This paper initially presents the results of several studies concerning what kind of writing mathematics teachers assign and what kind of writing mathematics textbooks assign. By far, report-research was the most popular type of writing assigned in the surveyed textbooks. The types of reports students were asked to write include biography, history, exposition, analysis, and cause and effect. Several exemplary paragraphs are given and analyzed to demonstrate the different organization and structure required by the different writing modes. Several teaching strategies are delineated for encouraging students to create quality writing: make the students aware of the type of reading, thinking, and writing the topic demands; give the students diagramed model paragraphs to show how ideas relate; and encourage students to compose and not copy by basing the assignment on more than one source, controlling the source, and conducting paraphrasing exercises in class. (TS)
Today most teachers realize that reading instruction doesn't stop at the primary grades, that, in fact, every teacher should be perceived as a teacher of reading—a content specialist. Could not the same case be made for writing? If content area teachers are the most appropriate ones to teach students to read texts, would they not also be the most appropriate ones to teach the content area writing that they or their textbooks assign?

Mathematics Teachers Do Assign Writing

A recent survey (Donlan, 1974) indicated that teachers in all content areas assign writing—even mathematics teachers. Almost half of the mathematics teachers (n = 11) surveyed assigned some sort of writing, other than essay tests and short answer questions. In fact three types of writing were assigned: narration (18% of teachers surveyed), exposition (35%), and reporting (45%), even though there was a tendency for these assignments to be brief (under 300 words) and infrequent (1-6/year). Generally, these surveyed mathematics teachers tended to assign writing as an extension of class work, rather than as extra credit or enrichment. And although 55 percent of the surveyed math teachers felt that writing was the responsibility of the English teacher, 45% felt that the teaching of writing was to be shared by the content area teacher.
Certainly a population of 11 mathematics teachers is an inadequate sample on which to justify or describe teaching or assigning composition in math classes, though the sample was only a subset of 123 teachers drawn from a large metropolitan area. However, there was evidence that some mathematics teachers assigned, and sometimes taught, writing in the content area of mathematics.

One might logically question the type of writing appropriate for a class primarily focused on quantitative, not verbal, skills. A survey of recent mathematics textbooks can supply specific types of assignments that can be made. Out of 195 mathematics textbooks housed at the two curriculum centers at UCR, 9 listed writing assignments that required composition skills (See Appendix for list of texts). Table 1 indicates the number of writing assignments by text by writing type:

<table>
<thead>
<tr>
<th>Text</th>
<th>Grade</th>
<th>Level</th>
<th>¥ of Assignments</th>
<th>Style</th>
<th>Report</th>
<th>Research</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7</td>
<td>1</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>18</td>
<td>6</td>
<td>6</td>
<td>0</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>18</td>
<td>8</td>
<td>6</td>
<td>2</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
<td>6</td>
<td>6</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>18</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<td>7</td>
<td>10</td>
<td>10</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>18</td>
<td>9</td>
<td>9</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>7</td>
<td>7</td>
<td>7</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>16</td>
<td>7</td>
<td>7</td>
<td>0</td>
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</tr>
<tr>
<td>10</td>
<td>4</td>
<td>15</td>
<td>58</td>
<td>47</td>
<td>11</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>2-10</td>
<td>0-10</td>
<td>0-6</td>
<td>5.2</td>
<td>1.2</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>6.4</td>
<td>0-6</td>
<td>0-6</td>
<td>5.2</td>
<td>1.2</td>
<td></td>
</tr>
</tbody>
</table>
Types of Writing/Content of Writing

Of the 58 assignments, 47 were labeled as "reports": e.g., "Report on the contributions of Blaise Pascal," "Report on the abacus and other ancient computing devices." An additional 11 assignments were labeled "research" or "investigate": e.g., "Research the Babylonian numerical system." "Investigate how ancient Egyptians, Babylonians, Greeks, and Romans represent rational numbers." Although one can argue the semantic differences between "report" and "research," it is this investigator's impression that the texts were asking the students to do the same thing: read secondary sources and summarize the findings. Nevertheless, Table 2 lists the assignments by type (report-research) and by subject matter (people, concepts, other).

Table 2. Writing Assignments from Nine Mathematics Textbooks/Type/Subject Matter

<table>
<thead>
<tr>
<th>Reports</th>
<th>Textbook</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. People</td>
<td></td>
</tr>
<tr>
<td>A. Phyllotaxis</td>
<td>1</td>
</tr>
<tr>
<td>B. George-Cantor</td>
<td>3</td>
</tr>
<tr>
<td>C. Rene Descartes</td>
<td>3, 4, 7</td>
</tr>
<tr>
<td>D. John Napier</td>
<td>4, 5</td>
</tr>
<tr>
<td>E. Carl Frederick Gauss</td>
<td>4</td>
</tr>
<tr>
<td>F. Eratosthenes</td>
<td>4</td>
</tr>
<tr>
<td>G. Blaise Pascal</td>
<td>5, 7, 9</td>
</tr>
<tr>
<td>H. Henry Briggs (with Napier)</td>
<td>5</td>
</tr>
<tr>
<td>I. Zeno</td>
<td>6, 8</td>
</tr>
<tr>
<td>J. Euclid</td>
<td>6, 7, 8</td>
</tr>
<tr>
<td>K. Thales</td>
<td>6, 8</td>
</tr>
<tr>
<td>L. Archimedes</td>
<td>6, 8</td>
</tr>
<tr>
<td>M. Pierpont (with Lobachevski)</td>
<td>7</td>
</tr>
<tr>
<td>N. Pythagoras</td>
<td>7, 9</td>
</tr>
<tr>
<td>O. Leonard Euler</td>
<td>7, 9</td>
</tr>
<tr>
<td>P. George Mendal</td>
<td>9</td>
</tr>
</tbody>
</table>
II. Mathematical Concepts

A. Perfect numbers, amicable numbers, Fibonacci sequence of numbers 3, 6

B. Abacus and other ancient computing devices 3

C. History of our numerals 3

D. Ancient numbers system 3

E. History of negative numbers 3

F. Representation of rational numbers by Egyptians, Babylonians, Greeks, and Romans 3, 8

G. History of equations 3

H. Polar coordinates 3

I. Fractions: Babylonians, Greeks, and Egyptians 3

J. Probabilities of height when dwarf and giant pea plants are crossed 3

K. Use of Roman numerals in modern times 4

L. Egyptians' contributions to geometry 4

M. Greeks' contributions to geometry 4

N. Septimal (base seven) system 4

O. History of standards of measure 4

P. History of one or more units of measure 4

Q. Differences among avoirdupois, troy and apothecaries' weights 4

R. History of the thermometer 4

S. Pascal's triangle and relation to study of probability 5

T. Number e/history of development of logarithms 5
I. History and development of Hindu-Arabic system of numeration

II. Prime numbers

III. Names and symbols for negative numbers used in the past

IV. Symbols for zero through the ages

V. Egyptian symbols for fractions

VI. History of the calendar

VII. Triangular and square numbers

VIII. "Squaring a circle" and its relationship to \(\pi\)

IX. Babylonians, Egyptians, and Hindu's contribution to development of algebra

X. History and development of decimal

XI. Modular systems and their independence from fractions

XII. Decimal and decimal point

XIII. Irrational numbers/decimal system

XIV. Transfinite number (\(\aleph_0\))

III. Other

a. Review Flatland, by E. A. Abbott

b. Functions and activities of the U.S. Bureau of Standards

c. Modern electronic computers

d. History of calendar

e. Careers in math

f. Collect information on interest rates for large companies buying on time
### People

<table>
<thead>
<tr>
<th>A. George Cantor</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>B. Rene Descartes</td>
<td>2</td>
</tr>
<tr>
<td>C. George Boall</td>
<td>2</td>
</tr>
<tr>
<td>D. Pierre Fermat</td>
<td>2</td>
</tr>
<tr>
<td>E. Henri Poincare</td>
<td>2</td>
</tr>
<tr>
<td>F. Leonhard Euler</td>
<td>2</td>
</tr>
<tr>
<td>G. Blaise Pascal</td>
<td>2</td>
</tr>
<tr>
<td>H. Karl Gauss</td>
<td>2</td>
</tr>
<tr>
<td>I. Pierre de Laplace</td>
<td>2</td>
</tr>
<tr>
<td>J. Daniel Bernoulli</td>
<td>2</td>
</tr>
<tr>
<td>K. Abraham Denaire</td>
<td>2</td>
</tr>
</tbody>
</table>

### Mathematical Concepts

<table>
<thead>
<tr>
<th>A. Babylonian numerical system</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>B. Greek numerical systems</td>
<td>2</td>
</tr>
<tr>
<td>C. Mayan numerical systems</td>
<td>2</td>
</tr>
</tbody>
</table>

### Other

<table>
<thead>
<tr>
<th>A. Maps, types of projections</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>B. U.S. Geodetic Survey</td>
<td>1</td>
</tr>
<tr>
<td>C. U.S. Office of Statistical Standards, Bureau of Census and federal agencies</td>
<td>2</td>
</tr>
</tbody>
</table>

### Assigning vs. Teaching Writing

Although an examination of the nine mathematics textbooks indicates that writing is assigned, there is little or no evidence to indicate that writing is, or should be, taught. For example, out of 58 assignments, there are only 7 instances where "helps," or enabling suggestions are made, but none of the suggestions concerning the form of the writing—merely the content: e.g., giving a source to read or posing a provocative question. Asking the students in mathematics to write reports without enabling suggestions is comparable to requiring students in geography to compute average daily rainfall without providing a formula: in both cases students are presumed to have skills they might not really have.
In examining Table 2, one can see that students are asked to write biographies (e.g., "report on the life of Leonardo da Vinci"), history (e.g., "report on the history of one or more units of measurement"), exposition (e.g., "report on Pascal's triangle and its relation to the study of probability"), analysis (e.g., "report on the differences among avoirdupois, troy, and apothecaries' weights"), cause and effect (e.g., "report on modular systems and their independence from fractions"),--to mention a few modes of thought and discourse. In reality, each mode demands a different organization, perhaps even a different paragraph structure. Consider these two paragraphs from mathematics content writing:

(1) Biography

Fermat, Pierre de (1601-1665), a French mathematician, won fame for his work in the theory of numbers or integers. He also shared in the invention of analytic geometry and calculus. He formulated the least-time law to explain the diffraction (bending) of light, and also developed an equation for the graph of a straight line. His "last theorem" has never been proved or disproved. Fermat knew integral solutions of the equation \( x^2 + y^2 = z^2 \) (for example, \( 3^2 + 4^2 = 5^2 \)). His theorem held that there was no whole number solution of \( x^n + y^n = z^n \) if the exponent, \( n \), is larger than 2. Fermat, along with Blaise Pascal, is credited with originating the theory of probability, now widely used in insurance and statistics. Fermat practiced law in Toulouse and studied mathematics only as a hobby. He was born in Beaumont-de-Lompagne.

Between 600 and 300 B.C., the Greeks took the next great step in mathematics. They inherited a large part of their mathematical knowledge from the Babylonians but they became the first people to separate mathematics from practical problems. For example, they separated geometry from practical applications and made it into an abstract exploration of space. They based this study of points, lines, and figures, such as triangles and circles, on logical reasoning rather than on facts found in nature. Thales of Miletus (c. 640-546 B.C.), a philosopher, helped begin this viewpoint of geometry. The philosopher Pythagoras (c. 580-c. 500 B.C.) and his followers explored the nature of numbers. Euclid (c. 300 B.C.), one of the foremost Greek mathematicians, organized geometry as a single logical system. His book, The Elements, remains one of the basic works in studying mathematics. (From World Book Encyclopedia (c. 1972), Vol. 13, p. 242.)

On the surface both paragraphs appear to be similar in length and content. However, a closer examination of the paragraphs (after Christenson, 1967) indicates that the paragraphs are quite different:

1. Biography
   - Fermat, Pierre de (1601-1655), a French mathematician, won fame for his work in the theory of numbers or integers.
   - He also shared in the invention of analytic geometry and calculus.
   - He formulated the least-time law to explain the diffraction (bending of light, and also developed an equation for the graph of a straight line.
   - His "last theorem" has never been proved or disproved.
Fermat knew integral solutions of the equation $x^2 + y^2 = z^2$
(for example, $3^2 + 4^2 = 5^2$).

His theorem held that there was no whole number solution of
$x^n + y^n = z^n$ if the exponent, $n$, is larger than 2.

Main 1. Fermat, along with Blaise Pascal, is credited with originating the
idea
theory of probability now widely used in insurance and statistics.

Main 1. Fermat practiced law in Toulouse and studied mathematics only as a
idea
hobby.

Main 1. He was born in Beaumont-de-Lomagne.

(2) History

Main 1. Between 600 and 300 B.C., the Greeks took the next great step
idea
in mathematics.

2. They inherited a large part of their mathematical knowledge
from the Babylonians.

3. But, they became the first people to separate mathematics from
practical problems.

3. For example, they separated geometry from practical applica-
coordinate
ations and made it into an abstract exploration of space.

4. They based this study of points, lines, and figures,
coordinate
such as triangles and circles, on logical reasoning
rather than facts found in nature.

5. Thales of Mileus (c. 640-546 B.C.), a philosopher,
coordinate
helped begin this viewpoint of geometry.

5. The philosopher Pythagorus (c. 580-c. 500 B.C.) and
his followers explored the nature of numbers...
5. Euclid (c. 300 B.C.), one of the foremost Greek mathematicians organized geometry as a single logical system.

6. His book *The Elements* remains one of the basic works in studying mathematics.

Passage 1--biography--contains 7 main ideas with little or no subordinating statements. Whereas passage 2--history--contains only one main idea with 8 subordinating sentences involving 5 levels of subordination. Passage 1 is almost a list; whereas passage 2 is a fairly in-depth discussion. Therefore, "report" is not a functional word, but list and discuss are. Consequently, "reporting" on X's life and contributions will involve "lower level," more general writing than will "reporting" on the historical development of concept Y.

Writing From Text, Not Copying

If it is understood that (1) reporting includes many modes of discourse, from low level composing, i.e., listing, to high level composing, i.e., analyzing and synthesizing and (2) more precise guide words (e.g., list, discuss, show causes and effects) can give students a clearer understanding of what they are to do, the mathematics teachers problem is to keep students from copying directly from encyclopedias, or other secondary/tertiary sources. Since under such circumstances (1) students are not in all probability learning any content and (2) they might as well xerox the source, hand it in, or read from it. Both mastery of content and acquisition of writing skill would be best served if the assignment were made in such a way as to preclude copying. Here are some methods for ensuring original writing.
The mathematics teacher might--

1. **Base the assignment on more than one source** so the student will have to assimilate the material.

2. **Control the sources**, that is, limit the references to only a few sources that every student has access to; even xerox the sources for the entire class.

3. **Conduct brief paraphrasing exercises** in class in which students synthesize two sentences relating identical or similar content into one original sentence.

**Conclusion**

The purpose of this article has not been to defend writing as part of the mathematics curriculum. However, the author has noted that some mathematics teachers and some mathematics textbooks assign writing as an extension of classwork. If writing is assigned, it might also well be taught. Several teaching strategies that are not time-consuming would insure higher quality writing and, thus, more effective learning:

1. **Make the student aware** of the type of reading, thinking, and writing the topic demands.

2. **If possible give students model paragraphs**, diagrammed, to show how ideas relate in, for instance, biography, history, and exposition.

3. **Encourage students to engage in original writing by** (a) basing the assignment on more than one source, (b) controlling the source, (c) conducting paraphrasing exercises in class.
REFERENCES


APPENDIX

Mathematics Texts Used in Survey


5. Smith, Eugene P.; Calder, George A.; Mehl, William G.; Rasmussen, Dean S.; and Steinen, Ramon F. Discoveries in Modern Mathematics, Course 2. Columbus, Ohio, Charles E. Merrill Books, Inc., 1968.


