DOCUMENT RESUME

ED 128 707 CG 010 815

AUTHOR	Noddings, Nel
TITLE	Cognitive Premises and Curriculum Construction.
PUB DATE	21 Apr 76
NOTE	53p.; Paper presented at the annual meeting of the American Educational Research Association (San Francisco, California, April 19-23, 1976)
EDRS PRICE	MF-$0.83 HC-$3.50 Plus Postage.
DESCRIPTORS	*Cognitive Processes; *Curriculum Design; *Curriculum Development; *Learning Theories; *Psychoeducational Processes; Speeches; State of the Art Reviews

ABSTRACT

Two sets of cognitive premises are identified. An attempt is made to state the premises of each set precisely, to draw conclusions, and to explore their implications for curriculum construction. The perspective is that of cognitive psychology, and the idea is to connect certain definite curriculum arrangements to stated cognitive premises. The study employs the methods of conceptual research and uses as its data source the writings of prominent cognitivists. It culminates in specific recommendations for the organization of certain topics in standard curricula and suggestions for the placement and use of "discovery exercises" in the mathematics curriculum. (Author)

ED128707

COGNITIVE PREMISES AND CURRICULUM CONSTRUCTION

Nel Noddings

The University of Chicago

(Paper for presentation at 1976 meeting of AERA, San Francisco, Division B, Session 10.05, April 21.)

Cognitive Premises and Curriculum Construction

NEL NODDINGS

The University of Chicago

(Abstract)

Two sets of cognitive premises are identified. An attempt is made to state the premises of each set precisely, to draw conclusions, and to explore their implications for curriculum construction. The perspective is that of cognitive psychology, and the idea is to connect certain definite curriculum arrangements to stated cognitive premises. The study employs the methods of conceptual research and uses as its data source the writings of prominent cognitivists. It culminates in specific recommendations for the organization of certain topics in standard curricula and suggestions for the placement and use of "discovery exercises" in the mathematics curriculum.

COGNITIVE PREMISES AND CURRICULUM CONSTRUCTION

Nel Noddings
The University of Chicago

My purpose in this paper is to explore some basic premises of cognitive psychology, particularly those which are relevant to complex human learning; to examine in the light of these basic premises certain other premises about curriculum which have been advanced under the cognitivist banner--to see, in particular, whether the curriculum premises can be in any way "legitimized" by the first set of premises; and, finally, to make some suggestions about curriculum construction which seem to be compatible with the original set of premises.

1.

It should be made clear at the outset that I am not attempting, here, to lay the groundwork for a postulational treatment of cognitive psychology. To undertake such a task--if we deemed it profitable at all to do so--we would have to be much further along in our empirical and intuitive development of the subject than we are today. Further, we are aware of the limits of axiomatization. Our system could only yield significant meaning as it is embedded in a meta-psychology which makes reference to, at least, the psychology of motivation and personality. The premises we seek, then, will represent the working assumptions of a paradigm and not the axioms of a rigorous postulational system.

The first thing we must do is to clarify what we mean by the term "cognitive psychology," which is used in at least two ways in the literature of psychology. First, it may refer in a global way to the study of "cognitive processes," those processes we usually associate with thinking, problem solving and complex learning in general; obviously, many schools of psychology may be concerned with the "cognitive" in this sense. Since the focus of attention here will be curriculum construction, it is clear that this sense of "cognitive psychology" is relevant to the present discussion. Second, "cognitive psychology" may be used to refer to a rather specific school of thought in psychology, and this is, of course, the important sense in which we shall use the term.

According to Neisser (1967):

> The central assertion is that seeing, hearing, and remembering are all acts of construction, which may make more or less use of stimulus information depending on circumstances. The constructive processes are assumed to have two stages, of which the first is fast, crude, wholistic, and parallel while the second is deliberate, detailed, and sequential. (p. 10)

The first, "constructive," assumption is fundamental. In making it, we are consciously and deliberately rejecting hypotheses which suggest that visual perception involves making "copies" of external reality or that remembering can be described in terms of the "reappearance" of faithfully, and passively, recorded events. We hold, with Bartlett, that "the description of memories as 'fixed and lifeless' is merely an unpleasant fiction ..." (1932, p. 311). Acceptance of this assumption will force us, of course, to discuss the nature of "constructions" and the existence and nature of structures which perform the constructions.

It should be noted immediately, however, that acceptance of the "constructive" assumption casts into doubt the usefulness of certain popular psychological terminology. The term "stimulus," for example, which has always depended on its effect for its psychological reality, becomes even more vague and ambiguous. For what acts as a stimulus on a particular human organism must depend partially on the constructive acts of the organism. Realization that stimulus and response are "contemporaneous"--each part of a "forming coordination" (Dewey, 1896)--has led cognitivists to concentrate on the <u>response</u>. As Piaget has quoted a colleague: "In the beginning was the Response!" (1971a, p. 8) Even here, it is important to realize that this shift, which attempts to place emphasis on the activity of the organism rather than upon its reactivity, is not drastic enough to provide a proper base for cognitive psychology, for it says nothing about the structure which generates the response. If we try to amend the $S \longrightarrow R$ configuration to any of a variety of suggested forms, say $S \leftrightarrows R$ or $S \longrightarrow (A) \longrightarrow R$, where A stands for assimilation to a schema (Piaget, 1971a, p. 8), we are still left with an imprecise and, at best, loosely descriptive terminology. What is needed, of course, is a much more complex description of the mechanisms which mediate among and transform potential stimuli and which generate behaviors we take to be "responses."

One suspects that a cognitive theory of complex learning might be developed more quickly if we were to drop the stimulus-response talk entirely. This is not to say that such terminology is <u>everywhere</u>

inappropriate. In some fields, e.g., physiological psychology, it seems sensible to amend the framework in which S - R talk takes place, substituting feed-back mechanisms for the reflex arc (Pribram, Miller, and Galanter, 1960). But language that is appropriate in one domain of discourse may be hopelessly inadequate for communication in another. Talk of "responses" will, nevertheless, be with us, out of habit, for some time. If we realize that the "response" of educational psychology is just a bit of common language meaning, roughly, answer or act following another in such a way that the two seem transactively related, we shall, perhaps, shift our attention from the external conditions in which the "response" occurs to the internal mechanisms which might generate the response.

It seems natural, while we are discussing difficulties with S - R terminology, to examine the second premise in Neisser's statement: that cognitive processes have two stages, the first of which is "parallel." The question of whether or not cognitive processes may be organized in parallel has been around for a long time. William James (1890) criticized the atomistic position which denied parallel processing; its thesis that it "is impossible for the mind to attend to more than one of these points (of a figure) at once" was denounced by James as "glaringly artificial" (1890, p. 406). But he could not, of course, settle the question.

It was simpler, certainly, in an S - R framework to postulate serial organization of mental processes, chains of S - R events. Even today, there is a temptation to prefer serial organization, because much of our study of human mental processes is being conducted, by metaphor as it were, through corresponding studies of artificial intelligence, and computers are, for the most part, organized serially and sequentially.

But that statement, "computers are organized ..." hints at the difficulties and probable inadequacies of theories which postulate completely serial organization. Obviously, the programs and circuitry of computers are organized by the integrative efforts of human beings, and the question quite naturally arises: Can serial organization explain the necessary integrations which must prepare and guide speech, problem solving, and other complex cognitive processes?

Is the question of parallel processing vs. serial or sequential processing important for the study of complex learning? It could be. An adequate theory of complex learning will have to account for its relative rarity. No existing theory of learning does this; indeed, one would suppose in reading the literature of school learning, that one needs only to find the right conditions in order to teach anyone anything. Intuitively, this is nonsense. Deep and creative understanding of any subject matter is rare. Even when a learner has mastered the routines of a subject matter, he may be unable to gain access to routines which are called for by the objective elements of a problematic situation. It may be, indeed, that only serial and sequential routines are "learnable" and that the integrative processes responsible for "transfer" are products of abstraction from learning, or development. The question for us in education, then, becomes: Shall we teach the obviously learnable or is there some way in which we can teach for development construed as cumulative learning?

There are other reasons for including a parallel processing assumption among our basic cognitive premises. First, the brain itself seems to be organized in a highly parallel fashion (Arbib, 1972). Second, the assumption of totally serial and sequential organization seems to lead to the necessity of postulating enormously complicated "switching" mechanisms to enable us to move from one topic of attention to another (Norman, 1969). And, third, the assumption of parallel processing seems to yield the best description and explanatory framework for the discussion of topics which remain fascinating for the cognitivist: intuition, abduction of hypotheses, heuristic choice, incubation and illumination. Finally, the notion of parallel processing will provide us with a powerful metaphor when we turn to the discussion of organizing subject matter for instruction.

Let's return, now, to our search for basic premises. Our first premise which assumes that cognitive processes are "constructive" forces us to postulate the existence of structures which perform the constructions. Further, we shall have to say something about their nature and the raw material used in construction.

Our next basic assumption is, then, that <u>there exist cognitive structures</u> which guide human behavior by providing the frameworks within which we perceive, interpret, and organize reality and by performing the actual short-term constructions which comprise perception and cognition. According to Neisser:

> ..., a cognitive structure may be defined as a non-specific but organized representation of prior experiences. (1967, p. 287)

It is clear that, for Neisser at least, "prior experiences" must encompass the possible inclusion of genetically transmitted information. Indeed, he asserts that the proferred definition "is meant to leave the question of empiricism and nativism open" (1967, p. 287). For Piaget, the difference is, more precisely, between <u>constructivism</u> and nativism

(1970, pp. 88-92), but he, too, while arg strongly for constructivism, recognizes the empirical openness of the question. Must a cognitivist interested in complex learning concern himself with this question? The answer, I think, is yes. If he is a thorough-going nativist, for example, believing that all of the integrative and generative structures are built-in, it might be very sensible for him to seek the most sophisticated existing structures of subject matters as "best-fit" reflections of those built-in, skeletal structures nicely fleshed out. If, to the contrary, he is a constructivist, he must consider the possible ways in which reflective abstraction contributes to the building of higher and more powerful integrative structures. Since the constructivist view gives us some reason to suppose that structures can be built and not simply "activated" or "fleshed out," it is the richer view upon which to build a theory of complex learning and, ultimately, a theory of pedagogy.

We postulate, then, cognitive structures whose existence (in an evolutionary sense) and whose growth (in an individual sense) are a matter of construction. How this construction is accomplished we must discuss shortly.

What shall we say concerning the <u>form</u> of these structures? Perhaps the most reasonable approach is simply not to restrict our conception of form artifically. We want to include as "cognitive structures" both the exquisitely abstract integrative and generative structures whose existence we postulate for theoretical reasons--and whose operations cannot be discerned by conscious thought--and those structures, e.g., the steps involved in solving quadratic equations, of which we are fully conscious and whose origins are certainly in learning. The temptation today, again possibly because of the high interest in computers, is to represent all structures as "trees," but this representation is surely too simplistic. Finally, in this connection, it should be emphasized that we are not here talking about physical patterns in the brain or central nervous system. We might seek empirical descriptions or metaphorical descriptions for these patterns, e.g., the hologram, but that job is for the physiological psychologist. The structures we seek are of two sorts: One kind we may infer from the structure of observed behavior; we might, for example, infer that a student's correct response to the question, What is the quadratic formula? has been generated by a weak associational structure, if we are unable to discern in his discussion of the matter any sense of development, or any special heuristic for "compiling" the formula. The second kind must be postulated and described in an entirely abstract fashion; the legitimacy of such theoretical structures must rest on 1) their logical consistency and 2) their psychological reality as demonstrated through derived empirical studies (Piaget, 1953, p. 26). Examples of such structures are transformational grammars as models of the mental organization capable of accounting for language production and comprehension and Piaget's "elementary groupements" as a model of the mental structure of children at the stage of concrete operations. These theoretical structures which attempt to link mind and external reality are known, generally, as competence theories, and I have tried elsewhere to describe their possible usefulness in educational science (Noddings, 1974).

It seems necessary, now, to say something about how cognitive structures function--how we obtain access to certain sets of structures and what general purposes the highest integrative processes serve. When we ask the questions:

How are our integrative structures "set?" How are they oriented? we are asking questions which form natural bridges between the psychology of cognitive processes and the psychology of motivation. For the theorist interested in complex learning, the building of such bridges is a necessity.

In this matter, we might again look for help from brain theorists:

> ..., in relation to perception in particular, I have been suggesting that we look for the physical correlate of perceiving not in the receptive area of the brain, but rather in the organizing system which prepares and maintains the 'conditional readiness' of the organism for the state of affairs giving rise to the received signals. On this basis to perceive something (say an object in one's path) is to respond by internal adaptation to it (setting up conditional readiness to avoid it, grasp it, describe it, etc., in case of need); and the unity of 'wholeness' of an object as perceived reflects the integration of the 'state of conditional readiness' to match the stimuli from it. (MacKay, 1969, p. 483)

Now, if cognition, like perception, involves the organizing systems--if it moves the organism from 'conditional readiness' to a decision to act, or if its central purpose is to strive for such movement, then we can see that the states selected by perception are those we have access to for the cognitive processes.

For the theorist interested in complex learning, the cognitive correlates of these physical processes are of enormous interest. We should like to be able to account for the avoidance behavior elicited in many students by the introduction of mathematical problems. In general, we should like to be able to account for the widely observed phenomenon described succinctly by John Holt: "To a very great degree, school is a

place where children learn to be stupid." (1964, p. 157).
And the clue is in what the student perceives the problem or goal to be; if he perceives the goal to be finding an answer to satisfy the teacher, then it may very well be the case that he cannot obtain access to sub-routines which would enable him to confront the objective elements of the problematic situation. Hence, when a student swings wildly from one absurd response to another equally absurd, he may still be behaving with internal consistency. The structures which he has activated are simply not the ones relevant to the situation as we perceive it.

The 'action orientation' of the organism is discussed also by Arbib. The assumption is that perception involves a transformation of internal mechanisms of selection and choice in keeping with the organism's need to be 'ready to act' (Arbib, 1972, p. 182-83). This assumption is interesting from a historical perspective, also. Not only is it poles removed from notions of 'photographic images' and the 'reappearance hypothesis' but it also suggests a significant refinement of the Gestaltist notions of developing a 'best' or 'structurally simplest' form. It suggests, indeed, that structure might be accounted for not so much by innate factors as by the continuous adaptations of internal mechanisms induced by functional needs. The ability to discern structure in an objectively useful way seems to depend on our prior resolution of a tacit question: What shall I do? If we "see" that a situation is not fraught with immediate danger, we can, perhaps, activate mechanisms which permit us to ask: What <u>is</u> it? Hence, if we have resolved the first level of action-decision problems in such a way that we need not prepare for flight or other defense, we may continue to alternate between the orientations illustrated by the questions, What shall I do? and What is it? in order to produce an objectively useful model of the problematic situation. It does not seem, in other words, that we are naturally organized to answer the intellectual question: What is it? While consciously asking the question, What is it? may be an intellectually superior move (Wertheimer,1959), we need to ask under what external and internal conditions it becomes possible for the organism to make this move. How do we control the ways in which we allow the world to come in upon us? How do we switch from a controlled acceptance of stimuli to an orderly imposition of hypothetical structure upon the situation? We are, here, asking questions which can profitably be asked and answered in several domains and, when we try to answer them in the cognitive-pedagogic domain we must be careful to use physiological terms metaphorically if we use them at all. We may use them as springboards to speculation but not as precise descriptions. We are not going to see in the classroom adaptation or assimilation, redundancy enhancement or redundancy reduction, but we are going to see behaviors, (some facilitative, some not) which indicate openness to experience, a willingness to defer the What shall I do? decision, and we are going to see behaviors indicative of intense concentration--some which are facilitative and some which are convergent to the point of perversity.

Before leaving the discussion of how our higher processes might be oriented, we should mention another important and well-documented function: regularization. It seems that our integrative structures do, indeed tend to organize our experiences in patterns which are compatible with previously formed schemata. Piaget (1971a, p. 250) has described the regularizing function of perceptual mechanisms, and Bartlett (1932) has described what seems to be a similar function in the workings of memory. A question for the learning theorist is this: How extensive is this regularizing function? Does it extend to the structures which are directly products of learning? Does it extend to all of the structures which are, as organizations of reflective abstraction, products of cumulative learning or development? If the regularizing function is pervasive, it is clear that we have built-in resistance to the sort of learning which is represented in discovery and invention. (We must not be carried away by all this, but one cannot help but speculate that this regularizing function may provide a partial explanation for a variety of phenomena interesting to educators, e.g., the relative youth of creative mathematicians, the temporary nature of liberalizing effects on college students, even the distinctive styles of writers and painters.)

Now, finally, we must consider one more area in which the cognitivist needs to make some working assumptions. We have talked about the constructive nature of cognitive acts, the possiblity of some parallel processing in perception and cognition, the <u>form</u> of cognitive structures, the action-orientation of the higher process structures, and their regularizing function. We must now say something about the formation of cognitive structures and the building blocks of construction.

We have already rejected the nativist explanation of the existence of higher-process structures and have embraced, instead, Piaget's (1971a) constructivist view. It is interesting to note that brain theorists, too, speak of "schemata," sometimes as "output feature clusters" (Arbib, p. 211) and differentiate between <u>learning</u> as the accumulation of more cues to gain access to existing routines and <u>development</u> as the long-term effects of learning which yield new schemata. A precise description of the construction of new schemata is not available. Possibly Piaget's description of the role of reflective abstraction in the formation of new structures is the best we have. By this account, it is clear that actions induced by a functional need--actions performed successfully in pursuit of a perceived goal--lead to internalized systems of potential action, that is, schemata which can be activated when the appropriate goal is perceived.

Here, again, we see a connection between the discourse of motivation and that of cognitive processing. If Piaget is right, then development is dependent on an accurate perception of what one is trying to do and repeated successes in doing it. What is abstracted from this repeated trying is an organized scheme for achieving a particular sort of goal. It would seem that, if the goals are broad, the resulting schemata might have wide application and multiple accesses; hence, their activation would be naturally frequent. Inversely, if the goals are narrow, the resulting schemes might have narrow application and their persistence in the behavioral repertoire might depend upon their frequent activation in the narrow context.

One is reminded here of Simon's "blackboard" metaphor (1966 , p.33). His contention is that, in the course of problem solving, the information which we gather both from the environment and long-term memory in connection with particular goals and sub-goals is fixated in permanent memory. The goals themselves may, ultimately, be forgotten--or they may be "erased" as they are achieved, but the writing on the "blackboard" remains as a permanent addition to cognitive structure.

It is this "writing on the blackboard," the continual updating of long term memory, the encoding of objects in a goal-action-oriented frame, which we see as the building blocks of cognitive structure. In Neisser's words:

> Stored information consists of traces of earlier constructive acts, organized in ways that correspond to the structure of those acts (1967, p. 279).

It is suggested, further, that the motivation of the cognizing subject, his goal perception, provides the guide to the "structure of those acts" and thus to the organization of cognitive structure.

II.

Our next task is to try to identify some premises about curriculum which are distinctively cognitivist. This, it turns out, is difficult to do, for there seems to be no one in curriculum work who has attempted to base his work on the sort of premises we have already discussed. To identify curriculum workers in the cognitivist mode, we can depend partially upon a general affiliation with the "activist" tradition (as opposed to the "reactive") and partially upon the nature of their concerns, e.g., "intuition," "discovery."

We might properly ask why we should concern ourselves with this particular group of curriculum workers. After the enormously productive effort of the sixties, influence in curriculum seemed to move from workers who were concerned with actual curriculum construction to the "reconceptualists"

(Pinar, 1975) and the theorists interested in identifying and clarifying variables for research. But there was something both intellectually stimulating and pedagogically intriguing in the sixties movement. Which of its recommmendations have the support of assumptions at a more general theoretical level? Which are worth another try?

It should be clear that some of the premises we shall explore may be first-order premises--premises which must depend entirely on confirmation through hypotheses derived from them. This discourages us, because the empirical evidence we have at present is not favorable to a continuation of sixties-like projects. But <u>some</u> of the premises may be second-order, deriving respectability from higher order premises to which they may be anchored. These are worth further study, further imaginative speculation.

In this section, I want to examine premises in three major areas: structure, intuition, and discovery. Obviously, there can be no attempt to discuss these concepts fully; that is not my purpose. The notion of structure, for example, has been discussed at length elsewhere. What I want to do here is to separate out some stated notions in the three categories which seem incompatible with our basic premises--hence, to point out possible mistakes we've made in fundamental assumptions. Then, in the last section, I shall try to make some suggestions which I feel <u>are</u> compatible with the basic premises.

There is one difficulty--a conflict of sorts within the activist camp--which should be mentioned immediately. The cognitivist curriculum writers to whom we shall refer rarely give a defintion of curriculum. They seem to assume, however, that "curriculum" refers primarily to pre-planned materials. The definition implicit in most of the curriculum work under examination seems to be well expressed in a statement by Keislar and Shulman:

> ..., a curriculum refers to the organization and sequence of a subject matter in which statements about that subject, methods of teaching, and the activities of the learners are intricately interrelated to form a single entity (1966, p. 190).

That this organization is to be pre-planned is affirmed by Zacharias and White:

> ..., "curriculum revision" is intended to signify the entire process of the preparation of educational materials for use in the formal school system, or in direct association with the formal school system (1964, p. 68).

The conflict, it seems to me, lies between the Deweyan philosophical roots of educational cognitivism (the "active" orientation) and the notion of curriculum as primarily preactive (Jackson, 1966). There is an insistence on understanding (Beberman, 1964) and a conviction that intellectual activity is essentially the same "whether at the frontier of knowledge or in a third-grade classroom" (Bruner, 1960, p. 14), and these emphases are reminiscent of Dewey's view of the child as scientist, but the notion of a comprehensive curriculum embodying subject, methods, and learner activities seems contrary to Dewey's view of curriculum as interactive. The question is, Is it also contrary to the premises we have already laid down?

I suspect it is. If we accept the premise that all experience is mediated by what we have called " cognitive structures," it seems clear that the activities planned for learners will be substantially different from the experiences learners actually undergo. Yet the curriculum developers of the sixties put a surprising emphasis on the pre-packaged activity. It is as though there were a conscious attempt to combine the Deweyan notions of activity and reflection with the traditional emphasis on disciplined subject matter. In the last section, I shall attempt to describe "curriculum" from a cognitivist view point, but it is enough for now to note that there is a serious inconsistency involved in attempting to pre-package curricula based on our cognitive premises.

The first curriculum-cognitivist premise that we challenge, then, is one which assumes that curriculum can be totally pre-planned. Now, of course, "pre-planning" is not an assumption that is distinctively cognitivist. Behaviorist educators are even more insistent on the construction of pre-stated objectives (Mager, 1962). But the behaviorist generally makes a sharp distinction between ends and means, between objectives and methods, and for him objectives and curriculum are synonymous (Gagné, 1967). The blending of activities, methods, and subject matter does seem to be distinctively cognitivist, and what we are challenging is that this blend can be effectively prescribed beforehand. Indeed, since cognitivists generally applaud the idea that students must be instructed in ways compatible with their cognitive states, it must be that they embrace premises concerning learning abilities which permit them to justify this comprehensive pre-packaging. Ausubel, for example, says:

> If I had to reduce all of educational psychology to just one principle, I would say this: The most important single factor influencing learning is what the learner already knows. Ascertain this and teach him accordingly (1968, p. vi).

How is this dictum, which _is_ compatible with our first set of basic premises, reconciled with a recommendation for comprehensive pre-planned curricula?

The cognitivist educator uses several basic premises to justify his attempts at pre-packaging curricula. First among them is a premise, or set of premises concerning the structure of subject matter and learning facility. Speaking of the curriculum projects of the sixties, Bruner says:

> The main objective of this work has been to present subject matter effectively--that is, with due regard not only for coverage but also for structure (1960, p. 2).

The emphasis on structure is pervasive:

> We hope to make each student in the early grades truly familiar with the structure of the real number system... Moreover, we want to make students familiar with part of the global structure of mathematics (Goals for School Mathematics, 1963, p. 8).

Now, we do not want to review all of the general objections raised against the structure of the disciplines approach; we want to be careful to separate the elements of the structure of the disciplines movement which seem to grow out of Platonism and Hebartianism from those which have a Deweyan heritage (Bruner, 1960; Schwab, 1962, 1964), for the criticisms which we shall make are leveled at _premises_ which seem to be operating in this school. Many writers have questioned the premise that there _is a_ defining structure for each discipline (e.g., Mason, 1972; Hullett, 1974), but we are concerned here with whether or not there is reason to expect that subject matter ordered to reveal such a structure (in the Brunerian sense) will be somehow more comprehensible. We are concerned with claims of the following sort:

> The first is that understanding fundamentals makes a subject more comprehensible (Bruner, 1960, p. 23).

This claim appears in a variety of forms in all of the work we are considering. Whether we define "fundamentals" as "underlying principles," "basic methods of investigation," "unifying ideas," or "basic concepts," we face the same question. What reason have we to expect that presentation of these fundamentals will make a subject more comprehensible to the novice?

Bruner admits:

> ... argument for such an approach is premised on the assumption that there is a continuity between what a scholar does on the forefront of his discipline and what a child does in approaching it for the first time (1960, p. 28).

But he also observes:

> The experience of the past several years has taught at least one important lesson about the design of a curriculum that is true to the underlying structure of its subject matter. It is that the best minds in any particular discipline must be put to work on this task (1960, p. 19).

And Zacharias says of the scholar engaged in curriculum reform:

> Before he can hope to make a matter clear to the student, he must make it clear to himself (1964, p. 77).

There is something revealing in these comments. If the fundamental ideas of a subject are so sophisticated that their explication and exposition require the best efforts of the "best minds," how can the same material be suitable for beginning students? The assumption is at odds with several of our basic cognitive assumptions. It is, fundamentally, at odds with our premise that cognitive structures are <u>constructed</u> through a self-regulatory process of reflective abstraction. As I have already pointed out, a nativist view of cognitive structure is compatible with a structure of the disciplines approach for, if we believe that outline structures are innate, it is natural to suppose that the most sophisticated existing structures approximate the innate structures "filled out." It follows, then, that presentation of subject matter should conform to this natural structuration.

But if we believe that cognitive structure develops through distinct stages or levels--each with its characteristic generating rules, guiding principles, and heuristic procedures--then we should have serious doubts about using the most sophisticated structure of a discipline as a pedagogic structure. Its very assessment as "fundamental" warns us that the novice will not possess the structure to grasp it, for the understanding of <u>fundamentals qua fundamentals</u> is the hallmark of mastery. It requires a continuous nesting of one level of structure in the next. One uses the skills acquired at one level to perform, fumblingly, the operations at the next and, as proficiency develops at this level, new light is shed upon the level below. Thus, all through the educative process, we are in the strange position of saying, <u>Now</u>, I understand (the work of last year, the previous unit, the previous lesson)...

But can we not use the unifying concepts of discipline to construct activities which both require the use of basic principles and reveal their nature? It certainly seems to make sense to induce the use of commutativity, for example, by presenting youngsters with examples of the following sort: $2 + 11 = ?$ $3 + 15 = ?$ $2 + 23 = ?$ The hope is, of course, that the child will find it easier to convert $2 + 11$ to $11 + 2$, saying 11, 12, 13. But this seems to happen and, when it does not, we cannot use a statement of the commutative property to teach the child to make the reversal. We have to persist with examples and practice until the child "sees"; this, of course, we expect, if we take reflective abstraction seriously.

Next, in the same context, we might ask how well the child understands commutativity when he begins to make reversals in the addition of integers easily. The answer seems to be that he anticipates such results as $\frac{1}{2} + \frac{1}{4} = \frac{1}{4} + \frac{1}{2}$, but scarcely one student in a hundred can answer the following question at the eleventh year of high school: If $a * b = (a + b) + 2$, where a and b are integers and "+" is the usual operation of addition, prove that the operation $*$ is commutative. Further, very few students can even express symbolically what it is they are expected to prove ($a * b = b * a$, for all a,b). They ask instead, what is $*$? Can I use numerical examples? Suppose they are told that $*$ is commutative under this definition and are then asked the following: If $a *' b = (a + b) - 2$ where a,b are integers and "+" and "-" are the usual operations of arithmetic, is $*'$ commutative? Almost invariably, the student will say, No. And his reason will be that "addition is commutative, but subtraction is not." My point here is that students (all of us) learn to generalize over a limited context, and it takes much intensive and extensive experience to understand fundamental principles as fundamental principles.

The use of a basic structure of a discipline as a pedagogic structure is sometimes justified on the basis of a structural view of memory:

> The second point relates to human memory. Perhaps the most basic thing that can be said about human memory, after a century of intensive research, is that unless detail is placed into a structured pattern, it is rapidly forgotten (Bruner, 1960, p.24).

This we agree with readily, but detail cannot be placed into a structure which is not present, and we must point out again that providing the structure externally is useless if the internal structures do not closely resemble the desired external structure. Examples, of course, abound in child language. We can say clearly to a young child, "Daddy went to the office," and the child may "repeat," "Daddy goed office." He cannot use a structure, certainly not consistently, until he possesses it. To say that detail must be placed into a structure is correct, but it must be thus placed by the student; it is the student who must perform the cognitive acts by which information is stored, and to perform those acts he must use structures already present.

Let's see where we stand. We've claimed that, unless one is a nativist, one cannot justify the use of a sophisticated structure of a discipline as a pedagogic structure. But we have not yet entirely discarded the very intriguing idea that such a sophisticated structure may yield, upon analysis, a set of unifying concepts and working rules which may be used to guide curriculum construction. The question, of course, is, How?

But our purpose, here, is to state our objections, as carefully as possible, to how it was done in the sixties curricula. Our major objection seems to be this: They tried to teach the unteachable. The major ideas of a discipline, the unifying concepts in particular, have to be abstracted through level after level of successful activity.

Max Beberman (1964, p. 14), in advocating the use of precise language in mathematical pedagogy, gives this example:

> Take the prime number 13 in the decimal system. In the octic system this is the number 15. Is it also a prime number in the octic system?
>
> Any hesitation at all in giving an answer to this question is a sign that the answerer's understanding is not all it should be. Of course, if the question is posed in language which shows a recognition of the distinction between number and numeral, the "problem" vanishes:
>
> Take the prime number whose decimal name is '13' and whose octic name is '15'. Is it a prime number?

Now, all of my own experience in mathematics and that of many mathematics teachers contradict Beberman in this. It seems as though the problem should vanish, but it does not. Students still say, No. Reason? Because $15 = 3 \cdot 5$. The distinction between number and numeral is just too subtle for most students. Similarly, although Beberman puts the blame on faulty teaching, most of the difficulty with 0 and "0" is natural; the concept of zero is sophisticated--steeped in convention. Most teachers do not understand it; and few understand the set theoretic definition of number. Again, one might try the following with a class of graduate students in education: Suggest that there are a priori truths in mathematics. Suggest as an example the immutable truth of $2 + 2 = 4$. Almost certainly someone will say, But that's not true in Base 3! We might, as Beberman and others bravely did, find this condition "deplorable" and blame it on the sloppy language of instruction. I am suggesting that it is not deplorable but predictable and that it arises not so much from sloppy language use as from lack of experience and a quite natural lack of comprehension.

The use of precise language, a requisite of structure identified by Beberman, is supposed to clarify instruction. Instead, our premises suggest that its effective use <u>depends</u> on the prior construction of assimilatory structures. This suggests a multitude of activities from which principles can be abstracted (but may <u>not</u> be) and a very careful watch by the teacher for the time when the student shows that he possesses a structure which can profit from nice distinctions. For those students whose relational thinking is non-verbal (Einstein was an example), these structures appear at a very late stage--if ever.

Structure is built by abstraction from purposeful doing. When we try to decide whether an activity should be included in the curriculum, we should ask what might be abstracted from it. We should not ask only whether it <u>illustrates</u> a basic concept, or <u>uses</u> a basic concept, or leads to the <u>discovery</u> of a generalization. Consider proving identities in introductory trigonometry. This "old" topic has been greatly minimized in modern texts on the grounds that there are so many important concepts to be learned that we should not spend time on manipulations which are highly redundant. But it is that very redundancy which encourages internalization. As we solve an identity, we perceive a sub-goal ("I need the double angle formula"), we ask ourselves how to achieve the goal ("What <u>is</u> the formula?"), and we play about with familiar objects (30°, 60° angles) in order to <u>concoct</u> the formula.

We do not always _derive_ the needed formula and we do not always prod our memories by looking it up. Frequently, we devise a heuristic by which to _compile_ it. And, when we do this oten enough, it is ours forever. The goal, solving the identity, is--admittedly--not important; it is erased upon completion. But the "writing" on the blackboard remains and is added to permanent structure.

Next, we want to examine some claims concerning intuition. The principal claims can be stated as follows:

(1) Intuitive thinking can be taught.

(2) Emphasis on the structure of knowledge in teaching increases the facility of students in intuitive thinking.

(3) Presentation of subject matter in an intuitive mode increases the likelihood of a student's understanding the underlying structure and, thus, increases his ability to think analytically.

Before we can sensibly examine any of these claims, we need some clear definition of what intuition _is_. Again, we find that curriculum-cognitivists do not _define_ intuition carefully. Bruner suggests that we

> ... begin with Webster: "immediate apprehension or cognition." "Immediate" in this context is contrasted with "mediated"--apprehension or cognition that depends on the intervention of formal methods of analysis and proof. Intuition implies the act of grasping the meaning, significance, or structure of a problem or situation without explicit reliance on the analytic apparatus of one's craft (1960, p.60).

But, here, we must be careful, for one of our basic cognitive assumptions asserts that _all_ of perception and cognition is mediated. On this view, evidence of "intuition" constitutes evidence that relevant structures exist.

Even here we must proceed with caution, for it is likely that "intuitions" are of concrete objects and events and not disembodied abstractions. Thus, when a child consistently uses transformations of the sort $2 + 11 = 11 + 2$, he does not have an intuitive grasp of commutativity; he has, rather, an implicit understanding that addition of integers is commutative. And that is a very different matter. He is able to use his powers of reason (implicitly) to predict what will happen when he does something to familiar objects.

For constructivists this is a matter of great importance, since we are trying to explain how the human organism comprehends and interacts with the world of objects and events. We reject intuition as a "third source" separable from reason and experiment and see its products as constructions of reason. Its products are analyzable; their mode of production is not. We are striving to pass from a traditional split in our view of man to a unified theory. Perhaps it will help to contrast a typical idealist view with the constructivist position:

> But between the physical processes which are released in the terminal organ of the nervous conductors in the central brain and the image which thereupon appears to the perceiving subject, there gapes a hiatus, an abyss which no realistic conception of the world can span. It is the transition from the world of being to the world of appearing image or consciousness. Here we touch the enigmatic two-fold nature of the ego, namely that I am both: on the one hand a real individual which performs real physical acts, the dark, striving and erring human being that is cast out into the world and its individual fate; on the other hand light which beholds itself, intuitive vision, in whose consciousness that is pregnant with images and that endows with meaning, the world opens up (Weyl, 1934, pp. 19-20).

It is that hiatus which the constructivist would bridge by connecting the world of action with the world of perception and cognition through the integrative processes of the organism. To do this, we must sacrifice some of the loveliness and poetry of Weyl's vision of the human organism as "light that beholds itself." We give up the notion of intuition as a <u>source</u>:

> It will be said that by separating intuition into experimental verification and deduction, we dissociate the interaction of the subject and object, acknowledged as indissociable. This is not the case: but we replace as the analysis of the phenomenon itself requires, the idea, completely arbitrary today, of an absolute beginning, by the dialectical idea of a constant becoming. The history of science as much as the study of individual development shows that this interaction, while remaining indissociable, passes from an undifferentiated phase to one of coordination. Starting from a state of centration on a self uncognizant of itself and in which the subjective and objective are inextricably intermingled, the progressive decentration of the subject leads to a two-fold movement, of externalization, tending to physical objectivity, and internalization tending to logico-mathematical coherence (Piaget, 1971b., pp. 114-15).

For the cognitive-constructivist, then, intuition must represent a functioning of the higher, integrative structures, a rapid scanning of accessible sub-routines which may be relevant. It is entirely likely that intuitive thinking is a parallel process. If it is, it is clear that we can decompose and reorder its <u>products</u> serially or sequentially, but we cannot describe the process of production itself in step wise fashion.

What can we say, then, of the claim that intuitive thinking can be taught? In fairness to Bruner and others who have emphasized the value of intuitive thinking, we should point out that the claim is rarely made so explicity. Bruner does speak of "the training of hunches" (1960, p. 13), however, and he certainly urges us to teach in a way which encourages intuitive thinking. In the last section, I shall make some suggestions as to how we might encourage intuitive thinking, but our purpose, here, is to point out that, in an important sense, learning to think intuitively is "learning the unteachable" (Hawkins, 1960). There is no subject matter which can be labeled "intuitive thinking," and teachers who attempt to induce such thinking should be careful not to deceive themselves. Imaginative answers, free-wheeling guesses, high level responses in class discussions are not necessarily manifestations of intuitive thinking as we are describing it. Imaginative responses may be generated by irrelevant structures; free-wheeling guesses may be produced by simple associative processes; and high level responses in a class discussion may be only a manifestation of the connective structure of the lesson and not of individual thinking.

This last point is important for the curriculum theorist who leans toward "inquiry training" and other classroom techniques which depend on raising the level of group responses. For, if our conception of intuition is accurate, it should be entirely possible to produce product-responses which are properly labeled "high level" and which are, at the same time, the product of no one student mind. Such sessions are demonstrably valuable for the products they generate, but they may be entirely irrelevant to the creation of cognitive structure. The external structure of the discussion cannot be equated with the cognitive structures of students.

The second claim, that emphasis on the structure of knowledge increases the facility of students in intuitive thinking, we have already denied on the grounds that structure presented is not necessarily structure available for assimilation. It seems more reasonable to suggest that manifestations of intuitive thinking reveal that assimilatory structures are available. Thus, intuitive responses on the part of students can be used as a valuable aid in diagnosis.

The last claim, that presentation of subject matter in an intuitive mode increases a student's perception of structure and, thus, his ability to think analytically is two-fold. It seems right, given our premises, that an "intuitive presentation" might be used to guide the student's thinking in such a way that an outline structure will be added to his cognitive structures. But if this addition of structure is to play a role in facilitating analytic thinking, the presentation must raise questions which require analytic thinking as a complement. The outcome of the presentation should be a need to <u>try</u> something; to polish a technique for which a proof is not yet understood, to test a plausible conjecture. Such a lesson should, ideally, result in a productive tension between subjective certainty and objective uncertainty. Without this tension, this need to act, the intuitive presentation may induce unwarranted certainty and over-simplification.

We thus conclude that intuitive thinking cannot be taught in any straight forward, learning-objective way, that presentation of "structure" can do little to encourage intuitive thinking, and that intuitive presentations must create a need to analyze, to do, to test.

We turn now to an examination of discovery learning. Again, we face the problem of definition. While it has been recognized that "discoveries" are of various sorts and qualities (Cronbach, 1966), "discovery learning" generally refers to something of the following sort:

> First, a learning-by-discovery sequence involves induction. This is the procedure of giving exemplars of a more general case which permits the student to induce the general proposition involved. ...
>
> Discovery sequences can generally be characterized by these two properties: one, inductive sequences, and, two, trial and error or errorful learning in various degrees (Glaser, 1966, p. 15).

It is usually claimed that discovery learning instills an attitude toward learning which increases the self-confidence of a student in his ability to guide his own learning. Further, it is claimed that a student should be able to use principles he discovers for himself in a wider range of meaningful contexts than he would if he were simply given the principle and asked to apply it to definite cases. The claim concerning attitude is important, but since we are here examining cognitive premises, we shall pass it over without comment.

Bruner says of the act of discovery:

> It is, if you will, a necessary condition for learning the variety of techniques of problem solving, of transforming information for better use, indeed for learning how to go about the very task of learning. Practice in discovery for oneself teaches one to acquire information in a way that makes that information more readily viable in problem solving. So goes the hypothesis. (1960, p. 26)

The act of which Bruner speaks must be somewhere between two extreme views of dicovery acts--neither of which is useful pedagogically. On the one extreme, it might be said that every act of learning is one of discovery, for we might "discover" (even with appropriate surprise) that something or other is true by reading it or by being told by someone. On the other hand, genuine discovery--discovery of something which is not confirmable by reference to existing bodies of knowledge is extremely rare (Simon, pp. 24-25). Bruner must be referring, then, to a significant act of "finding out" which can, generally, be confirmed as to accuracy of result by reference to existing materials.

But upon what does the significance of the act depend? And why should it contribute to one's ability to learn?

We would expect, on the basis of our fundamental premises, that any acts which contribute to the construction of new operational structures might increase the ability of the organism to learn. Thus if, after many exercises in which he moves from exemplars to generalization, the student concludes that "trying examples" is a powerful heuristic, he has added to his store of operational structures. But in what contexts would it be a good idea to try examples? Well, obviously, the technique is useful when one wants to test a generalization. How could one devise examples if he did not have a trial generalization in mind?

But suppose we lead students through a set of examples without suggesting the generalization which is under test. What happens to the student internally? It would seem that one of several things might happen. First, since the answer to the orienting question, What shall I do? is clear, he may just plod along following directions. The question, What do you conclude? may come as a rude shock to such a student. He concludes, usually, that he doesn't know what he was doing or why. Alternatively, the student may ask, What is it? or Why am I doing this? In this case, what he gains from the experience, the cognitive acts which he records in the doing, are connected to the perceived goal. If this goal is <u>not</u> the one envisioned by the curriculum maker, the student is likely, again, to be thrown into a state of disorientation.

Now, I am not suggesting that a clear statement of the goal, however it might be worded, will aid the student in performing significant cognitive acts. On the contrary, knowledge of exactly what he must <u>do</u> may short circuit the student's cognitive activity. What is needed is a <u>clear sense of purpose</u>. As the student struggles with a problem, as he perceives a goal to be a potential solution, he performs cognitive acts which are connected in a network unified by the perceived goal. We are all familiar with a strange sensation which sometimes overtakes us during problem solving. We pause, baffled, and ask ourselves, why was I doing this? (What was I trying to accomplish?) This seems to represent a deliberate attempt to restore the goal to mind, to review the activities which have been performed in pursuit of it, and, perhaps, to replace the goal with an extended set of sub-goals.

I cannot, in the light of our premises, see why the discovery of a generalization should in itself, be a significant act. The discovery of a <u>way</u> to demonstrate it, to compile it, or to make it plausible to oneself or others <u>is</u> a significant act. It requires the positing of a goal ("Here is what I want to do ..."), the quest for means ("How shall I do this?"), and the confirmation of the chosen means.

There are many times when we "discover" generalizations and feel a fleeting sense of enlightenment. The idea flickers on and off like a faulty light bulb. It is what we do with it after that first flicker, sometimes over a period of years, that finally makes it our own.

In this section, I have suggested that the following curriculum-cognitivist premises are independent of--and perhaps incompatible with--our basic cognitivist premises:

1) The structure of a discipline is a suitable pedagogic structure.
2) Preciseness in language is a valuable pedagogical tool at all levels.
3) Presentation of the structure of a discipline increases intuitive thinking.
4) Intuitive thinking can be taught.
5) Practice in discovering generalizations increases the ability of students to learn.

III.

Now we are ready to speculate a bit by making suggestions about curriculum construction which are compatible with the initial set of premises. First, we must settle the question whether "curriculum" is properly described as preactive or as interactive. As we have noted, the idea of curriculum as preactive is at odds with our basic premises, because there is no way that a given structure of subject matter can be significantly learned by a student who does not possess the required assimilatory structures. But an interpretation of curriculum as entirely interactive seems at odds with common usage; materials usually referred to as "curriculum" must be labelled "potential curriculum." Further, this interpretation suggests greater classroom freedom of choice with respect to subject matter than most of us would be comfortable in endorsing.

The answer seems to be that curriculum construction is properly characterized by two main stages: a preactive stage in which subject matter, methods, and activities are delimited and an interactive stage in which actual choices, constructions, revisions, and matches among these categories are made. The entire product, initial delimitation and record of choices, is the curriculum. The teacher, then, is necessarily a part of the team of curriculum workers. Further, it should be clear that the teacher is, necessarily, an integral and permanent part of the team. He is involved in the actual creation of materials, the planning of strategies which are compatible with both the material and the state of student structure, and the planning of student activities which may transform student structure in the desired direction.

Have we, now, thoroughly confused "curriculum" and "instruction" or, at least, "curriculum construction" and "instruction?" I don't think so. The two are, of course, interactive and, in practice, not easily separable. But I think we have maintained the theoretical separation. "Instruction" still refers to a process of interaction with students; it still refers to an implementation of choices and plans. "Curriculum construction" still refers to an activity which can be undertaken in solitude, in the quiet of one's office. I am suggesting that "teaching" includes both instruction and activities we ordinarily associate with curriculum construction.

Is it possible that what I am describing as the teacher's role in curriculum construction might better be called, simply, "planning for instruction?" Well, no, because in this scheme of things instruction of some sort must precede curriculum construction. And this difference sharply demarcates the role of the teacher in curriculum construction from the role of the to enhance or establish rapport.

Let's consider the matter of objectives, since this topic is traditionally central to curriculum development. The worker at the pre-active stage may use a process very like that described by Tyler (1950) in generating objectives. But it is important to realize that this scheme is <u>essentially</u> preactive. When Tyler speaks of the learner as a potential source of objectives, he is referring to the learner as developmental entity--not to the individual learner. The worker at the interactive stage (the teacher) refers to the <u>individual student</u> as a source of objectives. He plans materials with an eye toward helping the student to achieve his own objectives. Our basic premises suggest that it is absolutely essential that the student have a clear sense of purpose if learning is to be significant.

The teacher may, then, use the objectives identified at the preactiv stage as "dummy objectives" in instruction. He may, that is, set out to teach objective A with the intention of using A only to guide his efforts at demonstration and diagnosis. His <u>real</u> purpose is to construct, with the student, an objective which is relevant to the subject matter and which is <u>realistic</u> in the sense that it represents an end which seems attainable in light of the teacher's diagnosis of cognitive structure. The initial objective

<u>Goals for School Mathematics</u>. The Report of the Cambridge Conference on School Mathematics. Boston: Houghton Mifflin Company, 1963.

..awkins, D. "Learning The Unteachable." In E. R. Keislar and L. S. Shulman, Eds., <u>Learning By Discovery</u>. Chicago: Rand McNally & Company, 1966,

A is, in a mathematical sense, a dummy variable, an objective to be replaced by a particular, diagnostically appropriate objective (which may, of course, turn out to be A itself).

If, as our premises suggest, both stages of curriculum construction are essential, then we can see why wholly pre-packaged curricula are likely to fail. There are no instructional methods which can bring about in the student learning of subject matter for which he possesses no appropriate assimilatory structures. And methods which seem feasible given a particular subject matter may be entirely inappropriate given a proper diagnosis of student structure. Further, there is the well known "press" of such curricula. Everything--sequence, methods, activities, tests--is designed to be compatible with the subject matter. The natural flexibility of the teacher in his relation with individual students is lost as both labor under the press of prescribed materials.

How can workers at the preactive stage facilitate the work of teachers at the interactive stage? That is the question I'd like to explore in the rest of this paper as we discuss assumptions concerning the role of structure, intuition, and discovery in the construction of curriculum.

First, let's consider "structure." I've been claiming that no teaching methods can bring a student to learn that for which he does not possess an appropriate assimilatory structure. One can agree with this and yet protest that, after all, a major task of teaching is the construction of the needed structures. But under what conditions is this possible? Under what conditions is it feasible, economical?

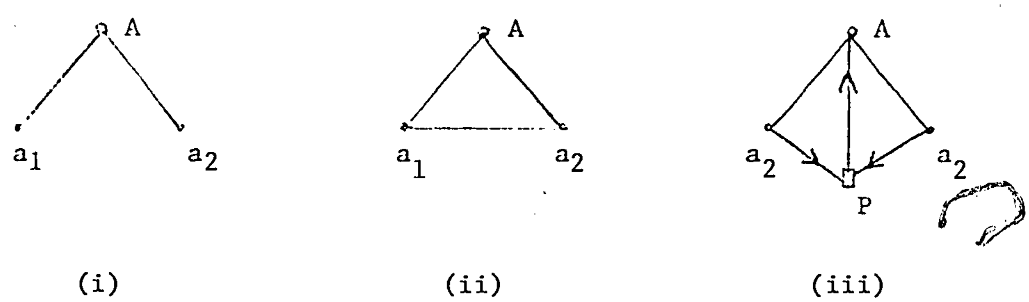
Let's suppose that we accept some well-defined, professionally respectable structure of a discipline as an end to be sought. We need to ask ourselves, next, what possible intermediate and initial structures might look like. Now, this is different from asking how we might translate or interpret the major ideas and unifying concepts of the discipline into language and activities which are appropriate for students at various of development. We do not ask: Can I teach the functional idea to elementary school students? How best might I do this? We ask, instead, how a child at a given stage of development looks at mathematics. Which of its objects interest him? Which of its operations can he perform? What principles does he employ? What heuristic procedures are characteristic of him? And, then, we look at our end-structure. What similarities are there? Where might we make rather natural extensions? If we succeed in creating an extension, how can we stabilize it?

An illustration might help here, and the one I'd like to offer leads nicely into a major recommendation. Let's consider an extension, the formation of a superordinate idea. Suppose I have been growing plants and among those I've worked with are African violets and gloxinias. Suppose, further, that it has not dawned on me that they are similar in any important ways. Since I have perceived no significant relation, my cognitive state might be pictured as in Figure 1, where "a_1" and "a_2" are African violets and gloxinias.

$\cdot a_1 \qquad \cdot a_2$

Fig. 1. Unstructured view

Now, I might notice a relationship myself (perhaps that they respond to the same horticultural treatment) or someone might tell me that both are "gesneriads." "Gesneriad" is a name which can be attached to a superordinate idea, but that idea is not singular; that is, I must, in an important sense, construct it from a multiplicity of components myself. This doesn't mean that I have to "discover" it; rather, it means that I must reflect upon it, record it. If I learn "gesneriad" as a name only, I am likely to discard it; if I see that the name implies <u>some</u> relationship among its sub parts, I may still "forget" it, since it is not useful for me. But if I can describe or use the relationship, the <u>idea</u> becomes part of a stable structure (Fig. 2, iii).



a_1 African violet

a_2 Gloxinia

A Gesneriad

P Set of horticultural operations I perform if I know x is a_1 or a_2

Fig. 2. (i) Name attached to both a_1 and a_2 (A)

(ii) Name attached to both a_1 and a_2 which are known to be related

(iii) Relation explicit; recognition that an operation $P_i \in P$ is properly applied to any $a \in A$.

When I realize that a set of operations P which I have abstracted from my work on violets and gloxinias may properly be applied to something called "gesneriad," I quite naturally wonder whether there are other gesneriads and whether there are further properties of gesneriads which I may profitably use to conduct my operations with violets and gloxinias. I construct an outline structure which is ready to receive new elements (Fig.3).

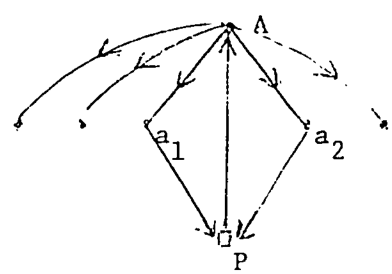


Fig. 3. Extended outline structure with direction

"Gesneriad" is still an inclusive name which is, properly, distinct from P, but it is P which has conferred meaning upon A. Gradually, however, I see that there are other ideas clustering about A; I begin to notice similarities, for example, in leaf shape, leaf arrangement, and flowering habits. I am thus prepared to do two things: (1) to look for more gesneriads and to try P on them and (2) to construct further sets like P which can serve as unifying ideas. Hence, eventually, A itself represents a composite superordinate idea. It is doubtful whether A would ever achieve significance for me if I could not fill in the structure with new gesneriads and learn more about violets and gloxinias through learning about gesneriads (Fig. 4).

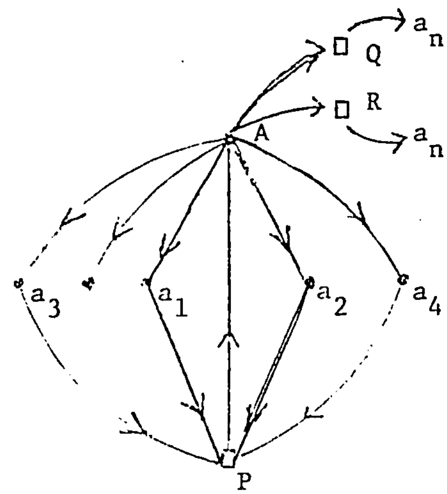


Fig. 4. Extended structure with cluster of superordinate concepts, Q, R

What seems to emerge from all this is a need for what might be termed a cyclic or reflexive curriculum. As Piaget suggests, a superordinate level is not just a named concatenation of lower levels; it is not simply an achievement built out of prior achievements. It sheds light on the levels below or, at least, it has the potential to do this. Learning and development are best represented, then, not as simple upward spirals but as nests of cyclic structures each of which is expanded and strengthened by the next level on the spiral. Frequently, we conceive the educational process as a continuous upward movement. This is a mistake, if we are correct in assuming that cognitive structure is formed through a process of reflective abstraction which depends upon action-goal orientation and regularization.

Can these ideas give us specific guidance in selecting items for the preactive curriculum? I think they can. I would not, for example, make a great fuss about "commutativity" at the elementary school level. Children would use the notion that addition is commutative, of course, but I would not focus on the priniciple itself until the students were ready for the introduction of further frequently used exemplars, e.g., matrix addition, addition of functions. I would, on the other hand, restore the working of identities to the trigonometry curriculum, because the process is rich in opportunities for the student to construct superordinate or unifying ideas. There is no harm in suggesting superordinate ideas and in presenting activities which are likely to make them useful, but there must be a multiplicity of activities from which students may (it is not guaranteed) either embrace the notion we have suggested or create their own. And this creation, I suggest, takes place cyclically: We apply old processes to new objects; apply, shakily, new processes to the new objects; move, wonderingly, to apply the new processes to our old objects; and, then, more firmly and easily, we move on to further explorations of our new process with other objects. There is a great deal of redundancy in this process, but redundancy is what an action-oriented mechanism depends upon in order to use its internal schemata.

One final word (although there is much more to be said) on structure, and then we shall move on to a discussion of intuition. The mathematical notion of isomorphism might be very useful to us in planning both curriculum and instruction. An isomorphism is a two-way mapping from one domain to another which preserves the important characteristics (operations, identity, reciprocity) of the domains. Yet, in our pedagogical theories, we rarely recognize the twowayness of structure building. In language instruction, for example, we frequently ask students to translate from, say, Latin into English, but we almost never suggest that the student then close his book and try to translate his own translation back into Latin. Similarly, after we have taught children to add fractions, we rarely go back in a deliberate and intensive way to the addition of whole numbers rewritten as fractions, e.g., $4 + 5 = \frac{4}{1} + \frac{10}{2} = \frac{4(2)+10}{2} = \frac{18}{2} = 9$. Again, in moral or ethical education we sometimes, over lengthy periods, build grand schemes. But we rarely return to basic events in an attempt to see whether our abstractly held principles apply without inconsistency to situations we once dealt with easily and concretely. Wittgenstein spoke of climbing the steps of a ladder to understanding and recommended throwing away the ladder after understanding was achieved. (The steps are recognized as "nonsensical.") I am suggesting that we use what we gain at each step to enlighten what we perceived, what we thought we knew, at the previous step.

Let's consider, now, intuitive thinking. In our framework, evidence of intuitive thinking is a mark of competence--the non-deliberate functioning of higher order structure. Intuitive thinking, then, cannot be taught. Can we do anything to encourage it?

We have assumed that the human organism is action-oriented, "set," in a sense, to respond internally to the question: What shall I do? It would seem that this question must be answered in a fundamental and satisfactory way before the organism is free to explore intelligently. John Holt (1964) describes school as a place where children "learn to be stupid." How, we ask, does this happen? If a child cannot, when presented with a problematic situation, answer the What shall I do? question with an easy: <u>Well, let's</u>

see. I need not flee, or fight, or hide. I've got time ... If the child cannot resolve this initial impulse to act into a deliberate scanning of options, he may (unless he is already highly competent) gain access to irrelevant routines. What appears to us, then, as "irrational" behavior in response to a mathematical situation is, possibly, just irrelevant behavior.

If this conception of what happens in "learning to be stupid" is accurate, we need to help the child to defer his action decisions. We want him to explore freely, to make mistakes without adverse criticism, to move calmly to the intellectual question: What is it? But when he has done this (and, hence, we claim, has gained access to potentially relevant structures), he must return to a refined version of the question: What shall I do? He now asks: What shall I do to find out what it is?

Here, it seems, teaching can help. It is clear that the student needs a set of heuristic procedures. It is, perhaps, less clear that he also needs algorithms. For suppose he hits upon the notion of using a particular procedure but then realizes that he cannot handle that procedure well? He may discard it as a possibility and move on to something less promising which he does well. ("It is easier to add than subtract, so I'll try adding.") Hence, it is imperative that students be taught algorithms, that they acquire a large repertoire of skills that they can use easily and accurately.

There is no guarantee, of course, that, having learned them, the child will apply them appropriately. But it is clear that, if he does not learn them, he cannot use them in testing hypotheses which seem promising to him. It is important to note that I am not claiming that the student will achieve some sort of deep understanding of the underlying process by using an algorithm repeatedly. He may not--probably will not. What he gains is the possibility of making useful abstractions from a wide variety of situations in which he uses the algorithms. He does not have to avoid the process and, therefore, can engage in potentially fruitful activities which require its use.

A preactive curriculum which is designed to increase intuitive thinking would not, then, be stripped of algorithms; it would be rich in them. It would, preferably, present several for each important skill. The idea, here, is not only to give the teacher a variety of algorithms to accommodate various student styles; the idea is for the student, eventually, to learn all of them. We deliberately increase redundancy in order to increase familiarity. We give the student means to check his own work and to proceed, when his memory fails on the use of one algorithm, to use another. (He can, then, go back and reconstruct the first in addition to solving the problem at hand.)

Let me try to give an example of the way in which the use of algorithms can contribute to what we call "intuition." A topic which, along with trigonometric identities, has been greatly minimized in the modern mathematics curriculum, is the straightedge and compass construction. Now, there are good reasons for this: The proofs are difficult and not very informative; we are trying to develop postulational thought and not the ability to make designs; instructional time is short, and the curriculum is growing. But, consider what happens as a student painstakingly constructs the given parts of a geometric problem. He sees before him empirical evidence that the suggested theorem is true. But more importantly, the feeling grows in him-- as he tries a variety of dimensions for the given--that the theorem has to be true. This is important, of course, because it represents the "feel" for logical necessity. Some youngsters enter geometry with a well developed sense of logical necessity.

They seem able to predict the consequences of certain projected actions mentally. Others are not able to do this. We would have to say, in accord with our premises, that they do not yet possess the internal schemata for achieving this projection. It follows, then, that we must try to develop the schemata through successful action. A correctly executed algorithm guarantees its result and, if applied often enough, it should induce the formation of a schema which can carry out the operation mentally. When the problem is too complicated (the addition too long, the geometric figure too complex), we return to the algorithm.

There are all sorts of things we might suggest in this vein--some of which seem terribly old-fashioned. Can practice in penmanship contribute to an "intuition" in spelling? Can exercises in spelling contribute to a sense of sentence structure? Can practice in punctuating complex sentences increase the likelihood that students will use complex sentences and, thereby, contribute to a sense of more balanced writing? I suspect the answer is yes. But, if we re-institute these practices, we must be careful to reduce the well known, unpleasant side-effects of boredom, tension, and recrimination for failure.

I am suggesting, then, that intuition is a product of competence as well as a sign of what may be possible as a next step. Understanding does not precede doing; it emerges out of the mastery of practical tasks. And, of course, for some people and some concepts, it may never occur.

In the first part of our discussion of intuition, I suggested that students need both heuristics and algorithms in order to be able to answer the question: What shall I do to find out what it is? It is clear that algorithms can be taught. But can we teach heuristics? Probably not directly.

We can, however, share our thinking with students, and this can be helpful. The whole orientation of modern education is away from this practice. We are counseled to state unambiguous objectives, to present rigorous demonstrations, to give deep, conceptually satisfying reasons for operational moves. But, perhaps, we need to share with our students the process of thinking in various disciplines. This is different, of course, from teaching "the processes." I am suggesting that we share the false starts, the dead-ends, the means by which we decide whether to entertain or discard a hypothesis, the means by which we concoct formulas, chronologies, arrangements--for, of course, we do not, in practice, derive every conclusion, or consider every logical possibility, or decide on the form of each poem before we start to write. The classroom performance of teachers makes everything look, at once, too easy and too hard.

How can preactive curriculum workers help? I would think by adding a great deal of developmental material, spending more time on the context of problems, considering the historical settings of various proposed treatments. But, also, we need to provide opportunities for the development of heuristics. This is the only meaningful sense of discovery I can find, given our premises. After a generalization has been given (discovered, presented, stumbled upon), the student must engage in much free play with appropriate objects. He must have an opportunity to create and test his own superordinates, to test his use of algorithms and correct it, to develop a host of techniques which enable him to answer the question: What shall I do to find out what it is? He needs opportunities to work at making hypotheses plausible; we need to quit doing that for him, for our reasons--satisfactory though they may be, logically--will not necessarily be sufficient for him. And if his reasons are wrong and his heuristics faulty? Face him, in time, with situations which may induce corrections, but do not short-circuit the process of heuristic development.

We need, I think, curricula rich with algorithms and conventions, and preactive curriculum workers can supply these and appropriate tests to accompany them. We need, also, curricula rich with opportunities and suggestions for exploration, free play with significant objects, and shared experiences. These can be suggested by the preactive curriculum worker, but they should not be so integrally related to the rest of the curriculum that they cannot be omitted, replaced, revised, or supplemented by the interactive curriculum worker. Rather, the preactive curriculum worker might provide the teacher with guidelines for the writing of suitable curriculum materials in this area. The teacher must be free, as he is responsible, to do what needs to be done for each student within the delimited subject matter assigned to him.

REFERENCES

Arbib, M.A. The Metaphorical Brain. New York: John Wiley and Sons, Inc., 1972.

Ausubel, D.P. Educational Psychology, A Cognitive View. New York: Holt, Rinehart and Winston, Inc., 1968.

Bartlett, F. C. Remembering. Cambridge (England): Cambridge University Press, 1932.

Bartlett, F.C. Thinking. New York: Basic Books, 1958.

Beberman, M. "An Emerging Program of Secondary School Mathematics." In R. W. Heath, Ed., New Curricula. New York: Harper & Row, Publishers, 1964, pp. 9-34.

Bruner, J.S. The Process of Education. New York: Vintage Books, 1960.

Colodny, R.G., Ed. Mind and Cosmos. Pittsburgh: Pittsburgh University Press, 1966.

Cronbach, L.J. "The Logic of Experiments on Discovery." In E. R. Keisler and L. S. Shulman, Eds., Learning By Discovery. Chicago: Rand McNally & Company, 1966, pp. 77-92.

Dewey, J. "The Reflex Arc Concept in Psychology." The Psychological Review, 1896, July, 357-70.

Dewey, J. Experience and Education. New York: The Macmillan Company (Collier Books Edition, 1963), 1938.

Gagne, R. "Curriculum Research and the Promotion of Learning." In R. Taylor, R. Gagne, and M. Scriven, Perspectives of Curriculum Evaluation, AERA Monograph 1., Chicago: Rand McNally & Company, 1967.

Glaser, R. "Variables in Discovery Learning." In E. R. Keislar and L. S. Shulman, Eds., Learning By Discovery. Chicago: Rand McNally & Company, 1966, pp. 13-26.

Goals for School Mathematics. The Report of the Cambridge Conference on School Mathematics. Boston: Houghton Mifflin Company, 1963.

Hawkins, D. "Learning the Unteachable." In E. R. Keislar and L. S. Shulman, Eds., Learning By Discovery. Chicago: Rand McNally & Company, 1966, pp. 3-12.

Heath, R.W., Ed. New Curricula. New York: Harper & Row Publishers, 1964.

Holt, J. How Children Fail. New York: Dell Publishing Company, 1964.

Hullett, J., "Which Structure?" Educational Theory, 1974, 24(1), 68-72.

Jackson, P.W., "The Way Teaching Is." In The Way Teaching Is, Washington,D.C.: ASCD, 1966, 7-27.

James, W. The Principles of Psychology. New York: Dover, 1950 (1890).

Keisler, E.R. and Shulman, L.S., "The Problem of Discovery: Conference in Retrospect." In E. R. Keislar and L. S. Shulman, Eds., Learning By Discovery. Chicago: Rand McNally & Company, 1966, pp. 181-99.

Lashley, K.S., "The Problem of Serial Order in Behavior." In L. A. Jeffress, Ed., Cerebral Mechanisms in Behavior. New York: John Wiley and Sons, Inc., 1951, pp. 112-36.

MacKay, D.M., "A Mind's Eye View of the Brain." In K. H. Pribram, Ed., Brain and Behavior 4: Adaptation. Middlesex, England: Penguin Books, 1969, pp. 478-93.

Mager, R. Preparing Instructional Objectives. Palo Alto, California: Fearon Publishers, 1962.

Mason, R.E. Contemporary Educational Theory. New York: David McKay Company, Inc., 1972.

Miller, G., Galanter, E., and Pribram, K. Plans and the Structure of Behavior. New York: Holt, Rinehart and Winston, 1960.

Neisser, U. Cognitive Psychology. New York: Appleton-Century-Crofts, 1967.

Noddings, N., "Competence Theories and the Science of Education." Educational Theory, 1974, 24(4), 356-64.

Norman, D.A. Memory and Attention: An Introduction to Human Information Processing. New York: John Wiley and Sons, Inc., 1969.

Piaget, J. Logic and Psychology. Manchester, England: Manchester University Press, 1953.

Piaget, J. Structuralism. New York: Basic Books, Inc., 1970.

Piaget, J. Biology and Knowledge. Chicago: University of Chicago Press, 1971a.

Piaget, J. Insights and Illusions of Philosophy. New York: Meridian Books, 1971b.

Pinar, W., Ed. Curriculum Theorizing: The Reconceptualists. Berkeley, California: McCutchan Publishing Corporation, 1975.

Schwab, J., "The Concept of the Structure of a Discipline." Educational Record, 1962, 43 (July), 197-205.

Schwab, J., "The Structure of the Natural Sciences." In G. W. Ford and L. Pugno, Eds., Education and the Structure of Knowledge. Chicago: Rand McNally & Company, 1964.

Simon, H.A., "Scientific Discovery and the Psychology of Problem Solving." In R. G. Colodny, Ed., Mind and Cosmos. Pittsburgh: Pittsburgh University Press, 1966, pp. 22-40.

Tyler, R.W. Basic Principles of Curriculum and Instruction. Chicago: University of Chicago Press, 1950.

Wertheimer, M. Productive Thinking. New York: Harper and Brothers Publishers, 1959.

Weyl, H. Mind and Nature. Philadelphia: University of Pennsylvania Press, 1934.

Zacharias, J.R. and White, S., "The Requirements for Major Curriculum Revision." In R. W. Heath, Ed., New Curricula. New York: Harper and Row Publishers, 1964, pp. 68-81.