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ABSTRACT

This report describes the results of preliminary work by the Human Resources Research Organization to develop a comprehensive theory for structuring subject matter. The report focuses on the first three of five components that any comprehensive model of instruction should include: (1) a representation of the subject matter to be taught; (2) a representation of the educational goal; (3) a representation of the initial or starting state of the student entering the system; (4) a representation of the current state of the student; and (5) a representation of the teaching system including teaching strategies. The instructional theory presented is axiomatic in nature--the primary concepts, axioms, definitions, and theorems are presented in order to clearly delineate the assumptions and the starting points of the theory itself. Within the axiomatically based structure, subject content and task components are distinguished. In order to specify tasks, the behavioral objectives of the instruction must be known. These objectives include terminal behavior, the specification of which has been referred to as the beginning of any design of programmed instruction. The notion of dependency between the content and task components is defined and its properties investigated. This relation is shown to be important because it restricts the order in which content components can be introduced during instruction. A comprehensive detailed example serves to illustrate the theoretical concepts, results, and procedures of this model for instruction. It is expected that this work will culminate in a comprehensive axiomatically based structure for individualized instruction. (MM)

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An Axiomatic Theory of Subject Matter Structure

John Stelzer and Edward H. Kingsley

HUMAN RESOURCES RESEARCH ORGANIZATION
300 North Washington Street • Alexandria, Virginia 22314

June 1974

Prepared for

U.S. Army Research Institute for Behavioral and Social Sciences
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This report can be viewed as a first step toward development of a formal theory of instruction. An axiomatic theory of subject matter structure was formulated, including both content and task components. The subject-matter content is analyzed in terms of constituents and their relations. Underlying the content structure is the task structure that is related to the content position via coordinating relations. The notion of dependency is introduced and investigated. Dependency leads to precedence in an instructional sense
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and this notion is also discussed. Formal procedures are developed that can be utilized to formulate the complete dependency relationship for particular subject matters. A comprehensive detailed example is developed that serves as a vehicle to illustrate the theoretical concepts, results, and procedures.

PROBLEM

The present work can be seen as being a first step toward the development of a formal theory of instruction. Any comprehensive model of instruction must include a representation of the subject matter to be taught.

Besides the subject matter representation, a representation of the state of the student at any point in time, and a representation of the goals of instruction are needed, at a minimum, to formulate a model of instruction. The development of these latter theoretical components is beyond the scope of this report. However, the theory presented below has the characteristic of being the basic building block on which other consistent, theoretical results can rest.

APPROACH

The theory of instruction to be presented herein is axiomatic in nature. This means that primary concepts, axioms, definitions, and theorems are presented in order to clearly delineate the assumptions and starting points of the theory itself from the derived results. The axiomatic method has a long and distinguished history dating back some 2,000 years to the time of Euclid. Axiomatic approaches are well suited for characterizing empirical phenomena. Instruction can be viewed as an empirical phenomena, in the sense that the process of instruction has developed over time in a trial-and-error, empirical manner. This being the case, the axiomatic method appeared to be well suited for the precise characterization of this phenomena.

RESULTING THEORY

An axiomatically based structure for subject matter is developed herein. Within this theory, subject content and task components are distinguished. Dependency between content constituents is defined and investigated as part of the theoretical framework being developed. The notion of dependency is crucial in sequencing instruction. That is,

dependency relations between content constituents can be seen to restrict the order in which content constituents can be introduced during instruction.

IMPLICATIONS

A comprehensive theory for structuring subject matter has been developed. The other components of a theory of instruction are still to be developed. This work has been started, and it is expected that the effort will culminate in a comprehensive, axiomatically based instructional theory.

PREFACE

This report describes the results of preliminary work by the Human Resources Research Organization to develop a comprehensive theory for structuring subject matter. The subject-matter structure that is formulated includes both content and task components. The notion of dependency between content components is defined and its properties investigated. This relation is shown to be important because it restricts the order in which content components can be introduced during instruction. The theory proposed for structuring subject matter is axiomatic in nature. A detailed example is given to illustrate the theoretical concepts, results, and procedures.

The present work is a first step toward developing a theoretical basis for individualized instruction.

The work was performed during April 1972-March 1973, by HumRRO Division No. 1 (System Operations), Alexandria, Virginia. Dr. J. Daniel Lyons is Director of the Division. Dr. Robert J. Seidel was Project Director, with Dr. John Stelzer and Mr. Edward H. Kingsley the principal investigators.

The work was begun under Army Contract DAHC 19-73-C-0004. Further development took place under National Science Foundation Grant GJ-774 and Air Force Contract No. F41609-73-C-0020.

Meredith P. Crawford
President
Human Resources Research Organization

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Chapter 1

INTRODUCTION

Pask (1) asserts that any comprehensive model of instruction should include five components:

(1) A representation of the subject matter to be taught.

(2) A representation of the educational goal.

(3) A representation of the initial or starting state of the student entering the system.

(4) A representation of the current student state, or the state of the student at any time.

(5) A representation of the teaching system including teaching strategies.

This paper focuses on the first of these components representing subject matter. In order for subject matter to be represented, it is clear that it must be structured. Accordingly, a general theory will be developed that provides the framework for structuring subject matter. A means of representing and describing such structures will also be given.

Briefly, the theory of subject matter structure presented herein is inspired by the example of axiomatic mathematics. This does not imply that every subject matter is reducible to a mathematical, let alone an axiomatic mathematical, equivalent. We simply believe that the axiomatic method itself provides an example of an approach that can be more generally applied to nonmathematical topics. This paradigm seems most reasonable since axiomatized theories, especially axiomatized mathematical theories, are subject matters that have been rigorously structured. Implicitly, to axiomatize a theory is to structure it, but the converse is not necessarily so.

Historically, attempts to characterize subject matter structures seem to fall into one of two classes. On the one hand, theories such as Gagne's (2, 3, 4) have focused on task structures. Gagne's formula, "What must one be able to do before . . . ?" leads to structures that focus on the tasks that a student must learn how to perform. Pask (1), on the other hand, has focused on the content of subject matter. Pask structures subject matter along the lines of the formula: "What must a person know before he can learn . . . ?"

It seems clear that an adequate theory of subject matter structure must include both task- and content-oriented components. Consider the problem of specifying tasks. In order to specify tasks, the behavioral objectives of the instruction must be known. These objectives include terminal behavior, the specification of which has been called "the beginning of any design for programmed instruction" (Tennyson and Boutwell, 5). Tuckman and Edwards (6) refer to the process of specifying behavioral objectives in terms of tasks or task analysis.

In general terms, the specification of terminal behavior, behavioral objectives, and so forth, can be done only with reference to the *context* of the instruction. That is, in order to specify the behavioral objectives and tasks, information concerning certain aspects of the instructional environment must be known. Relevant mediating considerations include the following: the target population, abilities that can be assumed on the part of the target population, the general purpose of the instruction, the general interests of the target population, and so forth. We refer to this aspect of instruction as the *Context* of the instruction.

In addition to the context, a general specification of the subject matter, *General Subject Matter*, must be given. In this sense the statement of the General Subject Matter begins to delineate the content of a discipline. A discipline can be denoted by general terms, such as the discipline called differential equations. In this case, the content of the discipline is large and varied. If a more specific discipline is being considered, such as the discipline called linear differential equations, the content is smaller and less varied. Although a General Subject Matter has some structure, the structure is difficult to define. As will be seen, it begins to take on more definite structure after other factors (the Context) are considered.

Applying the General Subject Matter to a specific Context results in a specification of the content of the instruction: the *Context Conditioned Subject Matter.* As an example, suppose the General Subject Matter is taken to be the content of an undergraduate first course in statistics. Then, the content of such a course might differ when given under the contexts associated with a group of English majors in contrast to a group of electrical engineering students. Such a course can thus be viewed in the context of the disciplines: mathematics, psychology, economics, business, engineering, biology, medicine, and so forth. Each of these disciplines modifies the content of the General Subject Matter "statistics" to make it relevant to the discipline and to the target population. In many instances, the differences in content result from using the terminology peculiar to a discipline in presenting examples and exercises. Other less trivial differences arise by

including special techniques useful in particular disciplines. Specifying contexts is thus seen to partially define subsets of the General Subject Matter. In a general way, the boundaries and content of these subsets begin to be identified.

So far, we have argued that the content of a General Subject Matter must be considered in contexts and never in isolation. A Context Conditioned Subject Matter begins to have structure if, and only if, its constituents are listed and the ways that they are interrelated are displayed. Structure always involves one or more relations. A mere listing of constituents as a class has no structure. Thus, to say a Context Conditioned Subject Matter has structure is first to identify and list its constituents, and second to specify the relations and their type between constituents.

Because the properties of constituents from a Context Conditioned Subject Matter are related to each other by one or more relations, it is useful to think in terms of graph theory and to depict the structure of a subject matter by graphs. The nodes of the graph correspond to constituents or to their properties, and the arcs of the graph represent relations among the nodes. Because several relations among the properties are possible, we follow Hohn, *et al.* (7) and call such a diagram a *net* instead of a graph. We thus define a *General Cognitive Net* (GCN)[1] as the graphic representation of the constituents and their relations from a Context Conditioned Subject Matter.

Given the General Subject Matter and Context of the instruction, it is also possible to specify the tasks to be performed as a result of the instruction: the *Context Conditioned Behavioral Objectives*. These objectives include both the enabling and terminal objectives of the instruction. The behavioral objectives are stated by explicitly defining the tasks that the instructor expects the students to perform, and the criteria for each task.

The set of tasks defined by the behavioral objectives are not totally unrelated to one another, and, thus, have structure. A student may have to demonstrate ability to perform certain previous tasks before he is permitted to undertake other tasks. For example, consider a mathematical course on Boolean Algebra. In such a course, a student might be expected to be able to perform to some criteria level of competence tasks concerning shading Venn Diagrams on the "union" operation before undertaking tasks on the "intersection" operation. In such cases, the respective tasks are related by a precedence relationship. A further relation can be defined between the subtasks of a task by

[1] Dr. Robert Seidel of HumRRO contributed significantly to the original versions of the content/task (GCN/TICS) analysis of subject matter structure.

including considerations of subtask complexity. Hence, the tasks are structured through relationships, and we call the task structure the *Task In Context Structure* (TICS).

This discussion is summarized in Figure 1. The six upper blocks in the diagram correspond to the six concepts described above. An arrow goes from one block to another block if, and only if, a change in the state of the first block has an immediate effect on the state of the second block. The diagram is related to Ashby's (8) "Diagram of Immediate Effects."

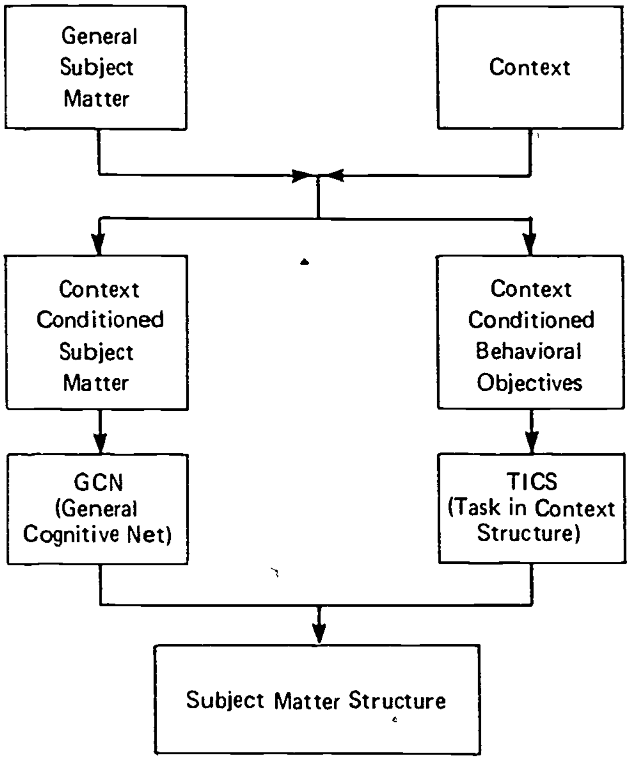

Figure 1. **Factors contributing to subject matter structure**

We have made a distinction between properties of constituents (the GCNs) and learning these properties, on the one hand, and tasks and problem solving (the TICS) on the other hand. In short, properties of constituents are defined independently of their utility in solving specific problems, while tasks refer to the groupings of properties of constituents into utilitarian (pragmatic) units to solve particular problems.

Using pictorial terms, visualize at least two planes in space, with the net-like structure of properties of constituents (the GCNs) drawn on one plane and the net-like

task structure (the TICS) on the other. Lines go from single elements of the GCNs to elements of the TICS. These lines depict the relation between the two structures, and their aggregate is termed the Coordinating Relations. We call the entire complex of nets and Coordinating Relations between the nets the *Subject Matter* structure. Both the cognitive structured understanding (obtained from the GCNs) and the applications (obtained from the TICS) are important for proper problem-solving behavior. Stated in another way, together they represent the necessary and sufficient conditions for solving problems pertinent to a given suitably restricted subject matter.

As an example, consider a programming language, such as COBOL, as the subject matter. Suppose it has been modified to reflect a context and a set of behavioral objectives. Then, to have had instruction solely on how various COBOL terms relate to one another without actually having written specific programs would result in one's "knowing" COBOL (understanding the relations among the terms), but not in one's being able to perform programming tasks adequately. On the other hand, performing program tasks without also obtaining instruction on the cognitive structure would probably lead to failure to perform as soon as some verbal categories were changed to the unfamiliar. Both skills involve information processing. The processing of the task-specific requirements results in many connections relating verbal (cognitive) representations to other specific, physical acts.

In the general setting of a model for individualized instruction, Figure 1 and the above discussion can be seen to be basically a prescriptive aspect of the model. Figure 1 summarizes guidelines to be followed in the development of subject matter structures. As such, it is not descriptive because it does not describe in detail the actual subject matter structure. In the next section the problem of developing the descriptive portion of the model for structuring subject matter is discussed.

Chapter 2

AXIOMATICALLY STRUCTURED SUBJECT MATTER

This chapter consists of the development of a theoretical model for structuring subject matter. Two examples of the theory are presented in the next chapter.

THE AXIOMATIC METHOD IN MATHEMATICS

The theory we propose for structuring the GCN parallels closely the axiomatic method in mathematics. The axiomatic method has a long and distinguished history beginning, at least, with Euclid some 2,000 years ago. As a discipline, in itself, axiomatics has only begun to be appreciated by mathematicians during this century. The pioneering efforts of mathematicians Russell and Whitehead, in England, and the logician Padoa, in Italy, at the turn of the century, began to focus the attention of mathematicians on axiomatics as a separate discipline. At this point, it is appropriate to discuss briefly the axiomatic process as it is used in scientific endeavors.

Assume a theory associated with a specific discipline. For convenience, let T represent this theory. An axiomatization of T begins with the selection of a small number of properties of T that are judged to be basic in the theory. The selected properties are termed variously: primitive terms, undefined terms, primitive notions, or primitive concepts. The primitive terms are taken to be undefined; the only requirement is for them to be recognizable and distinguishable by their appearance. No attempts are made to analyze them further. They are taken to be basic intuitive notions on which the theory is built. The primitive terms of the axiomatization are usually described, or their intended meaning or interpretation is given, in general terms. The primitive terms are usually assumed to be independent of each other.

Considerable freedom often exists in the choice of the primitive notions for formalizing a theory. In other words, the intuitive content of a theory does not determine uniquely the primitive notions used in the axiomatization of the theory. In fact, it is usually possible to provide more than one axiomatization of the same theory. Each axiomatization may rest on a completely different set of primitive notions.

Euclid's geometry is used as an example throughout this discussion. Euclidean geometry can be axiomatized with three primitive concepts: the notion of a point, the notion of a line, and the notion of a point lying on a line. These are taken to be the intuitive, undefinable basis for geometry by Euclid.

Other terms, concepts, and so forth, of a general discipline T are introduced into the axiomatization of T by definitions. An explicit definition relates the term to be defined (the definiendum) to other terms already available (the definiens). The definiens can consist solely of primitive terms, terms already defined, or a mixture of the two. However, the first new term defined in the axiomatization of T is given a meaning by relating it to some or all of the primitive terms. The second new term can be defined similarly or by relating it to some of the primitive terms and the first definition. New terms defined in the axiomatization of T are thus individually introduced and generate a fixed sequence of definitions. It is, therefore, sensible to speak of the definitions preceding a particular definition.

It is usually required of any new term introduced in the theory by definition, that the new term be replaceable ultimately by a combination of the primitive terms in the theory. In a sense, this requirement asserts that each defined term is a shortened notation for a longer expression. To construct an axiom system without this assumption would lessen the power that definitions have for facilitating deductive reasoning. Put another way, and using G.A. Miller's (9) term, definitions perform "chunking." This follows because a definition has at least two properties of chunking; it is a many-to-one process, and it is retrievable. In other words, a definition replaces a combination of many terms by a single term, and a defined term can always be decomposed into primitive terms. Questions of what logical conditions definitions must satisfy (the Theory of Definitions) are discussed in detail by Suppes (10).

An example of a Euclidean concept defined solely in terms of primitive concepts is that of intersecting lines. Two lines intersect if there is a point lying on both lines. An example of a definition in terms of primitive notions and defined concepts is that of a triangle. A triangle is a closed figure formed by three intersecting lines.

Primitive statements, also called axioms, are usually constructed from the set of primitive notions. They can also be formed by using defined terms or by a combination of primitive and defined terms. While this approach is not usual, it often makes the axioms simpler because of the use of defined terms. Axioms are thought of as statements fundamental to the theory being axiomatized, and form the basis for deducing other statements of the theory. In general, there is a choice to be made between various sets of

axioms, with the normal requirement being that the axioms for a theory must be independent.

Axioms can be viewed as determining the meaning of the primitive terms of the theory by stating relations between the terms. Thus, the meaning, or sense, of the primitive terms is determined by the use made of them in the axioms. Such a view of specifying the meaning of a term is not an explicit definition, for it does not establish a relation between the new term (the definiendum) and older terms (the definiens). It can be regarded as an implicit definition, because it acts like a definition by delimiting the meaning of a term.

Derived statements (also called theorems, deductions, propositions, and implications) are those statements that follow as a logical consequence of the primitive terms, defined terms, axioms, and previously derived statements. To be more precise, a statement is derived in the axiomatized version of T by a formal proof. A formal proof is a finite sequence of statements of the axiomatized theory such that each statement is either an axiom or is deducible from one or more preceding statements by logical rules of inference. Thus, a theorem is the last statement of some proof. An axiom can be viewed as a theorem with a one-step proof.

All axiomatizations of a theory are given within a body of presupposed knowledge. This aspect of axiomatization, although discussed last, is used throughout the axiomatization procedure. This, it is important to identify what other theories are assumed as known, even when selecting the primitive notions of the theory. Ordinarily, general set theory and some system of logic are presupposed as known in contemporary axiomatic formulations in mathematics. Standard portions of mathematics are assumed as known, in addition to set theory and logic, when axiomatizing an empirical science. The resulting axiomatization is called informal when standard logic and set theory are presupposed. The axiomatization of T is called formal when the rules of deduction and the system of logical axioms are not presupposed but are given in a completely and explicitly formalized language. Further discussion of the axiomatic method can be found in Suppes (10), Wilder (11, 12), and Blanche (13).

APPLICATION OF THE AXIOMATIC PARADIGM TO SUBJECT MATTER STRUCTURE

As mentioned above, the theory presented herein is based on the paradigm of axiomatics. It is not suggested that it is possible to axiomatize the content of any discipline in which instruction might be offered. What is suggested is that in order to

structure subject matter, a level of clarity concerning the subject matter must be obtained that approximates the level of clarity found in axiomatic systems. On the other hand, some characteristics of axiomatic systems are so restrictive that, if incorporated, they might hinder effective instruction. For example, in axiomatics it is usually required that the primitive notions be independent and the axioms be independent. In fact, one of the main goals of axiomatics is to exhibit the most parsimonious structure in order to eliminate redundancy completely. From an instructional point of view parsimony can be counterproductive, since repetition and redundancy are often both necessary and desirable. Our model does not require analogous restrictions that tend to create parsimony.

The GCN will be seen to contain elements analogous to primitive and defined concepts, axioms, and theorems in axiomatics. In order to discuss these elements while at the same time distinguishing them from components in axiomatic systems, specialized terminology is necessary. To this end, the categories of GCN elements are referred to as Primary Notions, Secondary Notions, Basic Principles, and Established Principles. The correspondence between axiomatic and GCN terminology is displayed in Table 1.

Table 1

Terminological Correspondence

Axiomatics	Subject Matter
Primitive Concept	Primary Notion
Defined Concept	Secondary (Defined) Notion
Axiom	Basic Principle
Theorem	Established Principle

In the developments that follow we will continuously refer to the paradigmatical aspects of axiomatics. This will be done to clarify and support the theoretical formalization to be presented. From a formal point of view the axiomatic method is the intended interpretation of the axiomatic theory developed herein. However, as will be seen, the present theory, since it deals with instruction only, is not sufficient as a formal treatment of axiomatics. The distinction between axiomatics as the paradigm, and axiomatic methodology as *used* herein must be maintained. For example, when "definitions" are referred to, the context of the discussion will determine whether definitions as part of the paradigm, or definitions *within* the theory below, are being referenced.

The formal theory is based on seven primitive concepts. Three of the seven are concerned with the GCN and will be discussed first. The GCN individuals are referred to as *constitutents* and are represented by the set C. In the axiomatic interpretation, the constituents will be seen to correspond to primitive concepts, defined concepts, axioms, and theorems. Constituents are assumed to be linguistic expressions and will be categorized as Primary Notions, Secondary Notions, Basic Principles, and Established Principles.

Constituents are either *formulated*, in terms of other constituents, or they are not formulated. Fxy will be written to indicate that Constituent x is used to formulate Constituent y. For example, a Constituent may be the Primary Notion (primitive concept) of an electron. This notion in turn may be used to formulate a Secondary Notion (defined concept) that consists of a linguistic expression or sentence that serves to introduce and define the concept of current. In this case, it can be said that the Primary Notion of an electron is used to formulate the Secondary Notion of current.[1] In the expression Fxy, x can be a Primary or Secondary Notion while y can be a Secondary Notion, a Basic Principle, or an Established Principle.

Constituents are also either *established* on the basis of other constituents or they are not established. Exy will be written to indicate that Constituent x is used to establish Constituent y. In the expression Exy, x can be a Secondary Notion or either a Basic or an Established Principle, while y must be an Established Principle.[2] Thus, the concept of a constituent being established is a primitive one for the theoretical developments to follow. In effect, this means that it is assumed that it is known what it means to establish a result, or that it is known what it means to prove a statement. The concept of proof can, as is well known, be developed within the context of Logic. Since Logic (and

[1] In a rigorous treatment of axiomatics itself it seems that the theoretical entities must be distinguished as either symbols or statements made up of symbols. For our purposes we will treat the symbols (expressions) as statements containing only one expression. That is, the distinction between a symbol and a statement is of minimal importance for instructional purposes. Notice, however, that this approach leaves us in the position of asserting that one statement is used to formulate another. While this is recognized as being somewhat awkward it is felt that the resulting simplification in the theoretical developments justifies this approach. If our goal was to rigorously investigate axiomatics, then this approach would not be taken.

[2] It may be felt that according to the convention of not distinguishing symbols and statements, when x in Exy refers to the Secondary Notion, the symbol defined and the statement that defines it are confused. That is, it may be felt that the symbol, not the definition, is used in a proof. However, as will be seen, we perceive definitions as auxiliary theoretical suppositions. Thus, when a defined symbol is used in a proof, the definition of the symbol (a statement) and not the symbol itself is being used to justify the step in the demonstration.

Set Theory) form the basis of the theoretical developments to follow, the assumption concerning an understanding of the concept of proof seems reasonable for our purposes.

Now consider the primitive concepts that will be used in order to characterize the TICS and its relationships to the GCN. First, it is assumed that a set of tasks is available. Let $\mathcal{T}$ denote this set of tasks. The set $\mathcal{T}$ is intended to serve as a pool of tasks from which criterion tests on constituents can be formulated.

A set of individuals to represent students will also be required, represented by I.

In order to judge whether a student can pass criterion for a constituent it is necessary to determine whether the student can perform the tasks in at least one of the associated criterion tests. This leads to the requirement for a binary relation between the set of students and the set of tasks. Pxy will be written to assert that student x can perform task y.

Finally, another relation is required that relates the constituents to the tasks. That is, is to be used as a pool of tasks for structuring criterion tests and, hence, some method of associating constituents and tasks is needed. To this end, T will represent the relation that serves this purpose. Each constituent is to be associated with a unique collection of task sets. Hence, T will be a function. In general, as discussed below, T will be a function from C into $\mathcal{P}\mathcal{P}(\mathcal{T})$ where $\mathcal{P}(A)$ denotes the set of all subsets of a set A (power set of A). Let $T(x)$ denote the image of x under the function T.

The logical symbols used herein are "$\neg$", "&", "$\rightarrow$", "$\leftrightarrow$" for "not", "and", "if, then", and "if and only if" respectively. "$\exists$" and "$\forall$" are used for the existential and universal quantifiers, respectively. In addition, standard set-theoretical symbols are also used with (A) denoting the Power Set (set of subsets) of a given set A. $T(x)$ denotes the image of x under function T. Set complement, set union, and set intersection are denoted by $\sim$, $\cup$, and $\cap$ respectively. $\subseteq$ denotes the subset relationship.

Definition 1. $\langle C, \mathcal{T}, I, F, E, P, T\rangle$ is an axiomatically structured subject matter if and only if the following five axioms hold.

(i) C is finite, and C and $\mathcal{T}$ are non-empty.

(ii) $(\forall x \in C)\,[\neg(\exists y \in C)\,Fyx \rightarrow (\exists z \in C)\,[\neg(\exists w \in C)Ewz \,\&\, Fxz]]$

(iii) $(\forall x \in C)\,[(\exists y \in C)Eyx \rightarrow (\exists z \in C)Fzx \,\&\, (\forall w \in C)\neg Fxw]$

(iv) There are no elements $x_1, \ldots, x_n$ in C such that $R_1x_1x_2$ & $R_2x_2x_3$ & ... & $R_{n-1}x_{n-1}x_n$ & $R_nx_nx_1$ holds when $R_1, \ldots, R_n$ are instances of F or E.

(v) $(\forall x,y \in C)[Fxy \vee Exy \rightarrow (\forall \alpha \in I)\,[(\exists T \in T(y))(T \neq \emptyset \,\&\, (\forall t \in T)\,P\alpha t) \rightarrow (\exists T' \in T(x))\,(T' \neq \emptyset \,\&\, (\forall t' \in T')\,P\alpha t')]]$

The five parts of Definition 1 will be referred to as axioms (i) to (v) respectively.

As asserted in axiom (i), C must be finite. Otherwise a student would be faced with learning an infinite number of subject matter components. From the authors' point of view, C must also be finite since an author cannot write down an infinite number of constituents.[1] C is assumed to be non-empty, because if it is empty there is nothing for the student to learn. The set of tasks, $\mathcal{J}$, is also assumed to be non-empty. This seems reasonable since there is at least one constituent to learn so there will be at least one task designed to determine whether or not a student has learned that constituent. Since the structure of the subject matter is independent of the question of the number of students, no assumptions are made concerning the size of the set I.

Axiom (ii) asserts that every nonformulated constituent is used to formulate at least one constituent that in turn is not established. In terms of the axiomatic paradigm, axiom (ii) asserts that every primitive concept must be used to formulate at least either a definition or an axiom. Axiom (ii) cannot be strengthened to an "if and only if" condition. Consider a defined concept, d, within an axiomatic theory that is used to formulate another defined concept, d'. Now d' is not established and hence d and d' satisfy the consequent of axiom (ii) ($x=d, z=d'$). However, d cannot satisfy the antecedent of axiom (ii) since d is a defined concept and hence formulated.

Axiom (iii) asserts that every established constituent is formulated but is not used to formulate other constituents. In the axiomatic paradigm, axiom (iii) asserts that theorems are formulated but not used to formulate other axiomatic entities. Axiom (iii) also cannot be strengthened to an "if and only if" condition. Consider an axiom, a, within an axiomatic theory. In this case, a is formulated and is not used to formulate other entities. Thus a satisfies the consequent of axiom (iii) but not the antecedent of (iii) since a is assumed to be true or is not established.

Axiom (iv) rules out cyclical chains of "used to formulate" and "used to establish" relationships and combinations of these relationships. In the axiomatic paradigm, axiom (iv) eliminates undesirable situations such as the following: Let $t_1, \ldots, t_n$ be theorems such that t_{i-1} is used to establish t_i for $i=2, \ldots, n$. Now suppose, contrary to axiom (iv), that t_n is used to establish t_1. This means that we have a theorem t_n whose proof

[1] Notice that potential subject matters such as set theory may contain axiom schema. Such a schema may be conceived as providing an infinite number of constituents. However, the schema itself is but *one* constituent, while each of its instances constitues another constituent, viz., an Established Principle. An author can only write down a finite number of schema instances. Thus, even subject matters that contain elements such as axiom schemas satisfy axiom (i).

depends on t_n itself. Obviously, this is contrary to accepted canons of logic and axiom (iv) effectively rules out such situations.

It can also be shown that axiom (iv) is required as well for the "used to formulate" relationship. Let $d_1, \ldots, d_n$ be definitions in an axiomatic theory. Further, suppose that d_{i-1} is used to formulate d_i for $i=2, \ldots, n$ and that d_n is used to formulate d_1,[1] which is contrary to axiom (iv). Recall that a defined concept must be eliminable from an axiomatic theory by the criterion of Eliminability (10). In the example under consideration we must be able to eliminate d_n (i.e., the symbol or expression defined in d_n), leaving at least d_{n-1} to be eliminated. Continuing, the point is reached when d_1 remains to be eliminated. However, d_n is used to eliminate d_1 since d_n is used to formulate d_1. As a result, there is again the problem of eliminating d_n. This means that the criterion of eliminability has been violated. Therefore, the criterion of eliminability implies that axiom (iv) is required for "used in the formulation of" relationships as well as for "used to establish" relationships.

In general, cycles of mixed F and E relationships must be eliminated to avoid other undesirable circularities. This is done in axiom (iv) where the R_i in the axiom refer to either instances of F or E.

The content of axiom (iv) can be depicted graphically. Constituents can be represented as nodes and directed lines can be used to represent the F and E relationships. The general form of the cyclic F, E relationships that are excluded by axiom (iv) is depicted in Figure 2. Extensive utilization of graphical representations such as the one shown in Figure 2 will be used throughout this paper.

Axioms (ii), (iii), and (iv) are concerned with the GCN structures while axiom (v) relates the GCN to the TICS. The function T from the set of constituents into $\mathcal{P}\mathcal{P}(\mathcal{J})$ provides the coordinating relations referred to in the introduction. This function serves to relate the components of the GCN (constituents) to components of the TICS (collections of sets of tasks). T must be a function into $\mathcal{P}\mathcal{P}(\mathcal{J})$, as the following discussion shows. Consider a constituent x. In general, an instructor will have more than one criterion test associated with x, especially in individualized instructional settings such as in computer-administered instruction. In such an environment remedial instruction may be scheduled

[1] Without the convention of not distinguishing symbols from statements, this example would have to be formulated as follows: $d_1, \ldots, d_n$ are definitions introducing defined concepts $d'_1, \ldots, d'_n$ where d'_{i-1} is used to formulate d_i for $i=2, \ldots, n$ and d'_n is used to formulate d_1. The agreement proceeds as above with "d_j'" replacing "d_j" wherever the symbol or expression rather than the statement of the definition is referenced.

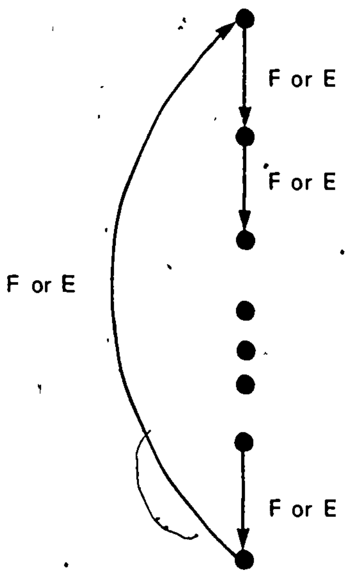

Figure 2. **Non-admissible F,E relations**

each time a student fails to pass criterion on a constituent. Usually multiple criterion tests are utilized so that a student does not repeat the same test more than once after remediation. That is, an effort is made to avoid letting the student become familiar with test items through multiple encounters. While such tests may be similar, each is in fact composed of discreet tasks and hence these tests must be discriminated.

For simplicity, suppose the given constituent, x, is associated with two discreet criterion tests, T_1, T_2. Tests T_1 and T_2 are both composed of individual tasks. Let $T_1 = \{t_1, t_2\}$ and let $T_2 = \{t_3\}$. Now x is associated by T with *both* T_1 and T_2. That is, $T(x) = \{T_1, T_2\} = \{\{t_1, t_2\}, \{t_3\}\}$. Thus, $T(x) \subseteq \mathcal{P}(\mathcal{T})$ or $T(x) \in \mathcal{P}\mathcal{P}(\mathcal{T})$ as assumed in axiom (v). T is assumed to be a function so that every element of C will be associated with a unique element in $\mathcal{P}\mathcal{P}(\mathcal{T})$. This justifies the use of the notation $T(x)$ where $x \in C$.

Now consider axiom (v) and assume x is used to formulate y for $x, y \in C$. Intuitively, since x is used to formulate y, y in some sense must depend on x. Suppose an individual α can perform all tasks in some criterion test for y. This means that α can pass criterion on y. Since y depends on x it must follow that α can also pass criterion on x. That is, α can perform all tasks associated with *at least one* criterion test for x as asserted in the final consequent of axiom (v).

In the formal developments that follow we deal first with the structure of the subject matter content (GCN). We lead up to the notion of dependency in the subject matter with a procedure being developed that permits an instructor to elucidate the

entire complex of dependency relationships. The TICS structure is examined next, with the final results relating to GCN and TICS structures through the coordinating function T.

The set of Primary Notions in a subject matter will be denoted by PN. The elements of PN are distinguished by the fact that they are not formulated in terms of other constituents. The meaning of any Primary Notion in a subject matter is usually assumed to be self-evident. Primary Notions provide the conceptual basis for the subject matter and in this sense they provide the conceptual starting point for a subject matter. The formal definition of PN is as follows:

Definition 2. $\mathrm{PN} = \{x \epsilon \mathrm{C}: \neg (\exists y \epsilon \mathrm{C}) Fyx\}$

Not only are elements of PN unformulated in terms of other constituents, but they are also unestablished. That is, Primary Notions (primitive concepts) are not established (proved) on the basis of other constituents. This result is stated in the first theorem below.

Theorem 1. $(\forall x \epsilon \mathrm{PN})\ [\neg(\exists y \epsilon \mathrm{C}) Eyx]$

To prove Theorem 1, note that if x is established on the basis of some y then by axiom (iii) x is formulated in terms of some constituent. Assuming that $x \epsilon \mathrm{PN}$ leads to a contradiction since elements of PN are not formulated. Thus, $x \epsilon \mathrm{PN}$ implies that x is not established as required.

Within axiomatics, definitions and axioms are similar in that they are both formulated in terms of other constituents. Furthermore, both axioms and definitions are not established. Axioms function as suppositions from which theorems are proved. Following Suppes (10, p. 142), definitions serve a similar role in that they serve as additional suppositions in an axiomatic development. Definitions differ from axioms in that the former satisfy the criteria of non-creativity while the latter do not.

As in the axiomatic paradigm we consider Secondary Notions (SN) and Basic Principles (BP) together as Suppositions (S) from which other statements (Established Principles) may be established. Elements of SN and BP are both formulated but Secondary Notions satisfy the criteria of non-creativity while Basic Principles do not. Elements of BP do not satisfy the criteria of non-creativity since elements of BP (axioms in the paradigm) are intended to serve the role as starting points for derivations and hence are inherently creative. Formally, there are the following distinctions where the statement of C1 closely follows Suppes' formulation (10):

C1: Criteria of Non-creativity.

A statement s introducing a new expression satisfies the criteria of non-creativity if, and only if, there is no statement s′ in which the new expression does

not occur such that $s \to s'$ can be established from the Basic Principles and preceding Secondary Notions of the subject matter but s' cannot be so established.

The statement of C1 makes use of the concept of preceding Secondary Notions in a subject matter. Secondary Notions are introduced in a specific sequence, with the first making use only of Primary Notions, the second of Primary Notions and perhaps the first Secondary Notion, and so forth. Thus, the phrase "preceding Secondary Notions in the subject matter" has a precise meaning Justifying its use in C1.[1]

Definition 3. Suppositions

$$S = \{x \epsilon C: (\exists y \epsilon C) Fyx \;\&\; \neg(\exists w \epsilon C) Ewx\}$$

Definition 4. Secondary Notions

$$SN = \{x \epsilon S: x \text{ satisfies C1}\}$$

Definition 5. Basic Principles

$$BP = S \sim SN$$

Consider some element, say, x where $x \epsilon PN$. By axiom (iii) we know there is some y such that y is not established and x is used to formulate y. Notice that axiom (iv) assures that $x \neq y$, that is, x is used to formulate at least one supposition in the subject matter. This is intuitively correct since it is expected that a primitive concept in the axiomatic paradigm will be used for some purpose. The appropriate way to bring a primitive concept into theoretical use is to use it to formulate either an axiom or a definition. These remarks serve to establish the next theorem.

Theorem 2. $(\forall x \epsilon PN)(\exists y \epsilon S)[x \neq y \;\&\; Fxy]$

Established Principles (EP) correspond to theorems and are characterized by the fact that they are established.

Definition 6. $EP = \{x \epsilon C: (\exists y \epsilon C)\, Eyx\}$.

Consider an element, x, such at $x \epsilon EP$. From axiom (iii) it is known that x is formulated in terms of other constituents. It is also known that x is not used to formulate other constituents. Theorem 3 formalizes these results.

[1] These ideas concerning preceding Secondary Notions, and so forth, are certainly amenable to formal treatment. However, the attendant formal mechanisms have minimal utility for our present purposes. The notion of precedence, in fact, as used in C1 must not be confused with dependence as a relationship between constituents. Neither should precedence in this sense be thought to imply requisite instructional precedence. The use of precedence in C1 simply means that subject matter components can be arranged in some linear order. However, to determine relationships of instructional importance one must look deeper than this linear order. One must take into account how constituents are utilized.

Theorem 3. (i) $(\forall x \epsilon \mathrm{EP})(\exists y \epsilon \mathrm{C})[x \neq y \,\&\, Fyx]$

(ii) $(\forall x \epsilon \mathrm{EP}) \neg (\exists y \epsilon \mathrm{C}) Fxy$.

Notice that $\mathrm{PN} \cup \mathrm{SN} \cup \mathrm{BP} \cup \mathrm{EP} \subseteq \mathrm{C}$. On the other hand, suppose $x \epsilon \mathrm{C}$. Either $(\exists y \epsilon \mathrm{C}) Fyx$ or $\neg(\exists y \epsilon \mathrm{C}) Fyx$. In the latter case, $x \epsilon \mathrm{PN}$, in the former either $(\exists w \epsilon \mathrm{C}) Ewx$ or $\neg(\exists w \epsilon \mathrm{C}) Ewx$. If $(\exists w \epsilon \mathrm{C}) Ewx$, then since $(\exists y \epsilon \mathrm{C}) Fyx$ we know that $x \epsilon \mathrm{EP}$. If $\neg(\exists w \epsilon \mathrm{C}) Ewx$ then $x \epsilon \mathrm{S}$, that is $x \epsilon \mathrm{SN} \cup \mathrm{BP}$. Therefore, $\mathrm{C} = \mathrm{PN} \cup \mathrm{SN} \cup \mathrm{BP} \cup \mathrm{EP}$.

Notice that the elements of PN are not formulated while the elements of SN, BP, and EP are all formulated. Thus, $\mathrm{PN} \cap \mathrm{SN} = \emptyset$, $\mathrm{PN} \cap \mathrm{BP} = \emptyset$ and $\mathrm{PN} \cap \mathrm{EP} = \emptyset$. $\mathrm{SN} \cap \mathrm{BP} = \emptyset$ be definition and $\mathrm{SN} \cap \mathrm{EP} = \emptyset$ since Secondary Notions are not established but Established Principles are established. Similarly, $\mathrm{BP} \cap \mathrm{EP} = \emptyset$. Hence, PN, SN, BP, EP are mutually exclusive. We have established that PN, SN, BP, and EP partition C, as stated in the next theorem.

Theorem 4. PN, SN, BP, EP partition C.

It would be expected in an axiomatic theory that there be at least one primitive concept. Without at least one primitive concept it would be impossible to make any theoretical developments. It would also be expected that there would be at least one supposition. Otherwise, the theory would be trivial in that it would consist of, at most, primitive concepts only. The next two theorems establish these results for an axiomatically structured subject matter.

Theorem 5. $\mathrm{PN} \neq \emptyset$

Theorem 6. $\mathrm{S} = \mathrm{SN} \cup \mathrm{BP} \neq \emptyset$

To prove Theorem 5, suppose $\mathrm{PN} = \emptyset$. By axiom (i) we have some element, say x, such that $x \epsilon \mathrm{C}$. By Theorem 4, since $\mathrm{PN} = \emptyset$ either $x \epsilon \mathrm{S}$ or $x \epsilon \mathrm{EP}$. In either case $(\exists y \epsilon \mathrm{C}) Fyx$ since elements of S and EP are formulated. Axiom (iv) assures us that $x \neq y$. $y \notin \mathrm{PN}$ since $\mathrm{PN} = \emptyset$. $y \notin \mathrm{EP}$ since Established Principles cannot be used to formulate other constituents by axiom (iii). Thus, $y \epsilon \mathrm{S}$, implying the existence of another constituent z such that Fzy. As before, $x \neq z$, $y \neq z$ and $z \epsilon \mathrm{S}$. Continuing, we have a nonending chain of distinct elements in S of the following form:

$$\ldots \,\&\, Fzy \,\&\, Fyx$$

This is contradictory since it implies that C is infinite. This establishes Theorem 5.

To establish Theorem 6, note that Theorem 5 implies the existence of an element $x \epsilon \mathrm{PN}$. By axiom (ii) there is an element $z \epsilon \mathrm{C}$ such that:

$$\neg(\exists w \epsilon \mathrm{C}) Ewz \,\&\, Fxz.$$

This implies $z \notin \mathrm{EP}$. Furthermore, $z \notin \mathrm{PN}$ since z is formulated in terms of another constituent, viz., x. Thus $z \epsilon \mathrm{S}$ as required in Theorem 6.

Theorems 5 and 6 serve to insure us that our axioms satisfy our intuitions concerning subject matters. An author must begin with at least one Primary Notion (Theorem 5). The author must also formulate at least one Supposition. That is, the subject matter must contain at least one other element, either a Secondary Notion or a Basic Principle (Theorem 6). If this were not the case, the subject matter would consist only of the Primary Notions and would be entirely trivial since Primary Notions are assumed to be intuitively comprehensible. In addition, for every constituent that is not a primary Notion, we can show there is a finite chain of "used to formulate" relationships starting from a Primary Notion and leading up to the constituent. Thus, every constituent that is not a Primary Notion is based on or founded on some Primary Notion through a finite chain of "used to formulate" relationships. This result is proved in Theorem 7, which follows one simplifying terminological definition.

Definition 7. For all x,y in C F^*xy if and only if there is a sequence (possibly empty) of constituents $x_1, \ldots, x_n$ such that

$$Fxx_1 \,\&\, Fx_1x_2 \,\&\, \ldots \,\&\, Fx_ny$$

holds.

In Definition 7, the reference to an empty sequence of constituents means that F^*xy includes Fxy as a member. That is, if Fxy holds then F^*xy also holds.

Theorem 7. $(\forall x \epsilon C)\,[x \epsilon S \cup EP \leftrightarrow (\exists y \epsilon PN) F^*yx]$

The proof that $x \epsilon S \cup EP$ is a sufficient condition for F^*yx for some $y \epsilon PN$ is as follows. Let $x \epsilon S$ or $x \epsilon EP$. In either case there is a $y_1 \epsilon C$ such that $y_1 \neq x$ and Fy_1x. From axiom (iii) we know $y_1 \notin EP$. Thus, either $y_1 \epsilon PN$ or $y_1 \epsilon S$. If $y_1 \epsilon PN$, then set $y=y_1$ so that F^*yx holds as required. If $y_1 \epsilon S$, then we have a $y_2 \epsilon C$ such that $y_2 \neq y_1$, $y_2 \neq x$ and Fy_2y_1. As before, $y_2 \epsilon PN$ or $y_2 \epsilon S$. If $y_2 \epsilon PN$ set $y=y_2$ so that we have both Fyy_1 and Fy_1x, or F^*yx as required. If $y_2 \epsilon S$ we repeat this process for a third constituent y_3. Continuing we know this process must terminate, say after n steps, since C is finite. This means that eventually we have a sequence $y_1, \ldots, y_n$ where $y_n \epsilon PN$ and Fy_ny_{n-1} & $Fy_{n-1}y_{n-2}$ & $\ldots$ & Fy_1x holds. Taking $y=y_n$ we now have F^*yx as required.

For the necessity portion of the proof of Theorem 7, let $y \epsilon PN$ and assume F^*yx for $x \epsilon C$. This means that we have two possibilities. In one case Fyx holds. By Definition 2, $x \notin PN$ and hence $x \epsilon S$ or $x \epsilon EP$ as required. In the other case we have a sequence of constituents $x_1, \ldots, x_n$ such that Fyx_1 & Fx_1x_2 & $\ldots$ & Fx_nx holds where $n \geq 1$. Since Fx_nx holds for some x_n we again know that $x \notin PN$, or $x \epsilon S \cup EP$ as required.

In the most general case any element in either S or EP is based on a subset of PN. That is, Theorem 7 simply asserts that at least one element in PN is related through a

sequence of F relationships to any element in S or EP. In general, however, there may be several elements in PN to which the element in S or EP is related via F relationships. A general hypothetical example of this phenomenom is depicted in Figure 3, in which each of the constituents x_1, x_2, x_3 and x_4 are in PN. In addition, F^*x_1y, F^*x_2y, F^*x_3y and F^*x_4y all hold. The next definition provides a means of referring to the subset of PN to which an element of S or EP is related via F relation sequences.

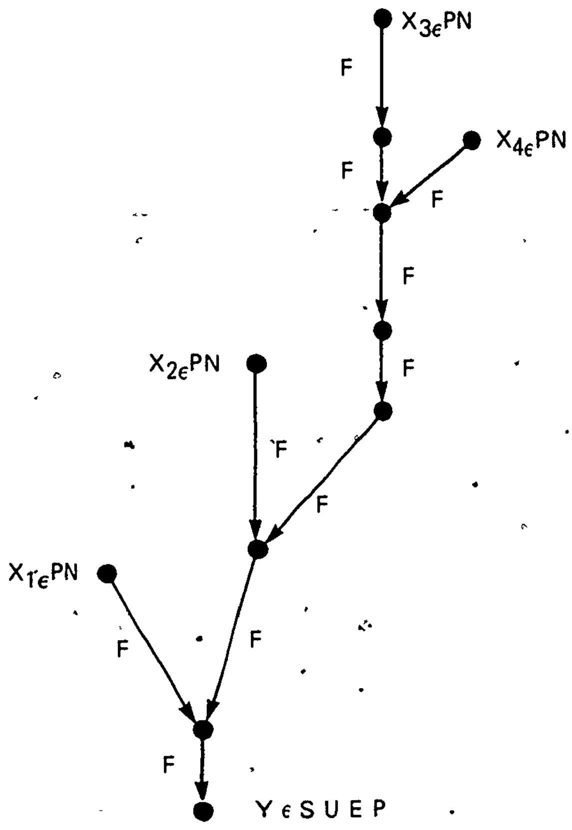

Figure 3. Hypothetical example of an element related to a subset of PN via sequences of F relationships

Definition 8. For all x in C, $F^*(x) = \{y \epsilon PN : F^*yx\}$

In the example presented in Figure 3, $F^*(y) = \{x_1, x_2, x_3, x_4\}$.

The next theorem follows immediately from Theorem 7 where it is assumed in both the theorem and its Corollary that $x \epsilon C$.

Theorem 8. $F^*(x) \neq \emptyset \leftrightarrow x \epsilon S \cup EP$

Corollary. $F^*(x) = \emptyset \leftrightarrow x \epsilon PN$

Definition 9. For all x,y in C, E^*xy if and only if there is a sequence (possibly empty) of constituents $x_1, \ldots, x_n$ such that

$$Exx_1 \;\&\; Ex_1x_2 \;\&\; \ldots \;\&\; Ex_ny$$

holds.

As in the case of F^*, Definition 8 is formulated in such a way that Exy implies E^*xy.[1]

Of primary importance from an instructional point of view is the notion of dependency. If one constituent is dependent on another, then before a learner can comprehend the latter the learner must comprehend the former constituent. In general, before a learner can comprehend a constituent the learner must comprehend all constituents the given constituent depends upon. Thus, for instructional purposes it is essential that a method be available by which we can identify all constituents depended upon by a given constituent.

To this end we next define the notion of dependency. The basic characteristics of dependency are investigated with particular emphasis on dependency, F^* and E^*. Finally, a method of determining all dependency relationships for a given Constituent is described.

Definition 10. For all x,y in C x is dependent on y (Dxy) if and only if there is a sequence (possibly empty) of constituents $x_1, \ldots, x_n$ such that

$$R_1yx \;\&\; R_2x_1x_2 \;\&\; \ldots \;\&\; R_{n+1}x_nx$$

holds when $R_1, \ldots, R_{n+1}$ are instances of F or E.

Notice that Definition 10 is formulated in such a way that both Fyz and Eyx imply that x is dependent on y (Dxy).

Theorem 9. Dependency is irreflexive, asymmetric and transitive.

The proof of Theorem 9 is straightforward with the asymmetric condition following from axiom (iv).

Theorem 10. For all x,y in C Dxy if and only if either

(i) F^*yx, or

(ii) E^*yx, or

(iii) $(\exists z \epsilon C)\,(F^*yz \;\&\; E^*zx)$.

[1] We have not formulated theorems analogous to theorems 7 and 8 for E^*, nor have we formulated a definition analogous to Definition 8 for E^*. In an axiomatic theory it would be expected that all theorems would be provable ultimately from axioms alone—the criteria of non-creativity. In the subject matter theory this means that all Established Principles must be ultimately established on the basis of Basic Principles and Primary Notions alone. We could introduce the formal machinery to establish this result. To do so, however, would require an extensive and unwarranted digression into the concept of a proof.

That Dxy is a necessary condition for (i), (ii), or (iii) in Theorem 10 follows from Definition 10.

To see that Dxy is also a sufficient condition assume Dxy. That is for some sequence (possibly empty) of constituents $x_1, \ldots, x_n$ we have

$$R_1yx_1 \,\&\, R_2x_1x_2 \,\&\, \ldots \,\&\, R_{n+1}x_nx$$

where $R_1, \ldots, R_{n+1}$ are instances of F or E. In the case that the sequence of constituents is empty we have

$$R_1yx$$

where R_1 is either F or E. This immediately yields either F^*yx or E^*yx as required.

Now suppose the sequence is not empty and there are in fact n constituents satisfying the above conjunction. Suppose for some $i\,(1 \leqslant i \leqslant n+1)$ R_i is an instance of E. This implies that $Ex_{i-1}x_i$ holds and thus $x_i \epsilon$EP. Since $x_i \epsilon$EP, R_{i+1} must also be an instance of E by Theorem 3. That is, $\neg(\exists z)Fx_iz$ leaving only E as a possibility for R_{i+1}. Continuing in this way we see that $R_{i+2}, \ldots, R_{n+1}$ must all be instances of E. In general, it can be asserted that for all $i\,(1 \leqslant i \leqslant n+1)$ if R_i is an instance of E, then $R_{i+1}, \ldots, R_{n+1}$ must also be instances of E. We now have three cases.

Case 1. R_1 is an instance of E. Then it follows that E^*yx holds.

Case 2. R_i is an instance of E where $1 < i \leqslant n+1$ and for all $j < i$, R_j is not an instance of E. In this case we have

$$Fyx_1 \,\&\, Fx_1x_2 \,\&\, \ldots \,\&\, Fx_{i-2}x_{i-1} \,\&\, Ex_{i-1}x_i \,\&\, Ex_ix_{i+1} \,\&\, \ldots \,\&\, Ex_nx$$

That is,

$$F^*yx_{i-1}$$

and

$$E^*x_{i-1}x.$$

In other words there is some z in C (viz.: x_{i-1}) such that

$$F^*yz \,\&\, E^*zx$$

holds.

Case 3. None of the R_i is an instance of E, in which case we have F^*yx.

Either Case 1, Case 2, or Case 3 must obtain, whence the proof is complete.

Theorem 10 means that there are three, and only three, forms chains of dependency relations can take. These forms are displayed diagramatically in Figure 4, where dxy is assumed to hold.

The next theorem establishes a minimal structure that an axiomatically structured subject matter must exhibit.

Figure 4. The three possible forms for dependency chains assuming X is dependent on Y

<u>Theorem 11</u>. If $\langle C, \mathcal{J}, I, F, E, P, T\rangle$ is an axiomatically structured subject matter, then there are elements x, y in C such that Dxy holds.

To prove Theorem 11 notice that we know there is some $y \in PN$ by Theorem 5. This yields an x in C such that Fyx by axiom (ii). Thus, by definition, Dxy as required.

While we have identified four major subsets of the set of constituents for a subject matter, we have not discussed the way in which elements of these sets are selected. Consider for example, the Primary Notions, PN. In selecting Primary Notions for instructional purposes there is a great deal of freedom of choice. The notions that are chosen as primary vary depending on the bias of the individual structuring the subject matter, the applications of the knowledge to be learned, and so forth. Consider, for

example, Set Theory as a potential instructional subject. One instructor might select the following Primary Notions:

set
set membership
the empty set
set complementation
set intersection.

while another instructor might select as a possible set of Primary Notions:

set
set membership
the universal set
set complementation
set union.

Each of these classes of notions can serve as the basis for instruction in Set Theory. Notions present in one class but absent from the other can be introduced by way of definition. For example, set intersection in the first class can be defined on the basis of the second class by means of set union and set complementation. The point is, there is no compelling abstract reason to select one class of Primary Notions over the other.

Similar remarks apply to the freedom that can be exercised in the selection of Secondary Notions, Basic Principles, and Established Principles. In the context of the Set Theory example, one person may introduce the Secondary Notions of

ordered pair
cross product
relation
function.

Another instructor, however, may forego the relation concept and introduce the notion of function based on an understanding of ordered pairs and cross product. Thus, the class of Secondary Notions is reduced in this case to

ordered pair
cross product
function.

The difference in approach might be dictated by the context of the instruction, with the first class being more appropriate for audiences such as social scientists where relations are of importance. The second class of Secondary Notions may be more appropriate for mathematically oriented instruction emphasizing the concept of functions.

These comments apply as well to the axiomatic paradigm for structuring subject matter. That is, a great deal of freedom of choice can be exercised in selecting primitive and defined concepts, axioms, and theorems when axiomatizing a theory. Examples of this latitude are evident when one considers the number of axiomatic approaches to a

common subject such as Symbolic Logic. In fact, there does not seem to be any agreed upon or standard axiomatic treatment of Logic. The test, of the adequacy of any particular treatment is not in what concepts are primitive or defined, or even what axioms are chosen. The adequacy of a particular axiomatization of a discipline such as Logic is judged, in part, on the basis of whether or not the accepted canons of Logic can be accounted for within the formalization.

The point of view expressed herein is that once the constituents in use, or to be used, have been identified they can be analyzed in terms of the F and E relationships. This in turn leads to a categorization of constituents along the lines of the definitions of PN, SN, BP, and EP. In addition, it is possible to elucidate the complete set of constituent interdependencies. This can be done through the following formal procedure:

It is clear that both F and E are irreflexive. Furthermore, from axiom (iv) we know that neither F nor E contains loops. The graph of either F or E does not contain parallel arcs and therefore these graphs are digraphs.[1] The adjacency matrix for a relation R (F or E) is defined as the n x n square matrix $A(R)=[a_{ij}]$ where $a_{ij}=1$ if constituent i is related to constituent j by R, and $a_{ij}=0$ otherwise. We assume here that there are a total of n constituents.

For convenience we introduce the notion of Boolean arithmetic. The only difference between Boolean and non-Boolean arithmetic is that in Boolean arithmetic 1+1=1. Following Harary *et al.* (14) we denote Boolean arithmetic by writing $(n+m)\#$. So that $(1+1)\# = 2\# = 1$, and, in general, it follows that if $n \geqslant 1$, then $n\#=1$.

Definition 11. The adjacency matrix for the dependency relation, DA, for the GCN is defined as follows:

$$DA = [A(F) + A(E)]\#$$

According to Definition 11, to form the adjacency matrix for the GCN dependency relation, the adjacency matrix for each of F and E is constructed first. These adjacency matrices are then Boolean added element by element. This is normal matrix addition except that the individual sums are Boolean sums so that each entry in DA is either a 1 or a 0. It follows that each element a_{ij} of DA is such that

$a_{ij} = 1$ if Constituent i is related to Constituent j by *at least one* of F or E.

and

$a_{ij} = 0$ if neither F nor E relates Constituent i to Constituent j.

[1] The cited concepts, definitions, etc. are modifications of those presented in F. Harary, *et al. Structural Models*, (14).

Whence DA depicts the dependency relationship to the extent of showing all dependency chains of length 1. The matrix DA does not necessarily depict *all* the dependency relations, since dependency may be established through relational chains of length greater than 1. The next theorem provides the rationale needed to generate the complete dependency matrix.

Theorem 12. (Harary, *et al.*, 1965, 14). In DA^n (DA raised to the nth power) the i,j^{th} entry is the number of F,E relational sequences of length n leading from Constituent i to Constituent j.

It can be shown that for some $m > 1$, $DA^{m+1} = [0]$. This follows, since there are no loops in the dependency relation. Thus, since the number of constituents is finite, there is an upper limit on the length of possible relational sequences leading from one constituent to another. This upper limit is denoted by m and is called the *maximal length dependency chain* for the GCN. It follows then that $DA^{m+1} = [0]$, or the matrix containing only entries of 0.

Definition 12. The *dependency* matrix for a GCN is defined as

$$D = [DA^1 + DA^2 + \ldots + DA^m]\#$$

where m is the maximal length dependency chain for the GCN.

Suppose D is the dependency matrix for a GCN and let x,y be GCN constituents. Within D, x will correspond to some row, and y will correspond to some column. We will write $a_{x,y}$ to indicate the entry *in* D consisting of the row and column corresponding to Constituents x and y, respectively. If x is represented by the i^{th} row in D, and y by the j^{th} column in D, then $a_{xy} = a_{ij}$.

On the basis of the foregoing concepts, we have the following result:

Theorem 13. Let x,y be GCN constituents. Dxy if and only if $a_{yx} = 1$.

X is dependent on y if and only if there is a sequence of F,E relations leading from y to x. Such a sequence will be composed of, say, n steps or links. Thus, the entry representing the y,x relation in the n^{th} power of the adjacency matrix DA will be 1. When the dependency matrix is formed by Boolean adding all nonzero adjacency matrices, this entry will carry over, causing a_{xy} to be set to 1. Conversely, if $a_{yx} = 1$ the i,j^{th} entry in some power of the adjacency matrix, DA, must be nonzero. Thus, there must be an F,E sequence leading from y to x. That is, Dxy.

Thus, Theorem 13 and the concepts on which it is based provide a direct method of developing the dependency relationship from the GCN. From an instructional point of view the dependency relationship is crucial to the sequencing of instructional material. A student cannot comprehend a constituent in a subject matter GCN until the student

comprehends all constituents upon which the first constituent depends. In other words, if a constituent, c, depends on each element in a set of constituents, γ, then the student cannot develop a comprehension of c until the student is known to have developed, or has developed, a comprehension of all elements in γ.

By comprehension of a constituent x we mean that a student can perform all tasks in at least one of the task sets associated with x. That is, if an individual comprehends x, then the individual must be able to perform all the tasks in at least one of the sets of tasks associated with x via T. From comprehension of x we cannot necessarily infer which set of tasks the individual can perform, unless, of course, the task set associated with x is a unique set of tasks (i.e., $T(x)$ contains one subset of $\mathcal{J}$). Conversely, if the individual can perform all tasks in at least one of the task sets associated with x, then it seems reasonable to infer that the individual comprehends x. At least such performance provides positive evidence of constituent comprehension without indicating any negative evidence that would lead one to suppose that the individual did not comprehend the constituent. Thus we have the following definition:

Definition 13. For all α in I and all x in C, α is said to comprehend x if and only if $(\exists T \epsilon T(x))[T \neq \emptyset \;\&\; (\forall t \epsilon T)P\alpha t]$

Theorem 14. For all α in I and for all x in C, if α comprehends x, then α comprehends all constituents on which x depends.

The proof of Theorem 14 is as follows: Suppose α comprehends x and Dxy where $\alpha \epsilon I$ and $x, y \epsilon C$. By Definition 13 we know

$$(\exists T \epsilon T(x))[T \neq \emptyset \;\&\; (\forall t \epsilon T)P\alpha t]$$

By Theorem 10 we have three cases.

Case 1. F^*yx holds. In this case we have Constituents $x_1, \ldots, x_n$ where

$$Fyx_1 \;\&\; Fx_1x_2 \;\&\; \ldots \;\&\; Fx_nx$$

holds. Fx_nx in conjunction with the above result yields

$$(\exists T_n \epsilon T(x_n))(T_n \neq \emptyset \;\&\; (\forall t \epsilon T_n)P\alpha t).$$

We also have $Fx_{n-1}x_n$ so by using the last result with axiom (v) yields

$$(\exists T_{n-1} \epsilon T(x_{n-1}))(T_{n-1} \neq \emptyset \;\&\; (\forall t \epsilon T_{n-1})P\alpha t).$$

Continuing after n+1 steps we have

$$(\exists T_y \epsilon T(y))(T_y \neq \emptyset \;\&\; (\forall t \epsilon T_y)P\alpha t)$$

or in other words α comprehends y as required.

Case 2. E^*yx holds. The proof in this case is the same as the proof in Case 1.

Case 3. $(\exists z \epsilon C)(F^*yz \ \& \ E^*zx)$ holds. Cases 1 and 2 can be generalized by asserting that if either F^*yx or E^*yx, then if α comprehends x, then α comprehends y: Thus, E^*zx and the fact that α comprehends x implies that α comprehends z. Thus, since F^*yz we also know that α comprehends y as required.

Notice that Theorem 14 cannot be strengthened to an "if-and-only-if" condition. That is, from the fact that an individual comprehends all constituents a Constituent x depends on, it does not necessarily follow that the individual comprehends x. This is intuitively correct, since the purpose of instruction is to permit the individual to synthesize knowledge he possesses in order to develop more understanding. That is, when an individual does comprehend all constituents x depends on, then and only then is the individual prepared to go on to comprehension of x. There is simply no guarantee that an individual will transfer comprehension of all constituents a constituent depends on to comprehension of the latter constituent itself.

In the next chapter we present two examples of the concepts introduced in the foregoing section. The first example, unicorns, consists of a trivial, fanciful subject matter, presented to provide a simple overview of the concepts. In the second example a subject matter is analyzed in detail and *all* GCN constituents and relationships are identified and displayed. This subject matter is a significant portion of a formalization of Boolean Algebra. The initial portion of the development of the Algebra is completely analyzed, including a complete representation of the dependency relationship for the GCN. For completeness, a hypothetical TICS is also developed for this example.

Chapter 3

TWO EXAMPLES OF THE THEORETICAL MODEL

EXAMPLE I: UNICORNS—GCN ONLY

For the first example, suppose the instructional problem is to provide an explication of the meaning of the term "unicorn." The General Subject Matter is that concerning unicorns. Assume the target population consists of a group of youngsters, or one youngster, reading a fairy tale. The Context Conditioned Subject Matter consists of explicating the concept of unicorn with respect to the knowledge the youngsters possess and perhaps the particular fairy tale being read. Let us assume that the students know what a horse, a stag, a lion, and horns on animals are.

The instructor in this case may proceed as follows. The Primary Notions can be chosen as being the notions of animal, horse, stag, lion, and horn. From these, Secondary Notions can be introduced. First is that of an animal with a horse's head and body. Second is that of an animal with a horse's head, horse's body, and stag's legs. Third is that of an animal with a horse's head, horse's body, stag's legs, and lion's tail. Finally, this notion can be extended to that of a creature with a single horn in addition. Therefore, the GCN is structured as shown in Figure 5.

The relation E in this example is simply the null relation since no constituents are established. Notice that in this example, the same GCN net depicted in Figure 5 would emerge if we proceeded by asking "What must one know before one can learn ____?" For example, in terms of the available Primary Notions we could ask, "What must one know before one can learn about animals with horses' heads and bodies?" The response that emerges is "animals" and "horses."

As will be seen, the analysis for the next example depends explicitly on both F and E. In this next case to use the "What must one know before one can learn ____" question is to sacrifice information that is instructionally important. The problem is that this general relationship will not suffice to develop both of the underlying ordering relations, and hence relevant instructional information does not manifest itself.

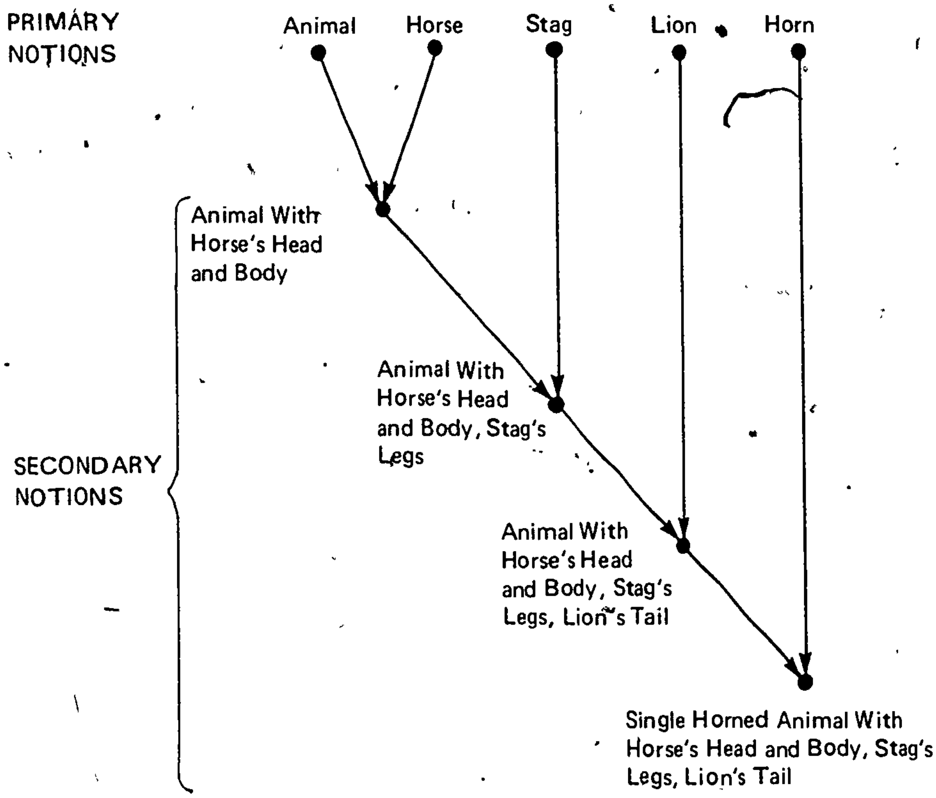

Figure 5. **GCN for explication of concept of a unicorn**

EXAMPLE II: BOOLEAN ALGEBRA

As the second example that illustrates a multiple relational GCN, Boolean algebra is chosen as the General Subject Matter. The structure of Boolean algebra is transparent since it has been axiomatized. The content of the basic theory is fairly limited so that the structure nets that will be developed can be exhibited completely. A more complex subject such as COBOL programming (an experimental subject taught at HumRRO) could have been chosen, but to do so would require hundreds of pages of complex graphs to exhibit its internal structure. While the ideas underlying Boolean algebra are simple to grasp and few in number, the theory is extremely rich with respect to results and applications.

Boolean algebra is named after its originator, George Boole, who studied it in detail in 1847. Interest in this type of algebra became very active with the advent of modern digital computers. It can be introduced and developed in a variety of ways because of its many practical applications. For example: Boolean algebra is used extensively as a tool in the design of switching circuits and computers.

One way of describing Boolean algebra is to say it is the algebra of all the subsets of some universal set. Another way is to state it is the algebra of sentences in the sense of symbolic logic. More detailed, nonaxiomatic descriptions of Boolean algebra would require considerable space; the interested reader can find such descriptions and applications in Hohn (15) and Mendelson (16).

As discussed above, the general attributes of the target population within the Context helps to delimit the General Subject Matter to be taught. To justify our selection of an *axiomatic* version of Boolean algebra for presentation to students one can, for example, assume a target population with the following general attributes: at least college sophomores, at least one year of college-level mathematics, and an interest in the methodology of science. Such a target population could include individuals whose main areas of interest are in mathematics, symbolic logic, philosophy, or computer science.

We further assume that the interests of the target population and the instructor are in the role of axiomatic methods in scientific methodology. These target population characteristics and interests determine the Context of a possible course in scientific methodology. An axiomatized version of Boolean algebra could be included as a part of such a course. It furnishes an introduction to axiomatic structures, elementary, simple, and rich in results. This suffices to define the Context Conditioned Subject Matter.

Let us now turn to the GCN. The axiomatized version of Boolean algebra to be discussed is the version presented by Rosenbloom (17). His version has three undefined terms and three definitions. We discuss the first 22 theorems given by Rosenbloom. These theorems represent only a small portion of the theorems of Boolean algebra. In our presentation we increase the number of primitive terms to four. There are four definitions instead of three because one of Rosenbloom's definitions is multiple. Finally, 29 theorems are derived instead of Rosenbloom's 22. The extra theorems result from counting multiple theorems separately. Our numbering system for the axioms, definitions, and theorems corresponds roughly to Rosenbloom's.

The four Primary Notions in the GCN of the chosen axiomatization of Boolean algebra are as follows:

PN1: C
PN2: $\cap$
PN3: $'$
PN4: $=$

While parentheses are used throughout the theory, it will be assumed that the student population is conversant with standard methods of grouping. Similarly, it will be assumed that the student population understands the meaning of the equality relation "=". Since

grouping conventions are assumed to be known, and since it is assumed that "=" represents the standard equivalence relation, no axioms, explication, and so forth, of these concepts are required. Thus, the target population and the context of instruction have affected the GCN structure by eliminating the need for some elements.

An interpretation of Boolean algebra, of help in understanding the axiomatization, is obtained by assuming C to be the class of all the subsets of a given universal set. In this interpretation, "$\cap$" represents set intersection, and "$'$" represents set complementation with respect to the universal set. The GCN Basic Principles are as follows:

BP1. If α is in C, then α' and $\alpha \cap \beta$ are uniquely determined members of C.

BP2. If α and β are in C, then $\alpha \cap \beta = \beta \cap \alpha$.

BP3. If α, β, and γ are in C, then $(\alpha \cap \beta) \cap \gamma = \alpha \cap (\beta \cap \gamma)$.

BP4. If α, β, and γ are in C, and $\alpha \cap \beta' = \gamma \cap \gamma'$, then $\alpha \cap \beta = \alpha$.

BP5. If α, β, and γ are in C, and $\alpha \cap \beta = \alpha$, then $\alpha \cap \beta' = \gamma \cap \gamma'$.

BP6. If α and β are in C, and $\alpha = \beta$, then $\alpha' = \beta'$.

BP7. If α, β, and γ are in C, and $\alpha = \beta$, then $\alpha \cap \gamma = \beta \cap \gamma$ and $\gamma \cap \alpha = \gamma \cap \beta$.

It is important to notice that while BP1-BP7 are Basic Principles for the GCN, they are also axioms for the Boolean algebra.

The GCN contains four Secondary Notions:

SN1 : $\subset$ - proper subset
SN2 : 0 - the empty set
SN3 : 1 - the universal set
SN4 : $\cup$ - set union.

The equality sign in these phrases is used in a definitional sense and thus should be read "is defined as."

As Established Principles, the GCN contains the theorems of the Boolean algebra. In the deduction of the theorems in the axiomatization, it is assumed that the standard development of logic and set theory is known. For completeness, and for reference purposes, the subset of 29 theorems from the set of all possible theorems of Boolean algebra is given below. They are labeled EP1, EP2, . . . ; EP29. To save space the Green letters α, β, γ, δ are all elements of the set C in all the theorems in which these letters appear. This assumption merely reduces the length of the theorems as formulated, but for structuring purposes it must be remembered that C is used implicitly in each theorem. Quantifiers are also suppressed.

EP1: $\alpha \cap \alpha = \alpha$

EP2: $\alpha \cap \alpha' = \gamma \cap \gamma'$

EP3: $\alpha \subset \beta$ if and only if $\alpha \cap \beta' = 0$

EP4: $\alpha \subset \alpha$

EP5: If $\alpha \subset \beta$ and $\beta \subset \gamma$, then $\alpha \subset \gamma$

EP6: $\alpha \cap \beta \subset \alpha$

EP7: If $\alpha \subset \beta$ and $\beta \subset \alpha$, then $\alpha = \beta$

EP8: $\beta \cap 0 = 0$

EP9: $0 \subset \beta$

EP10: $\alpha'' = \alpha$

EP11: $\alpha \cap \beta = (\alpha' \cup \beta')'$

EP12: $\alpha \subset \beta$ if and only if $\beta' \subset \alpha'$

EP13: $\alpha \subset \beta$ if and only if $\alpha \cup \beta = \beta$

EP14: $\alpha \cup \beta = \beta \cup \alpha$

EP15: $(\alpha \cup \beta) \cup \gamma = \alpha \cup (\beta \cup \gamma)$

EP16: $\alpha \cup \alpha = \alpha$

EP17: $\alpha \cup \alpha' = 1$

EP18: $\alpha \subset \alpha \cup \beta$

EP19: $\alpha \cup (\alpha \cap \beta) = \alpha \cap (\alpha \cup \beta) = \alpha$

EP20: If $\alpha = \beta$, then $\alpha \cup \gamma = \beta \cup \gamma$

EP21: If $\alpha \subset \beta$, then $\alpha \cap \gamma \subset \beta \cap \gamma$

EP22: If $\alpha \subset \beta$, then $\alpha \cup \gamma \subset \beta \cup \gamma$

EP23: If $\gamma \subset \alpha$ and $\gamma \subset \beta$, then $\gamma \subset \alpha \cap \beta$

EP24: If $\alpha \subset \gamma$ and $\beta \subset \gamma$, then $\alpha \cup \beta \subset \gamma$

EP25: $\alpha \cap (\alpha' \cup \beta) = \alpha \cap \beta$

EP26: $\alpha \cap (\beta \cup \gamma) = (\alpha \cap \beta) \cup (\alpha \cap \gamma)$

EP27: $\alpha \cup 0 = \alpha$

EP28: $\alpha \cap 1 = \alpha$

EP29: $\alpha \cup 1 = 1$

At this point, the Primary Notions, Secondary Notions, Basic Principles, and Established Principles have been listed for the GCN. The GCN structuring relations remain to be described. We begin with F, the used in the formulation of relationship.

Consider first the Primary Notions. Figure 6 summarizes, in matrix form, the use of each of the Primary Notions in the formulation of each Basic Principle.

The information in Figure 6 is depicted graphically in Figure 7.

Primary Notions	Basic Principles						
	BP1	BP2	BP3	BP4	BP5	BP6	BP7
C	X	X	X	X	X	X	X
∩	X	X	X	X	X		X
′	X			X	X	X	
=		X	X	X	X	X	X

Figure 6. **Primary Notions used to formulate the Basic Principles**

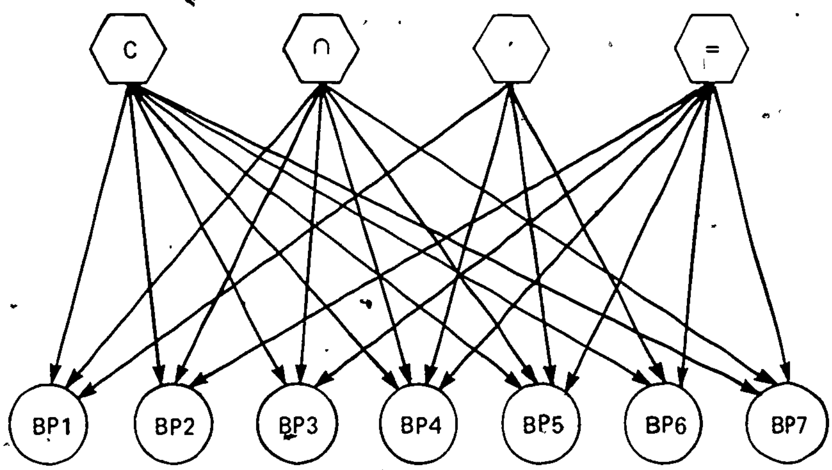

Figure 7. **Graphic representation of the use of Primary Notions in formulating Basic Principles**

Primary Notions are also used to formulate Secondary Notions, as shown in Figure 8. The Secondary Notions C, 0, ∪ are formulated solely in terms of Primary Notions, but Secondary Notion 1 uses another Secondary Notion, 0, in its formulation. For completeness, Figure 8 also shows how Secondary Notions are used to formulate other Secondary Notions. Figure 9 presents this portion of the relation F graphically. Secondary Notions are enclosed in triangles in Figure 9.

The final portion of F that needs to be depicted concerns the use of Primary and Secondary Notions in the formulation of the Established Principles. Figure 10 presents this information. Figure 11 combines the information contained in Figures 6, 8, and 10. Figure 12 presents a complete graphic representation of the "used in the formulation of" relationship. In Figure 12, a dot on the intersection of two lines indicates that they are connected. The lack of a dot indicates the lines merely cross and are not connected.

		Secondary Notions			
		⊂	0	1	∪
Primary Notions	c	X	X	X	X
	∩	X	X		X
	′		X	X	X
	=	X	X	X	X
Secondary Notions	⊂				
	0			X	
	1				
	∪				

Figure 8. **The use of Primary and Secondary Notions in the formulation of Secondary Notions**

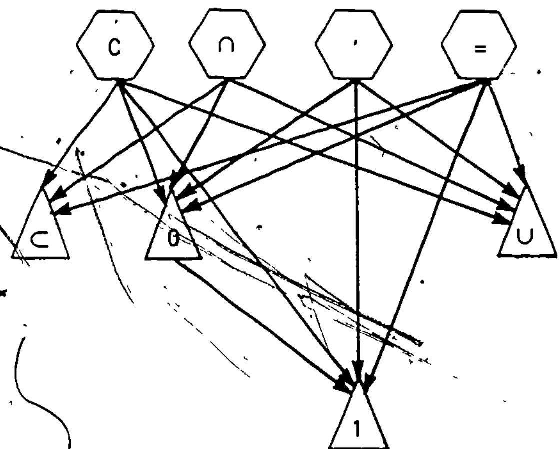

Figure 9. **Graphic representation of the use of Primary and Secondary Notions in formulating Secondary Notions**

To interpret Figure 12, consider, for example, the Primary Notion "′." Following down from "′" leads to a three-way connection. The left branch leads to the Basic Principles on the left. Following the connected lines leads to BP1, BP4, BP5, and BP6. Tracing the right branch below "′" leads to "0," "1," and "∪." Tracing the bottom line leads to EP2, EP3, EP10, EP11, EP12, EP17, and EP25. On the far right of Figure 12 we

Established Principles

		EP 1	EP 2	EP 3	EP 4	EP 5	EP 6	EP 7	EP 8	EP 9	EP 10	EP 11	EP 12	EP 13	EP 14	EP 15	EP 16	EP 17	EP 18	EP 19	EP 20	EP 21	EP 22	EP 23	EP 24	EP 25	EP 26	EP 27	EP 28	EP 29
Primary Notions	C	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
	∩	X	X	X			X		X			X								X		X		X		X	X		X	
	'		X	X							X	X	X					X								X				
	=	X	X	X				X	X		X	X		X	X	X	X	X		X	X					X	X	X	X	X
Secondary Notions	⊂			X	X	X	X	X		X			X	X					X			X	X	X	X				X	
	0			X					X	X																		X		
	1																	X												X
	∪													X	X	X	X	X	X	X	X		X		X	X	X	X		X

Figure 10. **The use of Primary and Secondary Notions in formulating Established Principles**

	BP 1	BP 2	BP 3	BP 4	BP 5	BP 6	BP 7	⊂	0	1	∪	EP 1	EP 2	EP 3	EP 4	EP 5	EP 6	EP 7	EP 8	EP 9	EP 10	EP 11	EP 12	EP 13	EP 14	EP 15	EP 16	EP 17	EP 18	EP 19	EP 20	EP 21	EP 22	EP 23	EP 24	EP 25	EP 26	EP 27	EP 28	EP 29
C	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
∩	X	X	X	X	X		X	X	X		X	X	X	X			X		X			X								X		X		X		X	X		X	
'	X			X	X	X			X	X	X		X	X							X	X	X					X								X				
=		X	X	X	X	X	X	X	X	X	X	X	X	X				X	X		X	X		X	X	X	X	X		X	X					X	X	X	X	X
⊂														X	X	X	X	X		X			X	X					X			X	X	X	X				X	
0										X				X					X	X																		X		
1																												X												X
∪																								X	X	X	X	X	X	X	X		X		X	X	X	X		X

Figure 11. **Constituents used to formulate Basic Principles, Secondary Notions, and Established Principles**

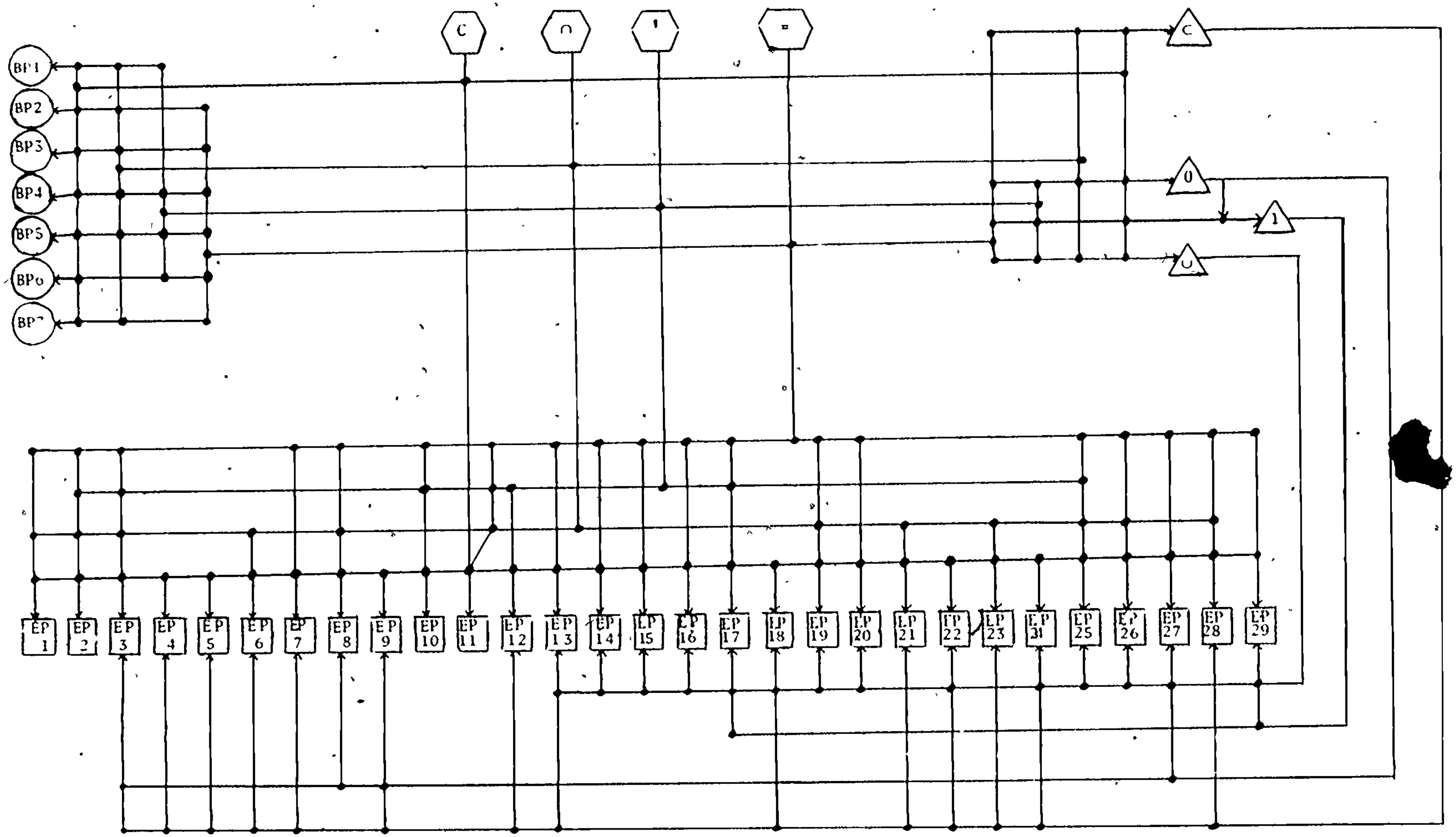

Figure 12. **Graphic representation of the "used in the formulation of" relation**

see that the defined concepts feed down into the theorems. For example, "1" is used in the formulation of EP17 and EP29.

Now consider the "used to establish" relationship. Since only theorems are established in the Boolean algebra, in the expression "*Exy*," only names of constituents that are Established Principles can be substituted for y, while x can be the name of any constituent (Primary Notion, Secondary Notion, Basic Principle, Established Principle). Since interest is in structure, concern is with the antecedents used in an Established Principle rather than with how the antecedents are usually used in a proof of the principle. To illustrate, consider Established Principle 8 of the Boolean algebra: "For all β in C, $\beta \cap 0 = 0$." The proof of the Established Principle can be arranged in steps with a listing of the Basic Principles, Definitions, and Established Principles used to justify each step. Thus, one possible proof of the Established Principle is as follows:

<u>EP8.</u> For all β in C, $\beta \cap 0 = 0$

<u>Proof:</u> $\beta \cap 0 = \beta \cap (\beta \cap \beta')$ D2

$= (\beta \cap \beta) \cap \beta'$ BP3

$= \beta \cap \beta'$ EP1

$= 0$ D2

Similar detailed proofs are presented in Appendix A for all 29 Established Principles of Boolean algebra. Figure 13 is one way of condensing this information. A column labeled by an Established Principle shows, by Xs in the rows, the Primary and Secondary Notions, Basic Principles, and Established Principles used in deriving the particular Established Principle.

In addition to the constituents cited above in the proof of EP8, the Primary Notions of C, $\cap$, $'$, and $=$ have also been used. Basic Principle 1 has been used also, and the Secondary Notion 0 plays a role in the proof. Thus, with respect to EP8 we have eight instances of the "used to establish" relation: viz: *Exy*, where x is one of C, $\cap$, $'$, $=$, BP1, BP3, 0, ~~or EP1,~~ and y is EP8.

As another example, consider the proof of Established Principle 9.

<u>EP9.</u> For all β in C, $0 \subset \beta$

<u>Proof:</u> $0 \cap \beta = \beta \cap 0$ BP2

$= 0$ EP8

$0 \subset \beta$ D1

As shown in Figure 13, the constituents used in this proof are C, $\cap$ $=$, BP1, BP2, $\subset$, and EP8. Notice in this proof that while the secondary Notion 0 is used, D2, the definition, is not cited explicitly. In a formalized subject matter such as Boolean algebra, the defined

	EP 1	EP 2	EP 3	EP 4	EP 5	EP 6	EP 7	EP 8	EP 9	EP 10	EP 11	EP 12	EP 13	EP 14	EP 15	EP 16	EP 17	EP 18	EP 19	EP 20	EP 21	EP 22	EP 23	EP 24	EP 25	EP 26	EP 27	EP 28	EP 29
G	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
O	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X		X		X	X	X	X	X
*	X	X	X					X		X	X	X	X	X	X	X	X	X		X					X	X	X	X	X
-	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
BP1	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X		X		X	X	X	X	X
BP2						X	X		X	X		X	X	X							X				X	X			
BP3					X	X		X							X						X		X		X	X			
BP4	X		X							X																			
BP5		X	X																										
BP6											X		X	X			X			X									
BP7					X	X					X	X			X		X				X		X		X	X			X
C			X	X	X	X	X		X	X		X	X					X	X		X	X	X	X	X	X			
0			X					X	X	X		X					X								X	X	X	X	X
1																	X											X	X
U											X		X	X	X	X	X	X	X	X		X		X	X	X	X	X	X
EP1	X	X		X		X		X								X					X	X							
EP2										X															X	X			
EP3										X		X													X	X			
EP4										X																			
EP5																													
EP6																		X	X										
EP7										X																			
EP8									X																				X
EP9																													
EP10											X	X	X	X	X	X	X								X	X		X	X
EP11														X	X											X		X	
EP12																		X											
EP13																			X			X	X						
EP14																			X			X				X			
EP15																						X							
EP16																													
EP17																													
EP18																			X							X			
EP19																									X		X		
EP20																													
EP21																										X			
EP22																													
EP23																													
EP24																										X			
EP25																										X			
EP26																													
EP27																												X	
EP28																													
EP29																													

Figure 13. **Constituents "used to establish" Established Principles**

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concepts are introduced as abbreviations for longer, more complex character strings. "0" is an abbreviation for $\gamma \cap \gamma'$, for example. Implicitly, to use "0" is to invoke definition D2 in the use of EP9. Thus the reference is suppressed in the proof, but Figure 13 asserts the use of 0 as appropriate.

Figure 14 provides a graphic representation of the information contained in Figure 13.

The adjacency matrices for the F and E relations are given in Figures 15 and 16 respectively.[1] The matrices in Figures 15 and 16 are combined by Boolean arithmetic to produce the adjacency matrix, DA for the dependency relation in the Boolean algebra GCN as shown in Figure 17.

Consider Established Principle 21. Reading down the EP21 column in Figure 17 one sees that EP21 is dependent on C, $\cap$ =, BP1, BP2, BP3, BP7, $\subset$, and EP1. Furthermore, this dependency is such that for each of the listed constituents there is a GCN relation leading from the constituent to EP21. From Figure 17, alone, it cannot be determined *which* GCN relation serves to relate each of these constituents to EP21. This information is contained in Figures 15 and 16.

In the case of the Boolean algebra, it was found that D^{10}=[0]. Thus, the maximal length dependency chain for the GCN is 9. That is, $m = 9$. For completeness, DA^2, DA^3, DA^4, DA^5, DA^6, DA^7, DA^8, and DA^9 are given in Appendix B. DA^9 is also given in Figure 18.

Notice that, according to Figure 18, EP28 is related to C by a nine-step chain of GCN relations. By way of an example this nine-step chain has been identified and is as follows.

EP28 depends on EP27
EP27 depends on EP19
EP19 depends on EP14
EP14 depends on EP11
EP11 depends on EP10
EP10 depends on EP4
EP4 depends on EP1
EP1 depends on BP4
BP4 depends on C

[1]The matrices presented in this example (including Appendix B) were taken from the output of a computer program especially prepared by Leslie Willis of HumRRO.

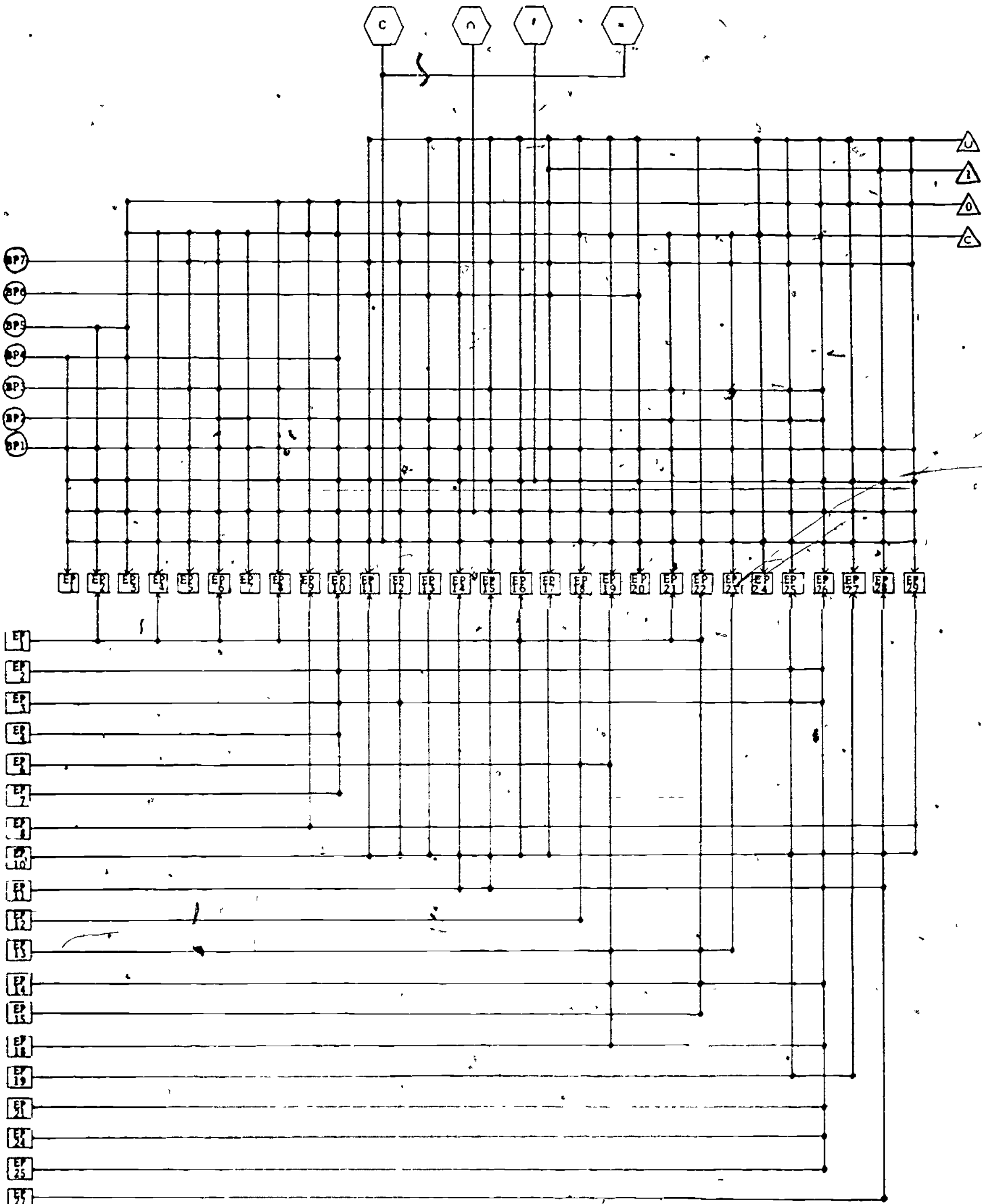

Figure 14. Graphic representation of the "used to establish" relation

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	C	∩	'	-	BP1	BP2	BP3	BP4	BP5	BP6	BP7	⊂	0	1	∪	EP1	EP2	EP3	EP4	EP5	EP6	EP7	EP8	EP9	EP10	EP11	EP12	EP13	EP14	EP15	EP16	EP17	EP18	EP19	EP20	EP21	EP22	EP23	EP24	EP25	EP26	EP27	EP28	EP29
C	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
∩	0	0	0	0	1	1	1	1	1	0	1	1	1	0	1	1	1	1	0	0	1	0	1	0	0	1	0	0	0	0	0	0	0	1	0	1	0	1	0	1	1	0	1	0
'	0	0	0	0	1	0	0	1	1	1	0	0	1	1	1	0	1	1	0	0	0	0	0	0	1	1	1	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0
-	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	1	1	0	1	1	0	1	1	1	1	1	0	1	1	0	0	0	0	1	1	1	1	1
BP1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
BP2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
BP3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
BP4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
BP5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
BP6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
BP7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
⊂	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0	1	0	0	1	1	0	0	0	0	1	0	0	1	1	1	1	0	0	0	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	[illegible]	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1
∪	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	0	1	0	1	1	1	1	0	1
LP1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
LP3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
LP4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
LP5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
LP6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
LP8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
LP9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
LP10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
LP11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
LP12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
LP13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
LP14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
LP15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
LP16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
LP17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
LP18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
LP19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
LP25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Figure 15. **A(F) Adjacency matrix for the "used in the formulation of" relation**

	C	∩	'	-	BP 1	BP 2	BP 3	BP 4	BP 5	BP 6	BP 7	⊂	0	1	∪	EP 1	EP 2	EP 3	EP 4	EP 5	EP 6	EP 7	EP 8	EP 9	EP 10	EP 11	EP 12	EP 13	EP 14	EP 15	EP 16	EP 17	EP 18	EP 19	EP 20	EP 21	EP 22	EP 23	EP 24	EP 25	EP 26	EP 27	EP 28	EP 29
C	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
∩	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	0	1	1	1	1	1
'	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	1	0	1	1	1	1	1	1	1	1	1	0	1	0	0	0	0	1	1	1	1	1
-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
BP1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	0	1	1	1	1	1
BP2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	1	1	0	1	1	1	0	0	0	0	0	0	1	0	0	0	1	1	0	0	0
BP3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	1	0	0	0	0	0	0	1	0	0	0	0	0	1	0	1	0	1	1	0	0	0
BP4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
BP5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	[illegible]	0	0	0	0	0	0	0	0	0	0	0	0	0	0
BP6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0
BP7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	1	1	0	0	1	0	1	0	0	0	1	0	1	0	1	1	0	0	1
⊂	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0	1	1	0	1	1	0	0	0	0	1	1	0	1	1	1	1	1	1	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	1	1	0	1	0	0	0	0	1	0	0	0	0	0	0	0	1	1	1	1	1
[illegible]	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	1
∪	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1	1	1	1	1	1	1	0	1	0	1	1	1	1	1	1
EP1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	[illegible]	1	0	1	0	1	0	0	0	0	0	0	[illegible]	1	0	0	0	0	1	1	0	0	0	0	0	0	0
EP2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	[illegible]	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0
EP3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0
EP4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0
EP7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
EP9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	0	0	0	0	0	0	0	1	1	0	1	1
EP11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	1	0	1	0
EP12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
EP13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	1	0	0	0	0	0	0
EP14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	1	0	0	0
EP15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
EP16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0
EP19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0
EP20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
EP22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
EP25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
EP26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
EP28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Figure 16. A(E) Adjacency matrix for the "used to establish" relation

	c	∩	•	•	BP 1	BP 2	BP 3	BP 4	BP 5	BP 6	BP 7	C	0	1	∪	EP 1	EP 2	EP 3	EP 4	EP 5	EP 6	EP 7	EP 8	EP 9	EP 10	EP 11	EP 12	EP 13	EP 14	EP 15	EP 16	EP 17	EP 18	EP 19	EP 20	EP 21	EP 22	EP 23	EP 24	EP 25	EP 26	EP 27	EP 28	EP 29
c	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
∩	0	0	0	0	1	1	1	1	1	0	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	0	1	1	1	1	1
•	0	0	0	0	1	0	0	1	1	1	0	0	1	1	1	1	1	1	0	0	0	0	1	0	1	1	1	1	1	1	1	1	1	0	1	0	0	0	0	1	1	1	1	1
•	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
BP1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	0	1	1	1	1	1
BP2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	1	1	0	1	1	1	0	0	0	0	0	0	1	0	0	0	1	1	0	0	0
BP3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	1	0	0	0	0	0	0	1	0	0	0	0	0	1	0	1	0	1	1	0	0	0
BP4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
BP5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
BP6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0
BP7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	1	1	0	0	1	0	1	0	0	0	1	0	1	0	1	1	0	0	1
C	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	[illegible]	0	1	1	1	1	1	0	1	1	0	1	1	0	0	0	0	1	1	0	1	1	1	1	1	1	0	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	1	1	1	0	1	0	0	0	0	1	0	0	0	0	0	0	0	1	1	1	1	1
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	1
∪	0	0	0	0	0	[illegible]	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1	1	1	1	1	1	1	0	1	0	1	1	1	1	1	1
EP1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	1	0	1	0	0	0	0	0	0	0	1	0	0	0	0	1	1	0	0	0	0	0	0	0
EP2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0
EP3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0
EP4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0
EP7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
EP9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	0	0	0	0	0	0	0	1	1	0	1	1
EP11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	1	0	1	0
EP12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
EP13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	1	0	0	0	0	0	0
EP14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	1	0	0	0
EP15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
EP16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0
EP19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0
EP20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
EP22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
EP25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
EP26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
EP28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Figure 17. **Adjacency matrix for the dependency relation in the Boolean algebra GCN**

	C	∩	'	=	BP1	BP2	BP3	BP4	BP5	BP6	BP7	⊂	0	1	∪	EP1	EP2	EP3	EP4	EP5	EP6	EP7	EP8	EP9	EP10	EP11	EP12	EP13	EP14	EP15	EP16	EP17	EP18	EP19	EP20	EP21	EP22	EP23	EP24	EP25	EP26	EP27	EP28	EP29
C	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0
∩	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0
'	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0
=	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0
BP1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
BP2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
BP3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
BP4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
BP5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
BP6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
BP7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
⊂	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
∪	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

D^9

Figure 18. **The Boolean algebra dependency relation adjacency matrix raised to the 9th power**

Figure 19 provides the complete dependency matrix for the Boolean algebra GCN. That is Figure 19 is the matrix for D, when

$$D = \left(\sum_{i=1,9} DA^i \right)^{\#}$$

Figure 19, then, presents a complete summary of the GCN dependency relationship. For example, as seen from Figure 19, EP20 is dependent on C, $\cap$, $'$, $=$, BP1, BP6, and $\cup$.

Our next task is to exhibit a TICS for the Boolean algebra. A subject such as Boolean algebra is usually explicated with the aid of a particular interpretation of the concepts, axioms, and so forth. The natural interpretation for the Boolean algebra is the algebra of subsets of a universal set. It will be assumed that the reader is familiar with this interpretation. Venn Diagrams will be assumed to be used to exhibit the union ($\cup$), intersection ($\cap$), and complement ($'$) of sets. We will exhibit tasks for each of the four types of GCN constituents: Primary Notions, Secondary Notions, Basic Principles, and Established Principles.

In the intended interpretation, C is conceived of as being a collection of sets. Tasks associated with C might be as follows:

t_1: Exhibit the set of positive odd integers less than 20.

t_2: Exhibit the set of positive primes less than 24.

t_3: Exhibit the set of English language vowels.

t_4: Exhibit the set of unicorns in the world.

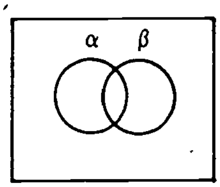
t_5: Let U = {a,b}. Exhibit all members of C.

t_6: Let U = {a,b,c}. Exhibit all members of C.

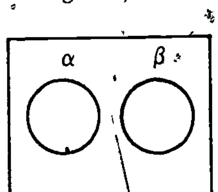
$\cap$ denotes set intersection, and as possible tasks we have the following:

t_7: State in words the meaning of $\alpha \cap \beta$.

t_8: In the following Venn Diagram, shade in $\alpha \cap \beta$.



t_9: In the following Venn Diagram, shade in $\alpha \cap \beta$.

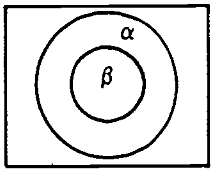


	C	∩	'	*	BP 1	BP 2	BP 3	BP 4	BP 5	BP 6	BP 7	C	0	1	∪	EP 1	EP 2	EP 3	EP 4	EP 5	EP 6	EP 7	EP 8	EP 9	EP 10	EP 11	EP 12	EP 13	EP 14	EP 15	EP 16	EP 17	EP 18	EP 19	EP 20	EP 21	EP 22	EP 23	EP 24	EP 25	EP 26	EP 27	EP 28	EP 29
C	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
∩	0	0	0	0	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
'	0	0	0	0	1	0	0	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
*	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
BP1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1
BP2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	0	1	1	1	1	1
BP3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	1	1	0	0	0	0	0	1	0	0	1	1	0	1	1	1	0	1	1	1	1	1
BP4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	1	0	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	0	[illegible]	1	1	1	1
BP5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	1	1	1	1	1
BP6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1	1	0	1	0	1	1	0	1	1	0	1	1	1	1	0
BP7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	1	1	0	1	1	0	1	1	1	0	1	1	1	0	1	1	1	1	1
C	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	1	1	1	1	1
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	1
∪	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1
EP1	0	0	0	[illegible]	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	1	0	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	0	1	1	1	1	1
EP2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	1	1	1	1	1
EP3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	1	1	1	1	1
EP4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	1	1	1	1	1
EP5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	1	1	1	1	0
EP7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	1	1	1	1	1
EP8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
EP9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	0	0	1	1	0	1	1	1	1	1
EP11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	1	0	0	1	0	0	1	1	1	1	0
EP12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	1	1	1	1	0
EP13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	1	0	1	1	1	1	0
EP14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	1	1	1	1	0
EP15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
EP16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	1	1	1	0
EP19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0
EP20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
EP22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
EP25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
EP26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
EP28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

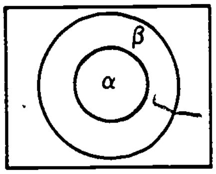
$$\underline{D} = \left(\sum_{i=1,9} \underline{DA}^{i}\right)^{*}$$

Figure 19. **Dependency matrix for the Boolean algebra GCN**

t_{10}: In the following Venn Diagram, shade in $\alpha \cap \beta$.



t_{11}: In the following Venn Diagram, shade in $\alpha \cap \beta$.



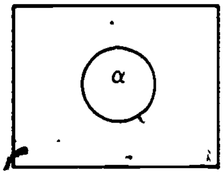
t_{12}: Let α be the set of all men and β be the set of all unmarried people. Describe $\alpha \cap \beta$.

t_{13}: Let α be the set of positive integers and let β be the set of all primes. Describe $\alpha \cap \beta$.

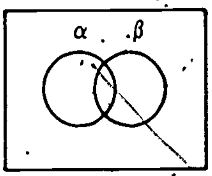
The symbol, $'$, denotes set complementation, and the tasks for $'$ also make use of Venn Diagrams.

t_{14}: State in words the meaning of α'.

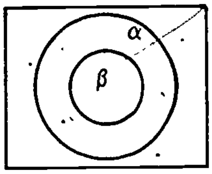
t_{15}: In the following Venn Diagram, shade in α'.



t_{16}: In the following Venn Diagram, shade in α.



t_{17}: In the following Venn Diagram, shade in β'.



t_{18}: Let the universal set be the set of all mammals, and let α be the set of four legged animals. Describe α'

t_{19}: Let the universal set be the set of all integers, and let α be the set of even positive numbers. Describe α'

We list four tasks associated with set equality, =.

t_{20}: Is the set of all mammals equal to the set of animals that bear their young alive?

t_{21}: Is $\{4,9,16,25\} = \{2^2,3^2,4^2,5^2\}$?

t_{22}: Is $\{a,b,c\} = \{c,b,a\}$?

t_{23}: Is $\{a,a\} = \{a\}$?

The notion underlying Basic Principle 1, BP1, is one of closure of the intersection and complement operations with respect to C.

t_{24}: Restate Axiom 1 of the Boolean algebra in words.

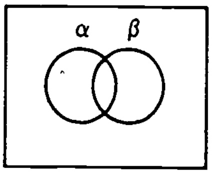
t_{25}: Let $U = \{1,2,3,4,5\}$ and let $\alpha = \{1,2\}$. What is α' and is it in C?

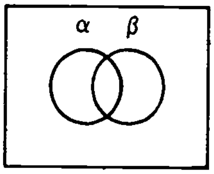
t_{26}: Let $U = \{a,b,c,\}$ and let $\alpha = \{a\}$. What is α' and is it in C?

t_{27}: Let $U = \{a,b,c,d\}$ and let $\alpha = \{a\}$, $\beta = \{a,b\}$. What is $\alpha \cap \beta$ and is it in C?

t_{28}: Let $U = \{1,2,3\}$ and let $\alpha = \{1,2\}$, $\beta = \{1,2\}$. What is $\alpha \cap \beta$ and is it in C?

It is not the purpose of this report to develop a complete set of tasks associated with Boolean algebra. Hence, we will not list tasks for the remaining six Basic Principles. It should be clear, however, that this could be done. These tasks would include describing the meaning of the principles in words and would rely to a great degree on Venn Diagrams. For example, to illustrate the principle behind commutativity, a student might be asked to shade in $\alpha \cap \beta$ in the following diagram, and to shade in $\beta \cap \alpha$ in the next diagram.





Now consider the Secondary Notion of subset, $\subset$.

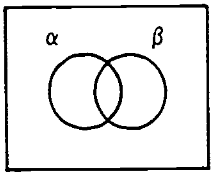
t_{29}: Interpret the meaning of $\alpha \subset \beta$, or α is a subset of β.

t_{30}: Illustrate the concept of a subset by Venn Diagrams.

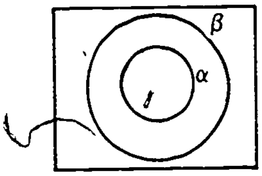
t_{31}: Is the set of animals a subset of the set of mammals?

t_{32}: Is the set of unicorns a subset of the set of horses?

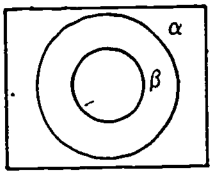
t_{33}: Is $\alpha \subset \beta$ in the following diagram?



t_{34}: Is $\alpha \subset \beta$ in the following diagram?



t_{35}: Is $\alpha \subset \beta$ in the following diagram?



t_{36}: Is the empty set a subset of any set?

t_{37}: Is any set a subset of itself?

t_{38}: Is any set except the empty set a subset of the empty set?

Similarly, tasks can easily be generated for the other three Secondary Notions: 0, 1, and $\cup$ (union). As before, such tasks would make use of verbal interpretations, Venn Diagrams, and direct questions. We will not develop these tasks herein.

Established Principles also have associated tasks. Established Principle 1 in the Boolean algebra asserts that $\alpha \cap \alpha = \alpha$.

t_{39}: State EP1 in words.

t_{40}: Prove EP1.

t_{41}: Using EP1 prove that $\alpha \cap (\beta \cap \alpha) = \alpha \cap \beta$.

t_{42}: Prove using EP1 that $(\alpha \cap \alpha)' = \alpha'$.

We will not develop tasks for each of the Established Principles. Notice that with these constituents, it is possible to not only have tasks concerned with the interpretation of the principle, but it is also possible to ask the individual to prove and apply the principle.

Thus, tasks t_1 through t_{42} represent a partial subset of the complete set of TICS Tasks. Notice that as in the choice of GCN constituents and relations, there is a great deal of latitude in the selection of tasks. As before, this seems reasonable since the individual preparing the instructional ~~material is free~~ to choose and formulate tasks as he desires.

To complete the example we will now consider how the Tasks can be structured into collections of tasks with each collection representing a criterion test. Consider the GCN constituent $\subset$ representing the subset notion and denote this constituent by c. Tasks t_{29} through t_{38} are concerned with this constituent. These tasks may be grouped into subsets as follows.

$$T_1 = \{t_{29}, t_{30}, t_{31}, t_{32}, t_{33}\}$$

$$T_2 = \{t_{29}, t_{30}, t_{33}, t_{34}, t_{35}\}$$

$$T_3 = \{t_{29}, t_{30}, t_{33}, t_{34}, t_{35}, t_{36}\},$$

$$T_4 = \{t_{29}, t_{33}, t_{34}, t_{35}, t_{36}, t_{37}, t_{38}\}$$

In this case, the set of criterions associated with c consists of T_1, T_2, T_3, and T_4; that is,

$$T(c) = \{T_1, T_2, T_3, T_4\}$$

Thus, if an individual can perform all tasks in one of T_1, T_2, T_3, *or* T_4, then it can be said that the individual comprehends the notion of subset. Conversely, if it is known that an individual comprehends the subset notion, then it is known that he can perform all tasks in at least one of the criterion sets T_1, T_2, T_3, or T_4. Furthermore, by EP9, we know that if an individual comprehends the notion of subset, then he also comprehends the notions of a set, intersection, and equals. This follows since subset depends on these notions as shown in Figure 19.

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AND
APPENDICES

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Appendix A

FORMAL PROOF OF GCN THEOREMS 1 THROUGH 29

FORMAL PROOF OF GCN THEOREMS 1 THROUGH 29

EP1. $\alpha \cap \alpha = \alpha$

$\alpha \cap \alpha' = \alpha \cap \alpha'$	BP4
$\alpha \cap \alpha = \alpha$	

EP2. $\alpha \cap \alpha' = \gamma \cap \gamma'$

$\alpha \cap \alpha = \alpha$	EP1
$\alpha \cap \alpha' = \gamma \cap \gamma'$	BP5

EP3. $\alpha \subset \beta$ if and only if $\alpha \cap \beta' = 0$

A. $\alpha \subset \beta$

$\alpha \cap \beta = \alpha$	SN1
$\alpha \cap \beta' = \gamma \cap \gamma'$	BP5
$\alpha \cap \beta' = 0$	SN2

B. $\alpha \cap \beta' = 0$

$\alpha \cap \beta' = \gamma \cap \gamma'$	SN2
$\alpha \cap \beta = \alpha$	BP4
$\alpha \subset \beta$	SN1

EP4. $\alpha \subset \alpha$

$\alpha \cap \alpha = \alpha$	EP1
$\alpha \subset \alpha$	SN1

EP5. $\alpha \subset \beta, \beta \subset \gamma$ implies $\alpha \subset \gamma$

$\alpha \cap \beta = \alpha$	SN1
$\beta \cap \gamma = \beta$	SN1.
$(\alpha \cap \beta) \cap \gamma = \alpha \cap (\beta \cap \gamma)$	BP3
$\alpha \cap \gamma = \alpha \cap \beta$	BP7
$= \alpha$	
$\alpha \subset \gamma$	SN1

EP6. $\alpha \cap \beta \subset \alpha$

$\alpha \cap \beta = \beta \cap \alpha$	BP2
$(\alpha \cap \beta) \cap \alpha = \alpha \cap (\beta \cap \alpha)$	BP3
$= \alpha \cap (\alpha \cap \beta)$	BP7
$= (\alpha \cap \alpha) \cap \beta$	BP3
$= \alpha \cap \beta$	EP1
$\alpha \cap \beta \subset \alpha$	SN1

EP7. $\alpha \subset \beta$ and $\beta \subset \alpha$ implies $\alpha = \beta$

$\alpha \cap \beta = \alpha$	SN1
$\beta \cap \alpha = \beta$	SN1
$\alpha \cap \beta = \beta \cap \alpha$	BP2
$\alpha = \beta$	

EP8. $\beta \cap 0 = 0$

$\beta \cap 0 = \beta \cap (\beta \cap \beta')$	SN2
$= (\beta \cap \beta) \cap \beta'$	BP3
$= \beta \cap \beta'$	EP1
$= 0$	

EP9. $0 \subset \beta$

$0 \cap \beta = \beta \cap 0$	BP2
$= 0$	EP8
$0 \subset \beta$	SN1

EP10. $\alpha'' = \alpha$

$\alpha'' \cap \alpha' = \alpha' \cap \alpha''$	BP2
$= 0$	EP2
$\alpha'' \cap \alpha = \alpha''$	BP4
$\alpha'' \subset \alpha$	SN1
* $\alpha''' \subset \alpha'$, $\alpha'''' \subset \alpha''$	BP6
$\alpha'''' \subset \alpha$	EP4
$\alpha'''' \cap \alpha' = 0$	EP3
$\alpha' \cap \alpha'''' = 0$	BP2
$\alpha' \subset \alpha'''$	EP3
$\alpha' = \alpha'''$	* and EP7
$\alpha \cap \alpha''' = 0$	EP2.
$\alpha \subset \alpha''$	EP3
$\alpha = \alpha''$	EP7

EP11. $\alpha \cap \beta = (\alpha' \cup \beta')'$

$\alpha' \cup \beta' = (\alpha'' \cap \beta'')'$	SN4
$(\alpha' \cup \beta')' = (\alpha'' \cap \beta'')''$	BP6
$= \alpha \cap \beta$	EP10, BP7

EP12. $\alpha \subset \beta$ if and only if $\beta' \subset \alpha'$

A. $\alpha \subset \beta$

$\alpha \cap \beta' = 0$	EP3
$\beta' \cap \alpha = 0$	BP2
$\beta' \cap \alpha'' = 0$	EP10, BP7
$\beta' \subset \alpha'$	EP3

B. $\beta' \subset \alpha'$

$\alpha'' \subset \beta''$	part A
$\alpha \subset \beta$	EP10

EP13. $\alpha \subset \beta$ if and only if $\alpha \cup \beta = \beta$

$\alpha \subset \beta$ if and only if $\beta' \subset \alpha'$	EP12
if and only if $\beta' \cap \alpha' = \beta'$	SN1
if and only if $\beta = \beta'' = (\beta' \cap \alpha')'$	EP10, BP6
if and only if $\beta = (\alpha' \cap \beta')'$	BP2
if and only if $\beta = \alpha \cup \beta$	SN4

EP14. $\alpha \cup \beta = \beta \cup \alpha$

$\alpha \cup \beta = (\alpha' \cap \beta')'$	SN4
$= (\beta' \cap \alpha')'$	BP2, BP6
$= (\beta'' \cup \alpha'')''$	EP11
$= \beta \cup \alpha$	EP10

EP15. $(\alpha \cup \beta) \cup \gamma = \alpha \cup (\beta \cup \gamma)$

$(\alpha \cup \beta) \cup \gamma = [(\alpha' \cap \beta')' \cup \gamma]$	SN4
$= [(\alpha' \cap \beta')'' \cap \gamma']'$	SN4
$= [(\alpha' \cap \beta') \cap \gamma']'$	EP10, BP7
$= [\alpha' \cap (\beta' \cap \gamma')]'$	BP3
$= [\alpha'' \cup (\beta' \cap \gamma')']''$	EP11
$= \alpha \cup (\beta' \cap \gamma')'$	EP10
$= \alpha \cup (\beta'' \cup \gamma'')''$	EP11
$= \alpha \cup (\beta \cup \gamma)$	EP10

EP16. $\alpha \cup \alpha = \alpha$

$\alpha \cup \alpha = (\alpha' \cap \alpha')'$	SN4
$= \alpha''$	EP1
$= \alpha$	EP10

EP17. $\alpha \cup \alpha' = 1$

$\alpha \cup \alpha' = (\alpha' \cap \alpha'')'$	SN4
$= (\alpha' \cap \alpha)'$	EP10, BP7
$= 0'$	SN2, BP6
$= 1$	SN3

EP18. $\alpha \subset \alpha \cup \beta$

$(\alpha \cup \beta)' = (\alpha' \cap \beta')''$	SN4
$= \alpha' \cap \beta'$	EP10
$\subset \alpha'$	EP6
$\alpha \subset \alpha \cup \beta$	EP12

EP19. $\alpha \cup (\alpha \cap \beta) = \alpha \cap (\alpha \cup \beta) = \alpha$

A. $\alpha \cap \beta \subset \alpha$	EP6
$(\alpha \cap \beta) \cup \alpha = \alpha$	EP13
$\alpha \cup (\alpha \cap \beta) = \alpha$	EP14
B. $\alpha \subset \alpha \cup \beta$	EP18
$\alpha \cap (\alpha \cup \beta) = \alpha$	SN1

EP20. If $\alpha = \beta'$, then $\alpha \cup \gamma = \beta \cup \gamma$

Let $\alpha = \beta$

$\alpha \cup \gamma = (\alpha' \cap \gamma')'$	SN4
$= (\beta' \cap \gamma')'$	BP6
$= \beta \cup \gamma$	SN4

EP21. If $\alpha \subset \beta$, then $\alpha \cap \gamma \subset \beta \cap \gamma$

$$\begin{aligned}
(\alpha \cap \gamma) \cap (\beta \cap \gamma) &= \alpha \cap [\gamma \cap (\beta \cap \gamma)] && \text{BP3}\\
&= \alpha \cap [(\gamma \cap \beta) \cap \gamma] && \text{BP3}\\
&= \alpha \cap [(\beta \cap \gamma) \cap \gamma] && \text{BP2}\\
&= \alpha \cap [\beta \cap (\gamma \cap \gamma)] && \text{BP3}\\
&= (\alpha \cap \beta) \cap (\gamma \cap \gamma) && \text{BP3}\\
&= (\alpha \cap \beta) \cap \gamma && \text{EP1, BP7}\\
&= \alpha \cap \gamma && \text{SN1, } \alpha \subset \beta\\
\alpha \cap \gamma \subset \beta \cap \gamma & && \text{SN1}
\end{aligned}$$

EP22. If $\alpha \subset \beta$, then $\alpha \cup \gamma \subset \beta \cup \gamma$

$$\begin{aligned}
(\alpha \cup \gamma) \cup (\beta \cup \gamma) &= \alpha \cup [\gamma \cup (\beta \cup \gamma)] && \text{EP15}\\
&= \alpha \cup [(\gamma \cup \beta) \cup \gamma] && \text{EP15}\\
&= \alpha \cup [(\beta \cup \gamma) \cup \gamma] && \text{EP14}\\
&= \alpha \cup [\beta \cup (\gamma \cup \gamma)] && \text{EP15}\\
&= (\alpha \cup \beta) \cup (\gamma \cup \gamma) && \text{EP15}\\
&= (\alpha \cup \beta) \cup \gamma && \text{EP1}\\
&= \beta \cup \gamma && \text{EP13, } \alpha \subset \beta\\
\alpha \cup \gamma \subset \beta \cup \gamma & && \text{EP13}
\end{aligned}$$

EP23. If $\gamma \subset \alpha$ and $\gamma \subset \beta$, then $\gamma \subset \alpha \cap \beta$

$$\begin{aligned}
\gamma \cap (\alpha \cap \beta) &= (\gamma \cap \alpha) \cap \gamma && \text{BP3}\\
&= \gamma \cap \beta && \text{BP7, D1, } \gamma \subset \alpha\\
&= \gamma && \text{SN1, } \gamma \subset \beta\\
\alpha \subset \beta \cap \gamma & && \text{SN1}
\end{aligned}$$

EP24. If $\alpha \subset \gamma$ and $\beta \subset \gamma$, then $\alpha \cup \beta \subset \gamma$

$$\begin{aligned}
(\alpha \cup \beta) \cup \gamma &= \alpha \cup (\beta \cup \gamma) && \text{BP3}\\
&= \alpha \cup \gamma && \text{EP13, } \beta \subset \gamma\\
&= \alpha && \text{EP13, } \alpha \subset \gamma\\
\alpha \cup \beta \subset \gamma & && \text{EP13}
\end{aligned}$$

EP25. $\alpha \cap (\alpha' \cup \beta) = \alpha \cap \beta$

$$\begin{aligned}
[\alpha \cap (\alpha' \cup \beta)] \cap \beta' &= [\alpha \cap (\alpha'' \cap \beta')'] \cap \beta' && \text{SN4}\\
&= [\alpha \cap (\alpha \cap \beta')'] \cap \alpha' && \text{EP10, BP7}\\
&= \alpha \cap [(\alpha \cap \beta')' \cap \alpha'] && \text{BP3}\\
&= \alpha \cap [\beta' \cap (\alpha \cap \beta')'] && \text{BP2}\\
&= (\alpha \cap \beta') \cap (\alpha \cap \beta')' && \text{BP3}\\
&= \alpha \cap \alpha' && \text{EP2}\\
&= 0 && \text{SN1}
\end{aligned}$$

$$\begin{aligned}
\text{Thus, } \alpha \cap (\alpha' \cup \beta) \subset \beta & && \text{EP3}\\
\alpha \cap (\alpha' \cup \beta) &= [\alpha \cap (\alpha' \cup \beta)] \cap \beta && \text{SN1}\\
&= \alpha \cap [(\alpha' \cup \beta) \cap \beta] && \text{BP3}\\
&= \alpha \cap [\beta \cap (\beta \cup \alpha')] && \text{BP2}\\
&= \alpha \cap \beta && \text{EP19}
\end{aligned}$$

EP26. $\alpha \cap (\beta \cup \gamma) = (\alpha \cap \beta) \cup (\alpha \cap \gamma)$

A.

*	$\beta \subset \beta \cup \gamma$	EP18
	$\gamma \subset \gamma \cup \beta$	EP18
**	$\gamma \subset \beta \cup \gamma$	EP14
	$\beta \cap \alpha \subset (\beta \cup \gamma) \cap \alpha$	EP21, *
	$\alpha \cap \beta \subset \alpha \cap (\beta \cup \gamma)$	BP2
	$\gamma \cap \alpha \subset (\beta \cup \gamma) \cap \alpha$	EP21, **
	$\alpha \cap \gamma \subset \alpha \cap (\beta \cup \gamma)$	BP2
	$(\alpha \cap \beta) \cup (\alpha \cap \gamma) \subset \alpha \cap (\beta \cup \gamma)$	EP24

B.

$\alpha \cap (\beta \cup \gamma) \cap [(\alpha \cap \beta) \cup (\alpha \cap \gamma)]'$	
$= \alpha \cap (\beta \cup \gamma) \cap [(\alpha \cap \beta)' \cap (\alpha \cap \gamma)']''$	SN4
$= \alpha \cap (\beta \cup \gamma) \cap [(\alpha \cap \beta)' \cap (\alpha \cap \gamma)']$	EP10, BP7
$= \alpha \cap (\beta \cup \gamma) \cap [\alpha' \cup \beta') \cap (\alpha' \cup \gamma')]$	EP11
$= (\beta \cup \gamma) \cap [\alpha \cap (\alpha' \cup \beta')] \cap (\alpha' \cup \gamma')$	BP2, BP3
$= (\beta \cup \gamma) \cap (\alpha \cap \beta') \cap (\alpha' \cup \gamma')$	EP25
$= (\beta \cup \gamma) \cap \beta' \cap [\alpha \cap (\alpha' \cup \gamma')]$	BP2, BP3
$= (\beta \cup \gamma) \cap \beta' \cap (\alpha \cap \gamma')$	EP25
$= \beta' \cap (\beta \cup \gamma) \cap (\alpha \cap \gamma')$	BP2
$= (\beta' \cap \gamma) \cap (\alpha \cap \gamma')$	EP25
$= (\beta' \cap \alpha) \cap (\gamma \cap \gamma')$	BP2, BP3
$= (\beta' \cap \alpha) \cap (\alpha \cap \alpha')$	EP2
$= (\beta' \cap \alpha) \cap 0$	SN2
$= 0$	EP8
$\alpha \cap (\beta \cup \gamma) \subset [(\alpha \cap \beta) \cup (\alpha \cap \gamma)]$	EP3
$\alpha \cap (\beta \cup \gamma) = (\alpha \cap \beta) \cup (\alpha \cap \gamma)$	EP7, part 1, part 2

EP27. $\alpha \cup 0 = \alpha$

$\alpha \cup 0 = \alpha \cup (\alpha \cap \alpha')$	SN2
$= \alpha$	EP19

EP28. $\alpha \cap 1 = \alpha$

$\alpha \cap 1 = \alpha \cap 0'$	SN3
$= (\alpha' \cup 0'')'$	EP11
$= (\alpha' \cup 0)'$	EP10
$= \alpha''$	EP27
$= \alpha$	EP10

EP29. $\alpha \cup 1 = 1$

$\alpha \cup 1 = \alpha \cup 0'$	SN3
$= (\alpha' \cap 0'')'$	SN4
$= (\alpha' \cap 0)'$	EP10, BP7
$= 0'$	EP8
$= 1$	SN3

Appendix B

MATRICES FOR THE POWERS OF THE DEPENDENCY ADJACENCY MATRIX DA

This appendix contains the matrices for the powers of the dependency adjacency matrix, DA, for the Boolean algebra. The eight matrices are for $\underline{DA}^2$, $\underline{DA}^3$, $\underline{DA}^4$, $\underline{DA}^5$, $\underline{DA}^6$, $\underline{DA}^7$, $\underline{DA}^8$, $\underline{DA}^9$.

	C	∩	'	=	BP 1	BP 2	BP 3	BP 4	BP 5	BP 6	BP 7	C	0	1	U	EP 1	EP 2	EP 3	EP 4	EP 5	EP 6	EP 7	EP 8	EP 9	EP 10	EP 11	EP 12	EP 13	EP 14	EP 15	EP 16	EP 17	EP 18	EP 19	EP 20	EP 21	EP 22	EP 23	EP 24	EP 25	EP 26	EP 27	EP 28	EP 29
C	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
∩	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
'	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
=	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
BP1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	1	0	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	0	1	1	1	1	1
BP2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	1	1	0	1	1
BP3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	1	0	0	1	0	0	0	1	0	0	1
BP4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	1	0	1	0	1	1	1	1	1	1	1	1	0	0	0	1	1	0	0	1	1	0	1	1
BP5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0
BP6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	1	0	0	1	1	0	0	1	0	1	0
BP7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	1	1	0	0	1	0	0	0	1	0	1	0
C	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	1	1	1	1	1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	1	1	0	1	1
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
U	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	1	0	0	1	1	0	1	1	1	1	0
EP1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	1	1	0	0	0	0	0	1	1	0	0	1
EP2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	0	0	0	0	0	0	0	1	1	0	1	1
EP3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0	0	0	1	1	0	1	1
EP4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	0	0	0	0	0	0	0	1	1	0	1	1
EP5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	1	1	0	0
EP7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	0	0	0	0	0	0	0	1	1	0	1	1
EP8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	0	1	0
EP11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	1	0	0	0
EP12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0
EP13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0
EP14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0
EP15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0
EP19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0
EP20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

$\frac{DA^2}{68}$

	C	∩	'	=	BP 1	BP 2	BP 3	BP 4	BP 5	BP 6	BP 7	⊂	0	1	∪	EP 1	EP 2	EP 3	EP 4	EP 5	EP 6	EP 7	EP 8	EP 9	EP 10	EP 11	EP 12	EP 13	EP 14	EP 15	EP 16	EP 17	EP 18	EP 19	EP 20	EP 21	EP 22	EP 23	EP 24	EP 25	EP 26	EP 27	EP 28	EP 29
C	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	1	0	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	0	1	1	1	1	1
∩	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	1	0	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	0	1	1	1	1	1
'	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	1	0	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	0	1	1	1	1	1
=	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	1	0	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	0	1	1	1	1	1
BP1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	1	1	1	1	1
BP2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	0	0	1	1	0	1	1	1	1	1
BP3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	1	1	0	0
BP4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	1	1	0	1	1
BP5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0	0	0	1	1	0	1	1
BP6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	1	1	1	0	0
BP7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	[illegible]	1	1	0	0
⊂	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	0	0	1	1	0	1	1	1	1	1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	0	0	1	1	0	1	1	0	1	1
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
∪	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	1	1	1	1	0
EP1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	0	1	0	0	0	0	0	1	1	1	1	1
EP2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	0	1	0
EP3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	0	1	0
EP4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	0	1	0
EP5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0
EP7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	0	1	0
EP8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	1	1	1	0	0
EP11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0
EP12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0
EP13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0
EP14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0
EP15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0
EP19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

	C	∩	·	=	BP 1	BP 2	BP 3	BP 4	BP 5	BP 6	BP 7	⊂	0	ı	∪	EP 1	EP 2	EP 3	EP 4	EP 5	EP 6	EP 7	EP 8	EP 9	EP 10	EP 11	EP 12	EP 13	EP 14	EP 15	EP 16	EP 17	EP 18	EP 19	EP 20	EP 21	EP 22	EP 23	EP 24	EP 25	EP 26	EP 27	EP 28	EP 29
C	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	1	1	1	1	1
∩	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	1	1	1	1	1
·	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	1	1	1	1	1
=	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	1	1	1	1	1
BP1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	0	0	1	1	0	1	1	1	1	1
BP2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	1	1	0	0	1	1	0	1	1	1	1	0
BP3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0
BP4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	0	0	1	1	0	1	1	1	1	1
BP5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	0	1	0
BP6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0
BP7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
⊂	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	1	1	0	0	1	1	0	1	1	1	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	1	1	0	0	1	1	0	1	1	1	1	0
ı	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
∪	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0
EP1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	1	1	0	0	1	1	0	1	1	1	1	0
EP2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	1	1	1	0	0
EP3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	1	1	1	0	0
EP4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	1	1	1	0	0
EP5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0
EP7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	1	1	1	0	0
EP8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0
EP11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0
EP12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0
EP13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

DA 4

	C	∩	·	=	BP 1	BP 2	BP 3	BP 4	BP 5	BP 6	BP 7	C	0	1	∪	EP 1	EP 2	EP 3	EP 4	EP 5	EP 6	EP 7	EP 8	EP 9	EP 10	EP 11	EP 12	EP 13	EP 14	EP 15	EP 16	EP 17	EP 18	EP 19	EP 20	EP 21	EP 22	EP 23	EP 24	EP 25	EP 26	EP 27	EP 28	EP 29
C	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	0	0	1	1	0	1	1	1	1	1
∩	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	0	0	1	1	0	1	1	1	1	1
·	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	0	0	1	1	0	1	1	1	1	1
=	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	0	0	1	1	0	1	1	1	1	1
BP1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	1	1	0	0	1	1	0	1	1	1	1	0
BP2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	1	1	1	1	0
BP3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	[illegible]	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0
BP4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	1	1	0	0	1	1	0	1	1	1	1	0
BP5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	1	1	1	0	0
BP6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0
BP7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0
C	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	1	1	1	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	1	1	1	1	0
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
∪	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	1	1	1	1	0
EP2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0
EP3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0
EP4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0
EP5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0
EP8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0
EP11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

	C	∩	'	•	BP 1	BP 2	BP 3	BP 4	BP 5	BP 6	BP 7	C	0	1	∪	EP 1	EP 2	EP 3	EP 4	EP 5	EP 6	EP 7	EP 8	EP 9	EP 10	EP 11	EP 12	EP 13	EP 14	EP 15	EP 16	EP 17	EP 18	EP 19	EP 20	EP 21	EP 22	EP 23	EP 24	EP 25	EP 26	EP 27	EP 28	EP 29
C	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	1	1	0	0	1	1	0	1	1	1	1	0
∩	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	1	1	0	0	1	1	0	1	1	1	1	0
'	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	1	1	0	0	1	1	0	1	1	1	1	0
•	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	1	1	0	0	1	1	0	1	1	1	1	0
BP1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	1	1	1	1	0
BP2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0
BP3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
BP4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	1	1	1	1	0
BP5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0
BP6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
BP7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
C	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
U	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0
EP2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0
EP3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0
EP4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0
EP5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0
EP8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

	C	∩		=	BP 1	BP 2	BP 3	BP 4	BP 5	BP 6	BP 7	⊂	0	1	∪	EP 1	EP 2	EP 3	EP 4	EP 5	EP 6	EP 7	EP 8	EP 9	EP 10	EP 11	EP 12	EP 13	EP 14	EP 15	EP 16	EP 17	EP 18	EP 19	EP 20	EP 21	EP 22	EP 23	EP 24	EP 25	EP 26	EP 27	EP 28	EP 29
C	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	1	1	1	1	0
∩	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	1	1	1	1	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	1	1	1	1	0
=	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	1	1	1	1	0
BP1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0
BP2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0
BP3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
BP4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0
BP5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0
BP6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
BP7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
⊂	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
∪	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0
EP2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

DA[7]
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	C	∩	'	=	BP1	BP2	BP3	BP4	BP5	BP6	BP7	⊂	0	1	∪	EP1	EP2	EP3	EP4	EP5	EP6	EP7	EP8	EP9	EP10	EP11	EP12	EP13	EP14	EP15	EP16	EP17	EP18	EP19	EP20	EP21	EP22	EP23	EP24	EP25	EP26	EP27	EP28	EP29
C	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0
∩	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0
'	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0
=	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0
BP1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0
BP2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
BP3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
BP4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0
BP5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
BP6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
BP7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
⊂	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
∪	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

	c	∩		=	BP 1	BP 2	BP 3	BP 4	BP 5	BP 6	BP 7	c	0	1	∪	EP 1	EP 2	EP 3	EP 4	EP 5	EP 6	EP 7	EP 8	EP 9	EP 10	EP 11	EP 12	EP 13	EP 14	EP 15	EP 16	EP 17	EP 18	EP 19	EP 20	EP 21	EP 22	EP 23	EP 24	EP 25	EP 26	EP 27	EP 28	EP 29
C	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0
∩	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0
=	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0
BP1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
BP2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
BP3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
BP4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
BP5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
BP6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
BP7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
c	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
∪	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

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