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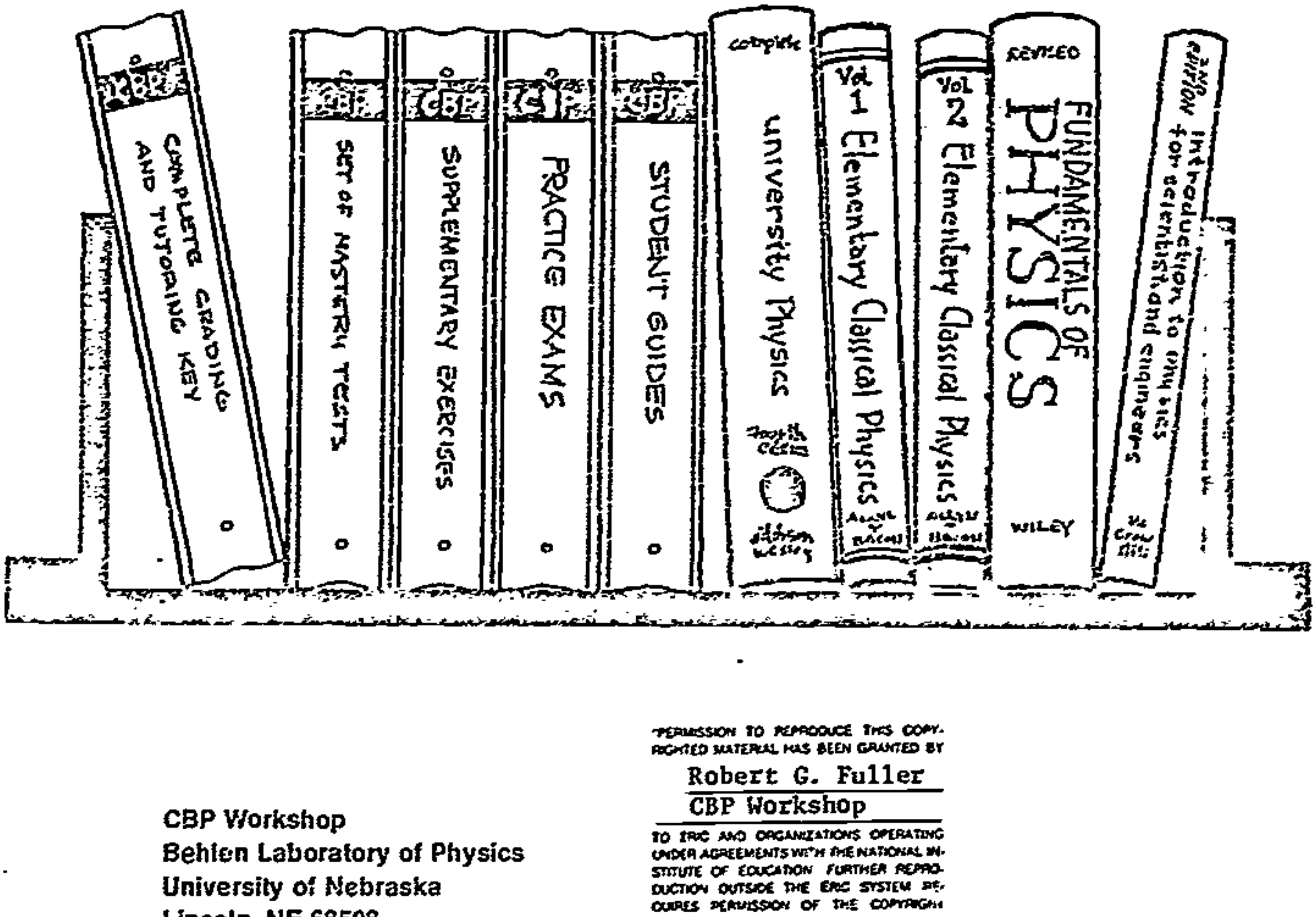
ABSTRACT

This is part of a series of 42 Calculus Based Physics (CBP) modules totaling about 1,000 pages. The modules include study guides, practice tests, and mastery tests for a full-year individualized course in calculus-based physics based on the Personalized System of Instruction (PSI). The units are not intended to be used without outside materials; references to specific sections in four elementary physics textbooks appear in the modules. Specific modules included in this document are: Module 24--Electric Potential, Module 25--Ohm's Law, and Module 26--Capacitors. (CP)

STUDY MODULES FOR CALCULUS-BASED GENERAL PHYSICS*



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Comments

These modules were prepared by fifteen college physics professors for use in self-paced, mastery-oriented, student-tutored, calculus-based general physics courses. This style of teaching offers students a personalized system of instruction (PSI), in which they increase their knowledge of physics and experience a positive learning environment. We hope our efforts in preparing these modules will enable you to try and enjoy teaching physics using PSI.

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COMMENT TO USERS

In the upper right-hand corner of each Mastery Test you will find the "pass" and "recycle" terms and a row of numbers "1 2 3 ..." to facilitate the grading of the tests. We intend that you indicate the weakness of a student who is asked to recycle on the test by putting a circle around the number of the learning objective that the student did not satisfy. This procedure will enable you easily to identify the learning objectives that are causing your students difficulty.

COMMENT TO USERS

It is conventional practice to provide several review modules per semester or quarter, as confidence builders, learning opportunities, and to consolidate what has been learned. You the instructor should write these modules yourself, in terms of the particular weaknesses and needs of your students. Thus, we have not supplied review modules as such with the CBP Modules. However, fifteen sample review tests were written during the Workshop and are available for your use as guides. Please send $1.00 to CBP Modules, Behlen Lab of Physics, University of Nebraska - Lincoln, Nebraska 68588.

FINIS

This printing has completed the initial CBP project. We hope that you are finding the materials helpful in your teaching. Revision of the modules is being planned for the Summer of 1976. We therefore solicit your comments, suggestions, and/or corrections for the revised edition. Please write or call

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STUDY GUIDE

ELECTRIC POTENTIAL

INTRODUCTION

You have no doubt noticed that TV sets, light bulbs, and other electric appliances operate on 115 V, but electric ovens and clothes dryers usually need 220 V. Batteries may be rated at a harmless 1.5, 6, 9, or 12 V, but a high-tension electric transmission line may provide electric power at 400 000 V. Now just what physical quantity is measured by all these volts? How do volts relate to force, energy, and power, about which you have learned in earlier modules? The answer is that volts measure electric potential difference (sometimes called "voltage"), which is derived from the potential energy acquired by electrically charged objects as a result of the electric forces they experience. Even though your familiarity with volts probably stems from electric power supplied to your household, your introduction to the concept of electric potential in this module will be in the context of the interaction of stationary (static) electric charges.

PREREQUISITES

Before you begin this module, you should be able to:	Location of Prerequisite Content
*Calculate work and apply the work-energy theorem in solving problems (needed for Objective 1 of this module)	Work and Energy Module
*Calculate potential energy and identify conservative forces (needed for Objectives 1 and 2 of this module)	Conservation of Energy Module
*Calculate electric force (needed for Objective 1 of this module)	Coulomb's Law and the Electric Field Module
*Interpret a line integral and find the derivative of a function of one variable (needed for Objectives 2 and 4 of this module)	Calculus Review
*Find the electric field using Gauss' law and describe the electric field near conductors (needed for Objectives 3 and 5 of this module)	Flux and Gauss' Law Module
*Find the electric field using Coulomb's law and the superposition principle, and use field lines to describe the electric field (needed for Objectives 3 and 5 of this module)	Coulomb's Law and the Electric Field Module Flux and Gauss' Law Module

LEARNING OBJECTIVES

After you have mastered the content of this module, you will be able to:

1. Definition - Relate electric potential to (a) work done on a displaced charge, (b) the electric field, and (c) electric potential energy. Use the electron volt to express energy and solve simple problems applying energy conservation.

2. Conservative field - State and interpret the conservative nature of the electrostatic field.

3. Finding potentials from charges - Use the definition and/or the superposition principle for finding the electric potential caused by (a) one or more given point charges, and (b) continuous charge distributions with planar, cylindrical, or spherical symmetry.

4. Finding fields from potentials - Determine the electric field when given an electric potential that is a function of one position variable only.

5. Equipotential surfaces - Use equipotential surfaces and field lines for describing the potential and field semiquantitatively near several given point charges and/or simply shaped metallic surfaces.

TEXT: Frederick J. Bueche, Introduction to Physics for Scientists and Engineers (McGraw-Hill, New York, 1975), second edition

SUGGESTED STUDY PROCEDURE

In the Table below, you can see that the reading in your text jumps around quite a bit. This is because your text alternates definitions and application, whereas the objectives of this module tend to group definitions together, separately from your ability to apply them. Depending on your preference, read Chapter 20 as printed (omitting Sections 20.8, 20.12, and 20.13), or follow the order given in the Table. Whichever you do, compare your reading with the statements of the objectives to see how it fits into your overall study plan. Pay special attention to the illustrations in the text and study Problems A through I carefully before working Problems J through R. Check your mastery by taking the Practice Test before attempting a Mastery Test.

The definition of electric potential and simple applications of energy conservation (Objective 1) are spread through Sections 20.1, 20.3, 20.6, and 20.10. Read these along with General Comments 1 and 2. Note how the emphasis changes from potential differences early in the chapter to absolute potential in Section 20.6.

BUECHE

Objective Number	Readings	Problems with Solutions		Assigned Problems	Additional Problems
		Study Guide	Text	Study Guide	
1	General Comments 1, 2, Secs. 20.1, 20.3, 20.6, 20.10	A, B	Illus.[a] 20.2, 20.7,	J, K	Chap. 20, Quest.[a] 1, 2, 4, 12, Probs. 1, 2, 7 to 9, 15, 17
2	General Comment 3	C			
3	Secs. 20.2, 20.4, 20.7, 20.11	D, E	Illus. 20.3, 20.4	L, M, N	Chap. 20, Quest. 7 to 9, 13, 14, Probs., 5, 6, 10 to 13, 14(c), 23 to 26
4	Sec. 20.9	F, G	Illus. 20.5, 20.1	O, P	
5	Sec. 20.5	H, I	Fig. 20.8	Q, R	Chap. 20, Quest. 5, 6, 10, 11

[a]Illus. = Illustrations. Quest. = Questions.

Read General Comments 1 and 2. Since few texts use the term "absolute potential" as carefully as yours, you should be prepared to recognize from the context whether a symbol refers to a potential difference or to the electric-potential function depending on distance from the reference point. See Eq. (20.5), where the absolute potential is a function of the radial distance r, and Illustration 20.5, where it is a function of y. Since the text does not use mathematical function notation, you have to infer the dependence from descriptive clues. You need not study the relativistic part of Illustration 20.7. Illustration 20.6(b) relates to Objective 1, but part (b) was done as Problem E in the module Coulomb's Law and the Electric Field.

Study General Comment 3 and the discussion of Figure 20.3 on p. 367, for Objective 2. Objective 3 is also distributed in several sections, as your text uses parallel metal plates, a single point charge, several point charges, and two concentric cylinders to illustrate the calculation of the electric potential for various geometrical arrangements of charges and conductors. Since Gauss' law enables you to find the electric field for planar, cylindrical, and spherical symmetries, these cases dominate the examples used in this module. Objective 4 is the subject matter of Section 20.9. If you can understand the explanation using partial derivatives or wish to go back and study the Partial Derivatives Review module, very good. If you have difficulty, however, consider the following simpler approach: Apply Eq. (20.1) to points A and B that are separated by the infinitesimal element $d\ell$ and therefore have closely equal values of potential,

$$V_B = V_A + dV.$$

The integral in Eq. (20.1) is then no longer necessary, and we have

$$dV = -\vec{E} \cdot d\vec{\ell} = -E\, d\ell \cos\theta, \qquad \text{(B1)}$$

where θ is the angle between $\vec{E}$ and $d\vec{\ell}$. To solve for $\vec{E}$, we hold $|d\vec{\ell}|$ constant and choose the direction of $d\vec{\ell}$ parallel to $\vec{E}$, so that $\cos\theta = 1$ (its maximum value) and dV has its maximum value

$$dV = -E\, d\ell. \qquad \text{(B2)}$$

After dividing by $d\ell$, we find

$$E = -dV/d\ell, \qquad \text{(B3)}$$

where the derivative is taken in the direction in which V changes most rapidly (maximum dV for fixed $d\ell$). If V depends on only one variable (radius r - distance from a point charge, distance d from a charged plate) then it changes most rapidly in the direction in which that variable changes. Because of the (−) sign in Eq. (B3), the electric field points in the direction in which the potential <u>decreases</u>. Illustration 20.5 actually uses a potential that depends on only one variable y. Applying our result in Eq. (B3) to this example, we find

$$E = -dV/dy = -k, \quad \text{which points in the negative } y \text{ direction,} \qquad \text{(B4)}$$

in complete agreement with the text. More complicated potential functions, such as that used at the top of p. 379, cannot be handled in the same general way by our method. See Problems F and G for other examples.

Objective 5 is taken up in Section 20.5. Supplement this with careful study of Problems H and I.

TEXT: David Halliday and Robert Resnick, Fundamentals of Physics (Wiley, New York, 1970; revised printing, 1974)

SUGGESTED STUDY PROCEDURE

When you look at the analysis of your text in terms of the objectives, you will see that the reading jumps around. This happens because the text combines definitions, theory, and applications, whereas the objectives tend to group the definitions and theory together, separately from your ability to apply them. We suggest that you read the text in the order it is printed, but keep referring to this Table to see how each section contributes to your overall study plan. Pay special attention to the examples in the text and to the Problems with Solutions in the study guide. After concluding the reading and problems, work out the Assigned Problems. Finally, check your learning by taking the Practice Test.

HALLIDAY AND RESNICK

Objective Number	Readings	Problems with Solutions		Assigned Problems	Additional Problems
		Study Guide	Text	Study Guide	
1	General Comments 1, 2, Secs. 25-1, 25-2, 25-6, 25-9	A, B	Ex.[a] 1, 2, 7, 8	J, K	Chap. 25, Quest.[a] 1 to 5, 7, 8, Probs. 3, 4, 24(b), (c), 25(b), 26 to 32, 33(b), 35, 40
2	General Comment 3	C			
3	Secs. 25-3, 25-4, 25-5	D, E	Ex. 3, 4, 5, 10	L, M, N	Chap. 25, Probs. 2, 7 to 10, 13, 15 to 20, 24(a), 25(a), 33(a), 49, 50
4	Sec. 25-7	F, G	Ex. 9	O, P	Chap. 25, Quest. 9, Probs. 45 to 47
5	Secs. 25-1, 25-7, 25-8	H, I	Fig. 25-15	Q, R	Chap. 25, Quest. 6, 11 to 14, Probs. 6, 11, 21, 22, 51

[a]Ex. = Example(s). Quest. = Question(s).

Read General Comments 1 and 2. Objective 1, dealing with the definition of electric potential and simple applications of energy conservation, is spread through four sections that also include references to Objectives 2 and 5. Note how the emphasis changes from potential difference at the beginning of the chapter to potential as function of position in Eqs. (25-6), (25-8), and later. This change is accomplished through the choice of a reference point at which the electric potential is arbitrarily defined to be equal to zero. Objective 2 is brought up on p. 466 in a mathematically incomplete way. See General Comment 3. Objective 3 is also distributed through several text sections. Here you are looking for the potential as a function of position and therefore have to choose a reference point in each example. Note that Example 6 is taken up in another module. The dipole in Section 25-5 is a special case of two closely spaced point charges. Objective 4 is taken up on p. 479. You have to assemble Objective 5 in three sections. The definition of equipotential surface is given on p. 467 and several very instructive diagrams are on p. 478.

TEXT: Francis Weston Sears and Mark W. Zemansky, University Physics (Addison-Wesley, Reading, Mass., 1970), fourth edition

SUGGESTED STUDY PROCEDURE

When you look at the analysis of your text in terms of the objectives, you will see that the reading jumps around. This happens because the text combines definitions, theory, and applications, whereas the objectives tend to group definitions and theory together, separately from your ability to apply them. We suggest that you read the text in the order it is printed (omit Section 26-7), but keep referring to this Table to see how each section contributes to your overall study plan. Pay special attention to the examples in the text and the Problems with Solutions in the study guide. After concluding the reading and problems, work out the Assigned Problems. Finally, check your learning by taking the Practice Test.

Read General Comments 1 and 2. Objective 1, dealing with the definition of electric potential and simple applications of energy conservation, is spread through four sections that also cover Objective 2. Note how, in Section 26-3, the potential at a point is contrasted with the potential difference, which is obtained directly from the electric field integral that represents work per unit electric charge. In Sections 26-5 and 26-6 the electric potential at a point P

SEARS AND ZEMANSKY

Objective Number	Readings	Problems with Solutions		Assigned Problems	Additional Problems
		Study Guide	Text	Study Guide	
1	General Comments 1, 2, Secs. 26-1, 26-2, 26-3, 26-8	A, B		J, K	26-1 to 26-4, 26-6, 26-10, 26-12, 26-14, 26-15, 26-27
2	Sec. 26-1, General Comment 3	C			
3	Sec. 26-4, parts 2, 3, 4, 26-5	D, E	Example (p. 364)	L, M, N	26-5, 26-7, 26-8, 26-18(a), 26-19(a), 26-20, 26-21
4	Sec. 26-6	F, G,	Examples 1, 2 (p. 365)	O, P	26-13
5	Secs. 26-1, 26-4, part 1	H, I	Fig. 26-3	Q, R	

in the field is considered relative to the potential at a reference point. This potential V_P is, of course, a function of the coordinates of the point P expressed in a convenient coordinate system. Each distance r in Eq. (26-13) can be computed from the coordinates of the charge and the coordinates of the field point P. Objective 2 is treated at the beginning of Section 26-1. Also see General Comment 3. Objective 3 is treated in Sections 26-4 and 26-5. Even though the former has a title referring to potential differences, the overall emphasis is on the potential at a point near certain electrically charged bodies, as a function of position of that point. Objective 4 is treated in Section 26-6. Note how the functional dependence of the potential is used to calculate the derivative and find the electric field. Equation (26-15) is easiest to use when you know the direction of the electric field from symmetry properties of the charges and need only to find the magnitude. For Objective 5, return to Sections 26-1 and 26-4. Figures 26-2 and 26-3 are especially instructive.

TEXT: Richard T. Weidner and Robert L. Sells, Elementary Classical Physics (Allyn and Bacon, Boston, 1973), second edition, Vol. 2

SUGGESTED STUDY PROCEDURE

As you read the text in the order printed, refer frequently to the Table below to identify the objective to which your reading relates. This is helpful because the objectives do not reflect the division of the chapter into sections. Pay special attention to the examples in the text and to the Problems with Solutions in the study guide. After concluding the reading and problems, work out the Assigned Problems. Finally, check your learning by taking the Practice Test.

Read General Comments 1 and 2. Objective 1, dealing with the definition of electric potential and simple applications of energy conservation, is spread over the first three sections. Your text gives rather more emphasis to the motion of particles in Section 25-1 than we have chosen to do, so you may skip or skim lightly the description of one charged particle orbiting about another (pp. 502-503). For Objective 2, consult General Comment 3; the very brief explanation given in the legend of Figure 25-1 is incomplete. Our derivation makes use of

WEIDNER AND SELLS

Objective Number	Readings	Problems with Solutions		Assigned Problems	Additional Problems
		Study Guide	Text	Study Guide	
1	General Comments 1, 2, Secs. 25-1, 25-2, 25-3	A, B	Ex.[a] 25-1, 25-2	J, K	25-1 to 25-3, 25-5, 25-6, 25-11 to 25-14
2	General Comment 3	C			
3	Secs. 25-4, 25-5	D, E	Ex. 25-3 to 25-5	L, M, N	25-9, 25-16
4	Sec. 25-6	F, G	Fig. 25-9	O, P	25-15
5	Secs. 25-6, 25-7	H, I	Ex. 25-6, 25-7	Q, R	25-10

[a]Ex. = Example(s).

the electric field $\vec{E}$ rather than the force field $\vec{F}_e$, but that does not affect the conclusion since the two fields only differ by the constant factor q [the charge of the particle being displaced, see Eq. (23-1)]. Objective 3 is discussed in two sections, one concerned with superposition (addition) of the potentials near point charges, the other with applying the definition of potential via potential energy and work done to electric fields obtained by Gauss' law. Keep in mind two facts that are somewhat hidden: (1) the potential V is a function of the coordinates of the point where the test charge is located [Eq. (25-10)] or of the point defining the upper limit of the integral [Eq. (25-11)]; (2) work leads to a difference of potential energy at two points, hence to a difference of potentials, and one must choose a reference point where the potential is arbitrarily defined to be zero in order to obtain the potential at a specified point. Objective 4 is treated in the second part of Section 25-6. The result is in Eq. (25-15), illustrated by Example 25-5. Please ignore Eq. (25-16) (unless you are familiar with partial derivatives) and Figure 25-10 (which has a nonsensical blue arrow labeled $\vec{E}$). The gravitational analogy in the last paragraph is useful for qualitative considerations. Objective 5 is discussed at the beginning of Section 25-6 and in Section 25-7. For additional examples of equipotential surfaces beyond Figure 25-9, see Problems F and G.

GENERAL COMMENTS

1. The Electron Volt

The electron volt (eV) is a unit of energy commonly used in atomic physics. It has the advantage that when an electron moves in an electric field across a potential difference of, say, 150 V, its kinetic energy changes by 150 eV. In other words, the potential difference gives the energy directly in electron volts, if you are dealing with a proton, electron, or other singly charged atomic particle. For household purposes, the electron volt is impractically small. The new system of International Units (SI) asks that the electron volt only be used in atomic physics. Otherwise it should be converted to joules ($1\ \text{eV} = 1.602 \times 10^{-19}$ J). Remember to convert to SI units before using masses in kilograms and velocities in meters per second with an energy originally given in electron volts.

2. Potential as a Function of Position

A certain point at infinity, at the coordinate origin, or at some other point having symmetry in relation to the charges and fields is chosen as the reference point in order to give potential as a function of position in relation to that point ("absolute potential"). Then the potential is simply a function of the coordinates of any given point with the reference point at zero.

3. Conservative Nature of the Electric Field

Are electrostatic forces conservative? You know that a force field is conservative (permits the definition of a potential energy function) if and only if (a) the work done by the force on a particle moved from point A to point B is independent of the path taken from A to B; or (b) the work done by the force on a particle moving through a closed path back to its starting point is zero. Since the electrostatic force at any point is directly proportional to the electric field at that point,

$$\vec{F}_e(\vec{r}) = q\vec{E}(\vec{r}), \qquad (1)$$

the two equivalent conditions on the work stated above lead to two equivalent conditions on the field:

(a) $$W_{A \to B} = \int_A^B \vec{F}_e(\vec{r}) \cdot d\vec{\ell} \quad \text{is independent of path from A to B} \qquad (2)$$

implies that

$$\int_A^B \vec{E}(\vec{r}) \cdot d\vec{\ell} \quad \text{is independent of path from A to B.} \qquad (3)$$

(b) $$W_{A \to A} = \int_A^A \vec{F}_e(\vec{r}) \cdot d\vec{\ell} = 0 \quad \text{by any closed path} \qquad (4)$$

implies that

$$\int_A^A \vec{E}(\vec{r}) \cdot d\vec{\ell} = 0 \quad \text{by any closed path.} \tag{5}$$

The line integral in Eqs. (4) and (5) along a closed path is sometimes indicated by an integral sign with a little circle:

$$\oint \vec{E}(\vec{r}) \cdot d\vec{\ell} = 0. \tag{6}$$

(Please do not confuse this symbol with the same symbol used by many texts to represent a surface integral over a closed surface, as in Gauss' law. You will have to distinguish the two symbols by looking carefully at the infinitesimal element under the integral sign and seeing whether it refers to a surface or to a displacement.)

We shall now show that the Coulomb field caused by a single point charge is conservative by evaluating the integral in Eq. (3) and observing that its value depends only on the end points, not on the path. The magnitude of the electric field caused by the point charge q depends only on the distances (not on the entire vector $\vec{r}$) from the charge and is

$$E(r) = kq/r^2, \tag{7}$$

where $k = 1/4\pi\epsilon_0$, and r is the distance. The direction of the field is radial. Figure 1 illustrates the geometrical relationships. In Figure 1(a) you can see points A and B, a particular path we have chosen, and arrows representing four electric field values $\vec{E}$ and four infinitesimal line elements $d\vec{\ell}$. We must now calculate $\vec{E}(\vec{r}) \cdot d\vec{\ell}$, which we shall do with the help of the enlarged diagram in Figure 1(b). We have indicated the angle θ_4 between the path and the radial

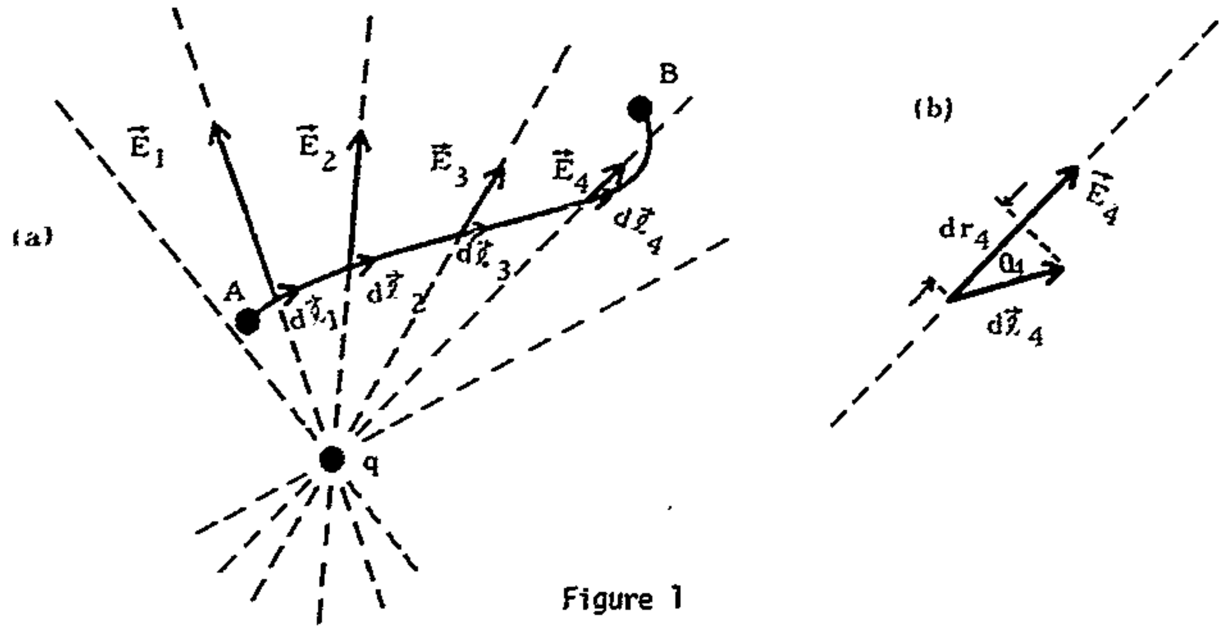


Figure 1

direction and also the radial increment $dr_4 = d\ell_4 \cos\theta_4$. Using the definition of the dot product, we find

$$\vec{E}(\vec{r}) \cdot d\vec{\ell} = E(r)\, d\ell \cos\theta = E(r)\, dr = (kq/r^2)\, dr. \qquad (8)$$

In other words, because the field is radially directed, only the radial increments and not the angular increments of the path contribute to the line integral. (In some texts, the curved path is "approximated" by a stair-step path. They then fail to show that the stair-step integral approaches the line integral along the curved path; assuming that this is true comes close to assuming that the integral is independent of path.)

With the integrand in Eq. (8), we can calculate the integral over the radius variable r, which varies from r_A to r_B:

$$\int_A^B \vec{E}(\vec{r}) \cdot d\vec{\ell} = \int_{r_A}^{r_B} E(r)\, dr = \int_{r_A}^{r_B} \left(\frac{kq}{r^2}\right) dr = kq(r_A^{-1} - r_B^{-1}). \qquad (9)$$

Evidently, this result depends only on the end points and not on the path. Our proof is hereby completed.

Our conclusion of path independence can be extended easily to an electric field that is caused by several point charges. Since such a field is obtained by adding the fields from the individual charges, the line integral can be expressed as a sum of line integrals like that in Eq. (9), for each of which path independence has been established. It is not so easy to extend the proof to the fields caused by continuous charge distributions. As a matter of fact, more advanced treatments of the theory of static electric fields always begin with the postulate that the fields are conservative.

PROBLEM SET WITH SOLUTIONS

A(1). Consider a region of constant electric field in the x direction, with the field having magnitude 400 N/C. The four points A, B, C, and D have the coordinates (5.0, 0.00, 0.00) m, (3.00, 2.00, 0.00) m, (9.0, 0.00, 2.00) m, and (3.00, 0.00, 0.00) m. There is no gravitational field.

(a) An external agent slowly moves a body with charge 0.35 C from A to B. How much work is done on the body by the electric field, and how much is done on the body by the external agent? Use a sketch to illustrate.

(b) Answer the questions in part (a) for displacement of the same body (i) from A to C, (ii) from A to D, and (iii) from B to D.

(c) A bead of mass 0.60 kg and charge -1.20 C is permitted to slide along a frictionless wire between points B and C. At which of the two points should it be released from rest in order that it will get to the other one, and with what speed will it arrive?

(d) Find the differences in electric potential between points B and A, points C and A, and points D and B; compare them with your answers to parts (a) and (b).

Solution

(a) See sketch in Figure 2. From the definition,

$$W_{AB} = \int_A^B \vec{F}_e \cdot d\vec{\ell}.$$

For a constant field, as in this case,

$$W_{AB} = \vec{F}_e \cdot \Delta\vec{\ell}_{AB} = q\vec{E} \cdot \Delta\vec{\ell}_{AB} = (0.35)(400)(-2.00) = -280\ \text{J}\ \text{ done by field.}$$

The external agent exerts force $-\vec{F}_e$, hence does +280 J of work.

(b) (i) $W_{AC} = q\vec{E} \cdot \Delta\vec{\ell}_{AC} = (0.35)(400)(+4.0) = +560\ \text{J};\ W_{AC(agent)} = -560\ \text{J}.$

(ii) $W_{AD} = q\vec{E} \cdot \Delta\vec{\ell}_{AD} = (0.35)(400)(-2.00) = -280\ \text{J};\ W_{AD(agent)} = +280\ \text{J}.$

(iii) $W_{BD} = q\vec{E} \cdot \Delta\vec{\ell}_{BD} = (0.35)(400)(0) = 0\ \text{J};\ W_{BD(agent)} = 0\ \text{J}.$

(c) The negative charge in this part experiences a force to the left, hence it must be released at C and will acquire speed sliding toward B. The kinetic energy gained is equal to the work done on it by the electric field, which is

$$W_{CB} = q\vec{E} \cdot \Delta\vec{\ell}_{CB} = -(-1.20)(400)(6.0) = 2880\ \text{J}.$$

Hence $K = (1/2)mv^2 = 2880$ J, and

$$v = \sqrt{2(2880)/0.60} = \sqrt{9600} = 98\ \text{m/s}.$$

(d) $V_B - V_A = -\vec{E} \cdot \Delta\vec{\ell}_{AB} = 800$ V. $V_C - V_A = -\vec{E} \cdot \Delta\vec{\ell}_{AC} = -1600$ V.

$V_B - V_D = -\vec{E} \cdot \Delta\vec{\ell}_{DB} = 0$ V. $W_{AB} = q(V_B - V_A)$. $W_{AC} = q(V_C - V_A)$. $W_{BD} = -q(V_B - V_D)$.

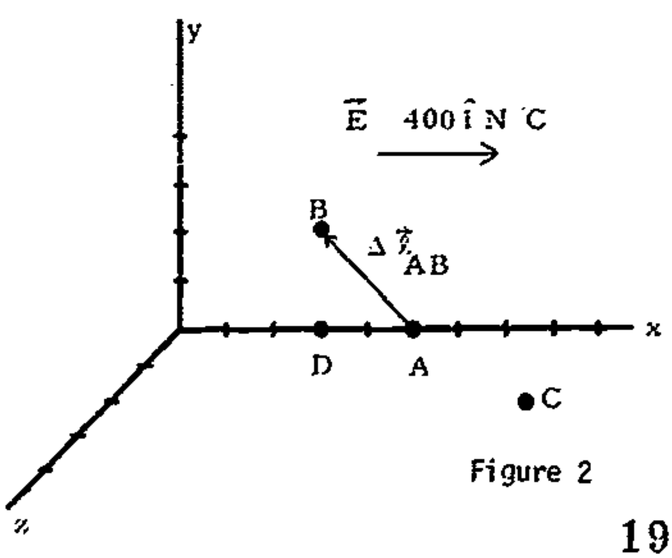


Figure 2

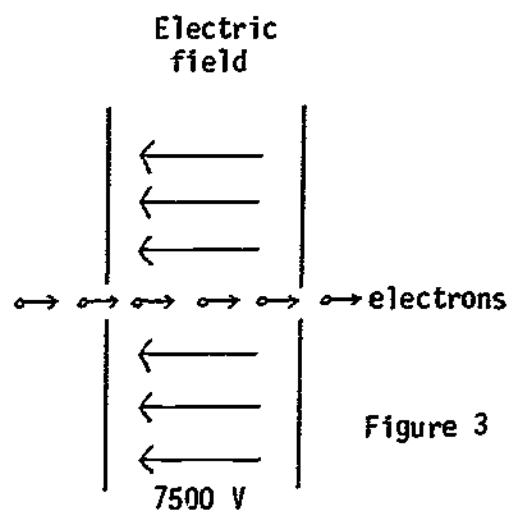


Figure 3

B(1). In Figure 3, electrons acquire an increased speed by moving in an electric field between two plates with slits through which the electrons can pass. The electric potential difference between the plates is 7500 V.

(a) By how much does the electric potential energy of the electrons change as they pass through the two-plate system? (Give answer in electron volts.)

(b) By how much does the kinetic energy of the electrons change as they pass through the two-plate system? (Give answer in electron volts.)

(c) With what speed do the electrons leave the space between the plates if they enter with speeds of 1.50×10^7 m/s?

Solution

(a) It decreases by 7500 eV. (b) It increases by 7500 eV.

(c) $7500 \text{ eV} = (7500)(1.60 \times 10^{-19}) = 1.20 \times 10^{-15} \text{ J} = \Delta U$. $U = K_2 - K_1$.

$K_2 = (1/2)mv_1^2 + \Delta U = 1.30 \times 10^{-15}$ J. $v_2 = 2K_2/m = 5.4 \times 10^7$ m/s.

C(2). Show that Eqs. (3) and (5) in General Comment 3 are equivalent.

Solution

(i) Assume Eq. (3) and prove Eq. (5): Draw two arbitrary paths from A to B and call them path 1 and path 2. Together they form a closed path. The line integral around this path is zero. Divide the integral into two parts, I_1 from A to B along one part of the loop and I_2 from A to B along the other, so that $I_1 + (-I_2) = 0$ by Eq. (3). This yields $I_1 = I_2$. Q.E.D.

(ii) Assume Eq. (5) and prove Eq. (3): Draw a closed path from A to A. Select any other point B on the path so that it is divided into paths 1 and 2. Calling the integrals from A to B along these two paths I_1 and I_2, we have from Eq. (5) that $I_1 = I_2$. But the integral around the closed path is $I_1 + (-I_2) = I_1 - I_2 = 0$. Q.E.D.

D(3). A point charge of magnitude +q is located at (a, 0, 0) and a point charge of magnitude -2q is located at (-2a, 0, 0) as in Figure 4. (a > 0, q > 0.)

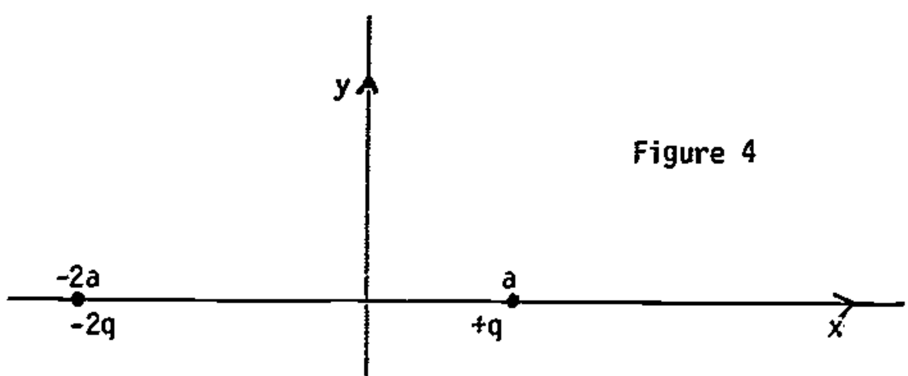


Figure 4

(a) Find a mathematical expression for the electric potential on the x axis, and sketch it on a graph. (Draw the sketch from about six to eight well-placed points.)

(b) Find a mathematical expression for the electric potential along the y axis, and sketch it on a graph. (Draw the sketch from about six to eight well-placed points.) How would you expect the potential to vary along the z axis? Explain.

(c) Find the locus of points where the electric potential caused by the two charges is equal to zero.

Solution

The electric potential caused by the two charges is

$$V(x, y, z) = kq(1/r_1 - 2/r_2),$$

where r_1 and r_2 are the distances from the locations of the charges to the point (x, y, z): $r_1 = [(x - a)^2 + y^2 + z^2]^{1/2}$, and $r_2 = [(x + 2a)^2 + y^2 + z^2)]$.

(a) Since y = z = 0 along the x axis, $r_1 = |x - a|$ and $r_2 = |x + 2a|$. The absolute values appear because square roots are always to be taken as positive - this is very important in avoiding algebraic mistakes. Hence

$$V(x, 0, 0) = kq(|x - a|^{-1} - 2|x + 2a|^{-1}).$$

Make a data table:

x =	-8a	-4a	-3a	-a	0	2a	4a	6a
(a/kq)V(x, 0, 0) =	-0.220	-0.80	-1.75	-1.50	0	0.50	0	-0.050

Note: We have chosen the combination (a/kq)V(x, 0, 0) because it is dimensionless. We shall plot this quantity in the graph. We also know, without calculating, that the electric potential approaches $+\infty$ as x approaches +a, and V approaches $-\infty$ as x approaches -2a, because the charges are located at these points. Now we can draw the graph as in Figure 5.

(b) Along the y axis $r_1 = (a^2 + y^2)^{1/2}$ and $r_2 = (4a^2 + y^2)^{1/2}$,

$aV(0, y, 0)/kq = a/(a^2 + y^2)^{1/2} - 2a/(4a^2 + y^2)^{1/2}$. Our data table is thus

y =	0	±a	±2a	±4a	±8a
aV(0, y, 0)/kq =	0	-0.190	-0.260	-0.200	-0.120

The graph is shown in Figure 6.

Comments: Since the charges are located on the x axis, the dependence of the potential on the y and z axes should be the same - they are located symmetrically in relation to the x axis. Note that the potential is positive along the x axis between 0 and 4a, but that it is negative elsewhere on the x axis and on the other

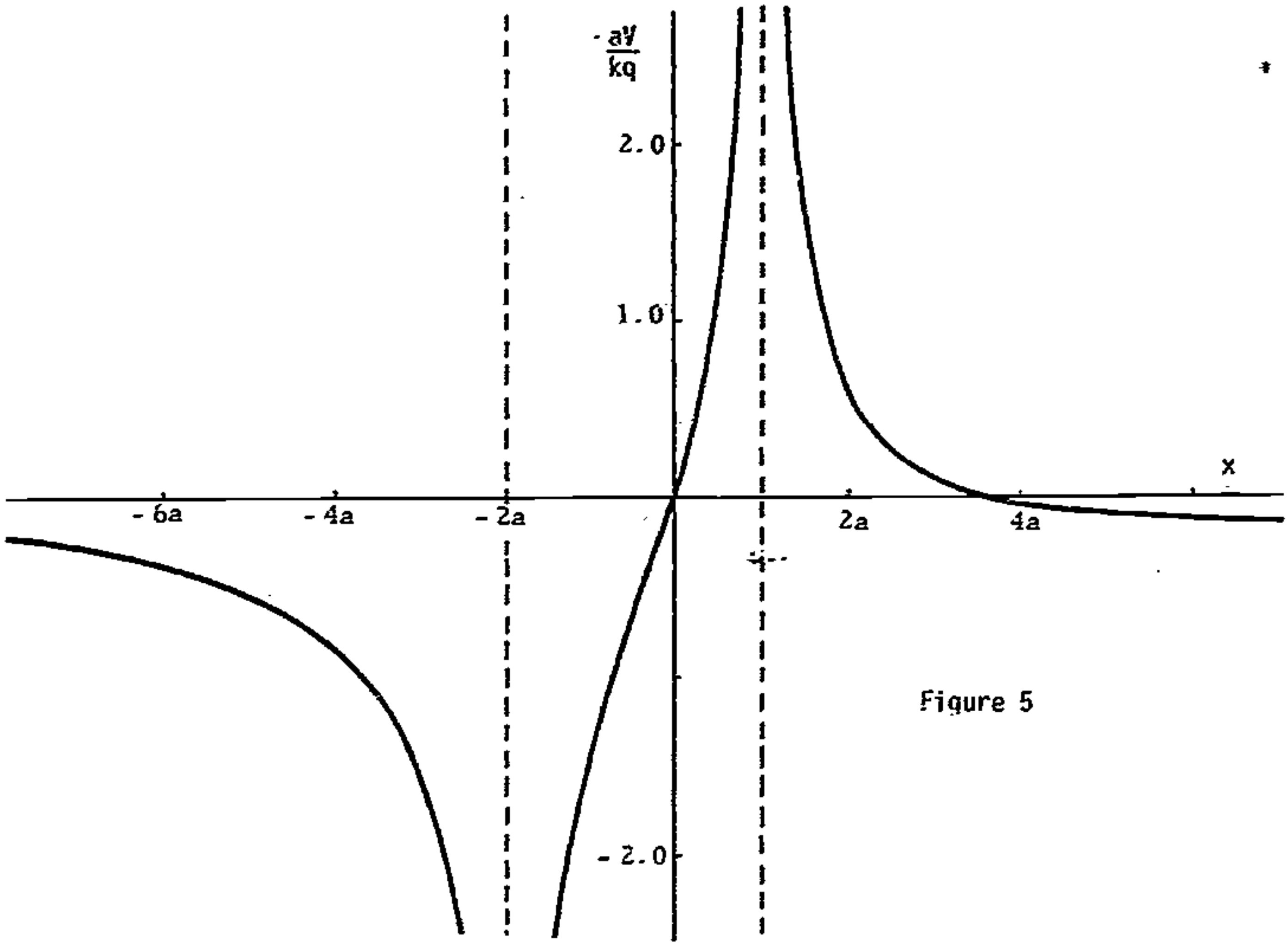


Figure 5

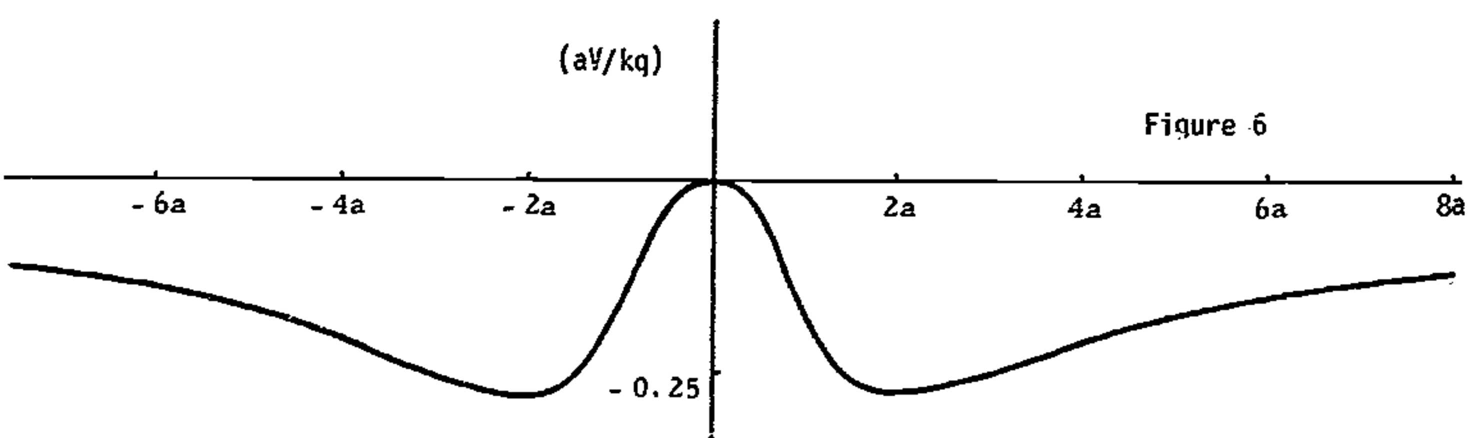


Figure 6

axes. This is to be expected, since the negative charge had the greater magnitude; hence the positive potential is confined to a region near the positive charge. At large distances from both charges, where the distance between them is negligible, the potential caused by the two-charge system resembles that caused by a charge -q at the origin. This charge establishes a negative potential.

(c) We saw from the graphs and qualitative reasoning that the potential would be positive near the positive charge and negative elsewhere. Therefore there should be a finite surface on which the potential is zero. Setting $V(x, y, z) = 0$, we find that

$$2r_1 = r_2 \quad \text{or} \quad 4(r_1)^2 = (r_2)^2.$$

In other words,

$$4(x - a)^2 + 4y^2 + 4z^2 = (x + 2a)^2 + y^2 + z^2.$$

By combining the x-dependent terms, we find

$$4x^2 - 8ax + 4a^2 - (x^2 + 4ax + 4a^2) = 3x^2 - 12ax = 3(x - 2a)^2 - 12a^2.$$

The other terms also combine and permit cancellation of a factor 3. The final equation is

$$(x - 2a)^2 + y^2 + z^2 = 4a^2,$$

which is a sphere of radius 2a about the point (2a, 0, 0). The sphere passes through the origin and the point (4a, 0, 0), where we had already found the potential to be zero. [Note: The electric potential of two point charges of opposite sign always is zero on a spherical surface of certain radius and center that surrounds the smaller of the two charges. You can prove this result without much difficulty by using charges q at the origin and -bq at the point (a, 0, 0), and proceeding as we did above. What do you expect for b = 1?]

E(3). A point charge of magnitude Q is located at the center of a hollow conducting spherical shell that is electrically neutral. Find the electric potential as a function of radius from the shell's center. The interior and exterior radii of the shell are R_i and R_e, respectively.

Draw a sketch of the dimensionless combination (R_eV/kQ).

Solution

Since the system of charges is spherically symmetric, the electric field outside the shell can be found from Gauss' law. It is directed radially and has the magnitude

$$E(r) = kQ/r^2, \quad r > R_e. \tag{10}$$

In the shell itself, the electric field is zero because it is a conductor,

$$E(r) = 0, \quad R_i < r < R_e. \tag{11}$$

Inside the hollow cavity of the shell, the field is again found from Gauss' law (or memory):

$$E(r) = kQ/r^2, \quad r < R_i. \tag{12}$$

To find the potential as a function of radius, relative to infinity as the reference point, we integrate the electric field:

$$V(r) = \int_{\infty}^{r} \vec{E} \cdot d\vec{\ell}. \qquad (13)$$

The important and new step now is to break up the integral into several parts because the electric field is given by differing mathematical expressions, depending on the radius. We also use the fact that E is radial, so that

$$\vec{E} \cdot d\vec{\ell} = E(r)\ dr.$$

First case, $r > R_e$ - one expression for $E(r)$:

$$V(r) = -\int_{\infty}^{r} E(r)\ dr = -\int_{\infty}^{r} \left(\frac{kQ}{r^2}\right) dr = \frac{kQ}{r}, \quad r > R_e. \qquad (14)$$

Second case, $R_i < r < R_e$ - break the integral into two parts, one for $r < R_i$ using Eq. (10) and a second for $R_i < r < R_e$ using Eq. (11):

$$V(r) = -\int_{\infty}^{R_e} E(r)\ dr - \int_{R_e}^{r} E(r)\ dr = -\int_{\infty}^{R_e} \left(\frac{kQ}{r^2}\right) dr + 0, \quad r < R_i,$$

$$V(r) = kQ/R_e, \qquad R_i < r < R_e. \quad (15)$$

Third case, $r < R_i$ - apply the result for $V(R_i)$ from Eq. (15) and integrate for the potential difference to $r < R_i$ using Eq. (12):

$$V(r) = V(R_i) + [V(r) - V(R_i)] = \frac{kQ}{R_e} - \int_{R_i}^{r} E(r)\ dr = \frac{kQ}{R_e} - \int_{R_i}^{r} \left(\frac{kQ}{r^2}\right) dr = \frac{kQ}{R_e} + \frac{kQ}{r} - \frac{kQ}{R_i}. \qquad (16)$$

We can now draw Figure 7, where we have used $R_i/R_e = 0.67$.

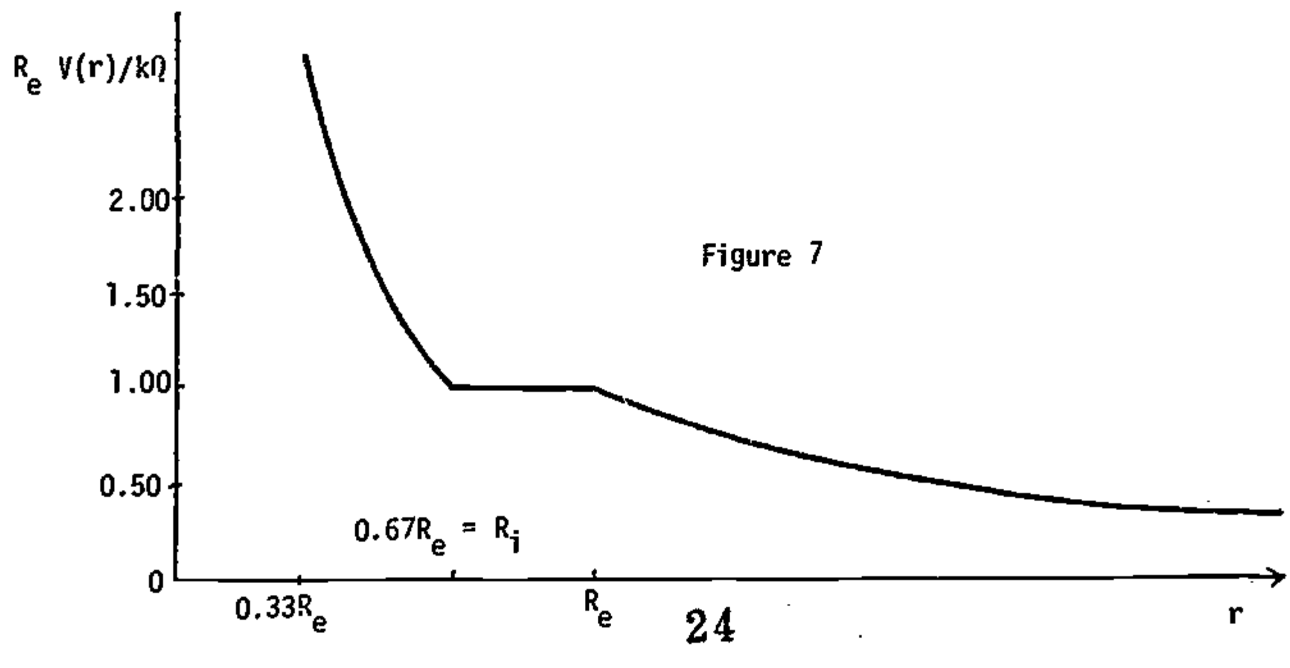


Figure 7

F(4). A certain spherically symmetric charge distribution generates the potential $V(r) = P/r^3$, where r is the radius from the center. Find the electric field.

Solution

Since V depends only on the radius, the electric field is radially directed and has the magnitude

$$E(r) = -dV(r)/dr = +3P/r^4.$$

If P is positive, E is directed radially outward. If P is negative, E is directed radially inward.

G(4). The symmetry of the problem of the two point charges in Problem D implies that on the x axis the electric field will be directed along the x axis. Find the electric field on the x axis, $E_x(x, 0, 0)$, using the result for $V(x, 0, 0)$ given in Problem D.

Solution

We must calculate $E_x(x, 0, 0) = -dV(x, 0, 0)/dx$ from $V(x, 0, 0) = kq(|x - a|^{-1} - 2|x + 2a|^{-1})$. To do this, the absolute-value signs must be eliminated. We therefore have three cases:

Case (i): $x > a$, $|x - a| = x - a$, $|x + 2a| = x + 2a$,

$V(x, 0, 0) = kq/(x - a) - 2kq/(x + 2a)$,

$E_x = -dV/dx = kq/(x - a)^2 - 2kq/(x + 2a)^2$.

Case (ii): $-2a < x < a$, $|x - a| = a - x$, $|x + 2a| = x + 2a$,

$V(x, 0, 0) = ?$ (student should complete - see Note below),

$E_x = ?$ (student should complete - see Note below).

Case (iii): $x < -2a$, $|x - a| = a - x$, $|x + 2a| = -x - 2a$,

$V(x, 0, 0) = ?$ (student should complete - see Note below),

$E_x = ?$ (student should complete - see Note below).

Note: Since the squares in the denominator of the electric-field terms are always positive, you can identify the sign of a term from the sign in front. The field caused by the charge at $x = a$ is positive in case (i) and negative in cases (ii) and (iii), always repelling another positive charge. The field caused by the charge at $x = -2a$ is negative in case (i) and case (ii) but positive in case (iii), thus always attracting a positive charge. The sign changes coming from the absolute values have these consequences.

H(5). Two equal positive charges are placed on the y axis, at equal distances above and below the x axis from the origin. Draw the equipotential lines and field lines caused by this two-charge system in the xy plane (i.e., trace the intersections of the equipotential surfaces with the xy plane).

Solution

For a problem like this, it is wise to exploit your knowledge of the potential of a single point charge. Thus

(a) Near each charge, where the potential increases without limit, the equipotential surfaces will be virtually unaffected by the presence of the other charge. The surfaces will approximate spheres, and their intersection with the xy plane will approximate circles centered around the charge.

(b) Very far from all the charges, the whole system will act like a single point charge whose magnitude is the algebraic sum of all the charges in the system. Equipotential surfaces will again approximate spheres, centered around the "center" of the charges.

(c) Field lines are perpendicular to the equipotential surfaces and can terminate only at charged bodies. After constructing the near- and far-field lines with the help of (a) and (b), you must connect the two parts of the diagram.

(d) At intermediate points, you may be able to use symmetry of the charges and clues from the need to connect field lines at small and large distances, as described in part (c). You may also be able to locate special points where the electric field is zero; at such points the field lines have no well-defined direction, and it is possible for two equipotential surfaces to intersect. (Since the surfaces are perpendicular to the field lines, and a field line can have only one perpendicular surface at a point, two equipotentials cannot ordinarily intersect.)

These ideas have been used to construct the diagram in Figure 8, appropriate to the two-charge system described in the problem. Note the small circular equipotentials near each charge, the large oval shapes that will become more and more circular at larger distances, the total number of field lines for the system (twice that of each charge separately), and the field-free symmetry point half-way between the charges where equipotentials intersect. One field line (heavy dotted) connects the two charges and another runs on the symmetry line between them; they cross at the field-free point.

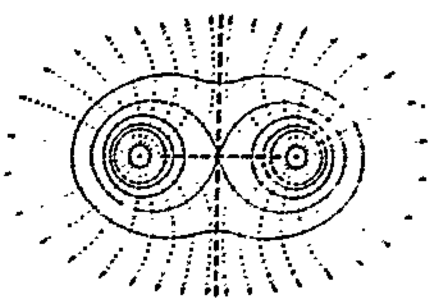


Figure 8

I(5). A point charge of magnitude +q is located at (a, 0, 0) and a point charge of magnitude -2q is located at (-2a, 0, 0), with both a and q positive. (See Problem D.) Draw approximate equipotential lines and electric-field lines in the xy plane for this system of charges.

Solution

We shall begin with an overall analysis, following the procedure of Problem H.

(a) Near each charge, there will be equipotential circles, representing positive potentials near $x = a$ and negative potentials near $x = -2a$.

(b) Far from the origin compared to a (perhaps with radius about 10a) there will be equipotential circles representing the negative potentials of a charge $-q$ located approximately at the origin.

(c) Field lines radiate out from the charge at $x = a$, and two times as many field lines radiate inward to the charge at $x = -2a$. All of the field lines going out from $x = a$ go in to the right hemisphere of the charge at $x = -2a$, whereas the lines coming in to the left hemisphere at $x = -2a$ come from infinity. The latter are uniformly distributed in angle at the large equipotential circles mentioned in (b).

(d) Since the charges are unequal, there is no simple right-left symmetry, but there is up-down symmetry. Since the charges have opposite sign, the field between them will be very strong. However, to the right of the positive charge, which is weaker than the negative charge, there must be a reversal of the field, because near $x = a$ there is repulsion (field to the right), whereas further away, where the negative charge dominates, there is attraction (field to the left). To find the zero-field position, use the field as calculated in Problem H, part (a): we need the root of $2(x - a)^2 = (x + 2a)^2$. This equation is solved most easily

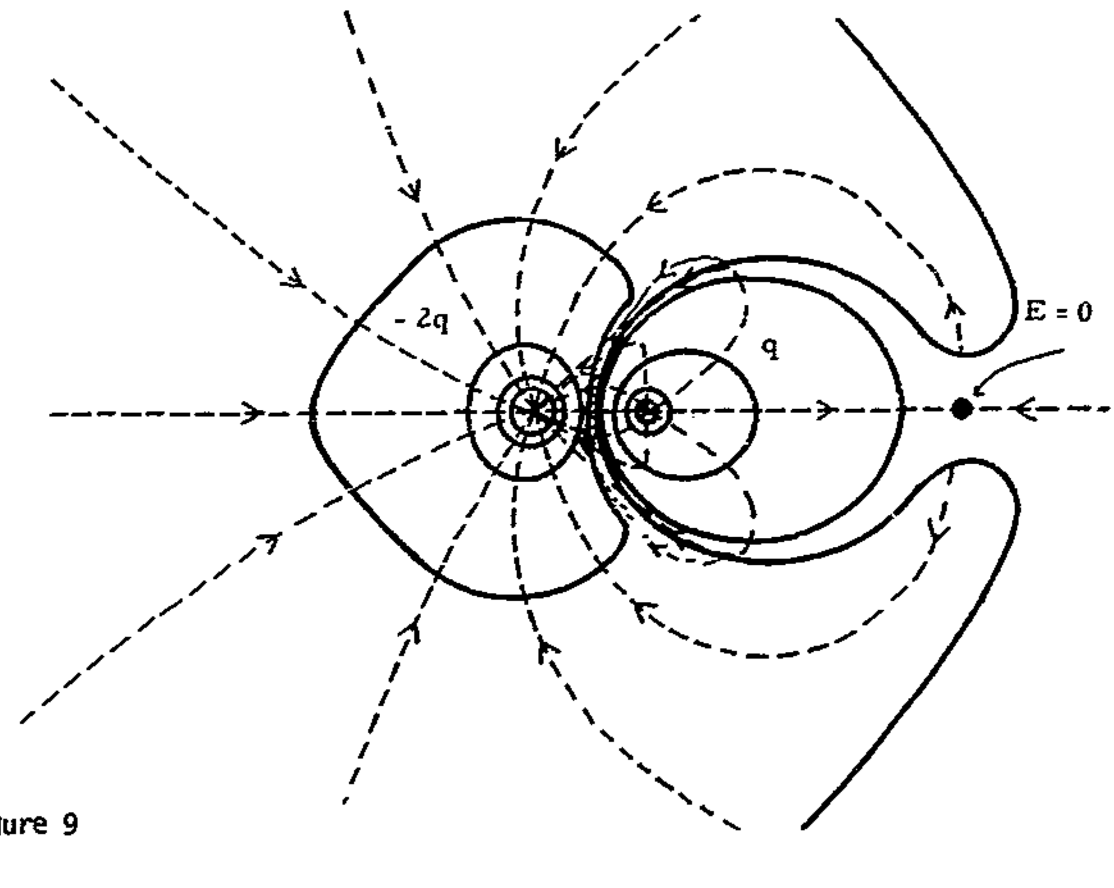


Figure 9

by taking square roots of both sides and then collecting terms of the linear equation, with the result that

$$x = (2 + \sqrt{2})a/(\sqrt{2} - 1) = 8.25a.$$

At this point equipotential lines may cross to make a transition from the behavior near the charges to the far behavior.

We can now draw Figure 9 as the solution to this problem.

Problems

J(1). An upward-directed electric field of 2.60×10^4 N/C exists in a certain region.

(a) What is the potential differences between a certain point 0 that we shall call the origin and the point (i) 0.80 m above? (ii) 1.20 m to the left? (iii) 2.50 m diagonally downward to the right at 45°?

(b) How much work does an external agent do when it moves a charge of 5.0×10^{-7} C very slowly from the origin to the point (i) in (a-i)? (ii) in (a-ii)? (iii) in (a-iii)?

Hint: Draw a diagram and remember the definitions.

K(1). An electron is placed at a distance of 5.0×10^{-10} m from a proton. It is then allowed to move freely until it is only 1.00×10^{-10} m from the proton.

(a) What is the potential difference between the two points in the proton's electric field?

(b) How much kinetic energy does the electron gain during its motion? Express the result in electron volts.

(c) With what speed does the electron arrive at the second point?

Hint: Draw a diagram and remember the definitions.

L(3). Two point charges are placed on the y axis, one of magnitude 3Q at the origin, and one of magnitude -Q at the point (0, a, 0).

(a) Find a mathematical expression for the electric potential on the y axis. Draw an approximate graph of your result.

(b) Find the point(s) on the y axis where the potential is zero.

(c) Find an approximate expression for the potential at large distances from the origin along the x axis and explain its physical significance.

(d) (Optional) Use the answer to part (b) to find the center and radius of the sphere on which the potential is zero; verify your result by checking the value of the potential at a point off the y axis.

Hint: Use Coulomb's law and the superposition principle. To check the algebraic sign and possible need for taking absolute values, think of the attraction or repulsion experienced by a test charge at various locations.

M(3). Figure 10 shows two large, thick, parallel metal plates. The lower and upper plates carry charge densities of -4.0×10^{-9} C/m and $+8.0 \times 10^{-9}$ C/m , respectively. Each plate is 0.100 m thick, and the space between them is 0.300 m high. Take the bottom of the lower plate (surface A) as the reference point of zero potential. Find the potential V(z) as a function of z between z = -0.50 m and z = 1.00 m. You may assume that the surface charges on the exterior surfaces A and D are equal, and that the surface charges on the interior surfaces B and C are opposite. (These conclusions can be derived from Gauss' law and symmetry, but are not a required part of this problem. Can you prove them?)

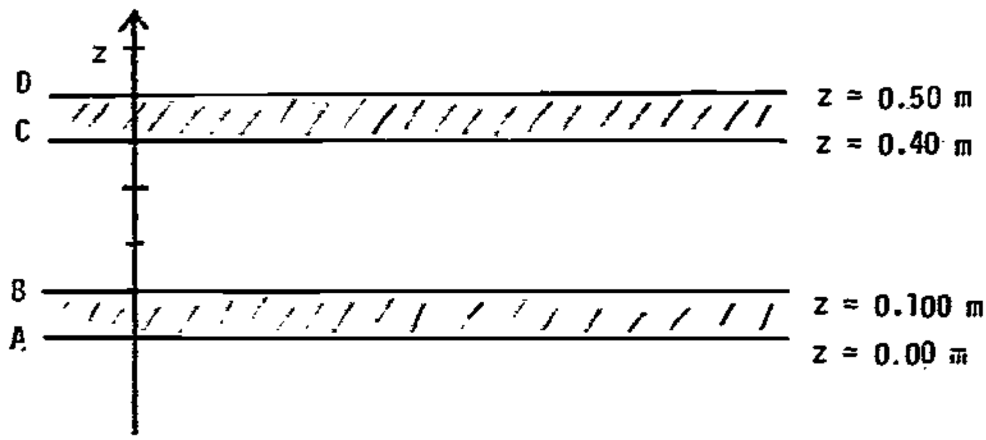


Figure 10

N(1, 3). A home electrostatic air cleaner has a wire of radius 0.50×10^{-4} m at the center of a metal cylinder of radius 2.00×10^{-2} m. The wire is at a potential of +7000 V relative to the cylinder.

(a) Find the electric charge per unit length needed on the wire to establish this potential difference.

(b) Show that the electric field at the surface of the wire is greater than 3.00×10^{6} V/m, a field at which air becomes ionized. (By the ionization of air, charged particles are produced that can attach themselves to the dust, thereby making the dust subject to removal by the action of the electric forces.)

O(4). A point charge of magnitude Q is placed at the point (0, 0, a). Its potential on the z axis is given by $V(0, 0, z) = kQ/|z - a|$. Find the z component of the electric field associated with this potential by differentiating, and justify your procedure. (Hint: Treat the absolute value carefully!)

P(4). A charge distribution at the coordinate origin gives rise to the potential

$V(r) = \vec{p} \cdot \vec{r}/r^3$, where r is the distance from the origin,

$r = \sqrt{x^2 + y^2 + z^2}$,

and $\vec{p}$ is a given constant vector called the electric dipole moment.

(a) Find the z component of the electric field for points on the z axis.

(b) Find the y component of the electric field for points on the y axis.

Hint: First state the potential on the axis.

Q(5). Draw approximate equipotentials and field lines in the xy plane for the two-charge system in Problem L.

Hint: Look at Solutions to Problems H and I.

R(5). In a region of electric field with cylindrical (not circular) symmetry, a cross section of equipotential surfaces is shown in Figure 11. Five points have been marked by letters.

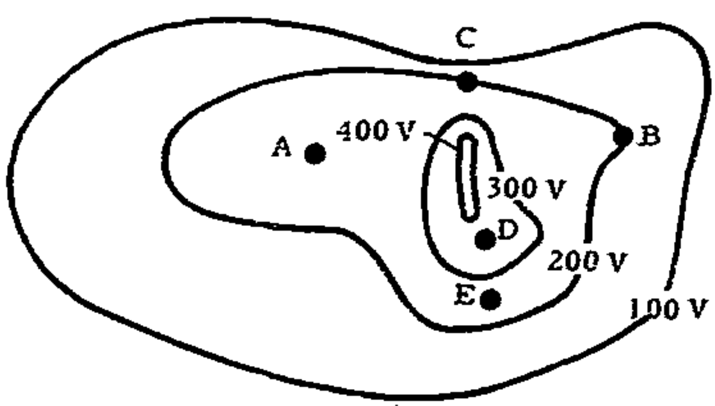
(a) At which point is the field strongest? Explain.

(b) At which point is the field weakest? Explain.

(c) Draw a few field lines in the figure.

Hint: Remember the relation between spacing of equipotentials and electric field. You can evaluate the rate of change of the potential approximately from the figure.

Figure 11



Solutions

J(1). (a-i) 2.10×10^4 V. (a-i) 0 V. (a-iii) 4.6×10^4 V.
(b-i) 1.04×10^{-2} J. (b-ii) 0 J. (b-iii) 2.30×10^{-2} J.

K(1). (a) 11.5 V. (b) 11.5 eV. (c) 2.00×10^6 m/s.

L(3). (b) 3a/4, 3a/2. (c) $2kQ/|x|$. (d) 9a/8, 3a/8.

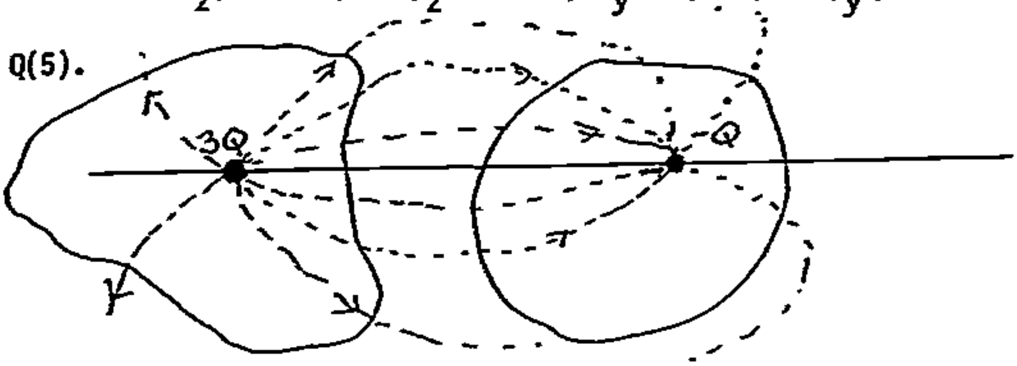
M(3). Surfaces A and D have surface charge each of $Q = 2.00 \times 10^{-9}$ C/m², B has -6.0×10^{-9} C/m , and C has $+6.0 \times 10^{-9}$ C/m . Now find the fields below the bottom plate, between the two plates, and above the top plate. Finally, integrate to obtain the potential. Remember the reference point. Check-point answers: V(-0.50 = -115 V. V(0.250) = 100 V. V(0.50) = 204 V. V(1.00) = 91 V.

N(1, 3). (a) 7.4×10^{-8} C/m. (b) 13.0×10^{6} V/m (much greater than required).

O(4). $E_z(0, 0, 0) = -kQ/a^2$.

P(4). (a) $E_z(0, 0, z) = 2p_z/z^3$. (b) $E_y(0, y, 0) = 2p_y/y^3$.

Q(5).



R(5). (a) C. (b) A.

PRACTICE TEST

1. A long, hollow, cylindrical conductor of radius 20.0 cm has a charge of -8.0×10^{-8} coulombs per meter of length. On one side it has a tiny hole through which electrons can emerge from the interior, but the hole has a negligible effect on the electric field due to the cylinder. Use the hole as the reference point for the electric potential.
(a) Find the electric potential $V(r)$ both outside and inside the cylinder (r is the radius from the cylinder axis).
(b) How much kinetic energy is acquired from the field by an electron that emerges from the hole and hits a screen 1.60 m from the axis? Express the answer in electron volts.
(c) With what speed does the electron hit the screen if it starts with negligible speed in the hole?
(d) Suppose you had a frictionless guiding mechanism for the electron that made it spiral around the cylinder rather than traveling straight from the hole to the screen. How would that affect the answers to parts (b) and (c)? Explain briefly.

2. A spherically symmetric charge distribution gives rise to the potential $V(r) = Kr/(r^2 + a^2)$, where r is the radius from the center of the charge, and K and a are constants having the dimensions of length and potential, respectively. Find the (vector) electric field.

3. Two point charges of magnitudes $2Q$ and $-3Q$ are placed with the separation L. Make an approximate drawing of several equipotentials and field lines in a plane including the two charges.

Practice Test Answers

1. (a) $V(r) = 1.44 \times 10^3 \ln(r/20)$ (V in volts, r in centimeters).
(b) $K = 3.00 \times 10^3$ eV.
(c) $v = 3.2 \times 10^7$ m/s.
(d) No difference, field conservation.

2. $E(r) = K[(r^2 - a^2)/(a^2 + r^2)^2]$.

3. See Figure 12.

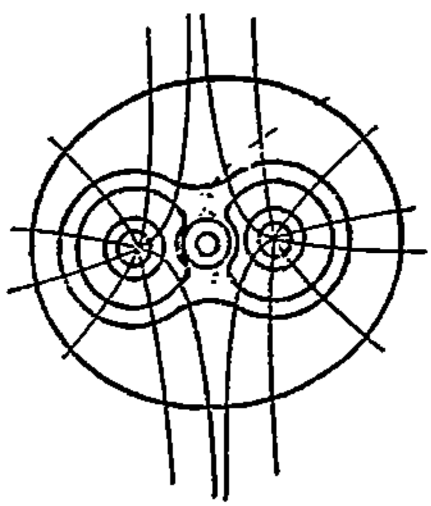


Figure 12

ELECTRIC POTENTIAL Date ________________

Mastery Test Form A

pass recycle

1 2 3 4 5

Name ____________________________ Tutor ____________________________

1. The electric field along the x axis in Figure 1 is given by

   $E_x = Fx + G$, where $F = 800\ V/m^2$, and $G = 80\ V/m$.

   The other components of $\vec{E}$ are not given. Two points, A and B, lie on the x axis.
   (a) What is the potential difference, $V_B - V_A$?

   (b) How much kinetic energy does an electron gain when released at one of these points and permitted to move along a straight, frictionless track toward the other? State at which point the electron must be released, and give the energy in both electron volts and joules.
   (c) How much work must be done on the electron by an external agent that moves it back to its starting point along the dashed path? Make any additional assumptions you need to answer this question; you may be asked to justify your result.

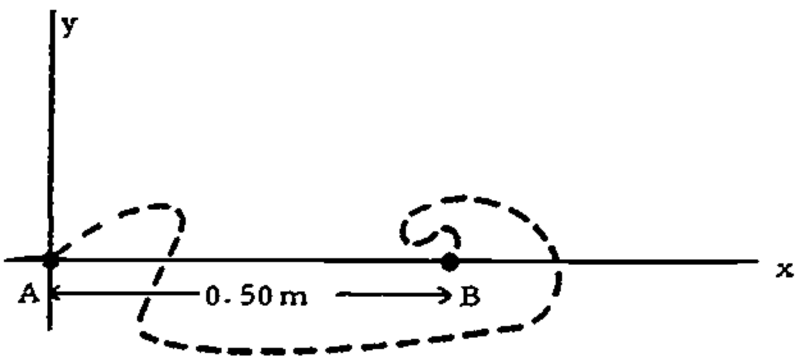


Figure 1

2. Two charges, one of magnitude Q and the other of magnitude 2Q are placed at the distance L from each other.
   (a) Find the potential along the line passing through the two charges as a function of distance from the smaller charge.
   (b) Use differentiation to find the component of the electric field along the line passing through the charges, and find the point(s) where the field is zero.
   (c) Draw a diagram showing the approximate equipotentials and field lines in a plane including the two charges; indicate any zero-field points that occur.

ELECTRIC POTENTIAL Date ________________

Mastery Test Form B

pass recycle

1 2 3 4 5

Name ____________________________ Tutor ____________________________

1. Three charges, one of magnitude Q at the origin, and two of magnitudes -4Q at (0, L, 0) and (0, -L, 0), establish an electric field.
(a) Find the potential along the x axis; graph the potential (very roughly).
(b) How much work is done by the field on a charge q that is permitted to move from the point ([3/4]L, 0, 0) to the point ([-4/3]L, 0, 0)? Describe the path that you choose for the motion and how this choice affects the work calculated.
(c) Find the (vector) electric field at the points ($L/\sqrt{3}$, 0, 0) and ($-\sqrt{3}L$, 0, 0).
(d) Draw a diagram showing the approximate equipotentials and field lines in the xy plane; indicate any zero-field points that occur.

ELECTRIC POTENTIAL Date ______________

Mastery Test Form C

pass recycle

1 2 3 4 5

Name ______________________ Tutor ______________________

1. A cylindrical metal shell with inner and outer radii of 0.200 m and 0.300 m, respectively, carries a net charge of 4.0×10^{-8} C/m. Along its center there is a wire with a uniform charge of -12.0×10^{-8} C/m. (This system is similar to an electrostatic air cleaner.)
(a) Find the electric potential caused by this system of electric charges, using the outer surface of the shell as reference point for the potential. Graph the potential (very roughly).
(b) An electron is formed by ionizing a gas molecule 0.00100 m from the wire. With what kinetic energy (in electron volts) and with what speed will the electron hit the metal shell, assuming it does not suffer any collisions enroute?
(c) Suppose the electron's path is a spiral rather than a straight line, but only the electric field does work on the electron. (This can be accomplished by providing a magnetic field - do not be concerned about the details.) With what energy will the electron then hit the metal shell? You may be asked to justify your conclusion.

2. Find the (vector) electric field associated with the potential $V(r) = p/r^2$, where p is a constant, and r is the distance from the coordinate origin.

3. Make a drawing of the approximate equipotential lines and field lines in the plane of the paper for the region between the two metallic shapes indicated in Figure 1. (The diagram shows a cross section of the system. $V_1 > V_2$.)

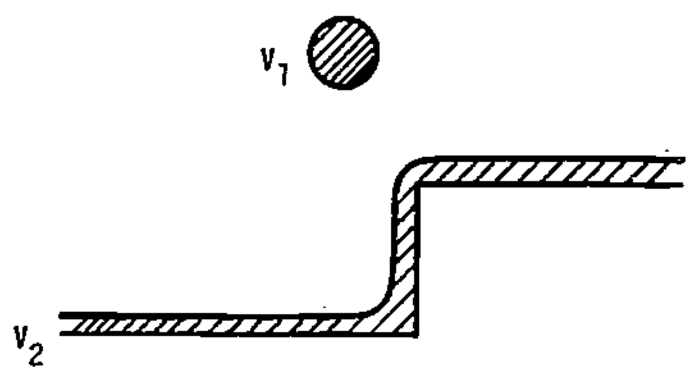


Figure 1

MASTERY TEST GRADING KEY - Form A

1. What To Look For: (a) Integral, sign, limits.
(b) Direction of motion, use of V from (a).
(c) Apply conservative-field idea. Work-energy theorem allows use of the result from (b).

Solution: (a)

$$V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{\ell} = -\int_0^{0.50} \vec{E}_x \, dx = -\int_0^{0.50} (800x + 80)\, dx = -400\, x^2 - 80x \Big|_0^{0.50}$$

$$= -140 \text{ V}.$$

(b) The electron has negative charge; since the field is positive to the right, the electron must be released at B in order that it will move to the left, to the region of higher V.

$K = 140 \text{ eV} = 2.24 \times 10^{-17}$ J.

(c) Since the field is conservative, the path along which the electron is moved is immaterial. No additional assumptions have to be made about the field off the axis, or other components. W_{AB} = 140 eV, just equal to the K gained and potential energy lost in part (b).

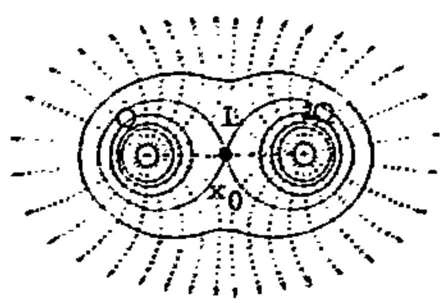


Figure 15

2. What To Look For: (a) Signs, locations, absolute values.
(b) Formula, sign, absolute values.
(c) Circles around charge locations, large "circles," assymetry in field near charges, symmetry far from charges, x_0 crossing point.

Solution: (a) Place the smaller charge at the origin and the larger one at x = L (see Figure 15). Then by using Coulomb's law for the potential of each charge, we find

$V(x) = kQ/|x| + 2kQ/|x - L|$.

(b) $E_x(x) = -dV/dx$. Note $|x| = x$, $x > 0$; $|x| = -x$, $x < 0$; $|x - L| = x - L$, $x > L$; $|x - L| = L - x$, $x < L$. Therefore

$$E_x(x) = kQ/x^2 + 2kQ/(x - L)^2, \quad x > L,$$

$$= kQ/x^2 - 2kQ/(x - L)^2, \quad 0 < x < L,$$

$$= -kQ/x^2 - 2kQ/(x - L)^2, \quad x < 0.$$

$$E_x(x_0) = 0, \quad 0 < x < L, \quad 2x_0^2 = (x_0 - L)^2 = (L - x_0)^2,$$

$$\sqrt{2}x_0 = L - x_0, \quad x_0 = L/(\sqrt{2} + 1) = 0.41\ L.$$

(c) See Figure 15.

MASTERY TEST GRADING KEY - Form B

1. What To Look For: (a) Note signs of terms, absolute value $|x|$, distance $(L^2 + x^2)^{1/2}$, symmetry of graph: (+) near x = 0, (-) near (x).
(b) Sign of potential difference, sign of * term with x negative, arithmetic, ask about path dependence.
(c) Symmetry, sign in formula, correct treatment of $1/|x|$ to give repulsion for (+) and (-)x values, correct substitutions to get the answers. If student uses E from Coulomb's law, ask orally about dV/dx.
(d) Small circles near charges, large "circle" far from charges, E = 0, two points equal to $\pm L/\sqrt{3}$, symmetry, no field lines from +Q.

Solution: (a) See Figure 16.

$$V(x) = kQ/|x| - 8kQ/(L^2 + x^2)^{1/2}, \quad k = 1/4\pi\epsilon_0.$$

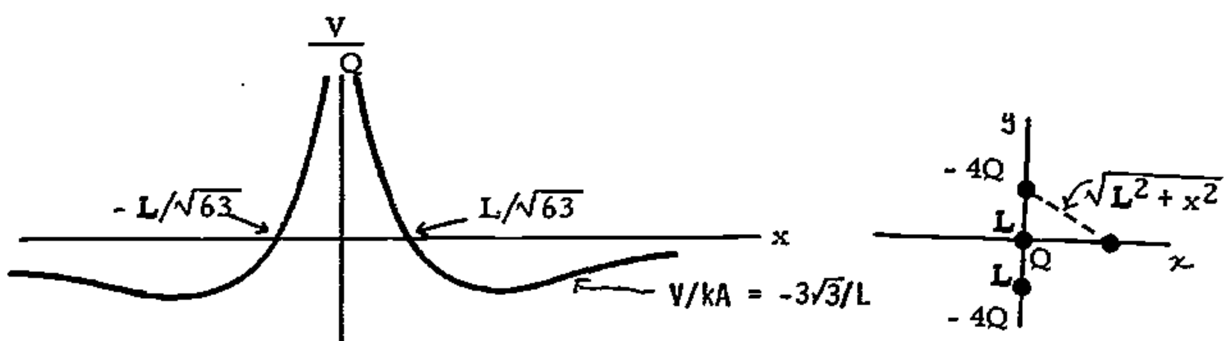


Figure 16

(b) $W = q[V(\frac{3}{4}L, 0, 0) - V(-\frac{4}{3}L, 0, 0)]$

$$= kqQ\left[\frac{4}{3L} - \frac{8}{[(9/16)L^2 + L^2]^{1/2}} - \left(\frac{3}{4L}\right)^* + \frac{8}{[(16/9)L^2 + L^2]^{1/2}}\right]$$

$$= \left(\frac{kqQ}{L}\right)\left[\frac{4}{3} - (8)\left(\frac{4}{5}\right) - \frac{3}{4} + (8)\left(\frac{3}{5}\right)\right] = \left(\frac{kqQ}{L}\right)\left[-\frac{61}{60}\right] = -1.020\left(\frac{kqQ}{L}\right),$$

independent of path.

(c) Because of symmetry, $\vec{E}(x, 0, 0) = E(x, 0, 0)\hat{i}$:

$$E(x, 0, 0) = -\frac{dV(x)}{dx} = \frac{kQ}{x^2} - \frac{8xkQ}{(L^2 + x^2)^{3/2}}, \quad x > 0,$$

$$= -\frac{kQ}{x^2} - \frac{8xkQ}{(L^2 + x^2)^{3/2}}, \quad x < 0.$$

Note: the +Q charge repels (+) test charges.

$$E(L/\sqrt{3}, 0, 0) = kQ\left[\frac{3}{L^2} - \frac{8(L/\sqrt{3})}{(L^2 + L^2/3)^{3/2}}\right] \qquad (x > 0)$$

$$= \frac{kQ}{L^2}\left[3 - \left(\frac{8}{\sqrt{3}}\right)\left(\frac{3}{4}\right)\left(\frac{3}{4}\right)^{1/2}\right] = 0,$$

$$E(-\sqrt{3}L, 0, 0) = \frac{kQ}{L^2}\left[\frac{1}{3} + \frac{8(3)^{1/2}}{4(4)^{1/2}}\right] = \frac{kQ}{L^2}(1.40), \qquad \vec{E}(-\sqrt{3}L, 0, 0) = (1.40\hat{i})Qk/L^2.$$

(d) See Figure 17. From part (c), $\vec{E} = 0$ and $x = \pm L/\sqrt{3}$.

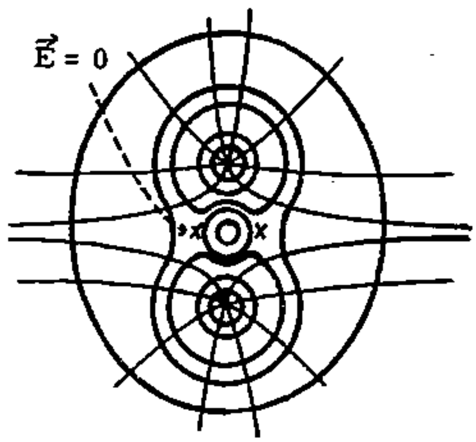


Figure 17

MASTERY TEST GRADING KEY - Form C

1. What To Look For: (a) Gauss' law to find $\vec{E}$, λ_1 is charge in hollow space. $\lambda_1 + \lambda_2$ = charge for outside. Sign in integral, break up integral, use properties of metal. Substitute in graph: V = 0 inside metal, + inside, + outside, because (-) charge on shell affects (+) test charges.

   (b) Note sign, (-) charge of electron e. Use data from (a).

   (c) Conservative force.

   Solution: (a) See Figures 18 and 19. $\vec{E}(r)$ is radial:

   $E = 2\lambda_1 k/r, \quad r < R_i,$

   $E = 0, \quad R_i < r < R_e,$

   $E = 2(\lambda_1 + \lambda_2)k/r, \quad R_e < r,$

   $V = -\int \vec{E} \cdot d\vec{\ell} = -\int_{R_e}^{r} E(r)\, dr,$

   $V(r) = 0, \quad R_i < r < R_e,$

   $$V(r) = -\int_{R_i}^{r} E(r)\, dr = -\int_{R_i}^{r} \frac{2\lambda_1 k}{r}\, dr = -2\lambda_1 k \ln\left(\frac{r}{R_i}\right) = 2\lambda_1 k \ln\left(\frac{R_i}{r}\right), \quad r < R_i,$$

   $$V(r) = -\int_{R_e}^{r} E(r)\, dr = -\int_{R_e}^{r} \frac{2(\lambda_1 + \lambda_2)k}{r}\, dr = -2(\lambda_1 + \lambda_2)k \ln\left(\frac{r}{R_e}\right),$$

   $2\lambda_1 k = 2(4.0)(10^{-8})(9.0)(10^{9}) = 720$ V,

   $2(\lambda_1 + \lambda_2)k = 2(4.0 - 12.0)(10^{-8})(9.0)(10^{9}) = -1440$ V.

   $V(r) = 720 \ln(R_i/r), \quad r < R_i,$

   $\quad = 0, \quad R_i < r < R_e,$

   $\quad = 1440 \ln(r/R_e), \quad R_e < r.$

   (b) $K_e = q[V(0.200) - V(0.00100)] = e[720] \ln(0.200/0.00100)$

   $\quad = 3800 \text{ eV} = 6.1 \times 10^{-16}$ J.

   $v = \sqrt{2K/m} = \sqrt{[(2)(6.1) \times 10^{-16}]/(0.91 \times 10^{-30})} = 3.7 \times 10^{7}$ m/s.

(c) Same K, v, independent of path.

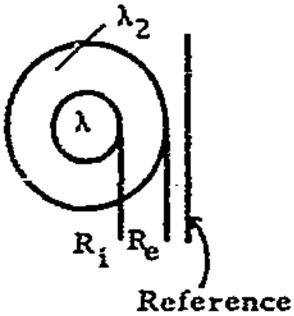


Figure 18

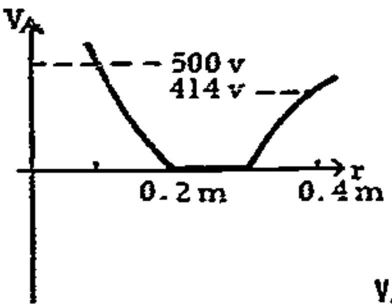


Figure 19

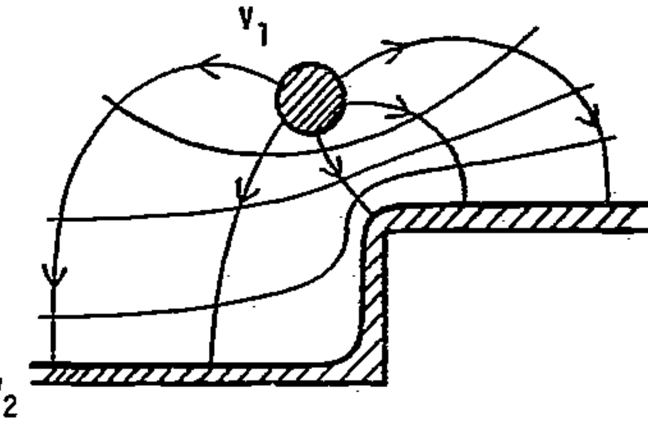


Figure 20

2. What To Look For: Note sign.

   Solution: Symmetry: $\vec{E}$ is radial,

   $E = -dV/dr = 2p/r^3$, outward.

3. What To Look For: Field lines perpendicular to metal. Equipotentials parallel to metal.

   Solution: See Figure 20.

OHM'S LAW

INTRODUCTION

Recently you have heard many ways of reducing energy consumption in the home. One of the suggested ways is to use 60-W light bulbs rather than 100-W bulbs; another is to cut back on the use of electrical appliances. You readily identify these suggestions with decreasing the amount of "electricity" being transported through the wires coming into your home.

You were warned quite early in life not to stick a metal knife into the toaster to force out burning bread; you wither unplug the toaster or use a utensil with a wooden handle. Why? Because you were warned of the consequences of coming into electrical contact with the heating element - consequences most of us have experienced at one time.

Several of the topics covered in this module, such as electrical current and the energy associated with it, are familiar to us all. When you have finished studying this module, you will be able to relate the material to many every-day experiences.

PREREQUISITES

Before you begin this module, you should be able to:	Location of Prerequisite Content
*Understand the relationship between electrical charge and matter (needed for Objectives 1 and 2 of this module)	Coulomb's Law and the Electric Field Module
*Apply the concept of electric field to charges (needed for Objectives 1 and 2 of this module)	Coulomb's Law and the Electric Field Module
*Relate electric fields and potential to work and energy (needed for Objectives 3 and 4 of this module)	Electric Potential Module
*Apply the definition of power (needed for Objective 4 of this module)	Work and Energy Module

LEARNING OBJECTIVES

After you have mastered the content of this module, you will be able to:

1. Current - Apply the definition of current or current density to problems in conductors in which these are related to electric charge.

2. Drift speed - Apply the microscopic model for conduction in a metal to problems where you are given information concerning several of the quantities (drift speed, current, current density, charge, charge density, cross-sectional area) and are asked to solve for another.

3. Ohm's law - Solve problems using the relationships among resistance, potential difference, electric current, current density, electric field, resistivity, and the physical dimensions of a conductor.

4. Electric power - Apply the relation for instantaneous power ($P = IV$) to problems dealing with resistive elements obeying Ohm's law.

TEXT: Frederick J. Bueche, Introduction to Physics for Scientists and Engineers (McGraw-Hill, New York, 1975), second edition

SUGGESTED STUDY PROCEDURE

Read Chapter 21, Sections 21.2, 21.3, 21.4, and 21.9. The definition of current is given in Eq. (21.1) and generalized in the preceding sentence. Read General Comment 1 regarding the vector notation given to the current in this equation. Current density is defined in Eq. (21.3). An explanation of drift velocity is given in Section 21.2, p. 392. For a more expanded description of the microscopic model for drift speed, you might like to refer to Elementary Classical Physics or Fundamentals of Physics.* Equations (21.2) and (21.3) give the relationship of current or current density to the properties of the charge carriers as called for in Objective 2. Read General Comment 2. The definition of resistance is given in Eq. (21.7). Read General Comment 3. Ohm's law is sometimes stated in terms of microscopic quantities as in Eq. (21.6), where the resistivity ρ is assumed constant. The relation for the power input or output of an element in an electrical circuit is given in Eq. (21.14). In Illustration 21.4, Ohm's law is substituted into this power relation to give $P = I^2R$ or V^2/R. These two equations are well known as Joule's law and hold only for elements with constant resistance. After completing these readings, study Problems A through H, and work Problems I through L and Problems 1, 3, 6, 8, and 15 in Chapter 21. Take the Practice Test and if you are still having any difficulties, work the Additional Problems before attempting a Mastery Test.

BUECHE

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text	Study Guide	Text (Chap. 21)	(Chap. 21)
1	General Comment 1, Sec. 21.2	A, B	Illus.[a] 21.2	I	1, 3	2, 4
2	General Comment 2, Sec. 21.2	C, D	Illus. 21.2	J	6	
3	General Comment 3, Sec. 21.3	E, F	Illus. 21.3	K	8	
4	Sec. 21.9	G, H	Illus. 21.4, 21.5	L	15	14, 17

[a]Illus. = Illustration(s).

*Richard T. Weidner and Robert L. Sells, Elementary Classical Physics (Allyn and Bacon, Boston, 1973), second edition, Vol. 2, p. 544.

David Halliday and Robert Resnick, Fundamentals of Physics (Wiley, New York, 1970; revised printing, 1974), pp. 508, 514.

TEXT: David Halliday and Robert Resnick, Fundamentals of Physics (Wiley, New York, 1970; revised printing, 1974)

SUGGESTED STUDY PROCEDURE

Read all of Chapter 27. The general definition of current is given in Eq. (27-1). Read General Comment 1 and Section 27-1. Current density is introduced in non-vector form in Eq. (27-3) for the case where the current is distributed uniformly across the conductor. Read General Comment 2 and Example 2. The concept of drift speed is introduced in Section 27-1 (p. 508), and the relation of current density or current to the properties of the charge carriers is developed at the end of this section (p. 509). A very nice discussion of the microscopic model for metals appears in Section 27-4. Read General Comment 3. The definitions for resistance and resistivity are given in Eqs. (27-5) and (27-6), respectively. Compare these equations carefully to relate the macroscopic variables to their microscopic counterparts. For a homogeneous conductor of length ℓ and cross-sectional area A, the relation for resistance in terms of resistivity and the physical dimensions of the conductor is given by Eq. (27-8).

A printing error may be noticed in Table 27-1. The missing labels are Column 1: Metal; Column 2: Resistivity at 20°C (ohm-m); Column 3: Temperature coefficient of resistivity,* (a per C°); Column 4: Density (gm/cm^3); and Column 5: Melting point (°C). Compare the voltage-current plots in Figures 27-4 and 27-6 - these distinguish between ohmic and nonohmic devices. The general relation for the power, or rate of transfer of electrical energy to a device through which charges flow, is given by Eq. (27-13). If the device obeys Ohm's law, Eqs. (27-14) and (27-15) can be useful. After completing the readings, study Problems A through H, and work Problems I through L in the study guide and Problems 1, 3, 6, 7, 12, 16, 23, 25, and Questions 12 and 14 in Chapter 27. Take the Practice Test, and if you are still having any difficulties, work the Additional Problems before attempting a Mastery Test.

HALLIDAY AND RESNICK

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text	Study Guide	Text (Chap. 27)	(Chap. 27)
1	General Comment 1, Sec. 27-1	A, B	Ex.[a] 1	I	1, 3	4
2	General Comment 2, Secs. 27-1, 27-4	C, D	Ex. 2	J	6	
3	General Comment 3, Secs. 27-2, 27-3	E, F		K	7, 12, 16	8, 9, 11
4	Sec. 27-5	G, H	Ex. 3	L	Quest.[a] 12, 14, Probs. 23, 25	24

[a]Ex. = Example(s). Quest. = Question(s).

TEXT: Francis Weston Sears and Mark W. Zemansky, University Physics (Addison-Wesley, Reading, Mass., 1970), fourth edition

SUGGESTED STUDY PROCEDURE

Read Chapter 28, Sections 28-1 through 28-4, 28-6 (neglect p. 400), and 28-7, Part 1. The definition of current is given by Eq. (28-1). Read General Comment 1 and p. 389. Read General Comment 2. Equations (28-2) and (28-4) relate current and current density to the properties of the charge carriers. A nice discussion of the microscopic model for conduction in a metal appears in Section 28-3. Although the derivation of Eq. (28-8) is not essential to the objectives of this module, it demonstrates the beauty of developing models for various physical phenomena. Read General Comment 3. Equations (28-5)(scalar form) and (28-6)(vector form) are the defining equations for resistivity. A general relation is developed for resistance [Eq. (28-12)], but for our purposes Eq. (28-13) will suffice. A general relation between resistance and resistivity is given in Eq. (28-10); however, Eq. (28-11) is adequate for our use. If the resistance in Eq. (28-13) is constant then the relation is known as Ohm's law. Equation (28-6) is also called Ohm's law (in microscopic variables), if ρ is constant. The general relation for the power or rate of transfer of electrical energy to a device through which charges flow is given by Eq. (28-23). If the element obeys Ohm's law (called pure resistance on p. 401) then we can use Eq. (28-24), which is called Joule's law. If time permits you will find the material in Sections 28-8 and 28-9 on thermocouples very interesting, but you will not be held responsible for it in this module.

After completing these readings, study Problems A through H, and work Problems I through L and Problems 28-4, 28-5, 28-1, 28-9, 28-12, 28-20, 28-21 in your text. Take the Practice Test and if you are still having any difficulties, work Problem 28-3 before attempting a Mastery Test.

SEARS AND ZEMANSKY

Objective Number	Readings	Problems with Solutions: Study Guide	Problems with Solutions: Text	Assigned Problems: Study Guide	Assigned Problems: Text	Additional Problems
1	General Comment 1, Sec. 28-1	A, B		I	28-4, 28-5	28-3
2	General Comment 2, Secs. 28-1, 28-3	C, D	Sec. 28-1, Example (p. 390)	J	28-1	
3	General Comment 3, Secs. 28-2, 28-4, 28-6	E, F		K	28-9, 28-12	
4	Sec. 28-7	G, H		L	28-20, 28-21	

TEXT: Richard T. Weidner and Robert L. Sells, Elementary Classical Physics (Allyn and Bacon, Boston, 1973), second edition, Vol. 2

SUGGESTED STUDY PROCEDURE

Read Chapter 27, Sections 27-1 through 27-6. The definition of current is given by Eq. (27-1), and a very thorough discussion follows in Section 27-1. A relation for current density is given in Eq. (27-5), and a more general integral form is given in Eq. (27-7). Read General Comments 1 and 2. The concept of drift velocity is introduced in Section 27-1 (p. 536), and a very good discussion of the microscopic model for its development is given in Sections 27-4 and 27-5. The relation between current density or current tn the properties of the charge carriers appears in Eqs. (27-3) and (27-6). A macroscopic statement of Ohm's law is given by Eq. (27-12), but it should be carefully noted that in the sentence following this equation the authors state that this same relation when R is not necessarily constant is the defining equation of electrical resistance for any dissipative element. Therefore in Figure 27-6 you can determine a resistance value for any three of the curves, for a given voltage or current. But only for the conductor in part (c) does the resistance remain constant as the voltage or current varries. The definition of resistivity is given in Eq. (27-17), and if the resistivity is constant we have a microscopic statement of Ohm's law. The relation between resistance and resistivity for a conductor of length L and cross-sectional area A is given in Eq. (27-14). Read General Comment 3. The general relation for the power or rate of transfer of electrical energy to or from a device through which a charge flows is given by Eq. (27-10). If the device

WEIDNER AND SELLS

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text	Study Guide	Text	
1	Secs. 27-1, 27-2, General Comment 1	A, B		I	27-1, 27-4	27-2, 27-5
2	General Comment 2, Secs. 27-1, 27-4, 27-5	C, D	Ex.[a] 27-1	J	27-21	27-15(c)
3	General Comment 3, Secs. 27-4, 27-5	E, F		K	27-14, 27-17, 27-19	27-13, 27-15 (a), (b)
4	Secs. 27-3, 27-4	G, H		L	27-6, 27-11	27-9, 27-10

[a]Ex. = Example(s).

obeys Ohm's law the power relation is conveniently given by Eq. (27-13). This equation is a particular way of writing the conservation of energy principle for the special case in which electrical energy is transformed into internal energy.

After completing these readings, study Problems A through H, and work Problems I through L and Problems 27-1, 27-4, 27-21, 27-14, 27-17, 27-19, 27-6, and 27-11 before taking the Practice Test. If you have any difficulties, work the Additional Problems before attempting a Mastery Test.

GENERAL COMMENTS

1. Vector Notation - Current

Most physicists prefer not to treat current as a vector. A convention for labeling current is needed because charges of opposite sign move in opposite directions in an electric field. Hence for simplicity of treatment you may treat all charge carriers as positive and draw arrows (not representative of vectors) in the direction that these charges would move. This is usually termeo conventional current (the direction that positive charges would move in a wire). (For currents in wires a negative charge moving in one direction is equivalent to a positive charge moving in the opposite direction. See Problem A.) Current density is a vector that at any given point in the conductor has a direction determined by the volocity vector of a positive charge moving through that point. The current is the same everywhere along a wire. This follows from conservation of charge, i.e., electrons just do not appear and disappear along the wire. This is a very simple but very important concept and will be used in problem solving in the module Direct Current Circuits.

2. Thermal Speed

We would like to stress that the average thermal speed of the random moving electrons is on the order of 10^8 times larger than the drift speed of these electrons. Even though the drift speed is surprisingly small, a change in the force that produced this motion can propagate through the conductor at nearly the speed of light (3.00×10^8 m/s). This is why, when you turn on the light switch, the light goes on "instantaneously." The speed of the electrons in a conductor subjected to a potential difference is very small. Even though the electrons in a metal have such small drift speed, the phenomenon responsible for it - the electric field - travels along the wire at nearly the speed of light.

3. Ohm's Law

It should be stressed that only when the resistance R is constant is the expression V = IR called Ohm's law. In a current-voltage (IV) plot as in Figure 1, the resistance for a given value of voltage and current is the reciprocal of the tangent of the curve at that point, and if R is independent of current and voltage, then the curve is straight line. Ohm's law is sometimes stated in terms of microscopic quantities, where the resistivity is assumed constant. Ohm's law follows from the definition of resistance when the resistance is independent of current and voltage. In this module we confine our discussion to homogeneous conductors and electrostatic fields.

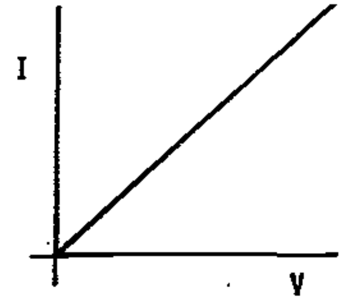


Figure 1

PROBLEM SET WITH SOLUTIONS

A(1). A current (Fig. 2) is established in a gas-discharge tube when a sufficiently high potential difference is applied across the two electrodes in the tube. The gas ionizes; electrons move toward the positive terminal and positive ions toward the negative terminal. What are the magnitude and sense of the current in a hydrogen-discharge tube in which 3.1×10^{18} electrons and 1.1×10^{18} protons move past a cross-sectional area of the tube each second?

Solution

The number of electrons passing cross-sectional area AA in 1.00 s is 3.10×10^{18}, and since the charge on an electron is -1.60×10^{-19} C, the negative charge crossing AA to the left per second is

$$I_- = (3.10 \times 10^{8} \text{ electrons/s})(-1.60 \times 10^{-19} \text{ C/electron}) = -5.0 \times 10^{-1} \text{ A}.$$

The number of protons crossing AA per second is 1.10×10^{18}, and since the charge on the proton is $+1.60 \times 10^{-19}$ C, the positive charge crossing AA to the right per second is

$$I_+ = (1.10 \times 10^{18} \text{ protons/s})(1.60 \times 10^{-19} \text{ C/proton}) = 1.76 \times 10^{-1} \text{ A}.$$

The negative charge moving to the left is equivalent to positive charge moving to right, thus

$$I_{total} = |I_-| + |I_+| = 6.8 \times 10^{-1} \text{ A} \quad \text{to the right.}$$

B(1). The current in a wire varies with time according to the relation

$I = 20 \sin(120\pi t)$,

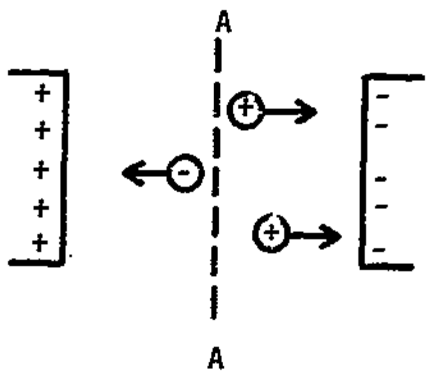


Figure 2

where I is in amperes, t in seconds, $120\pi t$ in radians, and the constants are 20 A and 120/s.

(a) How many net coulombs of charge pass a cross section of the wire in the time interval between t = 0 and t = 1/60 s?

(b) In the interval between t = 0 and t = 1/120 s?

(c) What constant current would transport the same charge in each of the intervals above?

Solution

(a) From the definition of current I = dQ/dt or dQ = I dt, thus

$$Q = \int_{t_i}^{t_f} I\,dt = \int_0^{1/60\text{ s}} 20\sin(120\pi t) = -\frac{20}{120\pi}[\cos(120\pi t)]\Big|_0^{1/60\text{ s}}$$

$$= -(1/6\pi)(\cos 2\pi - \cos 0) = -(1/6\pi)(1 - 1) = 0.$$

A plot of current versus time would look like Figure 3. Therefore at t = 0, I = 0; it increases to 20 A at t = 1/240 s and decreases to zero at t = 1/120 s. Now the current changes direction and increases to 20 A at t = 1/80 s in the opposite direction. One can therefore see that as much charge flowed to the right through our cross section during the first 1/120 s as flowed to the left through the cross section during the next 1/120 s. Thus the net charge passing the cross section in 1/60 s is 0.

(b) $$Q = \int_0^{1/120\text{ s}} 20\sin(120\pi t) = -\frac{20}{120\pi}(\cos\pi - \cos 0) = -\frac{1}{6\pi}(-1 - 1)$$

$$= \frac{2}{6\pi} = \frac{1}{3\pi} = 0.106\text{ C}.$$

(c) For a constant current I = Q/t. For 1/60 s, Q = 0, so I = 0. For 1/120 s, Q = 0.106 C, so

$$I = \frac{1/3\pi\text{ C}}{1/120\text{ s}} = \frac{40}{\pi}\text{ A} = 12.7\text{ A}.$$

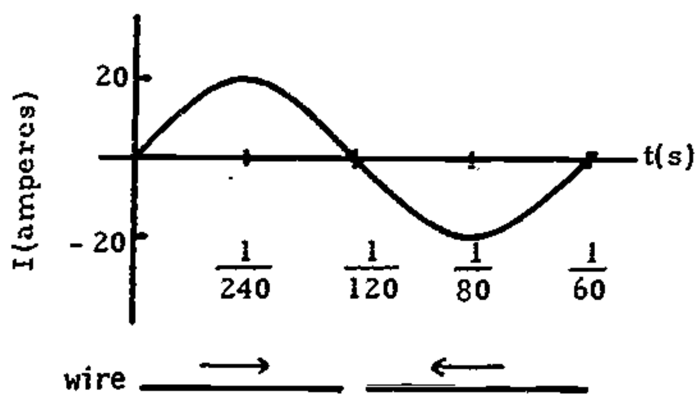


Figure 3

C(2). A small but measurable current of 1.00×10^{-10} A exists in a copper wire whose diameter is 0.250 cm. Calculate the electron drift speed.

Solution

$J = nev_d$, Current density = (Charge per unit volume)(electronic charge)(drift speed).

$v_d = J/ne$. N_0 is Avogadro's number, d is mass density, e_f is free electrons, and M is atomic weight:

$$n = \frac{N_0 d(e_f/\text{atom})}{M} = \frac{(6.0 \times 10^{26} \text{ atoms/kg mol})(9.0 \times 10^3 \text{ kg/m}^3)(1 \text{ electron/atom})}{64}$$

$$= 8.4 \times 10^{28} \text{ electrons/m}^3,$$

$$J = \frac{I}{\text{area}} = \frac{1.00 \times 10^{-10} \text{ A}}{\pi(0.250/2 \times 10^2 \text{ m})^2} = 2.00 \times 10^{-5} \text{ A/m}^2,$$

$$v_d = \frac{J}{ne} = \frac{2.00 \times 10^{-5} \text{ A/m}^2}{(8.4 \times 10^{28} \text{ electrons/m}^3)(1.60 \times 10^{-19} \text{ C/electron})} = 1.48 \times 10^{-15} \text{ m/s}.$$

D(2). A typical copper wire might have 2.00×10^{21} free electrons in 1.00 cm of its length. Suppose the drift speed of the electrons along the wire is 0.050 cm/s.

(a) How many electrons would pass through a given cross section of wire each second?

(b) How large a current would be flowing in the wire?

Solution

(a) nAv_d = (electrons/volume)(cross-sectional area)(length/second)

= [electrons/(length)(area)](area)(length/second)

= number of electrons crossing a cross-sectional area in one second

= (electrons/length)(length/second)

$= [(2.00 \times 10^{21} \text{ electrons})/(1.00 \text{ cm})]5.0 \times 10^{-2} \text{ cm/s} = 1.00 \times 10^{20}$ electrons/s.

(b) $e = 1.60 \times 10^{-19}$ C/electron,

$I = (1.60 \times 10^{-19} \text{ C/electron})(1.00 \times 10^{20} \text{ electrons/s}) = 16.0$ A.

E(3). A copper wire and an iron wire of the same length have the same potential difference applied to them. See Figure 4.

(a) What must be the ratio of their radii if the current is to be the same?

(b) Can the current density be made the same by suitable choices of the radii? Take

$$\rho_{Cu} = 1.70 \times 10^{-8}\ \Omega\ m, \qquad \rho_{Fe} = 1.00 \times 10^{-7}\ \Omega\ m.$$

Solution

V_{AB} = potential difference between A and B. Ohm's law, V = IR:

$$V = I\rho\ell/A = I\rho\ell/\pi r^2.$$

(a) $\dfrac{(V_{AB})_{Cu}}{(V_{AB})_{Fe}} = \dfrac{I_{Cu}\rho_{Cu}\ell_{Cu}/\pi r^2_{Cu}}{I_{Fe}\rho_{Fe}\ell_{Fe}/\pi r^2_{Fe}}$

where $\ell_{Cu} = \ell_{Fe}$, $(V_{AB})_{Cu} = (V_{AB})_{Fe}$, $I_{Cu} = I_{Fe}$

$$1 = \frac{\rho_{Cu}/r^2_{Cu}}{\rho_{Fe}/r^2_{Fe}} \text{ or } \frac{r^2_{Fe}}{r^2_{Cu}} = \frac{\rho_{Fe}}{\rho_{Cu}} \text{ or } \frac{r_{Fe}}{r_{Cu}} = \left[\frac{1.00 \times 10^{-7}}{1.70 \times 10^{-8}}\right]^{1/2} = 2.43.$$

(b) From above, $1 = \dfrac{I_{Cu}\rho_{Cu}r^2_{Fe}}{I_{Fe}\rho_{Fe}r^2_{Cu}}$. Therefore

$$I_{Fe}\rho_{Fe}r^2_{Fe} = I_{Cu}\rho_{Cu}r^2_{Cu} \text{ or } I_{Fe}\rho_{Fe}/r^2_{Fe} = I_{Cu}\rho_{Cu}/r^2_{Cu}.$$

Multiplying both sides by $1/\pi$ and noting that πr^2_{Fe} = $(\text{area})_{Fe}$ and $I_{Fe}/(\text{area})_{Fe}$ $= J_{Fe}$, we have $J_{Fe}\rho_{Fe} = J_{Cu}\rho_{Cu}$ or $J_{Fe}/J_{Cu} = \rho_{Cu}/\rho_{Fe}$.

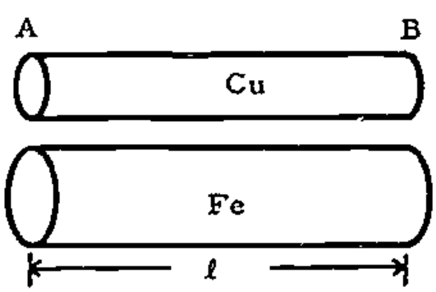


Figure 4

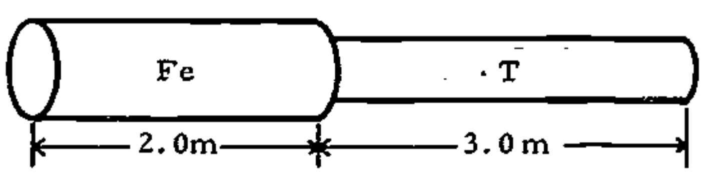


Figure 5

If $J_{Fe} = J_{Cu}$, then $\rho_{Fe} = \rho_{Cu}$. Since this is impossible, $J_{Fe} \neq J_{Cu}$. A shorter argument might be given by using the relation $E = \rho J$. Since $E = V/\ell$ is the same for both wires, J must be different for different values of ρ.

F(3). An iron wire of 1.00 m diameter and 2.00 m length is connected to a tungsten wire of 1.00 Ω resistance and 3.00 m lenath. A potential difference of 60 V is applied to the combined wire 5.0 in lenqth. The resistivity of iron is 1.00×10^{-7} Ω m.

(a) What is the current flowing in the irom wire? In the tungsten wire?

(b) What is the electric field in the iron wire?

Solution

(a) See Figure 5.

$$R_{Fe} = \rho_{Fe} \frac{\ell_{Fe}}{(\text{area})_{Fe}} = \frac{(1.00 \times 10^{-7}\ \Omega\ \text{m})(2.00\ \text{m})}{\pi(5.0 \times 10^{-4}\ \text{m})^2} = 0.254\ \Omega,$$

$R_{total} = R_{Fe} + R_T = 1.20$ Ω, $I = V/R = (60\ \text{V})(1.20\ \Omega) = 50$ A in both wires.

(b) $E = \rho J = 6.1$ V/m.

G(4). Comsumers of electric energy pay by the kilowatt-hour. What is the cost of operating a 10.0-Ω toaster across a potential difference of 110 V for 20 min at the rate of 3.00 cents per kilowatt-hour?

Solution

Power = (current)(potential difference) = $P = (I)(V) = V^2/R$,

where $I = V/R$ from Ohm's law.

$P = (110\ \text{V})^2/(10.0\ \Omega) = 1210\ \text{W} = 1.21$ kW, Work = $(P)(t) = (1.21\ \text{kW})(20/60\ \text{h})$
$= 0.40$ kWh,

\$ = (0.40 kWh)(0.030 dollar/kWh) = \$0.012 = 1.20 cent.

H(4). A clever student decides to defrost the windshield of her sports car by covering it with a transparent conductive coating. The coating is 1.40 m long, 0.30 m wide, and 5.0×10^{-4} m thick and has a resistivity of 7.0×10^{-5} Ω m. If the "defroster" is to be operated from the 12.0-V system of the car:

(a) What is the power dissipated by the defroster?

(b) How much current does the defroster utilize?

Solution

$R = \rho \ell / A = 0.65\ \Omega$. Assuming the defroster obeys Ohm's law,

(a) $P = V^2/R = 220$ W, where $V = 12.0$ V.

(b) $I = V/R = 18$ A.

Problems

I(1). The total charge that has passed a given cross section varies with time according to the relation

$$q(t) = K(1 - e^{-t/B}),$$

where $K = 3.00 \times 10^{-5}$ C and $B = 1.00$ s.

(a) Determine the current in the wire as a function of t.

(b) If the cross-sectional area of the wire is uniform and is 3.00×10^{-5} m^2, what is the current density in the wire as a function of t?

(c) When t = 1.00 s, how does the current compare to that at t = 0 s?

(d) Draw a rough sketch of I versus t.

J(2). A 10.0-m length of Number-18 copper wire is carrying a current of 5.0 A. If the diameter of the wire were doubled, with the same current, how would the drift speed change? Support your conclusion with a short physical argument.

K(3). Number-10 copper wire can carry a maximum current of about 30 A before overheating. Its diameter is 0.260 cm. The resistivity of copper is $1.70 \times 10^{-8}\ \Omega$ m.

(a Find the resistance of a 1.00-m length of the wire.

(b) How large a voltage drop occurs along it per meter when it carries a current of 30.0 A?

L(4). You decide to heat your office by placing a steel plate (resistivity = $1.80 \times 10^{-7}\ \Omega$ m) under the rug and passing a current through it from one side to the other. If the room is 4.0 m square and you want to get 4.0 kWatts by connecting it to a 120-V power line, how thick should the plate be? Assume the current is distributed uniformly in the plate.

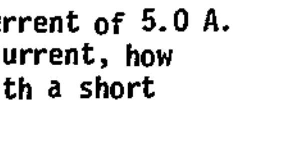


Figure 6

Solutions

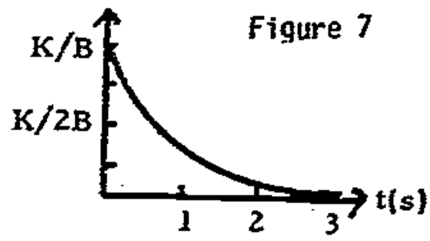


Figure 7

I(1). (a) $I = (K/B)e^{-t/B}$.

(b) $J = (1/B)e^{-t/B}$.

(c) $I(t = 1.00\ s)/I(t = 0\ s) = 0.37$.

(d) See Figure 7 for I versus t.

J(2). Decreases to 1/4; $J = I/A = nev_d$. $A = \pi d^2/4$; I and ne are constant. See how v_d changes when A changes.

K(3). (a) $3.20 \times 10^{-3}\ \Omega$. (b) 0.096 V.

L(4). Assuming Ohm's law; thickness = 5.0×10^{-8} m.

PRACTICE TEST

1. A bird whose feet are 5.0 cm apart perches on an aluminum power line that carries 1000 A. The power line is 2.54 cm in diameter, and the resistivity of aluminum is 2.80×10^{-8} Ω m. What is the potential difference between the bird's feet?

2. The current in a wire varies with time according to the relation

$$I = (4.0\ \text{A}) \div (2.00\ \text{A/s}^2)t^2,$$

where I is in amperes and t is in seconds.

(a) How many coulombs pass a cross section of wire in the time interval between t = 5.0 s and t = 10.0 s?

(b) What constant current would transport the same charge in the same time interval?

3. A 100-W light bulb is constructed to operate at 115 V.

(a) What will be the current in the filament?

(b) What is the resistance of the filament coil?

(c) How much energy is radiated in one hour?

4. A potential difference V is applied to a copper wire of diameter d and length ℓ. What is the effect on the electron drift speed by (a) doubling V? (b) doubling ℓ? (c) doubling d?

Practice Test Answers

1. $\Delta V = 2.80 \times 10^{-3}$ V.
2. (a) $Q = 6.0 \times 10^2$ C. (b) $I_{const} = 120$ A.
3. (a) I = 0.87 A. (b) R = 132 Ω. (c) 3.6×10^5 J.
4. (a) v_d doubles. (b) v_d is halved. (c) v_d is unchanged.

OHM'S LAW

Mastery Test Form A

Name ______________________

Date ______________________

pass recycle

1 2 3 4

Tutor ______________________

1. A nickel wire of length ℓ = 50 m and diameter d = 2.00 mm is joined to a wire of resistance R = 2.00 Ω to form a 100-m-long wire to which a potential difference V is applied. You wish to dissipate energy at the rate of 0.50 W in the combined wires.
   (a) What potential V is needed for this purpose?
   (b) How large is the electric field in the nickel?
   (c) How large is the drift speed of electrons in the nickel?

   Data: resistivity of nickel is 6.8×10^{-8} Ω m,
   charge of electron is 1.60×10^{-19} C,
   mass of electron is 1.00×10^{-30} kg,
   number of conduction electrons in nickel is 1.00×10^{28} electrons/m^3.

2. The current in a wire varies with time according to the relation

   $$I = (1.00 \times 10^{-6}\ A)\{1 + \cos[(2\pi/s \times 10^3)t]\},$$

   where I is in amperes and t in seconds. Find an expression for the charge that has passed a cross section of the wire as a function of time.

OHM'S LAW

Mastery Test Form B

Name ________________________________

Date ________________________

pass recycle

1 2 3 4

Tutor ________________________

1. A silver wire 1.00 mm^2 in cross-sectional area carries a charge of 10.0 C uniformly over a period of 30 min. The resistivity of silver is 1.60×10^{-8} Ω m. The number of conduction electrons in silver is 6.0×10^{28} electrons/m^3.
   (a) What is the current density in the wire?
   (b) What is the electric field in the wire?
   (c) What is the potential difference across 5.0 m of the wire?
   (d) What is the power dissipated by the wire?
   (e) What is the drift speed of the electrons in the silver?

2. You want to make a 1600-W heater to operate from 120 V. The heater is to be made from 4.0 m of manganin wire. The resistivity of manganin is 4.4×10^{-7} Ω m.
   (a) What should be the resistance of the wire?
   (b) What should be the wire diameter?

OHM'S LAW

Mastery Test Form C

Name ______________________

Date ______________

pass / recycle

1 2 3 4

Tutor ______________

1. An aluminum rod of square cross section (area 16.0 mm^2) and length 2.00 m is joined to a square iron rod (area 36 mm^2) of length 0.50 m. The resistivity of aluminum is 2.80×10^{-8} Ω m, that of iron is 1.00×10^{-7} Ω m. The combined rod, 2.50 m long, carries a current of 200 A.

   (a) Find the ratio of current densities in the two rods.

   (b) Find the ratio of electric fields in the two rods.

   (c) Calculate the potential difference that must be applied to maintain the current.

   (d) Find the rate of energy dissipation in the aluminum.

   (e) The number of conduction electrons in iron is about 1.00×10^{29} electrons/m^3. The charge of an electron is 1.60×10^{-19} C, the mass is about 1.00×10^{-30} kg. Find the drift speed of the electrons in the iron rod.

2. The current in a wire varies with time according to the relation

   $$I = (6.0\ A)t,$$

   where I is in amperes and t in seconds. How many coulombs pass a cross section of the wire in the time interval between t = 0 s and t = 10.0 s?

MASTERY TEST GRADING KEY - Form A

1. What To Look For: (a) Joule's law. You are given P but need the total resistance of the combination. The resistance of the combination is the sum of the component resistances. (b) You are given ρ and A but need I. Use Ohm's law. Ask directions of E and J. (c) Relation for current density.

Solution: See Figure 8.

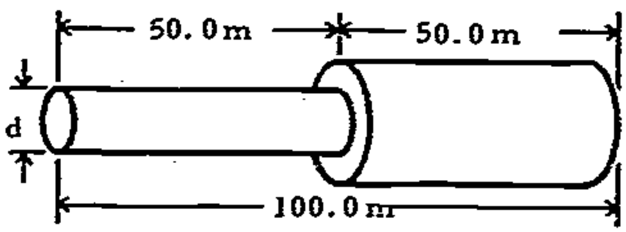


Figure 8

(a) $P = V^2/R, \quad V = \sqrt{PR}, \quad R = \rho\ell/A,$

$$R_{N_i} = \rho_{N_i}\ell_{N_i}/A_{N_i} = (6.8 \times 10^{-8}\ \Omega\ m)(50\ m)/[\pi(1.00 \times 10^{-3})^2\ m^2] = 1.10\ \Omega.$$

$$R_{total} = R_T = R_{N_i} + 2.00\ \Omega = 3.10\ \Omega. \qquad V = (0.50\ W)(3.10\ \Omega)^{1/2} = 1.20\ V.$$

(b)

$$E = \rho J, \quad J = i/A, \quad I = V/R_T = (1.20\ V)/(3.10\ \Omega) = 0.40\ A,$$

$$J = (0.40\ A)/[\pi(1.00 \times 10^{-3})^2\ m^2] = 1.27 \times 10^5\ A/m^2,$$

$$E = (6.8 \times 10^{-8}\ \Omega\ m)(1.27 \times 10^5\ A/m^2) = 8.6 \times 10^{-3}\ V/m.$$

(c) $J = nev_d$,

$$v_d = \frac{J}{ne} = \frac{(1.27 \times 10^5\ A/m^2)}{(1.00 \times 10^{28}\ electrons/m^3)(1.60 \times 10^{-19}\ C/electrons)} = 7.9 \times 10^{-5}\ m/s.$$

2. What To Look For: Definition of current. No limits of integration so we need a constant of integration.

Solution: $I = dQ/dt, \quad dQ = I\ dt,$

$$Q = \int I\ dt + const = (1.00 \times 10^{-6})\int[1 + \cos(2\pi \times 10^3 t)]\ dt + const$$

$$= (1.00 \times 10^{-6})\{t + [1/(2\pi \times 10^3)]\ \sin(2\pi \times 10^3 t)\} + const.$$

MASTERY TEST GRADING KEY - Form B

1. What To Look For: (a) Determine the current that is assumed to be constant. (c) From Ohm's law V = IR. Therefore we need to solve for resistance.

Solution: (a) $I = Q/t = 10C/1800s = 5.6 \times 10^{-3}$ A,

$$J = \frac{I}{A} = (5.6 \times 10^{-3}\ A)/(1.00 \times 10^{-6}\ m^2) = 5.6 \times 10^3\ A/m^2.$$

(b) $E = \rho J = (1.60 \times 10^{-8}\ \Omega\ m)(5.6 \times 10^3\ A/m^2) = 9.0 \times 10^{-5}$ V/m.

(c) $R = \rho\ell/\text{area} = (1.60 \times 10^{-8}\ \Omega\ m)(5.0\ m)/(1.00 \times 10^{-6}\ m^2) = 8.0 \times 10^{-2}\ \Omega$,

$$V = IR = (5.6 \times 10^{-3}\ A)(8.0 \times 10^{-2}\ \Omega) = 4.5 \times 10^{-4}\ V.$$

(d) $P = VI = (4.5 \times 10^{-4}\ V)(5.6 \times 10^{-3}\ A) = 2.52 \times 10^{-6}$ W. $J = nev_d$,

$$v_d = \frac{J}{ne} = \frac{5.6 \times 10^3\ A/m^2}{(6.0 \times 10^{28}\ \text{electrons}/m^3)(1.60 \times 10^{-19}\ \text{C/electron})} = 5.8 \times 10^{-7}\ m/s.$$

2. What To Look For: (a) Using Joule's law.

Solution: (a) $P = V^2/R$, $R = V^2/P = (120\ V)^2/(1600\ W) = 9.0\ \Omega$.

(b) $R = \frac{\rho\ell}{\text{area}} = \frac{\rho\ell}{\pi(d/4)^2}$, where the diameter of the wire is d.

$$d^2 = \left(\frac{4}{\pi}\right)\left(\frac{\rho\ell}{R}\right), \quad d = \left[\frac{(4)(4.4 \times 10^{-7}\ \Omega\ m)(4.0\ m)}{(3.14)(9.0\ \Omega)}\right]^{1/2} = 5.0 \times 10^{-4}\ m.$$

MASTERY TEST GRADING KEY - Form C

1. What To Look For: (a) $I = 200$ A in both wires.
(b) Relation for E. Ask: What is the direction of E and J. Total resistance of the combination is the sum of the resistances of the two blocks. Application of Ohm's law.
(d) Joule's law.
(e) Relation between current density and drift speed.

Solution: (a) $J = I/A$, $J_{Al} = I/A_{Al} = 200\ \text{A}/(16 \times 10^{-6}\ \text{m}^2) = 1.25 \times 10^7\ \text{A/m}^2$,

$$J_{Fe} = \frac{I}{A_{Fe}} = \frac{200\ \text{A}}{36 \times 10^{-6}\ \text{m}^2} = 5.6 \times 10^6\ \text{A/m}^2, \qquad \frac{j_{Al}}{j_{Fe}} = \frac{A_{Fe}}{A_{Al}} = \frac{36 \times 10^{-6}}{16.0 \times 10^{-6}} = 2.25.$$

$$\text{(b) } E = \rho J, \ \frac{E_{Al}}{E_{Fe}} = \frac{\rho_{Al} J_{Al}}{\rho_{Fe} J_{Fe}} = \left(\frac{2.80 \times 10^{-8}}{1.00 \times 10^{-7}}\right)2.25 = 0.63.$$

$$\text{(c) } R = \rho \ell/A, \ R_{Al} = \frac{(2.80 \times 10^{-8}\ \Omega\ \text{m})(2.00\ \text{m})}{(16.0 \times 10^{-6}\ \text{m}^3)} = 3.5 \times 10^{-3}\ \Omega.$$

$$R_{Fe} = \frac{(1.00 \times 10^{-7}\ \Omega\ \text{m})(0.50\ \text{m})}{36 \times 10^{-6}\ \text{m}^3} = 1.40 \times 10^{-3}\ \Omega. \qquad R_{total} = R_T = 4.9 \times 10^{-3}\ \Omega.$$

$$V = IR = (200\ \text{A})(4.9 \times 10^{-3}\ \Omega) = 0.98\ \text{V}.$$

$$\text{(d) } P = I^2R = (200\ \text{A})^2(3.5 \times 10^{-3}\ \Omega) = 140\ \text{W}.$$

$$\text{(e) } J = nev_d, \ (v_d)_{Fe} = \frac{J_{Fe}}{(n_{Fe})(\ell)} = \frac{(5.6 \times 10^6\ \text{A/m}^2)}{(1.00 \times 10^{29}\ \text{electrons/m}^3)(1.60 \times 10^{-19}\ \text{C/electrons})}$$

$$= 3.5 \times 10^{-4}\ \text{m/s} = 0.35\ \text{mm/s}.$$

2. What To Look For: Definition of current.

Solution: $I = dQ/dt$, $dq = I\,dt$,

$$Q = \int_0^{10.0\ \text{s}} 6.0t\,dt = (6.0t^2/2)\Big|_0^{10.0\ \text{s}}, \qquad a = (3.00)(100) = 300\ \text{C}.$$

CAPACITORS

INTRODUCTION

Capacitors are important components of electronic circuits and of electrical machinery and power grids. You can find large oil-insulated capacitors on power-line poles or small ceramic-insulated capacitors in a radio. In each application the capacitor is used to store electrical charge and electrical energy - for example, sometimes for a short time in an alternating-current cycle, sometimes for a long time until the energy is needed, as in a strobe light for a camera. Your body can be a capacitor, storing up enough charge and energy to cause a painful spark when the capacitor discharges.

Practical capacitors are basically two conducting plates or sheets separated by an insulator (dielectric) such as air, oil, paper, plastic film, or even the oxide layer on one of the conducting surfaces. This module will treat the basic physics of capacitors.

PREREQUISITES

Before you begin this module, you should be able to:	Location of Prerequisite Content
*Determine electric fields for charge distributions with planar, cylindrical, and spherical symmetries (needed for Objective 2 of this module)	Flux and Gauss' Law Module
*Determine electrostatic potentials for charge distributions with planar, cylindrical and spherical symmetries (needed for Objective 2 of this module)	Electric Potential Module

LEARNING OBJECTIVES

After you have mastered the content of this module, you will be able to:

1. Definitions - Define the terms "capacitor" and "capacitance" and use these definitions to relate capacitance, voltage difference, and charge in a capacitor.

2. Capacitance - Derive and use expressions for the capacitance of capacitors that have planar, cylindrical, or spherical symmetry.

3. Connected capacitors - Determine the equivalent capacitance of a set of capacitors connected together, and determine the charge and voltage on each capacitor of the set.

4. <u>Energy</u> - Determine the energy stored in a capacitor or combination of capacitors, and compute the energy stored per unit volume in a region where an electric field exists.

5. <u>Dielectrics</u> - Describe the effect on a capacitor's capacitance, voltage, charge, and stored energy, as well as the electric field in the capacitor, if the space between the conductors of the capacitor contains dielectric material; describe qualitatively the distribution of polarization charges that accounts for these effects.

TEXT: Frederick J. Bueche, Introduction to Physics for Scientists and Engineers (McGraw-Hill, New York, 1975), second edition

SUGGESTED STUDY PROCEDURE

Read Sections 21.5, 21.6, 21.7, 21.13, and 21.14 in Chapter 21. Then read Chapter 26, Sections 26.1 and 26.2 for perspective before reading Sections 26.3 and 26.4. Study Illustration 21.6 and Problems A through H before working Problems I through N.

Take the Practice Test, and work some Additional Problems if necessary, before trying a Mastery Test.

BUECHE

Objective Number	Readings	Problems with Solutions: Study Guide	Problems with Solutions: Text	Assigned Problems: Study Guide	Additional Problems (Chap. 26)
1	Sec. 21.5	A			
2	Secs. 21.6, 21.7	B, C		I	
3	Sec. 21.13	D, E, F	Illus.[a] 21.6	J	
4	Sec. 21.14	G		K, L	
5	Secs. 26.1, 26.2, 26.3, 26.4	H		M, N	1, 4, 7

[a]Illus. = Illustration

TEXT: David Halliday and Robert Resnick, Fundamentals of Physics (Wiley, New York, 1970; revised printing, 1974)

SUGGESTED STUDY PROCEDURE

Read Chapter 26. Section 26-5 is optional. Then study Problems A through H and Examples 1 through 5 before working Problems I through N and Problem 8 in Chapter 26.

Take the Practice Test, and work some Additional Problems if necessary, before trying a Mastery Test.

HALLIDAY AND RESNICK

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text	Study Guide	Text	(Chap. 26)
1	Sec. 26-1	A				
2	Sec. 26-2	B, C	Ex.[a] 1, 2	I	Chap. 26, Prob. 8	21
3	Sec. 26-2	D, E, F	Ex. 3	J		12, 13, 18
4	Sec. 26-6	G	Ex. 4	K, L		34, 37
5	Secs. 26-3, 26-4	H	Ex. 5	M, N		30

[a]Ex. = Example(s).

TEXT: Francis Weston Sears and Mark W. Zemansky, University Physics (Addison-Wesley, Reading, Mass., 1970), fourth edition

SUGGESTED STUDY PROCEDURE

Read Chapter 27. Sections 27-7 and 27-8 (through the first column of p. 284) are optional. Study the Examples in 27-3 and 27-4 (pp. 378, 379) and Problems A through H before working Problems I through N.

Then take the Practice Test, and work some Additional Problems if necessary, before trying a Mastery Test.

SEARS AND ZEMANSKY

Objective Number	Readings	Problems with Solutions		Assigned Problems	Additional Problems
		Study Guide	Text	Study Guide	
1	Sec. 27-1	A			
2	Sec. 27-2	B, C		I	
3	Sec. 27-3	D, E, F	Example (p. 378)	J	27-5, 27-11
4	Sec. 27-4	G	Example (p. 379)	K, L	27-4, 27-10
5	Secs. 27-5, 27-6, 27-8	H		M, N	27-15

TEXT: Richard T. Weidner and Robert L. Sells, Elementary Classical Physics (Allyn and Bacon, Boston, 1973), second edition, Vol. 2

SUGGESTED STUDY PROCEDURE

Read all of Chapter 26. Then study Examples 26-1 and 26-2 and Problems A through H before working Problems I through N. Take the Practice Test, and work some Additional Problems if necessary, before trying a Mastery Test.

WEINDER AND SELLS

Objective Number	Readings	Problems with Solutions		Assigned Problems	Additional Problems
		Study Guide	Text	Study Guide	
1	Sec. 26-1	A			
2	Sec. 26-2	B, C		I	26-2, 26-4
3	Sec. 26-3	D, E, F	Ex.[a] 26-1	J	26-3, 26-10
4	Secs. 26-6, 26-7	G		K, L	
5	Secs. 26-4, 26-5	H	Ex. 26-2	M, N	26-8

[a]Ex. = Example(s).

PROBLEM SET WITH SOLUTIONS

A(1). An initially uncharged 5.0-μF capacitor is charged to a potential difference of 200 V by transferring charge from one plate of the capacitor to the other. How much charge was transferred?

Solution

Capacitance is defined as the ratio of the magnitude of the charge on one or the other of the conductors to the potential difference between them: $C = Q/V$ (the larger the capacitance, the more charge the capacitor will hold for a given potential difference.) The answer is therefore

$$Q = CV = (5.0 \times 10^{-6}\ F)(200\ V) = 1.00 \times 10^{-3}\ C.$$

B(2). A spherical capacitor consists of two concentric spherical shells of radii a and b ($a < b$). Determine its capacitance.

Solution

Naturally, you can look up and memorize the formula, but you will clutter your mind with formulas if you do too much of this. So derive the capacitance: We know that if there is a charge Q on the inner conductor the electric potential in the space between the conductors depends on the distance from the center of the capacitor as $V = Q/4\pi\varepsilon_0 r$. Therefore the potential difference between the conductors is

$$V_a - V_b = \left(\frac{Q}{4\pi\varepsilon_0}\right)\left(\frac{1}{a} - \frac{1}{b}\right).$$

From the definition of capacitance $C = Q/(V_a - V_b)$ we obtain immediately

$$C_{sph} = \frac{Q}{(Q/4\pi\varepsilon_0)(1/a - 1/b)} = \frac{4\pi\varepsilon_0 ab}{b - a}.$$

C(2). The cylindrical capacitor in Figure 1, 2.00 cm long, consists of two conducting cylinders that are coaxial as shown. Determine the capacitance of this capacitor and compute the potential difference if the inner conductor is holding 2.00×10^{-9} C. ($R = 2.00$ mm; $r = 3.00$ mm.)

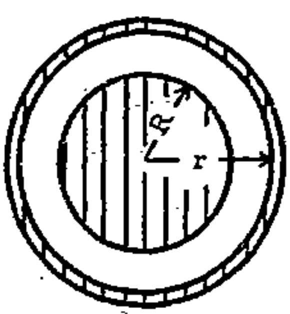


Figure 1

Solution

You should derive the expression for the capacitance: For a charge Q on the inner conductor, you can determine (do it!), using Gauss' law, that the electric field is given by

$$E = Q/2\pi\epsilon_0 \ell r.$$

Then

$$V_a - V_b = \int_a^b E\,dr = \frac{Q}{2\pi\epsilon_0 \ell}\ln\left(\frac{b}{a}\right).$$

Therefore

$$C_{cyl} = 2\pi\epsilon_0 \ell/[\ln(b/a)] = 2.74\ \text{pF} \quad \text{and} \quad \Delta V = \frac{Q}{C} = 730\ \text{V}.$$

D(3). Capacitors in parallel: Figure 2(a) shows three capacitors C_1, C_2 and C_3, connected in parallel (i.e., the plates of each capacitor are connected by wires to the same two terminals, a and b). Determine the equivalent capacitance of this arrangement [i.e., determine the size of the single capacitor C, Figure 2(b), which gives the same potential difference V between the terminals a and b if a given charge Q = CV is transferred from terminal b to terminal a and distributes itself among the three capacitors.]

Solution

Let the charges on C_1, C_2, and C_3 be Q_1, Q_2, and Q_3, respectively. The total charge Q is simply the sum of the charges on each capacitor: $Q = Q_1 + Q_2 + Q_3$. For this parallel connection, the potential difference V is the same for each capacitor (the upper plates are all at the same potential and the lower plates are all at some other potential), therefore

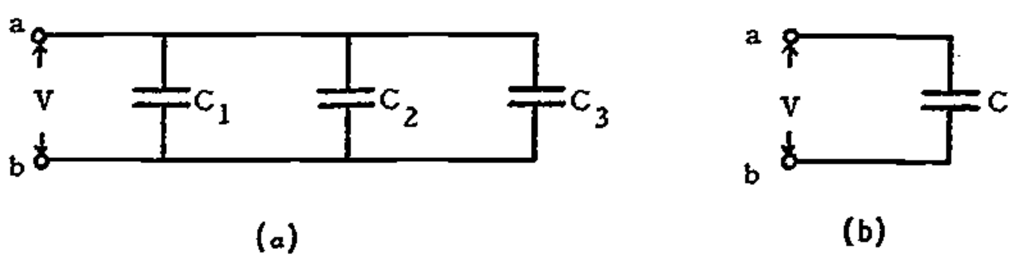


Figure 2

$$Q_1 = C_1V, \quad Q_2 = C_2V, \quad Q_3 = C_3V.$$

We can therefore immediately write

$$C = \frac{Q}{V} = \frac{Q_1 + Q_2 + Q_3}{V} = \frac{C_1V + C_2V + C_3V}{V} = C_1 + C_2 + C_3,$$

which says simply that "capacitors in parallel add." Your textbook treats the equivalent capacitor for capacitors connected in series.

E(3). Find the equivalent capacitance of the combination of capacitors shown in Figure 3 where $C_1 = 3.00\ \mu F$, $C_2 = 4.0\ \mu F$, and $C_3 = 5.0\ \mu F$.

Solution

In most practical cases, combinations of capacitors can be consolidated into groups of capacitors that are connected either in series or in parallel. The important thing is to be systematic as you work through a problem such as this. In this problem we first consolidate C_1 and C_2 into an equivalent capacitor $C_4 = C_1 + C_2$ to obtain the simplified Figure 4. Now we have C_3 and C_4 in series, which gives for the equivalent capacitance C the following relation:

$$\frac{1}{C} = \frac{1}{C_3} + \frac{1}{C_4}.$$

Inserting the values given for C_1, C_2 and C_3, we obtain $C = 2.92\ \mu F$.

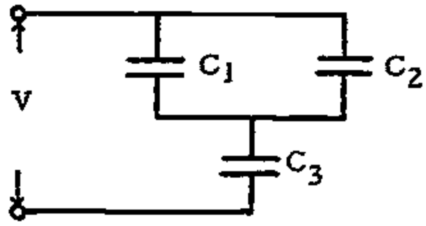


Figure 3

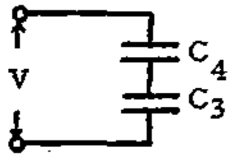


Figure 4

F(3). A potential difference of 300 V is applied to a 2.00-μF capacitor and an 8.0-μF capacitor connected in series.
(a) What are the charge and the potential difference for each capacitor?
(b) The charged capacitors are connected with their positive plates together and then with negative plates together, no external voltage being applied. What are the charge and the potential difference for each?
(c) The charged capacitors in part (a) are reconnected with plates of opposite sign together. What are the charge and the potential difference for each?

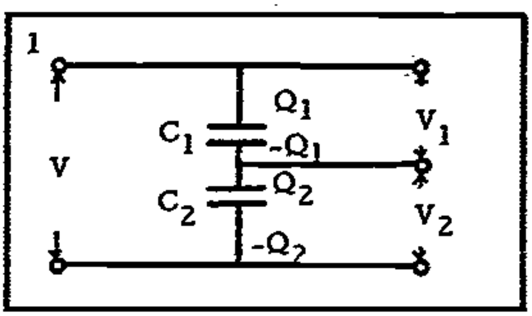
Solution

(a) You should be able to show that $Q_1 = 480$ C, $V_1 = 240$ V, $Q_2 = 480$ C, $V_2 = 60$ V.

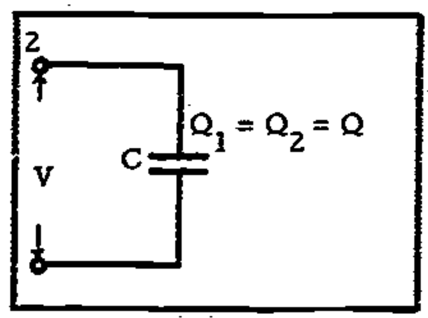
(b) The important part of this problem is to be sure that you can picture what is going on. For this purpose you may wish to illustrate each step of the problem in successive frames, as in Figure 5. Now, conservation of charge requires that the total charge on the upper plates in Step 4 of Figure 5 is

$$Q_1 + Q_2 = 2Q = Q_1' + Q_2',$$

and the negative of these on the lower plates. We can thus draw Step 5 showing

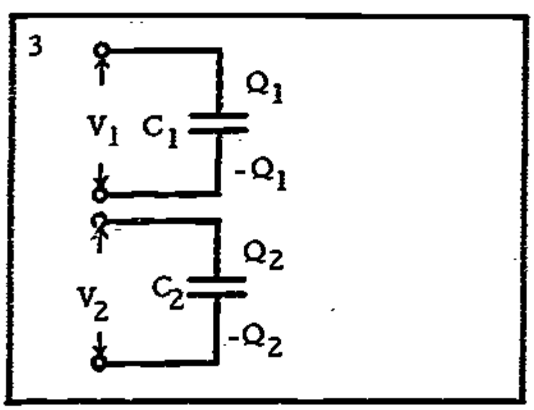


Step 1: Part (a) situation. Note that $Q_1 = Q_2 = Q$.

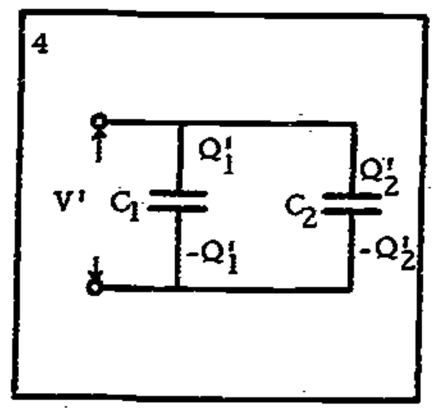


Step 2: Equivalent capacitor for Part (a).

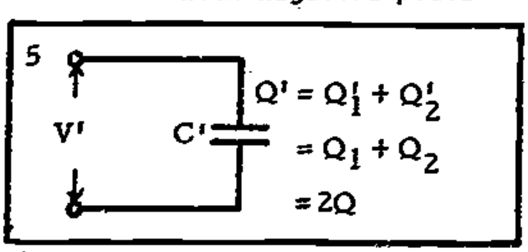
Figure 5



Step 3: Capacitors are disconnected from each other. $Q_1 = Q_2$.



Step 4: Capacitors are reconnected with positive plates together and with negative plates together.



Step 5: Equivalent capacitor for arrangement in Part (b).

an equivalent capacitance $C' = C_1 + C_2$, with charge $Q' = Q_1' + Q_2' = 2Q$ on it and potential difference $V' = Q'/C'$ across it. Having calculated V we can go back to Step 4 and determine the charges Q_1' and Q_2' on C_1 and C_2:

$$Q_1' = C_1V', \quad Q_2' = C_2V'.$$

Sometimes you may find that writing out the answer in terms of the given quantities gives a rather complicated expression, and it may be easier to compute intermediate numbers. A good rule of thumb is to give writing out an algebraic expression a try before you give up and plug in numbers along the way. In this case,

$$Q_1' = C_1V = C_1Q'/C' = C_1[2Q/(C_1 + C_2)]$$

$$= \frac{2C_1V}{(C_1 + C_2)(1/C_1 + 1/C_2)} = \frac{2C_1VC_1C_2}{(C_1 + C_2)^2} = \frac{2C_1^2C_2V}{(C_1 + C_2)^2};$$

also

$$Q_2' = C_2V' = 2C_2^2C_1V/(C_1 + C_2)^2 ,$$

which equations are not too complicated to evaluate. The potential across both capacitors is

$$V' = \frac{Q'}{C'} = \frac{2Q}{C_1 + C_2} = \frac{2}{C_1 + C_2}\frac{V}{(1/C_1 + 1/C_2)} = \frac{2C_1C_2V}{(C_1 + C_2)^2} .$$

(c) In this case we have a similar sequence of steps. The difference is that now we have $Q' = Q_1 - Q_2 = 0$.

G(4). Two capacitors, one of 1.00 μF and the other of 2.00 μF, are each charged initially by being connected to a 10.0-V battery. Then the two capacitors are connected together. What is the total electric energy stored in each capacitor, if the capacitors are connected such that

(a) both positive plates are brought together, and
(b) plates of opposite charge are brought together?
(c) Account for the lost energy in part (a).

Solution

This problem is very similar to earlier problems except that now you must also compute the energy stored in each capacitor, $(1/2)CV^2$, and add up all this stored energy. The energy that "disappears" in this problem goes into heating the wires that make the connections or into electromagnetic radiation.

H(5). A parallel-plate capacitor is half filled with a dielectric of dielectric constant κ, as shown in Figure 6. What is the capacitance?

Solution

You can consider this capacitor to be two capacitors in parallel, one filled with dielectric and the other one not. Thus, if C is the original capacitance,

$$C = (1/2)C + (1/2)C\kappa = (1/2)C(1 + \kappa).$$

Problems

I(2). A capacitor is made of two flat conducting circular plates of radius 12.0 cm, separated by 0.50 mm. Determine the capacitance.

J(3). In the circuit shown in Figure 7, the battery provides a constant potential difference of 50 V and C_1 and C_2 are initially uncharged. Switch S_1 is closed, charging the 100-μF capacitor C_1. Then S_1 is opened, disconnecting the battery from the circuit. Following this, S_2 is closed. The value of the potential difference V across C_2 is then measured to be 35 V. Determine the capacitance of C_2.

K(4). An isolated metal sphere whose diameter is 10.0 cm has a potential of 8000 V. What is the energy density at the surface of the sphere?

L(4). Compute the energy stored in the system of Problem E if V = 50 V.

M(5). A Geiger counter is made of two long, concentric metal cylinders with a gas of dielectric constant κ between them. Neglecting end effects, use Gauss' law to calculate the capacitance of this configuration. The center rod has a radius a, the surrounding tube a radius b, and the length $\ell = b$.

N(5). A parallel-plate capacitor of plate area A and separation d is charged by a battery to a potential B, and is then disconnected from the battery.
(a) Give expressions for the energy stored in and charge on the capacitor.
(b) A slab of dielectric with constant κ is then inserted into the capacitor, completely filling the space between the plates. Determine the capacitance, charge, potential difference, energy stored, and electric field in the capacitor. Describe qualitatively what happens to the "lost" energy, and the distribution of polarization charge in the dielectric.

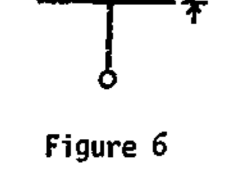


Figure 6

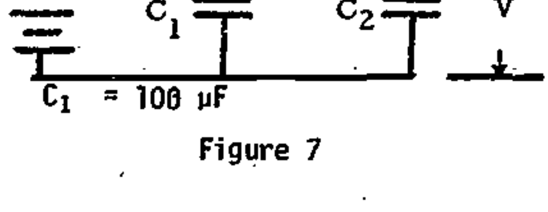


Figure 7

Solutions

I(2). Use the expression, which you should be able to derive, to determine that the capacitance is 8.0×10^{-10} F.

J(3). This problem is completely equivalent to a problem that reads: "A 100-μF capacitor is charged to 50 V, the battery then being disconnected. The capacitor is then connected across a second initially uncharged capacitor. If the potential difference drops to 35 V, what is the capacitance of the second capacitor?" Answer: 43 pF.

K(4). $0.100\ \mathrm{J/m^3}$.

L(4). 3.7 mJ.

M(5). $2\pi\varepsilon_0\kappa\ell/\ln\frac{b}{a}$.

N(5). Q does not change. $E = B/d = Q/\kappa C_0 d$. $C = \kappa C_0$. Charge is fixed. $V = Q/\kappa C_0$.

PRACTICE TEST

1. Derive the expression for the capacitance of a capacitor that consists of concentric cylindrical conducting shells of radius a and b ($a < b$) and of length L.

2. A 5.0-μF capacitor is charged to 10.0 V and an 8.0-μF capacitor is charged to 5.0 V. They are then connected in parallel (positive plate of one to the negative plate of the other) with an initially uncharged 4.0-μF capacitor. Determine the energy stored in the final configuration.

3. A parallel-plate capacitor with plate separation d has a capacitance C.
(a) If a dielectric slab of thickness d/3 and dielectric constant 2.50 is inserted between the plates and parallel to them, determine the ratio of the capacitance with the dielectric in place to the capacitance without the dielectric.
(b) Make a sketch showing the location of free and polarization charges (with their signs) when the capacitor is charged with the dielectric in place.

Answers--

1. $C = \frac{2\pi\varepsilon_0 L}{\ell n(b/a)}$; 2. 2.9×10^{-6} Joules ; 3. (a) 1.25.

CAPACITORS

Mastery Test   Form A

Name ______________________

Date ______________

pass   recycle

1 2 3 4 5

Tutor ______________

1. Two oppositely charged ($\pm Q$) parallel plates have an area A and are separated by a distance d in vacuum. The electric field between the plates is perpendicular to the surface and has a magnitude $E = Q/\varepsilon_0 A$.

   (a) Write the defining equation for capacitance.

   (b) Determine the potential difference between plates.

   (c) Use the results from parts (a) and (b) to obtain an expression for the capacitance of two parallel plates.

   (d) Determine the energy per unit volume between the plates.

2. Given the capacitor scheme in Figure 1, with the indicated capacitances:

   (a) Reduce the various capacitors to a single equivalent capacitor.

   (b) Determine the charge on, voltage across, and energy stored in the 1.00-μF capacitor.

3. Two capacitors are connected in series with a battery as shown in Figure 2. Describe quantitatively what happens to the charge, potential difference, capacitance, and energy of each before and after a dielectric slab of constant κ is inserted into the bottom capacitor.

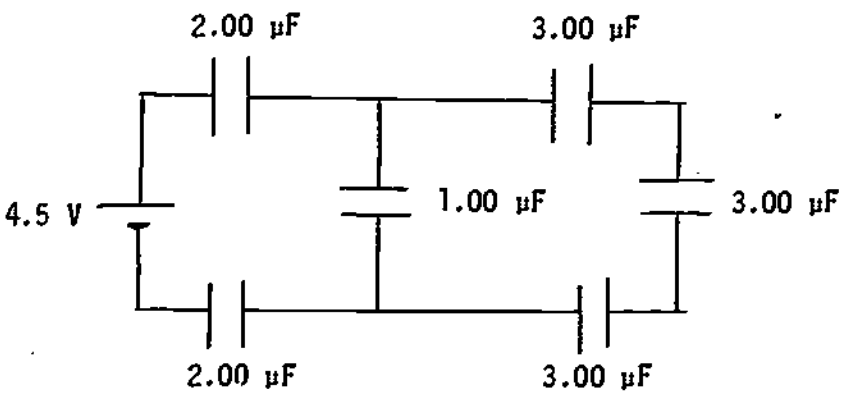


Figure 1

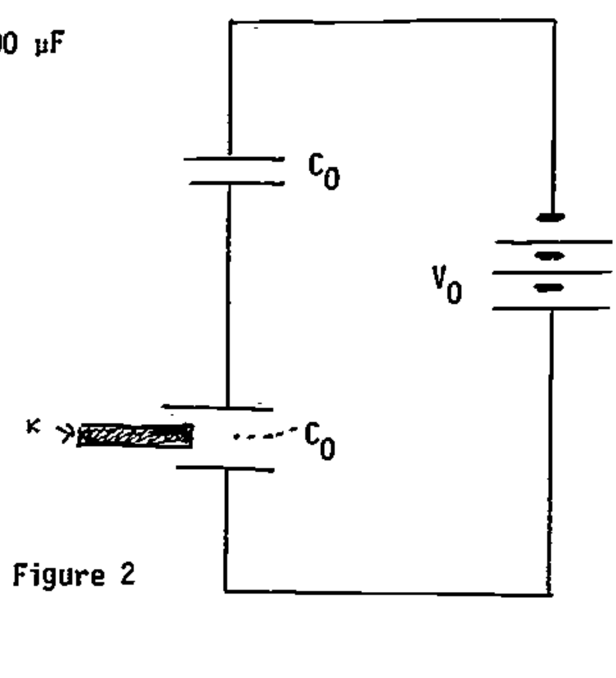


Figure 2

CAPACITORS

Date ______________

Mastery Test Form B

pass recycle

1 2 3 4 5

Name ______________________ Tutor ______________________

1. A spherical air capacitor consists of two concentric spherical shells. The inner sphere has a radius a and charge +Q. The outer sphere has a radius b and a charge -Q.

   (a) Write the defining equation for capacitance.
   (b) Determine the potential difference.
   (c) Use the results from parts (a) and (b) to obtain an expression for the capacitance of a spherical capacitor.
   (d) Determine the energy stored between the spheres.

2. Given the capacitor scheme shown in Figure 1:

   (a) Reduce the various capacitors to a single equivalent capacitor.
   (b) Determine the charge on, voltage across, and energy stored in the 3.00-μF capacitor.

3. The capacitor in Figure 2 is charged by using a battery, which is then disconnected. A dielectric slab that fills the volume between capacitor plates is then placed between them. Describe, quantitatively, what happens to the charge, capacitance, potential difference, electric field, and stored energy. Must you do work to insert the slab, or is work done on it by the field? Does the charge or the potential stay fixed as the slab goes in?

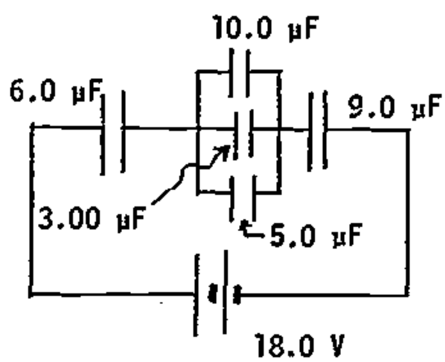


Figure 1

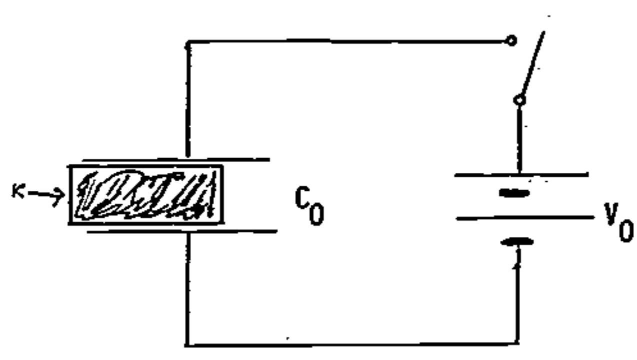


Figure 2

CAPACITORS

Date ____________________

Mastery Test   Form C

pass   recycle

1   2   3   4   5

Name ______________________________   Tutor ______________________________

1. A coaxial cable consists of an inner solid, cylindrical conductor of radius a supported by insulating disks on the axis of a thin-walled conducting tube of inner radius b. The two cylinders are oppositely charged with a charge per unit length of λ. The electric field between the cylinders is

$$\vec{E} = \frac{\lambda}{2\pi\varepsilon_0}\left(\frac{1}{r}\right)\left(\frac{\vec{r}}{r}\right) ,$$

where $\frac{\vec{r}}{r}$ is a unit vector in the r direction.

   (a) Write the defining equation for capacitance.
   (b) Use the expression for the electric field to determine the potential difference.
   (c) Use the results from parts (a) and (b) to obtain an expression for the capacitance of the coaxial cable.
   (d) Determine the energy stored per unit volume in the region $a < r < b$.

2. Given the capacitor scheme shown in Figure 1, (a) reduce the various capacitors to a single equivalent capacitor; and (b) determine the charge on, voltage across, and energy stored in the 6.0-μF capacitor.

3. In Figure 2, two identical capacitors of capacitance C_0 are connected in parallel and charged to a voltage V_0. A slab of dielectric constant κ is then inserted in one of the capacitors. Describe, quantitatively, what happens to the charge, capacitance, potential difference, polarization charge, and stored energy of each. The capacitors are disconnected from the battery (switch open) before the dielectric is inserted. First decide if the charge or the potential difference is fixed as the slab goes in.

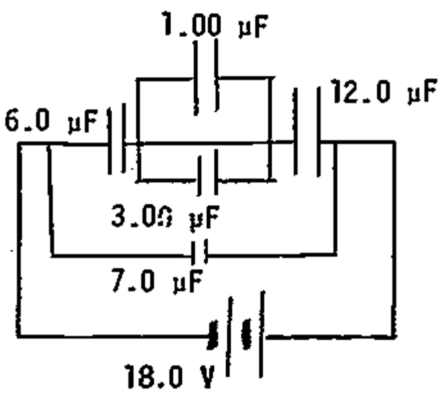


Figure 1

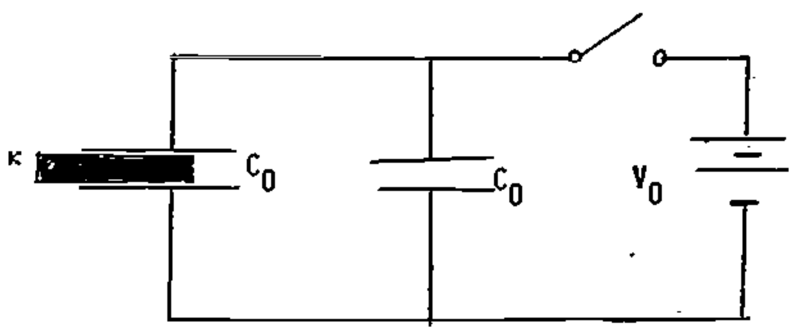


Figure 2

MASTERY TEST GRADING KEY - Form A

1. Solution: (a) $C = Q/V$. (b) $V = Ed = Qd/\varepsilon_0 A$. (c) $C = \varepsilon_0 A/d$.
(d) $U = (1/2)\varepsilon_0 E^2 = (1/2)\varepsilon_0 (Q^2/\varepsilon_0^2 A^2) = (1/2)(Q^2/\varepsilon_0 A^2)$.

2. Solution: (a) Find the equivalent capacitance:

$\frac{1}{C_1} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$, $C_1 = 1.00$ µF. $C_2 = 2.00$ µF.

$\frac{1}{C_T} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{3}{2}$, $C_T = 2/3$ µF.

(b) $V = 4.5$ V. $E = (1/2)CV^2 = (1/2)(10^{-6})(20) = 10^{-5}$ J.

$Q = CV = (10^{-6})(4.5) = 4.5 \times 10^{-6}$ C.

3. Before: $C_{eq} = C_0/2$. $Q = (C_0/2)V_0$ on each. $V = (C_0V_0/2C_0) = V_0/2$ on each.
$C = C_0$ on each. $E = (1/2)C_0V^2 = (1/2)C_0(V_0^2/4) = (1/8)C_0V_0^2$ on each.

After: $C_{top} = C_0$. $C_{bottom} = \kappa C_0$. $C_{eq} = C_0/(1 + \kappa)$. $Q = C_0V/(1 + \kappa)$ on each. $V_{top} = \kappa V/(1 + \kappa)$. $E_{top} = (1/2)C_0[\kappa/(1 + \kappa)]^2V^2$.

$V_{bottom} = V/(1 + \kappa)$. $E_{bottom} = [\kappa C_0/(1 + \kappa)][1/(1 + \kappa)]^2V^2$.

MASTERY TEST GRADING KEY - Form B

1. Solution: (a) $C = Q/V$.

   (b) $V = \frac{Q}{4\pi\varepsilon_0}(\frac{1}{a} - \frac{1}{b})$.

   (c) $C = 4\pi\varepsilon_0 ab/(b - a)$.

(d) $E = \frac{1}{2}CV^2 = \frac{1}{2}(\frac{4\pi\varepsilon_0 ab}{b - a})(\frac{Q^2}{(4\pi\varepsilon_0)^2})(\frac{(b - a)^2}{a^2b^2}) = \frac{(b - a)Q^2}{ab4\pi\varepsilon_0}$.

2. Solution: (a) $\frac{1}{C} = \frac{1}{6.0} + \frac{1}{18.0} + \frac{1}{9.0} = \frac{6}{18} = \frac{1}{3}$. $C = 3.00$ μF.

   (b) $Q_3 = (3.00 \times 10^{-6})(3) = 9.0 \times 10^{-6}$ C. $V_3 = 3.00$ V.

   $E = (1/2)CV^2 = (1/2)(3.00 \times 10^{-6})(9) = 13.5 \times 10^{-6}$ J.

   $Q = C_{eq}V = 54 \times 10^{-6}$ C. $V_6 = 54/6 = 9.0$ V.

   $V = 54/9 = 6.0$ V.

3. Solution: Q does not change. $E = V/d = Q/\kappa C_0 d$. $C = \kappa C_0$. Charge is fixed. $V = Q/\kappa C_0$. Work is done.

MASTERY TEST GRADING KEY - Form C

1. Solution: (a) $C = Q/V$.

(b) $V = \int \vec{E} \cdot d\vec{r} = \frac{\lambda}{2\pi\varepsilon_0} \ln \frac{b}{a}$.

(c) $C = \frac{2\pi\varepsilon_0 \lambda \ell}{\lambda \ln(b/a)} = \frac{2\pi\varepsilon_0 \ell}{\ln(b/a)}$.

(d) $E_{total} = \frac{1}{2}CV^2$,

$$U = \frac{E_{total}}{\pi(b^2 - a^2)\ell} = \frac{2\pi\varepsilon_0 \ell \lambda^2 \ln^2(b/a)}{2\pi(b^2 - a^2)\ell \ln(b/a)\ (4\pi^2\varepsilon_0^2)} = \frac{\lambda^2 \ln(b/a)}{4\pi^2\varepsilon_0(b^2 - a^2)}$$

2. Solution: (a) $\frac{1}{C_1} = \frac{1}{6.0} + \frac{1}{4.0} + \frac{1}{12.0} = \frac{6.0}{12.0}$. $C_1 = 2.00\ \mu F$. $C_{eq} = 9.0\ \mu F$.

(b) $Q = CV = 9.0 \times 18.0 \times 10^{-6} = 162 \times 10^{-6}$ C.

$Q_7 = 7.0 \times 18.0 \times 10^{-6} = 126 \times 10^{-6}$ C.

$Q_6 = 36 \times 10^{-6}$ C. $V = Q/C = 36/6.0 = 6.0$ V.

$E = (1/2)CV^2 = (1/2)(6.0)(36) = 108 \times 10^{-6}$ J.

3. Solution:

	Initial	Final
Free charge	$2C_0V_0$	$2C_0V_0$
Capacitance	$2C_0$	$C_0(\kappa+1)$
Voltage	V_0	$2V_0/(\kappa + 1)$
Polarization charge	0	$2V_0C_0(\kappa - 1)/(\kappa + 1)$
Stored Energy	$C_0V_0^2$	$2C_0V_0^2/(\kappa + 1)$