OF STICKS. SOME ARE SHORT AND SOME ARE TALL. FIND ME THE SHORTEST ONE. FIND ME THE TALLEST ONE.

Record child's responses.

Describe what the child does and draw a picture of the child's series. I WANT YOU TO PUT THE STICKS IN ORDER SO THAT THE SHORTEST IS HERE. MAKE THEM GO UP FROM THE SHORTEST TO THE TALLEST (give the child time to understand you and give examples if necessary).
If child did not form a series, arrange the sticks for him/her.

Bring out the 4 extra sticks (set 2).

Record child's responses.

If child didn't get it right, put the stick in the right location in the series and ask him/her to try another one.

Record child's responses.

I'm going to change the sticks like this...

Here are some more sticks that we didn't use. I want you to put one's one (one of the sticks) where it belongs in this row of sticks.

Look, if we put the stick here, the all the steps keep going up. See? (Show by pointing.) Here's another one. Put it in the staircase so that all the steps are going up.
ACTIVITY 9: Double Seriation and Tests of Ordinal Correspondences

This activity combines Activities 4, 5, 6 (pages 35 to 45) that you, role-played as an adult. To complete the activity, you should test two children between four and eight years of age. Interview children that you feel are capable of forming a single series. If they have difficulty, or you are uncomfortable, discontinue the interview. Use Activity Forms B-D. After the double seriation problem has been done (Form B), go on to a test of ordinal correspondences (Form C). If that task is comfortable for the child, go on to testing ordinal correspondences when the series has been destroyed (Form D).
ACTIVITY FORM B: Double Seriation

Your Name: ____________________  Child's Name: ____________________
Date: _________________________  Child's Age: ____________________
Name of Child's Teacher: ____________________

This form is to be used for Activities 4 and 9 (pages 35 and 65).

Do (Conditions)
Put out dolls and bats in a mixed up order.*

Say
HERE I HAVE SOME DOLLS AND STICKS (BATS). HOW ARE THE DOLLS DIFFERENT FROM EACH OTHER?
HOW ARE THE BATS DIFFERENT?
I WANT YOU TO GIVE EACH DOLL THE RIGHT BAT. WHICH BAT SHOULD THE SMALLEST DOLL HAVE?
WHICH BAT SHOULD THE TALLEST DOLL HAVE?

Move: child to an understanding of your desire to have him/her pair the smallest bat with the smallest doll and so on until the largest doll has the largest bat.

Once the child indicates an intention to give the small bat to the small doll...

O.K., GIVE ALL THE DOLLS THE RIGHT BATS.

* Dolls and bats will be used to illustrate the procedure.
Record what the child does.

When the child is finished...

ARE THEY ALL RIGHT NOW?

WHY DOES THIS DOLL GET THIS BAT? (pointing to one of the errors) IS IT RIGHT?

Record what child does and says.

When child is finished...

ARE THEY ALL RIGHT NOW?

VERY GOOD.
ACTIVITY FORM B: Double Seriation

Your Name: ____________________  Child's Name: ____________________

Date: ____________________  Child's Age: ____________________

Name of Child's Teacher: ______________________________________

This form is to be used for Activities 4 and 9 (pages 35 and 65).

Do (Conditions)                                                Say

Put out dolls and bats in a mixed up order.*

HERE I HAVE SOME DOLLS AND STICKS (BATs). HOW ARE THE DOLLS DIFFERENT FROM EACH OTHER?

HOW ARE THE BATS DIFFERENT?

Move child to an understanding of your desire to have him/her pair the smallest bat with the smallest doll and so on until the largest doll has the largest bat.

I WANT YOU TO GIVE EACH DOLL THE RIGHT BAT. WHICH BAT SHOULD THE SMALLEST DOLL HAVE?

WHICH BAT SHOULD THE TALLEST DOLL HAVE?

Once the child indicates an intention to give the small bat to the small doll...

O.K., GIVE ALL THE DOLLS THE RIGHT BATS.

* Dolls and bats will be used to illustrate the procedure.
Record what the child does.

When the child is finished...  ARE THEY ALL RIGHT NOW?

WHY DOES THIS CHILD GET THIS BAT? (pointing to one of the errors) IS IT RIGHT?

Record what child does and says.

When child is finished...  ARE THEY ALL RIGHT NOW?

VERY GOOD.
ACTIVITY FORM C: Ordinal Correspondences

Your Name: ___________________________ Child's Name: ___________________________

Date: ___________________________ Child's Age: ___________________________

Name of Child's Teacher: ________________________________________________

This form is to be used to complete Activities 5 and 9 (pages 41 and 65).

Do (Conditions) Say

After the child has formed a double series with or without your help (Form B), put the dolls closer together and the bats further apart, (or vice-a-versa).

After the dolls are bunched together.

O.K. I'M GOING TO TAKE THE DOLLS AND PUT THEM CLOSER TOGETHER.

NOW WHICH BAT SHOULD THIS DOLL HAVE (pointing to the tallest doll)?

WHICH BAT SHOULD THIS DOLL HAVE (pointing to the longest doll)?

Now pointing to a bat that is opposite to the 4th or 5th doll from the end.

WHICH BAT SHOULD THIS DOLL HAVE (pointing to a middle one)?

WHY?

}
If the child has been successful with the last two questions, switch the dolls around so that one series goes from short to tall and the other series goes in the same direction from tall to short.

Now the dolls are going to switch and line up in the opposite way.

What if a friend said you were wrong, how would you show them you were right?

Which bat should this doll have (pointing to the smallest)?

Which bat should this doll have (pointing to the tallest)?

Which bat should this doll have (pointing to the fourth or fifth doll in from the end)?

Why? Explain why that bat goes with that doll.
ACTIVITY FORM C: Ordinal Correspondences

Your Name: ___________________  Child's Name: ___________________
Date: ________________  Child's Age: ___________________
Name of Child's Teacher: ___________________

This form is to be used to complete Activities 5 and 9 (pages 41 and 65).

Do (Conditions) Say

After the child has formed a double series with or without your help (Form B), put the dolls closer together and the bats further apart, (or vice-a-versa).

After the dolls are bunched together.

O.K. I'M GOING TO TAKE THE DOLLS AND PUT THEM CLOSER TOGETHER.

NOW WHICH BAT SHOULD THIS DOLL HAVE (pointing to the tallest doll)?

WHICH BAT SHOULD THIS DOLL HAVE (pointing to the shortest doll)?

Now pointing to a bat that is opposite to the 4th or 5th doll from the end.

WHICH BAT SHOULD THIS DOLL HAVE (pointing to a middle one)?

WHY?
The child has been successful with the last two questions, switch the dolls around so that one series goes from short to tall and the other series goes in the same direction from tall to short.

WHAT IF A FRIEND SAID YOU WERE WRONG, HOW WOULD YOU SHOW THEM YOU WERE RIGHT?

Now the dolls are going to switch and line up in the opposite way.

Which bat should this doll have (pointing to the smallest)?

Which bat should this doll have (pointing to the tallest)?

Which bat should this doll have (pointing to the fourth or fifth doll in from the end)?

Why? Explain why that bat goes with that doll.
ACTIVITY FORM D: Ordinal Correspondences (Destroyed Series)

Your Name: ___________________ Child's Name: ___________________

Date: ___________________ Child's Age: ___________________

Name of Child's Teacher: ______________________________________

To be used for Activities 6 and 9 (pages 45 and 65).

Do (Conditions)

After child has formed a double series with or without your help, mess up the two series.

Say

NOW I'M GOING TO MAKE IT DIFFICULT. I'M GOING TO MESS UP YOUR NICE WORK AND SEE IF YOU CAN FIND THE RIGHT BAT FOR THIS DOLL (pick a doll that belongs somewhere in the middle of the series), FIND THE BAT THAT BELONGS TO THIS DOLL.

Record what the child does.

Does child reconstruct one series

both series

part of one series

part of both series?

Does the child find the right bat?
When child selects a bat...

HOW DO YOU KNOW THAT'S THE RIGHT BAT?

IF YOUR FRIEND THOUGHT THAT ANOTHER BAT WAS RIGHT, HOW WOULD YOU SHOW YOUR FRIEND THAT YOU ARE RIGHT?

Record what child does or says.
ACTIVITY FORM D: Ordinal Correspondences (Destroyed Series)

Your Name: _______________  Child's Name: _______________
Date: _______________  Child's Age: _______________
Name of Child's Teacher: ______________________

To be used for Activities 6 and 9 (pages 45 and 65).

Do (Conditions)  Say

After child has formed a double series with or without your help, mess up the two series.

NOW I'M GOING TO MAKE IT DIFFICULT. I'M GOING TO MESS UP YOUR NICE WORK AND SEE IF YOU CAN FIND THE RIGHT BAT FOR THIS DOLL (pick a doll that belongs somewhere in the middle of the series). FIND THE BAT THAT BELONGS TO THIS DOLL.

Record what the child does.

Does child reconstruct one series__________
both series_____________
part of one series ____________
part of both series ____________

Does the child find the right bat?
When child selects a bat...

HOW DO YOU KNOW THAT'S THE RIGHT BAT?

IF YOUR FRIEND THOUGHT THAT ANOTHER BAT WAS RIGHT, HOW WOULD YOU SHOW YOUR FRIEND THAT YOU ARE RIGHT?

Record what child does or says.
FOLLOW-UP TO ACTIVITIES 8 AND 9

The best follow-up to these activities with children is for you to discuss with other adults what you did and observed.

The following provides some focal points for your discussion:

1. Recall what you did, asked the child to do, and what the child did.

2. In what ways did the interview go well?

3. In what ways did it not go well?

4. If there were problems or misunderstandings, how could you make the interview better if you did it again?

5. How was the child’s performance similar to or different from how you would solve the problem?

6. How do you think a younger or older child might deal with the task?
CHAPTER 3:

ASSESSING STAGES OF INTELLECTUAL DEVELOPMENT

GENERAL REMARKS ON THE DEVELOPMENT OF SERIATION

The development of order relations concerns more than the seriation of sticks. It involves the whole question of mentally ordering or sequencing events and objects. Seriation tasks are only one context in which the child's understanding of order relations is expressed. They are significant however, in that they provide an easily administered and interpreted means of gaining insight into the child's understanding of order relations or sequences. The same reasoning that the child demonstrates in doing a seriation task is being used constantly. All mental activity requires some understanding of sequence. Some knowledge, such as concepts of time and measurement, are specifically and directly concerned with sequences.

To understand how a given child conceives of order relations requires more than simply observing a child building a series. Many four year olds, for example, can sequence nesting boxes, order rings on a self-correcting spindle, or even put Cuisenaire Rods in order from shortest to tallest. This in itself tells us little about the child's understanding. Is the child proceeding by rote memory or by reason? How such activity reflects an understanding of order relations requires further probing. To explore the child's understanding of order relations requires discovering the nature of the child's reasoning about relations of less- and greater-than, before and after, etc. To this end, it is important to know how to select
and administer tasks that demonstrate the nature of the child's understanding.

As adults, we understand that an element in a series comes before all the elements that follow it and after all the elements before it. In the case of seriated sticks, we recognize that a stick is both taller and shorter than its neighbors; taller than those that come before it and shorter than those that follow. Being able to coordinate concepts of less-than and greater-than allows you to reason through a problem such as:

Paul came to dinner after Sue; Paul came to dinner before Jack. Who came last to dinner? Young children do not understand the relations implied in such a problem, even if presented in a physical form. However, as the child develops, there is a progressive understanding of the relation less-than and greater-than, and by the concrete-operational stage these concepts are coordinated in a reversible organization.

THE PRE-CONCEPTUAL STAGE -- APPROXIMATE OR INCOMPLETE SERIES

(generally two-five years of age).

RE-READ PAGES 13 THROUGH 19

The material in Chapter 1 provides a good description of the pre-conceptual stage of seriation. Its main features are:

1. Pre-conceptual children from either approximate series, in which each successive element is not necessarily shorter (taller) than all those before it; Or

2. A series in which the tops or bottoms of the sticks are seriated, but not with a consideration of both the tops and bottom; Or

3. A number of small series not coordinated with each other.

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In addition, children of this period cannot insert new sticks into their proper place in the sequence.

Although the pre-conceptual child's inability to produce a series is, in itself, revealing, the character of a child's reasoning is revealed more in the method of construction than in the final product. The method of a child's construction is guided by an understanding, and, by observing the method, the reasoning is revealed.

The pre-conceptual child thinks of the sticks as either short or long. This is guided by a simple discrimination between one length and another. It does not take into account the relative length between a given stick and all the others, nor does it incorporate an understanding that a stick can be a long stick when compared to some and, at the same time, short when compared to others. A stick simply has the static appearance of big or small. However, most two and three year olds do not have even this understanding; most four year olds, when asked to select the shortest stick, are able to pick one that is generally shorter than all the others, although it is often not the shortest. When asked to select the shortest of two sticks, they usually give the correct response. In this sense, the pre-conceptual child understands the meaning of the words "shortest" and "longest."

On the other hand, if the pre-conceptual child had a clear understanding of the terms "shortest" and "longest," it would be possible to guide them to construct a series. Some guided practice with an emphasis on "taking the longest stick, then the next longest stick, and now the longest of all those that are left, etc." would produce a series. But two things characterize the pre-conceptual child in this regard. One is
that children in this stage do not have a systematic conception of "longest" or "shortest". This can be seen in the fact that, unlike older children, there is no attempt to make sure that a given stick is actually longer (or shorter) than all of some set of other sticks. The other point is that the idea of proceeding by taking the longest, next longest, etc., is foreign to the child. If the method is suggested, as it was for Darrilyn in the order-relations videotape, it is pursued without systematic consideration and, as a result, the series is approximate. In fact, prior to Darrilyn's scene in the film, she was led step by step to construct an ordered series by taking the longest, next longest, and so on. However, when left to her own, she treats the sticks as either generally long or short, and constructs a series that looks somewhat like a series, but is not.

Further evidence that the pre-conceptual child thinks of the sticks as short or long, is seen in their reaction when it is pointed out that a stick is too tall or too short in its present location. The typical response is to move the indicated stick to the tall or short extremes of the sequence. This is clearly revealed in Darrilyn's performance when asked to place new sticks in the series. It is difficult for the pre-conceptual child to conceive of where a new stick might go, even when it is clearly taller than the shortest stick and shorter than the tallest stick. When asked to do so, the child will oblige, but not without mistakes. Furthermore, when mistakes are made, they are not regarded as such. The fact that a stick may be taller, for example, than either of its neighbors, does not suggest to the pre-conceptual child that it should
be placed somewhere else.

If we were to look at the performance of pre-conceptual children on double seriation problems and on questions concerning ordinal correspondences, we would see similar inconsistencies. These inconsistencies are revealed in other areas of knowledge as well. For example, if a pre-conceptual child is shown a row of objects and asked to count them, s/he will not point to each object and say a number. Instead the child may point to one object and say three numbers or skip a few objects while only going up one numeral. The pre-conceptual child does not establish a one-to-one correspondence between the series of counting and the series of pointing. Similarly, a pre-conceptual child may think that as a row of blocks is spread out, its number increases. There is a lack of understanding of the co-seriation; that as the row gets longer, the space between its blocks increases. The pre-conceptual child's understanding still relies heavily on appearance. The extended row looks longer, therefore it must have more blocks in it.22

In closing, we should note the similarity between the pre-conceptual coordination of order relations and class relations. Darrilyn is a good example because she is our pre-conceptual star in both the classification and seriation videotapes. In classifying, she constructs graphic collections. The problem lies not in discriminating between two objects on the basis of some property, but rather, in coordinating similarities and differences among properties of all and some of the objects. For example, Darrilyn can put together a row of squares. This requires discriminating (seeing a difference) between squares and non-squares. The row of squares

is a result of matching one square to another, not thinking of the entire collection as united by the property "squareness," and thereby separated from shapes that are not squares." She is just as likely to form a graphic collection such as

![Shapes](image)

because each thing is in some way similar to one of its neighbors. Her performance in the seriation task is similar. Rather than thinking of "short and tall" in terms of all the sticks, she thinks of these characteristics only with respect to pairs. This results in an approximate series. She is pleased with her efforts, since its appearance is that of a series, and as a preconceptual child she reasons on the basis of appearances rather than an understanding of relations.

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THE INTUITIVE STAGE — SERIES CONSTRUCTED BY TRIAL AND ERROR

(Generally six-seven years of age)

RE-READ PAGES 19 THROUGH 22

The discussion of the intuitive stage in Chapter 1, provides a good description of this period. The main characteristics are:

1. The intuitive stage child can construct a series by trial and error because there is an understanding that each stick must be taller than all that come before it (or, conversely, each stick must be shorter than all those that come before it).
2. Children in the intuitive stage have difficulty inserting new
sticks into a series because they cannot yet simultaneously
consider the relations taller-than and shorter-than.

Again, the method of the child's construction is more revealing than
its result. In fact, the result of the intuitive child's efforts to ar-
range a single series is indistinguishable from that of an adult. In
Chapter 1, we've explored the character of the intuitive child's methods.
It is based upon an understanding that "shortest" and "tallest" mean
"shorter than all the others" or respectively, "taller than all the others."
This correct understanding is revealed in the child's concern that each
added stick be taller than all those already in the series. And this,
guided by visual judgments about how much taller each added stick should
be, is sufficient to construct a series. This trial and error approach,
in itself, however, is not what distinguishes the intuitive stage child
from children in the next stage.

The true character of a series is such that each added stick must
not only be taller than those before it, but also shorter than all those to
follow. This ability to coordinate shortest and tallest, and the re-
sulting understanding of its logical relationship to a sequence, is what
distinguishes intuitive from operational reasoning. The fact that this
coordination is absent in the intuitive child's reasoning is revealed in
a number of ways.

One is seen in the method of construction. Rather than proceeding
like a concrete-operational child, forming the series by taking the short-
est stick from the pile, then the next shortest, and so on, the intuitive
child simply starts adding sticks that have the appearance of being longer
than sticks already in the series. If one appears "too much longer", it
might be replaced with one that looks better. However, if the differences
between successive sticks are small, judgments based upon appearances will result in errors. These errors are only corrected after a situation arises in which none of the remaining sticks are longer than those in the series. When this occurs, the child destroys part of the series and begins again. Unlike the concrete-operational child, there is a lack of anticipation of the relations that will ultimately hold between members of the completed series; that each element will be taller than all those before it, and shorter than all those that follow.

A second indication of the intuitive child's inability to coordinate less-than and greater-than is the difficulty the child faces when asked to insert additional sticks into the series. Unlike the pre-conceptual child, children in the intuitive stage can reason that if a stick is too tall at one end of the series and too short at the other, it belongs somewhere in the middle. But this understanding is intuitive, and pursued only in one direction. The child's reasoning is something like the following: if a stick is "too short" it must come somewhere before the stick to which it's compared. The child looks for a place where the stick won't be "too short", but can't find it because what determines when the stick is no longer "too short" is the point at which it is longer than some other sticks. And this requires thinking of the stick as being, at the same time, both short and long with respect to some other sticks.

Both of the above are clearly illustrated by six-year-old Brock, in the order relations videotape.

Now, it is possible that our analysis is wrong; that the intuitive child understands the order relations of a series, but is simply confused by the task of inserting a new stick into the series. The double seriation problem, with its associated problems of ordinal correspondence, provides
the telling difference between intuitive and concrete-operational understanding.

Bebe, our other six-year old, can arrange the dolls in a series. If the series were constructed with a systematic understanding of order relations, it would also be understood that the double seriation task demands no more than constructing two series, lining them up so that the first member of one series matches the first member of the second, and so on. However, the intuitive series is not a result of reversible mental operations. It is something that is formed through trial and error. Therefore, Bebe pairs big-looking sticks with big-looking dolls, and ends up with an approximate correspondence between two series. When an obvious error is pointed out, such as the bigger of two dolls with the smaller of two bats, she recognizes the error and corrects it by exchanging bats. This trial and error approach, though adequate for a few members of the series, will not work for the total series for there are too many possible combinations. Thus, the intuitive child's actual lack of understanding becomes evident.

Further indication that the intuitive child's understanding of order relations is still incomplete is seen in the problem of ordinal correspondence. When the two series are rearranged so that the spatial location of corresponding members is changed, Bebe has no way of determining which bat goes with which doll. She matches a doll with the bat closest to it, demonstrating that like the pre-conceptual child, an understanding of order relations is still tied to appearances, rather than abstract concepts. The idea that the fifth largest doll must still be paired with the fifth largest bat is foreign to children of this period.
There are a wide variety of problems which likewise illustrate the intuitive child's incomplete understanding and use of order relations. Some of these problems involve an understanding of cause and effect relationships. For example, a child might be given a ramp with a number of different sized balls and asked to find what determines how far the balls will travel after they roll off the end of the ramp. Some children will think the size of the ball makes a difference: "Big balls jump further." Yet when they are questioned, they may believe that a ball that is a little bit bigger won't go further whereas a much bigger ball will. There is a lack of appreciation that if the size of the ball makes a difference, then each increase in the series of size should result in an increase in the series of distance jumped by the ball. 23

As has already been pointed out, the intuitive child still relies a great deal on appearances and cannot, as yet, coordinate order relations. Thus, the child's ability to co-seriate cause and effect is inconsistent. The cause, in most cases, must be visually obvious for the intuitive child to connect it to an effect, and even then she/he cannot use this empirical information to project the consequences of variations in the original experiment.

In concluding our remarks on the intuitive stage of seriation, it is interesting to note the similarities between the classification and seriation of children at this stage. In both, there is a systematic use of "all." The intuitive stage classes are composed of elements that are all the same in some way, and an intuitive understanding of order relations

Thcorporates the notion that "tallest" means taller than all of some set of elements. Further, while the intuitive stage child can form classes, the classes are still defined by maximal visual similarity among elements in the class. In other words, classificatory reasoning is still tied to appearances during the intuitive stage, even though the classes themselves are no longer graphic. Likewise, the intuitive series are still tied to the use of appearances as seen in its construction and in the confusion among spatial and ordinal correspondences. Lastly, while in both seriation and classification, intuitive children understand and can coordinate some of the relations, they do not yet conceive of these relations in reversible terms. That is, "similarities and differences" are not coordinated with "all and some", nor are notions of less-than and greater-than coordinated with one another.

THE CONCRETE-OPERATIONAL STAGE -- TRUE SERIATION

(generally occurring around eight years of age)

Chapter 1 provides a good description of the concrete-operational stage of seriation. Its main points are:

1. The child constructs the series with an obvious appreciation of order relations, systematically taking the longest (or shortest) stick from the pile and adding it to the series. This indicates an understanding that, the longest stick in the pile becomes the shortest stick in the series.

2. An ability to coordinate less-than and greater-than is also
revealed in the child's ability to insert additional sticks into an already formed series.

Ryan gives a beautifully clear example of the fact that inserting a stick into the series requires an understanding that the stick is, at the same time, shorter than those that follow and taller than those before it. 24

In Tanya's performance in the double-seriation and ordinal correspondence tasks, we find abundant evidence that the coordination of order relations has achieved a more consistent organization with entrance into the concrete-operational stage. Tanya's approach to the bats and dolls task is simple and logically straightforward. She constructs a series of dolls and then seriates the bats, placing the first bat on the first doll, and so forth. When the bats and dolls are separated and even reversed in order, she still knows which bat belongs with which doll because she knows that it is the order of the elements that defines the series and not its appearance. The fifth largest doll still gets the fifth bat, no matter what their arrangement may be. This understanding of order relations enables her to match the right doll to the right bat even after the two series have been destroyed.

The feature that distinguishes concrete-operational from pre-conceptual and intuitive seriation is the reversible organization of order relations. In concrete-operational thought, "less-than" is understood to be the opposite of "greater-than", and both are coordinated with each other to form a logical or structural whole. Recall, that with respect to

A similar reversible relationship holds among order relations, except that rather than subtraction, the reverse of the relationship "greater-than" is "less-than." For example, if A is greater than B, and B is greater than C, then it follows that A is also greater than C: \((A > B) + (B > C) = (A > C)\). If you reverse one of the relations, for example, making C greater than B \((C > B)\), then the formula becomes, \((A > C) + (C > B) = (A > B)\) or, if you reverse the relationship between A and B \((B > A)\), the formula becomes, \((A > C) + (B > A) = (B > C)\). This last formula simply expresses the understanding that if A is greater than C, and B is greater than A, then it follows that B is greater than C.

While the concrete-operational child does not think in terms of mathematical formulas or their abstract verbal expression, his/her ability to construct series, insert elements into the series, construct double series, and maintain ordinal correspondences, shows a flexibility in reversible understanding that is formally equivalent to the above mathematical expressions. It is this flexibility that enables the concrete-operational child to co-seriate the cause and effect elements of a problem such as the one involving the balls and ramp which the intuitive child could not do.

However, if this experiment were presented to the child without telling him/her which variables make a difference, he/she would not be able to determine whether it is the weight of the balls, their size, or the height of the ramp that makes the difference. It is not until the formal-operational stage that the child can work with the logic of
possibilities to formulate a precise experiment. Furthermore, the concrete-operational child's understanding of order relations is yet to be applied to purely abstract concepts. For instance, the concept of infinity, of boundless extension of a series, is not grasped until the formal-operational stage when concepts of order relations have developed more completely.

In summary, the concrete-operational child's performance on seriation tasks, is indistinguishable from how adults would perform. However, as pointed out with respect to classification and mental development in general, it is not the final stage of thought. During the years between eight and twelve, late childhood, there is a continued and progressive refinement of concrete-operational structures. During adolescence, there is the emergence and refinement of formal-operational structures and the ability to reason systematically with the abstract.

SOME CONCLUDING REMARKS ON STAGES OF SERIATION

Throughout this unit we have focused on three stages of seriation found in children (four-eight) years of age. Some form of seriation is going on during the first year of life, and its organization continues to develop through late adolescence.

One additional and important point should be made. Each of the stages covers a relatively long period of time: pre-conceptual stage from two to five years of age; intuitive stage, five to seven years of age; concrete-operational stage, eight-twelve years of age. By now you should be familiar with some of the things that are common to the seriation skills of children within each stage. However, you should also keep in mind that there is progress made by children within stages as well. The two-year-old pre-

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conceptual child, for example, is less sophisticated in his/her seriation than is the five-year-old pre-conceptual child. The same holds for children within all stages. In a sense, development is a series of numerous changes, organized around major changes in thinking. The three stages covered in this unit characterize the major changes. Within each stage there are subtle, but observable changes as well.

LOOKING FOR STAGES OF SERIATION

ACTIVITY 10: Reviewing the Videotape as a Review of Stages in Seriation

In this activity, you will review the videotape on seriation (order relations) in order to experience more directly the distinction among the ordinal reasoning of the pre-conceptual, intuitive, and concrete-operational stages.

INSTRUCTIONS FOR ACTIVITY 10

1. View the videotape with the following 10 questions in mind.
   - You may want to stop the tape, rewind, etc., to examine behaviors of which you are not sure.
2. Use Activity Form E.
3. Discuss the issues with others, so that you can share insights and profit from things seen by others that you might have missed.
4. You can use the transcript of the tape (Appendix B) to take notes.
ACTIVITY FORM E: Questions for Reviewing Seriation Tape.

To be used for Activity 10 (page 95)

1. What are some things that Darrilyn does that shows she has some appreciation of the concepts "short" and "tall" and their relationship to a series?

2. What evidence is there that Darrilyn is in the pre-conceptual stage?

3. What are some of the things that Bebe does that gives the impression that she is in the pre-conceptual stage?

4. What are some of the things that Brock and Bebe do that give the impression they are in the concrete-operational stage?

5. What evidence is there that Brock and Bebe are in the intuitive stage?
6. What evidence is there that Ryan is in the concrete-operational stage?

7. How does Tanya's solution to the double seriation problem differ from Bebe's?

8. How does Tanya's solution to the double seriation problem use the same approach that Ryan used to construct a single series?

9. How does Tanya's solution of the ordinal correspondence problem differ from Bebe's?

10. How does Tanya prove that she has found the right bat for the right doll when the two series have been destroyed?
FOLLOW-UP TO ACTIVITY 10:

You will gain the most from discussing your answers with others. You may also wish to look over these possible answers.

1. What are some things that Darrilyn does that shows she has some appreciation of the concepts "short" and "tall" and their relationship to a series?

She constructs an approximate series from tallest to shortest; when asked, she selects the smallest of two sticks; when inserting two sticks in the series, she makes the first one shorter than the second one.

2. What evidence is there that Darrilyn is in the pre-conceptual stage?

She does not construct an ordered series; she cannot insert new sticks and tends to put them at the ends of the series; she disregards the actual length of the sticks unless the interviewer points them out to her.

3. What are some of the things that Bebe does that gives the impression that she is in the pre-conceptual stage?

When Bebe is arranging the bats with the dolls, she does not arrange the bats in an ordered series.

4. What are some of the things that Brock and Bebe do that give the impression they are in the concrete-operational stage?

Both Brock and Bebe are able to construct an ordered series. Bebe does this with the dolls and Brock does it with the sticks.

5. What evidence is there that Brock and Bebe are in the intuitive stage?

While Brock can construct a series, it is done by trial and error and he has difficulty inserting sticks into the existing series. Bebe can order the dolls, but does not realize that the sticks must be seriated as well. Nor does she understand the order relations between the series of dolls and the series of bats, for when asked to find the bat for the fourth doll, she picks one that is opposite it, rather than selecting the fourth bat.

6. What evidence is there that Ryan is in the concrete-operational stage?

Ryan constructs the series by successively taking the longest stick in his hand and placing it at the developing end of the series. He also realizes that a stick in the series is shorter than one neighbor and taller than the next.
7. How does Tanya's solution to the double seriation problem differ from Bebe's?

Unlike Bebe, Tanya realized that the dolls and bats must each be put into a series and that the first member of one series must be paired with the first member of the second series, and so on. Bebe, on the other hand, pairs big-looking sticks with big-looking dolls, medium sticks with medium-size dolls, and small sticks with small dolls. She does not check to see that the biggest stick is paired with the biggest doll, and so forth down to the smallest stick for the smallest doll.

8. How does Tanya's solution to the double seriation problem use the same approach that Ryan used to construct a single series?

Tanya arranges the dolls and bats by taking the longest, next longest, and so on. This is the same approach Ryan used to seriate the sticks.

9. How does Tanya's solution of the ordinal correspondence problem differ from Bebe's?

When Tanya is asked to find which bat in one series goes with a given doll in the second, she counts to determine the ordinal position of the dolls and then counts to find the bat of the same ordinal position. Bebe does not consider the ordinal position, but rather, the appearances of the two series.

10. How does Tanya prove that she has found the right bat for the right doll when the two series have been destroyed?

Tanya proves that she has the right bat, by showing that the doll is the fifth in the series (that there are only four dolls that are smaller) and showing that the bat she selected is also the fifth in the series of bats. She realizes that she does not need to reconstruct both series in their entirety to solve the problem.
ACTIVITY 11: Sharing and Assessing Your Interviews of Children on Seriation Problems

In this activity, you will share with others what you did with children in Activities 8 and 9 (pp. 61 & 65). The purpose is to look at what kinds of evidence you have for the child's stage of seriation and to consider what additional information you could get to help you decide.

MATERIALS:
You will need the objects you used for interviewing children in Activities 3 and 9 and the records you kept on Forms A, B, C, D.

INSTRUCTIONS FOR ACTIVITY 11:
1. You and your fellow learners should work it out so that group members are able to share at least one of their interviews in Activity 8 (simple seriation with insertions) and Activity 9 (double seriation and tests of ordinal correspondence).
2. When demonstrating to other members in the group, use the actual material you used with children and use Activity Forms A, B, C, and D to remind you of what you did and how the child responded.
3. Discuss among each other what the evidence is that leads you to believe that a given child is in one of the three stages discussed in this unit.
4. Discuss the similarities and differences between children of the same approximate age and those of different ages.
5. Discuss how the interviews went well and not so well.
6. Discuss what kind of information you now realize you failed to get and how you will get this information next time you interview a child.
FOLLOW-UP TO ACTIVITY II:

By this point, you have gained a good understanding of some of the problems and tasks used to explore order relations. Furthermore, you have learned to recognize behaviors that reveal the character of the child's understanding. You have learned these through reading, through discussions, through interviewing children, and through viewing the videotape. In this section, we will briefly summarize the main things to look for in seriation tasks. For any of the tasks listed on the following pages, you should be able to answer all of the accompanying questions if you've done the interviews correctly and if you've kept records of the appropriate behaviors.

We've presented the tasks and their related questions in a way that should be useful for keeping records on individual children. The same sort of questions can be applied to any other seriation tasks you may wish to use. In a following section called "On-going Assessment," we'll discuss the use and importance of records in assessing a child's development.
SIMPLE SERIATION WITH INSERTION: CHILD RECORD

Your name______________________________ Child's name______________________________

Date______________________________ Child's age______________________________

Name of child's teacher______________________________

To be used as a record of a child's performance on a simple seriation tasks.

Simple Seriation

Does the child understand the terms "shortest" and "tallest"?
- When considering 2 or 3 sticks? Yes____ No____
- When considering all of the sticks? Yes____ No____

How does the child construct the series?
- Comparing one stick to some of the others? Yes____ No____
- Comparing one stick to all the others? Yes____ No____
- Adding sticks that are progressively taller (or shorter)? Yes____ No____
- Adding sticks only at the developing ends of the series? Yes____ No____
- Are mistakes recognized? Yes____ No____
- If so, how are mistakes corrected? ________________________________

What is the form of the completed series?
- A number of small series? Yes____ No____

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• One series using all the sticks? Yes___ No____
• Each element in the series is progressively taller (or shorter)? Yes___ No____
• Bottoms even? Yes___ No____

Placing New Sticks into the Series

How are new sticks inserted?
• Are sticks inserted in their right place? Yes___ No____
• Are new sticks put only at the ends of the series? Yes___ No____
• Are mistakes recognized? Yes___ No____
• If so, how are mistakes corrected? ________________________________

What stage do you think the child is in?
Pre-conceptual________________ Intuitive________________________
Concrete-Operational____________

Which other stage is the child closest to?

Comments:
DOUBLE SERIATION: CHILD RECORD

Your name ___________________________ Child's name ___________________________
Date ___________________________ Child's age ___________________________
Name of child's teacher ___________________________

To be used as a record of a child's performance on a double seriation task.

Double Seriation

Does the child understand the task?

- Know that the shortest stick goes with the shortest doll? Yes___ No____
- And that the biggest stick goes with the biggest doll? Yes___ No____

How does the child approach the task?

- Attempt to generally give longer bats to taller dolls? Yes___ No____
- Seriate the dolls or sticks? Yes___ No____
- Seriate both the dolls and sticks? Yes___ No____
- Create two series in which the shortest, next shortest, etc., stick is paired with the corresponding shortest, next shortest, etc., doll? Yes___ No____
- Are mistakes recognized? Yes___ No____
- If so, How are mistakes corrected? ___________________________

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What stage do you think the child is in? Which other stage is the child closest to?

Pre-conceptual

Intuitive

Concrete-operational

Comments:
TESTS OF ORDINAL CORRESPONDENCES: CHILD RECORD

Your name ___________________________ Child's name ___________________________
Date ___________________________ Child's age ___________________________
Name of child's teacher ___________________________

To be used as a record of a child's response to tests of ordinal correspondence.

To be done after double seriation problem.

How does the child approach the problem when the dolls and sticks are in the same order, but one row is spread out?

- Understand that the shortest doll gets the shortest stick, and the tallest doll, the longest stick? Yes ___ No ___
- Pick two that are closest to each other? Yes ___ No ___
- Count the positions (e.g., "first", "second", "third"...)? Yes ___ No ___

How does the child approach the problem when the order of the dolls and sticks is reversed?

- Understand that the shortest stick goes with the shortest doll and vice versa? Yes ___ No ___
- Pick the two that are closest to each other? Yes ___ No ___
- Count the positions? Yes ___ No ___
How does the child approach the problem when the two series are destroyed and messed up?

- Pick one that simply looks right? Yes ___ No ___
- Reconstruct one series? Yes ___ No ___
- Reconstruct both series? Yes ___ No ___
- Reconstruct only the necessary parts of one or both series? Yes ___ No ___

What stage do you think the child is in? Which other stage is the child closest to?

Pre-conceptual ____________________________
Intuitive ________________________________
Concrete-Operational ______________________

Comments: ________________________________

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ON-GOING ASSESSMENT

In Chapter 5, we talk about the broad educational implications for what you have learned about seriation and Piaget's theory in general. However, before concluding this chapter, we should discuss the importance of establishing an on-going program for assessing all the children in the classroom. By getting to know all your children and by assessing the same child a number of times during the year, you can expect to see a number of things.

- You will come to see a common pattern to the thinking of all your children.
- You will find that while there are similarities, children also differ from each other. For example, children will differ on how well they understand your instructions, and this will not always be a result of their level of development. How comfortable the child is with you, how familiar the child is with the expressions you use, etc., are all factors that will affect the child's performance.

Children will also differ in their level of development. Even if all the children are in the same stage, some will be just entering the stage and some ready to enter the next stage.

- You will find that children who are doing "equally well" in school are not necessarily at the same stage in their development. For example, two pre-school children may know their numbers, colors, alphabet, etc., equally well, but perform at somewhat different levels on seriation tasks.
Likewise, you will find that children may be at the same stage, even though one is doing well in school and the other not so well.

Maybe most importantly, you will find that children whom you regarded as less advanced, actually show themselves to be developmentally as far along as their classmates. On the other hand, you might come to realize that the problems a child is having with certain subject matters are the result of the child's level of development and not necessarily "an inability or unwillingness to learn" or an inability on your part to do a "good job" teaching.

Keeping Records

To get to know your children in the above terms is important. To better see and understand the similarities and differences among your children can only make you a better teacher. The first step for such insight comes from keeping records of your interviews with the children. This will allow you to compare children, to check your impressions against records of actual performance, to see how a child progresses through the year, to get a better feel for the relationships between the child's performance (or your expectations) and the child's stage of development, and to compile relevant information to share with parents.

A record for any child should consist of a description of the task you used, the materials, the questions you asked, and the child's responses. Such a record is called a "protocol." The various activity forms used while interviewing children are examples of protocol forms. However as you become a better interviewer, you will not want to restrict yourself to predetermined approaches. One way to keep a protocol record is simply to make notes of what you are doing and saying (as you do and say them) and...
record what the child does and says. Your protocol record might look like the following:

<table>
<thead>
<tr>
<th>Simple seriation - 10 blue sticks (9.0, 9.8, 10.6, 11.4, 12.2, 13.1, 13.9, 14.7, 15.5, 16.3, cm. long)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sally Miller</td>
</tr>
<tr>
<td>October 2, 1975</td>
</tr>
<tr>
<td>Age: 5 years, 2 months</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Examiners</th>
<th>Child</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longest stick?</td>
<td>Correct</td>
</tr>
<tr>
<td>Shortest stick?</td>
<td>Mistake</td>
</tr>
<tr>
<td>Give child two sticks - which is shortest?</td>
<td>Correct</td>
</tr>
<tr>
<td>Make series from shortest to longest.</td>
<td>Child makes three different series.</td>
</tr>
<tr>
<td>Make one series?</td>
<td></td>
</tr>
<tr>
<td>After I corrected the series to show child the correct series</td>
<td>Child puts the first three together</td>
</tr>
<tr>
<td>I asked: Can you do it like I did?</td>
<td>Child still makes an uneven series</td>
</tr>
<tr>
<td>And so on.</td>
<td></td>
</tr>
</tbody>
</table>
After you have interviewed the child and while your memory is still fresh, you should use your protocol to fill out a summary record of the important child behaviors. Record-forms such as those on pages 103-104 provide examples. With both the protocol and summary, you have a good record of the child's performance and a basis for making judgments about the child's stage of development and understanding of order relations.
All of the ways you've directly experienced children's understanding of order relations are only some of the ways such understandings can be explored. This simple fact is in itself one of the more important implications of Piaget's work. It suggests that there is a broad and common theme that underlies a wide variety of activities. We've called this theme "order relations." Order relations are involved in any activity -- either mental or physical -- that involves the sequencing of events or objects. This means that you can explore the logic of sequencing in an almost endless variety of ways. As we've discussed in a unit on concepts, all concepts involve the use of order relations, or class relations, or both. 25 As such, their expression is prevalent in much of the child's activity and thinking. As you develop more familiarity with their expression in a variety of physical seriation problems, you will become increasingly good at seeing their character in children's activities in general, including those that make up the subject matter of the classroom.

It is important to extend your exploration of order relations and their development to a broader range of tasks than those discussed in the preceding chapters. By experimenting with a variety of activities you will
see the variations that make order relations easier or more difficult to reason about. For example, while a child may not be able to arrange a series of sticks, he/she may be able to draw an accurate picture of the series. Experiences with a variety of seriation activities will also give you a better feel for the similarities between tasks used to explore the natural course of development and tasks that are a part of your teaching activity. Lastly, the more varied the tasks the better you will get at working with children, learning how they understand your requests and questions, the kinds of inconsistencies they recognize in their own behavior and judgments (and those not recognized as well), and how they deal with problems that are recognized but difficult to solve. In short, the better you understand order relations and their role in different types of activities, the better you will understand the child's understanding of the world and how that understanding is communicated.

A few remarks should be made regarding the following additional activities. Most of them are physical problems or ones requiring the production of pictures. Activities of this character make it easier for the children to express their understanding and easier for you to interpret a child's behavior. More importantly, physical activities involving concrete objects provide the context in which the child can do his/her best work and with the use of objects and pictures, you see most clearly what the child does or does not understand. Some of the problems have been used by Piaget and his colleagues for assessing intellectual development. Almost all of them are more difficult than the simple seriation tasks mentioned in the preceding chapters. The following is a very brief presentation. To get more information, look up the indicated references.
The standard seriation problem can be interestingly varied by using sticks that differ in length by a non-constant amount. They can be cut from balsa wood to form non-straight line series. For example:

Be careful not to use too many sticks (about 10 work well), or to make the differences in length too small to be noticed (vary between about 1/16" and 1/2").

A related task is to ask the child to copy more complicated designs, for example:
Piaget has also investigated the development of serial ordering when the objects cannot be seen.26 Objects of different dimensions (different sized circles could be tried) are placed under a cover or behind a screen and the child is asked to order them from largest to smallest. Because of the difficulty in identifying size differences without visual aid, there should be larger differences between the elements than there are for the standard seriation task (a minimum of 1/4 inch ought to do). The wooden table blocks referred to in Using Toys and Games with Children, Activity 5, can be easily adapted for use here.27

Children will have more difficulty when they are called upon to anticipate an ordering without actually arranging it. Here are two tasks that Piaget has used to study the roles of anticipation and perception. For the first task, a set of sticks, each with a different length and color, is mixed on a table. (Cuisenaire Rods can be used). The child is given a set of crayons with colors that match the colors of the sticks, and is then asked to draw a picture of how the sticks would look if they were ordered from largest to smallest. The problem can be made easier by giving the child an ordinary pencil, thereby eliminating the requirement of maintaining the correspondence between color and ordinal position.

The second task requires anticipation without visual cues. A set of sticks is screened. After examining them by touch, the child is asked to draw what they would look like if they were placed in order from largest to smallest.


27. This is another unit in the FLS.
smallest. For this kind of problem fewer sticks should be used (about five) with large differences in stick length. An interesting variation is to include some sticks of equal length.

Order relationships can also be studied that do not involve a single spatial dimension. For example, a child can be asked to copy an ordering of doll-clothes on a line.28

The child is given his/her own laundry and clothesline. Even young children find this task easy, so it is a good place to start. But it becomes more difficult if they are asked to produce a reverse ordering or one where both ends of the clothesline are connected.

The above kind of problem can also be presented using a collection of beads made into a necklace. And again, the problem can be made more difficult by eliminating perceptual cues. For example, a few of the beads can be placed on a string and drawn into a tunnel or behind a screen. The problem is to predict the order in which the beads will emerge. Or the beads can be inserted into a tube and the tube rotated 180 degrees. When asked to predict the order in which they will emerge, the child must reverse the original order.29 Variations include using more beads and/or

more rotations of the tube.

There is a more difficult variation of the single seriation problem that Piaget has not studied directly. This task also involves building staircases with blocks of different lengths. But to produce an even set of increments for each step, blocks of different lengths must be combined. The problem is described in *Using Toys and Games with Children*, (Part II, page 123).

In the same book can be found a series of sequencing games, some of which are quite difficult; all require a basic knowledge of ordering before the more difficult aspects of the problem can be considered (Part II, page 217).
A multiple seriation problem is one in which objects can be ordered with respect to two or more qualities simultaneously. When the ordering is completed, a partial or complete matrix is formed.

30. For a complete discussion of the development of the ability to perform matrix multiplication see The Early Growth of Logic in the Child, Chapter 10. Generally, the stage progression develops at the same rate as single seriation.
The example is a simple one. The number of objects forms one dimension, while the kind of object is ordered along the other dimension. A more standard set of objects usually consists of two quantitative dimensions like size and color hue. For example, the child is asked to arrange four to seven sets of different sized leaves, each set having leaves of from four to seven different shades of green. It is a good idea to show what is desired by starting the matrix arrangement for the child.

The illustrated balls of increasing size and darkness provide an additional example of a set of materials that could be used.
Matrix multiplication problems can be made more or less difficult by varying the number of categories along each dimension, and/or by making the differences along each dimension more or less distinct. A more interesting means for increasing the difficulty is to ask the child to complete an incomplete matrix.
With this kind of problem, the child is given the remaining cards for the entire matrix. From these s/he selects the appropriate single card and is asked to justify his/her choice. Note that the example problem is also difficult because the objects vary with respect to quantities that must be considered and ones that must be ignored.

Another possibility for a multiple series is a set of glass beakers that vary in height and width. But since it is difficult to find objects that vary systematically along two quantitative dimensions, appropriate objects can be molded from clay or plasticene.

All problems that require a sequencing also require an understanding of transitivity. For example, if stick "A" is longer than stick "B", and stick "B" is longer than stick "C", then the adult knows without having to compare them, that stick "A" is longer than stick "C". The
young child does not know this. S/he can be given "A" and "B" to compare, then "B" and "C" to compare, and even young children will make the correct judgments about relative height in each case. But when asked to make judgments about "A" and "C", without actually comparing them, the child may not be able to answer, or may merely guess.

A much more difficult transitivity problem is one where the information about the objects to be compared is presented entirely verbally, that is, without any objects to actually compare. We have already given several examples of these. For instance, if Sue is taller than Jane, and Julie is shorter than Jane, who is tallest?

The above problems should give you some additional feeling for the importance of understanding serial orderings. Serial ordering underlies all quantitative concepts such as height, length, force, hue, weight, and time. A good example of a physical concept that results from appropriate kinds of ordering is the concept of density. This concept is essentially an ordering of weight along one dimension and volume along another dimension. The density of any two substances can be compared, by comparing their weights for the same volume, or their volumes for the same weight. If two things weigh the same, but one has a larger volume, then it is less dense.
A is denser than B.
C is denser than B.
Piaget's first five books, published in the 1920's were enthusiastically received and widely discussed. In this book you have been introduced to some of the ideas that are discussed in those works: children's thinking progresses through stages; at each stage there is a common quality to much of their thought; and, prior to middle childhood, children's concepts are organized differently than those of adults. However, the initial popularity of these findings had little impact on American psychology or education, and between the 1930's and 1960's few people were familiar with his work.

This surprising indifference was the result of a predominate involvement with Behaviorism, according to which knowledge is a copy of external facts and can be taught through example, practice, correction and reinforcement. These views are older than Behaviorism itself, as evidenced by the traditional methods of education, such as rote learning, memory exercises and teacher-directed activities. Such practices reflect a belief that knowledge is memory of past events and that good teaching corrects or reinforces student responses to tasks and questions. Most educational practice in America today has its roots in history and its theoretical

31. Portions of this chapter also appear in Part 1 of ECT.

32. These same principles are explored in other areas of knowledge by the three remaining parts of the ECT series and by the FLS unit titled Working with Children's Concepts.

33. Langer, Jonas, Theories of Development.
rationale in Behaviorism. This, however, is rapidly changing and in large measure because of Piaget's work.

Towards the end of the 1950's a number of factors prompted a renewed interest in Piaget's studies. For one, his general findings seemed valid, yet unexplainable in behavioristic terms, thus providing a new focus for theoretical psychology. For another, the topic of mental development itself was virtually untouched by behavioral research. The breadth and richness of Piaget's preceding four decades of study opened an exciting and unexplored area of obvious importance. Lastly, Piaget brought to psychology a method of study and focus that was beginning to surface in a variety of disciplines. Piaget calls this approach "structuralism" and describes it as a study of how the parts of a system work together to form the system itself. What you have learned about the development of order relations is an example. In order relations the parts of the system are the mental activities of sequencing objects and events. These activities are organized to form a pattern or system. As you've learned, this system changes in systematic ways from one stage to the next.

Whereas behaviorists regard knowledge as a recording of external experience, Piaget has attempted to show that all knowledge has an underlying organization. This organization cannot be taught or recorded from examples. It is the result of the individual's constant regulation of physical and mental activities becoming increasingly systematic and better organized. The fact that an eight-year old reasons about order relations in adult-like fashion and the four-year old does not is not the result of

34. Piaget, Jean, Structuralism. 143
more experience with adult reasoning. By ordering objects and events in ways that are personally interesting and important, children come to organize their activities in more systematic terms. As they do, the ordering activity inevitably takes the form we regard as logical and adult-like.

Piaget has created a powerful picture of intellectual development but, for the most part, has left its educational implications to be explored by others. We now know rich and fascinating facts about what children do and do not believe. These facts have obvious educational implications, for education carries as its primary responsibility an understanding of what children regard as true. Piaget also provides a technique for exploring the common patterns of thought in general. As educators come to use his ideas for exploring thinking, they will be better at anticipating what children can and cannot do and more confident in stimulating and following children's thoughts.

Lastly, Piaget's theoretical account of how children progress in their mental development contradicts many commonly held assumptions about thinking. In its broadest terms, it suggests that thought is not derived from language, though language serves an important function; that thought is not abstracted from the environment, but actively constructed in a self-determined interaction with the environment; that the inability of children to understand certain truths is not because of a lack of relevant information, but an inability to coordinate information in the required fashion. These important additions to our understanding of thought, suggest educational practices that are quite different from traditional approaches. The theory suggests trusting children in more active and self-directed learning, asking their own questions, and finding means of answering them. It suggests that interactions between children are as important as those between adults.
and children, and that freely exploring the physical and social environment is as, or more, important than sitting quietly and doing one's lessons. It suggests that children be stimulated to reason rather than memorize and that they be encouraged to reflect upon their own views and their implications. However, because this psychological theory is relatively new to a majority of educators, it will take time to fully fashion it into educational practice.

THE DEVELOPMENT OF ORDER RELATIONS
AND ITS EDUCATIONAL IMPLICATIONS

When first introduced to the important relationship between seriation and thought, it is hard not to feel a responsibility for teaching it. The fact that young children seriate so differently than adults, and the presence of countless educational products designed to teach order relations, makes this seem all the more reasonable. In some respects there is justification in guiding children to specific ordering activities, and in other respects it is a misuse of energy and possibly even harmful to the child's overall education.

It now seems clear that all healthy and active children come to order in increasingly systematic ways and that by middle childhood these skills are logical from the adult perspective. It is also fairly well recognized that it is very difficult to teach children an understanding of order relations that is more advanced than one they naturally hold. Success in doing so is achieved only when children are at a transition point between one stage and the next, or, in other words, when they are close to understanding on their own what is being taught. Furthermore, once success is achieved, it by no means guarantees an overall improvement in the quality
of a child's thinking.

On what basis, then, is it reasonable to engage children in ordering activities such as seriation tasks? As long as the focus is on helping children explore their own understanding, rather than replacing it with advanced thought, seriation can be fun and rewarding for teachers and children. Moreover, by providing ordering activities for children, the teacher can get a clearer sense of their level of development and their understanding of the elements they are working with. However, if teachers attempt to enforce understandings that are more sophisticated than a child's level of development, there is risk that the child will divorce the activity from his/her own ability to reason, and replace it with attempts to imitate and to memorize what is being taught. To separate mental activity from reason replaces self-reliance with a dependency upon the teachings of others. More importantly, it may lead the child to conclude that his/her way of thinking and viewing the world is wrong when in fact it is the only way that s/he can reason.

It seems reasonable to state that, if anything, the implications of Piaget's theory are to avoid teaching seriation. Why replace vital aspects of education with activities designed to advance reasoning in the direction it is destined to take of its own accord? Rather, the effort should be directed towards providing children with opportunities to exercise whatever understanding of order relations they may have and to appreciate the results of their efforts. Children will become increasingly aware, but at their own pace; and soon enough, earlier ways of thinking will be replaced by more advanced reasoning.

Many American educators have come to the conclusion that if thinking
progresses through stages, then it is unreasonable to expect things from children that they are not yet ready to understand or do. The notion of "readiness" is a popular theme in American education. "Don't try to teach what the child is not ready to learn." Piaget's theory suggests that all knowledge has organization and that the organization advances through a sequence of orderly stages. Thus, the problem is not to avoid certain kinds of knowledge, but rather to examine all aspects of subject matter (curriculum) in terms of the level of understanding being demanded of the child. This is a difficult task because it requires understanding the underlying organization of thought during all of the relevant stages, and understanding how these issues bear on all the subject matter of the classroom.

This series of four units is an attempt to address part of this problem. By introducing you to the character of thought in its various forms of organization during the early childhood years, you will be better able to anticipate the kinds of problems your children will find difficult or easy to solve. But ultimately, the answer to the question "What is the child capable of doing or capable of learning?" is determined by the child. By coming to know how to explore a child's thinking and by respecting the integrity of the child's understanding you will also become more convinced that the child's own interest, likes and dislikes, best determine what s/he is ready to understand. Things that are too easy or too difficult do not interest the child nearly as much as the investigation of new problems and ideas suggested to the child by his/her own exploration. The natural course of intelligence is to find new problems suggested by the solution of old ones. These problems, and the knowledge necessary to finding their solutions, are what the child is ready to learn.
What you have learned about children's understanding of order relations between four and eight years of age provides a basis for anticipating what children can be expected to understand. For example, time is a continuous sequence. Because of the inability of pre-operational children to reason systematically and consistently about order relations, their understanding of time is incomplete and to the adult, illogical. Piaget has endeavored to show that concepts of space, number, measurement, chance, cause and effect, geometry, proportions, genealogical family relations, history, and so on, are likewise dependent upon an understanding of order relations. As a result, children generally do not form systematic understandings of these concepts until around eight years of age. The remaining three units in the ECT series explore some of these areas. The unit titled Working with Children's Concepts explores the relationship of classification and order relations to concepts in general.

OPEN-RESPONSIVE EDUCATION

Piaget's work has had an obvious and significant influence on the teaching of specific subjects. It is not in the least unusual to find developers of curriculum materials in areas as diverse as music, drama, art, social relations, math, science, and social studies, referring to Piaget as the foundation for their suggested practices. Yet a more significant impact is felt in the contemporary trends of open education. Educators are increasingly recognizing the need to turn more of the learning process over to the child, to get children out of their seats, inventing problems and solutions, talking freely with adults and each other, and, on the whole, actively participating in decisions as to what to study and how to study it.
A number of different educational approaches travel under the broad label of "open education." And there are disagreements among proponents of individual programs. But what is common to all is a concern with the character of the child's activities, and there are a number of ways in which Piaget's theory bears on this general question.

1. Central to the open-education philosophy is a belief that children learn best when they have a responsibility for their own learning. Because knowledge is constructed and made up by the learner, learners must develop skills in self-direction and they must be given opportunities to approach problems in ways that suit their individual character and disposition. Students should be involved in selecting projects and ideas to focus on; selecting resources to work with; determining when and for how long activities are pursued; participating in the evaluation of their learning; and in selecting the nature of feedback they receive.

2. Children's interests play an important role in the open-education environment. For one, it is recognized that interests are an indication of whether a child is developmentally ready for a subject--as presented. Secondly, it is understood that interest is a powerful source of energy for learning. Learning does not take place without an active involvement and this occurs more readily when there are genuine and personal interests.

3. In open-education classrooms the learning process is an active one in the sense that children can use a variety of resources and a variety of learning modes. Rather than learning arithmetic, for example, through rote-practice activities, workbook activities, and listening to the teacher, children may be involved in working
with physical objects that represent arithmetic concepts, playing games that involve mathematical concepts, making a movie, story, or play about a mathematical concept, keeping a diary of arithmetic used at home, and so on. Furthermore, the nature of the child's activities are guided to explore relationships rather than isolated facts. For example, rather than separately learning that $7+5=12$ and $2+10=12$, the child might be led to see the relationship between these two formulas; that, for example, if you take 5 from the first and add it to the second number in the first formula you get the second formula; $(7-5)+(5+5)=2+10=12$. What do you think? Would this principle apply to all formulas in which two numbers add up to a third?

4. All education concerns itself with developing an appreciation and skill in reasoning. Piaget's theory has provided a useful insight in this regard. He has shown that underlying all reasoning is a pattern of organization that can be identified as intelligence. As the child applies his/her interests and active consideration to events and objects, the pattern or organization undergoes changes and evolves towards a more satisfying and encompassing form. This course of emerging reason holds for any area of knowledge to which the mind may be applied. It is something that is neither learned nor acquired at birth. For reason to take its hold, the mind must be applied and challenged such that preliminary ideas are found to be false and wanting. The formulation of beliefs and the seeking of means to test beliefs is the exercise of intelligence. Asking children to imitate the patterns of reason imposed by others cannot serve this function. This principle is appreciated in open-
education and provides the basis for many of the practices that fall under that heading.

5. Open-education proponents stress the importance of social interactions among peers. Throughout his career Piaget has insisted that mutual cooperation and interaction among children sharing common goals and interests is essential to the development of reason. This means that children should be allowed to explore concepts through mutual play, through discussion, through sharing points of view, and through attempts to work together on common goals. The mutual interests of children are an important foundation for social and intellectual development and they should be built upon in the classroom.

6. Lastly, teachers in open-education classrooms must develop skills and talents that are different from those required of the traditional teacher. Open-education is not something that can be programmed or packaged in the sense of strict lesson plans or a guided series of instructions, such as in workbooks. Open-education requires a flexible approach in which individual needs can be met as they arise, where spontaneous questions can provide the focus for an extended project, where new challenges can be proposed as opportunities arise.

Education, as a question of learning and as a question of society, is enormously complex. It is something in which everyone has a share and a say, and it is something that affects and is affected by every factor in the human complex. The school itself is as much affected by education as it in turn affects the individual. Every intellectual revolution has left its imprint on the goals and values of the school. How these new goals are
achieved is a matter of technology; the developing of intelligence through doing. The issue of open-education, as with most other important educational issues, is strictly a question of what individuals in society want and value. The technology of producing competent students in open-education environments is a documented fact. The success of the British Infant School, the Bankstreet and Responsive Follow-Through Programs, testifies to the ability of open-education teachers to achieve the goals and values of society.\(^{35}\) The question is whether society wants to adapt the goals of open-education.

Piaget expresses his own view, which is often quoted by proponents of open-education:

The principle goal of education is to create men who are capable of doing things, not simply of repeating what other generations have done -- men who are creative, inventive, who discover.

The second goal of education is to form minds which can be critical, can verify, and not accept everything they are offered...we need pupils who are active, who learn to find out by themselves, partly by their spontaneous activity and partly through material we set up for them; who learn early to tell what is verifiable and what is simply the first idea to come to them.\(^{36}\)

Piaget is a scientist of immense stature. He is truly as much a source of intellectual revolution as were Freud, Galileo, and Aristotle. But science cannot, and should not, determine values. It can, however, educate


\(^{36}\) Ripple and Rockcastle (Eds.), Piaget Rediscovered.
our values and provide means of realizing the goals we set for ourselves. Through Piaget's work, extended by countless others who are concerned with the nature of intelligence, we can see that all children who are active come to develop increasingly refined expressions of intelligence. This happens irrespective of schools. It is the inevitable result of mental activity. But the question of school is critical and goes beyond the development of intelligence or reason in the abstract. School is properly concerned with the nature of subjects to which the intelligence is applied.

What Piaget suggests and practitioners have shown, is that classrooms and schools can be created in which students function with freedom in a climate of honesty and respect and still achieve what is expected by the society into which they must be absorbed. What is surprising to many seems obvious to others: intelligence directed and followed by interest leads to learning what society expects.

It is a myth to think that children are innately opposed and foreign to reading, writing, and arithmetic. These are simply society's ways of representing forms that intelligence naturally takes. Children develop in their linguistic intelligence, and reading and writing are simply shared ways of expressing this intelligence. Likewise, children develop in their understanding of class and order relations. Arithmetic is no more than socially shared ways of expressing this intelligence. These shared systems are conventions and not intelligence itself. Conventions must be learned from others. But is is the nature of people, and especially children, not to learn from others if in so doing one is demeaned, embarrassed, ridiculed, or demoralized. Only by personal interests free from fear and supported by others can children be expected to share and extend the exciting forms of personal and collective intelligence. Open-education at its best, sets a stage.
APPENDIX A:
SERIATION MATERIALS
APPENDIX A: SERIATION MATERIALS

The following is a description of materials to use in simple seriation tasks, insertion of new elements into a series, double seriation and tests of correspondence.

SIMPLE SERIATION: WITH INSERTION:

Dimensions

Set I consists of 10 sticks increasing in length by 8 mm. The sticks are 9.0, 9.8, 10.6, 11.4, 12.2, 13.0, 13.8, 14.6, 15.4, 16.2 cm. in length.

Set II consists of nine additional sticks that can be inserted among the 10 sticks in Set I. Each of the nine sticks is longer by 8 mm. The nine sticks are 9.4, 10.2, 11.0, 11.8, 12.6, 13.4, 14.2, 15.0, 15.8 cm. in length.

Materials

You can construct the sticks out of any material that is rigid and easy to handle. Wood is ideal. You can use molding from a lumber yard or other pieces of thin lumber. Hobby shops may have more appropriate materials. You can use other substances such as thick cardboard, pencils cut to length, etc. The sticks in set I are painted a different color than those in set II.

DOUBLE SERIATION:

Dimensions

Dolls: There are 10 dolls, each longer than the next by 8 mm.
The 10 dolls are 9.0, 9.8, 10.6, 11.4, 12.2, 13.0, 13.8, 14.6, 15.4, 16.2 cm. in length. They vary proportionately in width so that they maintain the same appearance even though their length changes.

Sticks: There are 10 sticks, one for each doll. Each stick is longer than the preceding one by 4 mm. The 10 sticks are 4.5, 4.9, 5.3, 5.7, 6.1, 6.5, 6.9, 7.3, 7.7, 8.1 cm. in length.

Materials

The dolls are cut with a jigsaw from 1/4" wood. Thinner material, such as cardboard, balsa wood, or linoleum will also work. The sticks can be made out of balsa wood used for constructing models. This can be found in any hobby shop. Other material can be used as well, such as pencils, pieces of wood, etc. The dolls are painted one color and the sticks another.
APPENDIX B:
TRANSCRIPT OF VIDEOTAPE

The Growing Mind: The Development
of Order Relations - Seriation.
In this film we examine the understanding and use of order relations. These relations are involved whenever objects, events, or experiences are sequenced. Some form of sequencing is involved in all reasoning.

The work of Jean Piaget shows that children progress through a number of stages in their understanding and use of order relations. Order relations always concern issues of less than and greater than. In this film they are revealed as children arrange objects in a series from shortest to longest.

Darrylyn is nearly five and is in the preconceptual stage. She considers the sticks, as either long or short. By starting with and continuing to add long looking sticks, Darrylyn forms an approximate series. To construct a uniform series, each stick must be shorter or longer than one and only one of its neighbors. Children in this stage cannot yet coordinate these relations.

After being helped to build a uniform series, Darrylyn is given three yellow sticks to be added. She thinks of the sticks as either short or long. If the stick appears long, it is put at the tall end of the series. It's exact position is not important.

Darrylyn shows some appreciation for order relations since she raises the sticks so that the first is shorter than the second. However, she disregards the length of the sticks.

When prompted to insert the two sticks inside the staircase, Darrylyn places the shorter of the two lower in the series. When dealing with two sticks she understands that the shorter one comes first.

If the stick is not the shortest, it's a tall one and belongs with the other tall sticks.

If it's too short, it's a short stick. The fact that it is both too long and too short, does not suggest to Darrylyn that it belongs in the middle.
Brock is one year older than Darrilyn, and in the intuitive stage.

Like Darrilyn, Brock realizes that if a stick is too short, it comes before the stick to which it's compared. However, he is different from Darrilyn in that he can place it so that it is also longer than all the sticks that come before it.

Brock removes the big ones and adds sticks only at the end of the developing staircase.

He proceeds by trial and error, children in the intuitive stage can produce a uniform series by trial and error as Brock does.

Each added stick is longer than all those already in the series. If it appears too long, he replaces it with a shorter one.

Brock's selection is based on whether the staircase looks even. This approach does not require an understanding that each stick is at the same time both longer and shorter than its neighbors.

Bebe is seven years old and also in the intuitive stage.

She has been asked to find the right bat for each baseball player.

This is a double seriation problem requiring the coordination of two series.

Without being told, Bebe has a general understanding that taller dolls get larger bats. But she does not understand that the tallest bat must be given to the tallest doll and so on down to the shortest bat for the shortest doll.

Like Brock, Bebe thinks of the materials only as big, medium sized and small. Consequently she pairs big, medium and small dolls with similar sized bats.
She can correct part of the series by giving the longer bat to the taller of two dolls.

Bebe can easily arrange the dolls in a uniform series. She uses the same trial and error method shown earlier by Brock.

After arranging the dolls, Bebe gives them their bats. Her effort yields an approximate double series.

Bebe's corrections are in the right direction; taller dolls for longer bats and so on. But they are only partial. The dolls and bats are not thought of as two series that can be aligned with each other.

Bebe has been helped to correct the double series. The dolls are now spread out and she is again asked to find the right bat for each doll.

Bebe's conception of the two series does not imply an ordinal correspondence such that the fourth doll gets the fourth bat. Instead, she picks the bat that is closest to the doll in question. This shows that the concept of order at the intuitive stage is still tied to spatial positions.

Ryan is eight years of age and at the concrete-operational stage of development. He approaches the task systematically, knowing that by placing the longest stick in his hand at the lower end of the series, he can construct an ordered relation of increasing and decreasing heights.

Ryan's conception of order relations leads him to note that the stick is both longer and shorter than its neighbors. He uses this information to determine its position.

While first making a mistake, he quickly corrects himself by again coordinating relations of less than and greater than. The concept of middle has now taken on a precise meaning.
Tanya is almost eight and also in the concrete-operational stage. She knows that to find the right bat for each baseball player, she must establish two series and then relate one to the other.

Having arranged the dolls, Tanya creates the double series by taking the longest of the remaining sticks and placing it on the tallest of the remaining dolls.

Having formed a double series with an understanding of ordinal position Tanya can easily determine which bat belongs to which doll even though the series are reversed.

Tanya realizes that the problem requires re-constructing the series of dolls and bats.

She mentally reconstructs the series of dolls without the need of physically arranging them.

Like Ryan, Tanya shows a mental flexibility typical of the concrete-operational stage. She reconstructs a portion of the two series to prove that she had chosen the right bat for the doll in question.

Most four- and five-year olds are in the pre-conceptual stage and consider the sticks as either long or short. By starting with and continuing to add long-looking sticks, Darrylyn constructs an approximate series. However, like all children in the pre-conceptual stage, she does not compare sticks to all those remaining or all those already in the series.

Because the sticks are considered as either long or short, children in this stage typically place new sticks only at the ends of series. By disregarding the actual length, the sticks are arranged to give the visual impression of a series.

Most six and seven-year olds are in the intuitive stage and know that each successive stick must be longer. This and a concern for evenness results in a uniform series. However, the series is built by trial and error and
not by an understanding that each added stick is both longer than all those before it and shorter than all those that will follow.

Brock is beginning to struggle with the relation of middle. Middle lies somewhere between two extremes. He does not understand that a middle stick is both longer and shorter than its neighbors.

The double seriation problem reveals more about the intuitive child's thinking. Medium sized sticks are matched by trial and error with medium sized dolls. Children in this stage do not grasp the necessity of constructing two series and matching one to the other.

Spatial and ordinal position are not distinct in the intuitive child's thinking. Bebe confuses the two and picks the bat that is closest to the doll.

Around eight years of age, most children enter the concrete-operational stage and can systematically coordinate relations of less than and greater than. Ryan understands that by taking the longest stick in his hand, it becomes the next shortest stick in the staircase. By this method he constructs an ordered series.

This method of seriating implies a systematic conception of order relations. This characteristic of concrete-operational thought is further revealed in Ryan's understanding that if a stick is shorter than one and taller than another, it belongs between the two.

Children in the concrete-operational stage approach the double seriation problem systematically. They think of the dolls and bats as two series and order them so that the first member of one series is matched with the first member of the other, and so on.

Children in this stage can also perceive the ordinal correspondence between the two series, even when they are reversed. The fifth longest stick goes with the fifth tallest doll. At this stage, ordinal and spatial positions are not
confused with each other.

Even when both series are destroyed, Tanya can maintain the ordinal correspondences between dolls and bats. To determine which bat goes with a randomly selected doll, she reconstructs the double series.

We have examined three stages in children between four and eight years of age. These stages are universal and passed in the same order by children from diverse cultures and backgrounds. Other stages occur through adolescence.

Before reasoning like an adult, the child must pass through earlier ways of thinking. These earlier ways seem incorrect from the adult view, but are perfectly reasonable to the child and must be experienced if more adequate forms are to emerge.

In this film, we have examined one area of development. Other films in this series cover additional topics on cognitive growth.
APPENDIX C:
REFERENCES AND ADDITIONAL RESOURCES
Some of Piaget's Translated Works

These earliest of Piaget's books concern the general character of children's thinking between three and eight years of age, as revealed in either natural observations or discussions with children. At the time of their writing, Piaget did not regard them as important works, and he later criticized them for their dependence on the child's verbal reasoning. However, these books set the stage for much of his later work and provided the foundation for public interest. Of his major descriptive works, these are probably the most readable. The works are listed in order of their original French publication dates.


These three books express Piaget's major observations on the mental development of infants. *The Origins of Intelligence* provides a theoretical model of sensory-motor intelligence. *The Construction of Reality* describes the first understandings of space, time, objects, and causality. The original French title, *Origins of the Symbol*, suggests the underlying focus of *Play, Dreams, and Imitation in Childhood*. Each of these three books will probably disappoint the casually interested reader. *The Construction of Reality* is the easiest of the three to read.


- These works provide information on the development of order, relations in children and/or seriation activities.
The following books make up the largest single focus in the study of cognitive development. In them, Piaget explores the development of logical and sub-logical thought between four and 12 years of age and its expression in a broad cross-section of knowledge. Each is composed of a rich array of concrete-manipulative experiments and the corresponding responses of children. The Growth of Logical Thinking is Piaget's major work on formal-operational thought. The beginning Piaget student will find the descriptions rich and readable, though tedious. The theoretical accounts are highly abstract and complex.


The following books provide an overview of Piaget's theory and his general views on the nature of knowledge. The Psychology of the Child provides his best introductory overview of development between infancy and late adolescence. As suggested by their titles, two of the books present Piaget's thoughts on education. They do not provide simple educational prescriptions.

Piaget as Seen by Others

These five books provide an overview of Piaget's theory and his main findings. Pulaski's and Phillips' works are probably the most readable by lay persons. The book by Ginsburg and Opper is an excellent overview of the main stages of development from infancy to late adolescence. The books by Boyle and Flavell focus more on the formal aspects of Piaget's theory and are probably more useful to the advanced student. Flavell's book is a classic American interpretation of Piaget's general theory.


The book by Isaacs is a good introductory presentation of quantitative concepts (number, measurement, time, etc.) between four and eight years of age. Brearley and Hitchfield provide a similar treatment of additional topics such as space, morality, and science.

Dasen's article explores the relationship between culture and knowledge. Furth provides a rich and insightful presentation of Piaget's general theory. He includes seven short papers by Piaget. Langer describes three predominant views on mental development: behaviorist, structuralist, and analytic. Ripple and Rockcastle edited the presentation of a large American conference on Piaget. They include four papers by Piaget, a number of theoretical papers on education, and a large number of papers concerning curriculum projects based on Piaget's theory. The papers by Piaget are informative and quite readable.


These works reflect some of the research studies directed toward refining and clarifying Piaget's theory and its implications.


Piaget's Theory and Education

The following books and papers present a range of views on the general implications of Piaget's theory for education. The book by Schweibel and Rath presents a number of readable and excellent articles by various Piagetian scholars.


These books reflect a number of efforts to prepare teachers to interview children in areas of cognitive development. Lavatelli's work is well known but criticized by Piaget for her suggestion that training children to perform on Piagetian tasks is an appropriate educational goal.


The following present some materials, activities, and suggestions for working with children.


Cuisenaire Company of America, Inc. 12 Church Street, New Rochelle, NY 10805.


Weikart and the Responsive Program Staff present two different broad applications of Piaget to early childhood education. Both are models for the National Follow Through Program. The Responsive Model Program has been implemented in hundreds of classrooms throughout the U.S. The paper by Rayder, et al., presents some of the findings on the effects of the program.


Responsive Educational Program Staff, A Description of the Responsive Education Program. San Francisco: Far West Laboratory, 1976.


Films and Videotapes

CRM Educational Films, Cognitive Development. (20 minutes)
Available from CRM Educational Films, 7838 San Fernando Road, Sun Valley, CA 91352.

Davidson Films, Piaget's Developmental Theory:
Classification. (19 minutes)
Conservation. (29 minutes)
Formal Thought. (33 minutes)
Growth of Intelligence in the Pre-school Years. (31 minutes)
Jean Piaget: Memory and Intelligence. (44 minutes)
Available through the University of California Extension Media Center, Berkeley, CA 94720.

Far West Laboratory, The Growing Mind: A Piagetian View of Young Children:
The Development of Classification. (30 minutes)
The Development of Order Relations -- Seriation. (27 minutes)
The Development of Quantitative Relations -- Conservation. (32 minutes)
The Development of Spatial Relations. (29 minutes)
Available through the Far West Laboratory, 1855 Folsom Street, San Francisco, CA 94103.

Phoenix Films, Learning About Thinking and Vice Versa. (32 minutes)
Available through Phoenix Films, 743 Alexander Road, Princeton, NJ 08504.

The Jean Piaget Society, Equilibration. (35 minutes) Available through The Jean Piaget Society, Box 493, Temple University, Philadelphia, PA 19122.
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