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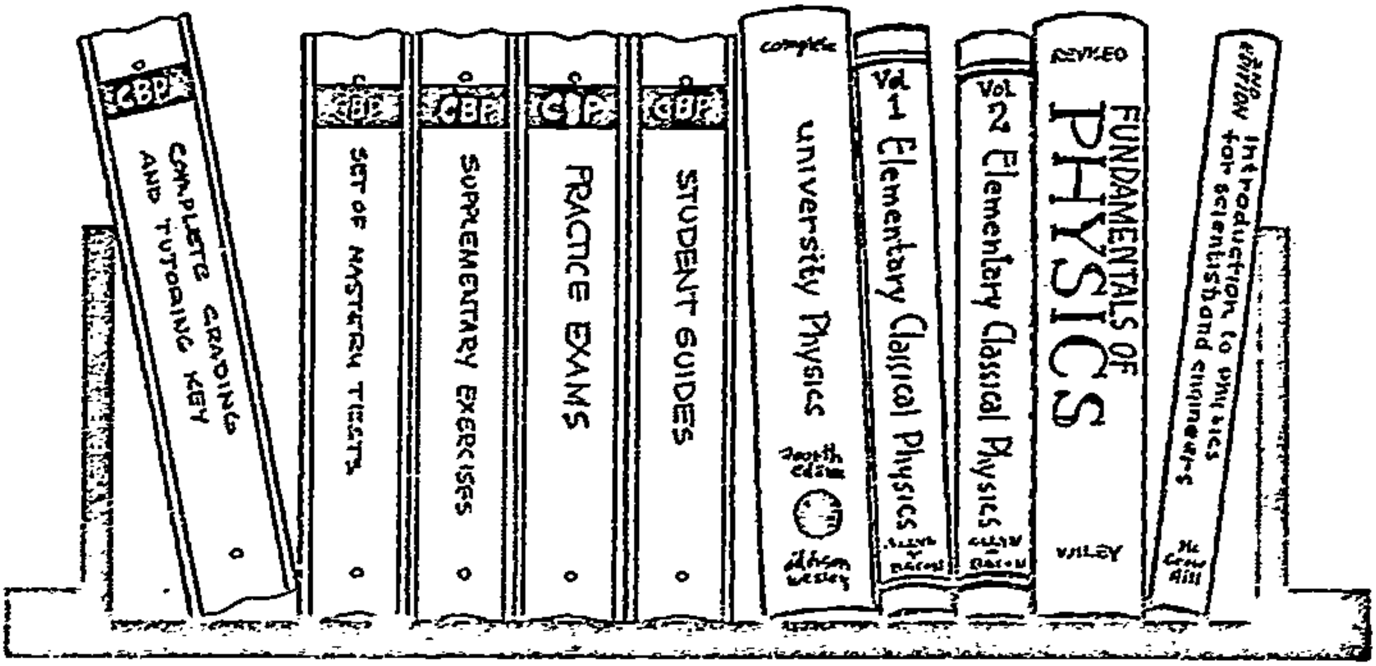
ABSTRACT

This is part of a series of 42 Calculus Based Physics (CBP) modules totaling about 1,000 pages. The modules indlude study guides, practice tests, and mastery tests for a full-year individualized course in calculus-based physics based on the Personalized System of Instruction (PSI). The units are not intended to be used without outside materials; references to specific sections in four elementary physics textbooks appear in the modules. Specific modules included in this document are: Module 27--Direct-Current Circuits, Module 28--Magnetic Forces, Module 29--Ampere's Law, and Module 30--Faraday's Law. (CP)

STUDY MODULES FOR CALCULUS-BASED GENERAL PHYSICS*



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Comments

These modules were prepared by fifteen college physics professors for use in self-paced, mastery-oriented, student-tutored, calculus-based general physics courses. This style of teaching offers students a personalized system of instruction (PSI), in which they increase their knowledge of physics and experience a positive learning environment. We hope our efforts in preparing these modules will enable you to try and enjoy teaching physics using PSI.

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These modules were prepared by the module authors at a College Faculty Workshop held at the University of Colorado - Boulder, from June 23 to July 11, 1975.

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COMMENT TO USERS

In the upper right-hand corner of each Mastery Test you will find the "pass" and "recycle" terms and a row of numbers "1 2 3 ..." to facilitate the grading of the tests. We intend that you indicate the weakness of a student who is asked to recycle on the test by putting a circle around the number of the learning objective that the student did not satisfy. This procedure will enable you easily to identify the learning objectives that are causing your students difficulty.

COMMENT TO USERS

It is conventional practice to provide several review modules per semester or quarter, as confidence builders, learning opportunities, and to consolidate what has been learned. You the instructor should write these modules yourself, in terms of the particular weaknesses and needs of your students. Thus, we have not supplied review modules as such with the CBP Modules. However, fifteen sample review tests were written during the Workshop and are available for your use as guides. Please send $1.00 to CBP Modules, Behlen Lab of Physics, University of Nebraska - Lincoln, Nebraska 68588.

FINIS

This printing has completed the initial CBP project. We hope that you are finding the materials helpful in your teaching. Revision of the modules is being planned for the Summer of 1976. We therefore solicit your comments, suggestions, and/or corrections for the revised edition. Please write or call

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DIRECT-CURRENT CIRCUITS

INTRODUCTION

One way to help you understand a new phenomenon is to show you that it is like something that you are already familiar with. This method is used very frequently in physics, e.g., the electric field is like the gravitational field. This module will introduce you to a simple class of RC circuits in which there are currents, charges, and voltages that decay exponentially. This may be your first detailed study of exponential decay, but it is like (analagous to) radioactive decay, Newton's law of cooling, the final depletion of a natural resource, the decrease in atmospheric pressure with altitude, and some other interesting phenomena. With a sign change, it is like a simple model of exponential growth, which is how population, energy consumption, and pollution generation seem to be growing. The module begins, however, with a few simple ideas applied to direct-current circuits. These are the basic ideas upon which you will later build an understanding of alternating-current circuits. This course will not cover electronics, but it will provide some of the introductory concepts that are needed for a study of electronic devices and circuits.

PREREQUISITES

Before you begin this module, you should be able to:	Location of Prerequisite Content
*Apply Ohm's law to find potential difference from current and resistance (needed for Objective 1 of this module)	Ohm's Law Module
*Find potential difference of a capacitor from charge and capacitance (needed for Objective 4 of this module)	Capacitors Module
*Find the power developed in a resistor (needed for Objective 1 of this module)	Ohm's Law Module

LEARNING OBJECTIVES

After you have mastered the content of this module, you will be able to:

1. Loop equation - Analyze a single-loop direct-current (dc) circuit consisting of resistances and a seat of emf to find the loop current, the power developed in the circuit elements, and the terminal potential difference of the seat of emf.
2. Equivalent resistance - Determine an equivalent resistance for a series or parallel combination of resistances.
3. RC-loop equation - Write the differential equation for a single RC loop, and verify that particular assumed solutions satisfy this equation.
4. Exponential decay - Write the equation for the current, charge or voltage of a capacitor as a function of time in a single RC loop, and manipulate this equation to determine the value of one of the parameters when an appropriate set of other values is given.

TEXT: Frederick J. Bueche, Introduction to Physics for Scientists and Engineers (McGraw-Hill, New York, 1975), second edition

SUGGESTED STUDY PROCEDURE

Study the text, Chapter 21, Sections 21.8 and 21.10, and Chapter 22, Sections 22.1, 22.2, 22.3, and 22.4. Then read General Comment 1. Study Problems A and B of this study guide; then work Problem F and Problems 2, 4, 7, and 9 of Chapter 22 in your text. Study Sections 21.11 and 21.12 of the text; then read General Comment 2. Study Problems C, D, and E and work Problems G and H. Then work Problems 18, 19, 20, and 21 in Chapter 21.

When you think you have mastered the four learning objectives, take the Practice Test. If you need more help, work the Additional Problems before taking a Mastery Test.

BUECHE

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text	Study Guide	Text	
1	Secs. 21.8 to 21.10, 22.2, General Comment 1	A	Illus.[a] 22.1	F	Chap. 22, Prob. 9	Chap. 22, Probs. 11, 14
2	Sec. 22.1, General Comment 2	B		F	Chap. 22, Probs. 2, 4, 7	Chap. 22, Probs. 1, 3, 5, 6, 20
3	Secs. 21.11, 21.12	C		G		
4	Secs. 21.11, 21.12	D, E		H	Chap. 21, Probs. 18, 19, 20, 21	I, J

[a]Illus. = Illustration(s).

TEXT: David Halliday and Robert Resnick, Fundamentals of Physics (Wiley, New York, 1970; revised printing, 1974)

SUGGESTED STUDY PROCEDURE

Study Chapter 28, Sections 28-1 through 28-5 in the text. Then read General Comment 1 of this study guide. Study Problems A and B of this study guide before working Problems F and 5, 6, 7, and 20 of Chapter 28. Then study Section 28-6 and read General Comment 2. Study Problems C, D, and E before working Problems G and H and 32, 35, 36, 37 in Chapter 28.

Try the Practice Test, and work some of the Additional Problems if you have any difficulty, before taking a Mastery Test.

HALLIDAY AND RESNICK

Objective Number	Readings	Problems with Solutions: Study Guide	Problems with Solutions: Text	Assigned Problems: Study Guide	Assigned Problems: Text (Chap. 28)	Additional Problems (Chap. 28)
1	Secs. 28-1 to 28-5, General Comment 1	A	Ex.[a] 2, 3	F	5, 6, 7	1, 2, 3, 4, 8, 9, 10, 11, 12, 13, 14
2	Secs. 28-3, 28-5, General Comment 2	B	Ex. 1, 4	F	16, 20	15, 17
3	Sec. 28-6	C		G		
4	Sec. 28-6	D, E	Ex. 5	H	32, 35 36, 37	31, 33, 34, 38, 39, 40, 41; I, J

[a]Ex. = Example(s).

TEXT: Francis Weston Sears and Mark W. Zemansky, University Physics (Addison-Wesley, Reading, Mass., 1970), fourth edition

SUGGESTED STUDY PROCEDURE

Study Chapter 28, Sections 28-5, 28-6, 28-7, and Chapter 29, Sections 29-1 through 29-6. Then read General Comment 1 of this study guide. Study Problems A and B and the Examples in Sections 28-5, 28-7, and 29-1 before working Problem F in this study guide and Problems 28-14, 28-15, 28-23, 28-24, 29-3, 29-4, and 29-6 in the text. Next study Section 29-7 (especially the Example), General Comment 2, and Problems C, D, and E before working Problems G, H, I, J, and 29-35.

Try the Practice Test, and work some of the Additional Problems if necessary, before taking a Mastery Test.

SEARS AND ZEMANSKY

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text	Study Guide	Text	
1	Secs. 28-5, 28-7, General Comment 1	A	Sec. 28-5, Ex.[a] 1 to 5; Sec. 28-7, Ex. 1 to 4	F	28-14, 28-15, 28-23, 28-24	28-16, 28-17, 28-18, 28-20, 28-21, 28-22
2	Sec. 29-1, General Comment 2	B	Sec. 29-1, Ex.	F	29-3, 29-4, 29-6	29-1, 29-2, 29-5, 29-7, 29-8, 29-9, 29-10, 29-11
3	Sec. 29-7	C		G		
4	Sec. 29-7	D, E	Sec. 29-7, Ex.	H, I, J	29-35	29-33, 29-34

[a]Ex. = Example(s).

TEXT: Richard T. Weidner and Robert L. Sells, Elementary Classical Physics (Allyn and Bacon, Boston, 1973), second edition, Vol. 2

SUGGESTED STUDY PROCEDURE

Study Chapter 28, Sections 28-1 through 28-5, and read General Comment 1. Study Problems A and B before working Problem F in this study guide and Problems 28-1 through 28-6 and 28-10 in your text. Next study Section 27-7, and read General Comment 2. Study Problems C, D, and E. Then work Problems G, H, I, and J and 27-24.

Try the Practice Test and work some Additional Problems if necessary, before taking a Mastery Test.

WEIDNER AND SELLS

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text	Study Guide	Text	
1	Secs. 28-1, 28-2, 28-4, 28-5, General Comment 1	A	Ex.[a] 28-1	F	28-1 to 28-5	
2	Sec. 28-3, General Comment 2	B	Ex. 28-2	F	28-6, 28-10	28-7, 28-8, 28-9
3	Sec. 27-7	C		G		
4	Sec. 27-7	D, E		H, I J	27-24	27-25, 27-26

[a]Ex. = Example(s).

GENERAL COMMENTS

1. The Loop Equation

The suggested procedure begins with a discussion of a seat of emf and the distinction between an emf and a potential difference. Next, the loop equation is introduced and a sign convention established so that the equation can be applied to particular circuits. The loop equation goes under a variety of aliases - loop theorem, Kirchhoff's loop rule, etc., and is an application of the conservation of energy. The main difficulty that you may have in applying the loop equation will probably be in correctly using the sign convention. There are four different directions (polarities) that are involved, and you need to be sure that you understand these differences among them. First, there is the direction of an emf which is independent of the direction of any current flowing though the seat of emf and may be represented by an arrow $\circ\!\longrightarrow$ and/or the battery symbol $\underline{+}\,|\,|\,\underline{-}$. The second direction is the direction of the potential difference across a resistor or between two points in a circuit. Note that $V_{ab} = V_a - V_b = -(V_b - V_a) = -V_{ba}$, and that the potential drop across a resistor does depend on the direction of current flow through the resistor. The terminal of a resistor at which the current enters is at a higher potential than the terminal from which it leaves. The third direction is that in which the current flows in a particular circuit element. Clearly, you must know the current direction in a resistor before you can determine the sign of the potential difference across a resistor, and just as clearly you frequently do not know the current directions at the beginning of solving a problem. The paradox is only apparent; simply assume a direction for each current and any wrong assumptions will simply result in a negative value for each wrong assumption. Be sure, however, to mark your assumptions on a circuit diagram so that you will use it consistently. The fourth direction is the one you adopt to traverse a loop in applying the loop equation. This direction is arbitrary and is not usually indicated on the circuit diagram. To make the opposite choice, simply change the sign of each term in the loop equation. If you do not understand the distinctions among these directions, return to your study of the text and examples until you do.

Your text discusses a number of applications of the loop equation, including many cases of two connected loops. These circuits add another equation that results from the conservation of charge. You should study these sections as applications of the loop equation, but you will not be tested on multiple-loop circuits requiring the solution of simultaneous equations. This exception does not include series and parallel resistance combinations, which you will be expected to reduce to a single equivalent resistance. The important thing to remember, both for deriving the expressions for equivalent resistance and for applying the expressions to simplify a circuit, is that a set of resistors is in series if the same current flows through each of them, and it is in parallel when the same potential difference appears across them.

2. Application of the Loop Equation

In this section we will consider a further application of the loop equation. When the loop equation is applied to a circuit with resistors and capacitors, the resulting equation is not simply an algebraic equation, but is a differential equation that has an implicit time dependence as a result of the interdependence of the charge (Q) and current ($I = dQ/dt$). The solution of this differential equation is not a number but a time-dependent function. In this course, we will not deal with the mathematics of solving such equations, but we will learn the general form of the solution and then check the solution to see if it satisfies the differential equation. "Satisfies" means that when we substitute all pertinent derivatives of the proposed solution into the differential equation we get an identity or an expression that can be made into an identity by renaming some constants, and we thus have verified the solution. This procedure was used in discussing traveling waves and in discussing simple harmonic motion.

The solutions of this module for RC circuits are of two types, one for charging a capacitor, the other for discharging a capacitor, respectively:

$$Q = \mathcal{E}C(1 - e^{-t/RC}) \quad \text{and} \quad Q = Q_0 e^{-t/RC}.$$

Each of these can be directly related to the voltage on a capacitor:

$$V_C = \mathcal{E}(1 - e^{-t/RC}) \quad \text{and} \quad V_C = V_0 e^{-t/RC}.$$

Both can also be related to a decreasing current

$$I = \frac{dQ}{dt} = \left(\frac{V_0}{R}\right)e^{-t/RC} \quad \text{and} \quad I = \frac{dQ}{dt} = -\left(\frac{V_0}{R}\right)e^{-t/RC}.$$

PROBLEM SET WITH SOLUTIONS

A(1). In the circuit shown in Figure 1 $\mathcal{E}_1 = 8.0$ V, $\mathcal{E}_2 = 6.0$ V, $r_1 = 1.00\ \Omega$, $r_2 = 2.00\ \Omega$, and $R = 4.0\ \Omega$. Find the terminal voltage and the output power of each battery and the heat generated in the 4.0-Ω resistor. (The resistances r_1 and r_2 represent the internal resistances of $\mathcal{E}_1$ and $\mathcal{E}_2$.

Solution

To begin the problem we need to find the current that will flow in the loop. This can be done by applying the loop equation starting at a and proceeding clockwise, noting that both emfs are in the positive sense as we traverse the loop and that we have assumed the current direction shown in the figure:

$$\mathcal{E}_2 - Ir_2 + \mathcal{E}_1 - Ir_1 - IR = 0.$$

This equation can be solved for the current:

$$I = \frac{\mathcal{E}_1 + \mathcal{E}_2}{r_1 + r_2 + R} = \frac{8.0 + 6.0}{2.00 + 1.00 + 4.0} = 2.00\ \text{A}.$$

Any real battery consists of a seat of emf and an internal resistance, and we cannot have access to these separately. The terminal voltage of $\mathcal{E}_2$ is the potential difference between a and b:

$$V_{ba} = V_b - V_a = \mathcal{E}_2 - Ir_2 = 6.0 - (2.00)(2.00) = 2.00\ \text{V}.$$

Similarly,

$$V_{dc} = V_d - V_c = \mathcal{E}_1 - Ir_1 = 8.0 - (2.00)(1.00) = 6.0\ \text{V}.$$

The power delivered to the external circuit is just the product VI, hence

$$P_1 = (6.0)(2.00) = 12.0\ \text{W}, \qquad P_2 = (2.00)(2.00) = 4.0\ \text{W},$$

and the heat generated in the 4.0-Ω resistor is

$$P = I^2R = (2.00)^2(4.0) = 16.0\ \text{W},$$

which is conveniently the sum of the power supplied by the two batteries. Note also that emf $\mathcal{E}_2$ is supplying $\mathcal{E}_2 I = (6.0)(2.00) = 12.0$ W, but $(2.00)^2(2.00) =$ 8.0 W is generated as heat within the battery.

Figure 1

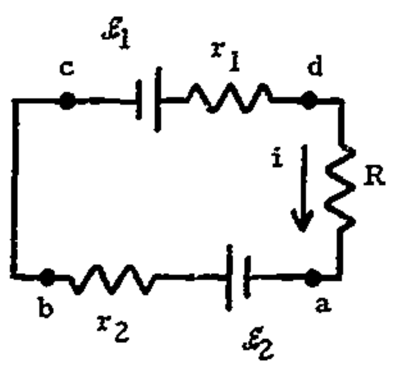
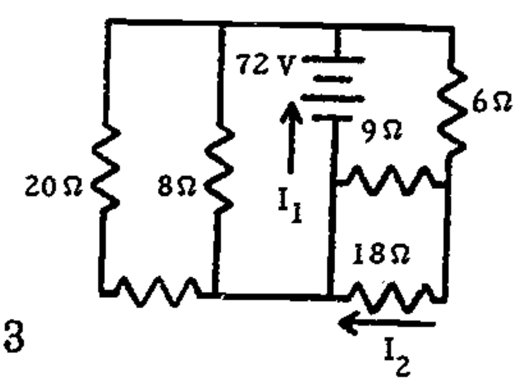


Figure 2



B(1, 2). For the circuit shown in Figure 2 find the equivalent resistance of the resistance network, and find currents I_1 and I_2.

Solution

Such network problems frequently appear more imposing than they really are. To solve this problem begin by combining any series or parallel combinations that you can identify and redraw the circuit as you go. Remember that the connecting lines represent wires with negligible resistance and can be rearranged any way that has an appearance that you like and preserves the electrical connections. This is not a general method, and there are resistance networks that cannot be reduced by these simple series-parallel equations.

You should find that the resistors to the left of the battery reduce to 6.0 Ω and those to the right to 12.0 Ω; these in turn are in parallel and equivalent to one resistor of 4.0 Ω. Thus, $I_1 = 72/4.0 = 18.0$ A. The parallel combination of 9.0 Ω and 18.0 Ω is equivalent to 6.0 Ω, which is in series with the original 6.0-Ω resistor across the 72-V emf. Thus the potential difference across the 9.0-Ω and 18.0-Ω parallel combination is 36 V, and the current $I_2 = 36/18.0 = 2.00$ A.

C(3). For the circuit shown in Figure 3 the loop equation gives $\mathcal{E} - IR - q/C = 0$, which with $I = dQ/dt$ leads to a differential equation: $\mathcal{E} = R(dQ/dt) + Q/C$. Show that $Q = K(1 - e^{-t/RC})$ is a solution.

Solution

To verify a solution we substitute it into the differential equation. If an identity results, the expression is a solution. First we find the derivative

$$dQ/dt = +K(1/RC)e^{-t/RC}$$

and substitute it and the expression for Q into the differential equation

$$\mathcal{E} = R[(K/RC)e^{-t/RC}] + (K/C)(1 - e^{-t/RC});$$

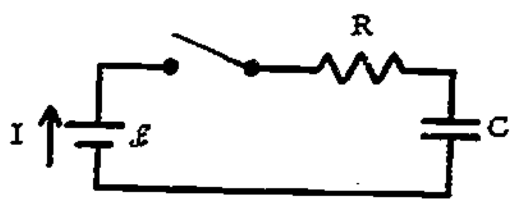
then

$$\mathcal{E} = (K/C)e^{-t/RC} + K/C - (K/C)e^{-t/RC},$$

which will be an identity if $K = \mathcal{E}C$. Thus,

$$Q = \mathcal{E}C(1 - e^{-t/RC}) \text{ is a solution.}$$

Figure 3



D(4). For the circuit shown in Figure 4 find the voltage across the capacitor at 1.20 s after the switch is closed.

Solution

From the previous example the charge on the capacitor at any time is

$$q = \mathcal{E}C(1 - e^{-t/RC}).$$

Since the voltage on the capacitor is $V_C = q/C$,

$$V_C = (\mathcal{E}C/C)(1 - e^{-t/RC}) = (100\ V)(1 - e^{-t/0.60\ s}),$$

where the time constant $RC = (3.00 \times 10^5)(2.00 \times 10^{-6}) = 0.60$ s. At $t = 1.20$ s,

$$V_C = (100\ V)(1 - e^{-1.20/0.60\ s}) = (100\ V)(1 - 1/e^2) = 8.6\ V.$$

E(4). For the circuit shown in Figure 5 the switch was placed in position a for a long time and then quickly moved to position b; 12.0 s later the voltage across the capacitor was 1.00 V. Find the value of the capacitor.

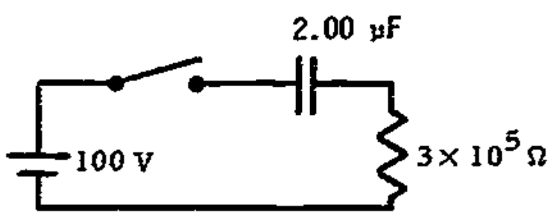


Figure 4

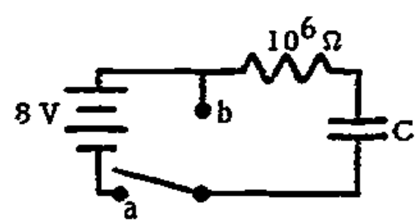


Figure 5

Solution

From the circuit we can conclude that the time constants for charging and discharging are about the same. Thus we can assume the capacitor was fully charged to 8.0 V when the switch was moved to b and we can write

$$V_C = \mathcal{E}e^{-t/RC} = 8.0e^{-t/10^6 C},$$

and for the problem values

$$1.00 = (8.0)e^{-12.0/10^6 C}.$$

To evaluate C we take the natural logarithm of this expression. Rewriting it slightly first we have

$$e^{-12.0/10^6 C} = 0.125, \qquad -12.0/10^6 C = \ln 0.125,$$

$$C = -12/(10^6 \ln 0.125) = 5.8 \times 10^{-6}\ F = 5.8\ \mu F.$$

With a scientific calculator this evaluation is straightforward. However, for purposes of approximation many physicists like to estimate exponentials in terms of half-lives, and we note here that the reduction from 8.0 to 1.00 V represents the passing of three half-lives:

$$8 \xrightarrow{t_{1/2}} 4 \xrightarrow{t_{1/2}} 2 \xrightarrow{t_{1/2}} 1.$$

Thus the half-life is 4.0 s in this problem. For the expression $V = V_0 e^{-t/RC}$, we take

$$V = V_0/2 \quad \text{at } t = t_{1/2}, \qquad V_0/2 = V_0 e^{-t_{1/2}/RC}$$

Again taking natural logarithms of both sides, we find

$$\ln(1/2) = -t_{1/2}/RC \quad \text{and} \quad t_{1/2} = RC \ln 2.0 = 0.69RC.$$

Thus in this case $RC = 4.0/0.69$ s and $C = 4/(10^6)(0.69) = 5.8 \times 10^{-6}$ F.

Problems

F(1, 2). For the circuit shown in Figure 6 find the current I, and the power dissipated in the 17.0-Ω resistor.

G(3). For the circuit shown in Figure 7 write the differential equation for the voltage on the capacitor, if the voltage was V_0 at $t = 0$. Show that $V = V_0 e^{-t/RC}$ is a solution for your differential equation.

Figure 6

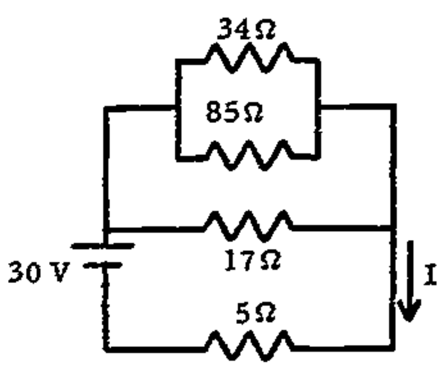
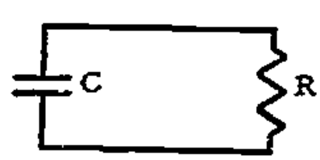


Figure 7



H(4). The switch in Figure 8 is left in position a for 2.00 h, then moved to position b.

(a) What is the charge in the capacitor 12.0 s after this is done?

(b) Two hours later, it is moved back to position a. What is the charge after 12.0 s now?

I(4). What is the current from the battery in Figure 9 5.0 s after the switch is closed?

J(4). Note that the circuit shown in Figure 10 can be analyzed in terms of series-parallel combinations of resistors, plus a capacitor.

(a) When the switch is first closed, the currents in various parts of the circuit change with time, as the capacitor charges up. But after a few seconds, the currents settle down to quite steady values. What is the steady current being drawn from the battery?

(b) What is then the potential across the capacitor?

(c) The switch is now opened. What is the potential across the capacitor after 30.0 ms?

Solutions

F(1, 2). $I = 2.00$ A. $P_{17.0} = 23.5$ W.

G(3). $R(dQ/dt) + Q/C = 0$. $Q = VC = V_0 e^{-t/RC}$.

H(4). (a) 22.0 μC. (b) 38 μC.

I(4). 28.0 μA.

J(4). (a) 0.300 A. (b) 18.0 V. (c) 12.0 V.

Figure 8

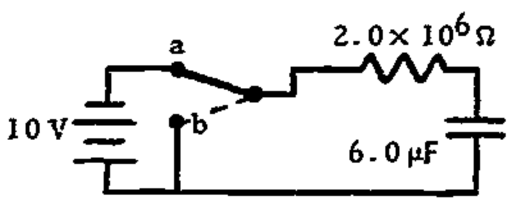


Figure 9

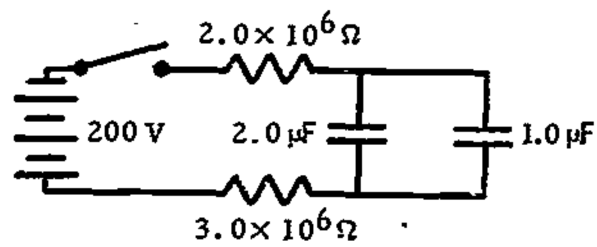
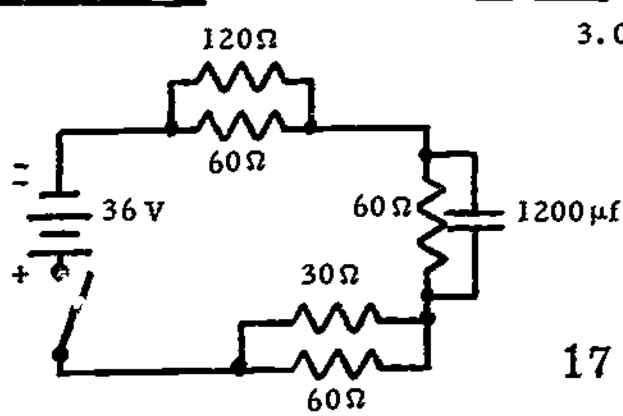


Figure 10



PRACTICE TEST

1. (a) For the circuit shown in Figure 11 find the current in the 5.0-Ω resistor.

   (b) Find the terminal voltage of the 24.0-V battery, V_{ab} (emf = 24.0 V; internal resistance = 1.00 Ω).

2. It has been suggested that an appropriate differential equation for the circuit shown in Figure 12 is $RQ = \mathcal{E} - (1/C)(dQ/dt)$. If this is true with the switch closed, show that $Q = Q_0(1 - e^{-t/RC})$ is a solution; if it is not true, correct the equation, and then show that $Q = Q_0(1 - e^{-t/RC})$ is a solution.

3. A capacitor is charged to 10.0 V and then connected in series with a resistor of 10^7 Ω. After 100 s the voltage on the capacitor is 2.50 V. Find the value of the capacitor.

Figure 11

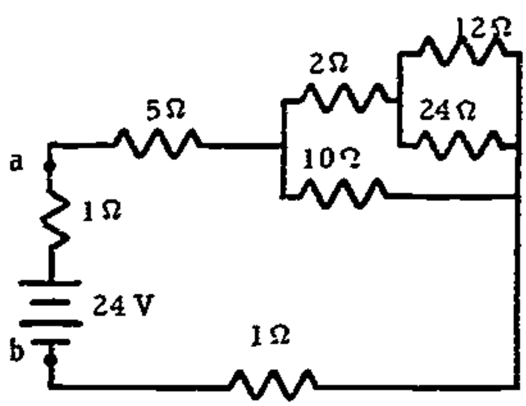
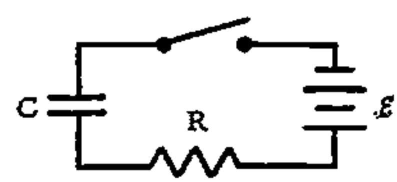


Figure 12



Practice Test Answers

1. (a) 2.00 A. (b) 22.0 V.
2. The Q and dQ/dt in the differential equation should be exchanged.
3. 7.2 μF.

DIRECT-CURRENT CIRCUITS

Mastery Test Form A

Name ______________________

Date ______________

pass recycle

1 2 3 4

Tutor ______________

1. For the circuit shown in Figure 1:

   (a) Find the current in the 10.0-Ω resistor.

   (b) Find the potential difference V_{ab}.

   (c) Find the total power generated in the 12.0- and 24.0-Ω resistors together.

Figure 1

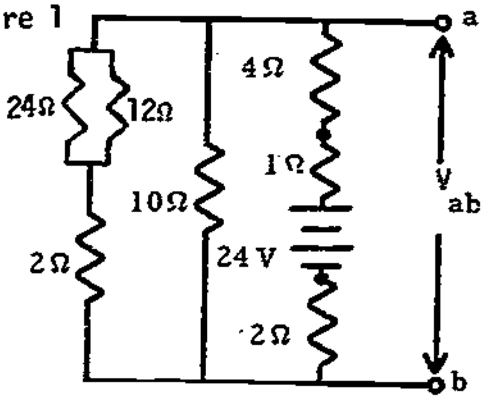
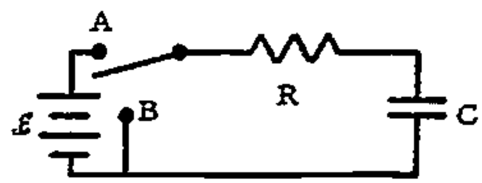


Figure 2



2. The circuit shown in Figure 2 had the switch in position A for a long time. At $t = 0$ it was moved to position B.

   (a) Is the following a correct differential equation for $t > 0$:

   $\mathcal{E} - R(dQ/dt) - Q/C = 0$?

   (b) If your answer was "yes," show that $Q = \mathcal{E}Ce^{-t/RC}$ is a solution. If your answer was "no," make the appropriate changes and show that $Q = \mathcal{E}Ce^{-t/RC}$ is a solution.

3. A 100-V battery, an uncharged 200-μF capacitor, and a 5.0×10^5 Ω resistor are connected in series at $t = 0$. Find the potential difference across the capacitor at $t = 200$ s.

DIRECT-CURRENT CIRCUITS

Mastery Test Form B

Name ______________________

Date ______________

pass recycle

1 2 3 4

Tutor ______________

1. For the circuit shown in Figure 1:

   (a) Find the current in the 1.00-Ω resistor.

   (b) Find the terminal voltage V_2 ($\mathcal{E}$ = 20.0 V, internal resistance 0.50 Ω).

   (c) Find the power supplied to the external circuit from terminals of V_1.

Figure 1

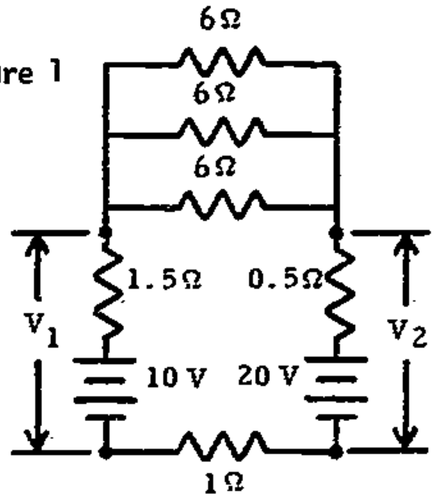
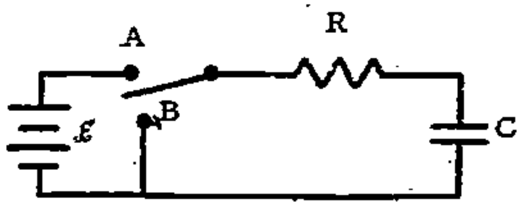


Figure 2



2. The circuit shown in Figure 2 had the switch in position A for a long time. At time = 0 it was moved to position B.

   (a) Is the following differential equation true for t > 0:

   $(1/C)(dQ/dt) + RQ = 0$?

   (b) Show that $V = V_0 e^{-t/RC}$ is a form of the solution of either the above equation or a corrected version of it.

2. A capacitor of 20.0 μF is fully charged to a potential difference V and quickly connected to a series resistor of 10^5 Ω. The first measurement of the current that can be made is 3.0×10^{-4} A at t = 4.0 s. Find the value V, the potential difference across the capacitor at t = 0.

DIRECT-CURRENT CIRCUITS

Mastery Test Form C

Name ______________________

Date ______________________

pass recycle

1 2 3 4

Tutor ______________________

1. For the circuit shown in Figure 1:

   (a) Find the indicated current I.

   (b) Find the terminal voltage V_1 ($\mathcal{E}$ = 24 V, internal R = 1.00 Ω).

   (c) Find the power generated in resistor R_1.

Figure 1

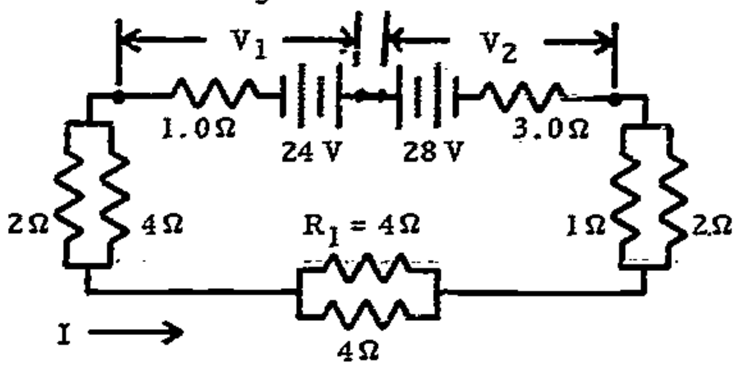
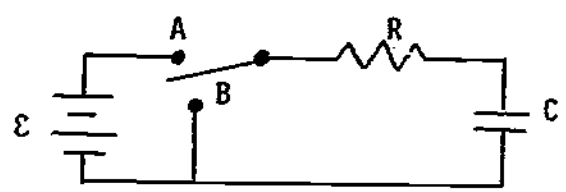


Figure 2



2. After a long time in position B the switch in the circuit shown in Figure 2 was moved to position A at t = 0.

   (b) Does the following differential equation represent the circuit for t > 0:

   $\mathcal{E} = -(dQ/dt)R - Q/C$?

   (b) Show that $Q = Q_0(1 - e^{-t/RC})$ is a solution of either the above equation or a corrected version of it.

3. A capacitor of 10.0 μF has a charge of Q_0 and is connected in series with a resistor R at t = 0. Find the value of the resistance R so that the charge on the capacitor will decrease to about $0.050Q_0$ within 10^{-3} s.

MASTERY TEST GRAOING KEY - Form A

1. What To Look For: (a) Correct answer. (b) You may note that the easiest way to get the answer is from an intermediate step at which it is clear that the current through the 10.0-Ω resistor is 1.00 A, hence $V_{ab} = 10.0$ V. The implied 24.0 - (7.0)(2.00) = 10.0 V is, of course, acceptable. (c) The simplest calculation is, of course, from the equivalent resistance, 8.0 Ω and current, 1.00 A.

Solution: (a) 1.00 A. (b) 10.0 V. (c) $P = I^2R = 1^2(8) = 8$ W.

2. What To Look For: (a) If the answer is simply "no" ask "why"? (b) Be sure the correct differential equation is given and that the derivative is correct and the substitution is correctly done.

Solution: (a) The equation is not correct. It is for the switch in position A. The equation for position B is

$$R(dQ/dt) + Q/C = 0.$$

(b) $Q = \mathcal{E}Ce^{-t/RC}$, $dQ/dt = -(C/RC)e^{-t/RC}$

Substituting in above equation, we find

$$(R\mathcal{E}C/RC)e^{-t/RC} + \mathcal{E}Ce^{-t/RC}/C = 0, \quad -\mathcal{E}e^{-t/RC} + \mathcal{E}e^{-t/RC} = 0,$$

which is an identity; therefore $Q = \mathcal{E}Ce^{-t/RC}$ is a solution.

3. What To Look For: If the answer is incorrect check to find out if the equation was wrong or if the student made an error in numerical computation.

Solution: $V = V_0(1 - e^{-r/RC}) = 100(1 - e^{-200/100}) = 86$ V.

MASTERY TEST GRADING KEY - Form B

1. What To Look For: (a) Check numerical answer. (b) The result should follow easily from (a). If part (a) is wrong, check work to see if a correct value of I would give the right answer. (c) Probably most easily calculated from $V_1 I$, but also acceptable is $\mathcal{E}I - I^2R$.

Solution: (a) 6.0 A. (b) 17.0 V. (c) 6.0 W.

2. What To Look For: (b) Check for the correct differential equation, correct derivative dQ/dt, and correct substitution of one into the other.

Solution: (a) The equation given is incorrect. It should be

$$R(dQ/dt) + Q/C = 0.$$

(b) $V = Q/C = V_0e^{-t/RC}$, $Q = CV_0e^{-t/RC}$, $dQ/dt = -(CV_0/RC)e^{-t/RC}$.

Substituting, we find

$$-(RCV_0/RC)e^{-t/RC} + (1/C)CV_0e^{-t/RC} = 0,$$

which is an identity; therefore $V = V_0e^{-t/RC}$ is a solution.

3. What To Look For: If the answer is incorrect check to find out if the equation used was wrong or if the numerical evaluation was incorrect.

Solution: $I = (V/R)e^{-t/RC}$,

$$V = IRe^{-t/RC} = (3.0 \times 10^{-4})(10^5)e^{4.0/2.00} = 220\ \text{V}.$$

MASTERY TEST GRADING KEY - Form C

1. What To Look For: (a) Correct answer. (b) Neither 24 nor 23.5 are correct. (b) and (c) If the answer in (a) is wrong, check to see if the work is correct.

Solution: (a) 0.50 A. (b) $V = \mathcal{E} + IR = 24.5$ V. (c) $P = I^2R = (1/4)^2(4) = 0.250$ W.

2. What To Look For: Check for a correct differential equation, correct differentiation and correct substitution into the differential equation.

Solution: (a) The equation does not represent the circuit. It should be

$$\mathcal{E} - R(dQ/dt) - Q/C = 0.$$

(b) $Q = Q_0(1 - e^{-t/RC})$, $\quad \frac{dQ}{dt} = -Q_0(-\frac{1}{RC})e^{-t/RC} = (\frac{Q_0}{RC})e^{-t/RC}$.

Substituting, we find

$$\mathcal{E} - \frac{RQ_0}{RC} e^{-t/RC} - \frac{Q_0}{C} + (\frac{Q_0}{C})e^{-t/RC} = 0,$$

which is an identity if $Q_0 = \mathcal{E}C$.

3. What To Look For: If the answer is incorrect, check to see if the equation was chosen or used incorrectly or if the numerical evaluation was wrong.

Solution: $Q = Q_0e^{-t/RC}$, $\quad -Q/Q_0 = e^{-t/RC}$.

$$\ln Q/Q_0 = -t/RC,\ R = -\frac{t}{C \ln Q/Q_0} = \frac{t}{C \ln(Q_0/0)} = \frac{10^{-3}}{10^{-5} \ln(1.00/0.050)} = 33\ \Omega.$$

Module ______

STUDY GUIDE

MAGNETIC FORCES

INTRODUCTION

It may surprise you to learn that the conversion of electrical energy to mechanical work in electric motors or stereo loud speakers is seldom done by electrostatic forces (Coulomb's law). Magnetic forces associated with moving charges (currents) are the basis of most electromechanical devices. In analogy with the electrostatic case, we introduce an intermediary called the magnetic field. This module considers the forces on currents or moving charges in a magnetic field; the module Ampere's Law will show how magnetic fields are generated by currents.

PREREQUISITES

Before you begin this module, you should be able to:	Location of Prerequisite Content
*Calculate vector products (needed for Objectives 1 through 4 of this module)	Vector Multiplication Module
*Define electric charge and field (needed for Objectives 1 and 4 of this module)	Coulomb's Law and the Electric Field Module
*Define current in terms of carrier density and velocity (needed for Objectives 2 through 4 of this module)	Ohm's Law Module

LEARNING OBJECTIVES

After you have mastered the content of this module, you will be able to:

1. Magnetic force - particles - Calculate the force on a moving charged particle in a uniform magnetic field; for the case of $\vec{v}$ perpendicular to $\vec{B}$, find the radius and/or frequency of the resulting circular orbit.

2. Magnetic force - wires - Calculate the force on a current-carrying wire in a uniform magnetic field.

3. Magnetic dipole - Calculate the magnetic moment of a current loop; use this to determine the torque on such a loop in a uniform magnetic field.

4. Hall effect - For problems with balanced electric and magnetic forces (Hall effect, velocity selectors) use the relation $v = E/B$ to relate the fields to parameters such as the Hall field or potential difference, current density, or charge sign and velocity.

GENERAL COMMENTS

Be sure that you are able to find the vector (cross) product of two vectors before reading any further. It is essential to everything in this module. Unlike electric forces and electric fields, magnetic forces are perpendicular to the magnetic field, a relation described mathematically by the cross product.

You will find that a closed circuit carrying a current I experiences a total force (not torque) equal to zero if it is in a uniform magnetic field. An interesting consequence is that the force on a part of such a circuit is independent of the path of the current, and hence is the same as the force on a straight wire connecting the end points.

Notice that although a magnetic dipole (a current-carrying loop) in a magnetic field behaves just like an electric dipole in an electric field, it is a very different object internally. There are no known magnetic charges; the magnetic dipole is a circulating current of electric charge (current loop), which produces the same fields far from the dipole and experiences the same torques as a dipole made from a charge pair.

We also consider the case in which $\vec{v}$, $\vec{B}$, and $\vec{E}$ form a right-handed set of orthogonal vectors like $\hat{i}$, $\hat{j}$, and $\hat{k}$. If $\vec{E} = \vec{B} \times \vec{v}$, then the total force on a charge moving with velocity $\vec{v}$ is zero. This is the basis of velocity filters for charged particles. It also occurs if a current-carrying conductor is placed in a magnetic field. The charge carriers are displaced in the direction $q\vec{v} \times \vec{B}$ until an electric force $q\vec{E}$ caused by the excess charge on one side makes the net transverse force on the charge carrier equal to zero. This field $\vec{E}$ is called the Hall field, and is used to investigate the sign and density of charge carriers in a conductor. One can also use it to measure the magnetic field.

TEXT: Frederick J. Bueche, Introduction to Physics for Scientists and Engineers (McGraw-Hill, New York, 1975), second edition

SUGGESTED STUDY PROCEDURE

Your text readings are from Chapter 23. Read the General Comments; then go through the textbook in the order given in the Table. As you complete the section for each objective, study the corresponding Problems with Solutions in the Problem Set; then work the Assigned Problems until you have mastered the associated objective. Then take the Practice Test.

An important application of the principles of Objective 1 arises from Illustration 23.5, where there is a calculation of the period of the circular orbit of a particle of mass m in a uniform field $\vec{B}$. The general result is $T = 2\pi m/qB$ independent of the particle energy. This fact is used in the particle accelerator called the cyclotron. Since the period is energy independent, an alternating electric field $E_y = E_0 \sin(2\pi t/T)$ will accelerate the particle. The energy, momentum, and orbit radius all increase as long as the particle remains in both the electric and the magnetic field. This principle is used to accelerate particles to high energies for nuclear research, to study electrons in solids, and to heat ionized gases in thermonuclear research.

BUECHE

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text	Study Guide	Text	
2	Secs. 23.1 to 23.4	B	Illus.[a] 23.1, 23.2, 23.3	E, F	1, 2, 3	4, 16
1	Sec. 23.5	A	Illus. 23.4, 23.5	D	5, 8, 9	10, 11
3	Secs. 23.7, 23.8	C		E	13, 14, 15	
4	Sec. 23.6	D	Illus. 23.6	G	7	

[a]Illus. = Illustration(s).

TEXT: David Halliday and Robert Resnick, Fundamentals of Physics (Wiley, New York, 1970; revised printing, 1974)

SUGGESTED STUDY PROCEDURE

All of your text readings are in Chapter 29. Read the General Comments; then go through the textbook in the order given in the Table below. As you complete the reading for each objective, study the corresponding Problems with Solutions in the Problem Set; then work the Assigned Problems until you have mastered the associated objective. Take the Practice Test and work some Additional Problems if necessary before trying a Mastery Test.

HALLIDAY AND RESNICK

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text	Study Guide	Text	
1	Secs. 29-1, 29-2, 29-6, 29-7	A	Ex[a]. 1, 4, 5	D	1, 5, 21, 25	4, 6, 23
2	Sec. 29-3	B		E, F	7, 9, 11	8, 10, 12
3	Sec. 29-4	C	Ex. 2	E	13, 15	16
4	Secs. 29-5, 29-8	D		G	19, 41	18, 20, 42

[a]Ex. = Example(s).

STUDY GUIDE: Magnetic Forces 3(SZ 1)

TEXT: Francis Weston Sears and Mark W. Zemansky, University Physics (Addison-Wesley, Reading, Mass., 1970), fourth edition

SUGGESTED STUDY PROCEDURE

Your text readings are from Chapters 30 and 31. Read the General Comments; then go through the textbook in the order given in the Table below. As you complete the section for each objective, study the problems with solutions in the Problem Set, then work the Assigned Problems until you have mastered the associated objective. Try the Practice Test and work some Additional Problems if necessary before taking a Mastery Test. For Objective 3, be sure to note the relation between direction of current and the direction of the magnetic moment for a current loop.

SEARS AND ZEMANSKY

Objective Number	Readings	Problems with Solutions	Assigned Problems		Additional Problems
		Study Guide	Study Guide	Text	
1	Secs. 30-1, 30-2, 30-4, 30-10	A	D	30-1, 30-3, 30-5	30-6, 30-13, 30-19
2	Sec. 31-1	B	E, F	31-1	
3	Sec. 31-3	C	E	31-5	31-4, 31-6
4	Secs. 30-5, 31-2	D	G	30-12, 31-3	31-2

TEXT: Richard T. Weidner and Robert L. Sells, Elementary Classical Physics (Allyn and Bacon, Boston, 1973), second edition, Vol. 2

SUGGESTED STUDY PROCEDURE

Your text readings are from Chapter 29. Read the General Comments; then go through the textbook in the order given in the Table below. As you complete the section for each objective, study the corresponding Problems with Solutions in the Problem Set; then work the Assigned Problems until you have mastered the associated objective. Take the Practice Test and work some Additional Problems if necessary before trying a Mastery Test.

WEIDNER AND SELLS

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text	Study Guide	Text	
1	Secs. 29-1, 29-3, 29-5	A	Ex[a]. 29-2, 29-3	D	29-1, 29-3, 29-11	29-4, 29-8
2	Sec. 29-6	B	Ex. 29-4	E, F	29-16[b]	29-17
3	Secs. 29-7, 29-8	C		E	29-18	29-20, 29-21, 29-23
4	Secs. 29-4, 29-9	F	Ex. 29-5	G	29-9, 29-24	

[a]Ex. = Example(s).

[b]$I = (mg/LB) \tan \theta$.

PROBLEM SET WITH SOLUTIONS

A(1). A particle of mass 10^{-27} kg is moving with speed 10^5 m/s perpendicular to a magnetic field $B = 5.0 \times 10^{-3}$ T* (charge of $+e = 1.60 \times 10^{-19}$ C).

(a) What is the force on the particle?

(b) What is the radius of the circle in which it moves?

(c) What is the period for its motion?

(d) If its speed were doubled, what would its period be?

Solution

(a) Use the magnetic force equation:

$$\vec{F} = q\vec{v} \times \vec{B} = (1.60 \times 10^{-19}\ \text{C})(10^5\ \text{m/s})(5.0 \times 10^{-3}\ \text{T}) = 8.0 \times 10^{-17}\ \text{N}$$

perpendicular to $\vec{v}$, $\vec{B}$.

(b) $\vec{F} = m\vec{a}$. $a = v^2/R$ for circular motion. $qvB = m(v^2/R)$.

$$R = \frac{mv}{qB} = \frac{(10^{-27}\ \text{kg})(10^5\ \text{m/s})}{(1.60 \times 10^{-19}\ \text{C})(5.0 \times 10^{-3}\ \text{T})} = 0.125\ \text{m}.$$

(c) $$T = \frac{2\pi R}{v} = 2\pi\left(\frac{m}{qB}\right) = 2\pi\left(\frac{10^{-27}\ \text{kg}}{(1.60 \times 10^{-19}\ \text{C})(5.0 \times 10^{-3}\ \text{T})}\right) = 7.9 \times 10^{-6}\ \text{s}.$$

(d) From part (c), T is independent of v. No change.

B(s). A square loop of wire, 30.0 cm on each edge, lies in the xy plane with edges parallel to the axes as in Figure 1, and carries a current of 4.0 A. It is in a uniform field $\vec{B} = (0.100\hat{j} + 0.173\hat{k})$ T.

(a) Draw $\vec{B}$ in a $B_xB_yB_z$ coordinate system.

(b) Calculate the force acting on each side of the square wire.

Add the forces on the four sides to get the total force.

Solution

(a) See Figure 2. $\tan(30°) = B_y/B_z = 1/1.73$.

(b) See Figure 3. Let L be the length of one side of the loop. Then

$$\vec{F}_1 = IL\hat{j} \times \vec{B} = IL\hat{j} \times (B_y\hat{j} + B_z\hat{k}) = ILB_z\hat{i}, \quad \vec{F}_3 = -IL\hat{j} \times \vec{B} = -\vec{F}_1,$$

*In SI units, the tesla is used for magnetic flux density. 1 T = 1 Wb/m^2.

$$\vec{F}_2 = -\hat{i}IL \times (B_y\hat{j} + B_z\hat{k}) = -ILB_y\hat{k} + ILB_z\hat{j}, \qquad \vec{F}_4 = \hat{i}IL \times \vec{B} = -\vec{F}_2.$$

Since $\vec{F}_3 = -\vec{F}_1$, $\vec{F}_4 = -\vec{F}_2$,

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = \vec{F}_1 + \vec{F}_2 - \vec{F}_1 - \vec{F}_2 = 0,$$

which is always the case in a uniform field.

$$ILB_y = (24\ \text{A})(0.300\ \text{m})(0.100\ \text{T}) = 0.120\ \text{N}, \qquad ILB_z = 0.210\ \text{N}.$$

Figure 1

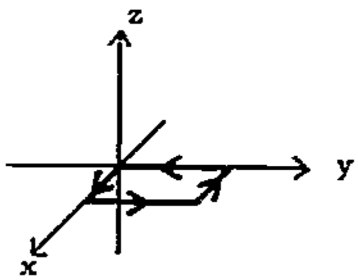


Figure 2

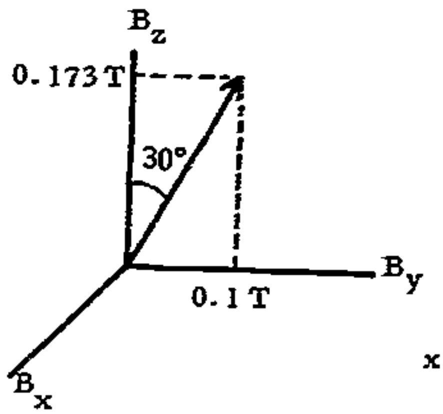
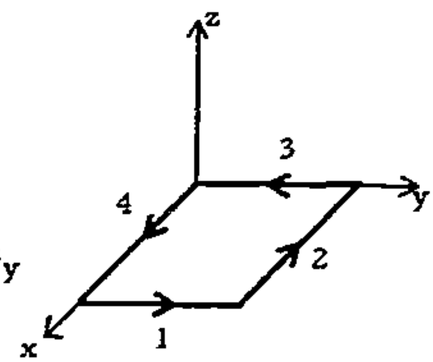


Figure 3



C(3). For the same loop as in Problem B (Figure 1),
(a) What is the vector dipole moment of the loop?
(b) Using the result of part (a), find the vector torque acting on the loop.

Solution

(a) $\vec{\mu} = IA\hat{k}$, where I is the current, and A is the cross-sectional area of the loop. The direction of $\vec{\mu}$ follows a right-hand rule; if fingers follow the current, the thumb picks the correct normal to the loop. Hence

$$\vec{\mu} = [(4.0\ \text{A})(0.300\ \text{m})^2]\hat{k} = 0.36\hat{k}\ \text{A m}^2.$$

(b) $\vec{\tau} = \vec{\mu} \times \vec{B} = \hat{k} \times (B_y\hat{j} + B_z\hat{k}) = -\mu B_y\hat{i} = -0.036\hat{i}$ N m.

D(1, 4). A particle in Figure 4 moving with speed 10^6 m/s along the y axis enters a magnetic field $\vec{B} = B_0\hat{k}$, $B_0 = 0.40$ T, $q = 1.60 \times 10^{-19}$ C, $m = 10^{-27}$ kg.

(a) What is the force on the particle?
(b) What will be the path of the particle if it stays in the field?
(c) What electric field $\vec{E}$ will result in zero net force on the particle?

Solution

(a) $\vec{F} = q\vec{v} \times \vec{B} = (1.60 \times 10^{-19}\ \mathrm{C})(10^{6}\hat{j}\ \mathrm{m/s})(0.40\hat{k}\ \mathrm{T}) = 6.4 \times 10^{-14}\hat{i}\ \mathrm{N}.$

(b) A circle parallel to the xy plane:

$$R = \frac{mv}{qB} = \frac{(10^{-27}\ \mathrm{kg})(10^{6}\ \mathrm{m/s})}{(1.60 \times 10^{-19}\ \mathrm{C})(0.40\ \mathrm{T})} = 1.56 \times 10^{-2}\ \mathrm{m}.$$

(c) $\vec{F} = q(\vec{v} \times \vec{B}) = 0.$

$$E = -\vec{v} \times \vec{B} = (-10^{6}\hat{i}\ \mathrm{m/s})(0.40\hat{k}\ \mathrm{T}) = -4.0 \times 10^{5}\hat{i}\ \mathrm{V/m}.$$

Problems

E(2, 3). A square loop as shown in Figure 5 is pivoted about the z axis and carries a current I = 10.0 A. The loop is in a uniform magnetic field B = 0.50 T parallel to the y axis.
(a) What is the magnitude and direction of force on the side labeled a?
(b) What torque is acting on the loop? (Give magnitude and direction.)
(c) What is the total force on the loop?

Figure 4

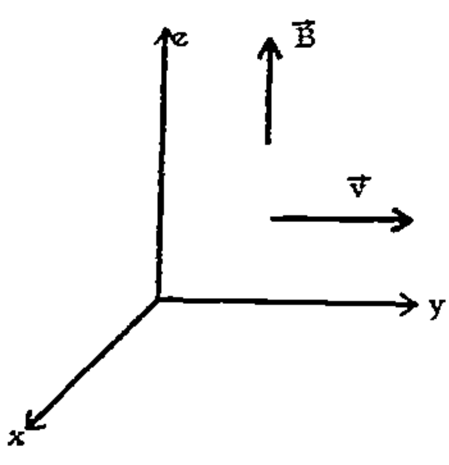
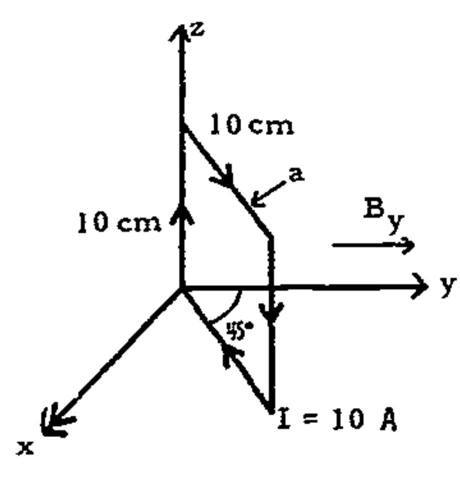


Figure 5



F(2). In Figure 6 a copper rod weighing 2.00 N rests on two horizontal rails 1.00 m apart and carries a current of 50 A from one rail to the other. The coefficient of static friction μ is 0.60. What is the smallest vertical magnetic field that would cause the bar to slide, and what is its direction? Remember that the frictional force = μ (normal force).

G(4). The conductor in Figure 7 with square cross section and side a = 0.0250 m carries a current I = 100 A. A magnetic field B_0 = 0.60 T is parallel to the positive z axis. The current is carried by electrons with $q = -1.60 \times 10^{-19}$ C.

(a) What is the direction of the Hall field?

(b) If the density of carriers is 4.0×10^{28} per cubic meter, what is the speed of the electron?

(c) What is the Hall voltage?

Figure 6

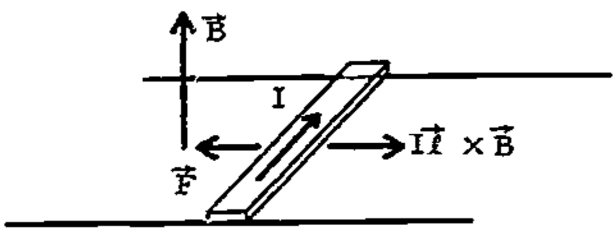
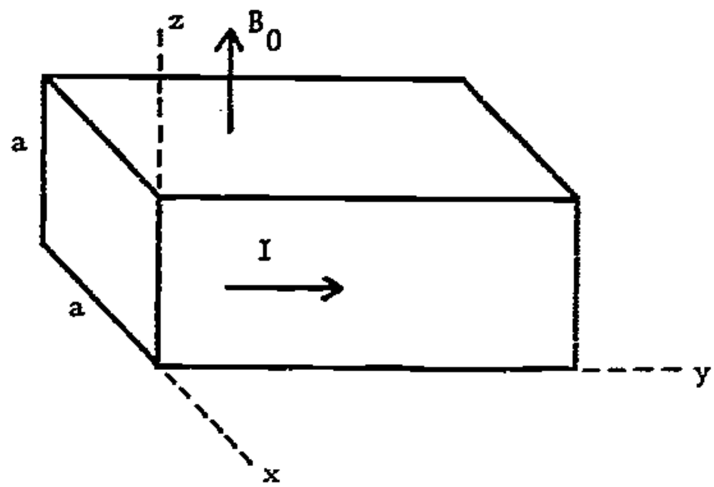


Figure 7



Solutions

E(2, 3). (a) $\vec{F} = I\vec{L} \times \vec{B} = (10.0\text{ A})(0.100\text{ m})(\hat{i}/\sqrt{2} + \hat{j}/\sqrt{2})(0.57)\hat{j} = 0.35\hat{k}$ N.

(b) $\vec{\mu} = IA(-\hat{i}/\sqrt{2} + \hat{j}/\sqrt{2}) = (0.100\text{ A m}^2)(-\hat{i}/\sqrt{2} + \hat{j}/\sqrt{2})$.

$$\vec{\tau} = \vec{\mu} \times \vec{B} = (0.100\text{ A m}^2)(-\hat{i}/\sqrt{2} + \hat{j}/\sqrt{2})(0.57)\hat{j} = -0.035\hat{k}\text{ N m}.$$

(c) $\vec{F} = 0$ for closed loop in uniform field.

F(2). B = 0.0240 T up. The rod would move to the right.

G(4). (a) Since carriers have negative charge, they go to the left if the current is to the right. Hall condition is $\vec{F} = q[\vec{E} + (\vec{v} \times \vec{B})] = 0$.

$\vec{E} = -\vec{v} \times \vec{B}$ points along the positive x axis.

(b) $I = nqvA$,

$$v = \frac{I}{nqA} = \frac{100\text{ A}}{(4.0 \times 10^{28}/\text{m}^3)(1.60 \times 10^{-19}\text{ C})(0.025\text{ m})^2} = 2.50 \times 10^{-5}\text{ m/s}.$$

(c) $V = Ea$ = Hall voltage,

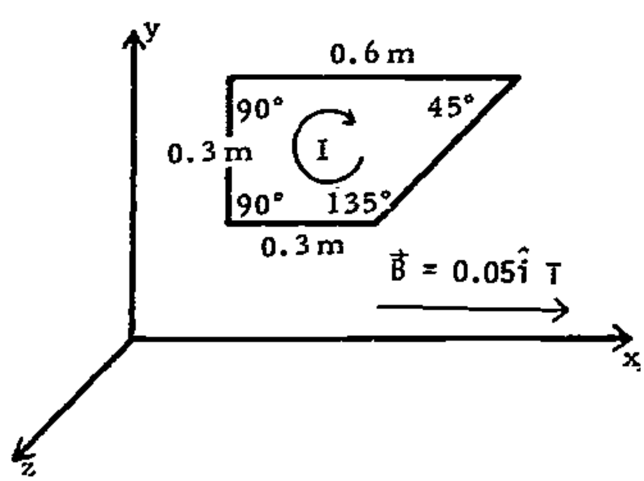
$E = vB = (2.50 \times 10^{-5}\ \mathrm{m/s})(0.60\ \mathrm{T}) = (1.50 \times 10^{-5}\ \mathrm{V/m})$,

$V = (1.50 \times 10^{-5}\ \mathrm{V/m})(0.0250\ \mathrm{m}) = 3.8 \times 10^{-7}\ \mathrm{V}$.

PRACTICE TEST

1. A four-sided wire loop as shown in Figure 8 (not rectangular) lies in a plane parallel to a uniform magnetic field $\vec{B} = 0.050\hat{i}$ T. A current of $I = 3.00$ A flows clockwise around the loop, as illustrated in the figure.
   (a) Calculate the magnitude and direction of the force acting on each straight-line segment of the loop.
   (b) Calculate the resultant force acting on the loop.
   (c) Calculate the magnetic moment of the loop.
   (d) Calculate the torque acting on the loop about any point in the plane of the loop.

Figure 8

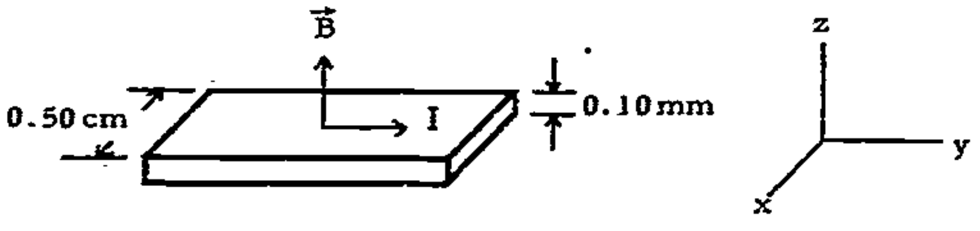


2. The silver wire in Figure 9 is in the form of a ribbon 0.50 cm wide and 0.100 mm thick. A 2.0-A current is passed through the ribbon perpendicular to a 0.80-T magnetic field. The number of charge carriers in silver is 6.0×10^{28} per cubic meter, thus the drift velocity is 4.2×10^{-4} m/s.
   (a) What is the magnetic force on an electron in the wire?
   (b) This force causes a charge accumulation on the sides of the wire until

the magnetic force is balanced by an equal electric force. What are the magnitude and direction of the electric field thus produced?

(c) What is the Hall voltage produced across the width of the ribbon?

Figure 9



3. A charged particle is accelerated through a potential difference of 5000 V, then enters a magnetic field of magnitude 0.300 T with its velocity perpendicular to the direction of the magnetic field. This particle has a charge-to-mass ratio of $q/m = 4.0 \times 10^6$ C/kg.
(a) Find its speed v.
(b) Use the basic magnetic force law to find the radius of its path.

<u>Practice Test Answers</u>

1. (a) See Figure 10. $\vec{F}_1 = 0$, $\vec{F}_2 = -0.045\hat{k}$ N,
$\vec{F}_3 = 0$, $\vec{F}_4 = 0.045\hat{k}$ N.

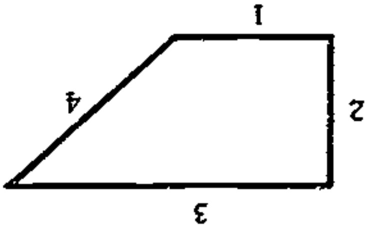


Figure 10

(b) $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = 0$ as always for uniform $\vec{B}$. (c) $\vec{\mu} = -0.41\hat{k}$ A m^2.
(d) $\vec{\tau} = -0.020\hat{j}$ N m.

2. (a) $q\vec{v} \times \vec{B} = -5.3 \times 10^{-23}\hat{i}$ N. (b) $\vec{E} = -3.3 \times 10^{-4}\hat{i}$ V/m.
(c) $V = 1.65 \times 10^{-6}$ V.

3. (a) $v = 2.00 \times 10^5$ m/s. (b) $R = 0.167$ m.

MAGNETIC FORCES | Date ______

Mastery Test Form A | pass recycle 1 2 3 4

Name ______ | Tutor ______

1. A gold strip 1.50 cm wide and 0.100 cm thick is placed in a $\vec{B}$ field of $2.00\ Wb/m^2$ at right angles to the strip. A current of 100 A is set up in the strip along its length. Let $n = 6.0 \times 10^{28}$ per cubic meter and $q = -1.60 \times 10^{-19}$ C.

   (a) Sketch the setup. Include a set of coordinate axes.

   (b) What is the velocity of the charge carriers? Assume they are electrons.

   (c) What is the magnetic force on an electron?

   (d) What Hall potential difference appears across the strip? Indicate the direction of the Hall field in your diagram.

2. A straight wire of length L carrying a current I is situated in a uniform $\vec{B}$ field directed at right angles to the wire segment. Draw a diagram showing the directions of the current in the wire, the $\vec{B}$ field, and the magnetic force acting on the wire. What is the magnitude of the magnetic force? Now imagine that the wire segment is bent into the form of a closed circular loop, the plane of which is oriented at an angle θ to the $\vec{B}$ field, as in Figure 1. What is the net force on the loop? What is the net torque of the loop? $(\sin\theta)(-\hat{i}) + (\cos\theta)\hat{j}$ is the normal to the loop.

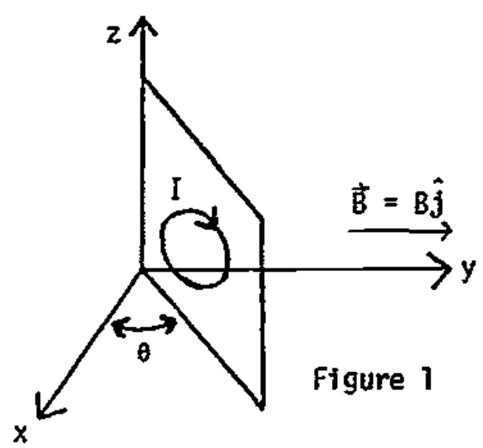


Figure 1

MAGNETIC FORCES Date ____________

Mastery Test Form B pass recycle

1 2 3 4

Name ______________________ Tutor ____________

1. At the instant a positive ion of charge +e and mass m traveling to the right with a velocity $\vec{v} = v_0\hat{i}$ passes through the origin of a coordinate system as shown in Figure 1, a uniform $\vec{B}$ field directed perpendicular to the xy plane is established and causes the ion to pass through the point x = R, y = R, z = 0.
   (a) Determine the magnitude of the $\vec{B}$ field in terms of e, m, and $\vec{v}$.
   (b) Is the $\vec{B}$ field directed into or out of this exam sheet?
   (c) What is the velocity (magnitude and direction) of the ion at (R, R, 0)?
   (d) How long does it take to make one turn?

   An electric field is introduced to cause a second ion entering the $\vec{B}$ field at the origin of the coordinate system to continue in a straight line with a velocity $\vec{v} = v_0\hat{i}$.
   (e) Indicate the direction of the $\vec{E}$ field.
   (f) Determine the magnitude of the $\vec{E}$ field in terms of the parameters given.

2. In Figure 2 a wire bent into a semicircular arc of radius R in the yz plane carries a current I. A uniform external magnetic field $\vec{B}_0$ is parallel to the x axis. Find the vector force acting on the wire. Remember that closed loops have zero net force.

3. A flat coil of 20 turns, with an area of 20 cm^2 is suspended in a magnetic field of strength B = 0.300 Wb/m^2. When the plane of the coil makes an angle of 45° with the field, the torque has a magnitude of 1.00×10^{-2} N m.
   (a) What is the magnetic moment of the coil?
   (b) What is the current flowing in the coil?

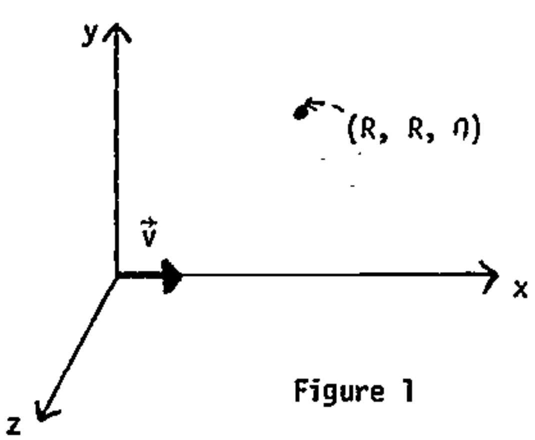


Figure 1

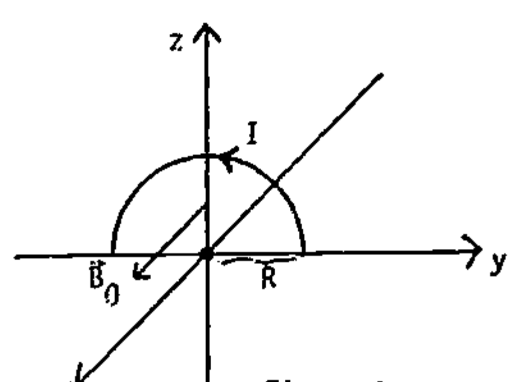


Figure 2

MAGNETIC FORCES — Date ____________

Mastery Test — Form C — pass 1 2 recycle 3 4

Name ______________________ Tutor ____________

1. (a) What is the speed of a proton (charge $q = 1.60 \times 10^{-19}$ C and mass $m = 1.70 \times 10^{-27}$ kg) that moves in a circle of 2.00 m radius in a $\vec{B}$ field of 0.200 Wb/m^2?

   (b) If the speed is doubled, how does the time for one turn change?

   (c) If the velocity of the proton is now along $\hat{i}$ and the magnetic field is along $\hat{k}$, what electric field (direction and magnitude) will cause the velocity to remain constant?

2. The wires of a high-voltage electric power transmission line experience a magnetic force from the Earth's magnetic field. Find the magnetic force per meter of its length if a wire carries a current of 500 A in the local (magnetic) northerly direction. The Earth's field is 2.50×10^{-5} Wb/m^2 and points northward and downward at 70° from the horizontal. Illustrate your solution with a sketch of the problem.

3. You wish to make a square coil out of a 1.00-m-long piece of wire such that it experiences a maximum torque of 0.200 N m when it carries a 4.0-A current and is placed in a uniform 0.84-Wb/m^2 $\vec{B}$ field. How many turns should it have?

MASTERY TEST GRADING KEY - Form A

1. What To Look For: (a) I along strip. B perpendicular to strip. Coordinate axes. (b) Current equation, A in square meters. Direction of v opposite to I (electrons are negative). (c) Lorentz equation. Direction. (d) Force balance or E = vB. Direction of $\vec{E}$ on sketch.

Solution: (a) See Figure 14.

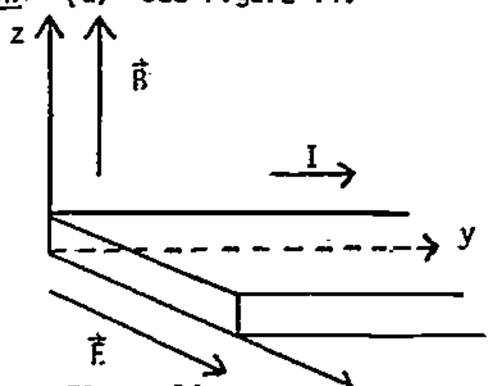


Figure 14

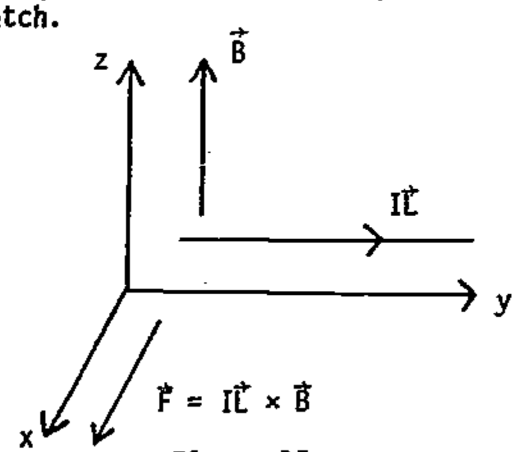


Figure 15

(b) $I = nqvA, \quad v = I/nqA,$

$v = 100/(6.0 \times 10^{28})(1.60 \times 10^{-19})(1.50 \times 10^{-5}) = 6.9 \times 10^{-4}$ m/s to left.

(c) $\vec{F} = q\vec{v} \times \vec{B} = -(1.60 \times 10^{-19})(6.9 \times 10^{-4})\hat{j}(2\hat{k}) = 2.20 \times 10^{-22}\hat{i}$ N.

(d) $\vec{F} + q\vec{E}_H = 0, \quad \vec{E}_H = -(1/q)\vec{F} = 1.38 \times 10^{-3}$ V/m,

$V_H = dE_H = (0.0150 \text{ m})(1.38 \text{ V/m}) = 2.07 \times 10^{-5}$ V.

2. What To Look For: Force equation, $\vec{B} \cdot I\vec{L} = 0$. Force zero in uniform field. Find area from length. Find magnetic moment. Torque equation.

Solution: See Figure 15. $\vec{F} = 0$ (uniform field).

$L = 2\pi R, \quad \mu = IA = I\pi(L/2\pi)^2 = IL^2/4\pi.$

$\vec{\tau} = \vec{\mu} \times \vec{B} = (IL^2/4\pi)[(-\sin\theta)\hat{i} + (\cos\theta)\hat{j}] \times B\hat{j} = -[(IL^2B\sin\theta)/4\pi]\hat{k}.$

MASTERY TEST GRADING KEY - Form B

1. What To Look For: (a) Determine radius. $\vec{F} = m\vec{a}$. (b) Get direction of $\vec{B}$ from force. (c) 1/4 turn on circle. (f) Force balance $v_0 = E/B$.

   Solution: See Figure 16. (a) Radius is R,

   $$mv^2/R = evB, \quad B = mv/eR.$$

   (b) Force is up, so $\vec{B} = -(mv/eR)\hat{k}$.

   (c) $\vec{v} = v_0\hat{j}$. (d) $T = 2\pi R/v_0$.

   (e) Electric force is down: $\vec{E} = -E_0\hat{j}$.

   (f) $q\vec{E} + q\vec{v} \times \vec{B} = 0$. $E = vB = mv_0^2/eR$.

2. What To Look For: F = 0 for closed loop. Force equation.

   Solution: See Figure 17. Force on half-loop same as force on line across diameter.

   $$\vec{F} = I\vec{L} \times \vec{B} = I(-2R\hat{j}) \times B_0\hat{i} = 2IRB_0\hat{k}.$$

3. What To Look For: (a) Torque equation. Definition of μ.

   Solution: (a) $\tau = \mu B \sin\theta$,

   $$\mu = \frac{10^{-2}\ \text{N m}}{(0.300\ \text{T})(1/\sqrt{2})} = 0.047\ \text{A m}^2 = NIA.$$

   (b) $I = \mu/NA = (0.047\ \text{A m}^2)/(20 \times 0.00200\ \text{m}^2) = 1.18\ \text{A}$.

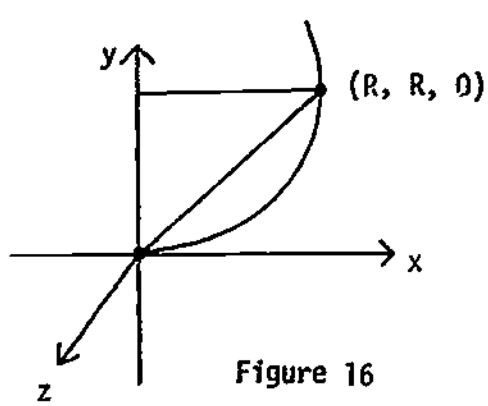


Figure 16

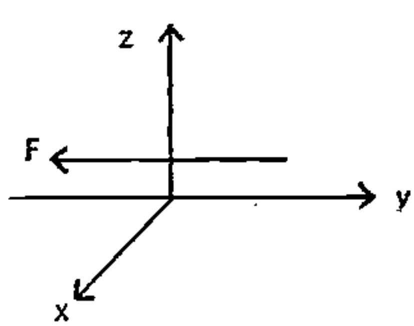


Figure 17

MASTERY TEST GRADING KEY - Form C

1. What To Look For: (a) $F = mA$. (b) T independent of v. (c) Force balance.

   Solution: (a) $mv^2/R = qvB$.

   $$v = qBR/m = (1.60 \times 10^{-19})(0.200)(2)/(1.70 \times 10^{-27}) = 3.8 \times 10^7 \text{ m/s}.$$

   (b) No change.

   (c) See Figure 18. $E = vB$ along $\hat{j}$.

   $$\vec{E} + \vec{v} \times \vec{B} = 0, \quad E = 0.76 \times 10^7 \text{ V/m}.$$

2. What To Look For: $\vec{F}/\ell = \vec{I} \times \vec{B}$.

   Solution: See Figure 19. $\vec{I} \times \vec{B}$ into paper (West).

   $$F/\ell = IB \sin\theta = (500 \text{ A})(2.50 \times 10^{-5} \text{ T})(\sin 70°) = 0.0117 \text{ N/m}.$$

3. What To Look For: Definition of μ. Relate area to length. Torque equation.

   Solution: $\mu = NIA$, $L = N4a$.

   $$A = a^2 = (L/4N)^2, \quad \tau_{max} = \mu B = t.$$

   $$NIAB = t, \quad NI(L/4N)^2 B = \tau,$$

   $$N = L^2BI/\tau 16 = (0.84)(4)/(0.200)(16) = 1.05 \text{ turns}.$$

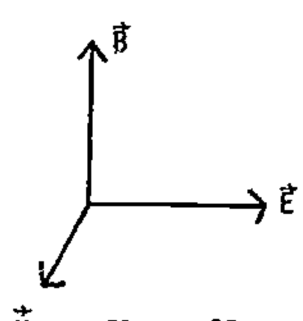


Figure 18

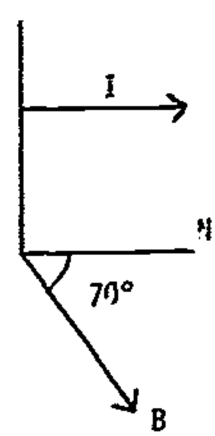


Figure 19

AMPÈRE'S LAW

INTRODUCTION

Everyone has seen a bar magnet in the form of a compass or a door catch. Anyone who has ever casually played with magnets or magnetic toys knows that magnets interact with other magnets; i.e., a magnet experiences a force caused by the presence of an external magnetic field produced by the other magnet. A wire carrying a current experiences a force caused by the presence of a nearby magnet (as you saw in the module Magnetic Forces). We then expect the converse to also hold true, i.e., that the bar magnet will also experience a force from the presence of the current-carrying wire. This expectation can be verified experimentally by putting a compass needle near a current-carrying wire.

Thus both a bar magnet and a current-carrying wire produce a magnetic field. A bar magnet, however, cannot be broken down into a single magnetic pole similar to the electric charge. Even on the atomic scale, there are always two magnetic poles similar to the two magnetic poles produced by a small loop of current-carrying wire. A bar magnet is really just a collection of atomic current loops or charges in motion. In this module (and the module Magnetic Forces), you are actually investigating the interaction at a distance of moving electric charges (e.g., electric currents). The intermediary in this interaction is the magnetic field $\vec{B}(r)$. By introducing $\vec{B}$, you separate the interaction into two parts: (1) creation of the $\vec{B}$ field by given currents (to be treated in this module), and (2) the action of this field on other given currents or moving charges (which you studied in Magnetic Forces).

Along beside the other memorable force relations involving magnetic fields, you will now add to your collection of fond memories the field relations known as Ampère's Law and the Biot-Savart Law. (Please note: The latter is pronounced Bee-oh Sah-var´.)

PREREQUISITES

Before you begin this module, you should be able to:	Location of Prerequisite Content
*Integrate simple polynomials (needed for Objectives 1 through 3 of this module)	Calculus Review
*Evaluate cross products (needed for Objective 3 of this module)	Vector Multiplication Module
*Define the magnetic field, and find the force on a current-carrying wire in a magnetic field (needed for Objectives 1 through 3 of this module)	Magnetic Forces Module

LEARNING OBJECTIVES

After you have mastered the content of this module, you will be able to:

1. Ampère's law - Write Ampère's law and use it to calculate the magnitude and direction of the magnetic field $\vec{B}$ caused by currents flowing in a conductor of cylindrical cross-sectional area with a simple symmetric shape, such as a long, straight wire, a solenoid, or a toroid, or a combination of these (principle of superposition).

2. Forces between currents - Given the currents in parallel conductors, solve for the force on one of the conductors.

3. Biot-Savart law - Write the Biot-Savart law and employ it to find the magnitude and direction of the magnetic field $d\vec{B}$ at a point P_1 caused by a current element at another point P_2; and/or find the magnetic field $\vec{B}$ at the center of a circular or semicircular loop of current-carrying wire.

TEXT: Frederick J. Bueche, Introduction to Physics for Scientists and Engineers (McGraw-Hill, New York, 1975), second edition

SUGGESTED STUDY PROCEDURE

Your readings are from Chapter 24. First read General Comments 1 and 2 and Sections 24.1 and 24.2. Then study Problem A before working Problem F in this study guide and Problem 2 in Chapter 24 of your text. Now read General Comment 3 and Section 24.2. Study Illustration 24.2 and Problem C before working Problems H and 15. Then go back to Objective 1 and read General Comment 4 and Sections 24.3 and 24.4. Study Problem B and Illustrations 24.1 and 24.4 before working Problem G and Problems 7 and 17 in Chapter 24. For Objective 3, read General Comments 5 and 6, Sections 24.5, 24.6, 24.8; study Illustrations 24.5 and 24.6 and Problems D and E; and work Problems I and J.

Try the Practice Test, and work some of the Additional Problems if necessary, before attempting a Mastery Test.

BUECHE

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text	Study Guide	Text (Chap. 24)	(Chap. 24)
1	Secs. 24.1, 24.2, 24.3, 24.4, General Comments 1, 2, 4	A, B	Illus.[a] 24.1, 24.4	F, G	2, 7, 17	1, 3, 4, 5, 6, 8
2	Sec. 24.2, General Comment 3	C	Illus. 24.2	H	15	
3	Secs. 24.5, 24.6, 24.8, General Comments 5, 6	D, E	Illus. 24.5, 24.6	I, J		

[a]Illus. = Illustration(s).

TEXT: David Halliday and Robert Resnick, Fundamentals of Physics (Wiley, New York, 1970; revised printing, 1974)

SUGGESTED STUDY PROCEDURE

Your readings are all from Chapter 30. First read General Comments 1 and 2 and Sections 30-1 and 30-2. Then study Problem A and work Problem F in this study guide, and Problems 1 and 23 in Chapter 30. Next read General Comment 3 and Sections 30-3 and 30-4, study Problem C, and work Problems H and 18. Return to Objective 1, reading General Comment 4 and Sections 30-3 and 30-5, studying Problem B and Examples 1 through 4 in Chapter 30, before working Problems G and 27. Then read General Comments 5 and 6, and Section 30-6, excluding Example 5. Study Problems D and E before working Problems I, J, and 33.

Take the Practice Test, and work some Additional Problems if necessary, before attempting a Mastery Test.

HALLIDAY AND RESNICK

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text	Study Guide	Text (Chap. 30)	(Chap. 30)
1	Secs. 30-1, 30-2, 30-3, 30-5, General Comments 1, 2, 4	A, B	Ex.[a] 1, 2, 3, 4	F, G	1, 23, 27	4, 5, 15, 19, 21, 22, 23
2	Secs. 30-3, 30-4, General Comment 3	C		H	18	16, 17, 24, 25
3	Sec. 30-6, General Comments 5, 6	D, E		I, J	33	29, 30, 34, 35(a)

[a]Ex. = Example(s).

TEXT: Francis Weston Sears and Mark W. Zemansky, University Physics (Addison-Wesley, Reading, Mass., 1970), fourth edition

SUGGESTED STUDY PROCEDURE

The learning objectives cover the material in the reverse order of your text. You may want to skim Chapter 32 before following this study procedure in detail. (Note: Your text changes notation from $4\pi k'$ to μ_0 in Section 32-6.)

For Objective 1, read General Comments 1 and 2 and Section 32-6. Then study Problem A and work Problems F and 32-3. Next read General Comment 3 and Section 32-4, study Problem C, and work Problems H and 32-11. Return to Objective 1, reading General Comment 4, and Section 32-7, studying Problem B, and working Problems G and 32-17. Next master Objective 3 by reading General Comments 5 and 6 and Section 32-2, studying Problems D and E, and working Problems I, J, 32-1, and 32-15.

Take the Practice Test, and work some Additional Problems if necessary, before attempting a Mastery Test.

SEARS AND ZEMANSKY

Objective Number	Readings	Problems with Solutions	Assigned Problems		Additional Problems
		Study Guide	Study Guide	Text	
1	Secs. 32-6, 32-7, General Comments 1, 2, 4	A, B	F, G	32-3, 32-17	32-2, 32-4, 32-6, 32-18, 32-19
2	Sec. 32-4, General Comment 3	C	H	32-11	32-7, 32-9, 32-10
3	Sec. 32-2, General Comments 5, 6	D, E	I, J	32-1, 32-15	

TEXT: Richard T. Weidner and Robert L. Sells, Elementary Classical Physics (Allyn and Bacon, Boston, 1973), second edition, Vol. 2

SUGGESTED STUDY PROCEDURE

The learning objectives cover the material in the reverse order of your text. You may want to skim through Chapter 30 and then come back and read the particular sections of the text as listed below in the table.

For Objective 1, read General Comments 1 and 2 and Section 30-7. Then study Problem A and work Problems F and 30-5. For Objective 2, read General Comment 3 and Section 30-5, study Problem C and Example 30-2, and work Problems H and 30-13. Return to Objective 1, reading General Comment 4 and Section 30-8, studying Problem B and Example 30-3, and working Problems G and 30-22(a), 30-23. Next read General Comments 5 and 6 and Sections 30-1 through 30-3 (pp. 604 and 605 only), for Objective 3. Study Problems D and E before working Problems I, J, and 30-6.

Take the Practice Test, and work some Additional Problems if necessary, before attempting a Mastery Test.

WEIDNER AND SELLS

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text	Study Guide	Text	
1	Secs. 30-7, 30-8, General Comments 1, 2, 4	A, B	Ex.[a] 30-3	F, G	30-5, 30-22(a), 30-23	30-1, 30-16, 30-17, 30-19, 30-20, 30-21
2	Sec. 30-5, General Comment 3	C	Ex. 30-2	H	30-13	30-3, 30-11, 30-14, 30-15
3	Secs. 30-1, 30-2, 30-3, General Comments 5, 6	D, E		I, J	30-6	30-4

[a]Ex. = Example(s).

GENERAL COMMENTS

1. Comparison of Ampère's Law and the Biot-Savart Law

The examples of the long wire, long solenoid, and toroid make very good illustrations of the use of Ampère's law. If you learn Ampère's law well, you will not need to memorize the expressions for $\vec{B}$ in these three cases (which are of some importance in themselves) - you will always be able to derive them! (Note that the long solenoid is the limiting case of a toroid as the radius approaches infinity.)

Ampère's law is completely general for steady currents, and can be used to calculate the magnetic field in certain symmetric situations in much the same way that Gauss' law can be used in electrostatics. However, many cases (e.g., the circular loop) cannot be handled with Ampère's law, since one must have a sufficiently simple situation so that $\vec{B}$ is constant and can be removed from the integral sign. For the circular loop this becomes very impractical. Therefore, we go to the Biot-Savart law for a more general approach. The Biot-Savart law, as stated, represents the magnetic field in terms of contributions from infinitesimal elements composing the current circuit. Ampère's law is still true in these cases - it just does not give any calculational help.

Note that Ampère's law and the Biot-Savart law are not independent of one another. Ampère's law can be derived from the Biot-Savart law and therefore does not contain more information than the Biot-Savart law. The Biot-Savart law is just a more general statement.

2. Straight Wires and Paths

The main thrust of Objective 1 is to familiarize oneself with Ampère's law:

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I.$$

On the right side of this equation, I is the net current passing through the area enclosed by the path of integration. In Figure 1 the net current through the area bounded by the curve C is $I_1 + I_2 + I_3$. The positive direction of the current through the area can be obtained by a right-hand rule - the fingers of the right hand curling in the direction of $d\vec{\ell}$ around the curve C and the thumb pointing in the positive direction of the current:

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 (I_1 + I_2 + I_3).$$

For an area pierced by no current, the line integral around the closed path is zero.

How do we know that the $\vec{B}$ field from a current in a long straight wire is circular around the wire? Since the wire looks the same from one side as from another, the magnitude of $\vec{B}$ at a point near the wire should remain unchanged as the wire is rotated. Thus $|\vec{B}|$ is constant on a circle of radius r centered on the axis of the wire. But what about the direction of $\vec{B}$? If we take a compass and move it around the circle of radius r, the needle will always be tangent to the circle. If the current in the wire is reversed, the needle will reverse end for end. The direction of $\vec{B}$ is taken to be the direction of the compass, and thus we have the right-hand rule. The Biot-Savart law may also be used to find the $\vec{B}$ field from a current-carrying wire and gives the same result.

To utilize Ampère's law to determine $\vec{B}$ for a given situation, we must have a high degree of symmetry just as we did to determine $\vec{E}$ from Gauss' law for the electric field,

$$\int \vec{E} \cdot d\vec{A} = q/\varepsilon_0.$$

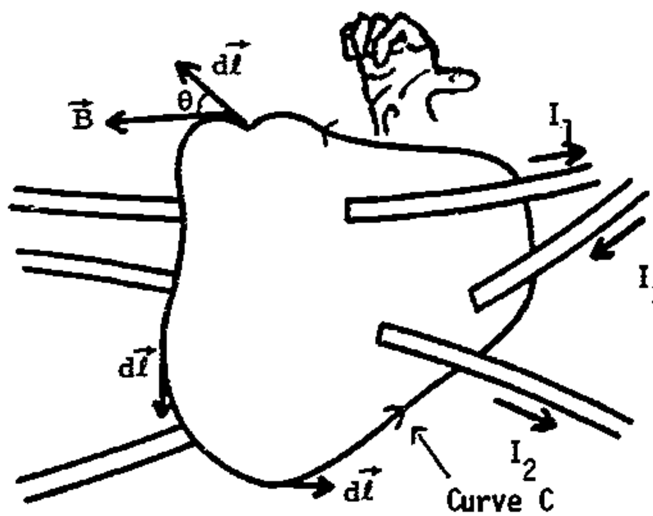


Figure 1

3. Forces

This comment treats problem solving for Objective 2. From the module Magnetic Forces, we know that a current-carrying wire in a magnetic field experiences a force, $\vec{F} = I\vec{\ell} \times \vec{B}$. To find the force, therefore, that one wire carrying a current I exerts on another parallel wire a distance d away and carrying current I_2, we find the $\vec{B}$ field of wire 1 at wire 2:

$$|\vec{B}_1| = \mu_0 I_1/2\pi d.$$

Then use this $\vec{B}$ field to find the force on wire 2:

$$\vec{F}_2 = I_2\vec{\ell} \times \vec{B}_1.$$

Note: This applies to parallel wires only.

4. Uniform Distribution of Current

To find the $\vec{B}$ field at a point inside a cylindrical wire as shown in Figure 2 that carries a current distributed uniformly over the cross section of the wire, we can use Ampère's law for a circular path about the axis of the wire:

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 i,$$

where i is the net current inside our circular path of integration (see Figure 2). A total current I is distributed uniformly over the cross section of the wire and into the page. The direction of $\vec{B}$ is determined by the right-hand rule.

Since $d\vec{\ell}$ and $\vec{B}$ are in the same direction all around the dotted circle, $\vec{B} \cdot d\vec{\ell} = B\, d\ell$, and symmetry suggests that $|\vec{B}|$ is constant along the circular path so that

$$\oint \vec{B} \cdot d\vec{\ell} = \oint B\, d\ell = B2\pi r.$$

Thus

$$B = \mu_0 i/2\pi r.$$

But i is the current inside the dotted circle:

$$i = I(\pi r^2/\pi R^2) = Ir^2/R^2.$$

Therefore,

$$|\vec{B}| = \mu_0 Ir/2\pi R^2.$$

See Figure 3. This reduces to the expected expression for $|\vec{B}|$ at $r = R$.

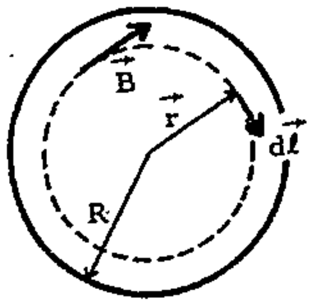


Figure 2

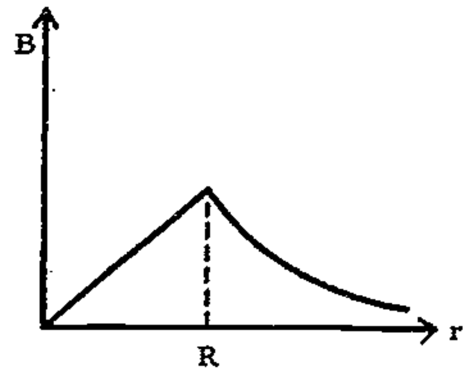


Figure 3

5. Current Elements

In electrostatics the fundamental law telling us how to calculate the electric field caused by an element of charge (a point charge) is Coulomb's law. The corresponding law for magnetic fields is called the Biot-Savart Law. The fundamental source of the magnetic field is a current element, which we can think of as a small length of wire $d\ell$ carrying a current I in the direction of $d\vec{\ell}$. The Biot-Savart law says that the magnetic field $d\vec{B}$ at a distance $\vec{r}$ caused by this incremental length is given by

$$d\vec{B} = \frac{\mu_0 I}{4\pi}\,\frac{d\vec{\ell} \times \hat{r}}{r^2} = \frac{\mu_0 I}{4\pi}\,\frac{d\vec{\ell} \times \vec{r}}{r^3},$$

where μ_0 is a constant ($\mu_0 = 4\pi \times 10^{-7}$ if the units for B, I, and r are teslas, amperes, and meters, respectively), and $\hat{r}$ is a unit vector in the direction of $\vec{r}$. See Figure 4.

Notice that the Biot-Savart law is an inverse-square law. It is more complicated than Coulomb's and the gravitational laws, however, because of the cross product: $d\vec{B}$ points not along $\vec{r}$ but in a direction perpendicular to both $\vec{r}$ and $d\vec{\ell}$.

Observe that the $d\vec{\ell}$ that occurs in Amoère's law has quite a different significance from the $d\vec{\ell}$ in the Biot-Savart law. In Ampère's law, it is along an imaoinary path that need bear no relation to the direction of current flow, whereas in the Biot-Savart law it is alonq the direction of the current flow in the wire.

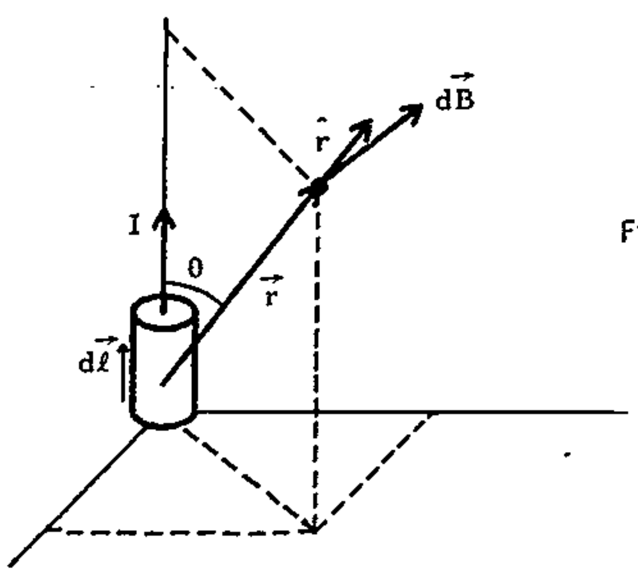


Figure 4

6. Semicircular Current Loops

The Biot-Savart law gives the magnetic field produced by an infinitesimal current, as we shall see in Problems D and I. However, no such isolated current element can exist by itself. There must always be a complete loop of current in any circuit.

The most general case of current-carrying wires involves a detailed integration of the Biot-Savart law, but there is a special case that requires only the interpretation of a line integral. This is the problem of finding the $\vec{B}$ field at the center of a circular or semicircular loop of current. Here, the current element $I\,d\vec{\ell}$ is always perpendicular to the position vector $\vec{r}$:

$I\,d\vec{\ell} \times \vec{r}$ = a vector whose magnitude is $Ir\,d\ell$, in a direction perpendicular to both $\vec{r}$ and $d\vec{\ell}$.

Since we are finding the field at the center of the loop, the distance r is a constant and equal to the radius a. Thus,

$$|d\vec{B}| = (\mu_0 I/4\pi a^2)d\ell.$$

In summing up or integrating the contributions from all the significant current elements, $\int d\ell$ is just the length of the semicircular loop.

PROBLEM SET WITH SOLUTIONS

A(1). Two long straight parallel wires both carry a current of 8.0 A, but in opposite directions as shown in Figure 5. Calculate the magnetic field at the indicated point P caused by these currents.

Solution

Divide this problem into two problems, and use Ampère's law in both. The $\vec{B}$ field caused by the lower wire is found by integrating $\vec{B}$ around a circle of radius 2.00 m that is concentric with the lower wire:

$$\oint \vec{B}_1 \cdot d\vec{\ell} = B_1 2\pi(2) = \mu_0(8), \qquad |\vec{B}_1| = 8.0 \times 10^{-7}\ \mathrm{T}.$$

According to the right-hand rule, $\vec{B}_1$ at the point of interest is directed out of the page.

To find $\vec{B}_2$, the contribution from the upper wire, we integrate $\vec{B}_2$ around a circle of 4.0 m radius that is concentric with the upper wire. Note that we include only the upper current element in the term for enclosed current, because we have divided this problem into two parts, each of which involves only one current-carrying wire. The result is

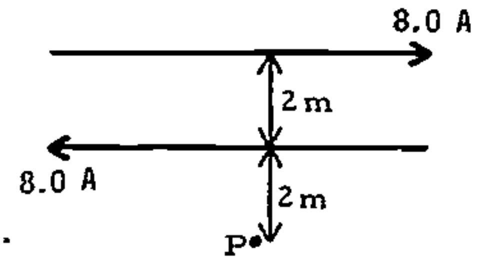
$$\oint \vec{B}_2 \cdot d\vec{\ell} = B_2 2\pi(4) = \mu_0(8), \quad \vec{B}_2 = 4.0 \times 10^{-7}\ \mathrm{T} \quad \underline{\text{into}} \text{ the page.}$$

Now to find the net field at the point indicated, we superimpose these two partial solutions (principle of superposition), obtaining

$$\vec{B}_{net} = 4.0 \times 10^{-7}\ \mathrm{T} \quad \underline{\text{out}} \text{ of the page.}$$

Note: The presence of a second wire does not destroy the symmetry of the $\vec{B}$ field from the first wire. The separate $\vec{B}$ fields just add together vectorially.

Figure 5



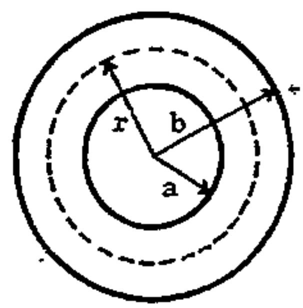


Figure 6

B(1). A hollow cylindrical conductor as shown in Figure 6 of radii a and b ($a < b$) carries a current I (out of the paper) uniformly spread over its cross section.
(a) Show that the magnetic field $\vec{B}$ for points inside the body of the conductor (that is, $a < r < b$) is given by

$$|\vec{B}| = [\mu_0 I/2\pi(b^2 - a^2)][(r^2 - a^2)/r].$$

Check this formula for the limiting case of $a = 0$, r.
(b) Make a rough plot of the general behavior of $\vec{B}(r)$ from $r = 0$ to $r \to \infty$.

Solution

Since the current I is uniformly spread over the cross section between a and b, J (current density) = I/A = const, where A is the area between a and b:

$$A = \pi b^2 - \pi a^2 = \pi(b^2 - a^2).$$

See Figure 7. Apply Ampère's law

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I$$

around a circle of radius r such that $a < r < b$. Since the current is uniform, there is no preferred point on the circle of radius r; therefore $|\vec{B}|$ must be constant on this circle. Since the lines of $\vec{B}$ are concentric circles outside the wire, there is no reason to suspect they are not likewise inside the wire. The direction of $\vec{B}$ is gained by the right-hand rule: the current coming toward you (thumb), fingers curl in the direction of $\vec{B}$.

Integrating around the circle in a counterclockwise direction dictates the current to be positive (right-hand rule).

$$\oint \vec{B} \cdot d\ell = \oint B\, d\ell \cos\theta = \oint B\, d\ell \cos\theta = B \oint d\ell,$$

since $\vec{B}$ and $d\vec{\ell}$ are in the same direction at every point. $\int d\ell = 2\pi r$ = circumference of the circle. $\oint \vec{B} \cdot d\vec{\ell} = B2\pi r$. The I in Ampère's law is the net current that passes through the circular area of radius r. Since $J = I/\pi(b^2 - a^2)$, the current interior to r is $[I/\pi(b^2 - a^2)]\pi(r^2 - a^2)$. Therefore

$$B2\pi r = \mu_0 \pi I(r^2 - a^2)/\pi(b^2 - a^2), \qquad |\vec{B}| = [\mu_0 I/2\pi(b^2 - a^2)][(r^2 - a^2)/r],$$

with the direction as shown in Figure 7. When a = 0,

$$B = \frac{\mu_0 I}{2\pi(b^2)} \frac{r^2}{r} = \frac{\mu_0 Ir}{2\pi b^2}.$$

Check this for the limiting case, r = a. Then let b = 2a, $K = \mu_0 I/6\pi a^2$. Then

$$|\vec{B}| = \left(\frac{\mu_0 I}{2\pi(b^2 - a^2)}\right)\left(\frac{r^2 - a^2}{r}\right) = K\left(\frac{r^2 - a^2}{r}\right) \quad \text{for } a < r < 2a.$$

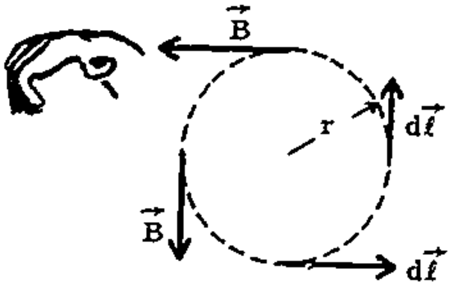


Figure 7

Figure 8

Figure 8 is a graph for $b = 2a$, which is not required for this problem. You could have chosen any other value for b as long as the point b is somewhat near the center of the graph so that the curve of $\vec{B}$ versus $\vec{r}$ is easily seen.

C(2). In Problem A, find the force per unit length on the bottom wire.

Solution

The $\vec{B}$ field caused by the top wire is given by $\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I$, as we found in Problem A:

$$\vec{B}_1 = -(\mu_0 I_1/2\pi d)\hat{k}$$

at the location of the bottom wire. The force on the bottom wire is given by $\vec{F}_2 = I_2\vec{\ell} \times \vec{B}_1$:

$$\vec{F}_2 = [-I_2\ell\hat{i}] \times [(\mu_0 I_1/2\pi d)(-\hat{k})] = -(I_1 I_2 \mu_0/2\pi d)\hat{j}.$$

Thus the force per unit length is

$$\vec{F}/\ell = -(I_1 I_2 \mu_0 \hat{j})/2\pi d = -6.4 \times 10^{-6}\hat{j}\ N/m \quad \text{repulsion.}$$

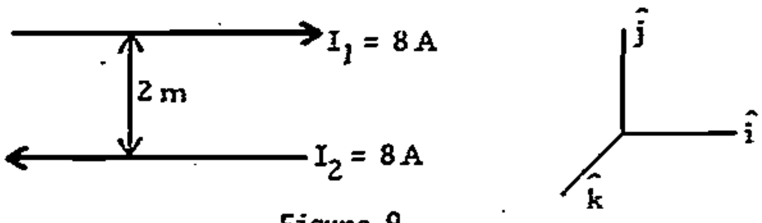


Figure 9

D(3). Imagine a current element $I\, d\vec{\ell}$ in which the current lies in the xy plane, and is directed to the right parallel to the x axis. What is the magnetic field at the origin if the current increment is located:
(a) at the point $(x, y, z) = (0, a, 0)$ m?
(b) at the point $(a, a, 0)$ m?
(c) at the point $(a, 0, 0)$ m?

Solution

(a) See Figure 10. $\hat{r} = -\hat{j}$.

$$d\vec{B} = \left(\frac{\mu_0 I}{4\pi}\right)\left(\frac{|d\vec{\ell}|[\hat{i} \times (-\hat{j})]}{a^2}\right) = \frac{\mu_0 I|d\vec{\ell}|}{4\pi a^2}(-\hat{k}).$$

Alternately we could use $|\hat{i} \times (-\hat{j})| = |\hat{i}|\;|-\hat{j}|\;[\sin(90°)]$ and the right-hand rule. From the right-hand rule, we would find, as above, that $d\vec{B}$ points in the -z direction (into the page).

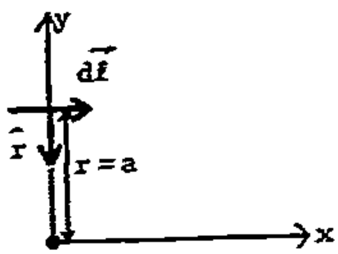


Figure 10

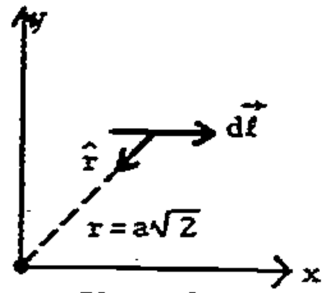


Figure 11

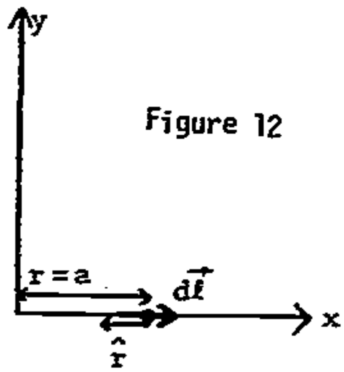


Figure 12

(b) See Figure 11, where $\hat{r} = (\sqrt{2}/2)(-\hat{i} - \hat{j})$. The $\sqrt{2}/2$ makes $\hat{r}$ a unit vector.

$$d\vec{B} = \left(\frac{\mu_0 I}{4\pi}\right)\left(\frac{|d\ell|[\hat{i} \times (-\hat{i} - \hat{j})(\sqrt{2}/2)]}{2a^2}\right) = \frac{\sqrt{2}\mu_0 I|d\vec{\ell}|}{16\pi a^2}(-\hat{k}).$$

(c) See Figure 12, where $\hat{r} = -\hat{i}$.

$$d\vec{B} = (\mu_0 I/4\pi)\{|d\vec{\ell}|[\hat{i} \times (-\hat{i})]/a^2\} = 0.$$

E(3). The wire shown in Figure 13 carries a current I. What is the magnetic field at the center C of the semicircle arising from:
(a) each infinite straight segment,
(b) the semicircular segment of R,
(c) the entire wire?

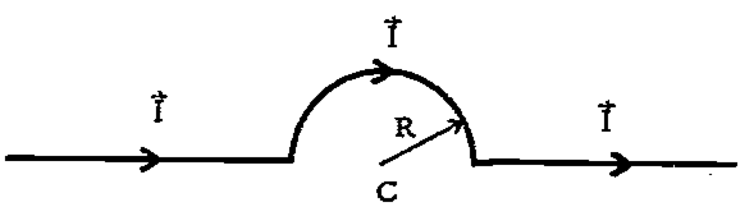


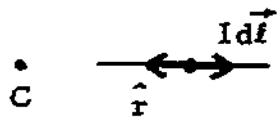
Figure 13

Solution

(a) For the left-hand segment $I\,d\vec{\ell}$ is in the same direction as r, thus $I d\vec{\ell} \times r = I\,d\ell \sin 0° = 0$. See Figure 14. For the right-hand segment $I\,d\ell$ is directly opposite of r, thus $I\,d\vec{\ell} \times \hat{r} = I\,d\ell \sin 180° = 0$. See Figure 15. Thus the straight segments contribute zero to the field at C, since C is in a direct line with each segment.

Figure 15

Figure 14



(b) For the circular portion, $I\,d\vec{\ell} \times \hat{r}$ = a vector whose magnitude is $I\,d\ell$ and whose direction is into the page. Therefore,

$$|d\vec{B}| = \left|\left(\frac{\mu_0 I}{4\pi}\right)\left(\frac{d\vec{\ell} \times \hat{r}}{r^2}\right)\right| = \frac{\mu_0 I\,d\ell}{4\pi R^2}.$$

Now since μ_0, I, and R are constants,

$$|\vec{B}| = \int dB = (\mu_0 I/4\pi R^2)\int d\ell.$$

The $\int d\ell$ is just the length of the circular segment, $(1/2)(2\pi R) = \pi R$. Thus $|\vec{B}|$ is $\mu_0 I/4R$ and the direction of $\vec{B}$ is into the page.

(c) The total $\vec{B}$ field is just the vector sum of parts (a) and (b), or $\mu_0 I/4R$, into the page.

Problems

F(1). Two infinitely long wires carry currents of 3.00 A and 10.0 A as shown in Figure 16. At what point(s) in the plane of the paper is $\vec{B} = 0$?

Figure 17

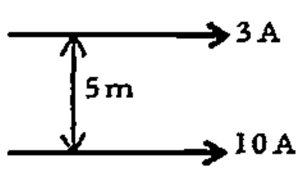
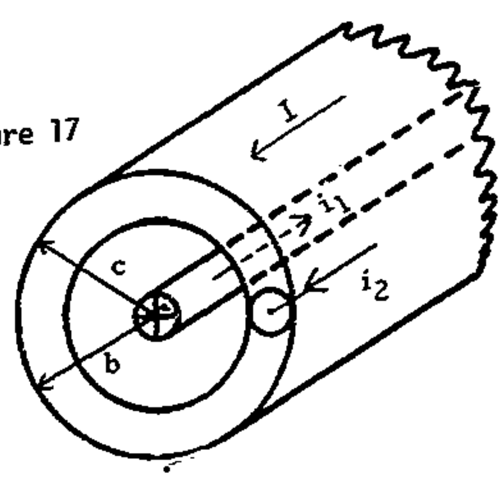


Figure 16



G(1). A long coaxial cable consists of two concentric conductors with the dimensions as shown in Figure 17 (a = radius of inner conductor). The conductors carry equal currents but in the opposite directions. Find the $\vec{B}$ field at a distance r from the center, where (a) $b < r < c$; and (b) $r > c$.

H(2). In Problem F, find the force per unit length on the top wire, carrying current I = 3.00 A, caused by the bottom wire, carrying current I = 10.0 A.

I(3). Imagine a current element $I\,d\vec{\ell}$ in which the current lies in the xy plane and is directed to the left parallel to the x axis. What is the magnetic field $d\vec{B}$ (magnitude and direction), at the point P = (0, 1, 0), if the current element is located at (a) (4, -1, 0) m? (b) (-2, 1, 0) m? See Figure 18.

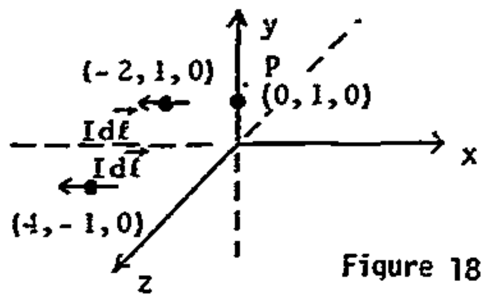


Figure 18

J(3). (a) A straight conductor is split in identical semicircular turns as shown in Figure 19. What is the magnetic field at the center of the circular loop? (Hint: This follows immediately from Problem E. Can you guess the answer?)

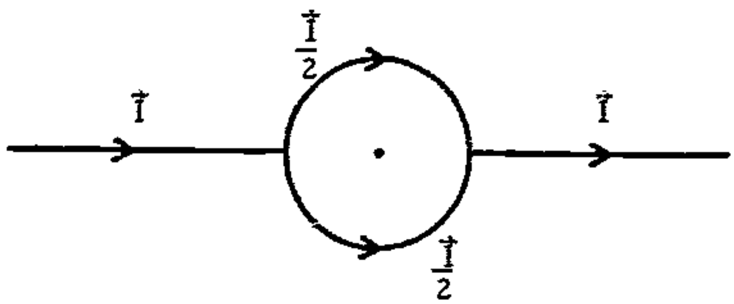


Figure 19

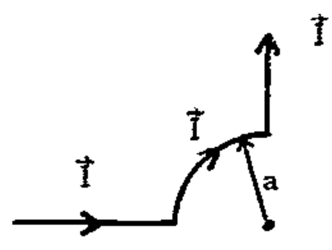


Figure 20

(b) A conductor as shown in Figure 20 consists of a circular arc of 90° and two long wires leading to this arc. Find the $\vec{B}$ field at the center of the arc.

Solutions

F(1). 1.15 m from the 3.00-A wire and toward the 10.0-A wire.

G(1). (a) $|\vec{B}| = (\mu_0 I/2\pi r)[(c^2 - r^2)/(c^2 - b^2)]$; direction is circular, clockwise. (b) $\vec{B} = 0$.

H(2). $\vec{F} = 1.20 \times 10^{-6}$ N/m, down (attraction).

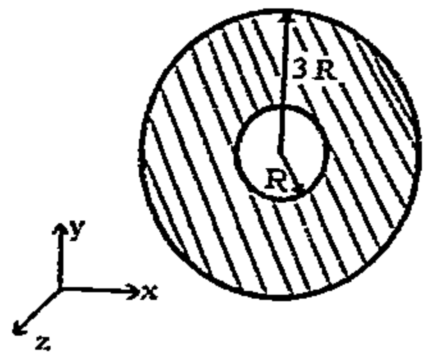
I(3). (a) $d\vec{B} = -\mu_0 I|d\vec{\ell}|\hat{k}/80\pi\sqrt{5}$. (b) $d\vec{B} = 0$.

J(3). (a) $\vec{B} = 0$. (b) $\vec{B} = \mu_0 I/8R$ into the page.

PRACTICE TEST

1. A long, straight hollow (inner radius R, outer radius 3R) conductor in Figure 21 carries a current I (out of paper) that is uniformly distributed over the cross section. If $\vec{r}$ measures the position from the axis of the conductor, determine $\vec{B}$ (magnitude and direction) for (a) $\vec{r} = -(R/2)\hat{i}$, (b) $\vec{r} = 2R\hat{j}$, and (c) $\vec{r} = 5R\hat{i}$.

Figure 21



2. A thin wire of length L carries a current 3I parallel to the conductor of Problem 1. The wire is positioned at $\vec{r} = 5R\hat{i}$ from the axis of the hollow conductor. Determine the force (magnitude and direction) on this wire.

3. Imagine a current element $I\, d\vec{\ell}$ in which the current lies in the xy plane and is directed downward and to the right at an angle of 45° to the x axis; i.e.,

$d\vec{\ell} = (\sqrt{2}/2)|d\vec{\ell}|(\hat{i} - \hat{j})$.

What is the magnetic field $d\vec{B}$ at the origin if the current element is located: (a) at the point (2, 0, 0)? (b) at (-4, 0, 0)?

Practice Test Answers

1. (a) 0. (b) $-(3\mu_0 I/32\pi R)\hat{i}$. (c) $(\mu_0 I/10\pi R)\hat{j}$.
2. $-(3\mu_0 I^2 L/10\pi R)\hat{i}$.
3. Let $C = (\mu_0 I/4\pi)|d\vec{\ell}|$: (a) $d\vec{B} = C(-\sqrt{2}\hat{k})/8$. (b) $d\vec{B} = C(\sqrt{2}\hat{k})/32$.

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1 2 3

Name ____________________ Tutor ____________________

1. A current I is distributed uniformly over the cylindrical region $2R < r < 5R$ in Figure 1. The current is in the $+\hat{k}$ direction.
(a) Determine the magnetic field at $\vec{r} = 3R\hat{i}$. Answer in terms of μ_0, I, R, $\hat{i}$, $\hat{j}$, $\hat{k}$.
(b) Determine the value of r for which $|\vec{B}|$ is a maximum, and calculate the magnitude of $\vec{B}$ at that point.

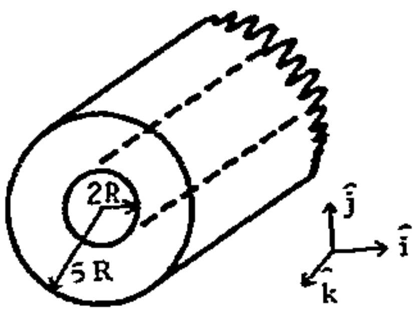

Figure 1

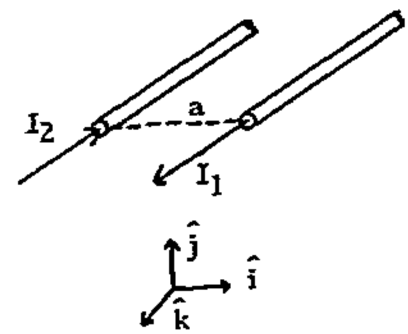

Figure 2

2. Two long parallel wires carry currents as shown in Figure 2. Determine the force on a 20.0-cm length of the wire carrying the I_1 current. $I_1 = I_2 = 60$ A, $a = 2.00$ cm.

3. A current element $I\,d\vec{\ell} = I\,d\ell(\sqrt{3}\hat{i} + \hat{j})/2$ lies in the xy plane and is directed upward and to the right at an angle of 30° with respect to the x axis. What is the magnetic field dB (magnitude and direction) at the origin if the current element is located at the point (2, 0, 0) m?

AMPERE'S LAW   Date ________________

Mastery Test   Form B   pass   recycle

                                           1   2   3

Name ______________________________ Tutor ______________________

1. (a) Currents of 6I and 8I are distributed uniformly in the cylindrical conductor as shown in Figure 1. Determine the magnetic field at point P in terms of I, R, μ_0, $\hat{i}$, $\hat{j}$, $\hat{k}$.

   (b) A thin wire of length L carries a current 3I <u>into the paper</u> at point P. It is "parallel" to the cylindrical conductors. Determine the force on this wire in terms of L, I, R, μ_0, $\hat{i}$, $\hat{j}$, $\hat{k}$.

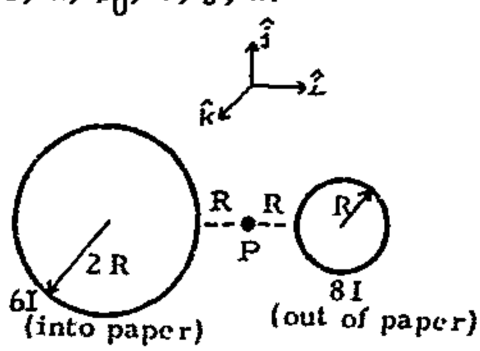

Figure 1

2. A current-carrying wire is bent in the shape shown in Figure 2.
   (a) Which of the four segments A, B, C, D contribute to the field at the point P?
   (b) What is the direction and magnitude of the field at P?

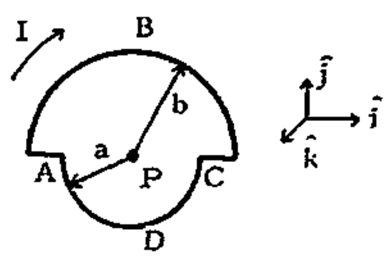

Figure 2

AMPÈRE'S LAW

Date ____________________

Mastery Test   Form C

pass   recycle

1 2 3

Name ____________________ Tutor ____________________

1. (a) A current of 3I is distributed uniformly in the cylindrical region $R < r < 2R$ as shown in Figure 1. A current of 2I is uniformly distributed in a cylindrical conductor of radius R as shown. Determine the magnetic field at points O and P in terms of I, R, and constants.
(b) A thin wire of length L carries a current 2I into the paper at point P. It is "parallel" to the two cylindrical conductors. Determine the force on this wire in terms of I, R, L, $\hat{i}$, $\hat{j}$, $\hat{k}$, and other constants.

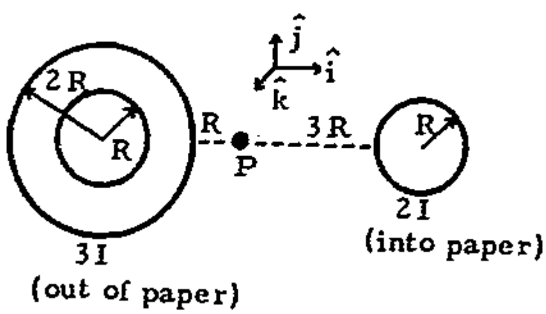

Figure 1

2. Imagine a current element $I\,d\vec{\ell}$ in which the current lies in the xy plane and is directed upward and to the right at an angle of 53° to the x axis, i.e.,

$d\vec{\ell} = d\ell(0.6\hat{i} + 0.8\hat{j})$.

What is the magnetic field $d\vec{B}$ (magnitude and direction) at the origin if the current element is located at the point (2, 0, 0) m?

AMPÈRE'S LAW

Date ________________

Mastery Test   Form D

pass   recycle

1   2   3

Name ____________________ Tutor ____________________

1. A solenoid consists of 500 turns of wire wound around a cylindrical shell 1.00 cm in radius and 10.0 cm long. What is the magnetic field at the center of the solenoid when a current of 6.0 A is sent through the solenoid?

2. Two wires carrying the same current I are bent into semicircles as shown in Figure 1. The ends are then overlapped closely but not touching to form a circle of radius R, as in Figure 2. What is the magnetic field at the center of the circle (direction and magnitude)?

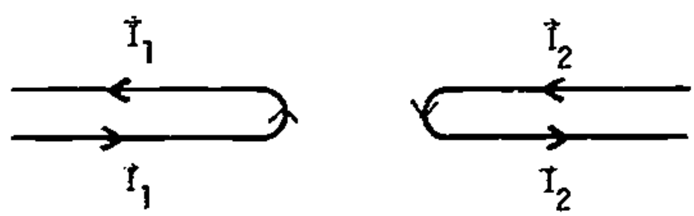

Figure 1

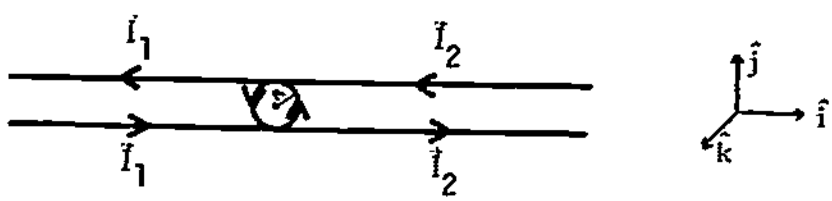

Figure 2

3. Two long wires carry currents of 3.00 A and 10.0 A, as in Figure 3. What is the force per unit length on the bottom wire carrying 10.0 A? (The wires are separated a distance of 5.0 m.)

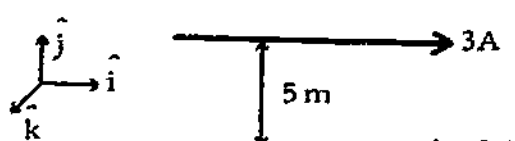

Figure 3

MASTERY TEST GRADING KEY - Form A

1. What To Look For: (a) Do they start off from Ampère's law? What is i? Fraction of I enclosed by circle of radius 3R. How do you determine direction? (Apply right-hand rule, have student do it.) (b) How do you know $\vec{B}$ is maximum at $r = 5R$? (See Figure 3D.)

   Solution: (a) $\vec{B}(\vec{r} = 3R\hat{i}) = ?$ $\oint \vec{B} \cdot d\vec{\ell} = \mu_0 i$,

   $$|\vec{B}| = \frac{\mu_0 i}{2\pi r}, \qquad i = \left(\frac{I\pi}{\pi}\right)\left(\frac{(3R)^2 - (2R)^2}{(5R)^2 - (2R)^2}\right) = I\left(\frac{5R^2}{21R^2}\right) = \frac{5I}{21},$$

   $\vec{B} = [5\mu_0 I/42\pi(3R)]\hat{j} = [5\mu_0 I/126\pi R]\hat{j}$.

   (b) See Figure 30. $\vec{B}$ is maximum at $r = 5R$. For $(2R < r < 5R)$,

   $|\vec{B}| = \mu_0 I(r^2 - 4R^2)/2\pi r(21R^2)$. For $5R < r$, $|\vec{B}| = \mu_0 I/2\pi r$. At $r = 5R$,

   $|\vec{B}| = \mu_0 I/10\pi R$.

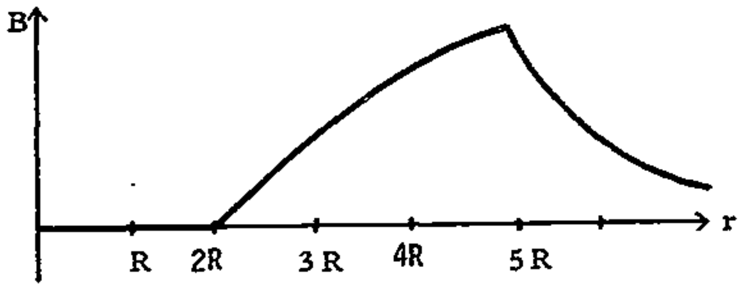

Figure ':

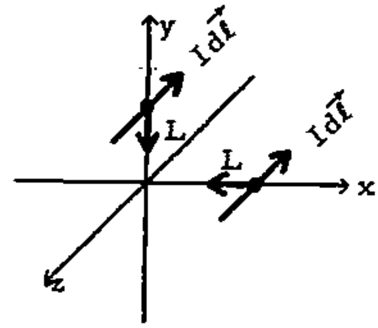

Figure 31

2. What To Look For: Are the directions for $\vec{B}$ from I_2 correct?

   Solution: From $\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I$, $\vec{B}$ at I_1 caused by I_2 equals $-(\mu_0 I_2/2\pi a)\hat{j}$.

   $$\vec{F} = i\vec{\ell} \times \vec{B}, \qquad I_1 = I_2 = (I_1 \ell\hat{k}) \times \left(-\frac{\hat{j}}{a}\frac{\mu_0 I}{2\pi}\right) = \left(\frac{\mu_0 I^2 \ell}{2\pi}\right)\left(\frac{\hat{i}}{a}\right) = \frac{(4\pi \times 10^{-7})(60)^2(0.200)\hat{i}}{2\pi(0.0200)}.$$

   $\vec{F} = (7.2 \times 10^{-3})\hat{i}$ N.

3. What To Look For: Do they start with the Biot-Savart law?

   Solution: See Figure 31.

   $$d\vec{B} = \frac{\mu_0 I\ d\vec{\ell} \times \hat{r}}{4\pi r^2} = \frac{\mu_0 I\ d\ell(\sqrt{3}\hat{i} + \hat{j}) \times (-\hat{i})}{4\pi 2(2^2)}, \quad \hat{r} = -\hat{i}, \quad |\vec{r}| = 2, \quad d\vec{B} = (\mu_0 I\ d\ell/32\pi)\hat{k}.$$

MASTERY TEST GRADING KEY - Form B

1. What To Look For: (a) Did they start from Ampère's law and get $|\vec{B}| = \mu_0 i/2\pi r$? What is i? What is r? (b) Can they evaluate the cross product correctly?

   Solution: (a) Add $\vec{B}$ fields from each wire at P:

   $\int \vec{B} \cdot d\vec{\ell} = \mu_0 i$, $\quad \vec{B}_{6I} = [\mu_0(6I)/2\pi(3R)](-\hat{j}) = -(\mu_0 I/\pi R)\hat{j}$,

   $\vec{B}_{8I} = [\mu_0(8I)/2\pi(2R)](-\hat{j}) = -(2\mu_0 I/\pi R)\hat{j}$, $\quad \vec{B}_{total} = -(3\mu_0 I/\pi R)\hat{j}$.

   (b) $\vec{F} = I\vec{\ell} \times \vec{B} = 3IL(-\hat{k}) \times (-3\mu_0 I/\pi R)\hat{j} = -(9LI^2\mu_0/\pi R)\hat{i}$.

2. What To Look For: (b) Do they start with Biot-Savart law or know where $|\vec{B}| = \mu_0 I/4R$ comes from?

   Solution: (a) Only B, D contribute. A and C have $d\vec{\ell} \times \hat{r} = 0$.
   (b) For a circle $d\vec{B} = (\mu_0 I\ d\vec{\ell} \times \hat{r})/4\pi r^2$.

   $|d\vec{B}| = (\mu_0 I\ d\ell)/4\pi r^2 \quad$ (r = radius).

   For a half-circle:

   $$\vec{B} = \left(\frac{\mu_0 I}{4\pi r^2}\right)\frac{1}{2}(2\pi r) = -\left(\frac{\mu_0 I}{4r}\right)\hat{k}.$$

   Therefore

   $$\vec{B} = -\frac{\mu_0 I\hat{k}}{4}\left(\frac{1}{a} + \frac{1}{b}\right).$$

MASTERY TEST GRADING KEY - Form C

1. What To Look For: (a) Do they start from Ampère's law? How do you know direction? (right-hand rule) Why can you add $\vec{B}_{2I}$ and $\vec{B}_{3I}$? (principle of superposition.)
(b) Did they evaluate cross product correctly?

Solution: (a) Field at 0 = ?

$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I$ or $|\vec{B}| = \mu_0 I/2\pi r$.

From the 3I current, $\vec{B} = 0$, zero radius. From the 2I current,

$\vec{B} = [\mu_0(2I)/2\pi 7R]\hat{j} = (\mu_0 I/7\pi R)\hat{j}$.

Field at P = ? From 3I current,

$\vec{B} = [\mu_0(3I)/2\pi 3R]\hat{j} = (\mu_0 I/2\pi R)\hat{j}$.

From 2I current,

$\vec{B} = [\mu_0(2I)/2\pi 4R]\hat{j} = (\mu_0 I/4\pi R)\hat{j}$, $\vec{B}_{total} = (3\mu_0 I/4\pi R)\hat{j}$.

(b) $\vec{F} = I\vec{\ell} \times \vec{B} = 2IL(-\hat{k}) \times \left(\frac{3\mu_0 I}{4\pi R}\right)\hat{j} = \left(\frac{3\mu_0 I^2 L}{2\pi R}\right)\hat{i}$.

2. What To Look For: What is the Biot-Savart law? Is the cross product evaluated correctly?

Solution: See Figure 32.

$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{\ell} \times \hat{r}}{r^2}$, $\hat{r} = -\hat{i}$, $r^2 = 4$,

$d\vec{B} = \frac{\mu_0 I\,d\ell(0.6\hat{i} + 0.8\hat{j}) \times (-\hat{i})}{4\pi(4)} = \left(\frac{0.8\mu_0 I\,d\ell}{16\pi}\right)\hat{k} = \left(\frac{\mu_0 I\,d\ell}{20\pi}\right)\hat{k}$.

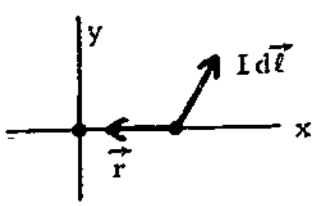

Figure 32

MASTERY TEST GRADING KEY - Form D

1. What To Look For: How do you know the direction of $\vec{B}$? (right-hand rule on one wire) Can they derive $\vec{B} = \mu_0 In\hat{i}$ from Ampère's law?

   Solution: See Figure 33. $\oint \vec{B} \cdot d\ell = \mu_0 I$ for dotted rectangle on sides D, C and $\vec{B} \cdot d\vec{\ell} = 0$. On side B, $\vec{B} = 0$. So for side A, $\oint \vec{B} \cdot d\vec{\ell} = B\ell$. Total I inside = $In\ell$, where n = turns/length and I = current in one turn. Therefore $B\ell = \mu_0 In\ell$.

   $\vec{B} = \mu_0 In\hat{i}$, n = 500 turns/0.100 m = 5000 turns/m.

   $\vec{B} = (4\pi \times 10^{-7})(6)(5000) = (3.77 \times 10^{-2})\hat{i}$ T.

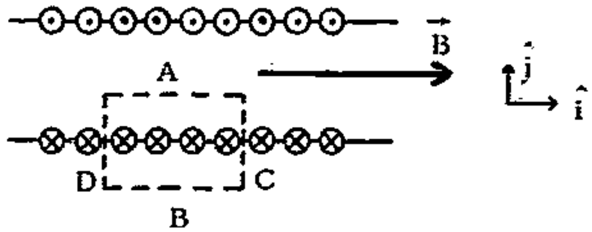

Figure 33

2. What To Look For: How can you determine direction? (right-hand rule) Can they derive each of these equations? (For the Biot-Savart law: r = const = R. $\int d\ell = 2\pi R$.)

   Solution: This is equivalent to two wires plus a current loop. From Ampère's law:

   $\vec{B} = (\mu_0 I/2\pi R)\hat{k}$, each straight wire.

   Circle (from Biot-Savart law):

   $\vec{B} = (\mu_0 I/2R)\hat{k}$. $\vec{B}_{total} = [(\mu_0 I/2R)(1/\pi + 1)]\hat{k}$.

3. What To Look For: How do you know direction of $\vec{B}$? (right-hand rule) Is the cross product evaluated correctly?

   Solution: $\vec{B}$ at bottom wire caused by top wire = $(\mu_0 I/2\pi r)(-\hat{r})$.

   $\vec{B} = (\mu_0 3/2\pi 5)(-\hat{k})$, $\vec{F} = I\vec{\ell} \times \vec{B} = (10\ell\hat{i}) \times [-(\mu_0 3/10\pi)\hat{k}]$,

   $\vec{F} = (\frac{3\mu_0 \ell}{\pi})\hat{j}$, $\frac{\vec{F}}{\ell} = (\frac{3\mu_0}{\pi})\hat{j} = (\frac{3(4\pi \times 10^{-7})}{\pi})\hat{j} = (1.20 \times 10^{-6})\hat{j}$ N/m.

Module ______

STUDY GUIDE

FARADAY'S LAW

INTRODUCTION

Consider the electric light you may be using to read this module by and the influence on your life style of the vast amounts of electrical energy produced in the United States. This module treats the fundamental principle that allows for the transformation of mechanical energy into electrical energy. The physical law that governs the production of electric current is named after its discoverer, Michael Faraday.

PREREQUISITES

Before you begin this module, you should be able to:	Location of Prerequisite Content
*Calculate scalar products (needed for Objectives 1 through 3 of this module)	Vector Multiplication Module
*Use Ohm's law to determine the current, knowing the resistance and voltage (needed for Objective 3 of this module)	Ohm's Law Module
*Given the current direction, determine the direction of the magnetic field; or given the direction of the magnetic field, determine the direction of the source current (needed for Objectives 2 through 4 of this module)	Ampère's Law Module

LEARNING OBJECTIVES

After you have mastered the content of this module, you will be able to:

1. Faraday's law - Write the equation for Faraday's law in the form $\mathcal{E} = -d\phi/dt$ and define all terms with correct units.

2. Magnetic flux - Determine the magnetic flux or the time rate of change of the magnetic flux for an area in a magnetic field.

3. Application of Faraday's law - Determine, using Faraday's law, the induced current and/or voltage for a situation involving either (a) a stationary circuit in a time-varying magnetic field, or (b) a conductor moving in a magnetic field.

4. Lenz's law - Apply Lenz's law to a situation as listed in Objective 3 to determine the direction of the induced current and explain your reasoning.

GENERAL COMMENTS

1. Faraday's Law

In this module you will be held responsible for the following form of Faraday's law:

$$\mathcal{E} = -d\phi/dt,$$

where the magnetic flux is

$$\phi = \oint \vec{B} \cdot d\vec{A}.$$

The induced emf is produced by the time rate of change of the magnetic flux. This is the fundamental principle of the generation of electrical energy. This form of Faraday's law allows us to calculate the induced emf caused by a time-varying magnetic field or a moving circuit in a magnetic field.

What is the induced emf? The voltage induced in a circuit is produced by an electric field (force per unit charge) acting over the current path (circuit):

$$\mathcal{E} = \oint \vec{E}_n \cdot d\vec{\ell}.$$

This is the work required to move a unit charge around a complete circuit. In this case, the force that produces the electric field is magnetic in origin and therefore produces a nonconservative force or electric field E_n. The integral of a nonconservative force per unit charge over a complete path is not equal to zero but, rather, is equal to the work done per unit charge, which is the induced emf. Mechanical work is required to produce a current in a magnetic field. The operation of the Betatron is based on this equation.

The complete form of Faraday's law is

$$\int \vec{E}_n \cdot d\vec{\ell} = -d\phi/dt,$$

where the left-hand side is the induced emf and the right-hand side tells you how to calculate the induced emf. This form suggested that there is a very intimate relationship between electric and magnetic fields.

Why have we chosen to test you on the first listed form of Faraday's law? In a certain sense, the decision is arbitrary because a relativistic treatment of electric and magnetic fields shows that they are merely different perspectives of the same quantity, which depends on the velocity of the observer. The reason for using the form $\mathcal{E} = -d\phi/dt$ is that it is more useful for a course at this level. A future module Inductance is based on this form of Faraday's law. The complete form of Faraday's law is very powerful, but its usefulness is more readily seen in an advanced course in electricity and magnetism.

2. Lenz's Law

The minus sign in Faraday's law can best be explained by the use of Lenz's law. Lenz's law, or the minus sign in Faraday's law, is merely a consequence of the conservation of energy. It tells us that we must do mechanical work to rotate a loop of wire in a magnetic field to produce electrical energy.

Objective 4 requires that you explain your reasoning for determining the direction of the induced current. It is suggested that you answer this question in terms of the first cause; for example, rotating a coil in a magnetic field requires a torque, and your answer should be in terms of the direction of the induced current that would oppose this rotation.

3. Units

You have experienced the confusion produced by different sets of units, and magnetic-field units are perhaps the worst. Textbooks use a variety of units, but this module (Problem Set, Practice Test, and Mastery Tests) will use webers (Wb) for magnetic flux, and teslas ($1\ T = 1\ Wb/m^2$) for magnetic fields. From Faraday's law it is possible to show that one volt = one weber per second. You may wish to prove this for yourself.

TEXT: Frederick J. Bueche, Introduction to Physics for Scientists and Engineers (McGraw-Hill, New York, 1975), second edition

SUGGESTED STUDY PROCEDURE

First read General Comments 1 through 3 and then read Sections 25.1 and 25.6 in Chapter 25 of your text. Sections 25.2 through 25.5 will be treated in the module Inductance.

The text's treatment of Faraday's law [Eq. (25.2)] and Lenz's law is brief, but adequate. Study Problems A through E and Illustrations 25.1 and 25.6 (first half only) before working Problems F through I and 1, 3, 8, 9, and 21 in Chapter 25. Take the Practice Test before attempting a Mastery Test.

BUECHE

Objective Number	Readings	Problems with Solutions		Assigned Problems	
		Study Guide	Text	Study Guide	Text (Chap. 25)
1	Sec. 25.1	A			
2	Sec. 25.6	B		F	21
3	Secs. 25.1, 25.6	C, D	Illus.[a] 25.1, 25.6 (first half)	G, H	1, 3, 8
4	Sec. 25.1	E		I	9

[a]Illus. = Illustration(s).

TEXT: David Halliday and Robert Resnick, Fundamentals of Physics (Wiley, New York, 1970; revised printing, 1974)

SUGGESTED STUDY PROCEDURE

First read General Comments 1 through 3 and then read Chapter 31, Sections 31-1 through 31-4. Equation (31-1) gives Faraday's law. Section 31-5 is not required but does have a limited discussion of the relationship between time-varying magnetic fields and electric fields as mentioned in General Comment 1.

Study Problems A through E and Examples 1 and 2 in Chapter 31 before working Problems F through I and 3, 9, 16(a), (b), 17, and 20 in Chapter 31. Take the Practice Test before attempting a Mastery Test.

HALLIDAY AND RESNICK

Objective Number	Readings	Problems with Solutions		Assigned Problems	
		Study Guide	Text	Study Guide	Text (Chap. 31)
1	Secs. 31-1, 31-2	A			
2	Secs. 31-2, 31-4	B	Ex.[a] 1	F	
3	Secs. 31-1, 31-2, 31-4	C, D	Ex. 1, 2(b)	G, H	3, 9(a), 16(a), (b), 17, 20
4	Sec. 31-3	E	Ex. 2	I	9(b)

[a]Ex. = Example(s).

TEXT: Francis Weston Sears and Mark W. Zemansky, University Physics (Addison-Wesley, Reading, Mass., 1970), fourth edition

SUGGESTED STUDY PROCEDURE

First read General Comments 1 through 3 and then read Chapter 33, Sections 33-1, 33-2, 33-4, and 33-5. See especially Eqs. (33-3) and (33-6). Your text derives Faraday's law by starting with the expression for the nonelectrostatic force per unit charge. Section 33-2, "The Search Coil," is an application of Faraday's law.

Study Problems A through E before working Problems F through I and the Assigned Problems from the text. Take the Practice Test before attempting a Mastery Test.

SEARS AND ZEMANSKY

Objective Number	Readings	Problems with Solutions	Assigned Problems	
		Study Guide	Study Guide	Text
1	Sec. 33-1	A		
2	Secs. 33-1, 33-2	B	F	33-8(a)
3	Secs. 33-1, 33-2, 33-4	C, D	G, H	33-6(b), 33-8(b), 33-9, 33-10, 33-15(c), (d)
4	Sec. 33-5	E	I	33-20, 33-21

TEXT: Richard T. Weidner and Robert L. Sells, Elementary Classical Physics (Allyn and Bacon, Boston, 1973), second edition, Vol. 2

SUGGESTED STUDY PROCEDURE

First read General Comments 1 through 3 and then read Chapter 31, Sections 31-1 through 31-4. See especially Eq. (31-5). Although not required, your text does have a partial discussion of the relationship between time-varying magnetic fields and nonconservative electric fields in Section 31-5 and in particular on p. 632.

Study Example 31-1 and Problems A through E before working Problems F through I and the Assigned Problems from your text. Take the Practice Test before attempting a Mastery Test.

WEIDNER AND SELLS

Objective Number	Readings	Problems with Solutions		Assigned Problems	
		Study Guide	Text	Study Guide	Text
1	Sec. 31-1	A			
2	Secs. 31-2, 31-3	B	Ex.[a] 31-1	F	
3	Sec. 31-3	C, D	Ex. 31-1	G, H	31-1(a), 31-2, 31-8, 31-10(a), 31-12
4	Sec. 31-4	E		I	31-5

[a]Ex. = Example(s).

PROBLEM SET WITH SOLUTIONS

A(1). Write the equation for Faraday's law and define all terms with correct units.

Solution

$\mathcal{E} = -d\phi/dt$ (Faraday's law),

where $\mathcal{E}$ is the induced voltage (volts), ϕ is the magnetic flux = $\int \vec{B} \cdot d\vec{A}$ (webers), B is the magnetic field (webers per square meter, or teslas), and t is time (seconds).

B(2). A square wire loop (edge d = 0.200 m) is hinged as in Figure 1 so it can rotate about one edge. This loop is then placed in a $\vec{B}$ field of magnitude B_0 = 1.50 T, directed perpendicular to the hinged edge. Express the flux through the loop as a function of the plane of the loop angle θ with the magnetic field direction.

Solution

The flux is

$$\phi = \int \vec{B} \cdot d\vec{A},$$

but $\vec{B} \cdot d\vec{A} = B\ dA \sin\theta$ (careful with the angle), and B and θ are constant; therefore

$$\phi = B \sin\theta \int dA = BA \sin\theta = (1.50\ \text{T})(0.200\ \text{m})^2 \sin\theta = (0.060 \sin\theta)\ \text{Wb}.$$

If θ was 0°, then $\vec{B}$ and the plane of A would be parallel and there would be zero flux because the projection of $\vec{A}$ on $\vec{B}$ would be zero.

C(3). Determine the magnitude of the induced emf as a function of time for Problem F.

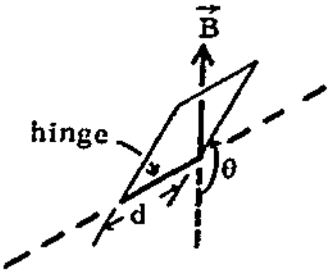

Figure 1

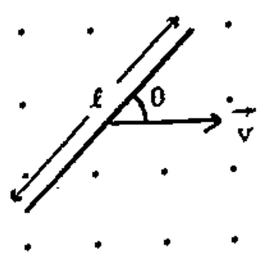

Figure 2

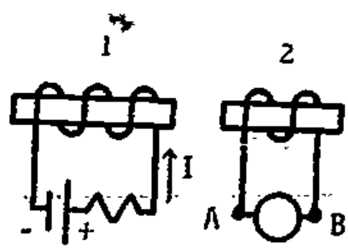

Figure 3

Solution

This is a situation of a time-varying magnetic field:

$$\mathcal{E} = -d\phi/dt.$$

We are interested in the magnitude of $\mathcal{E}$ and will therefore ignore the minus sign:

$$\phi = \int \vec{B} \cdot d\vec{A}.$$

$\vec{B}$ is a function of time but is uniform in space, therefore

$$\phi = \int \vec{B} \cdot d\vec{A} = B_0 L^2 e^{t/c},$$

$$\mathcal{E} = (d/dt)(B_0 L^2 e^{t/c}) = B_0 L^2 (d/dt) e^{t/c} \quad \text{or } \mathcal{E} = (B_0 L^2/c) e^{t/c}.$$

Check the units and remember that $e^{t/c}$ is dimensionless.

D(3). A straight wire, ℓ = 3.00 m long, which is part of a circuit, moves in the plane of Figure 2 as shown with a velocity $\vec{v} = 10.0\hat{i}$ m/s. If the angle θ is 30° and the magnetic field has a magnitude of B = 0.120 T and is directed perpendicularly out of the paper, what is the magnitude of the induced emf?

Solution

This is a situation of a moving conductor in a constant magnetic field. The conductor sweeps out an area per unit time of $\ell v \sin\theta$, therefore (ignoring the negative sign) Faraday's law is $\mathcal{E} = d\phi/dt$, where

$$\frac{d\phi}{dt} = \frac{d}{dt}\int \vec{B} \cdot d\vec{A} = B\ell v \sin\theta$$

and

$$\mathcal{E} = B\ell v \sin\theta = (0.120\ \text{T})(3.00\ \text{m})(10.0\ \text{m/s})(1/2) = 1.80\ \text{V}.$$

For $\theta = 0$, the wire would not sweep out an area. If you wish you may check how reasonable this answer appears in terms of the magnetic force on a moving charge $\vec{F} = q\vec{v} \times \vec{B}$ and determine whether $\theta = 0$ or $\theta = 90°$ would give the greater charge separation.

E(4). Use Lenz's law to determine the direction of the induced emf in coil 2 in the following cases and explain your reasoning (see Figure 3):

(a) 2 is moved toward 1.

(b) The current is decreased in 1.

Solution

(a) The cause of the induced current is the movement of coil 2 toward coil 1; therefore, the induced current must be in such a direction as to produce a force that will oppose this motion. Therefore, an induced emf from B to A through the meter will (using the right-hand rule) produce a magnetic field in coil 2 such that the magnetic interaction will be repulsive.

(b) The cause of the induced current is the decreasing current that produces a decreasing field in coil 2. The induced current must be in the direction to compensate for the decreasing current field produced by coil 1. This would be produced by an induced emf from A to B through the meter in coil 2 that would tend to compensate for the decreased field (using right-hand rule).

Figure 4

Problems

F(2). The uniform magnetic field in Figure 4 acts in the square shaded region and varies in time according to $\vec{B} = B_0 e^{t/c} k$, where $B_0 = 3.00$ T, $c = 0.0100$ s, $L = 0.40$ m, and $B = 0$ elsewhere. Determine the magnetic flux at $t = 0$ within the circular wire.

G(3). A magnetic field is uniform in space but varies with time as $\vec{B} = B_0(1 + t/c)\hat{k}$, where $B_0 = 0.300$ T, $c = 1.00$ T, and $\hat{k}$ points out of the paper. A wire with a 10.0-Ω resistance in the form of a square with sides $L = 0.50$ m is arranged such that the plane of the wire is perpendicular to $\vec{B}$. Determine the current in the wire at $t = 0$.

H(3). Determine the magnitude of the induced emf in Problem F at $t = c$.

I(4). Use Lenz's law to determine the direction of the induced emf in coil 2 for the same situation as Problem E in the following cases and explain your reasoning:
(a) Coil 1 is moved toward 2.
(b) Current is increased in 1.

Solutions

F(2). $\phi = B_0 L^2 = 0.48$ Wb (at $t = 0$). (Be careful: the magnetic field exists only in the area L^2.

G(3). $I = 0.0075$ A.

H(3). 131 V.

I(4). (a) From B to A through the meter.
(b) From B to A through the meter.

PRACTICE TEST

1. The magnetic field in a certain region of space is given by

$$\vec{B}(t) = 0.300 \sin(100t)\hat{k} \text{ T}.$$

A 50-turn loop of cross-sectional area 0.60 m^2 lies in the xy plane.

(a) Write Faraday's law and define all terms with correct units.

(b) Calculate the flux through the coil at $t = 0$.

(c) Use Faraday's law to calculate the magnitude of the induced emf in the coil at $t = 0$.

2. A current-carrying solenoid is moved toward a conducting loop as in Figure 5. Use Lenz's law to determine the direction of the induced current and explain your reason.

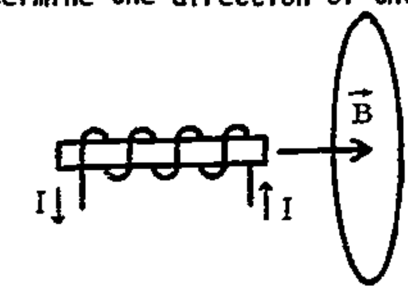

Figure 5

Practice Test Answers

1. (a) Faraday's law is $\mathcal{E} = -d\phi/dt$. The induced emf = $\mathcal{E}$(volts). The flux is $\phi = N\int\vec{B} \cdot d\vec{A}$(webers) = the magnetic field $\vec{B}$ (teslas) integrated over area A (square meters).

(b) $\phi = N\int\vec{B} \cdot d\vec{A} = NB\int dA = NBA = N(0.300)\sin(0)(0.60) = 0.$

(c) $\mathcal{E} = d\phi/dt = N(d/dt)(BA) = NA(dB/dt) = NA[d(0.300)\sin(100t)/dt]$
$= NA(0.300(100 \cos 100t)$. At $t = 0$, $\mathcal{E} = (50)(0.60\ m^2)(0.300\ T)(100\ s^{-1})$
$= 900$ Wb/s $= 900$ V.

2. The induced current will be counterclockwise. The cause of the induced current is the movement toward the ring. Therefore the current in the loop must be in such a direction as to oppose this motion. A counterclockwise current would produce a magnetic field that would repel the solenoid.

FARADAY'S LAW

Mastery Test   Form A

Date ______________

pass   recycle

1 2 3 4

Name ______________________________ Tutor ______________

A plane circular wire loop in Figure 1 of radius = 0.050 m and resistance = 2.00 Ω. lies in the plane y = 0 in a spatially uniform magnetic field $\vec{B} = 0.100 \cos(2\pi t/8)\mathbf{j}$.

1. Write Faraday's law and define all terms with correct units.
2. Calculate the magnetic flux through the loop at t = 1/6 s.
3. Use Faraday's law to determine the induced current in the loop.
4. Use Lenz's law to determine the direction of the induced current in the loop at t = 1/6 s and explain your reason.

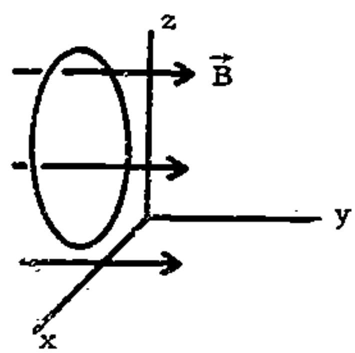

Figure 1

FARADAY'S LAW

Mastery Test   Form B

Name ______________________________

Date ____________

pass recycle

1 2 3 4

Tutor ____________

A conducting rod AB of length ℓ makes contact with the metal rails CA and DB in Figure 1. The apparatus is in a uniform magnetic field $\vec{B} = 0.50\hat{k}$ T(into the paper). The rod moves with a velocity $v = 4.0\hat{i}$ m/s.

1. Write Faraday's law and define all terms with correct units.

2. Calculate the time rate of change of magnetic flux swept out by the wire.

3. Use Faraday's law to calculate the magnitude of the induced emf.

4. Use Lenz's law to determine the direction of the induced current and explain your reason.

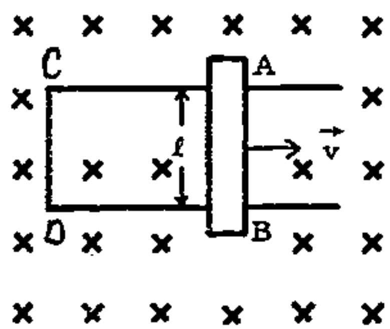

Figure 1

FARADAY'S LAW

Master Test   Form C

Name ______________________

Date ______________

pass   recycle

1   2   3   4

Tutor ______________

A circular wire coil of radius b in the plane of this page and resistance R is in a uniform magnetic field that varies with time as $B = B_0 e^{-t/t_0}$, where B_0 and t_0 are constant and the magnetic field $\vec{B}$ makes an angle of $\pi/3$ with the plane of the coil.

1. Write Faraday's law and define all terms with correct units.
2. Calculate the magnetic flux through the circular wire at $t = t_0$.
3. Use Faraday's law to determine the magnitude of the induced current in the coil.
4. Use Lenz's law to determine the direction of the induced current and explain your reason.

MASTERY TEST GRADING KEY - Form A

1. Solution: $\mathcal{E} = -d\phi/dt$ $\mathcal{E}$ is the induced voltage (in volts), ϕ is the magnetic flux and equals $\int \vec{B} \cdot d\vec{A}$ (in webers), and $\vec{B}$ is the magnetic field (in teslas).

2. What To Look For: $\vec{B}$ is perpendicular to $\vec{A}$.
   Solution: $\phi = \int \vec{B} \cdot d\vec{A}$. Since $\vec{B}$ is perpendicular to $\vec{A}$,
   $\phi = BA = (0.100\ \mathrm{T})(\cos \pi/6)(25\pi \times 10^{-4}\ \mathrm{m}^2) = 1.25 \times 10^{-4}\ \mathrm{Wb}$.

3. What To Look For: Differentiate $\vec{B}$.

   Solution: $\mathcal{E} = -d\phi/dt = -A(dB/dt) = -A[d(0.100 \cos 2\pi t)/dt]$

   $= -A(0.100)(2\pi \sin 2\pi t) = 5\pi^2 \times 10^{-4} \sin(2\pi t)\ \mathrm{Wb/s}$.

4. What To Look For: Cause is decreasing magnetic field.

   Solution: Magnetic field is decreasing in $\hat{j}$ direction; therefore the induced current must be counterclockwise (looking into the +y axis) to produce a magnetic field in the $+\hat{j}$ direction to compensate for decreasing the magnetic field.

MASTERY TEST GRADING KEY - Form B

1. Solution: $\mathcal{E} = -d\phi/dt$, where $\mathcal{E}$ is the induced voltage (in volts), ϕ is the magnetic flux = $\int \vec{B} \cdot d\vec{A}$ (in webers), and $\vec{B}$ is the magnetic field (in teslas).

2. What To Look For: Area swept out is ℓv.

   Solution: $d\phi/dt = B\ell v = (0.50\ T)(4.0\ m/s)\ell = 2\ell$ Wb/s.

3. Solution: $\mathcal{E} = d\phi/dt = 2\ell$ V.

4. What To Look For: Cause is moving wire. The force on the current-carrying wire in a magnetic field.

   Solution: The cause of the induced current is the motion of the wire. The induced current must be in such a direction as to produce a force that will oppose this motion. The induced current will be counterclockwise and will produce a force toward the left.

MASTERY TEST GRADING KEY - Form C

1. Solution: $\mathcal{E} = -d\phi/dt$, where $\mathcal{E}$ is the induced voltage (in volts), ϕ is the magnetic field = $\int \vec{B} \cdot d\vec{A}$ (in webers), and $\vec{B}$ is the magnetic field (in teslas).
2. What To Look For: $\vec{B}$ is perpendicular to $\vec{A}$.

   Solution: $\phi = \int \vec{B} \cdot d\vec{A} = BA = (B_0/e)\pi b^2$.
3. What To Look For: Differentiate $\vec{B}$.

   Solution: $\mathcal{E} = d\phi/dt = A(dB/dt) = A(dB_0 e^{-t/t_0}/dt) = (AB_0/t_0)e^{-t/t_0}$.

   $I = \mathcal{E}/R = (AB_0/t_0 R)e^{-t/t_0}$.
4. What To Look For: Decreasing magnetic field is the cause of the induced current.

   Solution: The cause of the induced current is the decreasing magnetic field. The induced current must be counterclockwise, which will produce a magnetic field out of the page that will compensate for the decreasing magnetic field.