This volume is the second in a series of 29 coordinated MINNEMAST units in mathematics and science for kindergarten and the primary grades. Intended for use by kindergarten teachers, this unit guide provides a summary and overview of the unit, a list of materials needed, and descriptions of four groups of activities. The purposes and procedures for each activity are discussed. Examples of questions and discussion topics are given, and in several cases ditto masters, stories for reading aloud, and other instructional materials are included in the book. The four activity groups in this unit are concerned with curves, regions, and boundaries. A variety of activities aimed at developing the notions of open and closed, simple and non-simple curves are described. Other activities are related to describing edges of objects as curves and to regions. (SD)
MINNESOTA MATHEMATICS AND SCIENCE TEACHING PROJECT

UNIT 2

CURVES & SHAPES
MINNEMAST COORDINATED MATHEMATICS-SCIENCE SERIES

1. WATCHING AND WONDERING
2. CURVES AND SHAPES
3. DESCRIBING AND CLASSIFYING
4. USING OUR SENSES
5. INTRODUCING MEASUREMENT
6. NUMERATION
7. INTRODUCING SYMMETRY
8. OBSERVING PROPERTIES
9. NUMBERS AND COUNTING
10. DESCRIBING LOCATIONS
11. INTRODUCING ADDITION AND SUBTRACTION
12. MEASUREMENT WITH REFERENCE UNITS
13. INTERPRETATIONS OF ADDITION AND SUBTRACTION
14. EXPLORING SYMMETRICAL PATTERNS
15. INVESTIGATING SYSTEMS
16. NUMBERS AND MEASURING
17. INTRODUCING MULTIPLICATION AND DIVISION
18. SCALING AND REPRESENTATION
19. COMPARING CHANGES
20. USING LARGER NUMBERS
21. ANGLES AND SPACE
22. PARTS AND PIECES
23. CONDITIONS AFFECTING LIFE
24. CHANGE AND CALCULATIONS
25. MULTIPLICATION AND MOTION
26. WHAT ARE THINGS MADE OF?
27. NUMBERS AND THEIR PROPERTIES
28. MAPPING THE GLOBE
29. NATURAL SYSTEMS

OTHER MINNEMAST PUBLICATIONS

The 29 coordinated units and several other publications are available from MINNEMAST on order. Other publications include:

STUDENT MANUALS for Grades 1, 2 and 3, and printed TEACHING AIDS for Kindergarten and Grade 1.

LIVING THINGS IN FIELD AND CLASSROOM (MINNEMAST Handbook for all grades)

ADVENTURES IN SCIENCE AND MATH (Historical stories for teacher or student)

QUESTIONS AND ANSWERS ABOUT MINNEMAST Sent free with price list on request

OVERVIEW (Description of content of each publication)

MINNEMAST RECOMMENDATIONS FOR SCIENCE AND MATH IN THE INTERMEDIATE GRADES (Suggestions for programs to succeed the MINNEMAST Curriculum in Grades 4, 5 and 6)
CURVES AND SHAPES

UNIT 2
Minnemast

Director
James H. Werntz, Jr.
Professor of Physics
University of Minnesota

Associate Director
Roger S. Jones
Associate Professor of Physics
University of Minnesota

Associate Director
Wells Hively II
Associate Professor of Psychology
University of Minnesota

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Third printing, 1971
CURVES AND SHAPES

This revision was developed by

ROY MUCKEY
Instructor
Hamline University

THOMAS ROCHAT
Research Assistant
University of Minnesota

STEPHEN C. LUNDIN
Research Fellow
University of Minnesota

on the basis of test results from the
MINNEMAST research and evaluation
staff and the experiences of the many
teachers who taught earlier versions by

LAURA M. PAGE
BARRIE G. KLAITS

EDMUND C. BRAY Assistant to Director
Curriculum Materials Development

PATRICIA A. ADAMS Editor

SONIA D. FORSETH Art Director
JUDITH L. NORMAN Artist and Photographer
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<th>Lesson</th>
<th>Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>Introduction to the Concept Tree</td>
</tr>
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<td>6</td>
<td>Locating Touch Points</td>
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<td>7</td>
<td>What Kind of a Curve -- Simple or Non-Simple?</td>
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<td>8</td>
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<td>9</td>
<td>Subsets of Curves</td>
</tr>
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<td>&quot;Berple the Bug&quot; (Story, Part 2)</td>
</tr>
</tbody>
</table>

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<table>
<thead>
<tr>
<th>Lesson</th>
<th>Title</th>
</tr>
</thead>
<tbody>
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<td>11</td>
<td>Sorting Curves by Two Properties</td>
</tr>
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<td>12</td>
<td>Regions and Boundaries</td>
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<td>Curves and Regions (Games)</td>
</tr>
</tbody>
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<table>
<thead>
<tr>
<th>Lesson</th>
<th>Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>How to Sort and Change Curves (Review)</td>
</tr>
<tr>
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</tr>
<tr>
<td>18</td>
<td>Edges of Objects as Curves</td>
</tr>
<tr>
<td>19</td>
<td>Review Activities with Curves and Shapes</td>
</tr>
<tr>
<td>20</td>
<td>Some Optional Games and Activities</td>
</tr>
</tbody>
</table>


Suggested Teaching Schedule for MINNEMAST Kindergarten Units

*Note on Unit 2: This unit should be taught in four sections at different times throughout the school year.
<table>
<thead>
<tr>
<th>Item</th>
<th>Total Number</th>
<th>Lessons in Which Item is Used</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Student Worksheets</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5-pronged chalk holder</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>chalk</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>chalk in 5 colors</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>* self-erasing slates</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>* 4-foot lengths of yarn</td>
<td>30</td>
<td>1, 3, 4, 7, 10, 12, 16</td>
</tr>
<tr>
<td>toy cars, trucks and other assorted small objects</td>
<td>5-10</td>
<td></td>
</tr>
<tr>
<td>dish containing watercolor paint</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>1/2 cup corn meal</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>* paper tray</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>mealworm larvae (borrow from Grade 3 teacher)</td>
<td>2-3</td>
<td></td>
</tr>
<tr>
<td>sheets of unlined paper</td>
<td>220</td>
<td>1, 2, 6, 7, 9, 11, 13</td>
</tr>
<tr>
<td>Two Little Trains by Margaret Wise-Brown (Wm. R. Scott, Inc., New York, 1949)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td><strong>curve cards</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>white and yellow chalk</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>red, blue and black crayons</td>
<td>30</td>
<td>2</td>
</tr>
<tr>
<td>small index cards</td>
<td>172</td>
<td>2, 5, 9</td>
</tr>
<tr>
<td>flannel board</td>
<td>1</td>
<td>3, 4, 10, 11</td>
</tr>
<tr>
<td><strong>&quot;Berple the Bug&quot; cut out figures</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>leaves</td>
<td>30-100</td>
<td>3, 4, 10</td>
</tr>
<tr>
<td>MINNEMAST handbook, Living Things in Field and Classroom</td>
<td>1</td>
<td>4</td>
</tr>
</tbody>
</table>

11
Unit 2 (cont.)

1. *20-foot length of rope
2. *sponge
30 boxes of crayons
30 envelopes
30 scissors
20 concept trees
243 small pieces of paper
1 ball of string or yarn
1 felt-tip pen
4 *balloons
30 clay or 4-foot lengths of yarn
10 *pipe cleaners
1-2 round containers, such as an oatmeal box
15 *sets of property blocks
2-3 rectangular objects, such as boxes
1-2 cone-shaped paper cups
1-5 Tinkertoys construction sets (optional)
30 *1-foot pieces of yarn
1 globe
1-5 maps

* kit items as well as
** printed materials available from Minnemath Center,
720 Washington Avenue S.E., Mpls., Minnesota 55455
*** available from The Judy Company,
310 North Second Street, Minneapolis, Minnesota 5540
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13
INTRODUCTION

This unit is designed:

- To help the children recognize several kinds of curves and some of their properties.
- To introduce a classification scheme for the sorting of curves that are drawn in a plane.
- To introduce the concepts of regions and boundaries.
- To draw attention to the fact that a curve can be transformed without changing some basic properties of the curve.

COMMENTARY

This unit places special emphasis on the use of a classification diagram called a "concept tree" as one method of classifying the members of a set—in this unit, a set of curves. Using a concept tree, curves can be assigned to different branches according to certain properties associated with each branch of the tree.

The children use concept trees to classify curves as being open or closed and simple or non-simple. (The descriptions for these classifications begin on page 7.) The diagrams on the bottom of the next page show only two of several possible ways the branches of the tree can be labeled. These trees show how a set of curves can be classified into four subsets: non-simple closed, non-simple open, simple closed and simple open. Mathematicians do not usually classify non-simple curves as being open or closed, but we do in this unit for the following reasons:

a) Our research evidence indicates that children psychologically treat both simple and non-simple curves as being either open or closed. This psychologically-based classification procedure is easily represented by a concept tree which possesses a form of bilateral symmetry in certain parts of the tree. (See diagrams on next page.)
b) We want to distinguish clearly between a boundary and the regions associated with that boundary. When a curve partitions a plane into more than one region, we say that the curve itself (or some part of the curve) is the boundary.

This curve does not partition the plane into more than one region. The curve is not a boundary.

This curve partitions the plane into two regions, here labeled "A" and "B." The curve is a boundary for both regions.

This curve partitions the plane into two regions, A and B. Only the dark part of the curve is considered as a boundary.

Two possible ways to develop a concept tree for classifying curves are shown below.

* There are no members. The set of simple closed curves made up of less than three line segments has no members. Therefore, this set is the empty set.
The concept tree can be applied to situations other than the sorting of a set of curves. Consider the fact that in most kitchens there is a special drawer for knives, forks, spoons, and other eating utensils. Some people literally toss all these utensils into the drawer, while other people systematically sort the tools into subsets of knives, forks, spoons, etc. In fact, most drawers are constructed with compartments of different sizes for different utensils. Such a drawer provides a means of partitioning the set of utensils into subsets. Membership in each subset is determined by certain properties possessed by the utensils. (Note the diagrams below.)

CONTENTS OF EACH SECTION OF THIS UNIT

Section I

The four lessons of this section should be taught early in the school year after Section I of Unit I.

In Section I of this unit the children must determine whether a curve is open or closed.
The test they employ to distinguish an open curve from a closed curve is the "cat and mouse" test. One child draws a curve and picks a spot where he, the cat, is sitting. Then the other child, who is the mouse, tries to find a place where he can sit and still be safe from the cat. The cat attempts to move from his spot to the place where the mouse is sitting. At no time may the cat cross the curve. If the cat cannot catch the mouse without crossing the curve, the curve is closed. A closed curve is a boundary that partitions a plane into two or more regions. With an open curve the mouse cannot find a place where he can be safe from the cat. An open curve is not a boundary because it does not partition the plane into more than one region.

Section 2

Section 2 contains six lessons that can be taught anytime after Lesson 11 of Unit 3.

The children determine whether a curve is simple or non-simple depending upon whether or not that curve has a touch point. A touch point is a point on a curve from which you can move in more than two directions and still remain on the curve. A simple curve has no touch points; a non-simple curve has at least one touch point. In the illustrations on the next page the arrows indicate the directions one could travel from the touch point.
Section 3

Section 3 contains five lessons that can be taught after Unit 5. The children pay particular attention to closed curves. They begin to visualize a closed curve as partitioning a plane into more than one distinct region. The curve is considered to be the boundary for the regions. They sort and classify curves on the concept tree, and experiment with curves to find ways of changing a curve's shape without changing its type.

Section 4

Section 4 contains the last five lessons of the unit and should be taught after Unit 6. The children again transform curves into different shapes and classify curves using the concept tree. One lesson focuses on simple closed curves, including some curves that have been given special names: circles, triangles, squares, and rectangles. The children trace the edges of objects and discover that the curves they trace have been given certain shape names. In the last lesson of this unit the children make maps and a model of their classroom using property blocks for furniture.
NOTES ON TEACHING THIS UNIT

This unit is divided into four sections, each of which should take about two weeks of class time. The chart below provides a suggested teaching schedule for each section of the unit. (See also the suggested teaching schedule for all MINNEMAST kindergarten units on page vii.)

<table>
<thead>
<tr>
<th>Section</th>
<th>Lessons</th>
<th>Use after</th>
<th>(     )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1-4</td>
<td>Section 1 of Unit 1</td>
<td>(September)</td>
</tr>
<tr>
<td>2</td>
<td>5-10</td>
<td>Lesson 11 of Unit 3</td>
<td>(November)</td>
</tr>
<tr>
<td>3</td>
<td>11-15</td>
<td>Unit 5</td>
<td>(February)</td>
</tr>
<tr>
<td>4</td>
<td>16-20</td>
<td>Unit 6</td>
<td>(April)</td>
</tr>
</tbody>
</table>

Scheduling the sections to be taught at different times of the year should help you meet the problem of individual learning rates because each section provides review of previous work. The children who are not mature enough to grasp the concepts the first time may have no trouble during later reviews.

Several weeks will pass between successive sections of the unit. During these interim periods when you are teaching other units, the children should be encouraged to review many of the activities from this unit.

Some activities in the last section of this unit require word recognition by the children. Not all children will learn to recognize the words "simple," "non-simple," "open" and "closed," but let them try to learn these words. Word recognition practice will help prepare the children for first grade reading. If you make use of opportunities to expose the children to these words during the first three sections of the unit, many children will know the words by the time you begin Section 4. Alternative procedures are provided in the word-recognition activity for those children who cannot recognize the words.

Explanations of terms used in this unit begin on the next page. You should review these terms before you begin teaching the unit.
EXPLANATION OF TERMS USED IN THIS UNIT

Plane Surface
A surface is considered to be a plane surface if a straight-edge (or ruler) moved across the surface in any direction always has all of its points in contact with the surface. Examples of plane surfaces are the top of a table and the wall of a room.

Curve
A curve can be thought of as a path made by moving a point. Such a path or a portion of it may be straight, angular, or rounded. The shape of a curve may be the outline of a familiar object, or it may be a random squiggle.

Curves exist apart from objects outlined by them. For example, it is possible to trace the rim of a paper cup. The path made by tracing is a curve. Because of its shape, such a curve has a special name: circle. Curves may be represented by objects. For example, children may join hands and stand in an arrangement that represents a circle. If they drop hands, the arrangement of children continues to represent the circle, but the circle is not to be confused with the objects that represent it. The children will not be asked to make this distinction. However, the idea involved is basic to material covered in this unit.

Open Curve
A curve that does not partition a plane into more than one distinct region is an open curve. An open curve does not form a boundary. The examples show open curves.
Closed Curve

A curve that partitions a plane into more than one distinct region is a closed curve. A closed curve is a boundary, dividing a plane into regions. The regions in the diagrams below are labeled A through I. These diagrams are examples of closed curves.

Touch Point

A touch point is a point on a curve that you can move toward or away from in more than two directions along the curve. The arrows in the diagrams below show the directions you can move away from each point.
### Simple Curve

A curve that has no touch points is a simple curve.

<table>
<thead>
<tr>
<th>Curve</th>
<th>Number of Touch Points</th>
<th>Type of Curve</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Simple Curve" /></td>
<td>0</td>
<td>Simple</td>
</tr>
<tr>
<td><img src="image" alt="Simple Curve" /></td>
<td>0</td>
<td>Simple</td>
</tr>
<tr>
<td><img src="image" alt="Simple Curve" /></td>
<td>0</td>
<td>Simple</td>
</tr>
<tr>
<td><img src="image" alt="Simple Curve" /></td>
<td>0</td>
<td>Simple</td>
</tr>
</tbody>
</table>

### Non-Simple Curve

A curve that has at least one touch point is a non-simple curve.

<table>
<thead>
<tr>
<th>Curve</th>
<th>Number of Touch Points</th>
<th>Type of Curve</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Non-Simple Curve" /></td>
<td>1</td>
<td>Non-Simple</td>
</tr>
<tr>
<td><img src="image" alt="Non-Simple Curve" /></td>
<td>5</td>
<td>Non-Simple</td>
</tr>
<tr>
<td><img src="image" alt="Non-Simple Curve" /></td>
<td>1</td>
<td>Non-Simple</td>
</tr>
<tr>
<td><img src="image" alt="Non-Simple Curve" /></td>
<td>3</td>
<td>Non-Simple</td>
</tr>
<tr>
<td><img src="image" alt="Non-Simple Curve" /></td>
<td>2</td>
<td>Non-Simple</td>
</tr>
</tbody>
</table>
## Examples of Curves

<table>
<thead>
<tr>
<th>Curve</th>
<th>Number of Regions</th>
<th>Number of Touch Points</th>
<th>Type of Curve</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="triangle.png" alt="Triangle" /></td>
<td>2 (closed)</td>
<td>0 (simple)</td>
<td>Closed Simple</td>
</tr>
<tr>
<td><img src="open_curve.png" alt="Open Curve" /></td>
<td>1 (open)</td>
<td>0 (simple)</td>
<td>Open Simple</td>
</tr>
<tr>
<td><img src="closed_curve.png" alt="Closed Curve" /></td>
<td>2 (closed)</td>
<td>1 (non-simple)</td>
<td>Closed Non-Simple</td>
</tr>
<tr>
<td><img src="open_curve.png" alt="Open Curve" /></td>
<td>1 (open)</td>
<td>1 (non-simple)</td>
<td>Open Non-Simple</td>
</tr>
<tr>
<td><img src="closed_curve.png" alt="Closed Curve" /></td>
<td>4 (closed)</td>
<td>2 (non-simple)</td>
<td>Closed Non-Simple</td>
</tr>
<tr>
<td><img src="open_curve.png" alt="Open Curve" /></td>
<td>1 (open)</td>
<td>1 (non-simple)</td>
<td>Open Non-Simple</td>
</tr>
<tr>
<td><img src="closed_curve.png" alt="Closed Curve" /></td>
<td>2 (closed)</td>
<td>0 (simple)</td>
<td>Closed Simple</td>
</tr>
<tr>
<td><img src="open_curve.png" alt="Open Curve" /></td>
<td>1 (open)</td>
<td>0 (simple)</td>
<td>Open Simple</td>
</tr>
</tbody>
</table>
SECTION I  OPEN AND CLOSED CURVES

PURPOSE

- To lead the children to recognize and to construct curves.
- To introduce the concepts of open and closed as properties of curves.

COMMENTARY

Section 1 of this unit should be taught early in the school year, before Unit 1 if you wish. Or, you may prefer to begin Unit 1 and then take a week or two to teach this part of Unit 2. The time spent on these first four lessons will depend on how quickly the children grasp the ideas presented, but should not be more than two weeks.

This section introduces the concept of a curve. The children find, identify and construct many curves and attempt to determine whether a curve is open or closed. A closed curve partitions a plane into more than one distinct region. If a curve does not separate a plane into more than one distinct region, that curve is open. The children use the words, "curve," "open" and "closed," in their discussions. Teach at least one activity from each lesson and as many others as you feel are necessary. Feel free to substitute similar activities.

Lesson 1 introduces curves. Lesson 2 introduces the open and closed properties of curves with the "cat and mouse" game. The concept of open and closed is very easy for the kindergarten child to grasp. To determine whether a curve is open or closed, the children ask, "Can the mouse be safe from the cat?" One child draws a curve. Another child picks a spot where the cat will sit and marks that place. Once the cat has picked his place, he cannot cross the curve. Then the first child tries to place the mouse where it will be safe from the cat.

If the mouse can hide so that the cat cannot catch him without crossing the curve, the curve is closed. With an open curve the mouse cannot hide from the cat. A closed curve forms a boundary that partitions a plane into two or more regions.
open curve is not a boundary; therefore the plane the curve is drawn in is thought of as being only one region. One activity in Lesson 2 relies on the children's intuitive feeling for regions and boundaries. The concepts of region and boundary will be dealt with more thoroughly in Section 3 of this unit.

"Berple the Bug," a story in Lesson 3, is a favorite with the children. They enjoy hearing the story again and again and acting out situations from the story.

The activities of Lesson 4 provide more practice with the concept of open and closed curves. Use as many of these activities as you think the children need and will enjoy. You may prefer to use these activities at different times while the children are working on Units 1 and 2, and until you begin Unit 3, Describing and Classifying.
Lesson I: CURVES

This lesson introduces curves drawn on the chalkboard and found in everyday life. Teach Activity A and only as many of the others as you feel are necessary to insure learning of the concepts involved.

MATERIALS

- five-pronged chalk holder (borrow from music teacher)
- several colors of chalk
- self-erasing slates
- 1-foot length of yarn for each child
- toy cars, trucks, and other small objects
- watercolor paint
- 1 cup cornmeal
- butcher tray
- mealworms (larval stage; borrow a few from the third grade)
- unlined paper (optional)
- Two Little Trains by Margaret Wise Brown (William R. Scott, Inc., New York, 1949.)

PROCEDURE

Activity A

1. Have the children gather in front of the chalkboard. (If you do not have a chalkboard, use a large piece of paper on the easel.) Draw several curves on the board similar to the examples below.
2. Ask:

WHAT WOULD YOU CALL THESE?

You will get many answers. Accept them all.

WE ARE GOING TO BE WORKING WITH LINES LIKE THESE ALL YEAR. I'M GOING TO GIVE YOU A NAME FOR THESE SO THAT WE WILL UNDERSTAND EACH OTHER WHEN WE TALK ABOUT THEM. THEY ARE CALLED "CURVES." WE SEE CURVES EVERYWHERE.

3. Have the children draw curves on the chalkboard. Suggest that they draw fancy and intricate curves.

4. The children might enjoy the story and pictures in Two Little Trains by Margaret Wise Brown (William R. Scott, Inc., New York; 1949). The illustrations show that very interesting pictures can be made from curves.

Activity B

Identify as curves the many different patterns you make with chalk. You can borrow the music teacher's five-pronged chalk holder and ask children how many curves the chalk points make on the board. Use five different colors of chalk.

Activity C

Have several self-erasing slates for the class. Identify as curves the lines the children draw with their fingernails or with a stylus. Ask the children to watch the curves as they slowly lift the oversheets and erase the lines.

Activity D

Give each of the children a four-foot length of yarn, and have them make a design by laying it on the floor or on paper. (If the children work on paper, shorter lengths of yarn are preferable.) There will be many different designs, and all will be curves. Identify them as such. If pasted on paper, yarn designs can be displayed.

Activity E

During their drawing periods, children can practice creating and recognizing the curves that appear in their drawings.
Activity F

Ask children to push small objects along the surface of the sand pile. The many paths made in the sand should be identified as curves.

Activity G

Dip the wheels of a toy car or truck in water or watercolor paint. When children run the car over the floor, chalkboard or paper, each wheel track will form a curve. Ask the children to think of other moving things that make curves. They may mention such patterns as the path cut out by a moving lawn mower, tracks made in the snow by sleds, skis, etc.

Activity H

The children can watch water trickle over a window pane, or down a melting icicle, and identify the path of water as a curve. Water moving to the gutter and the sewer after a rainstorm also makes a path that represents a curve.

Activity I

Sprinkle about \( \frac{1}{2} \) cup of corn meal on a butcher tray or similar low-walled, flat container. Spread the meal as evenly as possible over the surface and place a mealworm larva on it. Have the children observe the path made by the crawling worm.

Mealworm larvae can be purchased from a pet shop or biological supply house, or you can borrow a few from a third grade teacher who will be using mealworms in Unit 23. The larvae grow to about one inch long before they pupate. The pupae grow to adult beetles in one to three weeks. If you save the larvae, the children can watch them grow to adults through these stages. To find out how to rear the larvae, consult the MINNEMAST handbook, Living Things in Field and Classroom.
Lesson 2: OPEN-AND CLOSED CURVES

In this lesson the children will focus on the open and closed properties of curves. One test to determine whether a curve is closed is as follows:

Draw a curve in a plane. Then try to locate two points in the plane that you cannot connect without crossing the curve. If you can find two such points, as in the first example shown to the right, the curve is a closed curve. The second example shows an open curve:

The game of "cat and mouse" introduces the students to the concept of open and closed curves. If the mouse can be safe from the cat (find a place where the cat cannot reach him without crossing the curve) the curve is closed. If there is no place where the mouse can be safe the curve is open. This game is very popular with children. They will enjoy playing it many times.

After presenting Activity A, do only as many of the others as needed for the children to grasp the idea of open and closed curves. Activity E is recommended as an introduction to regions and boundaries, concepts which the children will study again in Section 3.

MATERIALS

- white and yellow chalk
- curve cards (provided with this unit)
  -- for each child --
- 8½" x 11" sheet of unlined paper
- red, blue and black crayons
- small index cards (optional)
PROCEDURE

Activity A

1. Give each of the children a sheet of unlined paper and a black crayon. Ask them to draw a curve on each side of the paper.

2. Tell the children that they are going to play the game "cat and mouse" with curves. In this game the cat tries to catch the mouse. The cat cannot cross the curve. The mouse tries to find a spot where the cat cannot catch him.

3. Ask one child to draw a curve on the chalkboard with
white chalk. This child will be the mouse. Ask another child to be the cat. He is going to try to catch the mouse. The cat marks a spot on the board (near the curve) with yellow chalk where he would like to sit.

4. Then the mouse marks a spot on the chalkboard where he would like to sit, realizing of course that the cat wants to catch him.

5. Tell the cat that the only rule he must follow is that he cannot cross the curve. Now the cat tries to catch the mouse by drawing a line from his position (the yellow spot) to the mouse. If the cat can get to the mouse without crossing the curve, he catches the mouse; if he cannot do it without crossing the curve, the mouse is safe.

6. Play this a few more times on the board until the children understand how to play. Leave the curves on the board. The cat and mouse can take turns drawing the curve, but the cat always picks his spot first. Pointing to the appropriate curves, say:

WITH THIS CURVE THE MOUSE CANNOT BE SAFE FROM THE CAT. WE CALL THIS CURVE AN OPEN CURVE.

WITH THIS CURVE THE MOUSE CAN BE SAFE FROM THE CAT. WE CALL THIS CURVE A CLOSED CURVE.

7. Organize the children in pairs and have them play "cat and mouse" using the curves they drew. They should be encouraged to say whether a curve is open or closed, and why they think so. The mouse might say, "This is an open curve because I could not be safe from the cat." For a different curve, the cat might say, "This is a closed curve. The mouse could be safe from me."
Lesson 3: "BERPLE THE BUG" (Part 1)

Before reading this story to the children, cut out the figures of Berple, Claudius and the jelly bean bushes provided with the unit. Color the jelly beans white, red, green, yellow and black.

MATERIALS

- flannel board
- cut-out figures of Claudius, Berple and the jelly bean bushes
- yarn, one 2-foot length, one 4-foot length
WANT TO HIDE FROM THE CAT. SHOW THEM WHERE THE CAT IS AND WHERE THEY CAN BE SAFE.

2. Tell the children who have closed curves to mark a spot for the cat, if they have not already done so, and then to color blue all the space where all the mice can hide and to color red where the mice would not be safe from the cat.

3. Now ask for volunteers to show open curves. Ask:

WITH THESE CURVES, CAN THE MOUSE FIND A PLACE FOR HIS FRIENDS TO HIDE? CAN THEY BE SAFE FROM THE CAT? (No.)

WHY NOT? (Because these are all open curves.)

4. Have these children color red the places where the mice will not be safe. Ask:

HOW MUCH OF YOUR PAPER DID YOU COLOR? (All of it.)

5. Save these curves for Activity E.

Activity C

1. Before class begins, sort out several open and closed curves from the curve cards provided with this unit.

2. Gather the children around the chalkboard. Draw an open curve and a closed curve and print the name above each. Pointing to each curve ask:

   CAN THE MOUSE BE SAFE FROM THE CAT? (Yes or no.)

   IS THE CURVE OPEN OR CLOSED? (Open, if the mouse cannot be safe from the cat; closed if he can.)

3. Now show one curve card at a time and ask the children if it is open or closed. Remind them that they must first determine if the mouse can be safe from the cat. Ask:
IS THE CURVE LIKE THIS ONE (pointing to the open curve on the board)? OR IS IT LIKE THIS ONE (pointing to the closed curve)?

4. Put each card in the tray under the appropriate curve drawn on the board as the children identify it.

5. After all the curves are sorted, mix them up and this time let the children place each curve in the correct place. Give each child a card to place in the tray (or tape to the board) after he determines if it is open or closed.

6. Encourage the children to play with the curve cards during free time. Although some cards are labeled "simple" and "non-simple," the children should only study the open and closed property at this time.

7. Some children may want their own sets of curve cards to take home. These could be made on small index cards. The children can also play "cat and mouse" with their friends on curves they draw themselves.
Activity D

The children might enjoy making curve trains to put on the walls, bulletin-boards, or in the corridor. One engine pulls cars of open curves, the other engine pulls cars of closed curves. Throughout the year, they can add new curves to each train. After teaching Section 2, the children can make four engines, each pulling a different type of curve: simple open, simple closed, non-simple open and non-simple closed.
Lesson 3: "BERPLE THE BUG" (Part I)

Before reading this story to the children, cut out the figures of Berple, Claudius and the jelly bean bushes provided with the unit. Color the jelly beans white, red, green, yellow and black.

MATERIALS

- flannel board
- cut-out figures of Claudius, Berple and the jelly bean bushes
- yarn, one 2-foot length, one 4-foot length
Berple the Bug

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Read the story aloud, referring to specific illustrations in the text.

Put Berple on the flannel board.

"Hi! I'm Berple. I live in Bugland. Bugland is a very special place where jelly beans grow on bushes. I eat jelly beans every day — red ones, green ones, white ones, yellow ones and black ones."

Put the bushes on the flannel board.
"I'm Claudius. I'm very fierce. I like to eat jelly beans, but most of all I'd like to eat Berple. Some day I'm going to catch him and then — yum, yum, that will be the end of him!"
Every day, all day long, Claudius tried to catch Berple. Every day, all day long, Berple ran away and hid from Claudius. Some days he did not even get a chance to eat any of the lovely jelly beans. Poor Berple!

One day Berple became very tired and very hungry. He decided that something must be done so that he could get some rest. He flopped down behind a tree where he would be safe for a few minutes.

"I must think of some way that I can be safe from Claudius and still be able to eat all of these yummy jelly beans. I'm so tired of running away and hiding. Oh, dear, oh dear!"

He thought and thought. "I know, I know! I'll build a fence. That's what I'll do."

So Berple built a fence. It was so high that Claudius could not climb over it. He built it beside some very good jelly bean bushes. The fence looked like this.

Put the two-foot piece of yarn on flannel board, as shown in Picture 2 on the next page.
COULD CLAUDIUS CATCH BERPLE NOW? HOW? (He could go around the fence.)

Using the four-foot piece of yarn, now extend the fence in both directions.

IS BERPLE SAFE FROM CLAUDIUS NOW? (No, Claudius could still come around the fence.)

What would happen if the fence were extended further and further in both directions?

COULD WE MAKE THE FENCE LONG ENOUGH SO THAT CLAUDIUS CANNOT CATCH BERPLE? (No, because no matter how long it was, Claudius could still go around it. Berple would probably get tired trying to build such a long fence anyway.)

The children may suggest ending the fence at a house or some obstruction. Such speculation is good, for it shows that they are trying to visualize the situation but remind them that Claudius could sneak around the obstruction.

Berple was resting, munching on some jelly beans after building his fence. He stopped eating and listened very carefully. What do you think he heard? That’s right, he heard Claudius sneaking through the jelly bean bushes. Poor Berple, he had to run and hide again.

He crouched behind some bushes and he thought very hard. "That fence was no good. It didn't keep me safe from Claudius. What shall I do now?"

Finally, he had another idea.
Berple built a fence that looked like this.

Make a fence like the one in Picture 3.

IS BERPLE SAFE NOW? CAN CLAUDIUS CATCH HIM?

Berple was very happy resting and eating jelly beans. As he reached for a large licorice bean (this was his favorite kind), he saw two feelers sticking up above one of the bushes. He knew it was Claudius. He got away just in time.

Once more poor Berple had to find a safe place to hide. He decided that he would need a very different kind of fence.

WHAT KIND OF FENCE DO YOU THINK BERPLE SHOULD BUILD SO THAT HE WOULD BE SAFE FROM CLAUDIUS?

Stop and let the children speculate about the kind of fence Berple could build. Do not turn the page until you are ready to show Berple's solution.
Picture 4
Finally, Berple built a fence just like this!

Demonstrate with yarn, as in Picture 4.

HAS BERPLE SOLVED HIS PROBLEM? IS HE SAFE FROM CLAUDIUS?

Berple was very happy for one whole day. He sat down and ate his favorite jelly beans.

End of Part 1 of "Berple the Bug." Save the cut-out figures for Part 2 in Lesson 10 of Section 2.
Lesson 4: REINFORCEMENT ACTIVITIES

Use as many of these activities as you feel the children need or will enjoy. Feel free to make up similar activities. Section 1 is completed by this lesson. The children should be able to recognize and talk about open and closed curves. Encourage practice with these concepts in everyday situations whenever possible. Section 2 can be taught during or after Unit 3.

MATERIALS

- 4-foot piece of yarn for each child
- leaves
- MINNEMAST handbook, Living Things in Field and Classroom
- construction paper
- paste
- space in the gymnasium or outdoors
- cut-out figures from "Berple the Bug" story in Lesson 3
- flannel board
- chalk
PROCEDURE

Activity A

Each day when the children come to school, point out the date on the calendar. Ask whether the numeral is an open or closed curve. If, the date is, say September 27, it is an "open day," and Claudius will be chasing Berple all day long. Some days will be both open and closed, such as September 28. Ask the children which curve Berple should choose to hide in. On a closed day, say October 6, Berple can find a place to hide and eat his jelly beans without worrying about mean Claudius.

Activity B

Give each child a piece of yarn about four feet long. Ask him to arrange his yarn into either an open or a closed curve. (If this is to be done on paper, a shorter length of yarn should be used.) If he is not sure how to do this, encourage him to discuss and compare curves with classmates. Encourage children to make curves of different shapes.

Ask the children who have made open curves to close them, and those who have made closed curves to open them.

Activity C

Sometimes children will sit or stand in a formation that they may recognize as an open curve. Ask them how they can close it. You might also ask them how to open a closed curve they have arranged.

Activity D

Children may bring leaves from many different trees into the classroom. Suggestions for examining and handling leaves can be found in the MINNEMAST handbook, Living Things in Field and Classroom. This gives you an opportunity to combine a MINNEMAST and an art lesson.

By pasting several different kinds of leaves on colored construction paper, children can make collage pictures. Before displaying these on a bulletin board, have the children trace each of several leaves with a finger to show you curves of different shapes. Some curves will be "smooth" and some will be "bumpy." An observant child may find other curves formed by the veins.
Activity E  "Berple the Bug"

The children may want to dramatize the story. Choose one child to be Berple and another to be Claudius. The remaining children are to join hands when Berple asks them to do so, in order to form a fence. Berple can place the "fence" of classmates in a variety of curves to protect himself from Claudius, who must try to tag Berple when he is not protected by his fence.

Activity F

Use the cut-out figures of Berple (B) and fierce Claudius (C), and yarn (on the flannel board) or chalk (on the board). Make one or several of the arrangements of fences and bugs shown below. Put Claudius at point C. Put Berple at point B.

WILL BERPLE BE SAFE HERE?

If so, the children remove the Berple figure and mark the point with a circular cutout (on the flannel board) or with a B (on the chalkboard). If Berple would not be safe, they are to leave the point unmarked. For variety, move Claudius sneakily around the fence and ask a child to guide Berple out of danger. Change the positions of Claudius and Berple. Have the children tell you whether Berple is safe when Claudius is at each of several different places.
Activity G

With chalk on the floor or playground, represent all of the fences in a field, as shown below. Choose one child to be Claudius and another to be Berple. Each child should move among the chalk fences in an adventure game. Claudius darts about and tries to sneak up on Berple, while Berple ducks behind fences to evade the wicked Claudius. There is one rule to this game: only Berple can step over a curve. Once Berple has crossed that curve, he cannot cross it again. The object of the game is to see if Berple can pick a safe (closed) curve from this variety of complex curves.

In each round of the game, Berple and Claudius are each assigned a starting position. For example, Berple may stand at point B and Claudius at point C. The chase may continue all over the field. It is concluded when (1) Claudius overtakes Berple or (2) Berple finds a safe place.
SECTION 2 SIMPLE AND NON-SIMPLE CURVES

PURPOSE

- To help children sort and classify curves using a concept tree.
- To help children identify a touch point.
- To help children recognize a curve as either simple or non-simple.

COMMENTARY

This section expands some of the concepts of Unit 3, Describing and Classifying and can be used any time after Lesson 11 of that unit. You may prefer to finish Unit 3 and then teach this section before going on to Unit 4, Using Our Senses. In this section the children use skills learned in Unit 3 to classify objects by properties into subsets. They classify and sort curves using a branched diagram called a "concept tree."

A curve is classified as simple or non-simple depending on whether or not it has a touch point. A touch point is a point on a curve that you can move toward or away from in more than two directions and still remain on the curve. A simple curve has no touch points; a non-simple curve has at least one touch point. The curves below are non-simple curves. Each curve has one touch point. The arrows and numerals show the directions you can move from each touch point.

The illustrations in this section show the movements along the curve as either all toward or all away from the point. A child may propose that he could make some moves away from the point and some toward the point. This is acceptable as long as only one move is made on each part of the curve adjacent to the point.
In Lesson 5 the children first sort themselves (by sex), and then sort property blocks and open and closed curves, using a concept tree. They will find the concept tree a useful device for sorting objects into subsets according to certain properties.

In Lesson 6 the children identify touch points on worksheets and on curves made of rope on the floor. They must be able to recognize a touch point before they can make the distinction between simple and non-simple curves in Lesson 7. Encourage the children to use the term "touch point."

![Diagram of simple and non-simple curves]

In Lesson 7 the children identify simple and non-simple curves. Most children will find it more difficult to distinguish between simple and non-simple curves than between open and closed curves. To identify a curve as simple or non-simple the children must first determine if the curve has any touch points. The children will identify simple and non-simple curves drawn on the chalkboard and on curve cards provided with the unit. They then make some curves of their own.

Lesson 8 is a game called "I'm A Curve," in which the children represent curves with their arms, legs, and bodies. The children can try this activity at any time when they are outdoors or in the gymnasium, but should not spend too much time on it in class.
In Lesson 9 the children draw curves and sort them into subsets of simple and non-simple curves.

Lesson 10 is the second half of the Berple story. At the end of the story, lead the children to see that the solution to Berple's problem is to build a closed curve fence around Claudius.

You do not need to present all the activities provided. Do at least one activity from every lesson and as many others as you think the children need. If you feel the children have not grasped the concepts, review some activities occasionally while you are teaching Units 4 and 5. Section 3 of this unit can be taught after Unit 5, Introducing Measurement.
Lesson 5: INTRODUCTION TO THE CONCEPT TREE

Before class begins, draw a 2-branch concept tree on a large sheet of newsprint (or make it with a rope on the floor). The children may want to examine it before the activity begins. Ask them what the tree could be used for. Remind them of the classifying activities in Unit 3 and lead them to suggest using the tree for classifying and sorting objects.

MATERIALS

- concept tree drawn on a large sheet of newsprint or made with rope or yarn on the floor
- several small, blank cards to label the branches of the concept tree
- curve cards (provided with the unit)
- construction paper

PROCEDURE

Activity A

1. Have the children sort themselves by sex, using the tree. The boys should walk up the tree to the top of one branch and the girls to the other. For more practice, have the children sort themselves by hair color. The children with blond hair go up one branch and those with not-blond hair up the other.

2. Now have the children label the branches with the names of two colors, say red and yellow. Suggest that the children put a red dot on one label and a yellow dot on the other. Then let them sort the property blocks, carrying all the red ones up one branch and yellow ones up the other.
3. With pieces of yarn, extend the tree to include branches for the properties circle and square on each original branch. Help the children sort the blocks according to two properties as they did in Lesson 10 of Unit 3.

4. Let the children sort many objects until they understand how to use the concept tree. When they are ready, review the concept of open and closed curves. Then distribute the curve cards (or let the children make new curves of their own) and have them sort open and closed curves with the tree. Remove the yarn pieces and label the branches "open" and "closed." Put an example of the correct curve on each branch label to guide the children in their sorting. (Save the tree for Activity B.)
Activity B

From construction paper, cut a leaf for each child. Have each child draw a curve on his leaf. Some should draw open curves and others should draw closed ones. Hang the concept tree used in Activity A on the bulletin board and let the children tape their leaves to the correct branches. You or the children may wish to paint or color the tree to make it look more realistic.
Lesson 6: LOCATING TOUCH POINTS

This lesson introduces touch points, and provides practice in locating and identifying touch points. The children must be able to identify a touch point to make the distinction between simple and non-simple curves in the next lesson. A simple curve has no touch points; a non-simple curve has at least one touch point. Do not identify curves as simple or non-simple to the children yet. In this lesson, the only distinction they make among curves is whether or not they have touch points.

To locate a touch point, first choose any point on a curve. If you can move along the curve in more than two directions away from this point, it is a touch point. The numerals and arrows in this diagram show the directions you can move away from the touch point.

Since most children find the distinction between simple and non-simple curves more difficult than that between open and closed, you may need to do most, if not all, of the activities provided. Since these activities are an important background for the material covered in the remainder of the unit, plan on spending at least two days with this lesson.

MATERIALS

- large sheet of newsprint or construction paper
- 6-foot length of rope
- sponge
- small pail of water
- Worksheets 1, 2 and 3
- crayons
- sheet of unlined paper
PROCEDURE

Activity A

1. Draw several curves on a large sheet of newsprint (or on pieces of construction paper). Some of the curves should have touch points and some should not. Do not mark the touch points. Place the paper on the floor.

2. Tell the children that they are going to examine these curves and try to find some touch points. Even though the children do not know what a touch point is, they may want to try to find some. If a child correctly locates a touch point, say:

   YES, THIS IS A TOUCH POINT.

   If a child finds a point that is not a touch point, say:

   NO, THIS IS NOT A TOUCH POINT.

3. After a few trial and error attempts at locating touch points, tell the children that there is a special way to find a touch point. Mark a point on one of the curves and ask the students to imagine that a child is standing on that point. Ask the class to find all the different routes the child could take if he walks along the curve away from that point. Let the children trace each route with a different color of crayon.

   ![Diagram of curves and routes]

   Do this for several curves. For each point, have the students color and then count the number of possible routes. If there are more than two routes, that point is a touch point. If there are only one or two routes, that point is not a touch point. After the children count the
routes, let them determine whether that point is a touch point.

4. Tape the curves to the chalkboard. Then draw these five curves on the board and mark the indicated points.

![Curves on a chalkboard with marked points]

5. Challenge a child to find a touch point on one curve. If he correctly locates a touch point, he tries to find another and another until all the touch points on that curve have been located. If he incorrectly identifies a point as a touch point, another child finishes that curve. If the children have difficulty locating touch points, have them study the curves that you drew on paper. If necessary, the children can trace and count the directions that they could walk away from each point.
Activity B

1. Have the children choose partners. (If necessary, three students can work together.) Give a copy of Worksheet 1 to each pair of students.

2. One student from each pair begins by finding what he thinks is a touch point on one of the curves drawn on the worksheet. Then he must prove that the point he chose is a touch point by using crayons to show the ways that he can move toward or from his chosen point. His moves must be either all away from or all toward the point. All moves must follow the path of the curve. By using a different color for each move, it is easy to count the number of possible moves. If more than two moves can be made from the point, he can be sure that it is a touch point. He and his partner take turns finding all the touch points on the curves.

3. The students should be encouraged to ask each other:

   HOW DO YOU KNOW THAT IT IS A TOUCH POINT?
   (Because, from this point, I can move in more than two directions along the curve.)

The children need not provide the exact answer given above.
4. Then give one copy of Worksheet 2 to each pair of children and have them take turns finding the touch points on that curve. Those children who can easily recognize touch points can go on to Worksheet 3 and, with a partner, find all the touch points. If some children are still having trouble finding the touch points, bring them to the chalkboard, draw a few curves and show them how to find the touch points.
Activity C

1. Help each child find a partner. Give each child a sheet of unlined paper and a crayon.

2. At a given signal, each student draws a curve on his paper. The curve can be as complicated as the child wishes. The only limitations are that he has only ten seconds to draw the curve and at no time can he lift his crayon from the page.

3. Then the children exchange curves and locate all the touch points on the partner's curve. Each touch point should be marked with a large dot in another color.

4. After locating every touch point, the children return the curves to their partners. The partner checks to determine whether all the points have been located.

5. This activity can be repeated as often as you think appropriate.

Activity D

1. Have one child draw a curve on the chalkboard with a wet sponge. Then he quickly calls on another child to tell him if the curve has any touch points. If this child can answer correctly before the curve "disappears" he comes to the board and draws a curve.

2. Another way the children can play this game is to have a child draw a curve, and the first child who can correctly answer "touch point" or "no touch points" gets to draw the next curve.

3. After completing Lesson 7, the children can play this game again, calling out "simple" or "non-simple" depending on whether the curve has any touch points.
Activity E

1. If necessary, review touch points by drawing curves on the chalkboard. Have a few children mark the touch points, and explain why these points are touch points.

2. Lay a long piece of rope on the floor in a curve. Choose a child to walk the curve. At each touch point, he must stop and say "touch point," and explain why it is a touch point. Then he continues walking.

3. If he misses a touch point, he is "out" and must sit down. If he finds and names every touch point, he chooses the next child to walk the curve.

4. Rearrange the rope into a different curve each time. You will need two pieces of rope for non-simple open curves. Use simple curves also. Some curves that might be fun to walk are shown below.

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\end{array} \]
Lesson 7: WHAT KIND OF A CURVE — SIMPLE OR NON-SIMPLE?

In this lesson, the children learn to classify curves into two new subsets: simple curves and non-simple curves. They will name the curves according to touch points. A simple curve has no touch points; a non-simple curve has at least one touch point. If the children still have difficulty identifying a touch point, review some activities from Lesson 6.

The children will observe and classify curves drawn on the chalkboard and on special curve cards. Also, take advantage of everyday situations to call attention to simple and non-simple curves. Playground games, art work and objects in the classroom lend themselves to your inquiry, "What kind of curve is this?"

MATERIALS

- curve cards (provided with unit)
- sponge
- small pail of water
- 8 1/2" x 11" unlined paper
- crayons
- 4-foot piece of yarn
PROCEDURE

Activity A

1. Have ready the curve cards provided with this unit. On the chalkboard, draw the following curves in the indicated sequence, one at a time. For each curve, ask:

**DOES THIS CURVE HAVE ANY TOUCH POINTS?**

Draw simple curves on the left and non-simple curves on the right. Some of the curves are similar to the ones the children walked in Lesson 6.

1.  
2.  
3.  
4.  
5.  
6.  
7.  
8.  

2. When all the curves are drawn, ask:

**WHAT DO WE CALL THESE FIGURES?** (Curves.)

Point to the left column:

THE CURVES I HAVE DRAWN HAVE NAMES. THESE CURVES THAT HAVE NO TOUCH POINTS ARE CALLED
"SIMPLE CURVES."

Point to the right column:

WHAT DO YOU THINK WE CALL THESE CURVES WITH TOUCH POINTS?

Encourage the children to guess. If they do not answer, repeat:

IF THESE ARE SIMPLE CURVES, WHAT KIND OF CURVES ARE THESE?

The expression, "non-simple curves," should become familiar to the children. Encourage them to use the words. Print "simple" and "non-simple" by each class of curves.

WHAT IS DIFFERENT ABOUT THE TWO KINDS OF CURVES? (Simple curves do not have touch points; non-simple curves have at least one touch point.)

3. Show the children the set of simple and non-simple curve cards and tell them that there is a picture of a curve on each. Mix up the cards so that there is no pattern to the sequence.
4. Hold up one card and ask:

**IS THIS CURVE SIMPLE OR NON-SIMPLE? HOW CAN YOU TELL?** (By identifying touch points.)

5. When the children have named the curve, have a child place the card on the chalk tray under the column for that kind of curve. Do the same with the next card.

6. Frequently review the idea that a simple curve has no touch points; a non-simple curve has at least one touch point.

7. Hold up each of the remaining cards in the set and ask individual children to place it in the correct column. As each card is placed in a column, ask the children if they agree that it is correctly placed. Identify by name the curves about which there is agreement. (Simple, non-simple.) Cards about which there is disagreement should be put in a third pile to be considered later. After the cards have been sorted, return to the doubtful ones and have the children try again to sort them. Where there is uncertainty, ask:

**DOES THIS CURVE HAVE ANY TOUCH POINTS?**

8. Some children may want to make their own set of curve cards to take home.

**Activity B**

Have each of the children draw a simple and non-simple curve on paper. Now ask them to make a picture of something interesting from their curves. If some children are slow to think of things, suggest specific things: a pumpkin face, a wagon, a clown, etc. The pictures might make an interesting bulletin board display.

**Activity C**

Have the children draw curves with a wet sponge on the chalkboard as they did in Activity D of Lesson 6. This time they call out "simple" or "non-simple" depending on whether the curve has any touch points.
Activity D

1. Give each child a piece of yarn several feet long, and ask him to arrange it on the floor. When you look at an arrangement, ask:

   IS THIS A SIMPLE OR NON-SIMPLE CURVE?

   HOW CAN YOU CHANGE IT TO THE OTHER KIND OF CURVE?

2. Variations include pasting yarn pictures on paper or arranging yarn curves on the flannel board, and then identifying the curves.

3. Ask the children to see how many different curves they can represent with a piece of yarn.

Activity E

Print the letters of the alphabet on the chalkboard and have the children identify each as simple or non-simple and open or closed. This activity can be repeated often during the school year to review curves and also to help prepare the children for first grade reading.
Lesson 8: "I'M A CURVE"

Suggest to the children that they can represent curves with their arms, legs and bodies. We ignore the fact that other parts of the body may distort the curve.

LET'S MAKE A SIMPLE CURVE WITH OUR ARMS; OUR LEGS; OUR BODIES.

MAKE A NON-SIMPLE CURVE WITH A LEG AND AN ARM.

Have the children represent a variety of curves using their bodies in many positions. They will think of many original curves and they will have fun doing it. Have other children identify the kind of curve a child is making.
Lesson 9: SUBSETS OF CURVES

This lesson provides more practice in sorting curves into subsets of simple curves and non-simple curves. Remind the children that a curve is classified as simple if it has no touch points, and non-simple if it has at least one touch point. More practice is provided in the language associated with simple and non-simple curves.

MATERIALS

-- for each child --
- 2 sheets of unlined paper
- used envelope
- crayons
- scissors
- index cards (optional)
- Worksheet 4

PROCEDURE

Activity A

1. Give each child a copy of Worksheet 4, and have the children cut apart the cards. There are six curves on this page.

2. Then have each child sort his set of six curves into two subsets, one of simple curves and the other of non-simple curves.

3. After the students have done this, let them check each other's subsets to see if they agree with the arrangements. They should be able to state that a curve is simple if it has no touch points and non-simple if it has at least one touch point.
4. Children who have difficulty locating touch points should use crayons to mark the touch points as they did in Lesson 7.

5. After the children have compared their sortings and all agree, they can paste the cards on paper, the simple curves on one side and the non-simple on the other side. Or, the children can paste each curve on an index card. They can take the curves home to show their parents or keep them at school to use for review. Curves pasted on paper can be displayed around the room.

6. The children might enjoy finding a partner and taking turns drawing simple and non-simple curves. They should be able to prove the type of curve according to the touch point test.

Activity B

Before teaching this activity, have each child bring a new or used envelope from home to keep his curves in.

1. Have each child make a deck of eight blank cards by folding a sheet of paper in half three times in succession. If the children are not yet adept at cutting, you may want to cut up the cards yourself before class, to save time. If the children are able to fold and cut, let them do it so that you can point out the geometric progression of 1, 2, 4, and 8 pieces as they fold.

2. Next have each child draw a curve on each of the eight cards. Using a crayon, he should be able to complete each curve in about ten seconds. You may want to give a starting and a stopping signal for each card. Tell the children to draw examples of both simple and non-simple curves.

3. Have the children cut along the creases in the paper and separate the eight cards. Then each student sorts his set of cards into two subsets (simple curves and non-simple curves). The children can check one another's sortings while you also move about checking.
4. Ask a student to pick one curve from his set and show it to the rest of the class. He should tell the class what type of curve he is holding and why he thinks it is that type. Examples:

"This is a simple curve because the curve has no touch points."

"This is a non-simple curve. It has a touch point."

5. Have the children keep the curve sets in their envelopes to use for review during free time or at home. In Section 3 of this unit the children again sort curves with the concept tree. They can use these curves because they do not need a new set for each activity. You may want to have the children exchange envelopes so that they get practice with a different set of curves. Occasionally let the children draw new curves to add to the envelopes.
Lesson 10: "BERPLE THE BUG" (Part 2)

This lesson is the second half of the Berple story. At the end of the story, lead the children to see that Berple's problem will be solved if he builds a closed curve fence around Claudius.

Section 2 is completed with this lesson. Section 3 can be taught after Unit 5.

MATERIALS

- flannel board
- cut-out figures of Berple, Claudius and jelly bean bushes
- yarn, one 4-foot length
Put Berple, the jelly bean trees and fence on the flannel board as shown in Picture 5. Do not put Claudius on the board yet. Remind the children that when they last left Berple he had built this fence and was safe from Claudius.

Berple was very happy for one whole day. He sat down and ate his favorite jelly beans.

Remove trees as Berple "eats" the jelly beans.

That night Berple dreamed of yummy jelly beans. In the morning Berple awoke and reached out sleepily to pick a jelly bean. He felt in the grass all around him. He opened his eyes wide, but he could not find even one white jelly bean! They were all gone. He had eaten every one. Berple was very hungry. Then he heard "munch, crunch," and when he peeked over the fence, what do you think he saw?

Put Claudius on the flannel board.

Claudius was eating a lovely breakfast of jelly beans. Berple felt like crying — the fence kept Claudius away from Berple, but it also kept Berple away from the jelly beans. He walked all along the fence, but there was no way for him to get at the jelly beans and still be safe from Claudius.

Berple thought and thought. If he made an opening so that he could get out to get some jelly beans, then Claudius could get in. So Berple started to build a new fence.
Make a fence as shown in Picture 6.

Claudius could not get through this fence. But Berple could see that he would soon be out of jelly beans again. And he couldn't get to the other section of the fence to eat the jelly beans that grew there. This fence was no good either!

At last, Berple had another idea. He thought of building a fence like this, a very long fence.

Show Picture 7 on the next page, but do not make the fence.
Picture 7
IF BERPLE BUILT A FENCE LIKE THIS ONE, WOULD IT SOLVE HIS PROBLEM? (Yes.)

IS THERE ANY WAY BERPLE COULD BUILD THIS FENCE WITHOUT CLAUDIUS SEEING HIM? REMEMBER, IT IS A VERY LONG FENCE.

Berple would have to build the fence while Claudius slept. But this was such a long fence that he would never get it built before Claudius woke up.

CAN YOU HELP BERPLE?

Claudius, Berple, and jelly bean trees should be arranged as in the pattern shown in Picture 7, but without the fence. Ask the children to put up a fence for Berple that will keep Claudius away from him but still allow Berple to eat jelly beans from all the bushes in Bugland.

Let the children discuss the variety of ways in which they might build a fence. Lead the children to suggest building a fence around Claudius. This, of course, allows Berple the freedom of eating from all the bushes but keeps him safe from Claudius. Let a child place the fence around the figure of Claudius on the flannel board.

And that's just the kind of fence that Berple built! Berple was very happy. He was safe from Claudius! He had all the jelly beans he could ever need or want! He felt so good about it that every night he threw two jelly beans over the fence for Claudius to eat.
SECTION 3  REGIONS AND BOUNDARIES

PURPOSE

- To give children more practice in sorting curves according to certain properties.
- To help children recognize and identify the regions and boundaries formed by closed curves.
- To give children practice in changing the shape of a curve without changing its type.

COMMENTARY

Section 3 reviews and extends some of the concepts of the first two sections of this unit, especially the ideas of regions and boundaries. The children also change the shapes of curves without changing the types of curves. These five lessons should take no longer than two weeks and can be presented after Unit 5, Introducing Measurement.

In Lesson 11 the children use the concept tree to sort curves into four subsets: simple open, simple closed, non-simple open and non-simple closed.

In Lesson 12 the children discover that closed curves are boundaries that partition a plane into more than one region. They see that with a closed curve the mouse can be safe if he is sitting in a different region from the cat. An open curve — no matter how small the opening — does not separate a plane into more than one region. An open curve does not form a boundary. The children know that the cat can catch the mouse without crossing the open curve.

In Lesson 13 the children play Claudius and Berple with non-simple closed curves. They discover that Berple does not need all of the closed non-simple curve to be safe from Claudius. The children then cut off the parts of the curve Berple does not need.

The children make curves from yarn or clay in Lesson 14. Then they change the shapes of their curves without changing the types of curves. In another activity they hold hands to form
a simple closed curve and then change the shape of the curve by moving to different positions.

Lesson 15 includes two games with regions and boundaries. The children can play these any time after Lesson 12.
Lesson 11: SORTING CURVES BY TWO PROPERTIES

The children again work with the concept tree, this time sorting simple and non-simple curves. When the children are proficient in using the tree, extend the branches to allow for classification by two properties into four subsets: simple open, simple closed, non-simple open and non-simple closed.

Kindergarten children sometimes have trouble working with two properties. They tend to concentrate on the last property named and forget, or don't even hear, the first. It will help if you occasionally change the order of the words when naming properties, such as saying "open simple curve" or "closed non-simple curve."

MATERIALS

- large sheet of newsprint
- several small cards to label branches of concept tree
  -- for each pair of children --
- concept tree drawn on an 18" x 24" sheet of newsprint
  -- for each child --
- sheet of unlined paper
- crayons
- scissors

PROCEDURE

Activity A

1. Before class begins, draw a 2-branch concept tree on a large sheet of newsprint, leaving room for the branches to be extended as shown on the next page. Label the branches "simple" and "non-simple." Also have ready labels for open and closed branches. It will be helpful if you draw an example of the curve on the branch label. Lay the tree on the floor for all the children to see.
2. Have each child fold a sheet of paper in fourths, draw a curve in each space, and cut apart the curves to form a deck of four cards.

3. Let a child pick one of his curves and bring it to the tree. Tell him to stand at the starting position and begin "climbing" the tree. To move his curve up the tree he must determine if it has any touch points. As he moves, ask him to tell the class what he is doing. He might say, "I have a simple curve because it has no touch points." Let each child bring one or two curves up the tree.

4. Now change the labels on the branches to open and closed. Have the children look at each curve to determine if the mouse can be safe from the cat and then move the curves up the correct branches. (Save the tree for Activity B.)

Activity B

1. Have the children look at the concept tree from Activity A, and tell them you are going to end the branches. Draw two more branches off each original branch, label them as shown on the next page, and draw a sample curve on each label.

2. Ask one child to bring a curve to the starting position and begin moving it up the tree. You may need to help him. Tell him to first decide if it is simple or non-simple, depending on whether it has any touch points. When he has decided, he moves up the correct branch. Then he determines if it is open or closed, according to whether or not the mouse can be safe from the cat. Then he moves up the correct branch.
3. Let another child bring a curve to the tree and begin climbing. He must follow the correct branches. The examples show the correct paths for four different curves. As the child moves his curves, ask him to tell the class what he is doing. He might say, "I have a simple, open curve because it has no touch points and the mouse cannot be safe from the cat." Give the children a chance to practice talking about their curves in this manner. You may need to help them until they understand what they should say about the curve.

4. Have each child move one curve up the tree, explaining what he is doing as he moves along. As each child reaches the top and has brought the curve up the correct branches, tell him to lay it down by the top branch. When each child has correctly brought one curve up the tree, look at the curves to determine whether there is any type of curve that has not been brought up the tree. If so, have the children draw a few examples of that type.
5. Each pair or small group of children should have a 4-branch concept tree drawn on an 18" x 24" sheet of newsprint or construction paper. Before handing out the trees, label the branches and draw an example of that curve on each label.

6. The children move their curves, one at a time, up the tree and place them by the top branch. After all the curves have been moved up the trees, tell the children to see if there are more of one type of curve than another. If some curves have not been made, have the children draw them.

7. If the children need more practice, have them exchange curves and move these up a tree drawn on the chalkboard or up one large tree made with yarn on the floor.

Activity C

Have the children make new cars for the curve trains they made in Section I. Label the engines: simple open, simple closed, non-simple open, and non-simple closed. You may want to draw a sample curve on each engine.
Lesson 12: REGIONS AND BOUNDARIES

In this lesson the children see that a closed curve forms a boundary partitioning a plane into distinct regions. They already have some familiarity with this concept from the cat and mouse activities in Section 1. Now they will begin to talk about a closed curve as being a boundary between regions.

The concepts of boundary and region should be familiar to the children, although they may not have used these words. At home they may have a fence all around the yard. The fence is a boundary that separates the yard from the street, the neighbors' yards, and perhaps mother's flower garden. The children cannot leave the yard without opening the gate or climbing over the fence.

To avoid confusion do not call the regions formed by a closed curve "inside" or "outside." The letter "O" partitions a plane into two regions. The letter "C" does not partition a plane; the plane is thought of as being one region. When we call the regions formed by the letter "O" inside and outside regions, the children become confused. They also want to name inside and outside regions for the letter "C," when there is actually only one region. A simple open curve (C) is not a boundary and does not separate a plane into more than one region; therefore, there is no inside or outside with a simple open curve. Encourage the children to call the regions by names such as, the cat region, the blue region, the mouse region.

MATERIALS

- 20-foot piece of rope
- several colors of construction paper
  -- for each child --
- crayons
- 3 pieces of paper, about 4" x 6"
- Plasticine, yarn, rope or clay
- small object (eraser, paper clip, toy car)
- 4-foot piece of yarn with ends taped together
PROCEDURE

Activity A

1. Give the children each two small pieces of paper. Have them draw (with a black crayon) an open curve on one piece and a closed curve on the other. Quickly review the cat and mouse activity from Section 1, in which the mouse had several friends who also wanted to hide from the cat. (The cat cannot cross a curve.)

2. On the paper with the closed curve the children should mark a spot for the cat and then color blue all the space where the mice could sit and be safe from the cat. Then they color red all the space where the mice would not be safe.

   THE PART OF YOUR PAPER THAT YOU COLORED BLUE IS CALLED A REGION. ALL THE MICE ARE SAFE AS LONG AS THEY SIT IN THE BLUE REGION. WE CAN CALL THIS THE MOUSE REGION.

   THE PART YOU COLORED RED IS ALSO CALLED A REGION. THE MICE CANNOT BE SAFE IF THEY SIT IN THE RED REGION. WHAT SHOULD WE CALL THE RED REGION? (The cat region.)

   HOW MANY REGIONS ARE THERE? (Two.)

   A CLOSED CURVE IS A BOUNDARY BETWEEN REGIONS. (Discuss the meaning of the word, "boundary.")

3. On the paper with the open curve the children should again color red the place where the mice would not be safe.

   THE PART OF YOUR PAPER THAT YOU COLORED RED IS CALLED A REGION. HOW MUCH OF YOUR PAPER IS COLORED RED? (All of it.)

   HOW MANY REGIONS ARE THERE? (Only one.)

   AN OPEN CURVE DOES NOT FORM A BOUNDARY.
4. On the chalkboard draw the letters O and C. Ask the children if the curves are open or closed, how many regions there are, and if the curves are boundaries.

5. Give each child another piece of paper, and then draw a figure 8 on the chalkboard. The children should identify this as a non-simple closed curve, and then copy it on their paper.

6. Tell the class that we now have a mouse, a cat, and a dog. The dog wants to catch the cat; the cat wants to run away from the dog and catch the mouse; the mouse wants to run away from the cat and the dog.

   Ask:

   WHERE CAN WE PUT THE DOG, THE CAT, AND THE MOUSE SO THAT THEY CANNOT CATCH EACH OTHER?

7. After the children have selected the regions for each animal, have them color the mouse region blue, the cat region red, and the dog region yellow.

   NOW EACH ANIMAL IS SAFE. HOW MANY REGIONS ARE THERE? (Three.)

   WHAT IS THE BOUNDARY BETWEEN THE REGIONS? (The closed curve.)

Activity B

Represent open and closed curves with "snakes" of Plasticine, yarn or rope, and place them on paper on a table or the floor. Place a small object near the curve. Choose two points in the plane and ask the children if they can move the object from one to the other without crossing the curve or lifting the object. Ask:

   IS THERE MORE THAN ONE REGION? IS THE CURVE A BOUNDARY? IS THE CURVE OPEN OR CLOSED?

Activity C

1. Give each child a four-foot piece of yarn with the ends taped together. The children should identify it as a...
simple closed curve. Have them lay the yarn on the floor, and ask:

HOW MANY REGIONS DO YOU HAVE? (Two.)

WHAT IS THE BOUNDARY BETWEEN THE REGIONS? (The yarn; the closed curve.)

2. Tell each child to stand with both feet in one region.

NOW YOU ARE IN ONE REGION. WHAT DO YOU HAVE TO DO TO GET TO THE OTHER REGION? (We must cross the curve.)

Remind the children that to step over the yarn is considered crossing the curve.

3. On the chalkboard draw the curve shown below. Mark an X and Y as indicated. Have the children use their yarn (they will have to untape the ends) to copy your curve. You may also want to make a curve, marked with an X and Y, on the floor. Ask:

CAN YOU GET FROM POINT X TO POINT Y WITHOUT CROSSING THE CURVE? (Yes.)

SHOW US HOW.

IS THERE MORE THAN ONE REGION? (No.) IS THE CURVE A BOUNDARY? (No.)

4. Ask the same questions for other curves, especially curves that are almost closed. Emphasize that if there is even a very small opening, the curve is not closed, is not a boundary and does not partition the plane into more than one region.
Activity D

1. With a long rope, make this curve on the floor.

   ![Diagram showing curved rope with regions labeled Green, Red, Orange, Yellow, Blue, Black, Purple.]

   Put a piece of construction paper of a different color in each region, including the floor surface region.

2. Tell the children they are going to play "Simon Says." Choosing one child each time, say:

   SIMON SAYS STAND WITH BOTH FEET IN THE BLUE REGION.

   SIMON SAYS STAND WITH BOTH FEET IN THE GREEN REGION.

   SIMON SAYS STAND WITH ONE FOOT IN THE ORANGE REGION AND ONE FOOT IN THE BLUE REGION.

3. Working in groups of three or four, have the children make similar curves with one long piece of yarn. One child is Simon; the others must follow his directions.
Activity E

1. With chalk, mark a starting point and a finishing point on the floor. Between these two points draw several simple closed curves. Tell the students that these simple closed curves represent stones. These stones are surrounded by water. The children are to walk from the starting point to the finishing point without getting their feet wet.

2. Now tell the students that things have changed. The simple closed curves now represent puddles of water. The rest of the floor is now land. They are to walk from start to finish without getting their feet wet.

3. Now have two children begin their journey at the same time. One child may be a person walking in such a way as to keep his feet dry, and another may be a frog jumping from puddle to puddle.

4. Tell the children the closed curves represent puddles and the floor represents land. Choose a child to walk from start to finish, keeping his left foot dry and his right foot wet.
Lesson 13: THE NON-SIMPLE CLOSED CURVE AS A BOUNDARY

This lesson further illustrates that a closed curve forms a boundary, and that this boundary partitions a plane into regions. The children discover that only part of a non-simple closed curve actually is the boundary which partitions the plane into regions. They do this by determining which part of the curve the mouse needs to keep him safe, and which part he does not need.

MATERIALS

-- for the class --

- ball of string or yarn

-- for each child --

- sheet of unlined paper
- red and blue color crayons
- several small pieces of paper
- Worksheet 5

PROCEDURE

Activity A

1. Make two simple-closed curves and two non-simple closed curves with yarn on the floor. Tape the ends of the yarn so that the curves are not open. Tell the children they are going to play the cat and mouse game with Claudius and Berple.

2. Choose one pair of children to be Claudius and Berple for each curve. Tell the Claudius in each pair to pick a spot near the curve where he would like to stand. Then tell each Berple to pick his spot, realizing that Claudius wants to catch him.

3. Pointing to the closed simple curves, ask:

   CAN BERPLE BE SAFE FROM CLAUDIUS? WHAT WOULD
CLAUDIUS HAVE TO DO TO CATCH BERPLE? (He would have to cross the curve.)

Ask the same for the closed non-simple curves.

4. Pointing to one closed simple curve at a time, ask:

WHAT PART OF THAT CURVE DOES BERPLE NEED TO KEEP HIM SAFE? (All of it.)

WHAT CAN WE CALL THIS CLOSED SIMPLE CURVE? (A boundary.)

WHAT WOULD HAPPEN IF WE CUT PART OF IT OFF? (Claudius could catch Berple.)

5. Next point to one of the non-simple closed curves and ask:

WHAT PART OF THAT CURVE DOES BERPLE NEED TO KEEP HIM SAFE?

The children may not immediately realize that Berple needs only part of the curve — that part which actually bounds the region he is in. Lead them to suggest part or parts of the curve that can be cut off. When a child discovers the part that can be cut off, let him cut it.
Then ask:

**IS THE CURVE STILL A BOUNDARY? IS IT CLOSED?**
(Yes.)

6. Ask the same questions for the other non-simple closed curve. If the children have difficulty, make a few more non-simple closed curves and let them cut off the parts that are not needed. Discuss with them that only part of a non-simple closed curve actually is the boundary for the regions. Ask:

**WHEN WE CUT OFF THE PARTS OF THE CLOSED NON-SIMPLE CURVE THAT BERPLE DOES NOT NEED, WHAT TYPE OF CURVE DO WE HAVE?** (A simple closed curve.)

Activity B

1. Have each child fold a sheet of unlined paper in quarters, thus providing four blank spaces on each side of the paper.

2. Make a large non-simple closed curve with yarn on the floor for the children to see.

3. Then have the students copy this curve in one of the boxes on their folded papers. After they have drawn the curve, ask them what type of curve it is. They must prove it is either simple or non-simple by whether it has a touch point. They must decide if it is open or closed by whether the mouse can be safe from the cat. Encourage them to place a dot on each touch point, a red dot where they want to place the cat and a blue dot where they want to place the mouse.
4. Tell the children the following:

ONE DAY WHEN THE MOUSE WAS HIDING FROM THE CAT HE DISCOVERED SOMETHING. THE MOUSE LOOKED AT HIS CURVE AND FOUND OUT THE SAME THING THAT BERPLE LEARNED IN OUR LAST ACTIVITY. HIS CURVE LOOKED JUST LIKE THE ONE YOU HAVE DRAWN ON YOUR PAPERS. WHAT DO YOU THINK THE MOUSE DISCOVERED ABOUT HIS NON-SIMPLE CLOSED CURVE?

Lead the children to discuss that the mouse, like Berple, needs only a part of his curve to be safe from the cat.

5. Then ask:

IF YOU WERE THE MOUSE, WHAT PART OF THE CURVE COULD YOU CUT OFF AND STILL BE SAFE?

Tell the students to color red the part of the curve that could be cut off. Then ask:

WHAT PART OF THE CURVE DO YOU NEED TO KEEP SAFE FROM THE CAT?

Then have the children color blue the part they need to be safe. Ask:

IS THE BLUE PART A BOUNDARY? (Yes.)

HOW MANY REGIONS ARE THERE? (Two.)

6. Have the children draw some more curves on small pieces of paper. Tell them to color the part of the curve the mouse needs to be safe in one color, and the part he does not need in another color.
Activity C

On Worksheet 5 the students play the cat and mouse game. Have the children cut apart the four curves. They should determine what part of the curve is necessary to keep the mouse safe and then cut off the parts of the curve not necessary for the mouse to be safe. After a part of a curve has been cut off, the children should no longer consider this as belonging to the curve. In this way, they should realize that all closed curves (simple and non-simple) form boundaries that partition a plane into more than one distinct region.
Lesson 14: CHANGING SHAPES

The children use yarn or clay and hand-holding to make curves in this lesson. Then they learn how to change the shapes of the curves without changing the kinds of curves they originally had.

MATERIALS

- felt-tip pen
- balloons

--- for each child ---

- clay or 4-foot length of yarn
PROCEDURE

Activity A

1. Have each child make a "snake" of clay or with a four-foot length of yarn on the floor or table. Ask everyone to make an open simple curve. Suggest that they try to make fancy ones that are nevertheless simple. Curves will differ in intricacy and design.

2. Choose two different-looking curves and ask:

   WHAT KIND OF CURVES ARE THESE? (Simple open curves.)

   ARE THEY BOTH THE SAME? HOW ARE THEY DIFFERENT? (They have different shapes.)

3. Change the shape of one curve and ask if it is still a simple open curve. Then have the child whose curve you changed, change it to look like the other child's curve. Ask:

   ARE THESE STILL OPEN SIMPLE CURVES? (Yes.)

4. Have the children change their curves to look like those of their classmates. In this way they should see the similarity among simple open curves even when the curves vary in intricacy and design.

5. Have the children make closed curves, compare them, and change them to look like a friend's curve. Then ask:

   CAN YOU CHANGE YOUR CURVE SO THAT IT LOOKS LIKE THE OUTLINE OF A VALENTINE? A PUMPKIN? AN APPLE?

Activity B

1. Ask the children to stand in a circle holding hands. Have them notice that they represent a simple closed curve. Two children who face each other should take two or three steps forward, while all children continue holding hands.

   WHAT DO WE CALL THIS CURVE? (A simple closed curve.)
IS IT JUST LIKE THE FIRST CURVE? HOW IS IT DIFFERENT? (It has a different shape.)

WE CHANGED ITS SHAPE, BUT IS IT STILL A SIMPLE CLOSED CURVE? (Yes.)

IS IT STILL A BOUNDARY? (Yes.)

HOW MANY REGIONS ARE THERE? (Two.)

Activity C

With a felt-tip pen draw a complicated simple open curve on a deflated balloon. Have the children identify the curve. Ask them to predict or guess what will happen to the curve when you blow up the balloon. Blow it up and ask the children to decide whether or not their predictions were correct. Help the children observe that the curve is a simple open curve both when the balloon is deflated and when it is inflated. Follow this procedure for simple closed curves and for several non-simple open and closed curves.
Lesson 15: CURVES AND REGIONS (GAMES)

These two games can be used any time the children are outdoors or in the gymnasium. This lesson completes Section 3. The last part of the unit, Section 4, can be used after Unit 6, Numeration.

Activity A: Bugs and Fences (Game)

This is played like the game known as "Squirrels in the Hollow Tree." About two-thirds of the children are arranged in pairs holding hands as for "London Bridge." By holding hands, each pair creates a boundary for a fenced region. They stand still during the game. The remaining children are bugs and will try to get to a fenced region. There should be one or two more bugs than fences. When you say "Bugeater," each bug tries to get to a safe region, but only one bug is allowed on each. Say "Bugeater" again. Each bug has to leave his fence and...
try to get to another one. (There will always be one or two "bugs" without fences.) After you have said "Bugeater" several times, have each bug within a boundary change places with one member of his fence. Repeat the game until each child has at least one chance to be a bug.

Activity B: Bugeater Tag (Game)

One child, called Bugeater, is "It." Half the class are bugs, half fenceposts. The children move around. "It" can move too. At the call, "Bugeater," all the bugs stop and crouch. The fence posts hurry to join hands. The shape and the number of fence posts in each of the closed curves doesn't matter, but the fences must enclose at least one bug. After a few seconds, call "Stop!" The bugeater taps each child who has not been enclosed by some fence. Tapped children become fence posts. Enclosed bugs are safe. Repeat. The game ends when all are fence posts, or all bugs are safe. A new "It" is chosen for another round.
SECTION 4  MORE WORK WITH CURVES

PURPOSE

- To give the children more practice in sorting and classifying curves.

- To help children recognize certain properties of simple closed curves, including circles, triangles, squares and other rectangles.

- To help children identify curves formed by objects and edges of objects.

- To give children experience in working with maps and models.

- To review many of the concepts presented in the first three sections of this unit.

COMMENTARY

In Lesson 16 the children transform one type of curve into different shapes without changing the type of curve. A game provides review of sorting curves with the concept tree. In the last activity each child labels the branches of his own concept tree in such a way that he can move any set of curves up the tree to the top branches.

Lesson 17 focuses on simple closed curves. The children study the similarities of all simple closed curves, including some that have been given special names: circles, triangles, squares, and other rectangles. The children sort the subset of simple closed curves according to a new property: the number of line segments that form each curve.

In Lesson 18 the children examine the edges of objects to locate and name the curves that can be traced.

Lesson 19 includes several activities that can be used to review concepts such as closed curves, boundaries, regions, edges of objects as curves, and shapes of objects.
Lesson 20 is the last lesson of the unit. The title lists it as optional, but this does not necessarily mean that you should omit it. If time permits and the children seem interested, let them try one of the map-making activities. In the first activity the children work with property blocks, putting them together to form many shapes. In another activity the children make a model of their classroom with property blocks.
Lesson 16: HOW TO SORT AND CHANGE CURVES (Review)

This lesson reviews the use of the concept tree to sort curves. The children take one type of curve and change its shape in many ways without changing the type. In the last activity, the children label the branches of their concept trees. They may need considerable help with this activity because of the need for word-recognition.

MATERIALS

-- for each child --
- 4-foot piece of yarn
- 3 small pieces of paper
- crayon
- 18" x 24" piece of newsprint
- Worksheet 6

-- for the class --
- several concept trees made with masking tape, chalk, or on newsprint

PROCEDURE

Activity A

1. Arrange the students in a large circle on the floor, and give each a four-foot piece of yarn.

2. Begin by asking all children to make simple open curves with their yarn. Then have the children form pairs and walk around the room, studying each curve to determine if all are simple open curves.

3. Have the children return to their own curves and change them so that the curves have new appearances but are still simple open curves. Tell the children that they can push or pull the curves along the floor, but must not lift them from the floor.
4. Now have the students walk around the room, investigating these curves. Encourage them to verbalize about these curves.

5. After the students have transformed several open simple curves into other open simple curves, have them try the activity with simple closed, non-simple open, and non-simple closed curves. The children should work in pairs with two pieces of yarn to make a non-simple open curve.

6. Each time the children change their curves, ask:

   DID YOU CHANGE THE APPEARANCE OF THE CURVE? (Yes.)

   DID YOU CHANGE THE CURVE TO A NEW TYPE OF CURVE? (No.)

For closed curves ask:

   IS THE CURVE STILL CLOSED? IS IT STILL A BOUNDARY? (Yes.)

   HOW MANY REGIONS ARE THERE?

Activity B: "Branch Tops" (Game)

1. Before class begins, use masking tape or chalk to mark several concept trees on the floor (or draw the trees on large sheets of newsprint or cardboard). All the branches of each tree should be named. (Some teachers like to place sample curves next to the names attached to each branch.)

2. Tell the students that they are going to play a game called "Branch Tops." Give each child two or three small pieces of paper and a crayon. Divide the class into groups of seven or eight.

3. Have each group take the paper and crayons with them as they line up at the starting position of one of the trees.
4. The first four students, one at a time, climb to the top of the tree. They do not draw any curves yet. As they walk they must tell the rest of the team where they are going. A child might say, "I'm going up the simple closed curve branch," as he walks. One child goes to each branch top.

5. The next child in line, the challenger, must study the tree closely and tell his team whom he is going to knock from the top branch. Then he draws a curve on one of his pieces of paper, and walks this curve up the tree. If he has drawn the correct curve, he gets to stay at that branch top. The person he knocked off goes to the end of the line. If he drew the wrong curve, he must go back to the end of the challenger's line.

6. All the curves that get to the top of the tree should be spread out near the correct branch.

7. Sometimes a child might want to challenge the challenger. When the challenger reaches the top branch, the student there might say, "What type of curve do you have? Prove it." Then the child who drew the curve must prove he knows his curve by saying, "Here is a touch point, so the curve is non-simple. The curve is closed because the mouse can be safe from the cat. This is a non-simple
closed curve." If the child who drew the curve cannot prove what type of curve he has, he must go back to the end of the challenger's line.

Activity C

This activity may be difficult for some children. Let all the children try it even though they may not succeed the first time. Read through the activity, especially the note on the next page, before presenting it to the children.

1. Give each child an 18" x 24" sheet of newsprint. On the chalkboard draw a concept tree and then have each child draw a tree on his paper.

2. Give each child a copy of Worksheet 6 and have each cut apart the cards. Some practice with recognition of the words on the cards will help prepare the children for reading. Encourage any child who can recognize the words to use the cards as they are. Those children who cannot recognize the words can draw an example of the curve on each card.

3. Have each child fold a sheet of paper in fourths, draw a curve on each piece, and cut these apart.

4. Each child takes his four curves and eight name cards to his tree. The object of this activity is to move the curves to the top branches of the tree. The children must label the branches themselves in such a way that every curve can be moved up the tree.
5. Have the children name each branch as they move their curves up the tree. The name of the branch is the same as the name of a property of a curve: simple, non-simple, open and closed. In some cases curves may not reach the top of the tree because the children have not labeled the branches correctly. If this happens, encourage the child to try changing the names of the branches. Let him discover how the names of the branches should be arranged to allow all the curves to reach the top.

NOTE: There are six branches on each tree and each child has eight cards. The children should notice that they will not need all the name cards provided, but do not tell them this.

The children should discover that the names they assign to the first two branches must allow them to separate their set of curves into two subsets (either simple and non-simple or open and closed). The next names they assign must allow them to partition each of these subsets into two more subsets.

The route each curve takes will depend on how each child labeled his branches. Do not discourage any arrangements the children make with the name cards, even though some may be wrong. Instead, encourage the children to keep moving the curves up the tree to determine whether the labels are in the correct places.
6. Even if a child labels his branches correctly and gets all the curves to the top of the tree, have him change the names of the branches to see what happens.

7. Save the branch label cards to be used again in Lesson 17.
Lesson 17: SORTING THE SET OF CLOSED SIMPLE CURVES

In this lesson the children extend the closed simple curve branch of the concept tree. They partition the set of simple closed curves into subsets according to the number of line segments used to form the curve. "Circle," "rectangle," "square," and "triangle" are some of the names assigned to these subsets of simple closed curves.

MATERIALS

- pipe cleaners
- rope or yarn
- labels for concept tree branches (from Lesson 16)

PROCEDURE

Activity A

1. Before class begins, make several simple closed curves from pipe cleaners, including at least two circles, two squares, two rectangles and two triangles. Also have ready a large concept tree taped to the floor. The branches should be labeled.
2. Give the pipe cleaner curves to several children and let them walk these curves up the correct branches of the tree. The children should soon notice that all the curves are simple closed curves.

3. Ask the children if the concept tree can be extended so that the set of simple closed curves could be partitioned into more subsets. Some children may suggest sorting the curves into subsets of those with special names, such as square and circle. If so, let them try it and when they are done ask what to do with the remaining curves that do not have special names.

4. Tell the children there is another way to sort these curves into subsets. Let the children study the curves and tell you specific properties of certain curves. The number of line segments which make up each curve is another property. If no one thinks of this, suggest that they sort the curves according to the number of line segments that make up each, such as the triangle which is made up of three line segments.

5. Save one set of branch labels to be used again in Lesson 18.
NOTE: The names "circle" and "triangle" are easily applied to two of these subsets of closed simple curves. All simple closed curves made up of three line segments are triangles. A simple closed curve made up of an infinite number of line segments is a circle. (The children do not have to know this definition.)

The subset of curves made up of four line segments includes both squares and rectangles which have the common property of all angles are right angles. If the children seem interested ask them to look closely at the subset of curves made up of four line segments and tell you how the square and the rectangle differ from the other curves and from each other. The square has a property which the rectangle does not have: all its sides are equivalent in length. The diagram below shows how the subset of simple closed curves made up of four line segments could be partitioned into subsets. You should go into this much detail only if the children show an interest.
Lesson 18: EDGES OF OBJECTS AS CURVES

In this lesson the children look for curves formed by edges of objects. They examine property blocks and other things in the classroom.

MATERIALS

- round container
- property blocks
- rectangular objects, such as boxes
- oatmeal box (optional)
- cone-shaped paper cup (optional)
- concept tree and branch labels from Lesson 17

PROCEDURE

Activity A

Show the children a round container (the oatmeal box if you have one). Have a child run his finger around the rim of the container. Ask him to name the curve he traced. Place the top of the container against the chalkboard and trace the rim with chalk. Have another child trace the curve of chalk with his finger, and tell you the name of that curve. This procedure should be followed with square and triangular property blocks and with rectangular objects such as a shoebox. Be sure that the children see that the chalk curves and the curves they traced with their fingers around solid objects have the same form.

A variation of this activity involves cutting a cylindrical cardboard container (such as an oatmeal box with its lid) down the middle vertically. Trace the rectangular curve formed by the cut edge of one of the halves with your finger. Now place the cut edge against the chalkboard, and trace the rectangular shape with chalk. The children may be surprised to find a rectangular shape as the cross-section of a cylinder. Ask the children to predict the shape of the cone-shaped paper cup if cut vertically in half. Cut the cup in half and trace its outline on the chalkboard. The shape of the cup resembles a triangle.
Activity B

Ask the children to make a stack of triangular-shaped property blocks. Have them trace with their finger around the stack horizontally at any height. They should be able to identify the shape they traced. (Triangular.)

Continue:

I can think of a way to trace around this stack and come out with a rectangle. Who can show me where it is by tracing it?
A similar procedure should be followed for the square property blocks. The vertical surface may be square or rectangular, while the horizontal surface is square.

Activity C

Have each child choose some object in the classroom. If he can find the shape of a closed simple curve on that object, have him move the object up the concept tree from Lesson 17. Before he moves it up the tree, he should trace the curve with his finger to show the other children which curve he is using.
Lesson 19: REVIEW ACTIVITIES WITH CURVES AND SHAPES

The first activity of this lesson shows the children that the curves with special names — circle, square, rectangle, and triangle — are boundaries that divide a plane into more than one region just as all closed simple curves do. The other activities can be used for reinforcement of concepts presented in this section.

MATERIALS

— construction sets, such as Tinkertoys (optional)
— several colors of construction paper

— 1 foot piece of yarn (most with ends taped together and a few with loose ends)
— sheet of construction paper
— crayon
— glue
— 2 small pieces of paper

PROCEDURE

Activity A:

1. Give each child a piece of yarn and a piece of construction paper. Draw these curves on the chalkboard.

2. Divide the children into eight groups and have each group copy one of the curves with yarn on their paper. The children who have yarn with loose ends should make the non-simple closed curves. It will probably be easier for the children to make an outline of the curve in glue first, and then lay the yarn down on the glue.
3. Tell the children that the curves they have made represent fences. The fence separates horses from people. On the chalkboard draw a simple outline of a horse. The children can copy this on a small piece of paper and then draw a man on another piece of paper. Tell the children that there is a horse region and a people region at this ranch. Each must stay in his own region. Have the children put the horse in one region and the man in the other region.

4. As the children compare their ranches, they will see that some children put the horse in the region bounded by the curve and some put it in the other region. Ask a child from each group to explain why his curve separates the people from the horses.

5. After each group has explained its curve, ask:

WHAT KINDS OF CURVES KEEP THE HORSES AWAY FROM THE PEOPLE? (Closed curves, including the four special ones.)

WHAT KINDS OF CURVES KEEP THE PEOPLE AWAY FROM THE HORSES? (Closed curves.)

ALL CLOSED CURVES—NON-SIMPLE AND SIMPLE, INCLUDING THE SPECIAL CURVES—KEEP THE HORSES AND PEOPLE AWAY FROM EACH OTHER.

Activity B

Either you or the children should select a piece of furniture and attempt to describe it in terms of the shapes you find. For example, the outline of the back of an upright piano is a rectangle and the tops of the black keys have rectangular shapes. The music rack may form a triangle with other parts of the piano. Always have the children trace the boundary with a finger to focus attention on the curve.

Activity C

You may want to plan Special Curve Days. On Circle Day, for example, children should be urged to keep their eyes open for circles in the classroom and on the playground. When a child finds a circle, he should show it to you and trace it with his finger. You might hide a new record, a bag of indi-
vidually-wrapped round hard candies, a ringtoss game, or a box of round cookies with the picture on the wrapper. Then tell the children there is a hidden circular-shaped surprise in the room, and that when they find it, they are to bring it to you. You may want to have Square, Rectangle, and Triangle Days as well as Circle Days.

Activity D

If construction sets, such as Tinkertoys, are available, have the children build structures with them. Ask them to discover — and trace with their fingers — curves that are found on the structures.

Activity E

Make several traffic signs from construction paper to hang up around the room. Discuss the shapes of these signs, noting the number of line segments which make up each curve that forms the shape of the sign. This activity presents an opportunity to discuss the meaning of the signs and to remind the children of traffic safety rules.
Lesson 20: SOME OPTIONAL GAMES AND ACTIVITIES

The activities of this lesson can be used in any way you wish. Some of them do not require much time and can be combined with activities in other subjects or used during playtime.

MATERIALS

- property blocks
- furniture in the classroom
- globe
- maps

PROCEDURE

Activity A: Property Blocks

The children can experiment with property blocks, objects in the classroom or shapes cut out of construction paper to discover that different combinations make a variety of shapes. Some examples are:

Two square shapes can be combined to make a rectangular shape.

A certain number of rectangular shapes combine to make a square shape.
Two equilateral triangles can be put together to make a diamond.

Two isosceles right triangles combine to make a square. Parquetry blocks may be available in these shapes. (The property blocks provided are equilateral triangles.) Or, you can cut the shapes from construction paper or have the children examine a floor tile to see how two triangles can form a square.

Four small equilateral triangles combine to form a larger triangle.
Activity B: Model Building

1. Make a curve which represents the classroom by laying a large closed curve of yarn on the floor and by asking the children to put corners in the curve so that it has the same outline as the classroom floor. Ask questions to be certain that children recognize that the simple closed curve represents the boundaries of the classroom floor. "Where are the edges of the floor in this model?" "Where is the boundary between our room and the hallway or outdoors?" "How many regions does this curve make?"

2. Ask the children how they can show some of the furniture in the classroom. Suggest using property blocks. Watch and assist as the children decide what size and shape blocks should represent the piano, your desk, their tables and other items. A possible motivation for making the model is a class discussion of ways to describe the classroom to their parents, brothers, or sisters.

3. After the children have made the model, suggest that they plan a new arrangement for the classroom. Have them move the blocks around and encourage them to consider the wisdom of the new positions. Will they be able to get the books from the bookcase? Will any furniture block the door or windows? Are too many pieces of furniture in one part of the room and not enough in another? When they have agreed upon an arrangement, let them try it in the actual room.
Activity C: Maps

Some time during the kindergarten year the children probably talked about and used the globe and maps. Now they may enjoy making some simple maps of their own. When using a map with kindergarten children, it should be laid on a table or floor, rather than hung on the wall. Also be sure to lay it down so that the north direction on the map is actually facing north in your classroom. Explain these directions to the children, pointing out north, south, west and east.

If your group shows interest in working with the globe and locating places, you may want to bring in roadmaps. Lay these on the floor and indicate such features as roads, lakes, rivers and historical sites. Ask questions about the map:

HOW DO YOU KNOW WHERE THIS LAKE STOPS?

IS THIS LAKE A REGION? DOES IT HAVE A BOUNDARY?

IS THE BOUNDARY A CLOSED CURVE?

WHAT KIND OF CURVES DOES THIS HIGHWAY FORM?

The children might enjoy making a map showing the route for a field trip the class went on. Children who walk to school could draw a map of the route they take. Some children could make a map showing how to get to a pirate's buried treasure, making sure there are many obstacles (buildings, trees, lake, rivers) along the way.

In your discussions of maps, be sure to point out the many curves and shapes which make up the map.

Activity D: Maps (Game)

Have small groups of children draw maps of hidden objects in the classroom. Two children in the group can draw the map for the others in the group who then try to locate the item. Encourage the children to put in enough reference objects, such as desks, tables, windows and chairs, to aid the others in their search. The children who draw the map might like to hide some real object ahead of time.