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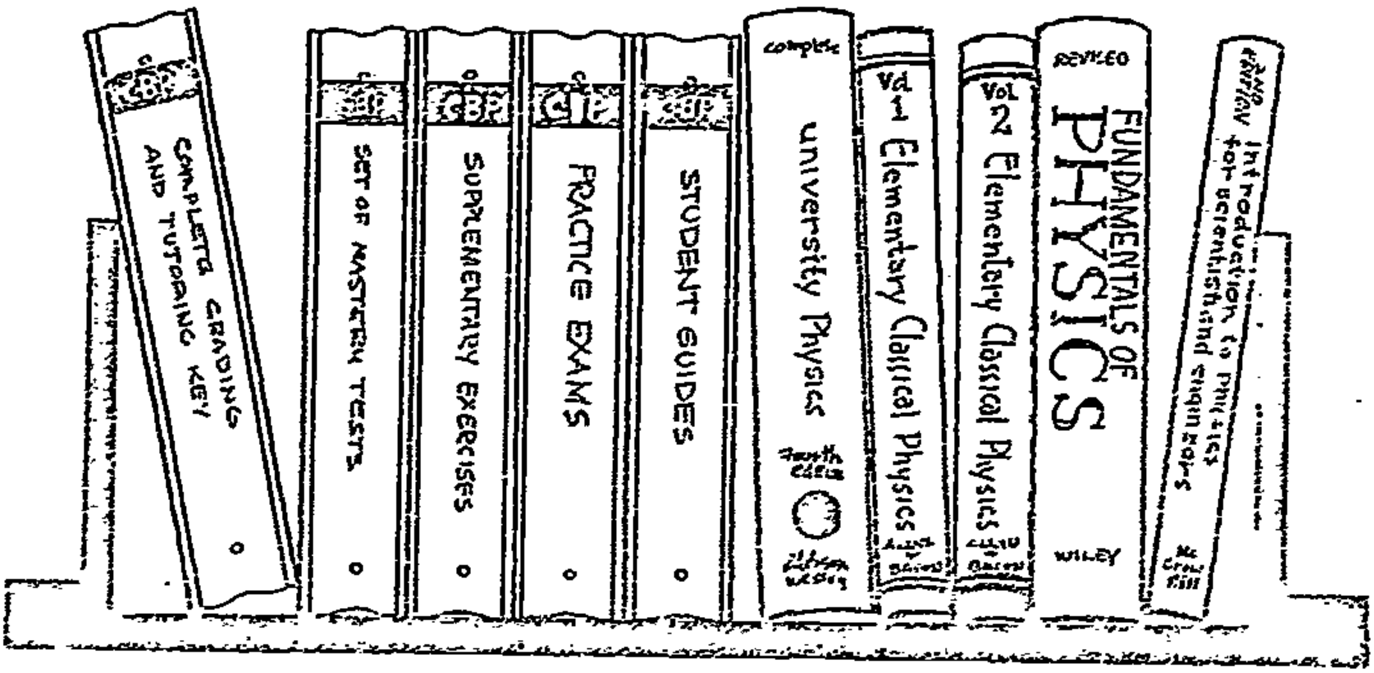
ABSTRACT

This is part of a series of 42 Calculus Based Physics (CBP) modules totaling about 1,000 pages. The modules include study guides, practice tests, and mastery tests for a full-year individualized course in calculus-based physics based on the Personalized System of Instruction (PSI). The units are not intended to be used without outside materials: references to specific sections in four elementary physics textbooks appear in the modules. Specific modules included in this document are: Module 21--Second Law and Entropy, Module 22--Coulomb's Law and the electric Field, and Module 23--Flux and Gauss' Law. (CP)

STUDY MODULES FOR CALCULUS-BASED GENERAL PHYSICS*



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Comments

These modules were prepared by fifteen college physics professors for use in self-paced, mastery-oriented, student-tutored, calculus-based general physics courses. This style of teaching offers students a personalized system of instruction (PSI), in which they increase their knowledge of physics and experience a positive learning environment. We hope our efforts in preparing these modules will enable you to try and enjoy teaching physics using PSI.

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These modules were prepared by the module authors at a College Faculty Workshop held at the University of Colorado - Boulder, from June 23 to July 11, 1975.

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COMMENT TO USERS

In the upper right-hand corner of each Mastery Test you will find the "pass" and "recycle" terms and a row of numbers "1 2 3 ..." to facilitate the grading of the tests. We intend that you indicate the weakness of a student who is asked to recycle on the test by putting a circle around the number of the learning objective that the student did not satisfy. This procedure will enable you easily to identify the learning objectives that are causing your students difficulty.

COMMENT TO USERS

It is conventional practice to provide several review modules per semester or quarter, as confidence builders, learning opportunities, and to consolidate what has been learned. You the instructor should write these modules yourself, in terms of the particular weaknesses and needs of your students. Thus, we have not supplied review modules as such with the CBP Modules. However, fifteen sample review tests were written during the Workshop and are available for your use as guides. Please send $1.00 to CBP Modules, Behlen Lab of Physics, University of Nebraska - Lincoln, Nebraska 68588.

FINIS

This printing has completed the initial CBP project. We hope that you are finding the materials helpful in your teaching. Revision of the modules is being planned for the Summer of 1976. We therefore solicit your comments, suggestions, and/or corrections for the revised edition. Please write or call

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SECOND LAW AND ENTROPY

INTRODUCTION

Suppose you get into your car and drive to ________.* During this trip, (1) the burning of the gasoline in the engine cylinders converts chemical energy into thermal energy of the gases - that is, they become very hot; (2) the expansion of these hot gases turns the crankshaft, performing work; and (3) this work, transmitted to the wheels, gives the car kinetic energy - which must be continually replenished, because of depletion by friction and air resistance. At the end of this trip, ________ gallons of gasoline have been converted into water vapor, carbon dioxide, and sundry less desirable vapors, scattered across _______ miles of countryside. In addition, the road, the air along it, and your engine have been heated up. No energy has been lost, but energy has merely been converted to different forms. So why not somehow collect all that energy again, and use it to drive another engine (presumably of a different kind)? That would not violate the first law of thermodynamics - but try to do it!

Well, then, perhaps we should concentrate just on the operation of the heat engine, step 2 above. Present-day gasoline engines convert only a small fraction of the "heat energy" released in their cylinders into useful work. Since we are being pinched by energy shortages, why not gear up a research program to develop engines with (say) 90% efficiency? Again, this too is impossible!

These are illustrations of the fact that energy is often unavailable (or only partially available) for conversion into work. There is a fundamental limit to the efficiency that can be obtained in this conversion - a limit that cannot be surpassed, regardless of technological developments. The basis of that limit is the subject of this module.

*Student should fill in the blanks as appropriate.

PREREQUISITES

Before you begin this module, you should be you should be able to:	Location of Prerequisite Content
*Write the equation of state of an ideal gas and find one parameter in terms of the others. Know that the internal energy is a function only of the absolute temperature for an ideal gas (needed for Objective 3 of this module)	Kinetic Theory of Gases Module
*Write and evaluate the expression for work done by or on an ideal gas by various processes (needed for Objective 3 of this module)	Temperature, Heat, and Thermodynamics Module
*State and use the first law of thermodynamics; and describe the absolute-temperature (K) scale and how to convert degrees Celsius to kelvins (needed for Objective 3 of this module)	Temperature, Heat, and Thermodynamics Module
*Calculate the specific heat at constant volume for an ideal gas (needed for Objective 3 of this module)	Kinetic Theory of Gases Module
*Solve problems involving the latent heats of fusion and vaporization and specific heat (needed for Objective 3 of this module)	Temperature, Heat, and Thermodynamics Module

LEARNING OBJECTIVES

After you have mastered the content of this module, you will be able to:

1. Definitions - Define the following: (a) reversible process, (b) irreversible process, (c) state variable, (d) cycle, (e) efficiency, (f) Carnot cycle, and (g) entropy.

2. Second law of thermodynamics - State the second law of thermodynamics as it relates to entropy on a macroscopic scale, and show that the second-law statements relating to heat flow satisfy this statement.

3. Entropy - Solve problems involving the following concepts: (a) efficiency and heat cycles from pV diagrams or from information necessary to construct pV diagrams; (b) Carnot cycles; and/or (c) entropy.

GENERAL COMMENTS

1. Definitions

The important definitions to know for this module are the following:

(a) Reversible process: a process that, by a differential change in the environment, can be made to retrace its path. Note: reversible processes can be represented graphically since at any time the system is (essentially) in equilibrium.

(b) Irreversible process: a process that, by a differential change in the environment, cannot be made to retrace its path. Note: irreversible processes cannot in general be represented graphically since some irreversible processes involve nonequilibrium states, which cannot be represented on a graph.

(c) State variable: a variable such that the integral over any closed path of the differential of that variable is zero, symbolically $\int ds = 0$, where s is the symbol representing the state variable. Note: recall the definition of a conservative force in Conservation of Energy and note the similarity between this definition and the definition of a state variable. State variables that you have encountered in thermodynamics so far are: U (internal energy), p (pressure), T (temperature), and V (volume). Differential increments of variables that are not state variables will be represented by slashes through the ds, e.g., $đQ$ and $đW$.

(d) Cycle: a sequence of processes that a system goes through, such that the system returns to its original equilibrium state. Note: cycles can be either reversible or irreversible.

(e) Efficiency: $e \equiv W_{out}/Q_{in}$ for a cycle.

(f) Carnot cycle: a cycle consisting of two reversible adiabatic processes and two reversible isothermal processes.

(g) Entropy: $dS \equiv đQ/T$, where S is the entropy, Q is the heat, and T is the absolute temperature. Note: The notation indicates that entropy is a state variable whereas heat ($đQ$) is not.

2. The Second Law of Thermodynamics

In your previous study of physics you have been concerned with conservation laws. i.e., conservation of energy (Conservation of Energy, Temperature, Heat, and Thermodynamics), conservation of linear momentum (Collisions), and conservation of angular momentum (Rotational Dynamics). Now we are going to consider a fundamental law of physics that is not a conservation law. In this case the law relates to a quantity that can never decrease when all parts of the interaction are considered. You already have considered a quantity that can proceed in one direction only, namely, time. You know from personal experience that people are born, age, and then die and not the other way around. This fundamental law we shall now study is necessary in order to understand why certain phenomena occur the way they do; for example, when we mix hot and cold water together the entire mixture comes to an equilibrium temperature somewhere in between the two initial temperatures. It never occurs (although according to statistical mechanics there is an infinitely small probability that it might) that the water that was initially hot gets hotter and the cold water gets colder. It would not violate the first law of thermodynamics (if the hot water did get hotter and the cold colder), thus we need another law to explain why this never happens. This law is simply called the "second law of thermodynamics" (indicating that physicists have very little imagination). Your text gives several equivalent statements of the second law. The one that you should memorize (since it is more general and the other statements can be obtained from this statement) is

SECOND LAW OF THERMODYNAMICS

> A process that starts in one equilibrium state and ends in another will proceed in the direction that causes the entropy of the system plus its environment to increase or to remain the same.

You will see that the second law of thermodynamics applied to reversible processes gives $\Delta S = 0$, and applied to irreversible processes gives $\Delta S > 0$. Since all <u>real</u> processes are irreversible, the second law should read something to the effect that for all real processes $\Delta S > 0$. However, since the analysis of reversible processes can yield much valuable information (like the analysis of projectile motion in the absence of perturbing effects like wind, etc.) we shall use the statement given above as our definition. Our analysis of the ramifications of the second law will for the most part (some of your texts briefly mention microscopic ramifications of the second law) be concerned with the macroscopic domain. For a more fundamental understanding of entropy and the second law, you are referred to the excellent text: <u>Statistical Physics</u> by Reif.*

3. Important Concepts

In addition to the second law of thermodynamics, the other important concepts in this module are

(a) <u>Efficiency</u>. Loosely speaking, the efficiency of a process is the measure of what we get out of the process divided by what we put into it. In thermodynamics we are concerned with heat engines (cycles involving an exchange of heat), and thus we define efficiency more precisely as given above in the definitions.

(b) <u>The Carnot cycle</u>. The Carnot cycle is important since the most efficient engine operating between two temperature reservoirs is that utilizing a Carnot cycle. The efficiency of the Carnot cycle is given by

$$e = 1 - T_c/T_h$$

and is independent of the working substance, e.g., an ideal gas or peanut butter. Since the efficiency is independent of the working substance we can define an absolute-temperature scale by use of the Carnot cycle. This temperature scale happens to be identical to the Kelvin scale introduced earlier in <u>Temperature, Heat, and Thermodynamics</u>. (This also shows that, whereas physicists may not be very imaginative, they do plan ahead.) Since they are easy to deal with, much of the material and many of the problems involve ideal gases; however, the concepts are general.

*X. X. Reif, <u>Statistical Physics</u> (McGraw-Hill, New York, 19XX), Berkeley Physics, Vol. V.

(c) Entropy. Entropy is a state variable of thermodynamic systems, and is involved in a fundamental law of thermodynamics, namely, the second law. It is also a measure of the disorder of a system; however, we shall not go into this aspect of it in the present module (see Reif* for elaboration).

(d) Adiabatic process. You need to know that for an ideal gas undergoing an adiabatic process, if $đQ = 0$, then

$$pV^{\gamma} = \text{const} \qquad (\gamma = C_p/C_v).$$

TEXT: Frederick J. Bueche, Introduction to Physics for Scientists and Engineers (McGraw-Hill, New York, 1975), second edition

SUGGESTED STUDY PROCEDURE

Read Chapter 17, Section 17.4. Then, read sections 17.5 through 17.11. Notes: The sentence in the first paragraph of Section 17.5 beginning "Let us consider..." is misleading: Real engines cannot be represented on a pV diagram, since in real engines one does not have an equilibrium situation at any time during the operation. However, one can approximate the operation of a real engine by a well-chosen reversible cycle that can be represented on a pV diagram, and which then can be analyzed according to well-known thermodynamic procedures.

In Section 17.6, in the second paragraph the sentence, "When the system reaches point B it is suddenly (and therefore adiabatically) expanded along curve BC," is misleading. A sudden expansion is adiabatic by its very nature since there is no time for heat to be transferred. However, a Carnot cycle is a reversible cycle - a sudden expansion is an irreversible process and therefore cannot be part of a reversible cycle. The statement should be reformulated to read: "...it is expanded adiabatically - that is, slowly enough to be considered reversible yet quickly enough so that a significant amount of heat is not transferred to the gas from the environment. Both of these conditions can be satisfied by the use of sufficient insulation. Similar statements apply to the statement in the next paragraph dealing with an adiabatic compression. Equation (17.15) in Section 17.7 applies to a Carnot cycle independently of the working substance even though it was derived for an ideal gas. In the first full paragraph under Eq. (17.15) the sentence should read: "Although it applies only to the Carnot cycle, it nevertheless sets an upper limit for all other cycles, since they are less efficient than the Carnot cycle."

BUECHE

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text	Study Guide	Text (Chap. 17)	(Chap. 17)
1	Secs. 17.2, 17.10, 17.6					
2	General Comments, Sec. 17.11					
3(a)	Secs. 17.5, 17.6	A		D	17	Quest.[a] 9 to 12
3(b)	Secs. 17.6, 17.7	B		E		Probs. 15, 18
3(c)	Secs. 17.10, 17.11	C	Illus.[a] 17.4, 17.5, Sec. 17.11, Cases 1, 2	F	19, 21	Probs. 20, 22, 23

[a]Illus. = Illustration(s). Quest. = Question(s).

You should be able to show that the statement of the second law of thermodynamics given at the beginning of Section 17.8 (p. 311) follows from the statement given in the last sentence of Section 17.11 (on p. 320 prior to the Questions) and from our formulation in General Comment 2. Work Problems 17, 19, and 21 in Chapter 17 of your text and Problems A through F in this study guide. When you think you know the material well enough to satisfy the objectives, take the Practice Test.

TEXT: David Halliday and Robert Resnick, Fundamentals of Physics (Wiley, New York, 1970; revised printing, 1974)

SUGGESTED STUDY PROCEDURE

Read Example 5 in Section 21-6 (Chapter 20, pp. 385, 386). Then read Sections 21-1 through 21-8. Work Problems 1, 5, 7, 11, 16, 21, and 33 in Chapter and Problems A through F in this study guide. When you think that you know the material well enough to satisfy the objectives, take the Practice Test.

HALLIDAY AND RESNICK

Objective Number	Readings	Problems with Solutions: Study Guide	Problems with Solutions: Text (Chap. 21)	Assigned Problems: Study Guide	Assigned Problems: Text (Chap. 21)	Additional Problems (Chap. 21)
1	General Comments, Secs. 21-2, 21-2, 21-6, 21-3					Quest.[a] 4, 7, 2, 5, 6, 10, 11, 18
2	General Comments, Secs. 21-1, 21-8					Quest. 3
3(a)	Secs. 21-3, 21-5	A		D	11, 16	Probs. 10, 14, 17, 18, 19
3(b)	Sec. 21-3	B	Ex.[a] 1, 2	E	1, 5, 7	Quest. 8, 9, 12, Probs. 2, 3, 8
3(c)	Secs. 21-6, 21-7, 21-8	C	Ex. 3, 4	F	21, 33	Quest. 14 to 17, Probs. 20, 23, 25 to 32

[a]Ex. = Example(s). Quest. = Question(s).

STUDY GUIDE: Second Law and Entropy 6(SZ 1)

TEXT: Francis Weston Sears and Mark W. Zemansky, University Physics (Addison-Wesley, Reading, Mass., 1970), fourth edition

SUGGESTED STUDY PROCEDURE

Read Section 19-13 in Chapter 19. Then read Sections 19-14 through 19-24. Note: Your text does not distinguish between state variables and nonstate variables. Make sure you understand this distinction from the material in General Comment 1. When you think that you do, write the first law of thermodynamics in differential form, distinguishing state variables from nonstate variables. (See answer at the bottom of this page.) If you missed this, review the material again - if you still do not understand this, ask your tutor for assistance.

You should be able to show that the statements of the second law of thermodynamics given in Sections 19-18 and 19-19 (pp. 274, 276) of the text follow from the statement given in Section 19-24 (p. 280) and General Comment 2. Work Problems 19-24, 19-26, 19-31, 19-32 in your text, and Problems A through F in this study guide. When you think that you know the material well enough to satisfy the objectives, take the Practice Test.

SEARS AND ZEMANSKY

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text	Study Guide	Text	
1	General Comments, Secs. 19-20, 19-14, 19-23					
2	General Comments, Sec. 19-24					
3(a)	Secs. 19-14 to 19-17, 19-19, 19-20	A		D	19-24	19-23, 19-25, 19-30
3(b)	Secs. 19-20 to 19-22	B		E	19-26	19-27 to 19-29
3(c)	Secs. 19-23, 19-24	C	Ex.[a] 1, 2, 3 (Sec. 19-23)	F	19-31, 19-32	

[a]Ex. = Example(s).

Answer: $dU = \text{đ}Q - \text{đ}W$.

TEXT: Richard T. Weidner and Robert L. Sells, Elementary Classical Physics (Allyn and Bacon, Boston, 1973), second edition, Vol. 1

SUGGESTED STUDY PROCEDURE

Read Chapter 18, Section 18-6, and Chapter 21, Sections 21-1 through 21-6. Notes: Your text does not define the concept of state variable explicitly but, rather, alludes to it in the discussion of Eq. (19-13) (pp. 428, 429). Study the definition given in General Comment 1. When you think that you understand this definition, write the first law of thermodynamics in differential form, distinguishing state variables from nonstate variables. (See answer at the bottom of this page.) If you missed this, review the material again - if you still do not understand this, ask your tutor for assistance.

In Section 21-2, on p. 430, the sentence beginning "Chemical or nuclear potential energy..." is misleading. Although it is theoretically possible to convert chemical or nuclear potential energy into thermal energy with 100% efficiency, the author of this module is unaware of any practical device that does so. Work Problems 21-5, 21-6, 21-9, 21-11 in your text and Problems A through F in this study guide. When you think that you know the material well enough to satisfy the objectives, take the Practice Test.

WEIDNER AND SELLS

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text	Study Guide	Text	
1	General Comments, Secs. 21-1, 18-6, 21-2, 21-4, 21-5					
2	General Comments, Sec. 21-6					
3(a)	Secs. 21-1, 21-2	A	Ex.[a] 21-1	D	21-5	21-2, 21-4
3(b)	Secs. 21-3, 21-4	B	Ex. 21-2	E	21-6	21-7, 21-8
3(c)	Secs. 21-5, 21-6	C	Ex. 21-4, 21-5, 21-6	F	21-9, 21-11	21-10, 21-12, 21-13, 21-14

[a]Ex. = Example(s).

Answer: $dU = đQ - đW$.

PROBLEM SET WITH SOLUTIONS

A(3). The gas in the internal combustion engine in your car undergoes a process that can be approximated by a cycle called the Otto cycle. The Otto cycle consists of (1) an adiabatic compression from V_1 to V_2, (2) a pressure increase at constant volume V_2, (3) an adiabatic expansion from V_2 to V_1, and (4) a pressure decrease at constant volume V_1 back to the original pressure.

(a) Why can we not analyze the real process that the gas undergoes in an internal combustion engine?

(b) Draw the Otto cycle on a pV diagram.

(c) Assuming that the gas is an ideal gas, show that the efficiency of the Otto cycle is

$$e = 1 - (V_2/V_1)^{\gamma - 1}$$

where $\gamma \equiv C_p/C_v$. For an ideal gas, $C_p = C_v + R$.

(d) What is the efficiency of an engine whose compression ratio V_1/V_2 is 10 and whose $\gamma = 1.40$?

Solution

(a) The gases in real engines undergo irreversible processes that cannot be plotted on a pV diagram, since the gases are never in an equilibrium state (and only equilibrium states can be represented on a pV diagram).
(b) See Figure 1.
(c) $e = W/Q_{in}$. Since work is done only during the adiabatic processes, let us find an expression for work for an ideal gas during an adiabatic process:

$$dW = p\ dV, \qquad pV^\gamma = C = p_iV_i^\gamma,$$

$$W = \int_{V_i}^{V_f} \frac{c}{V^\gamma}\ dV = c\left(\frac{1}{1-\gamma}\right)V^{1-\gamma}\Big|_{V_f}^{V_f} = \left(\frac{c}{1-\gamma}\right)[V_f^{1-\gamma} - V_i^{1-\gamma}]$$

$$= \frac{p_iV_i^\gamma}{1-\gamma}[V_f^{1-\gamma} - V_i^{1-\gamma}]. \tag{1}$$

Now the only time heat enters or leaves the gas is during the constant-volume process: $đQ = nC_v\ dT$. Q_{in} occurs during process $b \rightarrow c$. Since p is increasing as V remains constant, T increases, or heat is added to system:

$$Q_{in} = nC_v(T_f - T_i). \tag{2}$$

Now let us calculate the total work. Using Equation (1), we have

$$W = W_{cd} + W_{ab} = [p_c V_2^{\gamma}/(1 - \gamma)](V_1^{1 - \gamma} - V_2^{1 - \gamma}) + [p_a V_1^{\gamma}/(1 - \gamma)](V_2^{1 - \gamma} - V_1^{1 - \gamma}).$$

Now $Q_{in} = nC_v(T_c - T_b)$, thus

$$e = \frac{p_c V_2^{\gamma}(V_1^{1 - \gamma} - V_2^{1 - \gamma}) + p_a V_1^{\gamma}(V_2^{1 - \gamma} - V_1^{1 - \gamma})}{(1 - \gamma)nC_v(T_c - T_b)}$$

$$= \frac{p_c V_2[(V_1^{1 - \gamma}/V_2) - 1] + p_a V_1[(V/V_1)^{1 - \gamma} - 1]}{(1 - \gamma)nC_v(T_c - T_b)}, \quad \text{where } V_2/V_1 \equiv \alpha.$$

$$e = \frac{nRT_c(\alpha^{\gamma - 1} - 1) + nRT_a(\alpha^{1 - \gamma} - 1)}{(1 - \gamma)nC_v(T_c - T_b)}.$$

Now use the fact that $C_p = C_v + R$ for ideal gases to eliminate R:

$$\frac{T_c(\alpha^{\gamma - 1} - 1) + T_a(\alpha^{1 - \gamma} - 1)}{(1 - \gamma)[C_v/(C_p - C_v)](T_c - T_b)} = \frac{T_c(\alpha^{\gamma - 1} - 1) + T_a(\alpha^{1 - \gamma} - 1)}{T_b - T_c}.$$

Since $p_a V_1^{\gamma} = p_b V_2^{\gamma}$,

$$p_a V_1(V_1^{\gamma - 1}) = p_b V_2(V_2^{\gamma - 1}), \quad nRT_a(V_1^{\gamma - 1}) = nRT_b(V_2^{\gamma - 1}),$$

or

$$T_a V_1^{\gamma - 1} = T_b V_2^{\gamma - 1} \quad \text{and} \quad T_a = T_b^{\alpha^{\gamma - 1}}.$$

Thus

$$e = \frac{T_c(\alpha^{\gamma - 1} - 1) + T_b^{\alpha^{\gamma - 1}}(\alpha^{1 - \gamma} - 1)}{T_b - T_c} = \frac{T_c(\alpha^{\gamma - 1} - 1) + T_b(1 - \alpha^{\gamma - 1})}{T_b - T_c}$$

$$= 1 - \alpha^{\gamma - 1}.$$

Or $e = 1 - (V_2/V_1)^{\gamma - 1}$

(d) $e = 1.00 - (0.100)^{0.40} = 1.00 - 0.40 = 0.60.$

B(3). A Carnot engine is operated as a refrigerator. If the high-temperature reservoir is at 300 K and the low-temperature reservoir is at 270 K, find the amount of heat that can be removed from the low-temperature reservoir in 10.0 min if the power input is 200 w.

Figure 1

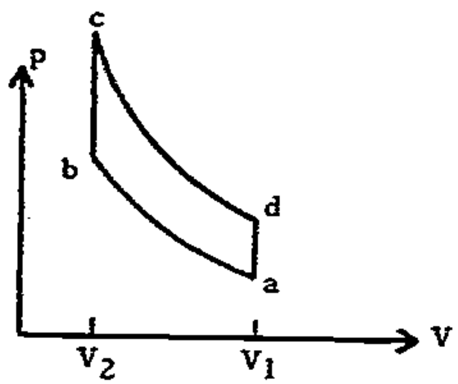
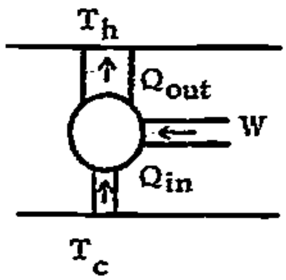


Figure 2



Solution

See Figure 2. We are given that $P = 200$ w, $T_c = 270$ K, and $T_h = 300$ K. We are to find $Q_{in} = ??$ in 10.0 min.

$$e = 1 - \frac{T_c}{T_h} = \frac{W}{Q_{out}}, \quad Q_{out} = W + Q_{in}, \quad 1 - \frac{T_c}{T_h} = \frac{W}{(W + Q_{in})},$$

$$W + Q_{in} = \frac{T_h W}{T_h - T_c}, \quad Q_{in} = W(\frac{T_h}{T_h - T_c}) - 1 = W(\frac{T_h - T_h + T_c}{T_h - T_c}) = W(\frac{T_c}{T_h - T_c}),$$

$$dQ_{in}/dt = (dW/dt)[T_c/T_h - T_c)], \quad P \equiv dW/dt.$$

$$dQ_{in}/dt = (200\text{ w})[270\text{ K}/(300\text{ K} - 270\text{ K})] = (200\text{ w})9.0 = 1800\text{ w}.$$

$$Q_{in} = \int \frac{dQ_{in}}{dt}\, dt = \int_0^{10.0\text{ min}} (1800\text{ w})\, dt = (1800\text{ w}) \int_0^{10.0\text{ min}} dt$$

$$= (1800\text{ w})t\Big|_0^{10.0\text{ min}} = (1800\text{ w})(10.0\text{ min})(60\text{ s}/1\text{ min}),$$

$$Q = 1.80(10^3)(10)(6.0)(10)\text{w} = 1.10(10^6)\text{ J - reasonable??}$$

C(3). (a) A mass m (specific heat C) is heated from T_1 to T_2. Show that the entropy change is given by $\Delta S = mc\ \ln(T_2/T_1)$.

(b) Does the entropy of this substance decrease on cooling? If it does, explain how this ·an happen in light of the second law of thermodynamics.

Solution

Given M, C, $T_i = T_1$, and $T_f = T_2$, we are (a) to show that $S_2 - S_1 = mc \ln(T_2/T_1)$; and (b) show if $đQ < 0$ whether $dS < 0$, and if so, show how this occurs in light of the second law.

(a) $đQ = mC\, dT$, $dS = đQ/T$, $dS = mC\, dT/T$,

$$\int_{S_2}^{S_1} dS = mC \int_{T_1}^{T_2} (dT/T), \qquad S_2 - S_1 = mC \ln T \Big|_{T_1}^{T_2},$$

$S_2 - S_1 = mC \ln(T_2/T_1)$ dimensions?? Is the answer reasonable? How does it depend on m? C? T_2? T_1?

(b) On cooling $T_2 < T_1$,

$$\ln(T_2/T_1) < 0, \qquad S_2 - S_1 < 0.$$

The total entropy will remain unchanged if the process is reversible, since the entropy of the environment will increase by just the right amount to maintain the entropy at a constant value. If the process is irreversible then

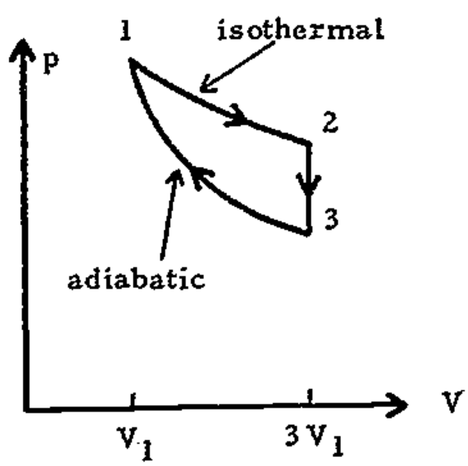
$$|ds_{\text{environment}}| > |dS_{\text{substances}}|.$$

The total entropy will increase.

Problems

D(3). One mole of an ideal gas passes through the cycle shown on the pV diagram in Figure 3. All answers are to be expressed in terms of p_1, V_1, T_1, and R. $\gamma = 5/3$.
(a) Find p_2. (b) Find p_3 and T_3. What is the net work done in one cycle?

Figure 3



E(3). (a) A Carnot engine operates with a hot reservoir at 400 K. If this engine has an efficiency of 30%, what is the temperature of the cold reservoir?
(b) A real steam engine operates with the same cold reservoir as in part (a) with the same efficiency; what do you know about its hot reservoir?

F(3). One end of a cylindrical rod is thermally in contact with a heat reservoir whose temperature is 127°C; the other end is thermally in contact with a heat reservoir whose temperature is 27°C. After the rod has achieved steady-state conditions: (a) Compute the change in the entropy of the universe due to 5021 J of heat flowing through the rod. (b) Does the entropy of the rod change during this process? Why or why not?

Solutions

D(3). (a) Use $T_1 = T_2$ to find $p_2 = p_1/3$.
(b) Use $V_2 = V_3$ and $p_3V_3^{\gamma} = p_1V_1^{\gamma}$ to find $p_3 = 0.160p_1$ and $T_3 = 0.48T_1$.
(c) Use $W = \int p\, dW$ to find $W = 0.320RT_1$.

E(3). (a) $e = 1 - T_c/T_h$, $T_c = 280$ K.
(b) $e_{\text{Carnot}} \geq e_{\text{any other device operating between the same two temperature reservoirs}}$
$T_{h(\text{steam})} \geq T_{h(\text{Carnot})}$.

F(3). (a) $\Delta S_{\text{total}} = 4.2$ J/K.
(b) No, the rod does not change its state. Since S is a state variable, $\Delta S_{\text{rod}} = 0$.

PRACTICE TEST

1. Define: (a) state variable, (b) efficiency, and (3) entropy.

2. State the second law of thermodynamics as it relates to entropy on a macroscopic scale, and show that the second-law statements relating to heat flow satisfy this statement.

3. (a) A Carnot-cycle heat engine operates between a thermal reservoir at a temperature of 140°C and a reservoir at 40°C. If the engine does 2092 J of work, calculate the heat taken in and the heat rejected.
(b) Calculate the change in entropy of the engine for each process of the Carnot cycle in part (a), and then calculate the total entropy change for the cycle.

Practice Test Answers

1. (a) A variable such that the integral over any closed path of the differential of the state variable is zero, symbolically $\oint dS = 0$.
(b) $e \equiv W_{out}/Q_{in}$ over a cycle.
(c) $dS \equiv đQ/T$, where S is the entropy, Q heat, and T the absolute temperature.

2. A process that starts in one equilibrium state and ends in another will go in the direction that causes the entropy of the system plus its environment to increase or to remain the same.

One of the second-law statements relating to heat flow is: "There can be no process whose sole effect is to convert a given amount of heat energy into work." Applying the second-law statement relating to entropy to an engine whose only effect is to convert heat into work we have $\Delta S < 0$, since an amount of heat is taken from the heat reservoir. This violates the entropy statement of our second law; therefore, the above process is impossible.

The other second-law statement relating to heat flow is "There can be no process whose sole effect is to transfer an amount of heat from a low-temperature reservoir to a high-temperature reservoir." Applying the second-law statement relating to entropy to an engine whose only effect is to transfer heat from a low-temperature reservoir to a high-temperature reservoir we have

$$S = -\frac{Q}{T_c} + \frac{Q}{T_n} = Q\left(\frac{1}{T_n} - \frac{1}{T_c}\right).$$

Now $1/T_n < 1/T_c$; therefore $\Delta S < 0$, which violates the entropy statement of the second law. Thus this process is also impossible.

3. (a) $e = \dfrac{W_{out}}{Q_{in}} = 1 - \dfrac{T_c}{T_h}$ for a Carnot cycle.

$$Q_{in} = \frac{W_{out}}{1 - T_c/T_h} = \frac{2092\ \text{J}}{1 - (313\ \text{K})/(413\ \text{K})} = 8.7 \times 10^3\ \text{J},$$

$$Q_{out} = Q_{in} - W_{out} = 6.6 \times 10^3\ \text{J}.$$

(b) $dS \equiv đQ/T$. For two adiabatic parts of cycle $đQ = 0$, $\Delta S = 0$. For a high-temperature isothermal process: T constant,

$$\Delta S = \Delta Q/T = (8.7 \times 10^3\ \text{J})/(413\ \text{K}) = 21.1\ \text{J/K}.$$

For a low-temperature isothermal process: T constant,

$$\Delta S = (-6.6 \times 10^3\ \mathrm{J})/(313\ \mathrm{K}) = -21.1\ \mathrm{J/K}, \qquad \Delta S_{total} = 0.$$

This is reasonable since the Carnot cycle is a reversible cycle and $\Delta S = 0$ for reversible cycles.

If you got 100% on this Practice Test, ask for a Mastery Test; if not, study the relevant material, work some more of the Additional Problems, and when you think that you understand that material ask for a Mastery Test.

SECOND LAW AND ENTROPY   Date ____________

Mastery Test   Form A   pass   recycle
                                                                          1   2   3

Name ______________________________   Tutor ______________________

1. Define (a) state variable, (b) cycle, and (c) entropy.

2. State the second law of thermodynamics as it relates to entropy on a macroscopic scale, and show that the second-law statements relating to heat flow satisfy this statement.

3. Four moles of an ideal gas is caused to expand from a volume V_1 to a volume V_2 $(= 2V_1)$.

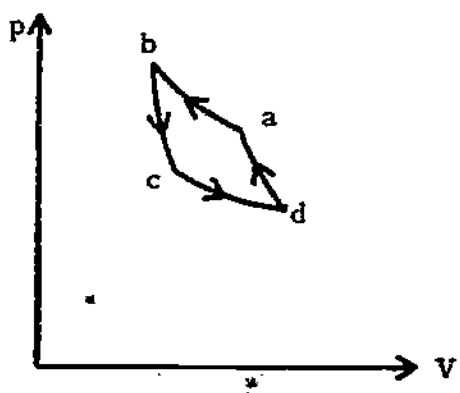
   (a) If the expansion is isothermal at the temperature $T = 400$ K, deduce an expression for the work done by the expanding gas.

   (b) For the isothermal expansion just described, deduce an expression for the change in entropy, if any.

   (c) If the expansion were reversibly adiabatic instead of isothermal, would the change in entropy of the gas be positive, negative, or zero?

4. A pV diagram of a Carnot cycle is sketched in Figure 1.
   (a) Is the device operating as an engine or a refrigerator?
   (b) In which part(s) of the cycle does heat flow into this device? Out of it?
   (c) In which part(s) of the cycle does the temperature of the working substance increase? Decrease?
   (d) Does the entropy of the working substance decrease in any part of this cycle? If not, prove that it cannot; if so, indicate in which part of the cycle.

Figure 1



SECOND LAW AND ENTROPY

Date ______________

Mastery Test Form B

pass recycle

1 2 3

Name ______________________ Tutor ______________________

1. Define (a) irreversible process, (b) efficiency, and (c) Carnot cycle.

2. State the second law of thermodynamics as it relates to entropy on a macroscopic scale, and show that the second-law satements relating to heat flow satisfy this statement.

3. A Carnot refrigerator takes heat from water at 0°C and discards heat to the room at a temperature of 27°C. If 100 kg of water is frozen to ice at 0°C by this refrigerator (the latent heat of fusion is 335 J/g.
   (a) How many joules are discarded to the room?
   (b) What is the required work?

4. One mole (63.5 g) of copper at 100°C is placed in a block of ice and remains until the whole comes to thermal equilibrium at 0°C. The latent heat of fusion (the amount of heat necessary to convert 1.00 g of ice to 1.00 g of water at 0°C of ice is 33 J/g. The specific heat of copper is 25.1 J/mol K.
   (a) How much water is formed?
   (b) What is the entropy change of the copper?
   (c) What is the entropy change of the water?
   (d) What is the total entropy change of the system?

SECOND LAW AND ENTROPY | Date ______________

Mastery Test   Form C | pass 1 2 recycle 3

Name ______________________ Tutor ______________________

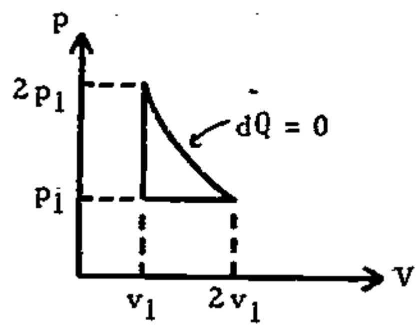
1. Define (a) state variable, (b) cycle, and (c) entropy.

2. State the second law of thermodynamics as it relates to entropy on a macroscopic scale, and show that the second-law statements relating to heat flow satisfy this statement.

3. (a) Calculate the efficiency of an engine using an ideal gas taken around the cycle shown in Figure 1, where $\gamma = 5/3$. For an ideal gas, $C_p = C_v + R$.

   (b) Draw a Carnot cycle on a temperature-entropy (TS) diagram. Work is just the area enclosed by the curve representing a cycle on a pV plot. Can a similar interpretation be made regarding a TS plot?

Figure 1



SECOND LAW AND ENTROPY   Date ____________

Mastery Test   Form D   pass   recycle

1   2   3

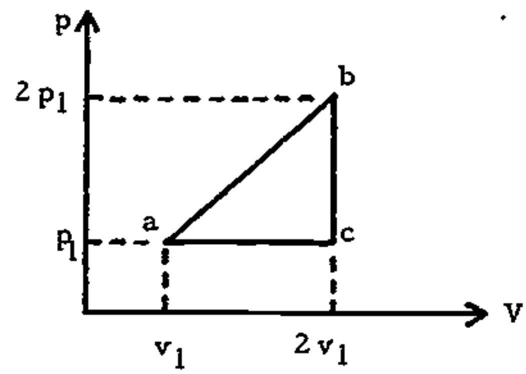
Name ______________________   Tutor ______________________

1. Define (a) irreversible process, (b) efficiency, and (c) entropy.

2. State the second law of thermodynamics as it relates to entropy on a macroscopic scale, and show that the second-law statements relating to heat flow satisfy this statement.

3. (a) Calculate the efficiency of an engine using an ideal gas taken around the cycle in Figure 1, where $U = (3/2)RT$, $C_v = (3/2)R$ and $C_p = (5/2)R$ for this gas.
(b) In a specific-heat experiment 100 g of lead (C_p = 0.144 J/g K) at 100°C is mixed with 200 g of water at 20°C. Find the difference in entropy of the system at the end from its value before mixing.

Figure 1



MASTERY TEST GRADING KEY - Form A

1. What To Look For: (a) $\oint dS = 0$. (c) Absolute temperature.

   Solution: (a) A variable such that the integral over any closed path of the differential of the state variable is zero, symbolically $\oint dS = 0$.
   (b) A sequence of processes that a system goes through such that the system returns to its original equilibrium state.
   (c) $dS \equiv đQ/T$, where S is the entropy, Q is the heat, and T is the absolute temperature.

2. What To Look For: $\Delta S_{universe} \geq 0$.

   Solution: A process that starts in one equilibrium state and ends in another will go in the direction that causes the entropy of the system plus its environment to increase or to remain the same.

   One of the second-law statements relating to heat flow is: "There can be no process whose sole effect is to convert a given amount of heat energy into work." Applying the second-law statement relating to entropy to an engine whose only effect is to convert heat into work we have $\Delta S < 0$, since an amount of heat is taken from the heat reservoir. This violates the entropy statement of our second law; therefore, the above process is impossible.

   The other second-law statement relating to heat flow is "There can be no process whose sole effect is to transfer an amount of heat from a low-temperature reservoir to a high-temperature reservoir." Applying the second-law statement relating to entropy to an engine whose only effect is to transfer heat from a low-temperature reservoir to a high-temperature reservoir we have

   $$\Delta S = -\frac{Q}{T_c} + \frac{Q}{T_n} = Q\left(\frac{1}{T_n} - \frac{1}{T_c}\right).$$

   Now $1/T_n < 1/T_c$; therefore $\Delta S < 0$, which violates the entropy statement of the second law. Thus this process is also impossible.

3. What To Look For: (a) $W = \int p\, dV$, $pV = nRT$. (b) Use of first law to deduce expression for entropy. (c) $dS = đQ/T$, $đQ = 0$.

   Solution: (a) $n = 4.0$ mol, $pV = nRT$.

   $$W = \int_{V_i}^{V_f} \frac{nRT}{V}\, dV = nRT \ln\left(\frac{V_f}{V_i}\right) = 3200 \ln(8.4\ J).$$

   (b) $dU = 0$ since $U = U(T)$ for an ideal gas. $đQ = đW$, $dS = \frac{đW}{T}$, or

$\Delta S = \int \frac{đW}{T} = \frac{1}{T}\int đW = 8 \ln(8.4\ J/K)$.

(c) $\Delta S = 0$.

4. What To Look For: (a) Use the fact that the Carnot cycle is independent of working substance. Now use an ideal gas as working substance. Recognize which are isothermal and which are adiabatic. Utilize first law and the fact that $U = U(T)$ for an ideal gas to deduce sign of $đQ$.
(b) Same as part (a).
(c) Utilize $pV = nRT$ to deduce regions where $\Delta T > 0$ and $\Delta T < 0$.
(d) $dS \equiv đQ/T$. If $đQ = 0$, $\Delta S = 0$. If $đQ < 0$, $\Delta S < 0$. If $đQ > 0$, $\Delta S > 0$.

Solution: (a) In a refrigerator, $a \to b$ high-temperature compression, $dU = 0$; therefore $đQ = đW$, $đW_{a \to b} < 0$; therefore $dQ < 0$.

Heat leaves system at the high-temperature reservoir, therefore this is a refrigerator.

(b) $c \to d$ is Q_{in}, $a \to b$ is Q_{out}.

(c) $d \to a$ is $\Delta T > 0$, $b \to c$ is $\Delta T < 0$.

(d) Yes, $a \to b$.

MASTERY TEST GRADING KEY - Form B

1. Solution: (a) A process that, by a differential change in the environment cannot be made to retrace its path.
   (b) $e \equiv W_{out}/Q_{in}$ for a cycle.
   (c) A cycle consisting of two adiabatic processes and two isothermal processes.

2. What To Look For: $\Delta S_{universe} \geq 0$.

   Solution: See solution to Problem 2 in Mastery Test Grading Key - Form A.

3. What To Look For: (a) Recognize that a Carnot cycle is a reversible cycle. $Q_c/Q_h = T_c/T_h$ for Carnot cycle. $Q = mL$.
   (b) $W = Q_h - Q_c$.

   Solution: (a) $Q_c = (10^5\ g)(335\ J/g) = 34 \times 10^6\ J$.

   $Q_h = (T_h/T_c)Q_c = (300/273)(34 \times 10^6\ J) = 34 \times 10^6\ J$.

   This is reasonable since $Q_h \quad Q_c$.
   (b) $W = 3.3 \times 10^6\ J$.

4. What To Look For: (a) Energy conserved.
   (b) $dS \equiv \text{đ}Q/T$.
   (c) $dS \equiv \text{đ}Q/T$.

   Solution: (a) $mL = m_{Cu}C_{Cu}\ \Delta T_{Cu}$,

   $$m = \frac{(100\ mol)(25.1\ J/mol\ K)(100\ K)}{335\ J/g} = 7.5\ g.$$

   (b) $\Delta S_{Cu} = mc\int(dT/T) = (25.1\ J/K)\ \ln(273/373) = -7.9\ J/K$.
   (c) $\Delta S_{Water} = mL/T = (7.5\ g)(335\ J)/(273\ K\ g) = 9.2\ J/K$.
   (d) $\Delta S_{total} \simeq 1.26\ J/K$.

MASTERY TEST GRADING KEY - Form C

1. What To Look For: (a) $\oint dS = 0$. (c) T = absolute temperature.

   Solution: (a) A variable such that the integral over any closed path of the differential of the state variable is zero, symbolically $\oint dS = 0$.
   (b) A sequence of processes that a system goes through such that the system returns to its original equilibrium state.
   (c) $dS \equiv đQ/T$, where S is the entropy, Q the heat, and T the absolute temperature.

2. What To Look For: $\Delta S_{universe} \geq 0$.

   Solution: See solution to Problem 2 in Mastery Test Grading Key - Form A.

3. What To Look For: $e = W/Q_{in}$, $\Delta Q = nC_v \, \Delta T$, $\Delta Q = nC_p \, \Delta T$. $W = \int p \, dV$.
   $\Delta Q = 0$, pV^{γ} = const for ideal gas. $pV = nRT$ for ideal gas.
   (b) Recognize definition of Carnot cycle two $\Delta Q = 0$ and two $\Delta T = 0$ processes.
   $dS \equiv đQ/T$.

   Solution: (a) For the work done by the adiabatic process:

$$W_{done\ by} = \int p \, dV = C \int_{V_i}^{V_f} \frac{dV}{V^{\gamma}} = C \frac{1}{1-\gamma} V^{1-\gamma} \Big|_{V_i}^{V_f} = \frac{2p_1 V_1^{5/3}}{-(2/3)} [(2V_1)^{-2/3} - (V_1)^{-2/3}]$$

$$= -3p_1V_1[2^{-2/3} - 1] = 3p_1V_1[1 - 2^{-2/3}] = 1.10 \, p_1V_1.$$

$W_{done\ on} = p_1V_1$.

Therefore, $W_{total} = 0.10 \, p_1V_1$.

$Q_{in} = nC_v \, \Delta T$, $\quad T_f = 2p_1V_1/nR$, $\quad T_i = p_1V_1/nR$.

Therefore $Q_{in} = nC_v(p_1V_1/nR) = p_1V_1/(\gamma - 1) = (3/2)p_1V_1$.

Therefore $e = 0.067$.

(b) See Figure 7. Area = $\int T \, dS$. $đQ = T \, dS$. Therefore, ΔQ = area on TS plot.

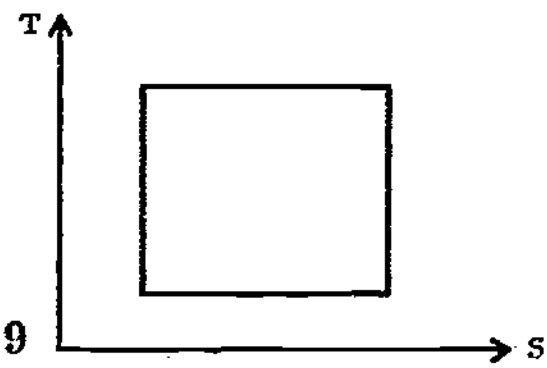


Figure 7

MASTERY TEST GRADING KEY - Form D

1. What To Look For: (c) T = absolute temperature.

   Solution: (a) A process that by a differential change in the environment cannot be made to return to its path.
   (b) $e = W_{out}/Q_{in}$ for a cycle.
   (c) $dS \equiv đQ/T$, where S is the entropy, Q is the heat, and T the absolute temperature.

2. What To Look For: $\Delta S_{universe} \geq 0$.

   Solution: See solution to Problem 2 in Mastery Test Grading Key - Form A.

3. What To Look For: (a) $e = W/Q_{in}$. $W = \int p\,dV$. Calculate heat on straight part by using first law and ΔW. $\Delta Q = nC_v\,\Delta T$, $\Delta Q = nC_p\,\Delta T$, $pV = nRT$.
   (b) $dS \equiv đQ/T$. $đQ = nc\,dT$. Conservation of energy.

   Solution: (a) By inspection of the figure, we see that

   $W_{total} = (1/2)p_1V_1$, $Q_{ab} = U_{ab} + W_{ab}$.

   $W_{ab} = (3/2)p_1V_1$, $U_{ab} = (3/2)R(T_b - T_a)$,

   $T_a = p_1V_1/nR$, $T_b = 4p_1V_1/nR$, $U_{ab} = (3/2)R(3p_1V_1/nR) = 9p_1V_1/2$,

   $Q_{ab} = 9p_1V_1/2 + (3/2)p_1V_1 = 6p_1V_1$ (it's positive)

   Q_{ab} is part of Q_{in}.

   Q_{bc}: $\Delta T < 0$, therefore $Q_{bc} < 0$,

   Q_{ca}: $\Delta T < 0$, therefore $Q_{ca} < 0$.

   Or

   $e = (1/2)(p_1V_1/6p_1V_1) = 1/12$.

   (b) $dS \equiv đQ/T = mc(dT/T)$. Therefore $\Delta S = mc\ \ln(T_f/T_i)$.

   $\Delta S_{sys} = \Delta S_{Pb} + \Delta S_w$, $m_{Pb}C_{Pb}\,\Delta T_{Pb} = m_wc_w\,\Delta T_w$, $m_{Pb}C_{Pb}(T_{Pb_i} - T) = m_wc_w(T - T_{w_i})$,

$$T = \frac{m_{Pb}C_{Pb}T_{Pb_i} + m_wC_wT_{w_i}}{m_{Pb}C_{Pb} + m_wC_w}$$

$$= \frac{(100\ g)(14.4 \times 10^{-2}\ J/g°C)(100°C) + (200\ g)(4.18\ J/g°C)(20°C)}{14.4\ J/°C + 836\ J/°C} = 21.4°C.$$

$\Delta S_{Pb} = (14.4\ J/°C)\ \ln(294/373) = -3.4\ J/°C,$

$\Delta S_w = (836\ J/°C)\ \ln(294.4/293.0) = 4.0\ J/°C.$

Therefore $S_{total} = 0.60\ J/°C.$

Module ______

STUDY GUIDE 1

COULOMB'S LAW AND THE ELECTRIC FIELD

INTRODUCTION

This module begins the study of electricity. Not only is it true that we see nature's gigantic electrical show in thunderstorm displays with lightning, but the very functioning of our smallest cells depends on the balance of electrically charged ions, and their movement through cell membranes. On a larger scale than cell membranes, water-purification studies with large membranes show promise of "electrically" removing undesired ions or debris from water. The electronic air cleaner is yet another direct application of the material to come: a 7000-V potential difference between a thin wire and flat collecting plates ionizes the air, and the "flying" electrons attach themselves to dust particles, which are then pulled to the collecting plates by strong electrical forces. Since forces that hold atoms together are ultimately electrical, the study of electricity is the study of one of nature's truly grand designs.

Later in your study of physics you will see the design unfold further; charges whose position is constant produce electric fields, charges whose velocity is constant produce magnetic fields as well as electric fields, and charges that accelerate produce that special combination of electric and magnetic fields we know as electromagnetic radiation (radio waves, x rays, microwaves, etc.).

PREREQUISITES

Before you begin this module, you should be able to:	Location of Prerequisite Content
*Add and subtract vectors (needed for Objective 2 of this module)	Dimensions and Vector Addition Module
*State Newton's law for linear motion (needed for Objectives 1 and 3 of this module)	Newton's Laws Module
*State the relation between work and energy (needed for Objective 3 of this module)	Work and Energy Module
*Analyze problems involving planar motion under constant acceleration (needed for Objective 3 of this module)	Planar Motion Module

LEARNING OBJECTIVES

After you have mastered the content of this module, you will be able to:

1. Conductors versus insulators - Make the distinction between insulators and conductors.

2. Electric forces and fields - Calculate, for a group of point charges at rest,
   (a) the resultant force on one of the charges caused by all of the others, and/or
   (b) the total electric field at some point in space caused by all of the charges.

3. Particle motion in electric fields - Apply the definition of electric field to solve problems involving a charged particle in an electric field, where
   (a) the particle is at rest under the influence of additional forces, like gravity or tension, and/or
   (b) the particle moves in a constant electric field.

   These problems will require you to calculate any of the following quantities: force, acceleration, time, position, velocity, work, kinetic energy. For vector quantities you must be able to calculate components, magnitude, and direction.

TEXT: Frederick J. Bueche, Introduction to Physics for Scientists and Engineers (McGraw-Hill, New York, 1975), second edition

SUGGESTED STUDY PROCEDURE

Read Chapter 18, Sections 18.1 through 18.5, and then Sections 18.9 through 18.12, and General Comments 1 to 4. Then study Problems A through F before working Problems G through J. Make your own decision about working some Additional Problems before taking the Practice Test and a Mastery Test.

BUECHE

Objective Number	Readings	Problems with Solutions		Assigned Problems	Additional Problems
		Study Guide	Text	Study Guide	(Chap. 18)
1	Secs. 18.1, 18.9, General Comment 1		Sec. 18.11		
2	Secs. 18.2, 18.3, 18.5, General Comments 2, 3, 4	A, B, C	Sec. 18.3, Illus.[a] 18.3	G, H	1 to 10
3	Sec. 18.12	D, E, F	Illus. 18.1	I, J	11, 12, 18, 19, 20

[a]Illus. = Illustration(s).

TEXT: David Halliday and Robert Resnick, Fundamentals of Physics (Wiley, New York, 1970; revised printing, 1974)

SUGGESTED STUDY PROCEDURE

Read over all of Chapters 22 and 23, for background. Then concentrate on a careful reading of Sections 22-2 through 22-4 in Chapter 22 and Section 23-5 in Chapter 23. Study General Comments 1 through 4 and Problems A through F before working Problems G through J and Problems 2 and 13 in Chapter 22, Problem 34 in Chapter 23. Take the Practice Test and decide whether to do some Additional Problems or take a Mastery Test. Note that calculating electric fields due to continuous-charge distributions using calculus is not an objective in this module, therefore you need not dwell on equations like (23-6) and (23-7).

HALLIDAY AND RESNICK

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text	Study Guide	Text	
1	Secs. 22-2, 22-3, General Comment 1					Chap. 22, Quest.[a] 3, 6, 7
2	Secs. 22-2, 22-3, 22-4, General Comments 2, 3, 4	A, B, C	Chap. 22, Ex.[a] 2, 3, 4	G, H	Chap. 22, Probs. 2, 13	Chap. 22, Probs. 3, 4, 5; Chap. 23, Probs. 14 to 17
3	Sec. 23-5	D, E, F	Chap. 23, Ex. 7, 8	I, J	Chap. 23, Prob. 34	Chap. 22, Probs. 14, 26; Chap. 23, Probs. 31(a), 35, 36, 37

[a]Ex. = Example(s). Quest = Question(s).

TEXT: Francis Weston Sears and Mark W. Zemansky, University Physics (Addison-Wesley, Reading, Mass., 1970), fourth edition

SUGGESTED STUDY PROCEDURE

Read over Chapter 24 and Chapter 25 through Example 1 in Section 25-2 (p. 343). Read General Comments 1 through 4 and study Problems A through F before working Problems G through J and Problem 24-1(a) and (b) in your text. Take the Practice Test, and decide whether to work some Additional Problems or take a Mastery Test.

Omit Example 4 in Section 25-1 (p. 341). Also note the text's interchangeable use of the terms "electric field," "electric field strength," and "electric intensity."

SEARS AND ZEMANSKY

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text	Study Guide	Text	
1	General Comment 1, Secs. 24-1, 24-4, 25-1		Sec. 24-5			
2	Secs. 24-6, 25-1, 25-2, General Comments 2, 3, 4	A, B, C		G, H	24-1(a), (b)	24-2(a), (b), 25-1, 25-6 to 25-9
3	Sec. 25-1	D, E, F	Sec. 25-1, Ex.[a] 1, 2, 3	I, J		24-3, 24-4, 25-2 to 25-5, 25-11

[a]Ex. = Example(s).

TEXT: Richard T. Weidner and Robert L. Sells, Elementary Classical Physics (Allyn and Bacon, Boston, 1973), second edition, Vol. 2

SUGGESTED STUDY PROCEDURE

Read over Chapter 22, except Section 22-6, then briefly read Chapter 23, except Section 23-6, without working through any examples. With this background, read General Comments 1 through 4, and study Problems A through F and Examples 22-1 and 23-3. Then work Problems G through J. Take the Practice Test and decide whether to take a Mastery Test, or work some Additional Problems. The Additional Problems on Objective 3 are likely to help you.

WEIDNER AND SELLS

Objective	Readings	Problems with Solutions		Assigned Problems	Additional Problems
		Study Guide	Text	Study Guide	
1	General Comment 1, Chap. 22, Introduction, Secs. 22-1, 23-4				
2	Secs. 22-2, 22-3, 23-1, General Comments 2, 3, 4	A, B, C	Ex.[a] 22-1	G, H	22-5, 22-7, 22-15, 23-1, 23-2
3	Sec. 23-5	D, E, F	Ex. 23-3	I, J	23-12, 23-15, 23-18, 22-9

[a]Ex. = Example(s).

GENERAL COMMENTS

1. Electric Field

The concept of the electric field is introduced in this module as a force per unit charge. In your text reading, you will find conductors mentioned as materials in which charges are free to move, whereas insulators are materials in which charges are not free to move. Let us now put these two statements together: When you establish an electric field in a conductor, the electrons (which are free to move) feel a force equal to their charge times the electric field and thus initially accelerate, and a current (movement of charge) gets started. If you establish an electric field in an insulator essentially no current flows, since the charges are basically not free to move in response to the field. (In an insulator, charges can move through distances like an atomic radius before they are stopped by forces within the atom. This movement in response to an electric field sets up "dipoles," about which you may learn in a later module.)

The results of this discussion are so important when using Gauss' law in connection with conductors that we have made it an objective in this module (Objective 1). You must keep in mind that if charges are moving in a conductor, they are responding to an electric field. If we find that the charges are at rest in a conductor, however, this means that there is zero electric field in that conductor.

2. Principle of Superposition

This principle is very simple, but very important. It says that if cause A has effect a, and cause B has effect b, then A and B taken together will have effect (a + b).

In this module, where charges exert forces on each other, this principle definitely holds; if a charge Q feels a force $\vec{F}_a$ when only charge A is present, and feels a force $\vec{F}_b$ when only charge B is present, it will experience a force that is the vector sum of $\vec{F}_a$ and $\vec{F}_b$ when both A and B are present. Since the electric field at the point where a charge Q is located is the force felt by this charge divided by the charge Q, the same superposition principle holds for electric fields: if charge A by itself causes an electric field $\vec{E}_a$ at some point in space when present by itself, and if charge B causes an electric field $\vec{E}_b$ at this same point in space when only B is present, the electric field at that point when both A and B are present will be the vector sum of $\vec{E}_a$ and $\vec{E}_b$.

3. Coulomb's Law

If we ask for the total force on point charge Q_1 in the presence of point charge Q_2 and point charge Q_3, the answer may be written

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13}, \tag{1}$$

where $\vec{F}_{12}$ is the force on Q_1 due to Q_2 and $\vec{F}_{13}$ is the force on Q_1 due to Q_3. From Coulomb's law, the force is

$$|\vec{F}_{ij}| = k(Q_iQ_j/r_{ij}^2)$$

directed along the line between Q_i and Q_j. Therefore,

$$\vec{F}_{12} = (9.0 \times 10^9 \text{ N m}^2/\text{C}^2)(Q_1Q_2/r_{12}^2)(\vec{r}_{12}/r_{12}),$$

where r_{12} is the distance between Q_1 and Q_2 and $\vec{r}_{12}/r_{12} = \hat{r}_{12}$ is a "unit" vector (length = 1) that points along the line in the direction from Q_2 to Q_1. In order to add F_{12} and F_{13} correctly, we may write

$$\hat{r}_{12} = \hat{i}(x_{12}/r_{12}) + \hat{j}(y_{12}/r_{12}) \quad .$$

When we also do this for $\vec{r}_{13}$ and add, Eq. (1) becomes

$$\vec{F}_1 = [(\frac{(9.0 \times 10^9 \text{ N m}^2/\text{C}^2)Q_1Q_2}{r_{12}^2})(\frac{x_{12}}{r_{12}}) + (\frac{(9.0 \times 10^9 \text{ N m}^2/\text{C}^2)Q_1Q_3}{r_{13}^2})(\frac{x_{13}}{r_{13}})]\hat{i}$$

$$+ [(\frac{(9.0 \times 10^9 \text{ N m}^2/\text{C}^2)Q_1Q_2}{r_{12}^2})(\frac{y_{12}}{r_{12}}) + (\frac{(9.0 \times 10^9 \text{ N m}^2/\text{C}^2)Q_1Q_3}{r_{13}^2})(\frac{y_{13}}{r_{13}})]\hat{j}. \quad (2)$$

PROBLEMS A AND B ARE GOOD EXAMPLES OF THIS PROCEDURE, AND YOU SHOULD GIVE THEM CAREFUL STUDY.

4. Electric Fields in Space

These remarks conclude your reading on the subject of the electric field in this module. The main point to be made is that for a single charge or for a group of charges, the electric field is not everywhere represented by a single number, or even one single vector. The magnitude and direction of the electric field depend on position. In Figure 1, fields $\vec{E}_1$, $\vec{E}_2$, and $\vec{E}_3$ are all different in both

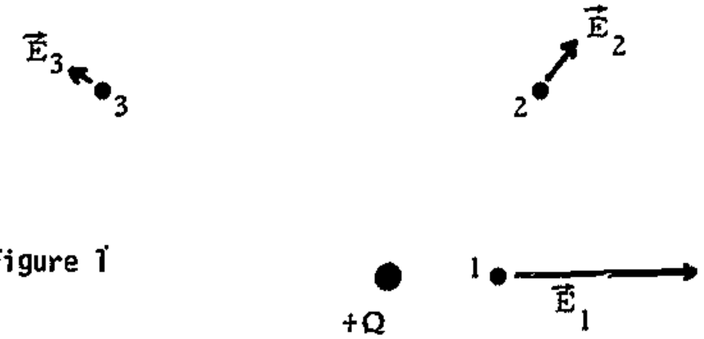


Figure 1

magnitude and direction because the points 1, 2, and 3 are in different locations in space: the electric field $\vec{E}$ is a function of coordinates like x, y, and z - it changes as x, y, and z change. This is again illustrated in Figure 2 for 6 points with respect to an assembly of 20 charges. The arrows are not rigorously correct, but they show what is going on generally: the electric field in the region between the lines of charge is fairly constant, and outside this region, the electric field is quite small. We often idealize this situation to say that the field between the "plates" is constant, and the field outside is zero. The point, however, is that even in the "ideal" situation, the electric field is a function of the coordinates: it depends on where you are located in space.

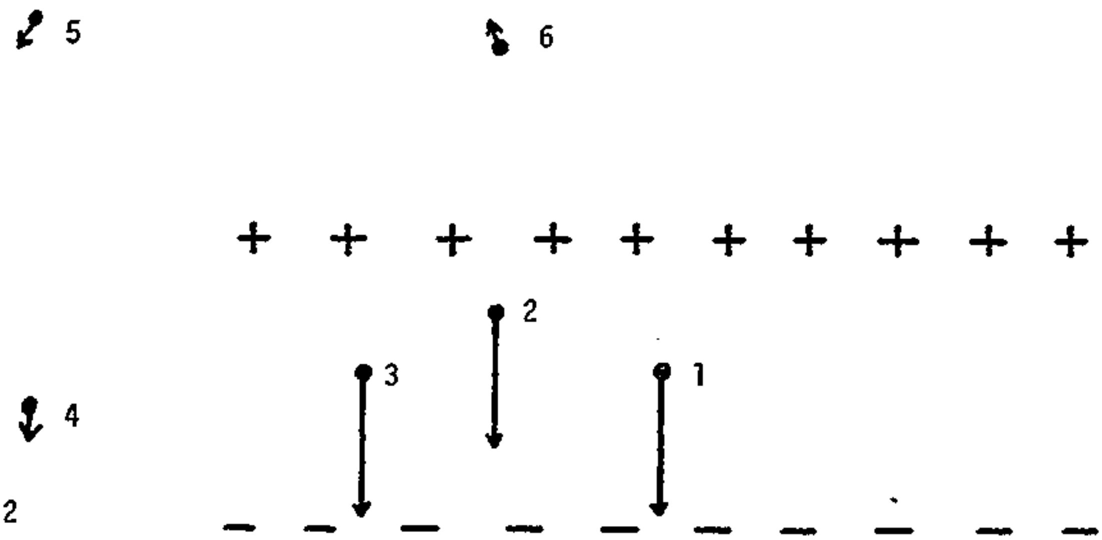


Figure 2

PROBLEM SET WITH SOLUTIONS

A(2). Calculate the total force on the -1.00-μC charge shown in Figure 3.

Solution

$$\vec{F}_{total} = \vec{F}_1 + \vec{F}_2.$$

Step 1: pick axes $\hat{i}$ and $\hat{j}$. Let $\hat{j}$ be positive along $\vec{F}_2$ and $\hat{i} \perp \hat{j}$ at q_1 in the plane of the paper.

Step 2: sketch all forces ($\vec{F}_1$ and $\vec{F}_2$). This gives the signs of the components right away:

$$\vec{F}_1 = -(F_1 \sin \theta)\hat{i} + (F_1 \cos \theta)\hat{j}, \qquad \vec{F}_2 = +F_2\hat{j}.$$

Step 3: Calculate force magnitudes from $F = q_1 q_2 / 4\pi\varepsilon_0 r^2$:

$$F_1 = \frac{(9.0 \times 10^{9}\ \mathrm{N\ m^2/C^2})(2.50 \times 10^{-12}\ \mathrm{C^2})}{(3^2 + 4^2)\ \mathrm{m^2}} = 9.0 \times 10^{-4}\ \mathrm{N},$$

$$F_2 = \frac{(9.0 \times 10^{9}\ \mathrm{N\ m^2/C^2})(2.00 \times 10^{-12}\ \mathrm{C^2})}{4^2\ \mathrm{m^2}} = 1.13 \times 10^{-3}\ \mathrm{N}.$$

Step 4: add components to get the resultant:

$$F_{total} \cong (9.0 \times 10^{-4}\ \mathrm{N})(3/5)\hat{i} + [(9.0 \times 10^{-4}\ \mathrm{N})(4/5) + (1.13 \times 10^{-3}\ \mathrm{N})]\hat{j}$$
$$= -(5.4 \times 10^{-4}\ \mathrm{N})\hat{i} + (18.5 \times 10^{-3}\ \mathrm{N})\hat{j}.$$

Figure 3

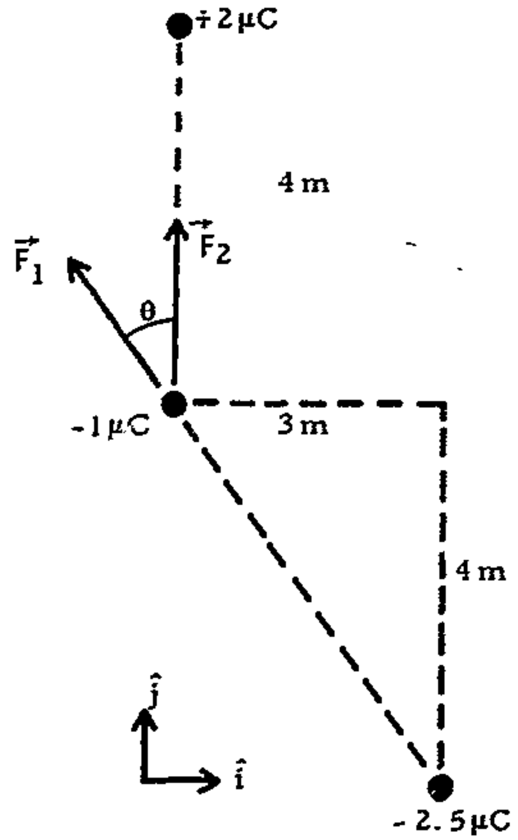


Figure 4

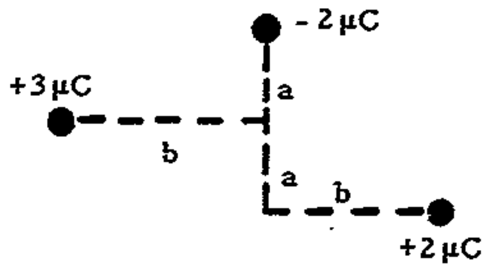
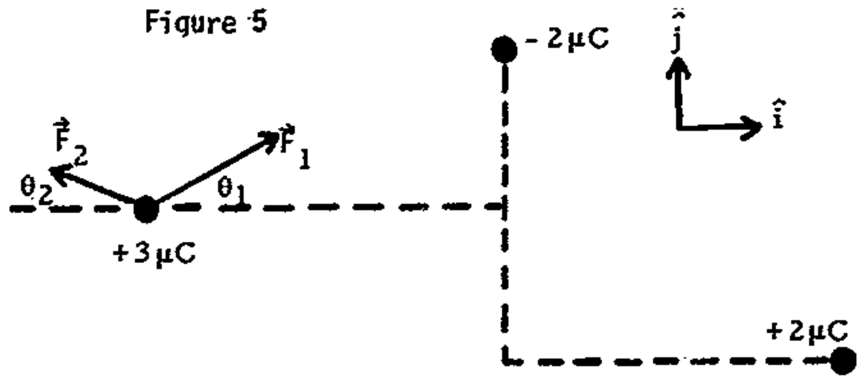


Figure 5



B(2). Calculate the total force on the +3.00-μC charge in Figure 4. (a = 1.00 m, b = 2.00 m).

Solution

Step 1: Choose axes as in Figure 5.
Step 2: Sketch in all forces. This gives component signs.

$$\vec{F}_{total} = (F_1 \cos \theta_1)\hat{i} + (F_1 \sin \theta_1)\hat{j} - (F_2 \cos \theta_2)\hat{i} - (F_2 \sin \theta_2)\hat{j}$$

$$= [F_1(2/\sqrt{1^2 + 2^2}) - F_2(4/\sqrt{4^2 + 1^2})]\hat{i} + [F_1(1/\sqrt{1^2 + 2^2}) + F_2(1/\sqrt{1^2 + 4^2})]\hat{j}.$$

Step 3: Calculate magnitudes from $F = q_1q_2/4\pi\varepsilon_0 r^2$:

$$F_1 = \frac{(9.0 \times 10^9 \text{ N m}^2/\text{C}^2)(6.0 \times 10^{-12} \text{ C}^2)}{(1^2 + 2^2) \text{ m}^2} = 1.08 \times 10^{-2} \text{ N},$$

$$F_2 = \frac{(9.0 \times 10^9 \text{ N m}^2/\text{C}^2)(6.0 \times 10^{-12} \text{ C}^2)}{(1^2 + 4^2) \text{ m}^2} = 3.18 \times 10^{-3} \text{ N}.$$

Step 4: Add components to get the resultant:

$$\vec{F}_{total} = (0.97 \times 10^{-2} - 0.310 \times 10^{-2} \text{ N})\hat{i} + (0.475 \times 10^{-2} + 0.077 \times 10^{-2} \text{ N})\hat{j}$$
$$= (6.6 \times 10^{-3} \text{ N})\hat{i} + (5.5 \times 10^{-3} \text{ N})\hat{j}.$$

C(2). Calculate $\vec{E}$ at point P in Figure 6 due to the -1.00-μC charge (a = 2 m, (a = 2.00 m, b = 3.00 m).

Solution

Step 1: Choose axes as in Figure 7. The direction of $\vec{E}$ at P is the direction in which a positive charge would move if placed at P. The magnitude of $\vec{E}$ at P is $(1/4\pi\varepsilon_0)$[1.00 μC/(distance)^2] (which is the ratio of force $\vec{F}_q$ on a very small test charge q divided by q: $E = F_q/q$).

Figure 6

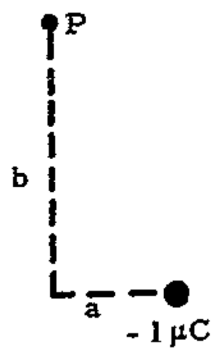
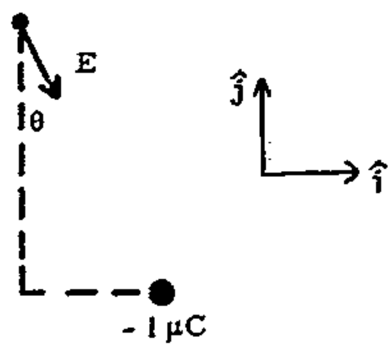


Figure 7



Step 2: Sketch $\vec{E}$ as in Figure 7 and calculate

$$\vec{E} = +(E \sin\theta)\hat{i} - (E \cos\theta)\hat{j}.$$

Step 3: Calculate the magnitude from $E = (1/4\pi\varepsilon_0)(q/r^2)$:

$$E = (9.0 \times 10^9 \text{ N m}^2/\text{C}^2)(10^{-6} \text{ C})/(2^2 + 3^2) \text{ m}^2 = 0.69 \times 10^3 \text{ N/C}.$$

Step 4: Calculate the result:

$$E = -[(0.69 \times 10^3 \text{ N/C})(2/\sqrt{13})]\hat{i} + [(0.69 \times 10^3 \text{ N/C})(3/\sqrt{13})]\hat{j}$$

$$= +(0.38 \times 10^3 \text{ N/C})\hat{i} - (0.58 \times 10^3 \text{ N/C})\hat{j}.$$

D(3). A stationary particle whose mass is 0.100 kg and whose charge is +0.300 C is suspended by a massless string under gravity in the presence of an electric field of magnitude 1.00 N/C as shown in Figure 8. Calculate the angle θ.

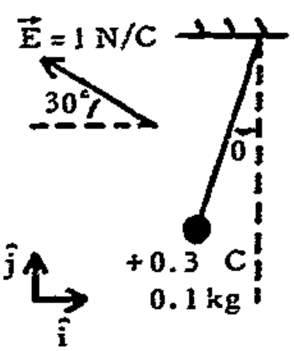


Figure 8

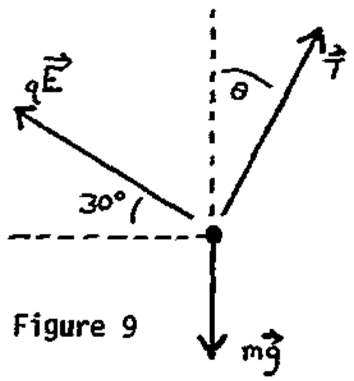


Figure 9

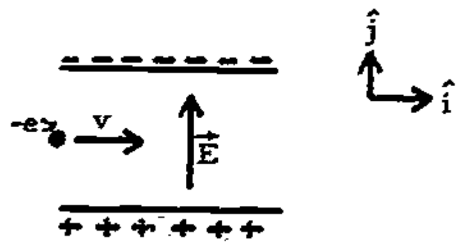


Figure 10

Solution

The sum of all forces must be zero for the particle at rest. First, we pick a coordinate system, taking +x to the right and +y upward, and draw a free-body diagram as in Figure 9. Then we add up all the forces acting on the particle:

	x component	y component
electrical ($\vec{F}_e = q\vec{E}$):	$-(0.300 \text{ C})(1.00 \text{ N/C})(\cos 30°)$	$+(0.300 \text{ C})(1.00 \text{ N/C})(\cos 60°)$
tension:	$T \sin\theta$	$T \cos\theta$
gravity:	0	$-(0.100 \text{ kg})(9.8 \text{ m/s}^2)$

Since the x and y components add to zero, we get

$$T \sin\theta = (0.300 \text{ N})(\cos 30°), \qquad T \cos\theta = (0.98 \text{ N} - 0.300 \text{ N})(\cos 60°),$$

$$\tan\theta = \frac{(0.300)(\sqrt{3}/2)}{0.98 - 0.300(0.50)} = \frac{(0.300)(0.866)}{0.83} = \frac{0.260}{0.83} = 0.310, \qquad \theta = 17.0°.$$

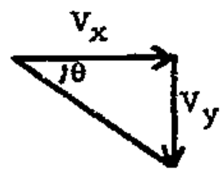
E(3). The diagram in Figure 10 shows an electron traveling with velocity 8.0×10^6 m/s in the x direction through a pair of deflecting plates 2.00 cm long. Assuming the electric field between the deflecting plates to be constant and equal to 9800 N/C in the +y direction, calculate
(a) the time the electron spends between the deflecting plates;
(b) the acceleration of the electron when between the plates;
(c) The electron's y component of velocity when it emerges from the plates;
(d) the angle between the electron's initial velocity and its velocity upon emerging;
(e) the amount the electron is deflected in the y direction when it emerges.
(f) Assuming the electron to enter the plates at x = 0, y = 0, find the equation for y(x).
The charge on an electron is -1.60×10^{-19} C; the mass of an electron is 9.1×10^{-31} kg.

Solution

(a) Since there is no acceleration in the x direction, the x component of velocity is constant at 8.0×10^6 m/s. Since only 0.0200 m needs to be traveled, the time is

$$t = \frac{2.00 \times 10^{-2}\ \text{m}}{8.0 \times 10^6\ \text{m/s}} = 2.50 \times 10^{-9}\ \text{s}.$$

Figure 11



(b) $a_x = 0$, $a_y = (\text{force})_y/m$,

$$a_y = \frac{qE_y}{m} = \frac{(-1.60 \times 10^{-19}\ \text{C})(9800\ \text{N/C})}{9.1 \times 10^{-31}\ \text{kg}} \cong 1.70 \times 10^{15}\ \text{m/s}^2.$$

(c) $v_y = v_{0y} + a_y t = 0 + (-1.70 \times 10^{15}\ \text{m/s}^2)(2.50 \times 10^{-9}\ \text{s}) = -4.3 \times 10^6\ \text{m/s}.$

(d) See Figure 11.

$$\tan\theta = \frac{v_y}{v_x} = \frac{-4.3 \times 10^6\ \text{m/s}}{8.0 \times 10^6\ \text{m/s}} = -0.54; \qquad \theta = 28.4° \text{ below the horizontal.}$$

(e) $y = y_0 + v_{0y}t + (1/2)a_y t^2 = 0 + 0 - (1/2)(1.70 \times 10^{15}\ \text{m/s}^2)(2.50 \times 10^{9})^2 \cong 0.53$ cm.

(f) $x = x_0 + v_{0x}t + (1/2)a_x t^2, \quad v_{0x} = 8.0 \times 10^6\ \text{m/s}, \quad a_x = 0, \quad x_0 = 0.$

$y = y_0 + v_{0y}t + (1/2)a_y t^2, \quad y_0 = v_{0y} = 0, \quad a_y = qEy/m, \quad x = v_{0x}t,$

$$t = \frac{x}{v_{0x}}, \qquad y = \frac{1}{2}a_y t^2 = \frac{1}{2}\left(\frac{a_y}{v_{0x}^2}\right)x^2 = \left(\frac{a_y}{2v_{0x}^2}\right)x^2 = \left(\frac{qE_y}{2mv_{0x}^2}\right)x^2,$$

and

$$\frac{qE_y}{2mv_{0x}^2} = \frac{(-1.60 \times 10^{-19}\ C)(9800\ N/C)}{2(9.1 \times 10^{-31}\ kg)(8.0 \times 10^6\ m/s)^2} = \frac{-1.60}{(128)(9.1)} \times 10^4 = -13.8\ m^{-1}.$$

Thus $y = -13.8x^2\ m^{-1}$.

F(3). An electron ($m = 9.1 \times 10^{-31}$ kg and $q = -1.6 \times 10^{-19}$ C) with $\vec{v} = 10^6\hat{i}$ m/s enters a region of space with uniform electric field $\vec{E} = 5.0\hat{i}$ N/C.

(a) How much time will it take for the electron to be stopped by the electric field?
(b) How far will it have traveled in coming to rest?
(c) How much work is done on the electron in bringing it to rest?
(d) What was the kinetic energy of the electron at the start of the problem?

Solution

(a) $$a_x = \frac{qE_x}{m} = \frac{(-1.60 \times 10^{-19}\ C)(5.0\ N/C)}{9.1 \times 10^{-31}\ kg} = -0.88 \times 10^{12}\ m/s^2;$$

$$t = \frac{v_x}{a_x} = \frac{10^6\ m/s}{0.88 \times 10^{12}\ m/s^2} = 1.12 \times 10^{-6}\ s.$$

(b) $x = x_0 + v_x t + a_x(t^2/2)$,

$$x - x_0 = (10^6\ m/s)(1.12 \times 10^{-6}\ s) + (-0.88 \times 10^{12}\ m/s^2)(1.12 \times 10^6\ s)^2 = 0.57\ m.$$

(c) For constant force

$$W = Fd = qEd = (-1.60 \times 10^{-19}\ C)(5.0\ N/C)(0.57\ m) = 4.6 \times 10^{-19}\ J.$$

(d) $$(KE)_i = mv^2/2 = [(9.1 \times 10^{-31}\ kg)/2](10^6\ m/s)^2$$

$$= 4.6 \times 10^{-19}\ kg\ m/s^2 = 4.6 \times 10^{-19}\ J.$$

Note that the work done equals the change in KE.

Problems

G(2). Calculate the total force on the -2.00-μC charge in Figure 12 ($a = 3.00$ m).

H(2). Calculate the electric field at point P in Figure 13 ($a = 1.00$ m, $b = 3.00$ m, $c = 2.00$ m). Hint: follow the steps in the Solution to Problem A using $E = q/4\pi\varepsilon_0 r^2$ (instead of F).

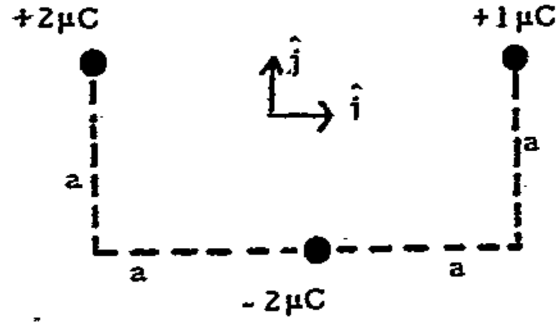


Figure 12

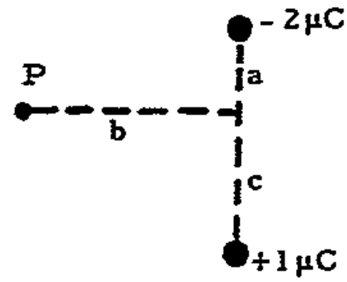


Figure 13

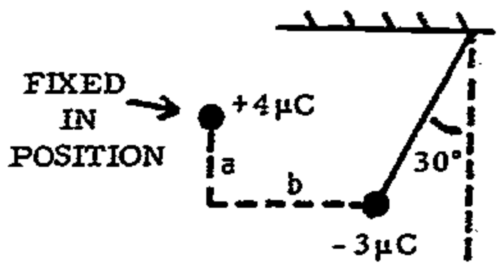


Figure 14

I(3). A particle of mass m and charge -3.00 μC is suspended at rest by a massless string as shown in Figure 14, in the presence of gravity; the fixed charge is +4.0 μC. Find the mass m (a = 2.00 m, b = 3.00 m).

J(3). An electron ($m = 9.1 \times 10^{-31}$ kg)($q = -1.60 \times 10^{-19}$ C) circles a stationary proton ($q = +1.60 \times 10^{-19}$ C) at a distance of 5.3×10^{-11} m. What is the electron's speed? Hint: Recall that particles traveling in a circle accelerate toward the center (worked out in the module Planar Motion). Solve by equating this "centripetal" acceleration times the mass to the attractive electrical force.

Solutions

G(2). $\vec{F}_{total} = [(-7.1 \times 10^{-4})\hat{i} + (2.10 \times 10^{-3})\hat{j}]$ N.

H(2). $\vec{E}_p = [(1.13 \times 10^{3})\hat{i} + (9.5 \times 10^{2})\hat{j}]$ N/C.

I(3). $m = 1.70 \times 10^{-3}$ kg.

J(3). $v = 2.20 \times 10^{6}$ m/s.

PRACTICE TEST

1. Calculate the total force on the -1.00-μC charge in Figure 15 ($a = 3.00$ m, $b = 5.0$ m, $c = 2.00$ m, $d = 4.0$ m).

2. A 0.400-kg mass has a charge +Q and is supported vertically by a massless string, with a massless spring attached on which a 3.00-N force is exerted, as in Figure 16. There is also a -5.0-μC charge, located as shown. Find the charge Q.

3. What is the main difference between a conductor and a perfect insulator?

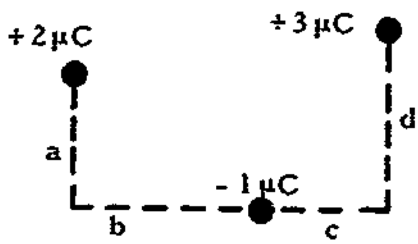


Figure 15

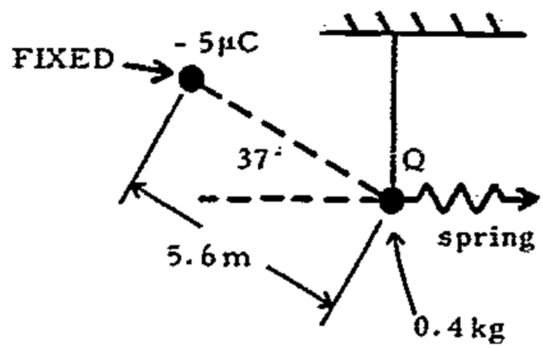


Figure 16

Practice Test Answers

1. $$F_{total} = [(9.0 \times 10^{-3})(-\frac{10}{34\sqrt{34}} + \frac{6}{20\sqrt{20}})\hat{i} + (9.0 \times 10^{-3})(\frac{+6}{34\sqrt{34}} + \frac{12}{20\sqrt{20}})\hat{j}]\ N$$

$$= [+(1.50 \times 10^{-4})\hat{i} + (1.50 \times 10^{-3})\hat{j}]\ N.$$

2. $$3.00\ N = \cos 37° (\frac{(5.0 \times 10^{-6}\ C)(9.0 \times 10^{9}\ N\ m^2/C^2)}{(5.6\ m)^2})Q, \quad Q = \frac{(5.6)^2(5/4)3}{45 \times 10^3}\ C = 2.61\ C.$$

3. In a conductor, charges are free to move in response to an electric field; in a perfect insulator, charges are not free to move.

COULOMB'S LAW AND THE ELECTRIC FIELD Date ______________

Mastery Test Form A pass recycle

1 2 3

Name ______________________________ Tutor ______________________

1. Determine the electric field at point P in Figure 1 (a = 3.00 m, 6 = 4.00 m). Show your work.

2. In Figure 2, a small sphere of mass 1.00 g carries a charge of 20.0 μC and is attached to a 5.0-cm-long silk fiber. The other end of the fiber is attached to a large vertical conducting plate that provides a uniform horizontal field of 10^3 N/C. Find the angle the fiber makes with the vertical.

3. What is the main difference between a conductor and a perfect insulator?

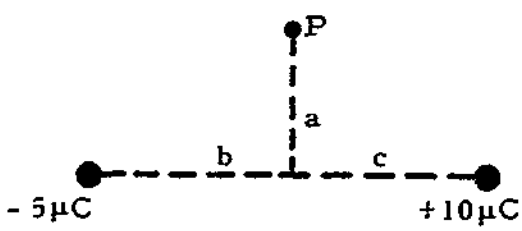


Figure 1

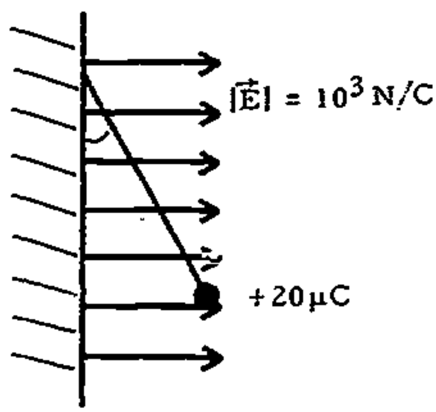


Figure 2

COULOMB'S LAW AND THE ELECTRIC FIELD   Date ______________

Mastery Test   Form B   pass   recycle

1   2   3

Name ______________________   Tutor ______________________

1. Determine the force on the +2.00-μC charge at point P in Figure 1. Show your work. ($Q = 4.0 \times 10^{-9}$ C, a = 0.80 m, b = 0.60 m.)

2. The object in Figure 2 with mass m = 0.100 kg and charge $q = +5.0 \times 10^{-3}$ C is fired upward at an angle of 30° with the horizontal with an initial speed of 400 m/s in a vertically downward electric field of 2.00×10^5 N/C. How high does it rise?

3. What is the main difference between a conductor and a perfect insulator?

Figure 1

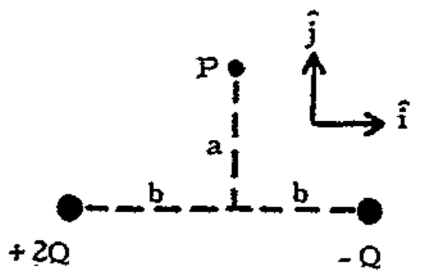
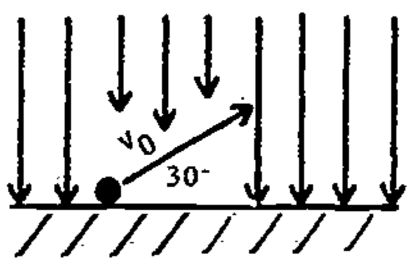


Figure 2



COULOMB'S LAW AND THE ELECTRIC FIELD | Date ________

Mastery Test   Form C

pass | recycle

1 2 3

Name ______________ Tutor ______________

1. Determine the electric field at point P in Figure 1. Show your work. ($Q_1 = 4.0 \times 10^{-10}$ C, $Q_2 = -8.0 \times 10^{-10}$ C, $Q_3 = 10.0 \times 10^{-10}$ C.)

2. A proton in Figure 2 ($m = 1.67 \times 10^{-27}$ kg) is projected horizontally with velocity $v_0 = 10^7$ m/s into a 10^5-N/C uniform field directed vertically between the parallel plates 20.0 cm long. What is the angle the proton velocity makes with the horizontal when it emerges from the other side? Neglect any fringing field effects. (Note: $\tan\theta = v_y/v_x$. For small θ, $\tan\theta \approx \theta$.)

3. What is the main difference between a conductor and a perfect insulator?

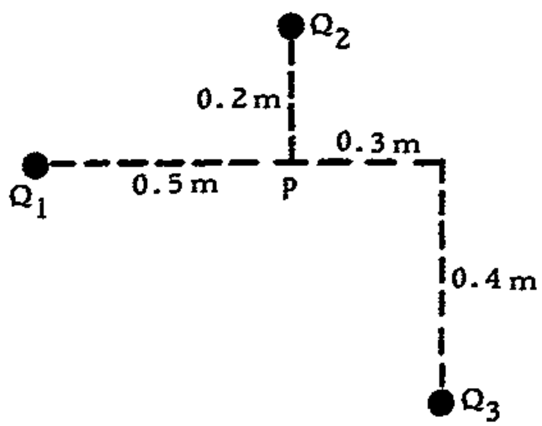


Figure 1

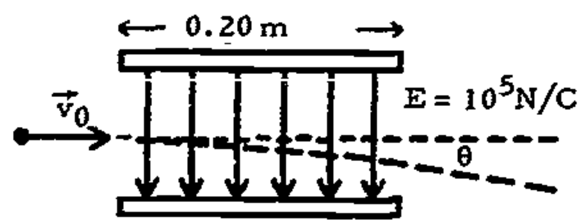


Figure 2

COULOMB'S LAW AND THE ELECTRIC FIELD Date ________________

Mastery Test Form D

pass recycle

1 2 3

Name ______________________________ Tutor ____________________

1. A -1.00-μC charge is at the center of a circular arc of radius $R = 3.00$ m in Figure 1. The +3.00-μC and -2.00-μC charges are located on the arc, as shown. Calculate the force on the -1.00-μC charge.

2. An electron ($q = -1.60 \times 10^{-19}$ C) experiences a force of $(+3.00 \times 10^{-16}\hat{i})$ N in a certain region of space.
(a) Find $\vec{E}$ in this region.
(b) Assuming that $\vec{E}$ is constant, and that the electron's motion takes it through the origin and also through the point $x = 5.0$ m, $y = 4.0$ m, how much work is done on the electron by the electric field in passing between these points?

3. What is the main difference between a conductor and a perfect insulator?

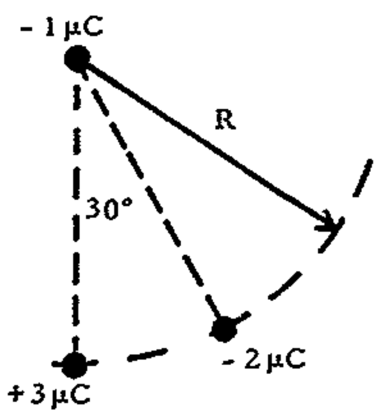


Figure 1

COULOMB'S LAW AND THE ELECTRIC FIELD Date ______________

Mastery Test Form E

pass recycle

1 2 3

Name ______________________ Tutor ______________________

1. Find the magnitude and direction of the field $\vec{E}$ at point A in Figure 1.

2. A charged object, with $q = +1.00 \times 10^{-4}$ C and $m = 0.50$ kg, is released at rest in a uniform electric field with intensity $E = 3.00 \times 10^4$ N/C, directed upward. This is done near the surface of the Earth.
   (a) In which direction does it move?
   (b) How long does it take to move 4.5 m?

3. What is the main difference between a conductor and a perfect insulator?

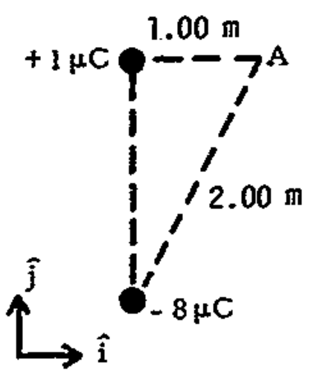


Figure 1

MASTERY TEST GRADING KEY - Form A

1. What To Look For: Invite the student to recheck his work if you spot a single sign error.

   Solution:

   $$\vec{E}_p = \frac{(9.0 \times 10^9)(10^{-5})(4/5)}{25}\hat{i} + \frac{(9.0 \times 10^9)(10^{-5})(3/5)}{25}\hat{j} - \frac{(9.0 \times 10^9)(5 \times 10^{-6})(4/5)}{25}\hat{i}$$

   $$- \frac{(9.0 \times 10^9)(5 \times 10^{-6})(3/5)}{25}\hat{j} \equiv (108/25 \times 10^3)\hat{i} + (27/25 \times 10^3)\hat{j}$$

   $$\vec{E}_p = [-(4.3 \times 10^3)\hat{i} + (1.08 \times 10^3)\hat{j}] \text{ N/C}.$$

2. Solution: $\Sigma F_x = 0 = (2.00 \times 10^{-5}\text{ C})(10^3\text{ N/C}) - T \sin\theta = 0$,

   $\Sigma F_y = 0 = T\cos\theta - (10^{-3}\text{ kg})(9.8\text{ m/s}^2)$.

   Thus $\tan\theta = 2.05$ and $\theta = 64°$.

3. Solution: In a conductor, charges are free to move in response to an electric field; in a perfect insulator, charges are not free to move.

MASTERY TEST GRADING KEY - Form B

1. Solution:

$$\vec{F}(\text{due to } 8.0\ \mu C) = (\tfrac{3}{5})\frac{(9.0 \times 10^9)(16.0 \times 10^{-15})}{1^2}\hat{i} + (\tfrac{4}{5})\frac{(9.0 \times 10^9)(16.0 \times 10^{-15})}{1^2}\hat{j}$$

$$= [(0.35/4 \times 10^{-3})\hat{i} + (0.46/4 \times 10^{-3})\hat{j}]\ N.$$

$$\vec{F}(\text{due to } -4.0\ \mu C) = (\tfrac{3}{5})\frac{(9.0 \times 10^9)(8.0 \times 10^{-15})}{1^2}\hat{i} - (\tfrac{4}{5})\frac{(9.0 \times 10^9)(8.0 \times 10^{-15})}{1^2}\hat{j}$$

$$= [(0.173/4 \times 10^{-3})\hat{i} - (0.231/4 \times 10^{-3})\hat{j}]\ N.$$

$$\vec{F}_{total} = (0.13 \times 10^{-3})\hat{i} + (0.06 \times 10^{-3})\hat{j} = [(1.3 \times 10^{-4})\hat{i} + (6.0 \times 10^{-4})\hat{j}]\ N.$$

2. What To Look For: $v_{0y} = v_0 \sin 30° = 200$ m/s. Student could also use $v_y^2 - v_{0y}^2 = 20.0_y y$, $v_y = 0$, $y_{max} = v_{0y}^2/(-20.0y) = 2.00$ m.

Solution: The time for a_y to take v_{0y} to zero is $|\frac{v_{0y}}{a_y}|$.

$$a_y = \mp\frac{qE}{m} - 9.8\ m/s^2 - \frac{(5.0 \times 10^{-3}\ C)(2.0 \times 10^5\ N/C)}{0.100\ kg} - 9.8\ m/s^2$$

$$= -9.8\ m/s^2 - 10^4\ m/s^2 \approx -10^4\ m/s^2,$$

$$t = \frac{200\ m/s}{10^4\ m/s^2} = 0.0200\ s,$$

$$y_{max} = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2 = (200\ m/s)(0.200\ s) - \frac{(10^4\ m/s)(0.0200\ s)^2}{2}$$

$$= 4.0\ m - 2.00\ m = 2.00\ m.$$

3. Solution: In a conductor, charges are free to move in response to an electric field; in a perfect insulator charges are not free to move.

MASTERY TEST GRADING KEY - Form C

1. Solution:

$$\vec{E}_P = \frac{(9.0 \times 10^9)(4.0 \times 10^{-10})}{(1/2)^2}\hat{i} - \left(\frac{3}{5}\right)\frac{(9.0 \times 10^4)(10^{-9})}{(1/2)^2}\hat{j} + \left(\frac{4}{5}\right)\frac{(9.0 \times 10^9)(10^{-9})}{(1/2)^2}\hat{i}$$

$$+ \frac{(9.0 \times 10^9)(8.0 \times 10^{-10})}{(1/5)^2}\hat{j} = (-7.2\,\hat{i} + 209\hat{j})\ \text{N/C.}$$

2. Solution: Travel time between plates is

$t = x/v_{0y} = 0.200\ \text{m}/10^7\ \text{m/s} = 2.00 \times 10^{-8}\ \text{s}.$

$a_y = qE/m = [(1.60 \times 10^{-19})/(1.67 \times 10^{-27})]\ (-10^5) = -0.95 \times 10^{13}\ \text{m/s}^2,$

$v_y = v_{0y} + a_y t = -1.90 \times 10^5\ \text{m/s},\ \tan\theta = \dfrac{1.90 \times 10^5\ \text{m/s}}{10^7\ \text{m/s}} = 1.90 \times 10^{-2},$

$\theta = 1.90 \times 10^{-2}$ rad = 1.10° below the horizontal.

3. Solution: In a conductor charges are free to move in response to an electric field; in a perfect insulator, charges are not free to move.

MASTERY TEST GRADING KEY - Form D

1. Solution: Choose axes x and y.

$$\vec{F} = -\frac{(9.0 \times 10^9)(3.00 \times 10^{-12})}{9.0\ m^2}\hat{i} + \frac{\sqrt{3}}{2}\,\frac{(9.0 \times 10^9)(2.00 \times 10^{-12})}{9.0\ m^2}\hat{j}$$

$$-\frac{(9.0 \times 10^9)(2.00 \times 10^{-12})}{9.0\ m^2}\frac{1}{2}\hat{i}$$

$$= [-10^{-3}\hat{i} - (1.30 \times 10^{-3})\hat{j}]\ N.$$

2. Solution:

(a) $\vec{E} = \vec{F}/q = (3.00 \times 10^{-16})/(-1.60 \times 10^{-19})\hat{i} = -(1.88 \times 10^3)\hat{i}$ N/C.

(b) For a constant force $W = Fd = (3.00 \times 10^{-16}\ N)(5.0\ m) = 1.50 \times 10^{-15}$ J.

3. Solution: In a conductor charges are free to move in response to an electric field; in a perfect insulator, charges are not free to move.

MASTERY TEST GRADING KEY - Form E

1. Solution:

$$E = -\left(\frac{1}{2}\frac{(8.0 \times 10^{-6})(9.0 \times 10^{9})}{4}\right)\hat{i} + \frac{(9.0 \times 10^{9})(10^{-6})}{1}\hat{i}$$

$$- \frac{(9.0 \times 10^{9})(8.0 \times 10^{-6})}{4}\left(\frac{\sqrt{3}}{2}\right)\hat{j} = -(1.56 \times 10^{4})\hat{j} \text{ N/C.}$$

2. Solution:

(a) $a_y = \frac{qE}{m} + g = \frac{(10^4 \text{ C})(3.00 \times 10^4 \text{ N/C})}{0.5 \text{ kg}} - 9.8 \text{ m/s}^2 = (6.0 - 9.8) \text{ m/s}^2 = -3.8 \text{ m/s}^2.$

It moves down.

(b) $y = y_0 + v_{0y} + \frac{1}{2}a_y t^2$, $4.5 \text{ m} = \frac{3.8t^2}{2}$, $t^2 = \frac{9}{3.8}$, $t = 1.54$ s.

3. Solution: In a conductor, charges are free to move in response to an electric field; in a perfect insulator, charges are not free to move.

FLUX AND GAUSS' LAW

INTRODUCTION

Charles Augustine de Coulomb (1736-1806) designed his famous experiment to measure the force relationships between charged bodies: Coulomb's law is the resulting empirical statement. Gauss' law (Karl Friedrich Gauss, 1777-1855), which you will learn in this module, has a more obscure origin. It was originally a mathematical theorem. Scientists in Gauss' nineteenth century were much more inclined than we are today to equate mathematical correctness with physical correctness. When it was realized that Gauss' (mathematical) theorem could be applied to the electric-field concepts of Faraday to produce Gauss' (physical) law, this extension was eagerly accepted. The origins of the law, however, continued and still continue to lie in the domain of pure logic; therefore they may be somewhat inaccessible to you in beginning physics courses. Your texts and this module will use both physical and mathematical arguments and examples to help you achieve a mastery of these ideas and their applications.

PREREQUISITES

Before you begin this module, you should be able to:	Location of Prerequisite Content
*Use vectors to represent certain quantities and add them (needed for Objectives 1 through 4 of this module)	Dimensions and Vector Addition Module
*Perform vector scalar multiplication (needed for Objectives 1 through 4 of this module)	Vector Multiplication Module
*State Coulomb's law (needed for Objective 3 of this module)	Coulomb's Law and the Electric Field Module
*Describe an electric field, conductors, and insulators (needed for Objectives 1 through 4 of this module)	Coulomb's Law and the Electric Field Module

LEARNING OBJECTIVES

After you have mastered the content of this module, you will be able to:

1. Statement of Gauss' law - State Gauss' law and explain all its symbols.
2. Limitations of Gauss' law - Recognize when Gauss' law cannot be used to determine the electric field caused by a static charge distribution, and explain why.

3. Applications of Gauss' law - Use Gauss' law to

   (a) determine the electric field due to certain symmetric charge distributions; or

   (b) determine the net charge inside volumes where the electric field is known everywhere on the surface of the volume.

4. Electric field, charge, and conductors - Given a conductor with a static charge distribution, use the properties of a conductor and/or Gauss' law to

   (a) explain why the electric field is perpendicular to the surface of the conductor;

   (b) explain why the electric field is zero inside the conductor.

   (c) explain why the excess charge is on the surface of the conductor.

TEXT: Frederick J. Bueche, Introduction to Physics for Scientists and Engineers (McGraw-Hill, New York, 1975), second edition

SUGGESTED STUDY PROCEDURE

Read General Comment 1 in this study guide and Sections 19.1 and 19.2 in Chapter 19 of your text. Then read General Comment 2, Section 19.3, and work through Problem A. Read Section 19.4 and work through Illustration 19.1. Read Sections 19.5 and 19.6 and work through Problem D and Illustration 19.2. Then read Sections 19.7 and 19.8, work through Illustrations 19.3 and 19.4 and Problem C. Read General Comments 3 and 4 and work through Problem B before working the Assigned Problems. Try the Practice Test.

BUECHE

Objective Number	Readings	Problems with Solutions: Study Guide	Problems with Solutions: Text	Assigned Problems: Study Guide	Additional Problems (Chap. 19)
1	General Comment 1, Secs. 19.1, 19.2, General Comment 2, Sec. 19.3	A			
2	General Comment 3	B			Quest.[a] 5, 7, 11, 12
3	Secs. 19.4, 19.6 to 19.8, General Comment 4	C	Illus.[a] 19.1, 19.2, 19.3, 19.4	E, F, G, H, I	Quest. 3, 6, Probs. 5 to 7, 9, 11, 13 to 15, 17, 18
4	Sec. 19.5	D			

[a]Illus. = Illustration(s). Quest. = Question(s).

TEXT: David Halliday and Robert Resnick, Fundamentals of Physics (Wiley, New York, 1970; revised printing, 1974)

SUGGESTED STUDY PROCEDURE

Read General Comments 1 and 2 in this study guide and Section 24-1 in Chapter 24 of your text before working through Example 1. Then read Section 24-2, and work through Problem A. Read Sections 24-3, 24-4, and 24-5, General Comment 3, and work through Problem B. Then read General Comment 4 and Section 24-6, and work through Examples 2 through 5. Do the Assigned Problems before attempting the Practice Test.

HALLIDAY AND RESNICK

Objective Number	Readings	Problems with Solutions: Study Guide	Problems with Solutions: Text	Assigned Problems: Study Guide	Additional Problems (Chap. 24)
1	General Comment 1, Sec. 24-1, General Comment 2, Sec. 24-2	A	Ex.[a] 1		Quest.[a] 3, 5
2	General Comment 3	B			Quest. 9, 11
3	Sec. 24-3, General Comment 4, Sec. 24-6	C	Ex. 2 to 5	E, F, G, H, I	Quest. 12 to 14, Probs. 8, 9, 12 to 35
4	Sec. 24-4	D			Quest. 7, 8

[a]Ex. = Example(s). Quest = Question(s).

TEXT: Francis Weston Sears and Mark W. Zemansky, University Physics (Addison-Wesley, Reading, Mass., 1970), fourth edition

SUGGESTED STUDY PROCEDURE

Read General Comments 1 and 2 in this study guide. Note that we use the symbol Φ for flux instead of ψ as used in your text. Then read Section 25-4 in Chapter 25 of your text and work through Problem A. Read General Comment 3 and work through Problem B; then read Section 24-5 and General Comment 4. Work through Problems C and D before trying the Assigned Problems. Try the Practice Test.

SEARS AND ZEMANSKY

Objective Number	Readings	Problems with Solutions		Assigned Problems	Additional Problems
		Study Guide	Text	Study Guide	
1	General Comments 1, 2, Sec. 25-4	A			
2	General Comment 3	B			
3	Sec. 25-5, General Comment 4	C	Sec. 25-5	E, F, G, H, I	25-12, 25-13, 25-16, 25-17(except d and e), 25-19, 25-20, 25-21, 25-22
4	Sec. 25-5, General Comment 4	D			

TEXT: Richard T. Weidner and Robert L. Sells, Elementary Classical Physics (Allyn and Bacon, Boston, 1973), second edition, Vol. 2

SUGGESTED STUDY PROCEDURE

Read General Comments 1 and 2 in this study guide and Section 24-1 in Chapter 24 of your text. Study Example 24-1. Then read Section 24-2 to p. 489. Read Section 24-3 and work through Problem A; read General Comment 3 and work through Problem B. Then read General Comment 4. Read the rest of Section 24-2 and all of Section 24-4 before working through Examples 24-2, 24-3, and 24-4, and Problem C. Read Section 24-5 and work through Problem D. The main ideas and equations in this module are presented in a summary at the end of Chapter 24 (p. 497). Work the Assigned Problems before trying the Practice Test.

WEIDNER AND SELLS

Objective Number	Readings	Problems with Solutions: Study Guide	Problems with Solutions: Text	Assigned Problems: Study Guide	Additional Problems
1	General Comments 1, 2, Secs. 24-2, 24-3	A	Ex.[a] 24-1		
2	General Comment 3	B			
3	General Comment 4, Secs. 24-2, 24-4	C	Ex. 24-2, 24-3, 24-4	E, F, G, H, I	24-6, 24-8(a), 24-9, 24-10, 24-11, 24-14(a), 24-15, 24-16
4	Sec. 24-5	D			

[a]Ex. = Example(s).

GENERAL COMMENTS

1. Electric Flux

You have seen from your reading that you can picture electric lines of force originating from each positive charge, bending around smoothly, and continuing on until they (possibly) end on some negative charge. You will recall that the direction of the line of force at any point gives the direction of the electric field $\vec{E}$ at that point. We now carry this pictorial representation of the electric field a little further, and specify how many lines shall be drawn. See Figure 1.

This is arbitrary, but if we make the number of lines that originate from or terminate on a charge directly proportional to the amount of charge, we shall be able to count the lines and determine the charge. In the SI system of units we draw $1/\varepsilon_0$ lines for each coulomb of charge:

$$\Phi = (1/\varepsilon_0)q. \qquad (1)$$

This quantity Φ is called the electric flux; and in this context the lines of force are often called lines of flux, or flux lines. Note that if q is negative, then Φ is negative, and the flux lines are drawn pointing inward (see Fig. 1).

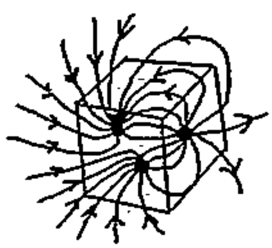


Figure 1

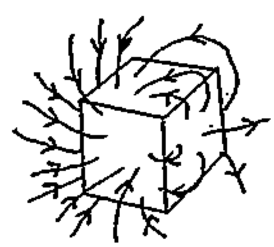


Figure 2

But now suppose you cannot actually see inside the box - that all you can see is the lines of flux leaving the box, as in Figure 2. Can you tell something about the charges contained inside? Yes, indeed, you can count up the total number of lines leaving the box, to find the total flux:

$$\Phi = \sum_i \Phi_i = \frac{1}{\varepsilon_0} \sum_i q_i, \qquad (2)$$

where Eq. (1) has been used to get the last equality. That is, the total charge $Q = \Sigma q_i$ contained within any surface S is related to the net amount Φ of flux lines passing outward through the surface by

$$Q = \varepsilon_0 \Phi. \qquad (3)$$

Note that the "counting up" of flux lines must be done in an algebraic sort of way, in which you add one for each line coming out, and subtract one for each line going in.

2. Surface Integral

In Figure 3 is shown a volume enclosed by a surface, much like the volume inside a distorted balloon. The surface can be divided into area elements $d\vec{A}$. You may have done this in your calculus class for regular shapes such as cylinders, spheres, and triangles. You probably have not treated these area elements as vectors, however, but just determined their magnitudes, dA. The direction of $d\vec{A}$ is outward from the volume and perpendicular to the surface.

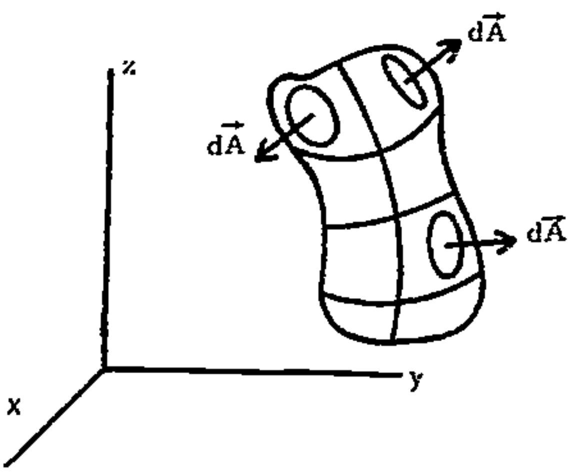


Figure 3

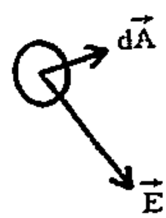


Figure 4

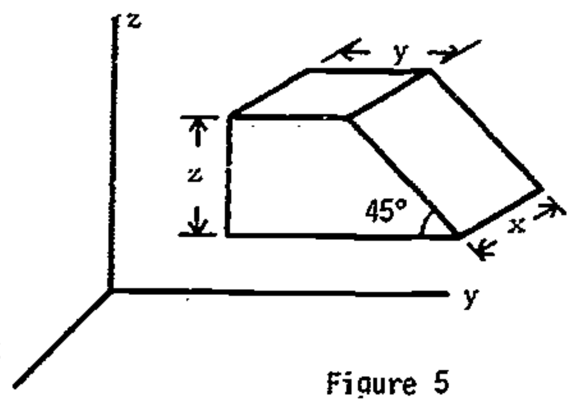


Figure 5

Now suppose you were told the value of the electric field $\vec{E}$ everywhere on the surface. Each surface area element dA would have its own value of $\vec{E}$, as shown in Figure 4. $\vec{E} \cdot d\vec{A}$ is a straightforward scalar multiplication of vectors, and the product will be an infinitesimal scalar quantity;

$$d\phi = \vec{E} \cdot d\vec{A}.$$

What if someone now asked you to find the total value of ϕ for the whole surface of the volume. You would have to add up all the $d\phi$'s associated with every $d\vec{A}$ on the surface. This sum is written as an integral:

$$\phi = \int_{\substack{\text{all surface}\\ \text{of volume}}} d\phi = \int_{\substack{\text{all surface}\\ \text{of volume}}} \vec{E} \cdot d\vec{A}.$$

This integral is called a <u>surface integral</u> and is represented by $\oint \vec{E} \cdot d\vec{A}$ or $\oint \vec{E} \cdot d\vec{S}$ in some texts and by $\int \vec{E} \cdot d\vec{A}$ or $\int \vec{E} \cdot d\vec{S}$ in others. It is important to remember that you must integrate over <u>all</u> of a completely closed surface.

<u>Exercise</u>: Here is a problem to check your ability to perform simple surface integrals: See Figure 5. The electric field in this space is uniform and equals $\vec{E} = E_0\hat{j}$. Calculate Φ for the surface shown in the figure. <u>Hint</u>: There are six sides. Calculate $\int \vec{E} \cdot d\vec{S}$ for each side and then add (scalarly) these answers. Do not forget that the scalar product of perpendicular vectors is zero. (Answers for the three sides shown are at the bottom of this page.)

3. Limitations

Although Gauss' law is always true it is not always useful. The integral

$$\oint \vec{E} \cdot d\vec{A} = \oint E \cos\theta \, dA = q/\varepsilon_0$$

can be solved for $\vec{E}$ only if $E \cos\theta$ can be factored out of the integral. Then you write Gauss' law as

$$E \cos\theta \oint dA = q/\varepsilon_0$$

and solve for E as

$$E = \frac{q/\varepsilon_0}{\cos\theta \oint dA} = \frac{q}{\cos\theta \, A\varepsilon_0} .$$

You can see that in order to factor out, $\vec{E}$ must have the same magnitude and the same direction with respect to every $d\vec{A}$ vector on the surface of integration. However, as indicated in the Exercise in General Comment 2, you might be able to divide this surface into smaller surfaces some of which have $\vec{E}$ perpendicular or parallel to $d\vec{A}$.

4. Use of Gauss' Law to Determine $\vec{E}$ for Symmetric Charge Distributions

When a charge distribution is known and possesses sufficient geometrical symmetry, one can use Gauss' law to deduce the resulting electric field. The procedure for doing so is pretty much the same in all cases and is outlined below.

<u>Answers</u>: $\int_{top} \vec{E} \cdot d\vec{A} = 0$, $\int_{front} \vec{E} \cdot d\vec{A} = 0$,

$\int_{\text{sloping side}} \vec{E} \cdot d\vec{A} = (E_0 \cos 45°) \dfrac{xz}{\sin 45°} = E_0 xz.$

Step 1. Deduce the direction of $\vec{E}$ from the symmetry of the charge distribution and Coulomb's law, e.g., for a spherically symmetric distribution $\vec{E}$ must be radial, i.e., must point away from (or toward) the symmetry center of the distribution.

Step 2. Use the symmetry of the charge distribution to determine the locus of points for which $\vec{E}$ must be constant in magnitude, e.g., for spherical symmetry the magnitude of $\vec{E}$ is necessarily the same at all points on the surface of a sphere concentric about the symmetry center.

Step 3. With Steps 1 and 2 as guides determine a closed (sometimes called Gaussian) surface such that at each point on the surface $\vec{E}$ is either (a) perpendicular to the surface and of constant magnitude E, or (b) in the plane of the surface, i.e., with no component normal to the surface.

Step 4. Let A be the area of that portion of the Gaussian surface for which $\vec{E}$ is normal and of constant magnitude E. The electric flux for this part of the surface is EA, although the flux for the remaining portion of the surface (if there is any such part) is zero since $\vec{E}$ has no normal component. Thus the surface integral over the Gaussian surface is EA.

Step 5. Now set the electric flux of Step 4 equal to the net charge q enclosed by the Gaussian surface multiplied by $1/\varepsilon_0$, i.e.,

$$EA = 1/\varepsilon_0 q \quad \text{or} \quad E = q/\varepsilon_0 A.$$

Thus at each point on that part of the surface for which $\vec{E}$ is perpendicular the electric field has a magnitude as determined here.

Example 1

Charge is distributed with a uniform density ρ (C/m^3) throughout a long (infinite) cylindrical rod of radius R as in Figure 6. Let r measure the distance from the symmetry axis of the cylinder to a point. Determine $\vec{E}(r)$.

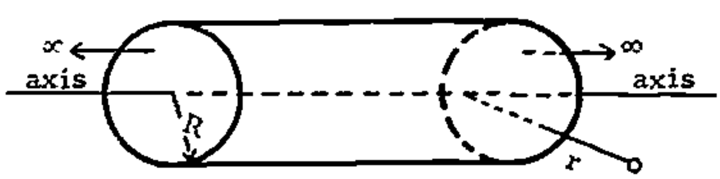


Figure 6

Step 1: From the symmetry of the charge distribution, $\vec{E}$ must be radial, i.e., perpendicular to the symmetry axis (see Fig. 7).

Step 2: The charge symmetry ensures that $\vec{E}$ will have the same magnitude at all points a distance r from the axis (see Fig. 7).

Step 3: The Gaussian surface is a cylinder of length L and radius r concentric with the symmetry axis. On the curved surface $\vec{E}$ is perpendicular (Step 1) and constant in magnitude (Step 2). On the flat ends $\vec{E}$ has no normal component (Step 1). (See Fig. 7.)

Step 4: The area of the curved part of the Gaussian surface is $A = 2\pi rL$. It is on this part that $\vec{E}$ is parallel to $d\vec{A}$ and constant in magnitude E. Therefore the electric flux for the curved part is

$$\phi_E = EA = 2\pi rLE.$$

Since $\vec{E}$ is perpendicular to $d\vec{A}$ on the two flat ends of the cylinder, the flux through these two parts is zero. Therefore $2\pi rLE$ is the total flux through the Gaussian surface.

Step 5: See Figure 7. Clearly the charge enclosed by the Gaussian surface depends upon whether $r < R$ or $r > R$. For $r > R$, the surface encloses all the charge in a length L of the rod, namely,

$$q = \rho\pi R^2 L \quad (r > R);$$

but for $r < R$, the surface encloses only that charge inside the Gaussian surface at radius r and length L, namely,

$$q = \rho\pi R^2 L \quad (r < R).$$

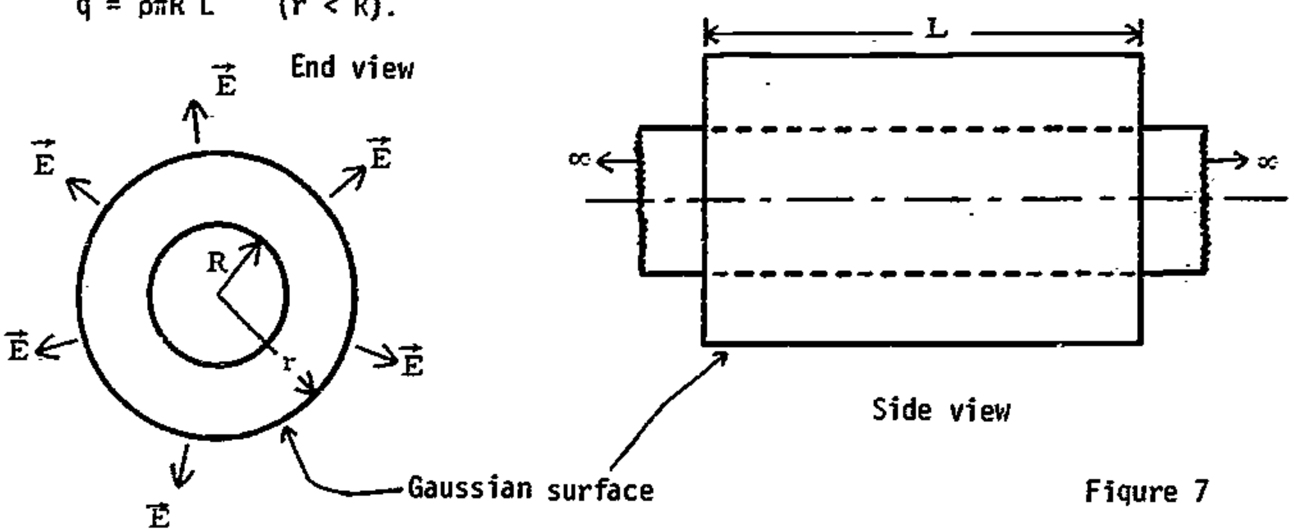


Figure 7

Now using Gauss' law and equating the electric flux to $1/\varepsilon_0$ times the net charge enclosed gives us

$$2\pi rLE = (1/\varepsilon_0)(\rho\pi r^2 L) \quad r < R$$

$$= (1/\varepsilon_0)(\rho\pi R^2 L) \quad r > R,$$

$$E = \rho r/2\varepsilon_0 \quad r < R$$

$$= \rho R^2/2\varepsilon_0 \quad r > R.$$

A graph of the magnitude of E versus r is shown in Figure 8. Remember this is just the magnitude. At each point $\vec{E}$ has this magnitude and points radially outward if $\rho > 0$ and radially inward if $\rho < 0$.

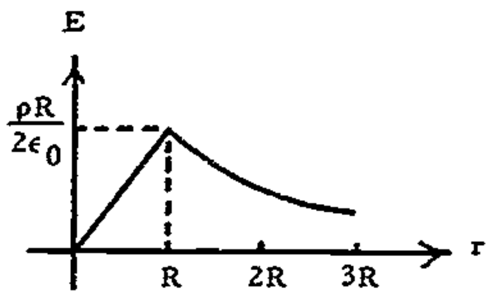


Figure 8

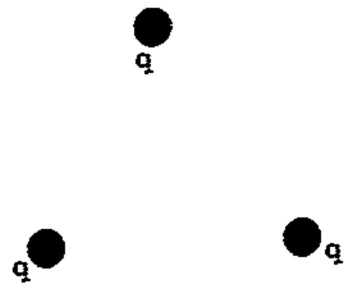


Figure 9

Gauss' law can be used to determine the electric field resulting from highly symmetric charge distributions. To do so, one infers from the charge symmetry a closed surface on which the electric field is either constant in magnitude and perpendicular to the surface, or in the plane of the surface. The electric flux is then easily determined in terms of the magnitude of the field and known geometrical quantities. Equating this flux to $1/\varepsilon_0$ times the known enclosed charge permits a determination of the magnitude of $\vec{E}$ at all points.

PROBLEM SET WITH SOLUTIONS

A(1). State Gauss' law and explain all its symbols.

Solution

Depending on your text's notations, Gauss' law is

$$\oint \vec{E} \cdot d\vec{A} = q/\varepsilon_0 \quad \text{or} \quad \int \vec{E} \cdot d\vec{A} = q/\varepsilon_0$$

$$\text{or} \quad \oint \vec{E} \cdot d\vec{S} = q/\varepsilon_0 \quad \text{or} \quad \int \vec{E} \cdot d\vec{S} = q/\varepsilon_0.$$

Given some volume V enclosed by surface A (or S), containing some net charge q, the surface integral of $\vec{E} \cdot d\vec{A}$ (or $\vec{E} \cdot d\vec{S}$) done over the whole surface equals the enclosed charge divided by ε_0. $\vec{E}$ is the electric field on the surface enclosing the volume; ε_0 is the permittivity of free space.

B(2). The three equal charges shown in Figure 9 are fixed at the points of an equilateral triangle. Explain why Gauss' law is not used to find $\vec{E}$ at any nearby point.

Solution

It is impossible to easily draw a Gaussian surface such that E cos θ is constant on it.

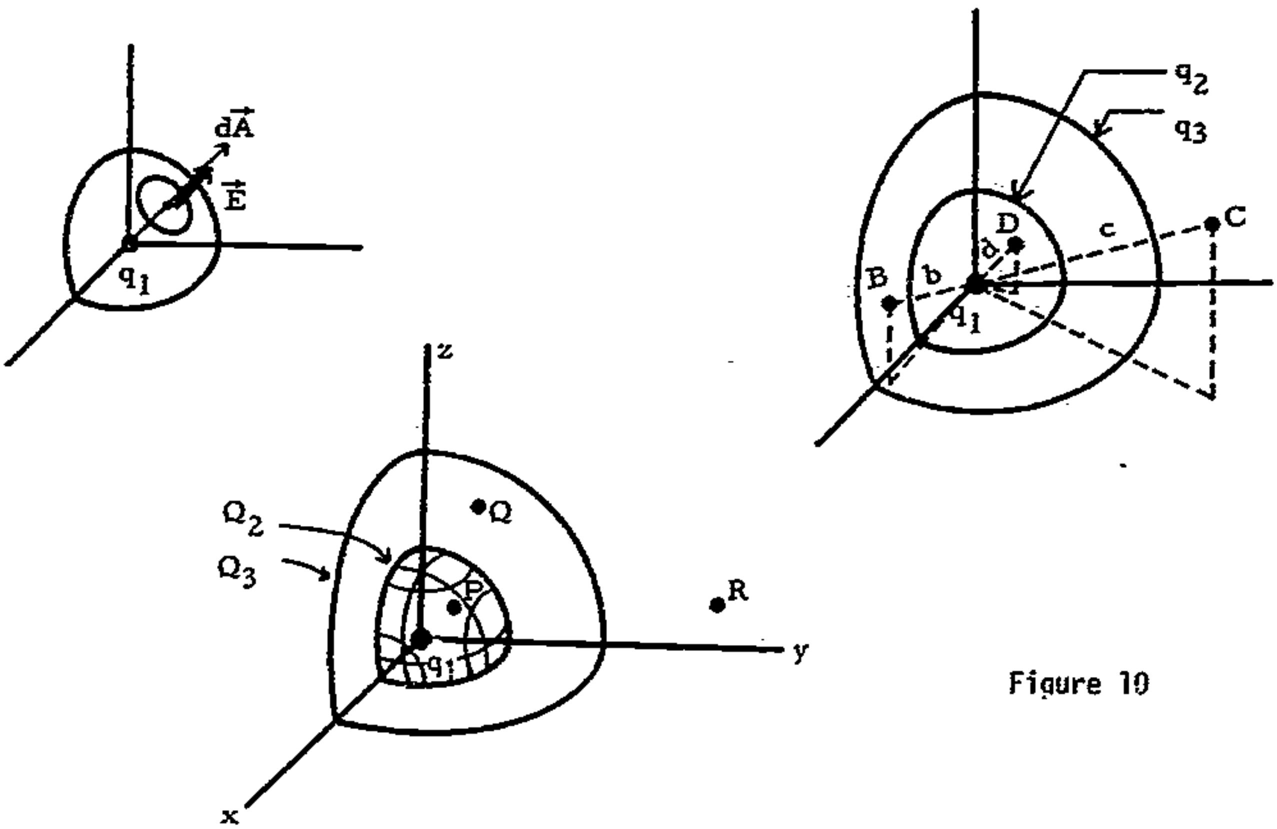


Figure 10

C(3). The point charge q_1 in Figure 10 is surrounded by two charged, spherical, thin metal shells as shown. Point P is inside the inner shell a distance 2.00 m from the origin. Point Q is between the shells a distance 3.00 m from the origin. Point R is outside the shells a distance 4.0 m from the origin.

$q_1 = 3.00 \times 10^{-6}$ C,

$Q_2 = -1.00 \times 10^{-6}$ C,

$Q_3 = -2.00 \times 10^{-6}$ C.

Use Gauss' law to find $\vec{E}$ at P, Q, and R. Show your Gaussian surfaces.

Solution

Point P: A spherical shell of radius 2.00 m will be a good Gaussian surface. Point P is located on this surface, it is the locus of points with constant E, and $\vec{E}$ is parallel to $d\vec{A}$ everywhere on this surface. $\vec{E}$ is outward since q_1 is positive. From

$$\oint \vec{E} \cdot d\vec{A} = q/\varepsilon_0, \quad \oint \vec{E} \cdot d\vec{A} = E\oint dA = E4\pi a^2.$$

Thus,

$$E = \frac{q_1}{4\pi\varepsilon_0 a^2} = \frac{3.00 \times 10^{-6}\ \text{C}}{4\pi(8.9 \times 10^{-12}\ \text{C}^2/\text{N m}^2)(2.00\ \text{m})^2} = 6.7 \times 10^3\ \text{N/C}.$$

Point Q: Since the charged shell will not alter the spherical symmetry of this problem, another spherical shell of radius 3.00 m is picked for the Gaussian surface. Now the total charge inside this surface is $(3.00 - 1.00) \times 10^{-6}$ C $= 2.00 \times 10^{-6}$ C. As before,

$$E = \frac{q_1 + Q_2}{4\pi\varepsilon_0 b^2} = \frac{2.00 \times 10^{-6}\ \text{C}}{4\pi(8.9 \times 10^{-12}\ \text{C}^2/\text{N m}^2)(3.00\ \text{m})^2} = 2.00 \times 10^3\ \text{N/C}.$$

The direction of $\vec{E}$ is again radially outward since the net charge in the Gaussian surface is positive.

Point R: Now the net charge inside a spherical Gaussian surface of radius 4.0 m is zero. $\vec{E}$ cannot be perpendicular to $d\vec{A}$; this would violate the spherical symmetry. Thus $\vec{E} = \vec{0}$.

D(4). (a) Do you need to use Gauss' law to show that $\vec{E}$ is perpendicular to the surface of a conductor with a static charge distribution?
(b) Do you need to use Gauss' law to show that $\vec{E}$ is zero inside a conductor with a static charge distribution?
(c) Do you need to use Gauss' law to show that the excess charge is on the surface of a conductor with static charge distribution?

Solution

(a) No. If $\vec{E}$ were not perpendicular to the surface it would have a component along the surface, and this would cause charge to flow. This would violate our assumption of a static charge distribution.
(b) No. If $\vec{E}$ were not zero inside the conductor, charge would flow, again violating the assumption of static charge distribution.
(c) Yes:

$$\oint \vec{E} \cdot d\vec{S} = q/\varepsilon_0,$$

and since $\vec{E}$ is zero inside the conductor [compare with part (b)], then for a Gaussian surface inside the conductor

$$\oint \vec{E} \cdot d\vec{S} = 0,$$

and q must be zero inside the conductor. The excess charge must be on the surface.

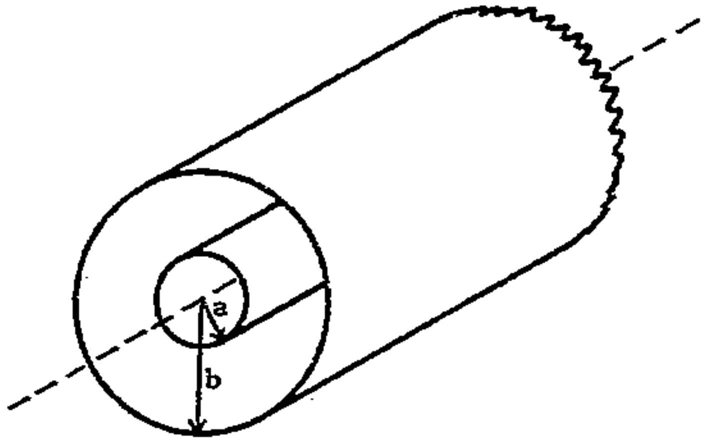


Figure 11

Problems

E(3). Two long statically charged thin coaxial cylinders are shown in Figure 11. The charge densities (in units of coulombs per square meter) have the relationship $\sigma_a/\sigma_b = -b/a$. Use Gauss' law to find $\vec{E}$:

(a) Between the cylinders. Show your choice of Gaussian surface.
(b) Outside the larger cylinder. Show your choice of Gaussian surface.

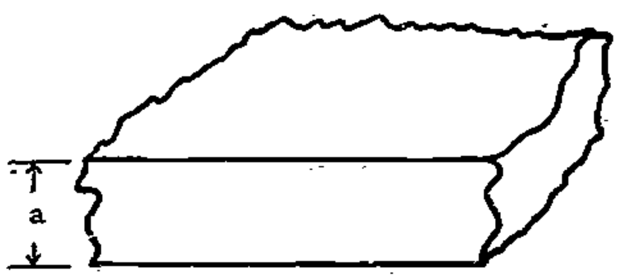


Figure 12

F(3). A large statically charged flat conducting plate is shown in Figure 12. The charge density is σ (in units of coulombs per square meter).
(a) Why is the charge density specified in units of C/m^2 instead of C/m^3?
(b) Use Gauss' law to find $\vec{E}$ outside the plate. Show your choice of Gaussian surface.

G(3). A long cylindrical uniformly charged insulator is shown in Figure 13. Its charge density is ρ (which has units of coulombs per cubic meter). A long, thin uniformly charged wire is coaxial to the cylinder as shown. Its charge density is λ (which has units of coulombs per meter). Use Gauss' law to find:

(a) $\vec{E}$ in the region $r < a$. Show your choice of Gaussian surface.
(b) $\vec{E}$ in the region $a < r < b$. Show your choice of Gaussian surface.
(c) $\vec{E}$ in the region $r > a$. Show you choice of Gaussian surface.

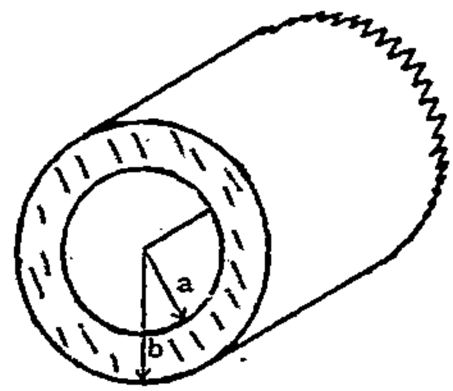


Figure 13

H(3). The infinite slab of insulating material in Figure 14 carries a uniform charge density ρ. There are no other charges in this region of space, so that the field must be symmetric about the plane $y = 0$, that is, $E(y = 0) = 0$. Use Gauss' law to find $E(y)$ for $-a < y < a$. Be sure to indicate clearly the (closed) Gaussian surface you are using. Which way does $\vec{E}$ point if $\rho < 0$?

I(3). A uniformly charged nonconducting sphere has a charge density of 3.00×10^{-12} C/m^3 and a radius of 1.00 m. Use Gauss' law to find $\vec{E}$

(a) 0.50 m from the center of the sphere. Show your choice of Gaussian surface; and

(b) 2.00 m from the center of the sphere. Show your choice of Gaussian surface.

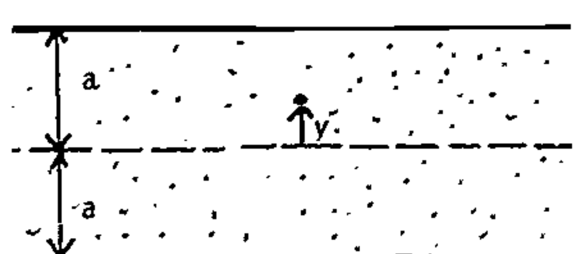


Figure 14

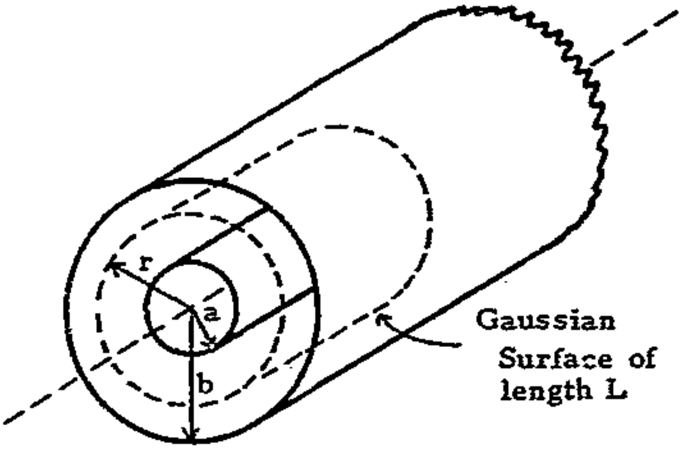


Figure 15

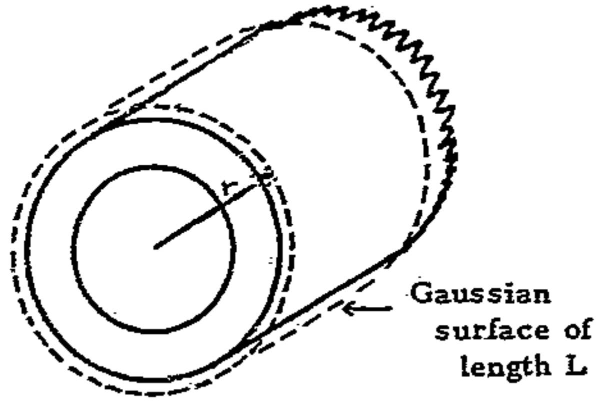


Figure 16

<u>Solutions</u>

E(3). (a) The charge inside the Gaussian surface of length L in Figure 15 is

$$q = 2\pi a L \sigma_a,$$

and the surface integral is

$$\oint \vec{E} \cdot d\vec{A} = \int_{\text{ends}} \vec{E} \cdot d\vec{A} + \int_{\substack{\text{curved}\\\text{side}}} \vec{E} \cdot d\vec{A} = 0(\vec{E} \perp d\vec{A}) + E2\pi rL.$$

Thus,

$$\vec{E} = 2\pi a L\sigma_a/\varepsilon_0 2\pi rL = a\sigma_a/\varepsilon_0 r; \quad \text{radially outward.}$$

(b) In the Gaussian surface of length L in Figure 16

$$q = 2\pi a\sigma_a L + 2\pi b\sigma_B L = 2\pi L(a\sigma_a + b\sigma_B) = 2\pi L(0).$$

Thus, $\vec{E} = \vec{0}$.

F(3). (a) Inside a statically charged conductor $\vec{E}$ is zero, and the excess charge resides on the surfaces (see Objective 4). In this case there are two equally charged surfaces.
(b) See Figure 17 for two choices for the Gaussian surface. In Figure 17(a), the charge inside the Gaussian surface is $q = \sigma A$. The surface integral is

$$\oint \vec{E} \cdot d\vec{A} = \int_{\text{top}} \vec{E} \cdot d\vec{A} + \int_{\text{sides}} \vec{E} \cdot d\vec{A} + \int_{\text{bottom}} \vec{E} \cdot d\vec{A} = 0(\vec{E} = 0) + 0(\vec{E} \perp d\vec{A}) + EA = EA.$$

Thus, $E = \sigma/\varepsilon_0$, perpendicular to the surface. In Figure 17(b), the charge inside the Gaussian surface is a = $2\sigma A$. The surface integral is

$$\oint \vec{E} \cdot d\vec{A} = \int_{\text{top}} \vec{E} \cdot d\vec{A} + \int_{\text{sides}} \vec{E} \cdot d\vec{A} + \int_{\text{bottom}} \vec{E} \cdot d\vec{A} = EA + 0(\vec{E} \perp d\vec{A}) + EA = 2\ EA.$$

Thus, $E = \sigma/\varepsilon_0$, perpendicular to the surface.

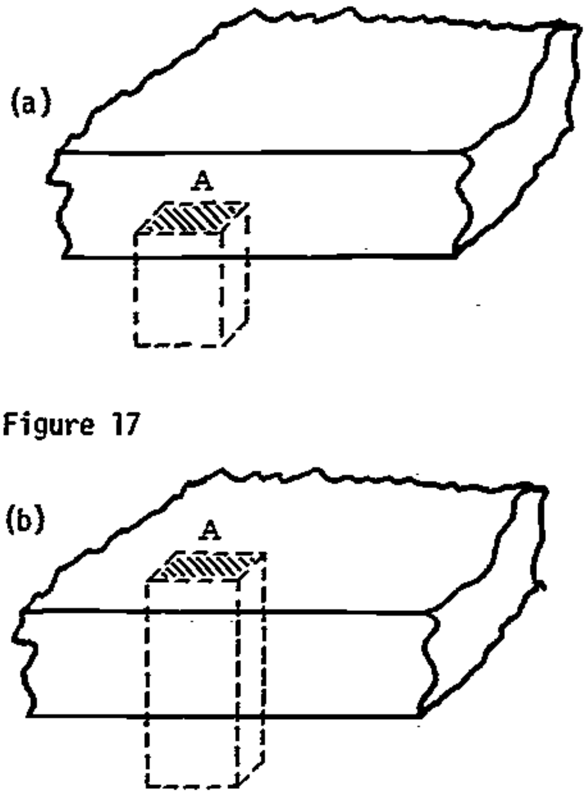


Figure 17

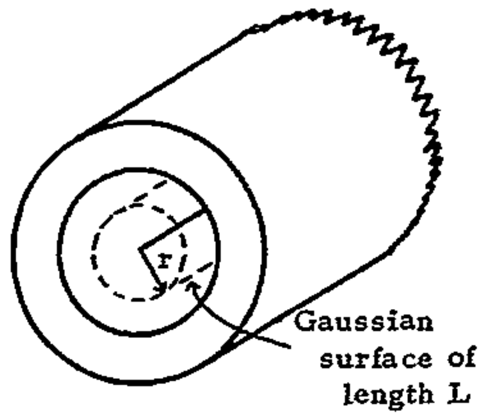


Figure 18

This shows that Gauss' law really works if applied correctly. Note that this result can be applied to any shaped surface if the point where you want to find the electric field is very close to the surface. If you are very close to a surface, it looks flat.

G(3). (a) See Figure 18, using

$$\oint \vec{E} \cdot d\vec{A} = q/\varepsilon_0$$

we find that the charge inside the Gaussian surface is $q = \lambda L$, and

$$\oint \vec{E} \cdot d\vec{A} = \int_{\text{ends}} \vec{E} \cdot d\vec{A} + \int_{\text{curved side}} \vec{E} \cdot d\vec{A} = 0(\vec{E} \perp d\vec{A}) + E2\pi rL.$$

Thus

$$E = \lambda L/\varepsilon_0 2\pi r L = \lambda/2\pi\varepsilon_0 r, \quad \text{radially outward.}$$

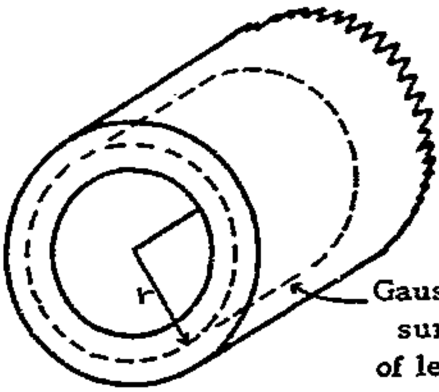


Figure 19

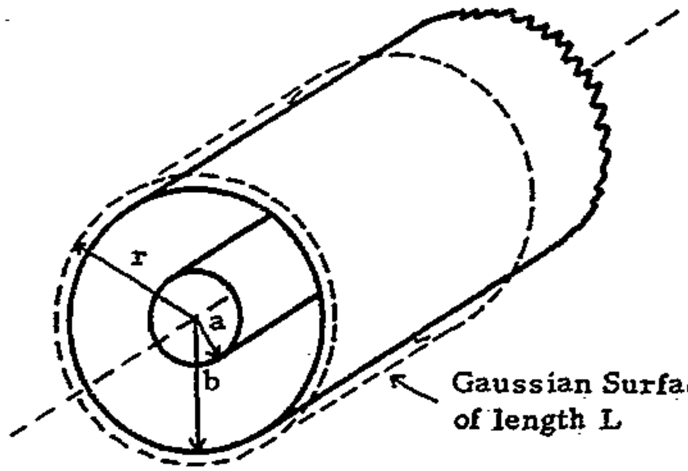


Figure 20

(b) See Figure 19, a Gaussian surface of length L. Now

$$q = \rho V + \lambda L = \rho(\pi r^2 - \pi a^2)L + \lambda L \quad \text{and} \quad \oint \vec{E} \cdot d\vec{A} = 2\pi r L E.$$

Thus,

$$E = \frac{[\rho\pi(r^2 - a^2) + \lambda]L}{2\pi\varepsilon_0 r L} = \frac{\rho\pi(r^2 - a^2) + \lambda}{2\pi\varepsilon_0 r}, \quad \text{radially outward.}$$

(c) See Figure 20, a Gaussian surface of length L, where

$$q = \rho(\pi b^2 - \pi a^2)L + \lambda L \quad \text{and} \quad \oint \vec{E} \cdot d\vec{A} = E2\pi r L.$$

Thus,

$$E = \frac{\rho\pi(b^2 - a^2) + \lambda}{2\pi\varepsilon_0 r}, \quad \text{radially outward.}$$

H(3). $E(y) = \rho_y/\varepsilon_0$, toward the center of the slab.

I(3). (a) $18\pi \times 10^{-3}$ N/C, radially outward. (b) $9\pi \times 10^{-3}$ N/C, radially outward.

PRACTICE TEST

1. State Gauss' law and briefly explain all its symbols.

2. Gauss' law is always true but not always useful. Explain why sometimes Gauss' law is a useful tool to determine the electric field caused by a static charge distribution.

3. Use Gauss' law to answer these questions:
(a) If a Gaussian surface encloses zero net charge, does Gauss' law require that $\vec{E} = 0$ at all points on the surface?
(b) If $\vec{E} = 0$ everywhere on a Gaussian surface, is the net charge inside necessarily zero?

4. The top and bottom of the cylindrical can in Figure 21 each have an area of $0.200\ m^2$. In this region of space there is a layer of charge, some of which is inside the can, so that E_B is larger than E_T; but the field points up everywhere. If $E_B = 5.0 \times 10^4$ N/C and $E_T = 2.50 \times 10^4$ N/C, how much charge is contained in the can?

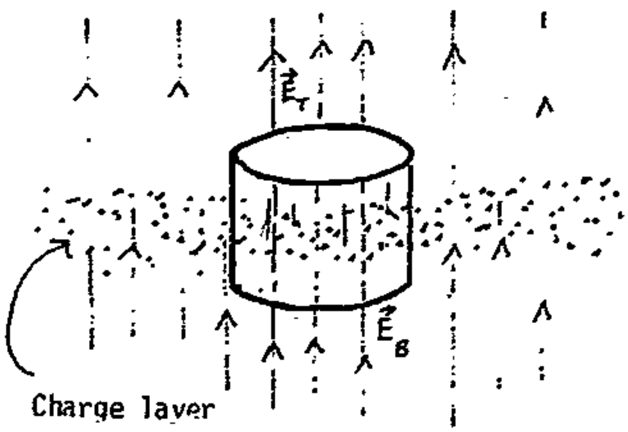


Figure 21

5. You are shown a conductor with a static charge distribution. Use the properties of a conductor and/or Gauss' law to:
(a) Explain why $\vec{E}$ is parallel or antiparallel to $d\vec{A}$ at the surface of the conductor.
(b) Explain why the excess charge lies on the surface of the conductor.

Practice Test Answers

1. $\oint \vec{E} \cdot d\vec{A} = q/\epsilon_0$ or $\int \vec{E} \cdot d\vec{A} = q/\epsilon_0$ or $\oint \vec{E} \cdot d\vec{S} = q/\epsilon_0$ or $\int \vec{E} \cdot d\vec{S} = q/\epsilon_0$.
The volume in Figure 22 contains a charge q. $d\vec{A}$ is an area element of the surface covering the volume, and $\vec{E}$ is the value of the electric field at that area element.

They are multiplied (scalar product) to determine the component of $\vec{E}$ in the direction of $d\vec{A}$ times dA. The integration must be performed over the complete surface covering the volume. ε_0 is a constant called the <u>permittivity of free space</u>.

2. Gauss' law is useful when $E \cos \theta$ can be factored out of the integral

$$\oint E \cos \theta \, dA = q/\varepsilon_0.$$

This can be accomplished when $E \cos \theta$ is a constant over the surface of integration.

3. (a) No. $\oint \vec{E} \cdot d\vec{A} = 0$ can be true without $\vec{E} = \vec{0}$. (b) Yes.

$$\int \vec{E} \cdot d\vec{A} = q/\varepsilon_0$$

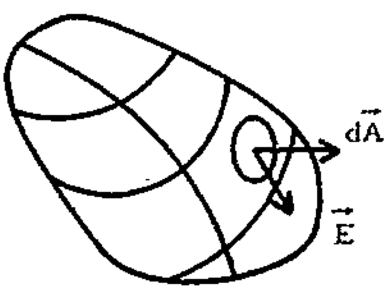
and if $\vec{E} = \vec{0}$ on the surface of integration then the left-hand side of the equation must be zero. Thus the right-hand side must also be zero and $q = 0$.

4. See Figure 23, where the dotted lines indicate the Gaussian surface.

$$\oint \vec{E} \cdot d\vec{A} = \int_{\text{top}} \vec{E}_T \cdot d\vec{A} + \int_{\substack{\text{curved}\\ \text{side}}} \vec{E} \cdot d\vec{A} + \int_{\text{bottom}} \vec{E}_B \cdot d\vec{A} = E_T A + 0 + (-E_B A) = (E_T - E_B)A = q/\varepsilon_0,$$

$$q = \varepsilon_0(E_T - E_B)A = -(8.9 \times 10^{-12}\ C^2/N\ m^2)(2.50 \times 10^4\ N/C)(0.200\ m^2) = -4.4 \times 10^{-8}\ C.$$

Figure 22



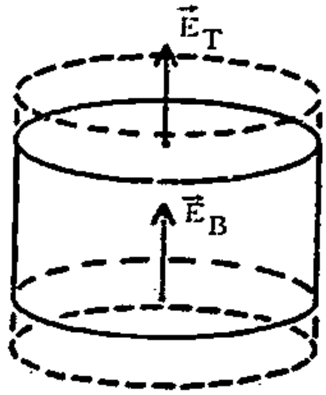


Figure 23

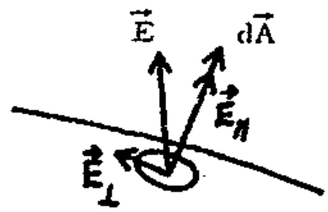


Figure 24

5. (a) See Figure 24. If $\vec{E}$ were not in the direction of $d\vec{A}$ it could be resolved into vectors parallel and perpendicular to $d\vec{A}$. From $\vec{F} = q\vec{E}$, the component of $\vec{E}$ along the surface would produce a force on the charges in the conductor. They would flow and our assumption about a static charge distribution would be false. Thus $\vec{E}$ must be in the direction of $d\vec{A}$.
(b) In the interior of a conductor $\vec{E}$ must be zero. Otherwise there would be charges moving, in contradiction to the assumption of a static charge distribution. Applying Gauss' law to a surface just inside the surface of the conductor shows us that q is zero inside this surface. The excess charge must therefore reside outside the Gaussian surface on the surface of the conductor.

FLUX AND GAUSS' LAW

Date ____________

Mastery Test Form A

pass recycle

1 2 3 4

Name ______________________ Tutor ______________________

Use $1/4\pi\epsilon_0 = 9.0 \times 10^9$ N m^2/C^2 in working these problems.

1. State Gauss' law and explain all its symbols.

2. For the following charged conductors EITHER

   Sketch the conductor and the Gaussian surface you would choose to find the electric field outside the conductor, OR

   Explain why Gauss' law cannot easily be used to find $\vec{E}$ outside the conductor.

   (a) a charged sphere,
   (b) a very long charged cylinder,
   (c) a very long charged wire,
   (d) two nearby point charges,
   (e) a large, flat, charged surface,
   (f) a charged cube.

3. What is the net charge on a statically charged conducting sphere of 2.00 m radius if $\vec{E}$ is 15.0×10^9 N/C in the radial direction toward the center of the sphere at a distance 3.00 m from the center of the sphere. Use Gauss' law.

4. Given a conductor with a static charge distribution:
   (a) Use the properties of a conductor and Gauss' law to explain why $\vec{E}$ is discontinuous across the surface of the conductor (why $\vec{E}$ changes abruptly from just inside to just outside the surface of the conductor).
   (b) Use the properties of a conductor to explain why $\vec{E}$ is perpendicular to the surface of the conductor.

FLUX AND GAUSS' LAW

Date ____________

Mastery Test   Form B

pass   recycle

1   2   3   4

Name ______________________   Tutor ______________________

Use $1/4\pi\epsilon_0 = 9.0 \times 10^9$ N m^2/C^2 in working these problems.

1. State Gauss' law and explain all its symbols.

2. Explain why Gauss' law is not always a useful tool with which to determine the electric field.

3. The long uniformly charged circular rod in Figure 1 with a radius of 2.00 m has a constant charge density of

$$\rho = (1/\pi) \times 10^{-12} \text{ C/m}^3.$$

(a) Use Gauss' law to find $\vec{E}$ at a radius of 1.00 m from the axis of the rod. Show your choice of a Gaussian surface.
(b) Use Gauss' law to find $\vec{E}$ at a radius of 3.00 m from the axis of the rod. Show your choice of a Gaussian surface.

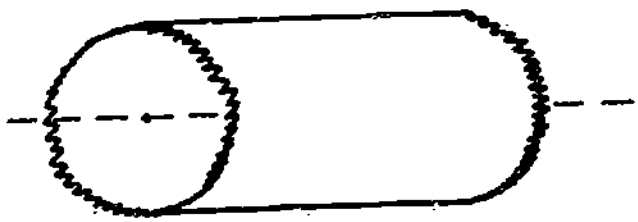


Figure 1

4. Given a conductor with a static charge distribution, use the properties of a conductor and/or Gauss' law to:
(a) Explain why $\vec{E}$ is perpendicular to the surface of the conductor.
(b) Explain why $\vec{E}$ is zero inside the conductor.
(c) Explain why the excess charge is on the surface of the conductor.

FLUX AND GAUSS' LAW Date ________________

Mastery Test Form C

pass recycle

1 2 3 4

Name ______________________________ Tutor ______________________

Use $1/4\pi\varepsilon_0 = 9.0 \times 10^9$ N m^2/C^2 in working these problems.

1. State Gauss' law and explain all its symbols.

2. For the following charged conductors EITHER

   Sketch the conductor and the Gaussian surface you would choose to find $\vec{E}$ outside the conductor, OR

   Explain why Gauss' law cannot easily be used to find $\vec{E}$ outside the conductor.

   (a) a charged disk,
   (b) a charged sphere,
   (c) a long, hollow cylinder,
   (d) three equal charges at the points of an equilateral triangle,
   (e) a large, thick, flat plate.

3. A large flat conducting plate is 0.50 m thick. It is statically charged, and the surface charge density σ is 2.50×10^{-15} C/m^2. Use Gauss' law to find $\vec{E}$ 1.50 m from the upper surface of the plate.

4. Given a conductor with a static charge distribution:
   (a) Use the properties of a conductor and Gauss' law to explain why $\vec{E}$ is discontinuous across the surface of the conductor (why $\vec{E}$ changes abruptly from just inside to just outside the surface of the conductor).
   (b) Use the properties of a conductor to explain why $\vec{E}$ is perpendicular to the surface of the conductor.

MASTERY TEST GRADING KEY - Form A

1. What To Look For: q causes $\vec{E}$. Integration is over a closed surface. q is inside the surface of integration. It's OK to write out $\vec{E} \cdot d\vec{A}$ as E cos θ dA, but now θ must be explained.

Solution:

$\oint \vec{E} \cdot d\vec{A} = q/\varepsilon_0$ or $\int \vec{E} \cdot d\vec{A} = q/\varepsilon_0$ or $\oint \vec{E} \cdot d\vec{S} = q/\varepsilon_0$ or $\int \vec{E} \cdot d\vec{S} = q/\varepsilon_0$.

Given some volume V enclosed by surface A (or S), and which contains some net charge q, the surface integral of $\vec{E} \cdot d\vec{A}$ (or $\vec{E} \cdot d\vec{S}$) over the whole surface equals the enclosed charge divided by ε_0. $\vec{E}$ is the electric field at area element $d\vec{A}$, and ε_0 is the permittivity of free space.

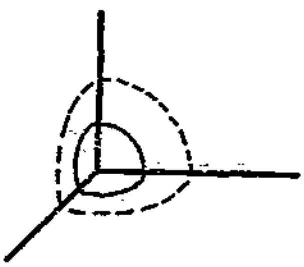


Figure 27

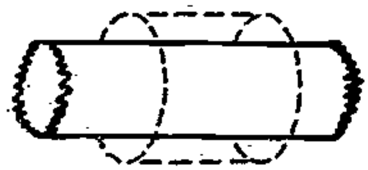


Figure 28

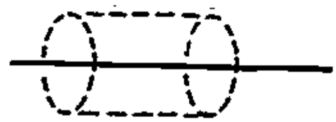


Figure 29

2. What To Look For: (a) See Figure 27, where the dotted lines indicate the Gaussian surface.
(b) See Figure 28, where the dotted lines indicate the Gaussian surface.
(c) See Figure 29, where the dotted lines indicate the Gaussian surface.
(d) Cannot easily draw a Gaussian surface over which E cos θ is constant.
(e) See Figure 30, where the dotted lines indicate the Gaussian surface.
(f) Cannot easily draw a Gaussian surface over which E cos θ is constant. The corners give trouble.

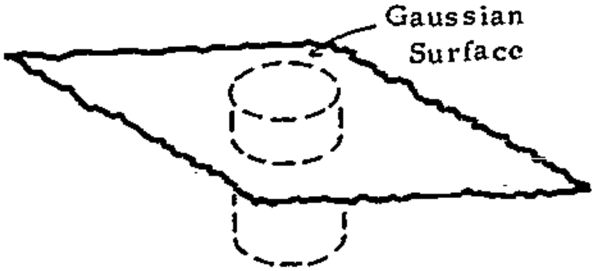


Figure 30

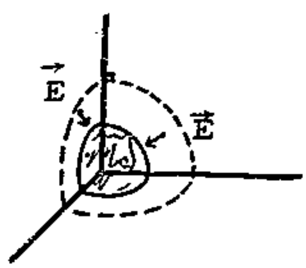


Figure 31

3. What To Look For: Correct Gaussian surface charge is negative.

Solution: See Figure 31, where the dotted lines indicate the Gaussian surface of radius 3.00 m. The spherical symmetry of the charge distribution causes spherical symmetry in $\vec{E}$.

$$q/\varepsilon_0 = \vec{E} \cdot d\vec{A} = -EA = -E4\pi r^2,$$

$$q = -\varepsilon_0 E 4\pi r^2 = \frac{(15.0 \times 10^9 \text{ N/C})(9.0 \text{ m}^2)}{9.0 \times 10^9 \text{ N m}^2/\text{C}^2} = 15.0 \text{ C}.$$

4. Solution: (a) Inside the conductor $\vec{E} = 0$. The static charge distribution of the movable charges indicates that $\vec{F}$ and therefore $\vec{E}$ is zero inside. Outside the conductor $\vec{E} \neq 0$. Gauss' law says that if q is not zero then $\vec{E}$ is not zero. Thus $\vec{E}$ changes from $\vec{0}$ to $\vec{E}$ as you go from inside to outside a conductor with static charge distribution.
(b) If $\vec{E}$ were not perpendicular to the surface there would be an electric field along the surface. This would contradict the static charge assumption.

MASTERY TEST GRADING KEY - Form B

1. What To Look For: q causes $\vec{E}$. Integration is over a closed surface. q is inside the surface of integration. It's okay to write out $\vec{E} \cdot d\vec{A}$ as $E \cos \theta \, dA$, but now θ must be explained.

   Solution:

   $\oint \vec{E} \cdot d\vec{A} = q/\varepsilon_0$ or $\int \vec{E} \cdot d\vec{A} = q/\varepsilon_0$ or $\oint \vec{E} \cdot d\vec{S} = q/\varepsilon_0$ or $\int \vec{E} \cdot d\vec{S} = q/\varepsilon_0$.

   Given some volume V enclosed by surface A (or S), and which contains some net charge q, the surface integral of $\vec{E} \cdot d\vec{A}$ (or $\vec{E} \cdot d\vec{S}$) done over the whole surface equals the enclosed charge divided by ε_0. $\vec{E}$ is the electric field at area element $d\vec{A}$, and ε_0 is the permittivity of free space.

2. Solution: It is not always possible to draw a Gaussian surface everywhere on which $E \cos \theta$ is the same, and also of which you know the area.

3. What To Look For: (a) Correct Gaussian surface. Correct charge inside Gaussian surface. Direction of $\vec{E}$.

   Solution: (a) See Figure 32, where the dotted lines indicate the Gaussian surface. The charge inside the Gaussian surface is $q = \rho\pi r^2 \ell$. The surface integral is

   $\int \vec{E} \cdot d\vec{A} + \int \vec{E} \cdot d\vec{A} + \int \vec{E} \cdot d\vec{A}$.

   On the ends $\vec{E} \perp d\vec{A}$:

   $0 + E2\pi r\ell + 0$.

   Thus

   $$E = \rho r/2\varepsilon_0 = \frac{(1/\pi \times 10^{-12}\ \mathrm{C/m^3})(1.00\ \mathrm{m})(4\pi)(9.0 \times 10^9\ \mathrm{N\ m^2/C^2})}{2}$$

   $= 1.80 \times 10^{-2}$ N/C, radially outwards.

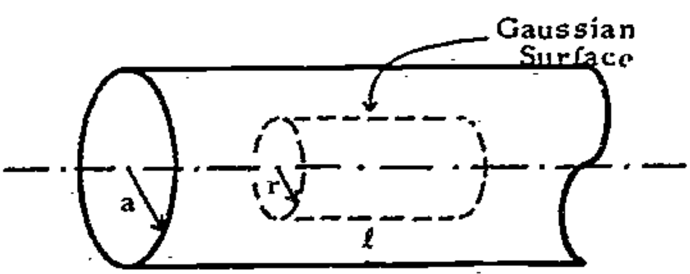


Figure 32

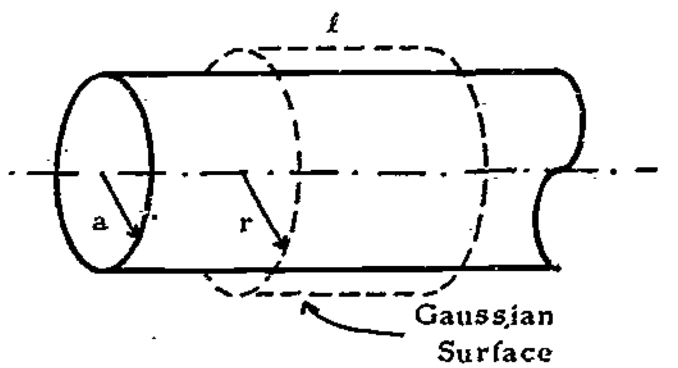


Figure 33

(b) See Figure 33, where the dotted lines indicate the Gaussian surface. Now the charge inside the Gaussian surface is $q = \rho\pi a^2 \ell$. The surface integral is treated as before to give

$$E = \rho a^2/2\epsilon_0 r = \frac{(1/\pi \times 10^{-12}\ C/m^3)(2.00\ m)^2(4\pi)(9.0 \times 10^9\ N\ m^2/C)}{2(3.0\ m)}$$

$= 2.4 \times 10^{-2}$ N/C, radially outward.

4. Solution: (a) If $\vec{E}$ were not perpendicular to the surface, there would be some electric field along the surface of the conductor, and charges would flow. This would contradict the static charge assumption.
(b) If $\vec{E}$ were not zero inside the conductor, charges would flow. This contradicts the static charge assumption.
(c) Use the results of (b) in Gauss' law. Draw a Gaussian surface just inside the surface of the conductor. The charge inside this surface is zero. Therefore the charge must be on the surface of the conductor.

MASTERY TEST GRADING KEY - Form C

1. What To Look For: q causes $\vec{E}$. Integration is over a closed surface. q is inside the surface of integration. It's OK to write out $\vec{E} \cdot d\vec{A}$ as E cos θ dA, but now θ must be explained.

   Solution:

   $\oint \vec{E} \cdot d\vec{A} = q/\varepsilon_0$ or $\int \vec{E} \cdot d\vec{A} = q/\varepsilon_0$ or $\oint \vec{E} \cdot d\vec{S} = q/\varepsilon_0$ or $\int \vec{E} \cdot d\vec{S} = q/\varepsilon_0$.

   Given some volume V enclosed by surface A (or S), and which contains some net charge q, the surface integral of $\vec{E} \cdot d\vec{A}$ (or $\vec{E} \cdot d\vec{S}$) done over the whole surface equals the enclosed charge divided by ε_0. $\vec{E}$ is the electric field at area element dA, and ε_0 is the permittivity of free space.

2. Solution: (a) Cannot easily draw a Gaussian surface over which E cos θ is constant. Corners give trouble.
   (b) See Figure 34, wherein the dotted lines indicate the Gaussian surface.
   (c) See Figure 35, wherein the dotted lines indicate the Gaussian surface.
   (d) Cannot easily draw a Gaussian surface over which E cos θ is constant.
   (e) See Figure 36, wherein the dotted lines indicate the Gaussian surface.

Figure 35

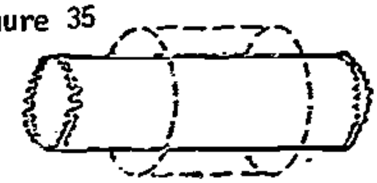
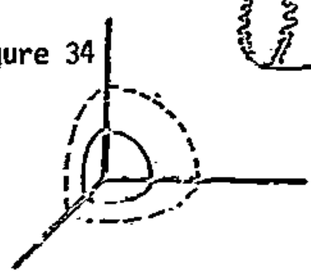


Figure 34



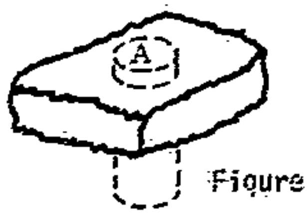


Figure 37

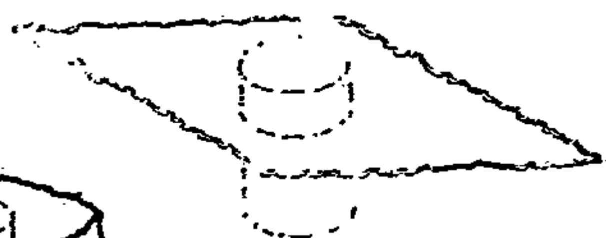


Figure 36

3. Solution: See Figure 37, where the dotted lines indicate the Gaussian surface. The charge inside the Gaussian surface is $q = 2\sigma A$. The surface integral is

   $$\oint \vec{E} \cdot d\vec{A} = \int \vec{E} \cdot d\vec{A} + \int \vec{E} \cdot d\vec{A} + \int \vec{E} \cdot d\vec{A} = EA + 0 + EO = 2EA.$$

   Thus $E = \sigma/\varepsilon_0$ which does not depend on the distance from the plate.

   $$E = (2.50 \times 10^{-15}\ \mathrm{C/m^2})(4\pi)(9.0 \times 10^9\ \mathrm{N\ m^2/C^2}) = 28.3 \times 10^{-5}\ \mathrm{N/C}$$

   outward perpendicular to the plate's surface.

4. Solution: (a) Inside the conductor $\vec{E} = 0$. The static charge distribution of the movable charges indicate that F and therefore E is zero inside. Outside the conductor $\vec{E} \neq 0$. Gauss' law says that if q is not zero then E is not zero. Thus $\vec{E}$ changes from 0 to $\vec{E}$ as you go from inside to outside a conductor with static charge distribution. If $\vec{E}$ were not perpendicular to the surface there would be an electric field and moving charge along the surface. This would contradict the static charge assumption.