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AUTHOR	Fuller, Robert G., Ed.; And Others
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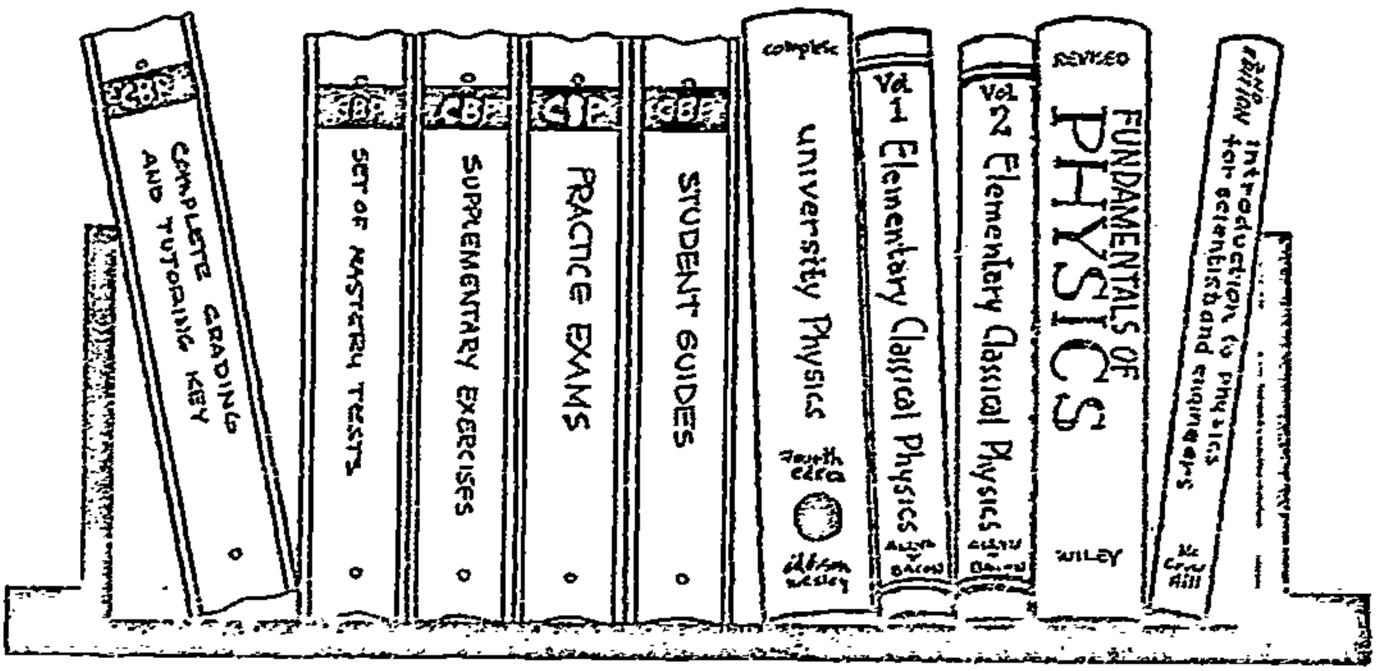
ABSTRACT

This is part of a series of 42 Calculus Based Physics (CBP) modules totaling about 1,000 pages. The modules include study guides, practice tests, and mastery tests for a full-year individualized course in calculus-based physics based on the Personalized System of Instruction (PSI). The units are not intended to be used without outside materials; references to specific sections in four elementary physics textbooks appear in the modules. Specific modules included in this document are: Module 41--Lenses and Mirrors, Module 42--Relativity, and an Appendix. (CP)

STUDY MODULES FOR CALCULUS-BASED GENERAL PHYSICS*



CBP Workshop
Behlen Laboratory of Physics
University of Nebraska
Lincoln, NE 68508

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Comments

These modules were prepared by fifteen college physics professors for use in self-paced, mastery-oriented, student-tutored, calculus-based general physics courses. This style of teaching offers students a personalized system of instruction (PSI), in which they increase their knowledge of physics and experience a positive learning environment. We hope our efforts in preparing these modules will enable you to try and enjoy teaching physics using PSI.

Robert G. Fuller
Director
College Faculty Workshop

MODULE AUTHORS

OWEN ANDERSON	Bucknell University
STEPHEN BAKER	Rice University
VAN BLEUMEL	Worcester Polytechnic Institute
FERNAND BRUNSCHWIG	Empire State College
DAVID JOSEPH	University of Nebraska - Lincoln
ROBERT KARPLUS	University of California - Berkeley
MICHAEL MOLONEY	Rose Hulman Institute of Technology
JACK MUNSEE	California State University - Long Beach
GARY NEWBY	Boise State University
IVOR NEWSHAM	Olivet Nazarene College
WILLIAM SNOW	University of Missouri - Rolla
WILLARD SPERRY	Central Washington State College
ROBERT SWANSON	University of California - San Diego
JAMES TANNER	Georgia Institute of Technology
DAVID WINCH	Kalamazoo College

These modules were prepared by the module authors at a College Faculty Workshop held at the University of Colorado - Boulder, from June 23 to July 11, 1975.

Workshop Staff

Albert A. Bartlett	University of Colorado
Thomas C. Campbell	Illinois Central College
Harold Q Fuller	University of Missouri - Rolla

Calculus-Based Physics (CBP) Modules Production Staff

Robert G. Fuller	Editor
Thomas C. Campbell	Assistant Editor
William D. Snow	Illustrator
Catherine A. Caffrey	Production Editor

COMMENT TO USERS

In the upper right-hand corner of each Mastery Test you will find the "pass" and "recycle" terms and a row of numbers "1 2 3 ..." to facilitate the grading of the tests. We intend that you indicate the weakness of a student who is asked to recycle on the test by putting a circle around the number of the learning objective that the student did not satisfy. This procedure will enable you easily to identify the learning objectives that are causing your students difficulty.

COMMENT TO USERS

It is conventional practice to provide several review modules per semester or quarter, as confidence builders, learning opportunities, and to consolidate what has been learned. You the instructor should write these modules yourself, in terms of the particular weaknesses and needs of your students. Thus, we have not supplied review modules as such with the CBP Modules. However, fifteen sample review tests were written during the Workshop and are available for your use as guides. Please send $1.00 to CBP Modules, Behlen Lab of Physics, University of Nebraska - Lincoln, Nebraska 68588.

FINIS

This printing has completed the initial CBP project. We hope that you are finding the materials helpful in your teaching. Revision of the modules is being planned for the Summer of 1976. We therefore solicit your comments, suggestions, and/or corrections for the revised edition. Please write or call

CBP WORKSHOP
Behlen Laboratory of Physics
University of Nebraska
Lincoln, NE 68588

Phone (402) 472-2790
(402) 472-2742

LENSES AND MIRRORS

INTRODUCTION

If you have ever worn glasses, used a magnifying glass, looked through a telescope, or looked in a mirror, you have some idea of the effect that transmitting and reflecting materials have on light. When light passes from one material to another it is refracted. It is this property of light that is used in making eye glasses and magnifying glasses. The laws of reflection and refraction have immediate application in the construction of optical instruments. Two main objectives of most optical devices are to increase the light-gathering area and to provide a magnified image. Magnification is not usually the only requirement. For instance, the important characteristic of a large astronomical telescope is its diameter, which allows the telescope to gather more light, allow shorter exposures, and give higher resolution.

In this module we shall apply the laws of reflection and refraction to two types of simple devices, thin lenses and spherical mirrors. Understanding of these two simple devices sets the stage for the module Optical Instruments, which utilizes various combination of lenses and mirrors to produce the instruments' desired effects.

PREREQUISITES

Before you begin this module, you should be able to:	Location of Prerequisite Content
*Solve problems using the law concerning the reflection of light rays from a surface (needed for Objective 1 of this module)	Reflection and Refraction Module
*Solve problems using Snell's law concerning a light ray passing from one substance to another (needed for Objective 2 of this module)	Reflection and Refraction Module

LEARNING OBJECTIVES

After you have mastered the content of this module, you will be able to:

1. Spherical mirrors - Solve problems involving a single spherical mirror by drawing ray diagrams and/or by applying the mirror equation. In addition you may be asked to calculate the lateral magnification.

2. Thin lenses - Solve problems involving a single thin lens by drawing ray diagrams and/or by applying the thin-lens equation. In addition you may be asked to calculate the lateral magnification.

GENERAL COMMENTS

In this module, you need to know the equations describing:

(1) The relationship between the image distance, the object distance, and the focal length for spherical mirrors.

(2) The relationship between the focal length and the radius of curvature of a spherical mirror.

(3) The lateral magnification of an object by a spherical mirror.

(4) The relationship between the image distance, the object distance, and the focal length for thin lenses.

(5) The relationship between the focal length, the radii of curvatures, and the indices of refraction of the lens material and the medium.

(6) The lateral magnification of an object by a thin lens.

You can accomplish this by memorizing the relevant equations along with the sign conventions, or you can study the derivations in order to understand them well enough to derive the equations yourself. We recommend this latter course since knowledge of the derivation of these equations can be used to consider more complex situations.

In addition, you will be expected to know and apply the rules for drawing ray diagrams for both spherical mirrors and thin lenses.

Lateral magnification is magnification in the direction perpendicular to the optical axis of the mirror or lens. Longitudinal magnification is magnification in the direction parallel to the optical axis of the mirror or lens.

To avoid confusion, note that this module uses the following notation:

n = index of refraction.	s = object distance.
i = size of image.	s' = image distance.
o = size of object.	R = radius of curvature.
M = magnification.	

TEXT: Frederick J. Bueche, Introduction to Physics for Scientists and Engineers (McGraw-Hill, New York, 1975), second edition

SUGGESTED STUDY PROCEDURE

Read Sections 30.4 through 30.6 and 30.9 through 30.11 in Chapter 30. Then study Problems A and B and Illustrations 30.2 through 30.4 and 30.6 through 30.10 before working Problems C and D and Problems 3, 9, 13, and 19 in Chapter 30.

The important equations, keyed to number in the General Comments, are:

(1) Eq. (30.4) - p. 589. (2) Eq. (30.3) - p. 591. (3) Eq. 30.5) - p.
Sign conventions on p. 591, ray-diagram rules on pp. 592, 593.

(4) Eq. (30.15) - p. 601. (5) Eq. (30.14) - p. 601. (6) Eq. (30.16) - p. 602.
Sign conventions on p. 599, ray-diagram rules on p. 602.

Statement 2 in Section 30.11 is misleading, as is Figure 30.21(e). The conclusion that a ray passing through the center of a thin lens is essentially undeviated is correct; however, it is not necessarily true that "a ray through the center of the lens enters and leaves through parallel faces." Nor is it in general true that "such a ray is undeviated." For thin lenses (with the assumption of small angles, etc.) the faces are nearly (if not exactly) parallel, and the deviation of the beam is negligible.

Take the Practice Test, and work some Additional Problems if necessary, before attempting a Mastery Test.

BUECHE

Objective Number	Readings	Problems with Solutions: Study Guide	Problems with Solutions: Text	Assigned Problems: Study Guide	Assigned Problems: Text (Chap. 30)	Additional Problems (Chap. 30)
1	Secs. 30.4 to 30.6	A	Illus.[a] 30.2, 30.3, 30.4	C	Quest.[a] 1, 12, Probs. 3, 9	Quest. 2, 5, 17, Probs. 4 to 8
2	Secs. 30.9 to 30.11	B	Illus. 30.6 to 30.10	D	Quest. 11, Probs. 13, 19	Quest. 3, 4, 7 to 10, Probs. 10, 11, 12, 17, 18, 20, 21

[a]Illus. = Illustration(s). Quest. = Question(s).

TEXT: David Halliday and Robert Resnick, Fundamentals of Physics (Wiley, New York, 1970; revised printing, 1974)

SUGGESTED STUDY PROCEDURE

Read Chapter 36, Sections 36-8 through 36-10. Answer questions 21, 24, and 30, study Problems A and B and Examples 6, 9, and 10, and work Problems C, D, and 33, 43, and 48 in Chapter 36.

The important equations, keyed to the numbers in the General Comments, are:

(1) Eq. (36-17) - p. 683. (2) Unnumbered equation preceding Eq. (36-17) - p. 683.
Sign conventions on p. 683, ray-diagram rules on p. 685.

(3) Eq. (36-18) - p. 686. (4) Eq. (36-31) - p. 693. (5) Eq. (36-30) - p. 692.

(6) Eq. (36-32) - p. 693. Sign conventions on p. 688, ray-diagram rules p. 693.

Take the Practice Test, and work some Additional Problems if necessary, before attempting a Mastery Test.

HALLIDAY AND RESNICK

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text (Chap. 36)	Study Guide	Text (Chap. 36)	(Chap. 36)
1	Sec. 36-8	A	Ex.[a] 6	C	Quest.[a] 21, Prob. 33	Quest. 19, 20, Prob. 34
2	Secs. 36-9, 36-10	B	Ex. 9, 10	D	Quest. 24, 30, Probs. 43, 48	Quest. 22, 23, 25 to 29, 31, 32, Probs. 40, 41, 42, 44, 47, 52

[a]Ex. = Example(s). Quest. = Question(s).

TEXT: Francis Weston Sears and Mark W. Zemansky, University Physics (Addison-Wesley, Reading, Mass., 1970), fourth edition

SUGGESTED STUDY PROCEDURE

Read Chapter 39, Sections 39-3 through 39-5 and Chapter 40, Sections 40-1 through 40-4, 40-6, and 40-8. Study Problems A and B and the Examples listed in the Table. Then work Problems C and D and Problems 39-8, 40-10, and 40-17.

The important equations, keyed to the numbers in the General Comments, are:

(1) Eq. (39-9) - p. 563. (2) Eq. (39-8) - p. 563. (3) Eq. (39-6) - p. 559.
Sign conventions, p. 558. Ray-diagram rules, p. 564.

(4) Eq. (40-3) - p. 575. (5) Eq. (40-2) - p. 574. (6) Eq. (40-4) - p. 576.
Sign conventions, p. 558. Ray-diagram rules, p. 578.

Take the Practice Test, and work some Additional Problems if necessary, before trying a Mastery Test.

SEARS AND ZEMANSKY

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text	Study Guide	Text	
1	Secs. 39-3 to 39-5	A	Examples 1 and 2 in Sec. 39-3, Example in Sec. 39-5	C	39-8	39-5, 39-6, 39-7, 39-9
2	Secs. 39-7, 40-1 to 40-4, 40-6, 40-8	B	Example in Sec. 40-2	D	40-10, 40-17	40-9, 40-11, 40-13, 40-14

TEXT: Richard T. Weidner and Robert L. Sells, Elementary Classical Physics (Allyn and Bacon, Boston, 1973), second edition, Vol. 2

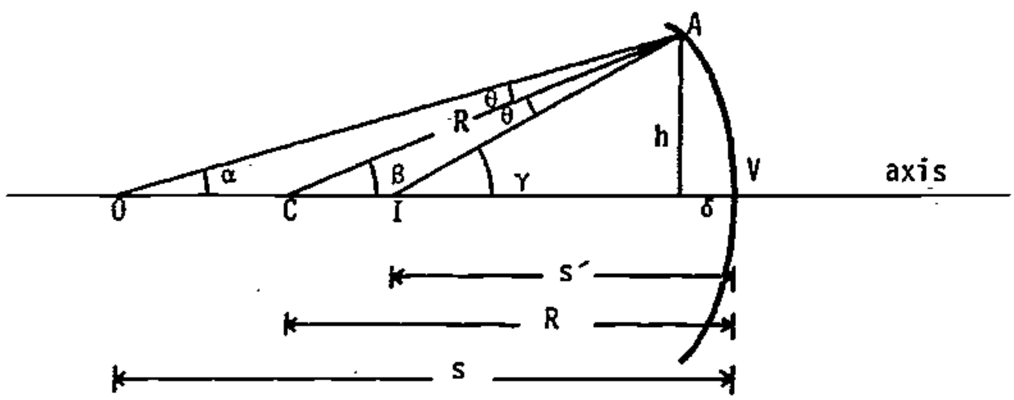
SUGGESTED STUDY PROCEDURE

Since your text does not treat spherical mirrors, you should read this study guide for the relevant development; then read Chapter 37, Sections 37-1, 37-2, 37-4. Study Problems A and B and Example 37-1, besides the problems below. Then work Problems C, D, and Problems 37-1 and 37-2 in your text, besides Problems 1 through 6 in the following discussion of spherical mirrors.

Spherical Mirrors

Consider a concave mirror of radius R as shown in Figure 1. Let there be a point source of light at O. One ray of light from O will go directly from O to V and be reflected back on itself. Pick another arbitrary ray emanating from O and hitting the mirror at A. It is reflected at A and crosses the first ray at I. Since the angle of incidence is equal to the angle of reflection, the angles made by the second ray with the radius R are equal. From the diagram we have

Figure 1



WEIDNER AND SELLS

Objective Number	Readings	Problems with Solutions: Study Guide	Problems with Solutions: Text	Assigned Problems: Study Guide	Assigned Problems: Text	Additional Problems
1	This study guide	A		C, 1 to 6		
2	Secs. 37-1, 37-2, 37-4	B	Ex.[a] 37-1	D	37-1, 37-2	37-3, 37-4, 37-7, 37-8, 37-10, 37-13, 37-15, 37-18 to 37-27

[a]Ex. = Example(s).

$\tan\alpha = h/(s - \delta)$, $\tan\beta = h/(R - \delta)$, and $\tan\gamma = h/(s' - \delta)$.

Since for small angles, $\tan\alpha \approx \alpha$,

$\alpha = h/(s - \delta)$, $\beta = h/(R - \delta)$, and $\gamma = h/(s' - \delta)$.

Looking at the geometry of the situation we also have (why?)

$\beta = \alpha + \theta$ and $\gamma = \beta + \theta$.

Eliminating θ we have $2\beta = \alpha + \gamma$. Combining this last equation with the first three and using the small-angle approximation, we get

$$\frac{h}{s - \delta} + \frac{h}{s' - \delta} = \frac{2h}{R - \delta}.$$

Since α, β, and γ are small, δ will also be small and can be neglected; thus,

$$1/s + 1/s' = 2/R. \qquad (1)$$

Now let $s \to \infty$, and Eq. (1) becomes

$$1/s' = 2/R.$$

As $s \to \infty$, the rays from 0 become parallel to the axis. When parallel rays strike the mirror they are focused at a point called the <u>focus</u> or focal point of the mirror. The distance of the focus from the mirror is called the <u>focal length</u> (f). Thus we can rewrite Eq. (1) as

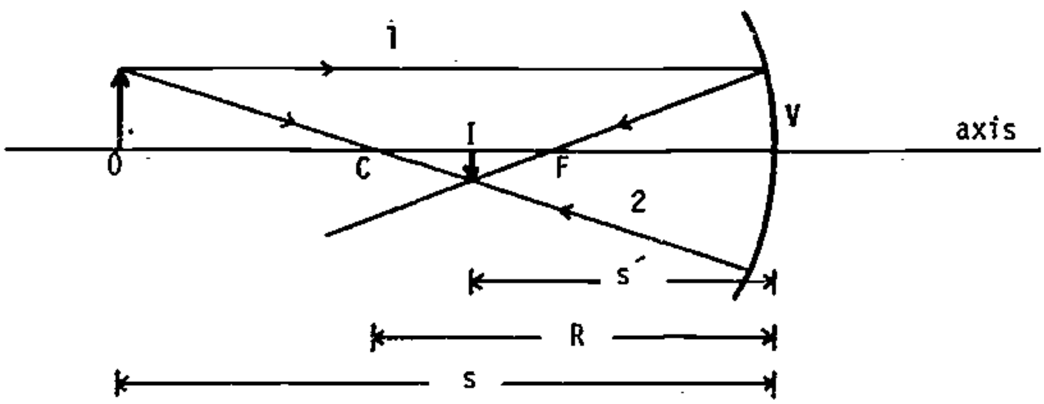
$$1/s + 1/s' = 1/f, \qquad (2)^*$$

where

$$f = R/2. \qquad (3)^*$$

Now consider an object (arrow) placed at 0 (see Figure 2). A ray (labeled 1) from the head of the object parallel to the axis will go through the focus. A ray [2] from the head of the object through C (the center of the spherical surface) must

Figure 2



be reflected back on itself. It can be shown that where these two rays cross all other rays emanating from the head of the object also meet, and an image of the object is formed at this distance from the mirror. Denote the distance from the mirror to this image by s', the distance from the object (o) to the mirror by s, and we have the mirror equation

$$1/s + 1/s' = 1/f.$$

Now let us solve for the magnification of the mirror. Using similar triangles,

$$i/(R - s') = o/(s - R),$$

where i is the size of the image and o is the size of the object. The magnification of the mirror is defined as the ratio of the image size to the object size (actually lateral extent from the axis), i.e.,

$$m \equiv i/o = (R - s')/(s - R).$$

Solving for R in terms of s and s' in Eq. (1) we have

$$R = 2ss'/(s + s').$$

Substituting this into the equation for m we have

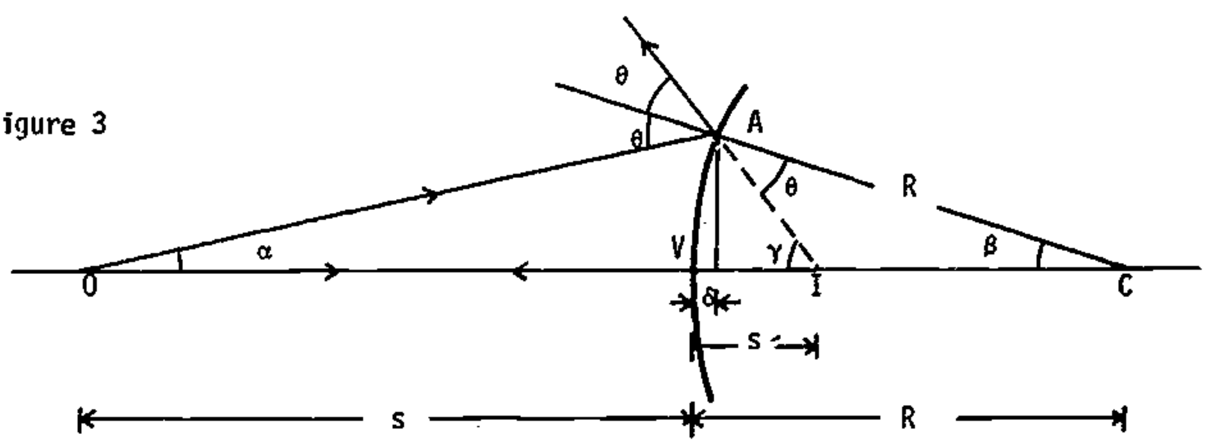
$$m = \frac{2ss'/(s + s') - s'}{s - 2ss'/(s + s')} = \frac{(2ss' - ss' - s'^2)/(s + s')}{(s^2 + s's - 2ss')/(s + s')} = \frac{ss' - s'^2}{s^2 - ss'}$$

$$= s'(s - s')/s(s - s') = s'/s. \qquad (4)*$$

Now consider what happens when we have a convex mirror, as in Figure 3. The ray from O to A will be reflected and appear to the observer to cross the axis at I. The ray from O to V will be reflected back on itself and appear to the observer to cross the extension of the reflected ray at I. Again we have

$$\tan \alpha = h/(s + \delta), \quad \tan \beta = h/(R - \delta), \quad \text{and} \quad \tan \gamma = h/(s' - \delta).$$

Figure 3



Using the small-angle approximation, we then have

$$\alpha = h/(s + \delta), \quad \beta = h/(R - \delta), \quad \text{and} \quad \gamma = h/(s' - \delta).$$

Again using geometry we have

$$\theta = \alpha + \beta, \quad \gamma = \theta + \beta, \quad \text{or} \quad \gamma = \alpha + 2\beta, \quad \text{and} \quad \frac{h}{s' - \delta} = \frac{h}{s + \delta} + \frac{2h}{R - \delta}.$$

Again ignoring δ because of the small-angle assumption we have $1/s - 1/s' = -2/R$. Notice how this equation compares with Eq. (1). We can make the equations identical if we adopt the following conventions for spherical mirrors:

(a) The object distance is always greater than zero, i.e., $s > 0$.

(b) For the image distance,
if $s' > 0$, then light rays actually pass through the image location;
if $s' < 0$, then light rays do not pass through the image location.

(c) For the radius of curvature,
$R > 0$ when the mirror is concave to the object;
$R < 0$ when the mirror is convex to the object.

As an exercise, you should show that the absolute value of the magnification of a convex mirror is $m = s'/s$.

In order to draw ray diagrams for spherical mirrors you can use the following:

Rule 1: An incident ray parallel to the axis is reflected through the focal point of the mirror (Ray 1 in Figures 4 and 5).

Rule 2: A ray along the radius of curvature is reflected back on itself (Ray 2 in Figures 4 and 5).

Rule 3: A ray through the focal point is reflected by the mirror parallel to the axis (Ray 3 in Figures 4 and 5).

The comment in Section 37-2 on lens aberration by and large can also be applied to mirrors for essentially the same reasons. Statement 1 on p. 756 is misleading and should read, "For all practical purposes, a ray passing through the center of the lens is essentially unchanged, inasmuch as its deviation is small (the lens

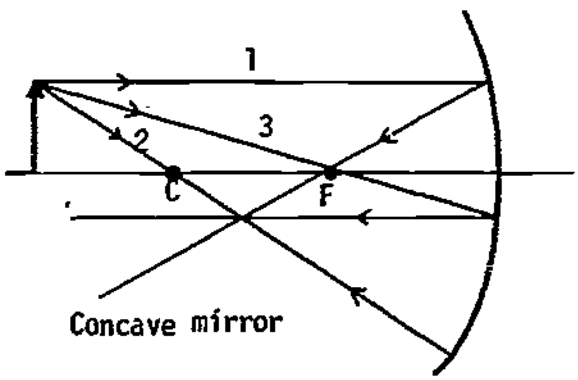


Figure 4

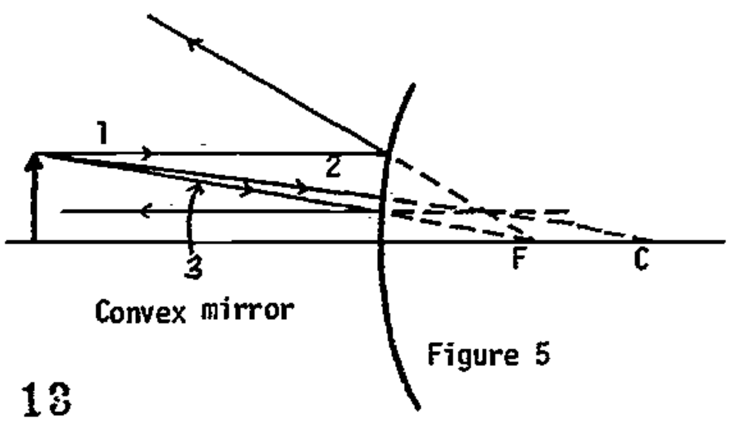


Figure 5

faces are parallel or are nearly parallel here) and it is not appreciably displaced laterally (the lens is very thin)."

The important equations keyed to the numbers in the General Comments are:

(1) Eq. (2)*. (2) Eq. (3)*. (3) Eq. (4)*. Signs and rules given above.

(4) Eq. (37-2) - p. 757. (5) Eq. (37-4b) - p. 768. (6) Eq. (37-1) - p. 753. Sign conventions given on p. 769; ray-diagram rules on p. 756.

Problems

1. In Figure 6, locate the image (a) graphically and (b) by use of the appropriate equation. Is it real or virtual? Erect or inverted? What is the lateral magnification?

2. In Figure 7, locate the image (a) graphically and (b) by use of the appropriate equation. Is it real or virtual? Erect or inverted? Lateral magnification?

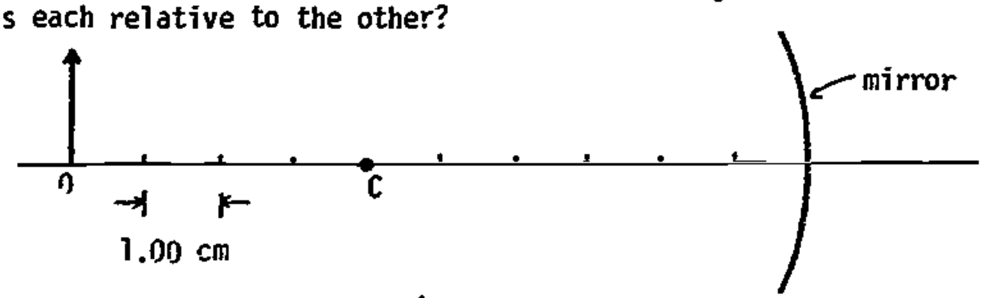
3. An object of height 6.0 mm is placed 60 cm from a concave mirror of radius of curvature 30 cm. What is the nature of the image and what is the image distance? Draw an appropriate ray diagram.

4. Where must the object be placed so that the image formed by a convex mirror is one-half as far from the mirror as the object? What is the magnification?

5. A concave mirror of radius 40 cm has an object located 50 cm in front of it. (a) Locate the image graphically. (b) Calculate the image location. (c) Calculate the magnification. (d) Is the image real or virtual?

6. Most truck drivers use both plane and convex mirrors for looking to the rear. What advantage has each relative to the other?

Figure 6



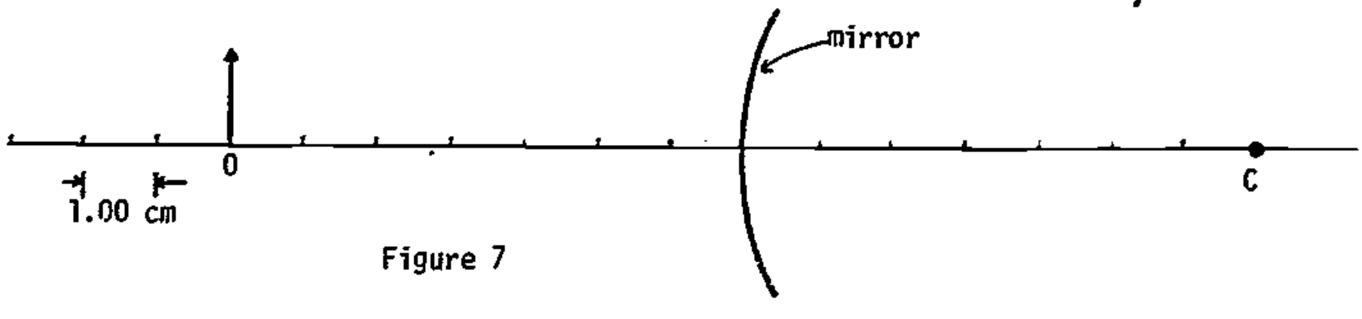


Figure 7

Answers to Problems

1. (b) 4.3 cm. Real. Inverted. 0.43. 3. 20.0 cm. Real. Inverted. 2.00 mm high. 5. (b) 33 cm. (c) 0.67. (d) Real.

PROBLEM SET WITH SOLUTIONS

A(1). Given a convex spherical mirror of radius R, discuss what happens to the image as the object starts at infinity and approaches the mirror.

Solution

For a convex mirror we have

$1/s + 1/s' = -2/R$ and $M = s'/s$. $1/s' = -2/R - 1/s$. Thus

$1/s' = -(2s + R)/sR$ and $s' = -Rs/(2s + R)$.

Since $s > 0$ and $R > 0$, $s' < 0$, the image is always virtual. As $s \to \infty$ we have

$$s' = \frac{-R}{2 + R/s} \to \frac{R}{2}$$ and the image size goes to 0.

When $s = R$, $s' = -R^2/3R = -R/3$, and the image is one-third the size of the object. As $s \to 0$, $s' \to 0$, and the image size goes to infinity. Actually, the small-angle approximation breaks down here and our formulas are no longer valid.

B(2). Given a diverging lens of focal length f, discuss what happens to the image as the object starts at infinity and approaches the lens.

Solution

For a diverging lens we have $1/s + 1/s' = -1/f$ and $h'/h = s'/s$. $1/s' = -1/f - 1/s$. Thus

$1/s' = -(f + s)/fs$ and $s' = -fs/(f + s)$.

Since $s > 0$ and $f > 0$, $s' < 0$. Therefore the image is always virtual. As $s \to \infty$ we have

$$s' = -\frac{f}{1 + f/s} \to -f,$$ and the image size $\to 0$.

When $s = f$, we have $s' = -f^2/2f = -f/2$, and the image is one-half the size of the object. As $s \to 0$, $s' \to 0$ and the image size $\to \infty$. Actually, the small-angle approximation breaks down and our formulas are no longer valid.

Problems

C(1). A spherical mirror has a focal length of +30.0 cm. An object 1.00 cm high is located 20.0 cm in front of the mirror.
(a) Locate the image graphically.
(b) Calculate the location of the image.
(c) Calculate the radius of curvature for the mirror.
(d) Calculate the size and orientation of the image.

D(2). Where is the image of an object that is placed at the focal point of a converging lens?

Solutions

C(1). See Figure 8, for part (a). (b) -60 cm. (c) 60 cm. (d) Image virtual, upright, and 3.00 cm high.

D(2). At infinity.

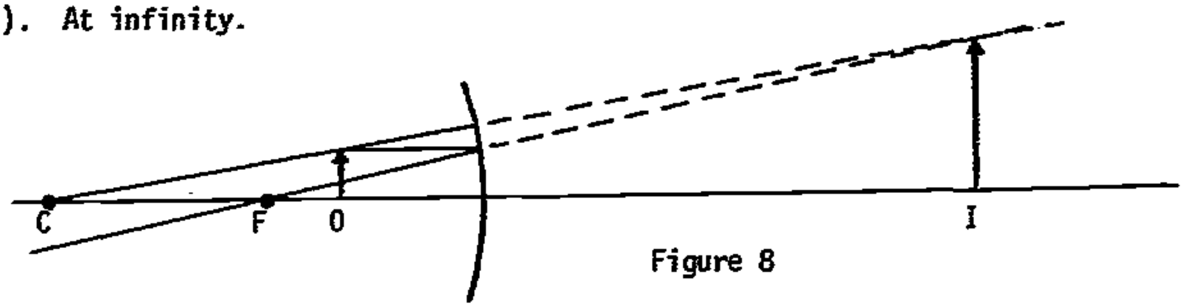


Figure 8

PRACTICE TEST

1. An object is located 20.0 cm in front of a convex mirror that forms a virtual image 5.0 cm behind the mirror.
   (a) Calculate the focal length of the mirror.
   (b) Draw a ray diagram and solve the problem graphically.

2. Explain how a magnifying glass works in terms of object and image relative to focal point.

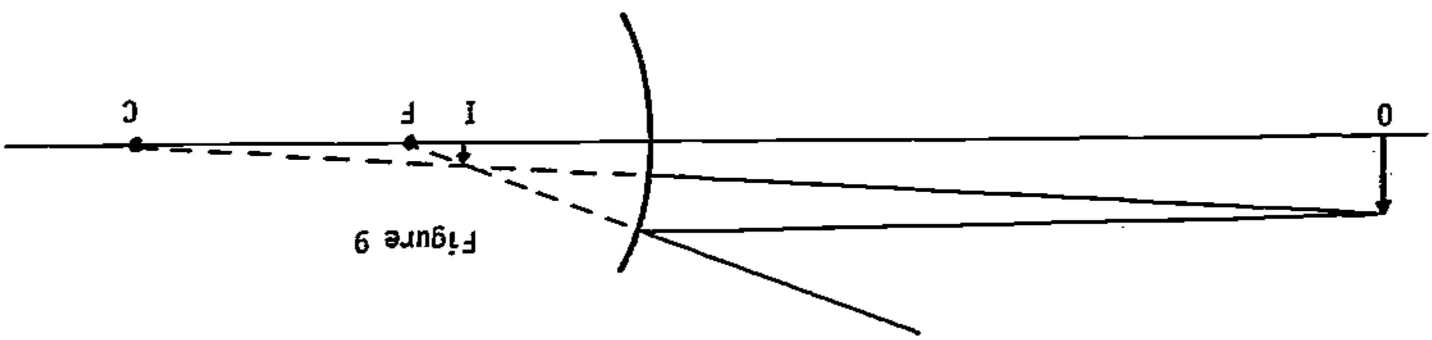


Figure 9

Practice Test Answers

1. (a) -6.7 cm. (b) See Figure 9.

2. $0 <$ (object distance)$/f < 2$ for magnification > 1.

LENSES AND MIRRORS

Date ____________________

Mastery Test   Form A

pass   recycle

1   2

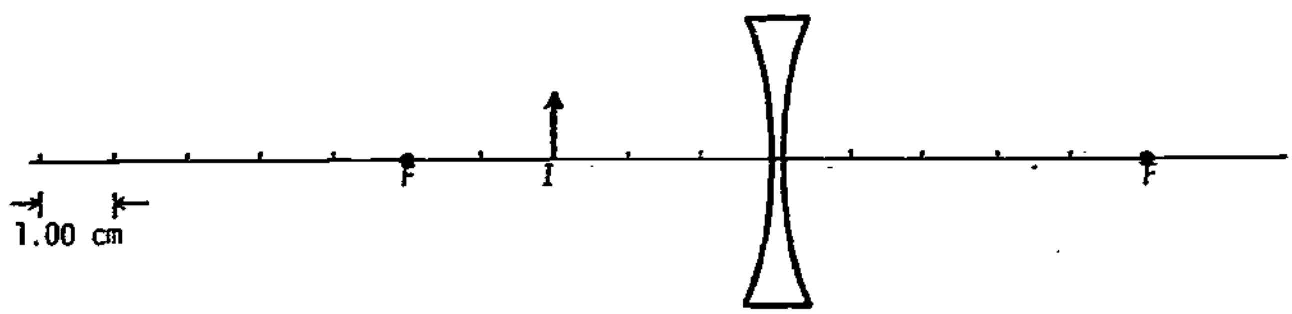
Name ____________________

Tutor ____________________

1. When is the image formed by a concave spherical mirror virtual? For those cases, discuss the size of the image and whether it is erect or inverted relative to the object.

2. Light enters from the left to form the image shown in Figure 1. Locate the object (a) graphically and (b) by use of the appropriate equation.

Figure 1



LENSES AND MIRRORS　　Date ______________________

Mastery Test　Form B　　pass　recycle

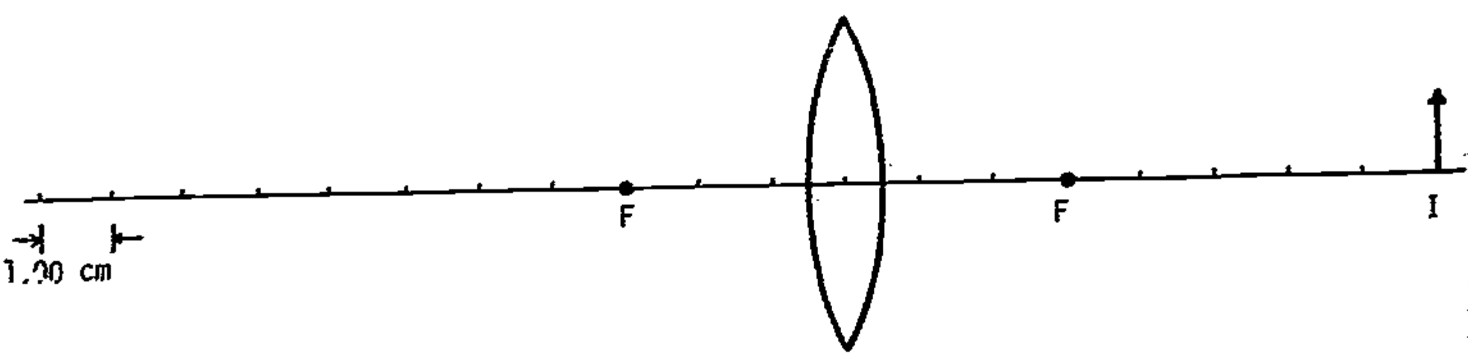
1　2

Name ______________________　　Tutor ______________________

1. When is the image found by a convex spherical mirror virtual? For those cases, discuss the size of the image and whether it is erect or inverted relative to the object.

2. Light enters from the left to form the image shown in Figure 1. Locate the object (a) graphically and (b) by use of the appropriate equation.

Figure 1



LENSES AND MIRRORS

Date ____________________

Mastery Test   Form C

pass   recycle

1   2

Name ____________________

Tutor ____________________

1. An object is located 10.0 cm from the center of a spherical silvered ball 5.0 cm in radius.
   (a) Calculate the position of its image.
   (b) What is its magnification?
   (c) Make a graphical solution.

2. When is the image formed by a converging thin lens virtual? For those cases, discuss the size of the image and whether it is erect or inverted relative to the object.

LENSES AND MIRRORS

Date ____________________

Mastery Test Form D

pass recycle

1 2

Name ______________________________ Tutor ____________________

1. A concave mirror is to form an image of the filament of a headlight lamp on a screen 10.0 m from the mirror. The filament is 5.0 mm high, and the image is to be 1.00 m high.
   (a) What should the radius of curvature of the mirror be?
   (b) How far in front of the vertex of the mirror should the filament be placed?

2. When is the image formed by a diverging lens virtual? For those cases, discuss the size of the image and whether it is erect or inverted relative to the object.

MASTERY TEST GRADING KEY - Form A

1. What To Look For: Spherical mirror formula. Condition when the image distance is negative, image virtual. Formula for magnification. Knowing whether image is erect or inverted.

Solution:

$$\frac{1}{\text{object distance}} + \frac{1}{\text{image distance}} = \frac{1}{f}. \qquad \text{image distance} = \frac{(\text{object distance}) \times f}{(\text{object distance}) - f}.$$

When f > object distance, the image is virtual.

$$|\text{magnification}| = \left|\frac{\text{image distance}}{\text{object distance}}\right| = \left|\frac{f}{\text{object distance} - f}\right| = \left|\frac{1}{\frac{(\text{object distance})}{f} - 1}\right|.$$

$f >$ object distance, therefore when $0 < (\text{object distance})/f < 1$, the

$|\text{magnification}| > 1$.

Image is erect as shown by ray diagram in Figure 12 if object distance is < f.

2. What To Look For: Graphical construction, thin-lens formula.

Solution: (a) See Figure 13. (b) Object distance = 7.5 cm.

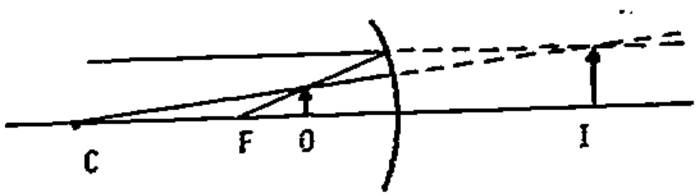
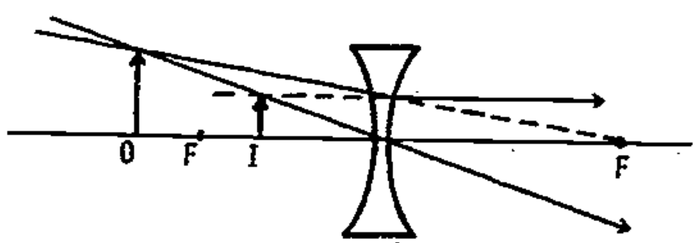


Figure 12

Figure 13



MASTERY TEST GRADING KEY - Form B

1. What To Look For: Spherical mirror formula. Condition when the image distance is negative, image virtual. Knowing whether image is erect or not.

   Solution:

   $$\frac{1}{\text{object distance}} + \frac{1}{\text{image distance}} = -\frac{1}{f}. \qquad \text{image distance} = -\frac{(\text{object distance}) \times f}{(\text{object distance}) + f}.$$

   Since object distance > 0 and f > 0 then image is always virtual.

   $$|\text{magnification}| = \left|\frac{\text{image distance}}{\text{object distance}}\right| = \left|\frac{1}{(\text{object distance})/f + 1}\right|.$$

   Therefore |magnification| < 1. Image is erect as seen from ray diagram in Figure 14.

2. What To Look For: Graphical construction, thin-lens formula. Student knows that left and right on lens can be interchanged.

   Solution: (a) See Figure 15. (b) image distance = 4.8 cm to left of lens.

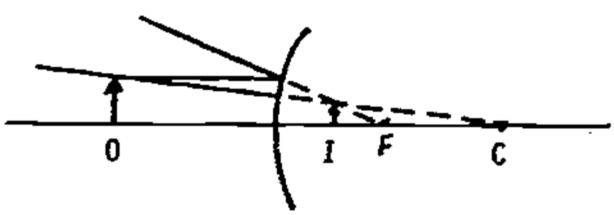


Figure 14

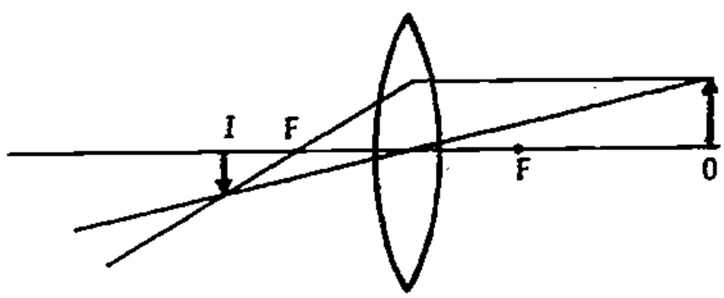


Figure 15

MASTERY TEST GRADING KEY - Form C

1. What To Look For: Spherical mirror equation. Equation for magnification. Graphical construction.

Solution: (a) 1/(object distance) + 1/image distance = -2/R.

Therefore,

1/(10.0 cm) + 1/(image distance) = -0.40 cm.

1/(image distance) = -(4 + 1)/(10.0 cm); image distance = -2.00 cm.

(b) |magnification| = |(image distance)/(object distance)| = 2.00/10.0 = 0.200.

(c) See Figure 14 in Solution 1 of Test B.

2. What To Look For: Thin-lens formula. Condition when image distance is negative, image virtual. Knowing whether or not image is erect.

Solution:

$$\frac{1}{\text{object distance}} + \frac{1}{\text{image distance}} = \frac{1}{f}. \quad \text{image distance} = \frac{(\text{object distance}) \times f}{(\text{object distance}) - f}.$$

Since object distance > 0 and f > 0, the image distance < 0 when object distance < f: image virtual.

$$|\text{magnification}| = \left|\frac{\text{image distance}}{\text{object distance}}\right| = \left|\frac{f}{\text{object distance} - f}\right|$$

$$= \left|\frac{1}{(\text{object distance})/f - 1}\right|.$$

Since object distance < f, |magnification| > 1. See Figure 16. Image is erect.

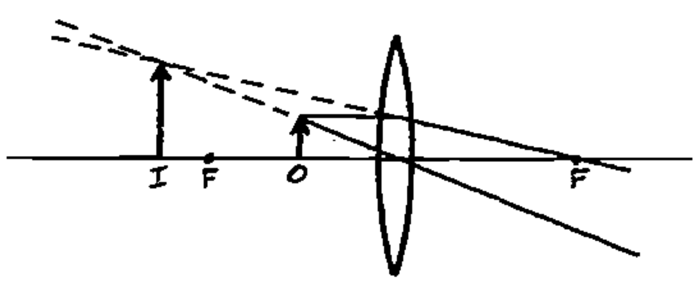


Figure 16

MASTERY TEST GRADING KEY - Form D

1. What To Look For: Spherical mirror equation. Magnification for a spherical mirror. Relationship between focal length and radius of curvature for a spherical mirror.

   Solution:

   (a) $\frac{1}{\text{object distance}} + \frac{1}{\text{image distance}} = \frac{1}{f}$.

   $|\text{magnification}| = |\frac{\text{image distance}}{\text{object distance}}| \doteq \frac{1\text{ m}}{0.005\text{ m}} = 200$. Therefore, object distance

   $= 10\text{ m}/200 = 0.05\text{ m}$, which is the answer to (b).

   $\frac{1}{f} = \frac{1}{0.05\text{ m}} + \frac{1}{10\text{ m}} \approx \frac{1}{0.05\text{ m}}$, therefore $f \approx 0.05$ m. Now $f = R/2$, therefore,

   $R \approx 0.100$ m, answer to (a).

2. What To Look For: Thin-lens formula. Condition when image distance is negative, image virtual. Knowing whether the image is erect or not.

   Solution:

   $\frac{1}{\text{image distance}} + \frac{1}{\text{object distance}} = -\frac{1}{f}$. image distance $= -\frac{(\text{object distance}) \times f}{(\text{object distance}) + f}$.

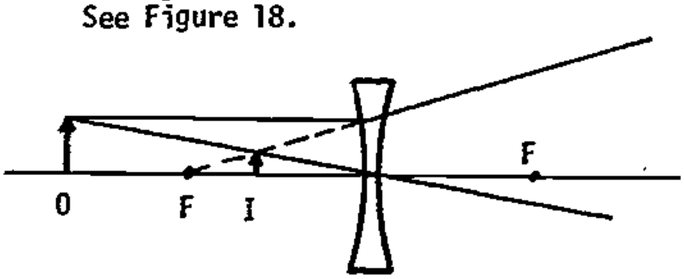
   Since object distance > 0 and f > 0 image distance < 0 for all object distances and f.

   $$|\text{magnification}| = |\frac{\text{image distance}}{\text{object distance}}| = |\frac{f}{\text{object distance} + f}|$$

   $$= |\frac{1}{(\text{object distance})/f + 1}|.$$

   Magnification < 1 for all cases. Image is erect as seen from ray diagram in Figure 17 if object distance > f. The image erect if object distance < f. See Figure 18.

Figure 17



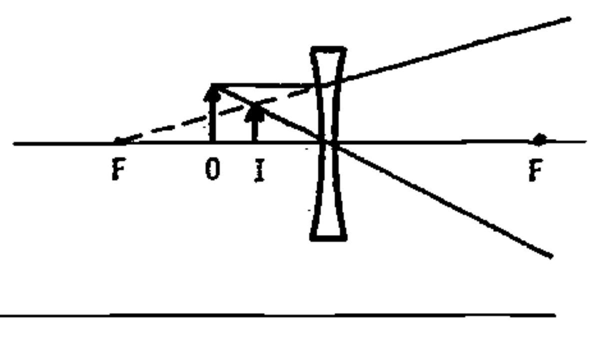


Figure 18

STUDY GUIDE

RELATIVITY

INTRODUCTION

Seldom has a development in science captured the attention of the general populace to the extent that Einstein's special theory of relativity did. Once, after giving a public lecture, Einstein was on the way to the railroad depot when he was asked to summarize his theory in one sentence, in a way the general public could understand. His reply: "When does the station get to the train?" Many of the predictions of the theory violate common sense - lengths change, times change, masses change, depending on who is looking - but the theory has been proved correct whenever it has been tested.

At the end of the nineteenth century, Newtonian mechanics and Maxwell's electromagnetic theory seemed to explain all physical phenomena, and some scientists believed that physics as a creative science was finished! Yet there was a serious gap: the ether, the postulated medium necessary for the propagation of electric and magnetic fields, appeared to have several incompatible properties. To resolve the ether problem, Einstein reformulated theoretical physics by introducing an operational approach that made use of light signals propagating at the speed $c = 3.00 \times 10^8$ m/s relative to all inertial frames. This approach replaced the view that space and time have certain absolute properties, as assumed by Newton and his successors.

Incidentally, the "special" in special theory of relativity refers to the fact that only uniform relative motion is considered. Accelerated reference frames, such as those attached to projectiles, are not treated in the special theory of relativity, but are the subject of Einstein's general theory.

PREREQUISITES

Before you begin this module, you should be able to:	Location of Prerequisite Content
*Use Cartesian coordinate systems in three dimensions (needed for Objectives 1 to 3 of this module)	Trigonometry Review
*Relate speed, distance, and time for motion in one dimension (needed for Objectives 1 to 3 of this module)	Rectilinear Motion Module

LEARNING OBJECTIVES

After you have mastered the content of this module, you will be able to:

1. Postulates - State the two postulates of special relativity and apply them in a descriptive way to simple phenomena.

2. Lorentz-Einstein transformation - State the Lorentz-Einstein transformation equations, compare them with the Galilean transformation equations, and apply them to simple problems involving two inertial reference frames in uniform relative motion.

3. Length contraction, time dilation - Apply the length-contraction and time-dilation formulas in simple situations to find their consequences, separately or in combination.

GENERAL COMMENTS

If your text does not have a substantial introduction to Einstein's special theory of relativity, we suggest that you consult one or more of the following references:

Adolph Baker, Modern Physics and Anti-Physics (Addison-Wesley, Reading, Mass., 1970).*

Frederick J. Bueche, Introduction to Physics for Scientists and Engineers (McGraw-Hill, New York, 1975), second edition, Chapter 6.

Albert Einstein, Relativity (Doubleday, Garden City, N.Y., 1947).*

Anthony P. French, Special Relativity (Norton, New York, 1968).

Carl Helmholz and Burton Moyer, Berkeley Physics Course: Mechanics (McGraw-Hill, New York, 1973), Vol. 1, Chapters 10, 11.

N. David Mermin, Space and Time in Special Relativity (McGraw, New York, 1968), Chapters 1 through 12.*

B. Russell, The ABC of Relativity (New American Library, New York, 1959; revised, 1970 or Humanities, revised 1969).*

Edwin F. Taylor and John A. Wheeler, Spacetime Physics (Freeman, San Francisco, 1966).

Hugh D. Young, Fundamentals of Mechanics and Heat (McGraw-Hill, New York, 1974), second edition.

We have prepared study guides to accompany the texts by Bueche and by Young. If you use the other readings, go through the entire book fairly quickly to locate the principal points, then review the module objectives and correlate them with the readings. We have purposely not included the relativistic theory of force, mass, acceleration, and energy for the sake of brevity, but you may wish to read these topics after you complete the module. After completing the reading, study Problems A through E and work Problems F through K. Finally, check your understanding by taking the Practice Test.

*These semipopular books are expecially worthwhile, including as they do presentations of the historical developments in physics that led to the formulation of the theory of relativity.

TEXT: Frederick J. Bueche, Introduct'on to Physics for Scientists and Engineers (McGraw-Hill, New York, 1975), second edition

SUGGESTED STUDY PROCEDURE

An introduction to Einstein's special theory of relativity involves many qualitative understandings of apparently paradoxical relationships. To supplement the text, which concentrates on the theoretical aspects, with historical and descriptive information we strongly recommend that you read one of the semipopular books listed in the General Comments.

Read Sections 6.1 through 6.13 in Chapter 6, studying especially Illustrations 6.1 through 6.4. Then study Problems A through E and work Problems F through K. Take the Practice Test, and work some Additional Problems if necessary, before trying a Mastery Test.

Objective 1 is taken up in the first six sections. Chapter 6 begins with a statement of Einstein's two postulates called the principle of relativity (a better statement is, "Accurate identical experiments performed in any two inertial reference frames in uniform relative motion will give identical results") and the principle of constancy. Most of the paradoxical results of the theory can be traced back to the principle of constancy, as you will discover in later sections.

Some of the surprising consequences are described in Sections 6.5 and 6.6. The discussion in these two sections makes use of "thought experiments," theoretical speculation about what would or would not happen under certain experimental conditions that are plausible to imagine but very difficult to create in reality. Of course, one does not get real data from a thought experiment, but one can get more concrete insights into the workings of a theory.

BUECHE

Objective Number	Readings	Problems with Solutions		Assigned Problems	Additional Problems
		Study Guide	Text	Study Guide	(Chap. 6)
1	Secs. 6.1 to 6.6	A, B			Quest.[a] 1, 2, 5, 6, 9, 10
2	Secs. 6.7 to 6.10, Appendix 8 (p. 845)	C		F, G	Probs. 16, 17
3	Secs. 6.11 to 6.13	D, E	Illus.[a] 6.1 to 6.4	H, I, J, K	Quest. 8; Probs. 1 to 9, 14

[a]Illus. = Illustration(s). Quest. = Question(s).

One difficulty with thought experiments is that one often does not know whether one has done them "right." For instance, what really happens to make the man in the car (p. 76) say that the light pulses hit the opposite ends of the car at equal times? Since he is in the middle, he might have mirrors at the ends and observe the two pulses returning to him at the same time. But then, would he still be sure that they had hit the ends simultaneously? Perhaps certain effects would cancel between the outward and return trip of each pulse. Einstein found it necessary to describe, by means of a thought experiment, how one observer would judge the simultaneity of events occurring at different places in his own inertial reference frame. This procedure justifies the example on p. 76, and is as follows:

Assume that there is an "observer" at each point in the inertial reference frame. Assume further that all the observers have identical clocks, running at exactly the same rate. To compare events, the clocks have to be set to the "same" time once, so none is permanently either early or late by a fixed amount. To set the clocks, the observer O_1 at the origin of coordinates sends out a light pulse that is reflected by a second observer O_2 back to O_1. Observer O_2 notes the time of reflection, and O_1 notes the times of emission and return. Now, in view of the principle of constancy, the time for the pulse to travel from O_1 to O_2 equals the time for return, since the speed of light always has the same value. This means that the time of reflection should be midway between the times of emission and return. O_1 signals this time to O_2 and O_2 can set his clock ahead or behind, depending on the observed reflection time. Einstein called the process the synchronization of clocks. Whenever coordinate and time measurements in a reference frame are mentioned, they are accomplished by means of synchronized clocks.

Sections 6.7 and 6.8 provide background for Objective 2 by describing in considerable detail the so-called Galilean transformation that relates observations in two coordinate systems in uniform relative motion according to ideas proposed by Galileo and in accord with our everyday experience. The crucial thought experiment in Section 6.9 makes use of the "light sphere," whose progress in the (x, y, z, t) system is observed by clocks synchronized reading t, and whose progress in the (x', y', z', t') system is observed by synchronized clocks reading t'. The discrepancy with the Galilean approach is evident.

The Lorentz transformation equations (Objective 2) can be derived from Eqs. (6.2) and (6.3) only by the addition of certain assumptions that are stated in Eq. (A8.1) in Appendix 8. You are not required to derive these, but you may enjoy playing with the algebra from Eq. (A8.1) on to see how far you get. You are expected to know Eqs. (6.4) and to be able also to state the inverse transformation leading from the (x', y', z', t') system to the (x, y, z, t) system.

Unlike the Galilean transformation, Eqs. (6.4) relate both x' and t' to x and t: the time t is not equal to the time t'. Look at it this way: At a certain instant, Observer S in the (x, y, z, t) system is just passing observer S′ in the (x', y', z', t') system; at that instant, Observer S looks at his own clock and at the clock of S′ and sees different readings. Just what time difference he sees will depend on the locations of the two observers (and their clocks) relative to the coordinate origins of the systems, as measured by the values of x and x'.

Objective 3 is treated in Sections 6.11 through 6.13. These topics really represent applications of the Lorentz transformation equations, but they are so important that we have stated them as a separate objective. You might begin with Section 6.13, which describes certain mathematical properties of the relativistic factor $\sqrt{1 - v^2/c^2}$, which is often represented by the symbol γ^{-1},

$$\gamma = (1 - \beta^2)^{-1/2}, \quad \beta = v/c. \tag{B1}$$

Remember that β is always less than one, and that γ is always one or greater. A useful identity (check this!) is

$$\gamma^2 = 1 + \beta^2\gamma^2. \tag{B2}$$

Regarding the Lorentz contraction, the point here is that "length" signifies the distance between the end point of the rod at the same time in the system in which length is measured - the (x', y', z', t') system for the proper length L_0 and the (x, y, z, t) system for the contracted length L. In finding $L = x_2 - x_1$, therefore, we use the same time t in the (x, y, z, t) system for both x_2' and x_1' in Eqs. (6.4c) and (6.4d):

$$x_2 = x_2'/\gamma + vt, \quad x_1 = x_1'/\gamma + vt, \quad L = (x_2' - x_1')/\gamma = L_0(1 - \beta^2)^{1/2}. \tag{B3}$$

Thus, the proper length of the rod, L_0 (i.e., the length in the coordinate system in which it is at rest), is longer than the length L measured in any moving frame.

In Section 6.12 the analysis of time dilation proceeds in a similar fashion to length contraction, but the text does not use the term "proper time" correctly. Just replace the phrase "proper time" on p. 85 by the phrase "time on stationary clocks" and every statement will be correct. The proper time interval of a phenomenon is the time interval in a reference frame in which the phenomenon stays at the same place. Thus, every clock shows its own proper time. A precise definition is "A time interval between two events is called the PROPER TIME interval if it is measured by one clock that is present at both events. If a time interval is measured by two different synchronized clocks, each present at one of the events but not at the other, it is not a proper time interval."

The statement about moving clocks running slowly can be stated as follows: "The proper time interval $\Delta\tau$ is the shortest possible time interval between two events. The proper time interval is related to any other time interval Δt between the same pair of events by the equation

$$\Delta\tau = \Delta t/\gamma = \Delta t(1 - \beta^2)^{1/2}. \tag{B4}$$

To give you practice with proper time: In Illustration 6.3 the proper half-life of pi-mesons is 2.00×10^{-8} s, the time after which half have decayed when observed at rest. In Illustration 6.4 the proper time of the one-way trip is the two and one-half months read by the spaceship clock that is physically present at the departure of the Earth and then the arrival of Alpha Centauri. The round trip in this illustration is not really suitable for discussion with special relativity, because the spaceship cannot be in uniform motion if it is to stop and return!

TEXT: Hugh D. Young, Fundamentals of Mechanics and Heat (McGraw-Hill, New York, 1974), second edition

SUGGESTED STUDY PROCEDURE

Your readings are from Chapter 14. As you do the readings, work the problems in this study guide. Then take the Practice Test.

Young does not state Einstein's assumptions explicitly. There are two:

(1) Principle of relativity: The fundamental laws of physics are identical for any two observers in uniform relative motion (p. 376, top).

(2) Principle of constancy: The observed speed of light is independent of the motion of the source (p. 377, bottom).

Most of the paradoxical results of the theory of relativity can be traced to the principle of constancy, which is contrary to everyday experience with water waves, sound propagation, and the motion of objects thrown from moving vehicles.

In Sections 14-3 and 14-4, Young investigates the consequences of the two principles for the measurement of time intervals and lengths by differing observers in uniform relative motion. The discussion of the synchronization of clocks (p. 385), though brief, is very important. It is essential for a good understanding of why the time intervals are dilated: As $0'$ moves relative to S, the clock at $0'$ is compared, not with the clock at 0, but with many different clocks along the path of the motion of $0'$ in S. In Section 14-5, the time-dilation and Lorentz-Fitzgerald contraction results are put together to lead to the Lorentz transformation equations.

PROBLEM SET WITH SOLUTIONS

A(1). State the two postulates on which the special theory of relativity is based.

Solution

(1) Principle of relativity: The fundamental laws of physics are identical for two observers in uniform relative motion.

(2) Principle of constancy: The speed of light in vacuum is the same when measured in any inertial reference frame, independent of the motion of the source.

B(1). Use Einstein's two postulates directly to show that the hands on a moving clock advance more slowly than those on a stationary clock. (Do not make use of the Lorentz transformation equations.) Hint: Analyze a thought experiment in which light from a source travels to a mirror and back, and is observed from a second frame moving parallel to the mirror.

Solution

A frame of reference S' moves with velocity u relative to a frame S. An observer O' in S' has a source of light that he directs at a mirror a distance d away, oriented so that the light is reflected back to him as shown in Figure 1. This observer measures the time interval $\Delta t'$ required for a light pulse to make the round trip to the mirror and back. The total distance as measured in S' is 2d, the speed is c, and the time required is

$$\Delta t' = 2d/c. \tag{1}$$

Consider how this experiment looks to an observer at O, with respect to which O' is moving with speed u. Let the time interval observed by O for the round trip be Δt. During this time, the source moves relative to O a distance $u\,\Delta t$, as shown in Figure 1(b). The total round-trip distance as seen by O is not 2d, but

$$2\ell = 2\sqrt{d^2 + [(1/2)u\,\Delta t]^2}\ . \tag{2}$$

According to the basic postulate of relativity, the speed of light c is the same with respect to both observers, and thus the relation in S analogous to Eq. (1)

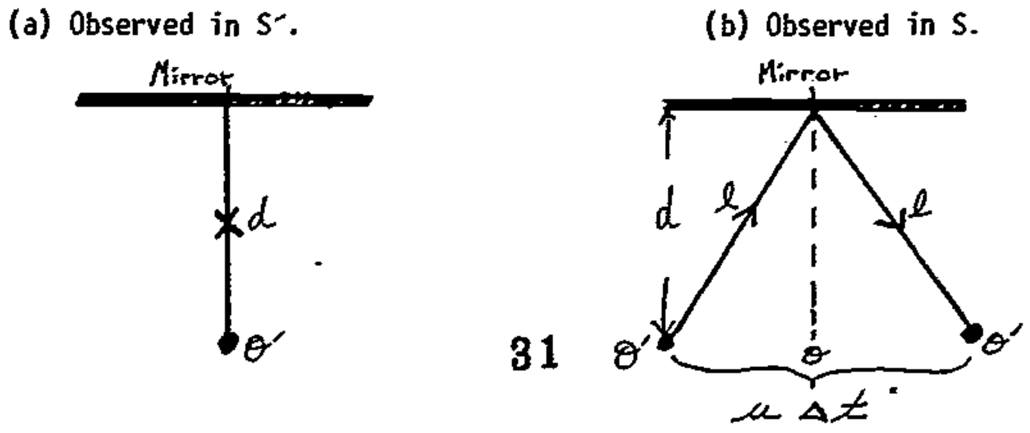


Figure 1

is

$$\Delta t = 2\ell/c = (2/c)\sqrt{d^2 + [(1/2)u\ \Delta t]^2}. \qquad (3)$$

Equations (1) and (3) can be combined to obtain a relationship between Δt and $\Delta t'$ that does not contain the distance d. We solve Eq. (1) for d, substitute the result in Eq. (3), and solve for Δt. The result of this algebraic manipulation is

$$\Delta t = \frac{\Delta t'}{(1 - u^2/c^2)^{1/2}}. \qquad (4)$$

We may generalize this important result: If two events occurring at the <u>same space point</u> in a frame of reference S' (in this case, the departure and arrival of the light signal at $0'$) are observed to be separated in time by an interval $\Delta t'$, then the time interval Δt between these two events as observed in the frame of reference S is <u>larger</u> than $\Delta t'$, and the two are related by Eq. (4). Thus a clock moving with S' appears to an observer in S to run at a rate that is <u>slower</u> than the rate observed in S'.

C(2). Two events take place at (x_1, y_1, z_1, t_1) and (x_2, y_2, z_2, t_2) in one coordinate system, and they take place at "primed" coordinates (x_1', y_1', z_2', t_2') in a second coordinate system. The second system, its axes parallel to corresponding axes of the first, moves relative to the first with constant speed u along their common x axes. Find the transformation equations relating the space and time intervals $\Delta x = x_2 - x_1$, $\Delta y = y_2 - y_1$, $\Delta z = z_2 - z_1$, and $\Delta t = t_2 - t_1$, to the corresponding primed space and time intervals. You may start from the Lorentz transformation equations.

<u>Solution</u>

The Lorentz transformation equations may be used to obtain the required intervals. Suppose that event 1 occurs at position x_1, time t_1, and event 2 occurs at position x_2, time t_2, in the unprimed frame. The spatial separation in the primed system is derived in the following manner:

$$x_1' = \gamma(x_1 - \beta ct_1), \qquad x_2' = \gamma(x_2 - \beta ct_2),$$

$$x_2' - x_1' = \Delta x' = \gamma(x_2 - x_1) - \beta c(t_2 - t_1),$$

$$\Delta x' = \gamma(\Delta x - \beta c\Delta t), \quad \Delta y' = \Delta y, \quad \text{and} \quad \Delta z' = \Delta z.$$

Similarly, the separation in time is

$\Delta t' = \gamma(\Delta t - \beta\,\Delta x/c)$.

Remember, $\beta = v/c$, $\gamma = (1 - \beta^2)^{1/2}$, and $\beta = 1 - 1/\gamma^2$.

D(3). The μ-meson is an unstable particle whose half-life (the time after which half of an original number have decayed radioactively) when at rest is approximately 2.30×10^{-6} s. These mesons are produced in bursts of many mesons in the upper atmosphere by the impact of energetic cosmic rays.
(a) What is the half-life, as measured by observers on Earth, of μ-mesons moving through the atmosphere with a speed $u/c = 0.99$?
(b) How far do the mesons move during the time of one half-life as measured by observers on Earth?
(c) How far has the ground moved during the time of one half-life, as observed in the rest frame of the mesons? Compare with the Lorentz-Fitzgerald contraction.

<u>Solution</u>

(a) A burst of μ-mesons traveling at the same velocity can be used to define two events: (1) their production and (2) the decay of half the original burst. In the rest frame of the mesons, the time between these events is 2.30×10^{-6} s, and this defines the proper time interval $\Delta t'$. In the other frame, relative to which the μ-mesons move at constant velocity u, the time between the two events is given by

$$\Delta t = \frac{\Delta t'}{[1 - (u/c)^2]^{1/2}} = \frac{2.30 \times 10^{-6}}{[1 - 0.99^2]^{1/2}} = \frac{2.30 \times 10^{-6}}{[0.020]^{1/2}}$$

$$= 2.30 \times 10^{-6}/0.14 = 1.60 \times 10^{-5}\ \text{s}, \quad \text{greater than } \Delta t'.$$

(b) Since the mesons are moving with the speed $u = 0.99c$, the distance Δx they travel in the time Δt is

$$\Delta x = u\,\Delta t = 0.99(3.00 \times 10^8)(1.6 \times 10^{-5}) = 4.8 \times 10^3\ \text{m}.$$

(c) In the mesons' rest frame the ground moves with speed $u = 0.99c$ in the direction opposite to the mesons' motion, and the motion occurs for the time interval $\Delta t'$; hence the distance $\Delta x'$ moved by the ground is

$$\Delta x' = u\,\Delta t' = 0.99(3.00 \times 10^8)(2.30 \times 10^{-6}) = 6.8 \times 10^2\ \text{m}.$$

Note that the answers in parts (b) and (c) are unequal. This is an example of the Lorentz-Fitzgerald contraction associated with the relative motion of the ground and meson reference frames:

$$\Delta x = \Delta x'(1 - u^2/c^2)^{1/2} = 0.14\,\Delta x.$$

There are several important points to remember: (1) In the time-dilation equation, the proper time interval $\Delta t'$ in the reference frame S′ denotes the time interval between <u>two events at the same position</u> in that frame. (2) In any inertial reference frame S in which these two events occur at different positions, the time interval

Δt separating them is longer than the proper time interval $\Delta t'$ between the same events. (3) In the Lorentz-Fitzgerald contraction equation, the proper length L' is measured in the frame of reference in which the object is at rest. (4) In any inertial reference frame S in which the object is moving parallel to its length, the length L must be measured by noting the positions of its ends at the same time in S, and L is smaller than the proper length L'.

Note. An experimental comparison of μ-meson flux and decay rates has been carried out by using Mt. Washington, NH and sea level as the two locations. See D.H. Frisch and J.H. Smith [Am. J. Phys. 31, 342 (1963)]. Also available is a film by Frisch and Smith, "Time Dilation, an Experiment with μ-mesons," Education Development Center, Newton, MA, 1963.

E(3). The Orient Express moves past a small station in Central Europe at a speed of 1.50×10^8 m/s. It has mirrors attached to both ends. The length of the train (in its rest system) is 100 m. After it passes, the stationmaster turns on a light. The light travels to both mirrors and is reflected back to him. How much times elapses between the arrival of the two reflected light beams at the station?

Solution

We consider two events: light striking the rear mirror and light striking the front mirror. We have to provide for the motion of the train as well as the motion of the light. Let the unprimed frame be that of the stationmaster and the track (see Figure 2), and let

$x = 0$	be the position of the rear mirror when light strikes;
$x = x_0$	be the position of the front mirror when light strikes;
$t = 0$	be the time when rear mirror is struck;
$t = t_0$	be the time when front mirror is struck;
v	be the speed of the train;
L_0	be the length of the train (proper); and
$L = L_0\gamma$	be the length of the train (moving).

Since the light reflected from the front has to travel the extra distance x_0 two times (forward and back), it will take the extra time $2t_0$, which will be the answer. The time for light to reach the train's rear is immaterial for the delay we are calculating, since both beams require that extra time. Since the train is moving while the light is propagating, we have

$x_0 = vt_0 + L$ (from motion and length of train)

and

$x_0 = ct_0$ (from motion of light).

We must solve these equations for t_0 in terms of v, which appears explicitly and concealed in $L = L_0(1 - v^2/c^2)^{1/2}$:

$$(c - v)t_0 = L = L_0(1 - v^2/c^2)^{1/2},$$

$$t_0 = \frac{L_0(1 - v^2/c^2)^{1/2}}{c\,(1 - v/c)} = \frac{L_0}{c}\left(\frac{1 + v/c}{1 - v/c}\right)^{1/2}$$

$$= \frac{100}{3 \times 10^2}\left(\frac{1 + 0.50}{1 - 0.50}\right)^{1/2} = 0.58 \times 10^{-6}\ \mathrm{s},$$

$$2t_0 = 1.16 \times 10^{-6}\ \mathrm{s}.$$

Comments: (i) Note how we have written t_0 in terms of L_0/c and the dimensionless ratio v/c. This makes it easy to check for dimensional consistency. (ii) In the train's frame, the light has traveled from back to front in the time $t_0' = L_0/c$, which is smaller than t_0. (iii) Proper time does not enter this problem because no clock is present at both events.

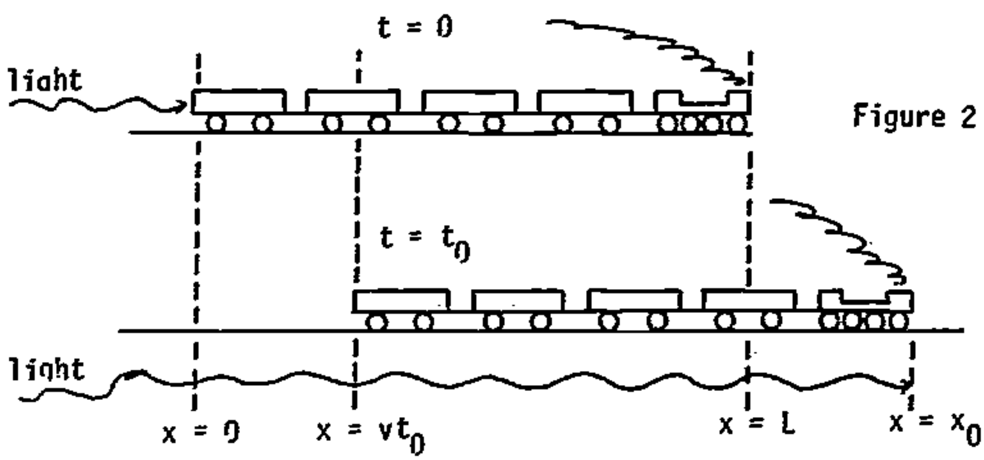


Figure 2

Problems

F(2). A very energetic proton interaction is observed at laboratory coordinates $x_1 = 0.100$ m, $y_1 = 0.100$ m, $z_1 = 0.100$ m, $t_1 = 0$ s. A mysterious second event occurs at $x_2 = 0.50$ m, $y_2 = 0.100$ m, $z_2 = 0.100$ m, and $t_2 = 1.00 \times 10^{-9}$ s.

(a) How far apart in space and time are the two events (see Problem C) as observed by a distinguished visitor passing through the laboratory on a brief inspection tour with a speed $v_x = +1.80 \times 10^8$ m/s?

(b) How fast must the visitor be moving with respect to the laboratory to observe the two events simultaneously in her reference system?

(c) How fast must the visitor be moving with respect to the laboratory to observe the two events at the same position?

(d) (Optional) What is the possibility that the proton interaction "caused" the mysterious event?

G(2). In the same experimental arrangement used in Problem F, a second energetic proton interaction and a particle decay are observed at the following positions and times:

proton: $x_1 = 0$, $y_1 = 0$, $z_1 = 0$, $t_1 = 0$.

particle decay: $x_2 = 1.00$ m, $y_2 = z_2 = 0$, $t_2 = 1.00 \times 10^{-8}$ s.

(a) How far apart in space and time are the two events for a distinguished visitor passing through the lab with a speed $v_x = 1.80 \times 10^8$ m/s?
(b) How fast must the visitor be moving with respect to the lab to observe the two events occurring simultaneously in her frame?
(c) How fast must the visitor be moving to observe the events at the same position in her frame?
(d) (Optional) What is the possibility that the decaying particle was created in the proton interaction?

H(3). David Winch rode his bicycle up a straight mountain road at a steady speed of 0.385 c. Stationary observers with synchronized clocks at the beginning and end of his 50-km course noted the times of his start and finish.
(a) What elapsed time did the observers record?
(b) What elapsed time did David record?
(c) Whose time was the proper time?
(d) What distance was the length of the course as observed by David?

I(3). A train measures 1000 m in length when stationary. The train's track goes through a tunnel that is 1100 m long when surveyed in its rest frame. The engineer does not want the train ever to be contained completely within the tunnel. Therefore he decides to go through the tunnel very fast, at 0.60 times the speed of light. That way, he figures, at least __(a)__ meters of the train will always be out in the sunlight. He tests this prediction with cameras on board the engine and the rear end of the train. The cameras are set up to take motion pictures of the surroundings at both ends of the train. The tests __(b)__ (choose "confirm" or "contradict") his prediction. The dispatcher alongside the track has a different opinion. Going fast, he asserts, will only make the situation worse, because it will make the train still shorter in comparison with the tunnel. He sets up cameras on the ground along the track (outside and inside the tunnel) to provide evidence. At the instant when the rear of the train enters the tunnel, all of his cameras take flash pictures. The camera that records the head of the train is __(c)__ (choose "inside" or "outside") the tunnel, __(d)__ meters from its exit. Do the two direct measurements by the engineer and the dispatcher: (A) confirm only one of the predictions, definitely settling the question as to whether the train is really ever entirely within the tunnel? or (B) confirm both predictions, leaving the engineer and dispatcher in disagreement? or (C) confirm neither prediction, because the train and the tunnel both shrink or expand by the same factor in any frame of reference? __(e)__. [Choose (A), (B), or (C).]

The dispatcher takes out his stop watch and determines that it takes __(f)__ seconds for the entire train to pass in front of him. The engineer determines this time interval by seeing how much time elapsed between the picture showing the dispatcher taken by the engine-mounted camera and the picture showing the dispatcher taken by the rear camera; he finds it to be __(g)__ ("longer" or "shorter"), namely __(h)__ seconds. Which of these is the "proper time interval"? __(i)__.

J(3). Two spaceships, each 120 m long in its own rest frame, meet in outer space. An observer in the cockpit at the front of one ship observes that a time of 1.80×10^{-6} s elapses while the entire second ship moves past her, heading in the opposite direction.
(a) What is the relative velocity of the two spaceships?
(b) How much time elapses on the clocks of the first ship while the front of the second ship moves from the front to the back of the first ship?

K(3). A student bit his tongue (by mistake) and opened his mouth to cry in pain 0.50 s later. A callous friend who passed by at a high speed (but did not stop to give aid and comfort) noted that 2.00 s elapsed between the two events in his reference frame.
(a) What are the space and time intervals (see Problem C) between the two events for the student in pain?
(b) What is the speed of the callous friend?
(c) What is the spatial separation of the two events in the reference frame of the callous friend?

Solutions

F(3). Hints: (a) Find the space and time intervals in the lab frame, then transform. (b) The space and time intervals in the visitor's frame must have $\Delta t' = 0$. Solve the transformation equation for v. (c) The space interval must have $\Delta x' = 0$. Solve for v. (d) The "cause" must (precede? follow?) the effect. Answers: (a) 0.275 m; 2.50×10^{-10} s. (b) $+2.25 \times 10^{8}$ m/s. (c) $v = 4.0 \times 10^{8}$ m/s $> 3.00 \times 10^{8}$ m/s, impossible! (d) Since an observer going faster than $v = 2.25 \times 10^{8}$ m/s will see the order of events reversed, such an observer cannot even agree with the laboratory observer on which event occurred first. Since a cause must precede an effect, no matter who is observing, we conclude that these two events cannot be related causally. That is true for any two events for which $\Delta x > c\,\Delta t$. Such a separation is called spacelike.

G(2). See Hints for Problem F. (a) 1.00 m, 1.00×10^{-8} s. (b) $v = 9.0 \times 10^{8}$ m/s, impossible! (c) 1.00×10^{8} m/s. (d) Since $c\,\Delta t > \beta\,\Delta x$, $\Delta t'$ is always positive. Thus any observer can agree with every other that the first event really was first. It could be the cause of the decaying particle, although it need not be. That is true for any two events for which $\beta\,\Delta x < c\,\Delta t$. Such a separation is called timelike.

H(3). Hints: (a) Remember in which frame (David's or observers') the 50 km is laid out. (b) Apply time-dilation equation. (c) What were the two events and whose clock was present at both events? (d) Two methods: Lorentz contraction applied to "course" or David's time applied to courses' speed relative to David. Answers: (a) 4.3×10^{-4} s. (b) 4.0×10^{-4} s. (c) David's. (d) 46 km.

I(3). <u>Hint</u>: Use two reference frames, one for the track, tunnel, cameras on the ground, and dispatcher; the other for the train, engineer, and cameras on the train. Identify the "events" that are being discussed. <u>Answers</u>: (a) 120 m. (b) confirm. (c) inside. (d) 300. (e) B. (f) 0.44×10^{-5}. (g) longer. (h) 0.55×10^{-5}. (i) (f) is proper time.

J(3). (a) 0.65×10^{8} m/s. (b) 1.85×10^{-6} s.

K(3). (a) $\Delta x = 0$, $\Delta y = 0$, $\Delta z = 0$, $\Delta t = 0.50$ s. (b) 2.904×10^{8} m/s. (c) 5.81×10^{8} m (too far to help).

PRACTICE TEST

1. State the two postulates of special relativity.

2. A cosmic-ray particle from space crosses the solar system, parallel to the plane of the system. The speed of the particle is $v = \beta c = 0.95c$. The diameter of the solar system is about 1.20×10^{13} m.

   (a) How long does it take for the particle to cross the solar system, as measured by observers in the solar system?

   (b) How long does it take to cross the solar system according to an observer riding with the particle?

   (c) What is the diameter of the solar system as measured by the observer riding with the particle?

   (d) Which observers measure a proper time for the trip? A proper distance for the diameter?

3. In a certain inertial reference frame, two explosions occur at the space-time points $(x_1, y_1, z_1, t_1) = (5.0\text{ m}, 0, 0, 0)$ and $(x_2, y_2, z_2, t_2) = (8.0\text{ m}, 1.00\text{ m}, 2.00\text{ m}, 1.00 \times 10^{-9}\text{ s})$. An observer in a different reference frame, moving with velocity $v = -2.40 \times 10^{8}$ m/s relative to the first, observes the same two explosions. Assume the origins of the two coordinate systems coincide at $t = t' = 0$.

   (a) Write the appropriate Lorentz-Einstein transformation equations to calculate the coordinates of the two events in the second (primed) system.

   (b) Calculate the positions and times of the explosions in the primed system.

<u>Practice Test Answers</u>

1. Identical experiments performed in any inertial reference frames give identical results. The speed of light in vacuum is independent of the motion of the source.

2. (a) 4.2×10^{4} s. (b) 1.30×10^{4} s. (c) 3.8×10^{12} m. (d) "Particle" observer records proper time. Solar-system observer measures proper diameter.

3. (a) $x' = \gamma(x - \beta ct)$, $y' = y$. $t' = \gamma(t - \beta x/c)$, $z' = z$.
(b) $x_1' = 8.33$ m. $x_2' = 13.7$ m. $t_1' = 2.22 \times 10^{-8}$ s. $t_2' = 3.72 \times 10^{-8}$ s. $y_1' = z_1' = 0$. $y_2' = 1.00$ m. $z_2' = 2.00$ m.

RELATIVITY Date ______________

Mastery Test Form A

pass 1 2 recycle 3

Name ______________ Tutor ______________

1. (a) Briefly name the two postulates of the special theory of relativity. You will be asked to describe them orally in more detail.

   (b) A friend argues that, since the measured speed of a horse depends on the relative motion of two observers, the principle of relativity must be incorrect. How do you reply?

2. Figure 1 represents the top view of two meter sticks, in relative motion with speed $v = \beta c$. The picture is the view by an observer in the unprimed reference frame. At $t = t' = 0$, ends A and A′ of the two meter sticks are together. Let $\beta = 12/13 = 0.92$.

   (a) At $t' = 0$, calculate the position of end B of the unprimed meter stick, as observed by someone in the primed reference frame. Draw a sketch, similar to the one in the figure, showing the sticks at $t' = 0$, as seen by the "primed" observer.

   (b) Calculate the time t'_C when ends B and B′ are together, as observed by the "primed" observer. Sketch the situation.

   (c) How would your answers to parts (a) and (b) differ in a universe that obeyed Galilean relativity?

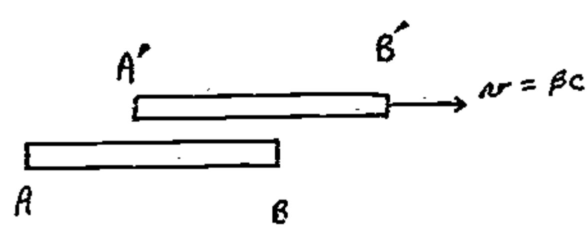


Figure 1

RELATIVITY

Date ________________

Mastery Test Form B

pass 1 2 recycle 3

Name ________________________ Tutor ________________________

1. (a) Briefly name the two postulates of the special theory of relativity. You will be asked to describe them orally in more detail.
   (b) A flashbulb is mounted on the side of a speeding train, and an identical bulb is mounted on a pole beside the track. When the two bulbs are side by side, they are fired. Which pulse reaches the engineer first?

2. Pioneer XLVI is an unmanned space probe, traveling from Earth to Sun in the year 1992. Its uniform speed relative to the Earth is 0.60c. Assume that it leaves Earth at $t = t' = 0$. The Earth-Sun distance is 1.50×10^{11} m in the rest frame of Earth and Sun. Neglect gravitation and rotation.
   (a) At what time, according to earthbound clocks, does it reach the Sun?
   (b) According to the clock on Pioneer XLVI, how long does the trip take?
   (c) Describe how an observer riding with Pioneer XLVI would determine the distance from Earth to Sun. What distance would the observer calculate?
   (d) Describe how your answers to parts (a), (b), and (c) would differ if Galilean relativity were Nature's way of operating.

RELATIVITY

Mastery Test Form C

Date ______________

pass 1 2 3 recycle

Name ______________ Tutor ______________

1. (a) Briefly name the two postulates of the special theory of relativity. You will be asked to describe them orally in more detail.

   (b) A friend argues that, since the measured speed of sound depends on the motion of the observer with respect to the source, the principle of relativity must be incorrect. How do you reply?

2. A train that is 300 m long in its rest frame is moving at a speed of $v = \beta c = 0.80c$. At $t = 0$, the engineer in the locomotive at the front of the train is adjacent to a switchman standing on the ground (event A). As the caboose at the rear of the train passes the switchman, he flashes a light signal forward (event B). The light strikes a rearview mirror beside the engineer (event C), and is reflected back to be detected by the conductor in the back of the caboose (event D), and ultimately by the switchman (event E).

   (a) State which time intervals between pairs of events are proper for the conductor in the caboose.

   (b) According to a system of clocks and measuring sticks on the ground, calculate the time and location of event B.

   (c) According to a system of clocks and measuring sticks on the ground, calculate the time and location of event D.

   (d) How would your answer to part (b) change if Galilean relativity applied?

MASTERY TEST GRADING KEY - Form A

1. What To Look For: Be sure student knows the difference between the invariance of physical laws, and, say, getting the same numbers for velocity in a given experiment.

   Solution: (a) "Laws of physics are the same in every inertial reference frame," or some equivalent version. "Speed of light is a constant in every inertial reference frame."

   (b) The relativity postulate does not say two measurements of speed must give the same number. It does say that in each frame $\vec{F} = d\vec{p}/dt$, so the horse must push with the same force to get the same momentum change.

2. What To Look For: (a) In the drawing, points A and A′ should be together, and AB should be shorter than A′B′. The solution should demonstrate ability to use the appropriate transformation equation, or the ability to reason from arguments about proper length. (b) In the drawing, points B and B′ should be together, and AB should be shorter than A′B′. Minus sign is important, since comparison of sketches in (a) and (b) shows that (b) occurred first. Ask student about the meaning of the minus sign. Problem solution should demonstrate ability to use Lorentz transformation correctly, or to reason from arguments about the proper length.

   Solution: See Figure 4.

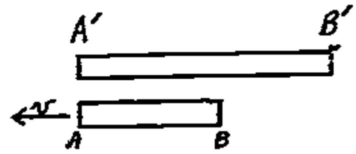


Figure 4

   $x_B = \gamma(x_B' + \beta c t_B') = \gamma x_B'$.

   $x_B' = x_B/\gamma = (1)\sqrt{1 - \beta^2} = 0.39$ m.

   Or: A′B′ is a proper length. AB is Lorentz contracted, so $AB = A'B'\sqrt{1 - \beta^2} = 0.39$ m. Since left end is at $x_A' = 0$, right end must be at $x_B' = 0.39$ m.

   (b) See Figure 5.

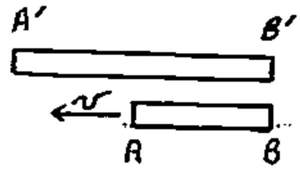


Figure 5

   $x_B = \gamma(x_B' + \beta c t_B')$;

   $x_B = x_B' = 1.00$ m.

   $t_B' = (1/\beta c)[(x_B/\gamma) - x_B'] = -0.222 \times 10^{-8}$ s.

   Or: Primed observer says AB is 0.39 m long. At t = 0, A and A′ were together. To get B and B′ together, move AB 0.62 m to the right. That occurred at earlier time

   $t = -8/13\beta c = 0.222 \times 10^{-8}$ s.

   (c) Parts (a) and (b) would have the same answer: $x_B = x_B' = 1.00$ m at $t' = 0$. The length AB is 1.00 m.

MASTERY TEST GRADING KEY - Form B

1. What To Look For: Student should know the postulate of constancy. Speed of source has no effect on speed of light.

   Solution: (a) See Grading Key form A, Solution 1.
   (b) Both reach the engineer at the same time.

2. What To Look For: Proper use of correct Lorentz transformation equation, or correct use of Lorentz contraction arguments, or correct use of time-dilation arguments. The distance must be less than the proper distance.

   Solution: (a) Call the Earth-Sun distance ℓ in the Earth-Sun frame. Then

$$t_s = \frac{\ell}{v} = \frac{1.50 \times 10^{11}\ \text{m}}{0.60(3.00 \times 10^{8}\ \text{m/s})} .$$

(b) $t_s' = \gamma(t_s - \beta\ell/c) = \gamma(t_s - \beta v t_s/c) = t_s(1 - \beta^2)/\sqrt{1 - \beta^2} = t_s\sqrt{1 - \beta^2}$.
$= 830(0.80) = 670$ s.

Or: Probe "sees" a Lorentz-contracted distance. The time to traverse that distance is

$t_s' = \ell'/\beta c$, thus

$$t_s' = \frac{\ell\sqrt{1 - \beta^2}}{\beta c} = \frac{(1.50 \times 10^{11})(0.80)}{(0.60)(3.00 \times 10^{8})} = 670\ \text{s}.$$

(c) Time the trip, and multiply by speed. The time (a proper time) is, from (b), 670 s. The Lorentz-contracted length is

$670(0.60)(3.00 \times 10^{8}) = 1.20 \times 10^{11}$ m.

(d) Answer to (a) would be unchanged. Part (b) would have same answer as (a), and answer to (c) would be 1.50×10^{11} m.

MASTERY TEST GRADING KEY - Form C

1. What To Look For: Student should know the difference between invariance of physical law, and getting the same number in a given experiment.

   Solution: (a) See Grading Key Form A, Solution 1.
   (b) The physical properties of the medium determine the speed of sound with respect to the medium. Those properties obey physical laws that are the same in any inertial frame.

2. What To Look For: Clear understanding of condition for proper time interval. Should recognize that $x_B = 0$, $x_B' = -300$ m, in coordinate frame given here. Sign of t_B must be positive. (You may ask student why.) Alternative solution to (d) is more difficult. Student should correctly use Lorentz transformation equation.

   Solution: (a) Conductor: $\Delta t_{BD}'$ is proper.

   (b) Train frame = primed frame, with origin at front end. Ground frame = unprimed frame, with origin at switchman. Set proper time interval:

   $t_{BD}' = 2(300)/(3 \times 10^8) = 2.00 \times 10^{-6}$ s, for light to go to engineer and back.

   Then $t_{BD} = \gamma t_{BD}' = (5/3)(2)(10^{-6}) = 3.33 \times 10^{-6}$ s.

   Distance train moves: $x_{BD} = (0.80)(3 \times 10^8)(3.33 \times 10^{-6}) = 800$ m.

   $x_B = 0$. $\beta = 0.80$, $\gamma = 1.70$. $x_B' = \gamma(x_B - \beta c t_B) = -\gamma\beta c t_B$.

   $$t_B = -\frac{x_B'}{\gamma\beta c} = \frac{-(-300)}{(5/3)(4/5)(3 \times 10^8)} = 0.75 \times 10^{-6} \text{ s.}$$

   Or: Switchman sees Lorentz-contracted train, $\ell = \ell_0\sqrt{1 - \beta^2}$. Time for train to pass is $t_B = \ell/v = 300(0.60)/0.80(3 \times 10^8) = 0.75 \times 10^{-6}$ s.

   (c) Get t_B' first; then $t_C' = t_B' + \ell'/c$, t_B is proper time interval between events A and B, so

   $t_B' = \gamma t_B = 1.25 \times 10^{-6}$ s. $t_C' - t_B' = \ell_0/c = 10^{-6}$ s. $t_C' = 2.25 \times 10^{-6}$ s.

   $x_C' = 0$. $t_C = \gamma(t_C' + \beta x_C'/c) = \gamma t_C' = 3.75 \times 10^{-6}$ s. $x_C = \beta c t_C = 9 \times 10^2$ m.

   Or: Engineer argues that event B occurs after the train has passed the switchman. That takes time $t_B' = \ell_0/\beta c = 1.25 \times 10^{-6}$ s.

   $t_C' = t_B' + \ell_0/c = 2.25 \times 10^{-6}$ s.

   t_C' is proper, so $t_C = t_C'/\sqrt{1 - \beta^2} = 3.75 \times 10^{-6}$ s. $x_C = \beta c t$, as before.

   (d) $x_B = 0$, as before. t_B is time for train to pass. $t_B = \ell_0/\beta c = 1.25 \times 10^{-6}$ s.

APPENDIX A: The Use of the Personalized System of Instruction (PSI)*

STUDY MODULES FOR THE USE OF THESE CBP MODULES

Recommended

1. What is the Philosophy of PSI?
2. How Do I Manage This System?
3. The Care and Feeding of Student Tutors

Optional

4. How to Plan the Content of a Keller Plan Course
5. How to Design a Study Module
6. How to Write Learning Objectives

REFERENCES (not included in this Appendix)

1. J.G. Sherman (ed.), Personalized System of Instruction, 41 Germinal Papers, A selection of Readings on the Keller Plan (Benjamin, 1974).

2. F.S. Keller and J.G. Sherman, The Keller Plan Handbook, Essays on a Personalized System of Instruction (Benjamin, 1974),

These references may be ordered from the Center for Personalized Instruction, 29 Loyola Hall, Georgetown University, Washington, D.C. 20007.

STUDY MODULE 1: WHAT IS THE PHILOSOPHY OF PSI?

INTRODUCTION

PSI (The Keller Plan) is based on learning theory. In this module you will read about "reward motivated learning": a method of making instruction more effective, relevant, and student-centered. Perhaps the most exciting way to become exposed to this revolution in learning is to read Fred Keller's article "Goodbye Teacher ...". The style is delightful and the content rich with information. (See p. 3 in Sherman's ...41 Germinal Papers, Reference 1.)

*Adapted from the PSI Workshop materials developed by Frank Six, Department of Physics and Astronomy, Western Kentucky University, Bowling Green, KY 42101. These materials were first used at the AAPT meeting, Reno, Nevada in June 1973.

LEARNING OBJECTIVES

When you have mastered the content of this module, you will be able to:

1. Keller plan - Given a list of descriptive terms, identify those that characterize the Keller Plan.

2. Success - List a minimum of two reasons why self-paced, supervised study is successful.

SUGGESTEO STUDY PROCEDURE

To master the first learning objective, begin by reading the one-page description of "The Keller Plan" on p. 5 of the Michigan Memo to the Faculty. You will find it attached to this study module.

To satisfy the second objective, read the article by Ben A. Green, Jr., "Physics Teaching 8y the Keller Plan at MIT" [Am. J. Phys. 37, 7641 (1971)], or see p. 71 of Reference 1, and list the reasons Professor Green gives for the success of PSI at MIT.

PHILOSOPHY OF PSI

All of the learning objectives specified in this study module can also be achieved by digesting Keller's article "Goodbye, Teacher." Now try the Practice Test below.

RESOURCE MATERIALS

1. F. S. Keller, "Goodbye, Teacher . . ." [J. App. 8ehav. Anal. 1, 79-89 (1968)]; or p. 3 of Reference 1.

2. Memo to the Faculty, No. 48, April 1972, Center for Research on Learning and Teaching, University of Michigan, Ann Arbor, page attached.

3. 8. A. Green, Jr., "Physics Teaching by the Keller Plan at MIT" [Am. J. Phys. 37, 764 (1971)], or p. 71 of Reference 1.

PRACTICE TEST

1. Which of the following features describe the Keller Plan?
   (a) Regular lectures over the course subject matter.
   (b) Course material subdivided into units.
   (c) Advancement dependent upon student's demonstration of mastery of the learning objectives.
   (d) All students proceed through the materials at the same rate (lock-step).
   (e) Course objectives and procedures are explained in written study guides that are given to the student.
   (f) Repeated testing without grade penalty.

(g) Each student proceeds at his own rate of learning.
(h) Grades based on student performance on three progress tests and a final exam.
(i) Use of student tutors to individualize the instruction.

2. List two reasons given to account for the success of Keller-type courses.

STUDY MODULE 2: HOW DO I MANAGE THE SYSTEM?

INTRODUCTION

The previous module emphasized the benefits of PSI that accrue to the learner and to the teacher. If we arm the learner with descriptive objectives, relevant materials, and meaningful activities, he is free to go at his own pace and take part in as much or as little of the learning activities as he desires in order to achieve the objectives. He is given responsibility and control over his own efforts, with the guidelines and resource materials that he needs to advance and profit from the experience.

This unit will deal with management skills and course policy. Samples and suggestions will be presented, but the instructor will have considerable latitude in setting up his own rules.

LEARNING OBJECTIVES

When you have mastered the content of this module, you will be able to:

1. Policy statement - Write the course policy statement for your PSI course.

2. Management problems - Devise plans to handle the management problems of your course.

Practice Test Answers

1. All of the features listed, with the exception of (a), (d), and (h), describe PSI. In addition, there are the following common characteristics: group instruction avoided, quizzes graded on the spot with oral tutor-student interaction (immediate feedback), student participation maximized, classroom becomes a study hall, instructor becomes a systems manager.

2. (a) Individual rates of learning, i.e., freedom; (b) frequent and positive feedback to the student, i.e., reward; (c) mastery of the units of study, i.e., success; (d) intensive involvement by the teacher and the students.

GENERAL COMMENTS

1. Aspects of PSI

Mastery: Demand excellence; settling for less degrades the spirit of the endeavor. Keller refers to the "Truman effect," that the job makes the man. The usual treatment of students that implies they are only worth a "C" is an insult. The expectation of excellence is a compliment, a challenge in response to which students do act in a new way (Sherman, p. 120, Reference 1).

Motivation: Make the first module appealing and easy to get the students started. After this the reinforcing nature of the system will take over. Make every effort to engage the learner in searching, discovering, and verifying. Provide a multi-media approach with student options, and insist on permanent storage characteristics and maximum accessibility.

Grades: Perhaps achievement of your core objectives might represent a three-credit "C" or a two-credit "B" (variable credit), with the core plus optional units for correspondingly higher grades. Isn't it more important to know a student's competencies in relation to desired accomplishments than in relation to other students' competencies?

2. System Management

Tutors: Prior students in the course can serve as tutors for academic credit plus the reward of learning the physics better. You may wish to consider choosing students who advance rapidly through the first several study units as tutors. Reward them for serving as tutors by excusing them from the final exam. Peers understand many of the difficulties better than do teachers, and praise from peers is probably more important than praise from teachers. Close supervision of tutors is recommended.

Procrastination: If you decide to hold quality constant and let time vary (the Keller tradition), some procrastination is bound to occur. Build in rewards for prompt performance. Give students graphs of the number of modules as a function of time with progress lines drawn on them.

Communications: With no attendance requirements, keeping in touch becomes a problem. Some have found it helpful to keep a "Keller Bulletin Board" in the learning center.

Logistics: Keep the learning center open as many hours as possible. Students should be able to obtain assistance and have quizzes graded without much delay.

SUGGESTED STUDY PROCEDURE

Read Chapter 4, Logistics, in The Keller Plan Handbook (see p. 24 of Reference 2). Then proceed to the Practice Test and answer the questions.

RESOURCE MATERIALS

1. The Keller Plan Handbook (Reference 2 on page 1).

PRACTICE TEST

1. How many students will be enrolled in your Keller class?
2. How many and what kinds of staff will you need?
3. How many hours per week will class be open?
4. When will students' quizzes be graded?
5. What type of classroom facilities/space will be desirable?
6. What will be your grading policy?
7. Do you intend to give a final exam?
8. Will your course really be self-paced?
9. How can you "help" students pace themselves?
10. What will you do with the procrastinators?
11. How will you handle incompletes?
12. How will you keep in touch with your students (communications)?
13. What functions will your tutors perform?
14. How will you reward the tutors?
15. What criteria will you use in choosing your tutors?
16. How will you (or will you) use the lecture mode?
17. How will you motivate the students?
18. What will you establish as the level of proficiency on test items (mastery)?
19. How will you maintain test question security and prevent cheating?
20. What kind of records will you maintain?

Practice Test Answers

Your answers to these questions will determine the kind of PSI course you offer. If you wish to have additional input, read the exhibits found in the CBP Orientation Module. You may wish to call or write to CBP Workshop, Behlen Laboratory of Physics, University of Nebraska - Lincoln, NE 68588 (402)472-2790, The Center for Personalized Instruction, 29 Loyola Hall, Georgetown University, Washington, D.C. 20007, or the CBP Module author who is nearest to you.

STUDY MODULE 3: THE CARE AND FEEDING OF STUDENT TUTORS

INTRODUCTION

Student tutors play an essential role in the success of a Keller Plan course. In fact, the first time you use the Keller Plan you will probably discover that the students in the course prefer to ask the tutors for help rather than to ask you. The students feel more comfortable in exposing their ignorance to other students than to the instructor, who finally decides their grade. It is important to provide your tutors with a positive learning experience so that they can function effectively in your Keller Plan course. Hence, you need to think seriously about the care and feeding of your student tutors.

LEARNING OBJECTIVES

When you have mastered the content of this module, you will be able to:

1. Use of tutors - Construct a Keller Plan system that makes effective use of student tutors.

SUGGESTED STUDY PROCEDURE

To satisfy Objective 1, you should read all of the materials in References 1 and 2 that discuss the roll and selection of proctors. In Reference 2, see pp. 20 through 23 and 33, 38, and 39. Read articles #1, #5, and #29 in Reference 1.

A wide variety of procedures have successfully been used to attract and hold Keller Plan tutors. You need to examine the variety of systems that have been used and decide upon a system that will work at your institution. The secret of a successful tutoring system is to have enough so that the individual tutoring load is not too heavy (not more than 10 students per tutor) and to provide meaningful reward for the tutors.

PRACTICE TEST

1. How many student tutors will I need?
2. What reward will the tutors get? money? credit hours? a better grade in the course? a cup of coffee each week with the instructor? snake oil?

Practice Test Answers

1. Number of students divided by 8 to 12.
2. Depends upon your institutional system.

STUDY MODULE 4: HOW TO PLAN THE CONTENT OF A KELLER PLAN COURSE

INTRODUCTION

In this module you will consider general planning considerations: how to get organized, where to start. The questions raised here will indicate that "Kellerizing" your course is not something to be undertaken lightly. The commitment is a major one in terms of time and effort. The rewards are worth it.

LEARNING OBJECTIVES

When you have mastered the content of this module, you will be able to:

1. Kellerizing - List three decisions that require considerable thought on the part of the instructor as he prepares to Kellerize his course content.

2. Learner objectives - Recall various methods you might employ to select student competencies (learner objectives) that should result from your course.

3. Activity sequence - Given a list of Keller-course planning activities, order them in a chronological sequence.

4. Consolidation - Describe, in writing, Keller's suggestion for avoiding undue fragmentation of the course (the result of too much modularization) and for consolidating what the student has learned.

SUGGESTED STUDY PROCEDURE

Joel Greenspoon (Reference 1), Associate Dean of the Faculty at Temple Buell College, recommends the following procedure: First, decide what the student is to learn in the course. This is not easy for many instructors because they have not asked themselves this kind of question. When deciding on the content of the course, the decision should be made in terms of what the student is to do or say. To state that a student is to know, understand, or appreciate the effects of Copernican theory on scientific developments in the early stage of the Renaissance does not describe what the student is to do. To ask the student to describe (under certain conditions) these effects is telling the student what he is to do. To analyze critically some development does not describe the student's behavior, because the determination of critical analysis still rests with the instructor. It would be better to instruct the student to describe, in writing, the arguments for and against a given position. The clear specification of learning objectives is crucial. Sequencing the objectives so that students can proceed systematically through the course involves some trial and error. Second, decide how you will determine when the student has learned it. This implies a clear statement of how he is to do it. The instructor should provide explicit criteria by which the student' .ork is judged. Some instructors prefer the use of a written exam; others prefer an oral or a paper. Laboratory reports and skill testing are other ways in which the student might satisfy objectives. Despite the fact that faculty members have been judging students' work for years, they find it difficult to state explicitly the criteria by which they judge or evaluate the students' efforts. Third, decide what sources of information are pertinent to the objectives. Choose learning activities that will facilitate learning by the students. Field trips, experiments, readings, lectures, and demonstrations should be considered. If self-pacing is one of your goals, lectures and demonstrations can be handled by making accessible audiotape and videotape, to ensure that the progress of any single student is not hindered. The above remarks should enable you to satisfy Objective 1.

Objective 2 is concerned with methods you might use to make the first decision - what should the student learn in the course. Remember the advice: think in terms of what the student is to do or say (student competencies). It is a simple matter to go through a book and identify all of the possible learning objectives. But which of those objectives define competencies (proficiencies, performances) that you want every student to take away from your course? Concentrate on the minimum learning objectives (core) as opposed to the "nice-but-not-necessary," enrichment-type objectives. Take a look at your old final exams. Are these competencies the most important? You might even go so far as to enlist the advice of colleagues (shudder). Deterline and Lenn (Reference 2) suggest performing a simulated task analysis: imagine the oral examination setting and develop a list of appropriate questions, problems, and tasks for each major course topic; sift out the core objectives by deleting the nonessential; consult other colleagues who teach the same course.

Your planning activities (Objective 3) might go like this:

(1) Identify competencies desired (simulated task analysis, text, final exams, colleagues).
(2) Determine criteria of evaluation (test items, practice exercises, study questions).
(3) Write the minimum learning objectives based on (1) and (2).
(4) Sequence the objectives and divide into study modules.
(5) Specify the resource materials (learning activities).

A simpler prescription has been offered by Dr. Keller. See the page of the University of Nebraska newsletter PSI "A Guide to What, How, Why & Why Not of PSI" (attached, also Reference 3), and read the section PSI Instructional Tasks.

The advice concerning review modules applies to Objective 4. The CBP modules are examples of a course that was planned according to Keller's recipe.

Now, proceed to the Practice Test.

RESOURCE MATERIALS

1. J. Greenspoon, Assoc. Dean of Faculty, Temple Buell College, Denver, Colorado (private communication).

2. W. A. Deterline and P. D. Lenn, Coordinated Instructional Systems, Study Resource Materials Book (Sound Education, Inc., Palo Alto, Calif., 1972), pp. 14-17.

3. "A Guide to What, How, Why & Why Not of PSI," Instructional Media Center, 421 Nebraska Hall, University of Nebraska, Lincoln, NE 68588.

PRACTICE TEST

1. Write three tasks that Greenspoon recommends an instructor deliberate over during the planning of a Keller course.

2. Name two procedures useful in determining just what the student is to learn (student competencies) in your course.

3. Order the following activities in the sequence you would perform them as you plan the content of a Keller course:
(a) Perform a task analysis to identify the competencies students should be able to demonstrate when they have completed the course.
(b) State explicitly the conditions and the criteria by which you intend to evaluate the students performance; in other words, write test items or practice exercises that identify the competencies you want all of your students to attain.
(c) Write the minimum learning objectives (core) in performance terms, e.g., what the student is to do or say.
(d) Sequence the objectives; group them in study modules (prepare a course outline).
(e) Specify learning activities that will enable the student to satisfy the learning objectives, e.g., select resource materials, write practice tests.

4. What is Keller's recipe for consolidating what the student accomplishes in the separate study modules?

Practice Test Answers

1. First, deciding what student competencies are to be attained; second, deciding what criteria you will use to judge the students competencies; third, deciding what learning activities will facilitate development of these competencies. These tasks are the foundation of a new surge in education circles - "competency-based education" (CBE), or "performance-based education" (PBE), and they are of central importance in the Keller method.
2. Perform a simulated task analysis, examine your old final exams, proceed through a textbook and identify learning objectives, talk with your colleagues.
3. The order is correct, as stated. Some might suggest that first, decide what material you want to cover and second, divide it into short units. But, do you know what your destination is? The sequence (a) through (e) stresses learner outcomes (competencies). We'll have more to say about this in succeeding modules.
4. Include review modules.

STUDY MODULE 5: HOW TO DESIGN A STUDY MODULE

INTRODUCTION

The preceding module dealt with deciding upon the goals you intend students to reach at the end of your course. It has been suggested that you select procedures, content, and methods appropriate to the objectives; cause students to interact with the subject matter in well-thought-out learning activities; and, evaluate the student's performance according to the objectives or goals that were selected. Assuming that you know what your destination is, this module concerns selecting the most efficient route to get the students there.

LEARNING OBJECTIVES

When you have mastered the content of this module, you will be able to:

1. Module components - List the components of a study module in the sequence they are presented to the student.

2. Function of components - Write descriptions of the function of each component of a study module and be able to distinguish between properly and improperly written components.

3. Instructor order - List the components of a study module in the order that they are developed by the instructor.

4. Developing modules - Describe a method of developing study modules that is initiated by constructing test items.

SUGGESTED STUDY PROCEDURE

The first objective refers to the order of presentation of the component information in a study module. If you read the CBP modules you will surely recognize the order: Introduction, Objectives, Procedure (including directions, resource materials, and finally, Practice Tests). The table below is patterned after material in Deterline and Lenn's manuals on Coordinated Instructional Systems (Reference 1). You should examine this table until you can satisfy Objective 2.

STUDY-MODULE COMPONENTS AND FUNCTIONS

Component	Function
Introduction	Tells the learner what the unit is about; identifies the topics; sets the scope; is brief, clear and interesting. Provides a concrete, familiar example of the physics contained in the module.
Learning Objectives	Tell the student what he will be able to do as a result of completing the unit; the competencies he will acquire and will be required to demonstrate.
Procedure	Facilitates learning by the student; directs the learner to the resource materials in the form of books, audio-visuals, lectures, demonstrations, models, and other instructional materials that have maximum accessibility; practice exercises and discussions identifying correct answers; identicates what to do, using which materials, located where, and why.
Practice Test	Provides a private progress evaluation to help the learner realize his own problems and decide what to do about them; measures the increase in observable competencies; provides practice in the use of the skills acquired.

The order in which the components of a study module are developed is not the same as the order of presentation to the student. It is suggested that you start by deciding what the successful learner is to be able to do at the end of the study module. If you follow the sequence below in developing study modules, then you are more apt to lead the student to the desired competencies.

Order of Development - Study-Module Components

(1) Mastery Tests
(2) Objectives
(3) Procedure (learning activities, resource materials, practice exercises)
(4) Introduction

Designing the mastery tests is an easier method of identifying objectives than the method of constructing learning objectives first. The test items will identify the competencies completely and provide a guide to show you what it is that you must prepare the student to do. In developing the test items, ask yourself, "What questions, problems or tasks do I want the learner to be able to answer, solve or perform; and what is the minimum acceptable performance?" Evans (Reference 2) put it this way: develop the test, teach the test, stop! (This should give you a hint about Objective 4.)

RESOURCE MATERIALS

1. Coordinated Instructional Systems, Lesson Book and Study Resource Materials Book (Sound Education, Inc., Palo Alto, Calif.), edited by W. A. Deterline and P. D. Lenn.

2. J. L. Evans, "Behavioral Objectives Are No Damn Good," Technology and Innovation in Education (Aerospace Education Foundation, Washington, D.C., 1967).

3. Sample Module: A CBP module, for example "Work and Energy."

PRACTICE TEST

1. List the components of a study module in the order they are presented to the learner.

2. What is the function of the Practice Test?

3. Does the Introduction in the CBP module "Work and Energy" fulfill the function of a properly written introduction?

4. List the components of a study module in the order of development by the instructor.

5. Describe the use of test items in identifying your learning objectives.

Practice Test Answers

1. Introduction, Objectives, Procedure, Practice Test.
2. To provide the student a basis for progress evaluation. The test items should require the same behaviors that you desire the student to be able to perform.
3. It does identify the topics of the module, i.e., the scope; but, is it interesting? Students' comments should be solicited (feedback).
4. Test, Objectives, Procedure, Introduction.
5. A good test question calls for the desired accomplishment of instruction. This accomplishment (student competency) is the Objective.

STUDY MODULE 6: HOW TO WRITE LEARNING OBJECTIVES

INTRODUCTION

The advantage of clearly defined learning objectives is that the student knows where he is going, can tell what progress he is making, and can tell when he gets there. He can direct his attention to the essential information that he needs; and he can make more efficient use of his time. Minimum learning objectives fortify the instructor. Without them, there is no basis for evaluating a course, for selecting materials, or for choosing instructional methods. The core objectives referred to in the previous study module are the essential objectives that every student should take with him from the course. Some will say that trial and error has a place in education because students must learn to achieve in that kind of real world environment. If that be the case, we should develop "survival objectives" rather than hope students will pick them up in courses designed to develop other competencies.

LEARNING OBJECTIVES

When you have mastered the content of this module, you will be able to:

1. Instructional intentions - Given a list of objectives distinguish between those that do and do not communicate instructional intentions.

2. Criteria for discrimination - Differentiate between learning objectives that do and do not describe (a) the kind of performance or learner competency to be demonstrated, (b) the conditions under which the competency is to be demonstrated, and (c) the minimum acceptable performance.

3. Write learning objectives - Use content material and a list of action verbs to write learning objectives that accomplish all of the following:
   (a) describe the learner's competency (behavior performance) to be demonstrated as evidence of accomplishment of the objective;
   (b) identify the conditions under which competency is to be demonstrated; and
   (c) define the acceptable standards of competency.

SUGGESTED STUDY PROCEDURE

Mager says an instructional objective is an intent communicated by describing a proposed change in a student. It is a statement of what the student is to be like when he has successfully completed a learning experience. In other words, it is a description of a pattern of behavior (performance) that we want the student to be able to demonstrate. Everyone who is interested in teaching skills and knowledge to others should read Mager's book, Preparing Instructional Objectives (Reference 1).

The objectives in this study module are "content" objectives as opposed to "attitudinal" objectives. Bloom's books (Reference 2) discuss the various kinds of objectives you can choose. Whatever the objective, you should be detailed enough so that others understand your intent as you understand it.

The first objective in this module is concerned with communicating your instructional intent. Mager suggests you pose three questions to ensure that you do communicate effectively:

(1) Does your statement describe the performance (behavior, competency) by name, that will be accepted as evidence that the learner has achieved the objective?

(2) Does your statement identify the conditions (givens, restrictions) under which the learner must demonstrate his competency?

(3) Does your statement describe how well the learner must perform to be acceptable (acceptable standards)?

The best way to couch your objectives in "performance terms" is to use action verbs that describe what the learner will be doing, e.g., to write, to identify, to discriminate, to solve, to repair, to construct, to draw, to list. Strive to avoid misinterpretation.

Often it is desirable to define the objective more precisely by stating the conditions that will be imposed on the learner when he is demonstrating his competency. Restrictions such as, "given such and such," "using only a voltmeter," and "without the aid of trig tables" might improve the ability of the objective to communicate. Ask yourself what the learner will be provided and what will he be denied. Does your objective exclude those learner responses that you do not want to elicit? If not, be more specific. One method of ensuring your objectives are understood is to provide examples or practice exercises. Note the use of this practice in the CBP modules.

Stating the minimum acceptable performance further specifies your learning goals. When appropriate, identify the standard or lower limit of acceptable performance, e.g., a time limit, a minimum number of correct responses, a percentage, or an accuracy. If you have followed the above remarks, then you should be able to satisfy the objectives of this study module. Try the Practice Test now.

RESOURCE MATERIALS

1. R.F. Mager, Preparing Instructional Objectives (Fearon, Palo Alto, Calif., 1962).

2. B.S. Bloom, Taxonomy of Educational Objectives, Handbooks I and II (David McKay, New York, 1956, 1964).

3. Verb List (attached).

4. CBP Modules, for example, "Work and Energy."

PRACTICE TEST

1. Which of the following learning objectives communicate instructional intentions?
   (a) To develop an understanding of motion with constant acceleration.
   (b) To appreciate the nature of physical law.
   (c) To recite from memory . . . .
   (d) To be able to repair a voltmeter.
   (e) To write Newton's three laws of motion.
   (f) To know how a laser works.

2. Which of the learning objectives in the "Work and Energy" Module are written in performance terms?

3. Which of the above objectives defines the standard of acceptable performance (criterion)?

4. Which of the above objectives specify the conditions under which the performance is to occur?

5. Do the three written objectives in this study module contain the items that are listed in Objective 3?

Practice Test Answers

1. (c), (d), and (e) communicate, in performance terms, the instructional intentions. If you chose (a) what is the learner doing when he demonstrates that he understands? The objective (b) does not preclude or define any behavior. If the learner remarked, "I dig that, daddy-oh," it would be necessary to accept this as evidence of appreciation. The word "know" in objective (f) doesn't explain much either.

2. All of them are written in performance terms. Each names an act the learner would perform as he demonstrates he has accomplished the objective.

3. None do. This could be remedied by adding the criterion, "allowable error ± 5%" or by specifying a time limit for solving the problems.

4. Most specify conditions, e.g., "on a scale diagram . . . ," "for example," "given a description of . . . ," "which includes . . . ," "when you are given a list of . . . ."

5. They (we hope) communicate the instructional intent by describing the performance, and by identifying the conditions. Only Objective 3 includes a criterion, "accomplish all the following." So, you see, there is room for improvement.

THE KELLER PLAN*

In a 1967 address Professor Fred Keller, a distinguished investigator of basic processes of learning, described to fellow educators a method of college teaching which breaks radically with past practices. In the five years since Keller's address, the method - sometimes known as "self-paced supervised study" but often called simply the Keller Plan - has been applied in numerous college courses around the country.

The work of a course taught by the Keller plan is divided into units. In a simple case, 15 units may be delineated which reflect the 15 chapters of the course text. A student starting the first unit is given a printed study guide that introduces the unit, describes its objectives, recommends procedures for studying to achieve these objectives, and includes sample questions. The student works individually on the unit, and must demonstrate his mastery of the material before moving on to the next unit in the sequence.

Mastery is ordinarily demonstrated by perfect or near perfect performance on a short-essay examination (Keller's preference for his introductory psychology course). The student may take an examination on a given unit whenever he feels ready, and failure to pass the test on the first try, the second, the third, or even later, is not held against him. However, he is given the study guide for the next unit only after he demonstrates mastery of the unit. Thus, students move at their own pace through a course from start to finish. A student may meet all course requirements in less than a semester, or he may not complete the course within the semester.

Throughout much of the course, the classroom simply functions as a study hall, where the student may read course material. Lectures and demonstrations are given less frequently than in a conventional course (perhaps six lectures in the course of a semester). In Keller's courses lectures and demonstrations were vehicles for motivation; they were not compulsory and no examination was based on them.

The staff for implementing the Keller Plan includes the instructor and undergraduate proctors. The instructor selects and organizes all study materials used in the course, and is responsible for the construction of examinations and for the final evaluation of each student's progress. A proctor is an undergraduate who has already completed the course and been chosen for his mastery of the course content and orientation, for his maturity of judgement, for his understanding of the special problems that confront students as beginners, and for his willingness to assist. The proctor receives academic credit for a significant learning experience. He provides students with study materials (except for textbooks) for the unit they are attempting and passes upon the successive mastery test as satisfactory or unsatisfactory. Course grades under the Keller Plan are usually based on the number of units of reading and the laboratory work successfully completed during the term and a final examination.

Keller (1968) summarizes those features of the plan that seem to distinguish it most clearly from conventional teaching procedures.

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*By Dr. James A. Kulik of CRLT, from the Memo to the Faculty, April 1972.

"(1) The go-at-your-own-pace feature, which permits a student to move through the course at a speed commensurate with his ability and other demands on his time.

"(2) The unit-perfection requirement for advance, which lets the student go ahead to new material only after demonstrating mastery of that which preceded.

"(3) The use of lectures and demonstrations as vehicles of motivation, rather than sources of critical information.

"(4) The related stress upon the written work in teacher-student communication; and finally:

"(5) The use of proctors, which permits repeated testing, immediate scoring, almost unavoidable tutoring, and a marked enhancement of the personal-social aspects of the educational process."

From letters received by the editors of a newsletter devoted to the Keller Plan (Sherman, 1971), it has been estimated that over 500 faculty members in a variety of disciplines have taught (or are about to teach) Keller-based courses. Not all the courses include all five of the features described by Keller; modifications have been introduced to fit a variety of local demands. A review (Kulik, 1972) of early reports on application of Keller-based plans makes several points:

1. Students taking Keller courses report spending a good deal of time on their studies. Several investigators report relatively high dropouts rates from Keller-based courses, and the most frequent comment from students who withdraw is that these courses are "too much work."

2. Students finishing Keller-based courses usually are given high grades. Since grades are assigned in a manner having little parallel in traditional courses, grade-distributions do not necessarily indicate that students learn more, but there are no reports of poorer learning under the Keller Plan.

3. In a number of comparisons, there are no significant differences on final-examination performances of students in Keller and conventional classrooms and in a few investigations, students studying under the Keller Plan do somewhat better on final examinations. Interpretation of these results must take into account dropout rates.

4. Most studies show that students completing Keller courses are highly satisfied with the learning method. In the University of Florida project, for instance, <u>all</u> students reported that they preferred the unit-performance format to typical course formats. Evidence showing strong student dissatisfaction with the Plan has not yet been presented. Interpretation of these results also must take into account dropout rates and grading practices.

5. There is some consensus among those who have used the Keller Plan that undergraduate students serving as proctors benefit especially from the method.

6. Several authors have noted the possible cost-savings to institutions using the Keller Plan. The use of undergraduate assistants is one basis for the economy.

VERB LIST*

SOME POSSIBLE VERBS FOR USE IN STATING COGNITIVE OUTCOMES†

Bloom's Taxonomy of Educational Objectives

Knowledge - to recall and recognize.
Comprehension - to translate from one form to another.
Application - to apply or use information in a new situation.
Analysis - to examine a complex and break it down into its parts.
Synthesis - to put together information in a unique or novel way to solve a problem.
Evaluation - to make a judgment about something in light of some criteria.

Knowledge	Comprehension	Application	Analysis	Synthesis	Evaluation
define	translate	interpret	distinguish	compose	judge
repeat	restate	apply	analyze	plan	appraise
record	discuss	employ	differentiate	propose	evaluate
list	describe	use	appraise	design	rate
recall	recognize	demonstrate	calculate	formulate	compare
name	explain	dramatize	experiment	arrange	value
relate	express	practice	test	assemble	revise
underline	identify	illustrate	compare	collect	score
	locate	operate	contrast	construct	select
	report	schedule	criticize	create	choose
	review	show	diagram	set up	assess
	tell	sketch	inspect	organize	estimate
			debate	manage	measure
			inventory	prepare	
			question		
			relate		
			solve		
			examine		
			categorize		

THREE KINDS OF OBJECTIVES

Cognitive - to promote abilities in thought and understanding.

Attitudinal - to promote changes in attitude, feeling, or emotion.

Psychomotor - to promote improvement in physical or manipulative skills.

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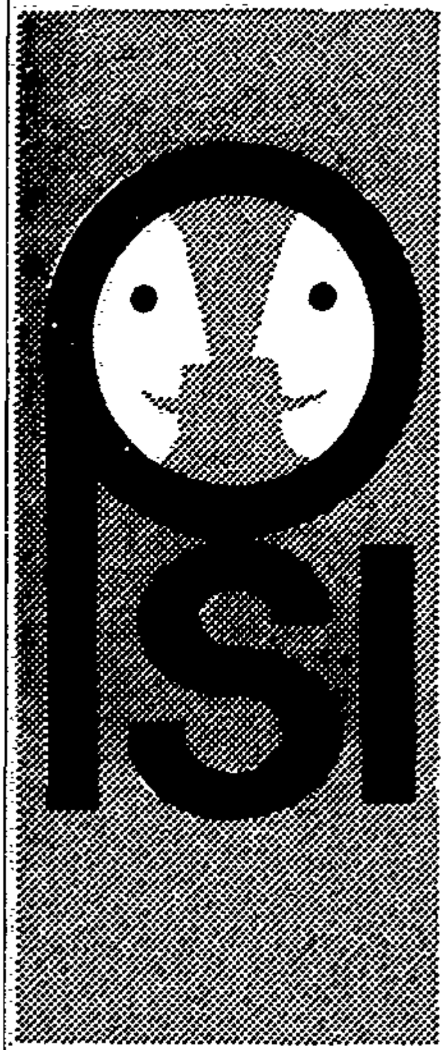
*R.B. & S.R. Johnson, Assuring Learning with Self Instructional Packages (Self Instructional Packages, Inc., Philippines, 1973).

†Compliments of Marybelle Savage.



PSI INSTRUCTIONAL TASKS

If you are interested in teaching a course using the Personalized System of Instruction, you need to give thought to at least three instructional tasks: development of the course content, devising a course policy, and formulating a strategy for managing the course.

The development of your course content may be facilitated if you look at what the other people have done in your academic discipline or any related content areas. Both the Proceedings of the Rice Conference[1] and the PSI Newsletter[2] are good sources of such information.

Most PSI courses use some form of written study guides. If you have not had experience in writing study guides you will find a booklet by Walbesser, et.al.[3] of some interest or perhaps the article by Speeth & Marguilles on "Techniques for Maintaining Student Motivation".[4]

Of course, you could hardly go wrong if you follow the prescription, prescribed by Dr. Keller as follows:

"The first thing to do is to break down your course material into the study units. Twenty to thirty, in a three-hour course, is my suggestion; but there's getting to be a lot of talk about the number of units, and I guess it depends on so many things that I'd better not stress that. It should include, however, three or four units of a review. That is to avoid undue fragmentation of the course and to consolidate what the student has learned.

Secondly, add to each unit a set of study questions and objectives and make up three or four equivalent tests to cover the same material.

Thirdly, put each student through the unit at his own pace, testing him as many times as needed, without penalty for failing, until each unit is mastered to perfection. I know perfection is a word that should be in quotations, but we know roughly what I mean.

Fourthly, throw in a few lectures or demonstrations during the new term for seasoning. But don't require your students to attend them. Their aim is to inspire, not to be remedial or to inform, and if you overdo the lecturing, you're taking the students away from things that would be more productive.

Fifthly, use well-prepared and carefully guided student proctors to grade the unit tests. One proctor to each ten students is about the right proportion.

Add a final examination, if you wish, when the units have all been completed. It may make your course smell better to your colleagues, and it may fortify the product. Give an "A" to everyone who completes the course requirements, early or late—roughly as you would award a Ph.D. Be generous with your Incompletes, but it is possible that you may want to stir things up a bit if you see too much procrastination taking place. And, finally, watch carefully, while cooking."

UNIVERSITY OF NEBRASKA-LINCOLN

In response to questions about PSI raised by our film, "Personalized System of Instruction: An Alternative", this guide has been compiled by Dr. Vernon Williams, Teaching and Learning Center; and Drs. Robert G. Fuller and David W. Joseph, Department of Physics.

Single copies of this guide are available for $.25 and 10 or more copies for $.10 each plus postage. Order from Dr. James G. Buterbaugh, Instructional ... Center, 421 Nebraska Hall, University of Ne..., Lincoln 68508.