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ABSTRACT

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Abstract

Strategies for Rotating Canonical Components

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Strategies for Rotating
Canonical Components

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When listening to discussions between investigators who use canonical analysis, one occasionally hears speculation about the possibility of rotating the canonical components to ease and clarify interpretation of the results. The realization of this possibility can probably be traced to the work of Bartlett (1948), who pointed out the similarities between factor analysis and canonical analysis, and of Meredith (1964), who developed the idea of canonical component structure coefficients. Since rotation is a standard and useful procedure in factor analysis and since there are substantial similarities between factor analysis and canonical analysis, it would seem that rotation procedures might fruitfully be applied to the canonical components as an aid to interpretation. (For the purposes of this discussion we will assume that the diagonal entries in the correlation matrix are all unity, so we are technically dealing with rotation of principal components.)

However, the problem of rotation is more complex in canonical analysis because there are two sets of variables. The concepts of simple structure and psychological meaningfulness are relatively straightforward in a principal components analysis because there is only one set of variables. The criteria for the location of factors, both original and rotated, are contained within the single set, and the interpretations refer only to that set. Canonical analysis provides two sets of component loadings, each determined by relations existing between the sets as well as within the sets. Interpretation of the

components ma, involve both between-set and within-set relationships.

A multitude of criteria and procedures have been advanced for the rotation of factors, but all have as their objective simplicity of structure and increased meaningfulness. These criteria might be applied to the problem of rotation in canonical analysis as well. However, none of the existing approaches address the problem of how to handle the relationships between the two sets of variables. Rotation in canonical analysis is logically different from the factor matching problem.

Before attacking canonical rotation, it is first necessary to consider some of the conditions which exist at the outset of any rotation problem. In both factor analysis and canonical analysis a decision must be made regarding the number of factors or components to retain for rotation. This decision determines the proportion of variance of each variable which is to be included in the analysis, and hence, the total variance available. A large array of rules of thumb, some statistical in nature, some psychometric, and some logical, exist to aid in making this important decision in factor analysis. In contemplating the rotation of canonical components, it would seem reasonable to retain only those pairs of variates which have statistically significant canonical correlations since one of the objectives of canonical analysis is to identify pairs of components which are correlated.

Using this decision rule has an important consequence. It limits the range of correlations among pairs of rotated components. Selecting the number components defines the space available for rotation, which of course, is also true in principal components analysis. We may view canonical analysis as similar to a principal components analysis in the sense that in either case the selection of

a particular number of components defines the common variance of each variable. The difference is that in canonical analysis there are two of these "communalities" associated with each variable, one given by the sum of squared loadings on the composites within its set (which I will call intraset communalities), and one given by the sum of squares of its loadings on the composites of the other set (which I christen interset communalities). Thus, if there are $\underline{p}$ variables in one set and $\underline{q}$ variables in the other, there are 2 ($\underline{p+q}$) constraints or constants restricting any rotation, with $\underline{p+q}$ of them affecting either set of components.

These communality conditions have an additional consequence for rotation. The total redundancy of the first set with the second and of the second with the first must remain constant under rotation because each is the mean of the sum of its respective interset communalities. Since an alternative definition of redundancy (Stewart and Love, 1968) is the sum of the means of the sums of the squared intraset loadings multiplied by their respective squared canonical correlations, and the intraset communalities must be constant, then the sums of squared canonical correlations must be constant. Given this restraint, and since the original canonical correlations are successively maximum correlations within the conditions of orthogonality, the canonical correlations between pairs of components after orthogonal rotation will fall in the range between the highest and lowest significant original correlations.

With these limits and assurances, we may consider three possible approaches to the rotation of canonical components. For purposes of illustration, let us assume that there are three significant canonical correlations, and, therefore, three pairs of components. The first approach might be to rotate the entire set of six components to some more satisfactory solution. Second, one might

rotate each set of three components independently to new locations. The third possibility would be to select one of the sets for rotation to an improved solution and then rotate the second set to the location which yields successive maximum correlations with the components of the first set. Let us consider each of these strategies in somewhat more detail.

When the components of both sets are rotated simultaneously to a single overall solution, the rotation is constrained by both the interest and intraset communalities. This complex set of constraints may make the rotation difficult. However, the more important question is whether this type [illegible] is logically defensible. The two sets of variables presumably have been kept separate for a reason. If an investigator is interested in the structure of the combined sets, then he probably should have performed a traditional factor analysis in the first place. Given that the sets are logically distinct, ignoring this distinction in the rotation step would seem to be an error. A combined simple structure for the canonical components is a logical contradiction.

The second strategy, that of rotating the components of each set to an independent solution, is logically consistent with the separation of sets in canonical analysis. Once a satisfactory structure is found within the set, the new canonical correlations may be computed. However, the new components in one set may now have complex relations with the components of the other set. The criterion of maximum relationship must be sacrificed for clarity of structure within set. One may reasonably ask whether it might not be better to factor analyze the two sets independently and, after separate rotations, compute the correlations between factor scores. The major difference between these two proposals is that in the first case the overlap between the sets is put to maximum use in determining the spaces defined by the composites, while in the

second it is ignored. Either procedure could be defended; however, the former would seem preferable where overlap between the sets is being considered.

In those situations where the structure of one of the sets is of primary interest and the structure of the other set is desired with reference to the first, and would seem advisable to use the third rotation strategy outlined above. Assume, for example, that one has data from several measures of achievement as one set of variables and some personality measures as the other set. It might be desirable to find an interpretable structure for the achievement measures and then determine the composites of the personality variables which best predict those components which were significantly correlated. Then, the components of the achievement set would be rotated, perhaps by varimax or some other algorithm, to their new positions. Finally, the components of the personality set would be rotated to new positions, subject to the constraint that each component have the highest possible correlation with one of the already rotated achievement components. The necessary equations are presented ~~in the handout~~.

There are two major advantages which would accrue from this procedure. First, although the criterion of maximum relationship between pairs of components would no longer hold precisely, this criterion would be met within the limitations imposed by the positions of the components in the first-rotated set. Second, giving the rotation of the first set, this procedure would provide a unique solution for the second set.

Of the three approaches discussed, the third seems most in keeping with the spirit of canonical analysis. The first may be rejected outright as logically inconsistent with fundamental assumption that two sets of variables are being analyzed. Traditional factor analysis of the combined data would be the appro-

priate procedure. The second approach disregards the criterion of maximum relationship. It may be appropriate in some situations while independent factorings may be the method of choice in others. Only the third strategy retains the fundamental principles of canonical analysis while at the same time seeking descriptive clarity.

The basic equation of canonical analysis is

$$(R_{22}^{-1} R_{21} R_{11}^{-1} R_{12} - \lambda_i I) v_i = 0$$

subject to the restriction that

$$v_i' R_{22} v_i = 1,$$

where v_i is the vector of canonical weights for the ith canonical variate of set 2. The vectors of weights, u_i, for the variables in set 1 are found from

$$u_i = \frac{R_{11}^{-1} R_{12} v_i}{\sqrt{\lambda_i}}$$

and the matrices of canonical loadings are obtained by

$$S_{11} = R_{11} U \quad \text{and} \quad S_{22} = R_{22} V.$$

Applying an orthogonal transformation T to S_{22} we obtain rotated component loadings

$$\hat{S}_{22} = S_{22} T$$

and

$$V = R_{22}^{-1} S_{22}$$

provides the weights for the rotated variates in set 2.

Letting $G = R_{22}^{-1} R_{21} R_{11}^{-1} R_{12}$, $G \hat{v}_i - \lambda_i' I \hat{v}_i = 0$

we can solve for λ_i'. Then

$$\hat{u}_i = \frac{R_{11}^{-1} R_{12} \hat{v}_i}{\sqrt{\lambda_i'}} \quad \text{and}$$

$$S_{11} = R_{11} U$$

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