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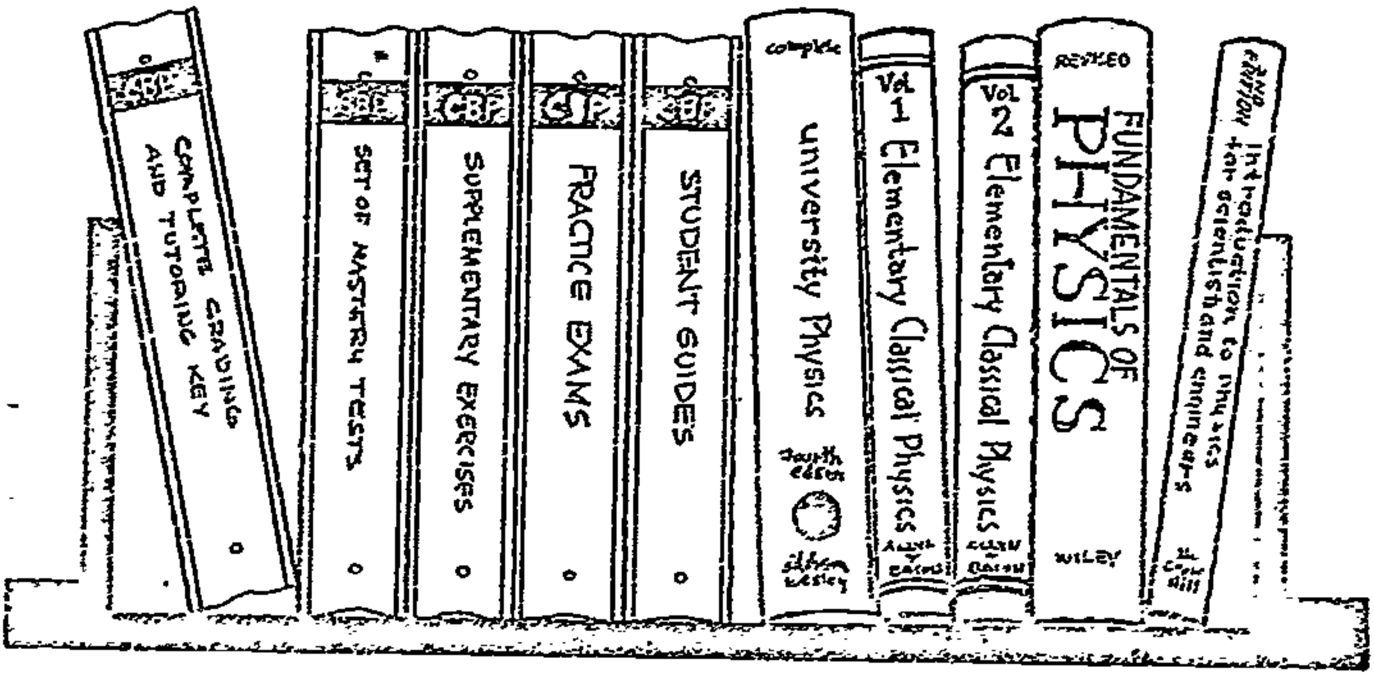
ABSTRACT

This is part of a series of 42 Calculus Based Physics (CBP) modules totaling about 1,000 pages. The modules include study guides, practice tests, and mastery tests for a full-year individualized course in calculus-based physics based on the Personalized System of Instruction (PSI). The units are not intended to be used without outside materials; references to specific sections in four elementary physics textbooks appear in the modules. Specific modules included in this document are: Module 11--Collisions, Module 12--Equilibrium of Rigid Bodies, Module 13--Rotational Dynamics, and 14--Fluid Mechanics. (CP)

STUDY MODULES FOR CALCULUS-BASED GENERAL PHYSICS*



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Comments

These modules were prepared by fifteen college physics professors for use in self-paced, mastery-oriented, student-tutored, calculus-based general physics courses. This style of teaching offers students a personalized system of instruction (PSI), in which they increase their knowledge of physics and experience a positive learning environment. We hope our efforts in preparing these modules will enable you to try and enjoy teaching physics using PSI.

Robert G. Fuller
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These modules were prepared by the module authors at a College Faculty Workshop held at the University of Colorado - Boulder, from June 23 to July 11, 1975.

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COMMENT TO USERS

In the upper right-hand corner of each Mastery Test you will find the "pass" and "recycle" terms and a row of numbers "1 2 3 ..." to facilitate the grading of the tests. We intend that you indicate the weakness of a student who is asked to recycle on the test by putting a circle around the number of the learning objective that the student did not satisfy. This procedure will enable you easily to identify the learning objectives that are causing your students difficulty.

COMMENT TO USERS

It is conventional practice to provide several review modules per semester or quarter, as confidence builders, learning opportunities, and to consolidate what has been learned. You the instructor should write these modules yourself, in terms of the particular weaknesses and needs of your students. Thus, we have not supplied review modules as such with the CBP Modules. However, fifteen sample review tests were written during the Workshop and are available for your use as guides. Please send $1.00 to CBP Modules, Behlen Lab of Physics, University of Nebraska - Lincoln, Nebraska 68588.

FINIS

This printing has completed the initial CBP project. We hope that you are finding the materials helpful in your teaching. Revision of the modules is being planned for the Summer of 1976. We therefore solicit your comments, suggestions, and/or corrections for the revised edition. Please write or call

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COLLISIONS

INTRODUCTION

If you have ever watched or played pool, football, baseball, soccer, hockey, or been involved in an automobile accident you have some idea about the results of a collision. We are interested in studying collisions for a variety of reasons. For example, you can determine the speed of a bullet by making use of the physics of the collision process. You can also estimate the speed of an automobile before the accident by knowing the physics of the collision process and a few other physical principles. Physicists use collisions to determine the properties of atomic and subatomic particles. Essentially, a particle accelerator is a device that provides a controlled collision process between subatomic particles so that, among other things, some of the properties of the target particle can be studied.

In addition the study of collisions is an example of the use of a fundamental physical tool, i.e., a conservation law. A conservation law implies that something remains the same, i.e., is conserved, as you have seen in a previous module, Conservation of Energy.

Conservation laws play an important role in physics. In the study of collisions in this module we are interested in one of the fundamental conservation laws, conservation of linear momentum. If the sum of the external forces is zero, then the linear momentum is conserved in the collision. This is fortunate since it provides a way around the analysis of the forces of interaction between two bodies as they collide, an otherwise formidable task. Thus the conservation-of-linear-momentum law allows one to analyze the effects of a collision without a detailed knowledge of the forces of interaction. One can deduce the converse also, as does the particle physicist in accelerator experiments, for example - some of the properties of the target particles may be deduced from the law of conservation of linear momentum and other laws of physics.

PREREQUISITES

Before you begin this module, you should be able to:	Location of Prerequisite Content
*Solve mechanical problems involving conservative and nonconservative forces, by applying the conservation-of-total-energy concept (needed for Objective 2 of this module)	Conservation of Energy Module
*Use the concepts of impulse and linear momentum to solve mechanical problems (needed for Objective 2 of this module)	Impulse and Momentum Module

LEARNING OBJECTIVES

After you have mastered the content of this module, you will be able to:

1. Conservation of linear momentum - Define or state: (a) elastic collision, (b) inelastic collision, (c) perfectly or completely inelastic collision, and (d) the law of conservation of linear momentum.

2. Collisions - Solve problems involving collisions between two or more bodies and/or the splitting up of a body into two or more fragments.

GENERAL COMMENTS

The important concepts presented in this module are

Elastic collision: a collision in which kinetic energy is conserved.

Inelastic collision: a collision in which kinetic energy is not conserved. Note: Kinetic energy may be either gained or lost during a collision.

Perfectly inelastic collision: a collision in which the colliding objects stick together after the collision.

Conservation of linear momentum: If the sum of the external forces acting on a system is zero, then the total linear momentum of the system remains constant. Or, during a collision, if the interaction impulsive force is very large in comparison to the sum of all external forces such as gravity, then it is a good approximation to say that linear momentum is conserved.

Remember: Momentum is a VECTOR quantity and must be treated as such.

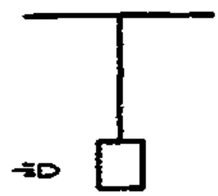
TEXT: Frederick J. Bueche, Introduction to Physics for Scientists and Engineers (McGraw-Hill, New York, 1975), second edition

SUGGESTED STUDY PROCEDURE

Read Sections 7.4, 7.5, and 9.3 through 9.6; work Problems 16 in Chapter 7, 15 in Chapter 9, and Problems A and B plus any two of the problems listed in the table below; answer Question 7 in Chapter 9. Also work the following problem:

A block of balsa wood whose mass is 0.60 kg is hung from a string of negligible weight. A bullet with a mass of 2.00 g and a muzzle velocity of 160 m/s is fired into this block at close range (horizontally) and becomes embedded in the block.

(a) Find the velocity of the block plus the bullet just after the collision.
(b) Calculate how high the block will rise.



When you think that you know the material well enough to satisfy the objectives, take the Practice Test.

BUECHE

Objective Number	Readings	Problems with Solutions	Assigned Problems		Additional Problems
		Study Guide	Study Guide	Text	(work any two)
1(a)	General Comments, Sec. 9.3				
1(b)	General Comments, Sec. 9.3				
1(c)	General Comments				
1(d)	General Comments				
2	Secs. 7.4, 7.5, 9.3-9.6	A, B	A, B	Chap. 7, Prob. 16; Chap. 9, Prob. 15	Chap. 7, Quest.* 5, 7, 13, Probs. 4, 9-18; Chap. 9, Quest. 4, 7, 10, Probs. 10-17, 26, 28

*Quest. = Question(s).

TEXT: David Halliday and Robert Resnick, Fundamentals of Physics (Wiley, New York, 1970; revised printing, 1974)

SUGGESTED STUDY PROCEDURE

Read Chapter 9, Sections 9-1 and 9-3 through 9-5; answer Question 6; work Problems 18, 22, 30, 40, plus Problems A and B.

Note: Definitions of elastic, inelastic, and completely inelastic collisions given in Section 9-4 apply to all collisions, not just to one-dimensional collisions.

When you think that you know the material well enough to satisfy the objectives, take the Practice Test.

HALLIDAY AND RESNICK

Objective Number	Readings	Problems with Solutions	Assigned Problems		Additional Problems
		Study Guide	Study Guide	Text* (Chap. 9)	(Chap. 9)
1(a)	General Comments, Sec. 9-4				
1(b)	General Comments, Sec. 9-4				
1(c)	General Comments, Sec. 9-4				
1(d)	General Comments, Sec. 9-3				
2	Secs. 9-1, 9-3 to 9-5	A, B	A, B	Quest. 6, Probs. 18, 22, 30, 40	Quest. 1-8, Probs. 14-17, 21, 24, 28, 34, 37, 44

*Quest. = Question(s).

TEXT: Francis Weston Sears and Mark W. Zemansky, University Physics (Addison-Wesley, Reading, Mass., 1970), fourth edition

SUGGESTED STUDY PROCEDURE

Read Chapter 8, Sections 8-2 through 8-6; work Problems 8-6, 8-10, 8-20, 8-25, 8-37 plus Problems A and B. Note: Conservation of linear momentum can be used to a good approximation when the external forces are small compared to the interaction forces during the collision. For example: when a bat hits a ball, the interaction forces are large (generally) compared to gravity and the force exerted by the batter; therefore, in this case gravity and the force exerted by the batter can be neglected during the interaction.

When you think that you know the material well enough to satisfy the objectives, take the Practice Test.

SEARS AND ZEMANSKY

Objective Number	Readings	Problems with Solutions	Assigned Problems		Additional Problems
		Study Guide	Study Guide	Text	
1(a)	General Comments, Sec. 8-3				
1(b)	General Comments, Sec. 8-4				
1(c)	General Comments, Sec. 8-3				
1(d)	General Comments				
2	Secs. 8-2 to 8-6	A, B	A, B	8-6, 8-10, 8-20, 8-25, 8-37	8-5, 8-11, 8-12, 8-16, 8-23, 8-28, 8-29, 8-30, 8-35

TEXT: Richard T. Weidner and Robert L. Sells, Elementary Classical Physics (Allyn and Bacon, Boston, 1973), second edition, Vol. 1

SUGGESTED STUDY PROCEDURE

Read Sections 5-5 through 5-7 in Chapter 5 and Section 10-6 in Chapter 10. Work Problems 5-1, 5-11, 5-12, and 10-32 in the text plus Problems A and B.

Note: Even though the statement of the law of conservation of linear momentum was deduced for a particular two-body collision, it is valid in general. Conservation of linear momentum can be used to a good approximation when the external forces are small compared to the interaction forces during the interaction. For example: when a bat hits a ball, the interaction forces are large (generally) compared to gravity and the force exerted by the batter; therefore in this case gravity and the force exerted by the batter can be neglected during the interaction.

Your text makes a distinction between types of inelastic collisions that is not generally made, i.e., $\Delta K < 0$ inelastic and $\Delta K > 0$ explosive, where ΔK is the change in kinetic energy during the collision. Generally $\Delta K \neq 0$ is classified as an inelastic collision, as is done in the General Comments.

When you think that you know the material well enough to satisfy the objectives, take the Practice Test.

WEIDNER AND SELLS

Objective Number	Readings	Problems with Solutions	Assigned Problems		Additional Problems
		Study Guide	Study Guide	Text	
1(a)	General Comments, Sec. 10-6				
1(b)	General Comments, Sec. 10-6				
1(c)	General Comments, Sec. 10-6				
1(d)	General Comments				
2	Secs. 5-5 to 5-7, 10-6	A, B	A, B	5-1, 5-11, 5-12, 10-32	5-1, 5-2, 5-5 to 5-12, 5-15 to 5-21, 10-1, 10-6, 10-8, 10-27 to 10-35

PROBLEM SET WITH SOLUTIONS

A(2). In the absence of any external forces a particle with mass m and speed v is incident on a particle of mass M initially at rest (see Fig. 1). After collision, particle m is observed to go off at an angle θ_2 with respect to the initial direction with speed v_f (see Fig. 2). M is observed to go off at an angle θ_1 with respect to the initial direction with speed V.

(a) Find V in terms of all the other parameters except θ_1.

(b) Let each parameter in turn approach zero and comment on the reasonableness of the answer.

(c) What is the maximum value for V? Is this reasonable?

(d) What happens as the magnitude of $M \to \infty$?

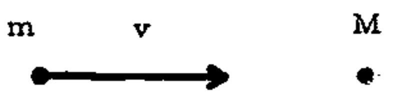


Figure 1

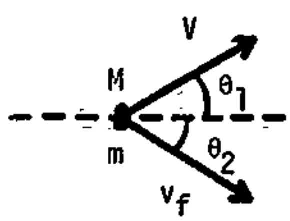


Figure 2

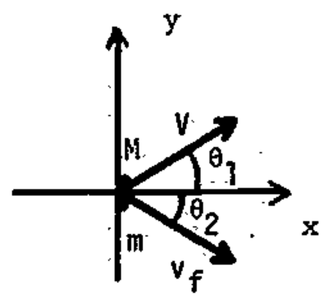


Figure 3

Solution

(a) Given m, M, v, v_f, and θ_2. Find V. Use momentum conservation (see Fig. 3). The x component of the linear-momentum-conservation equation is

$$mv = MV\cos\theta_1 + mv_f\cos\theta_2. \tag{1}$$

The y component is

$$MV\sin\theta_1 = mv_f\sin\theta_2. \tag{2}$$

Rearranging Eq. (1) we have

$$\begin{aligned} MV\cos\theta_1 &= m(v - v_f\cos\theta_2), \\ MV\sin\theta_1 &= mv_f\sin\theta_2. \end{aligned} \tag{3}$$

Squaring the above equations and adding we have

$$M^2V^2(\cos^2\theta_1 + \sin^2\theta_1) = m^2(v^2 - 2vv_f\cos\theta_2 + v_f^2\cos^2\theta_2) + m^2v_f^2\sin^2\theta_2.$$

Using the fact that $\sin^2\theta + \cos^2\theta = 1$ and doing some rearranging we have

$$V = (m/M)\sqrt{(v_f^2 - 2vv_f\cos\theta_2 + v^2)}.$$

(b) Now let

$m \to 0$ and $V \to 0$:

Reasonable - consider a ping-pong ball colliding with a bowling ball.

$M \to 0$, V becomes large:

Reasonable - consider a bowling ball colliding with a ping-pong ball.

$v_f \to 0$, $V \to mv/M$:

Reasonable - all linear momentum transferred from m to M.

$v \to 0$, $V \to mv_f/M$:

Reasonable - explosion, total linear momentum zero.

$\theta_2 \to 0$, $V \to (m/M)(v_f - v)$:

Reasonable - linear momentum lost by m given to M.

(c) V_{max} at $\theta_2 = \pi$:

Reasonable - since m has maximum change in momentum, i.e., it transfers maximum momentum to M.

$$V_{max} = (m/M)(v + v_f).$$

(d) As $M \to \infty$, $V \to 0$:

Reasonable - since as M goes to ∞, V has to become smaller in order to conserve linear momentum.

B(2). In the absence of external forces a particle of mass m collides elastically with another particle of the same mass initially at rest. Show that if the collision is not head-on the two particles go off so that the angle between their directions is $\pi/2$.

(a) State what is given and what you are to find symbolically.

(b) Draw a diagram.

(c) Write down the relevant equation or equations. In this case use the laws of ________________ and ________________.

(d) Solve the equations for the relevant unknown or unknowns.

Solution

(a) Given $m_1 = m_2 = m$, show that $\theta_1 + \theta_2 = \pi/2$.

(b)

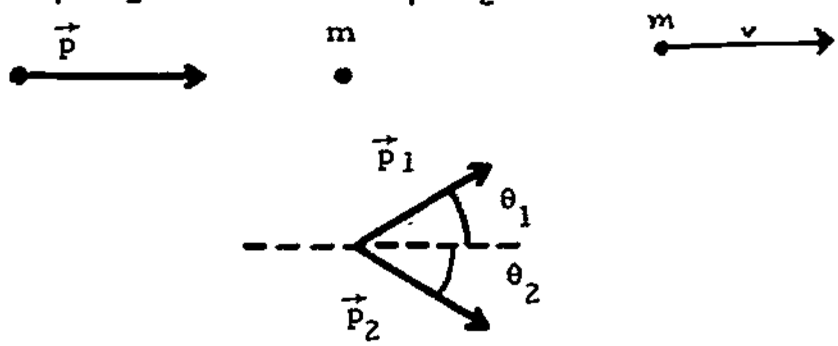


Figure 4

(c) Conservation of linear momentum:

$$\vec{p} = \vec{p}_1 + \vec{p}_2. \tag{4}$$

Conservation of kinetic energy:

$$K = K_1 + K_2, \tag{5}$$

where K stands for the kinetic energy of the incident particle before collision, etc.

(d) Now

$$K = p^2/2m. \tag{6}$$

Squaring Eq. (4) we have

$$p^2 = p_1^2 + p_2^2 + 2\vec{p}_1 \cdot \vec{p}_2.$$

Combining the above equation with Eq. (6) we have

$$2mK = 2mK_1 + 2mK_2 + 2\vec{p}_1 \cdot \vec{p}_2.$$

Combining the above equation with Eq. (5) we have

$$(\vec{p}_1 \cdot \vec{p}_2)/m = 0.$$

Assuming $p_1 \neq 0$; $p_2 \neq 0$ and $m \neq \infty$, all not very interesting cases, then $\vec{p}_1 \perp \vec{p}_2$, which was to be shown.

PRACTICE TEST

1. Define or state: (a) elastic collision; (b) inelastic collision; (c) perfectly inelastic collision; (d) the law of conservation of linear momentum.

2. A hockey puck B rests on a smooth ice surface and is struck by an identical puck A that was originally traveling at 60 m/s and that is deflected 30° from its original direction. Puck B acquires a velocity at an angle of 45° to the original velocity of A.
   (a) Compute the speed of each puck after collision.
   (b) Is the collision perfectly elastic? If not, what fraction of the original kinetic energy of puck A is "lost"?

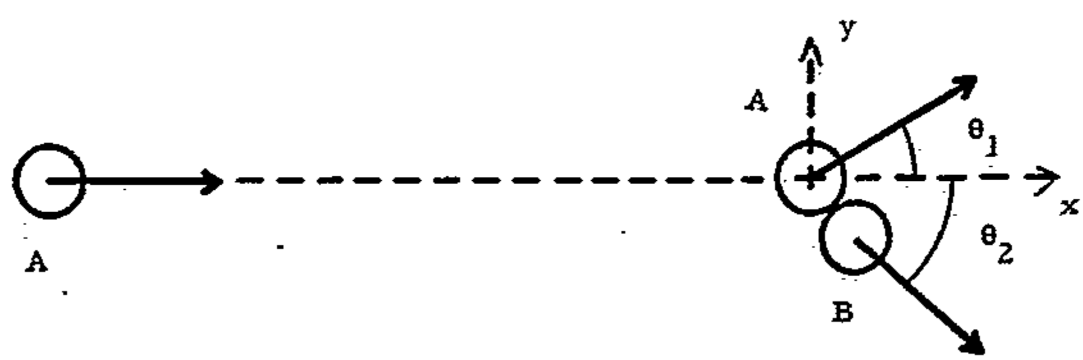


Figure 5

Practice Test Answers

1. (a) Elastic collision: a collision in which kinetic energy is conserved.
   (b) Inelastic collision: a collision in which kinetic energy is not conserved. Note: kinetic energy may be either gained or lost during a collision.
   (c) Perfectly inelastic collision: a collision in which the colliding objects stick together after the collision.
   (d) Conservation of linear momentum: If the sum of the external forces acting on a system is zero, then the total linear momentum of the system remains constant. Or, during a collision, if the interaction impulsive force is very large in comparison to the sum of all external forces such as gravity, then it is a good approximation to say that linear momentum is conserved.

   Note: If you missed any of these definitions, MEMORIZE the ones that you missed.

2. (a) $V_{BF} = V_{Ai}/(\sin \theta_2 \cot \theta_1 + \cos \theta_2)$.

   Check this answer for dimensions and reasonableness [see parts (b), (c), and (d) in the solution of Problem A for reasonableness check].

   $V_{BF} = 31$ m/s.

   $V_{AF} = V_{Ai} \sin \theta_2/(\sin \theta_2 \cos \theta_1 + \sin \theta_1 \cos \theta_2)$

   Check this answer for dimensions and reasonableness.

   $V_{AF} = 44$ m/s.

   (b) $\Delta K/K_i = 0.20$, or the collision is inelastic.

   Note: If you missed this problem, work some more of the optional problems in the text until you feel that you understand the material. When you understand the material, then ask for a Mastery Test. If you answered this Practice Test correctly, ask for a Mastery Test now.

COLLISIONS

Mastery Test Form A

Date ______________

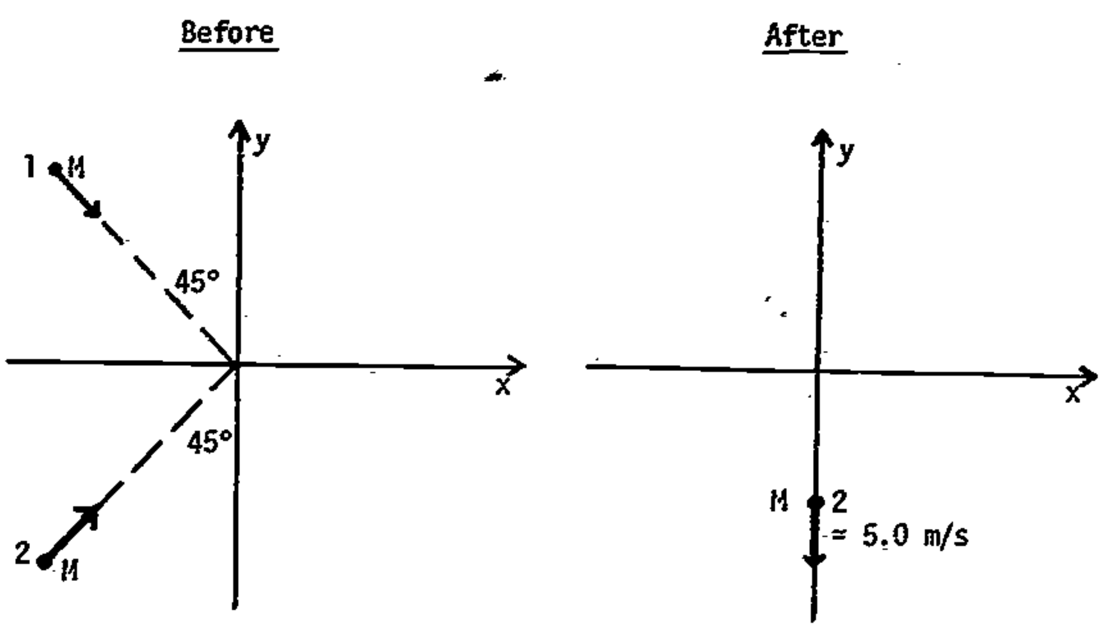
pass recycle

1 2

Name ______________ Tutor ______________

1. Define or state:
   (a) elastic collision;
   (b) inelastic collision;
   (c) perfectly inelastic collision;
   (d) the law of conservation of linear momentum.

2. Consider the collision as shown in the figure below. The colliding particles are identical and initially have a speed of 10.0 m/s. After the collision particle 2 moves as shown.
   (a) Find the velocity of particle 1 after collision.
   (b) Is this an elastic collision?



COLLISIONS

Mastery Test Form B

Date ______________

pass recycle

1 2

Name ______________________ Tutor ______________________

1. Define or state:

   (a) elastic collision;

   (b) inelastic collision;

   (c) perfectly inelastic collision;

   (d) the law of conservation of linear momentum.

2. A cannon mounted on a stationary railroad car fires a 100-kg projectile so the latter moves horizontally with a speed of 600 m/s at a sideways angle of 30.0° to the track. The car plus the cannon have a mass of 10 000 kg.

   (a) Make a sketch and describe in what way momentum conservation can be used to solve this problem, or explain why this is not the case.

   (b) At what speed will the railroad car recoil along the track? (Neglect friction with the track.)

COLLISIONS

Mastery Test Form C

Date ______________

pass recycle

1 2

Name ______________________ Tutor ______________________

1. Define or state:

   (a) elastic collision;

   (b) inelastic collision;

   (c) perfectly inelastic collision;

   (d) the law of conservation of linear momentum.

2. As you stand at a lightly traveled street intersection, you are startled to observe the collision of a fire engine (mass = 6000 kg), a house trailer (mass = 25 000 kg), a steam calliope (mass = 4000 kg), and a dump truck (mass = 8000 kg). The four vehicles are, respectively, traveling northeast at 30.0 m/s, west at 10.0 m/s, south at 20.0 m/s, and east at 25.0 m/s.

   (a) Make a diagram of the vehicles immediately before the collision, and indicate their masses and vector velocities.

   (b) If the entire junk pile sticks together after the collision, what is its velocity before it has been slowed down by friction?

   (c) Is this collision elastic?

COLLISIONS

Date ____________

Mastery Test Form D

pass recycle

1 2

Name ____________ Tutor ____________

1. Define or state:
   (a) elastic collision;
   (b) inelastic collision;
   (c) perfectly inelastic collision;
   (d) the law of conservation of linear momentum.

2. A radioactive nucleus, initially at rest, decays by emitting an electron and an electron antineutrino at right angles to one another. The momentum of the electron is 1.20×10^{-22} kg m/s and that of the electron antineutrino is 6.4×10^{-23} kg m/s.
   (a) Find the momentum of the recoiling nucleus.
   (b) If the mass of the recoiling residual nucleus is 5.8×10^{-26} kg, what is its kinetic energy of recoil?

COLLISIONS

Mastery Test Form E

Date ______________

pass recycle

1 2

Name ______________________ Tutor ______________________

1. Define or state:

   (a) elastic collision;

   (b) inelastic collision;

   (c) perfectly inelastic collision;

   (d) the law of conservation of linear momentum.

2. A body of mass $m_1 = 10.0$ kg moves to the right along a frictionless table top at a speed of 50 m/s and makes a head-on collision with another body whose mass m_2 is unknown, but which is originally moving to the left at a speed of 30.0 m/s. If the bodies stick together after the collision and move to the right at a speed of 20.0 m/s, what is the value of m_2? Is the collision elastic?

COLLISIONS

Date ____________

Mastery Test Form F

pass recycle

1 2

Name ____________________ Tutor ____________________

1. Define or state:
   (a) elastic collision;
   (b) inelastic collision;
   (c) perfectly inelastic collision;
   (d) the law of conservation of linear momentum.

2. A ball with speed 3.00 m/s and mass 1.00 kg strikes off-center a second ball of mass 3.00 kg initially at rest. The incident ball is deflected 90° from its incident direction, and the collision is completely elastic. In what direction, relative to that of the incident ball before the collision, does the second ball leave the collision?

MASTERY TEST GRADING KEY - Form A

What To Look For	Solutions
1.(a) $\Delta K = 0$.	1.(a) Elastic collision - a collision in which kinetic energy is conserved.
(b) $\Delta K \neq 0$.	(b) Inelastic collision - a collision in which kinetic energy is not conserved.
(c) Objects stick together after collision.	(c) Perfectly inelastic collision - a collision in which the colliding objects stick together after the collision.
(d) $\Delta\vec{P} = 0$ if $\Sigma\vec{F}_{ext} = 0$.	(d) Conservation of linear momentum: If the sum of the external forces acting on a system is zero, then the total linear momentum of the system remains constant.
2. $\Delta\vec{P} = 0$.	2.(a) $\vec{p} = 2mv\hat{i}$: total momentum of particles 1 and 2 before collision.
Vector nature of $\vec{p}$	$\vec{p}_2 = -mv/2\hat{j}$: momentum of particle 2 after collision.
Is answer dimensionally correct?	$\vec{p}_1 = \vec{p} - \vec{p}_2 = 2mv\hat{i} + (mv/2)\hat{j}$.
Is the answer reasonable?	$v_1 = 2v\hat{i} + (v/2)\hat{j} = (14.0\hat{i} + 5.0\hat{j})$ m/s.

(b) $K_i^2 = mv^2$,

$$K_f = \left(\frac{m}{2}\right)\left(\frac{v^2}{4}\right) + \left(\frac{m}{2}\right)v^2\left(2 + \frac{1}{4}\right) = mv^2\left(\frac{1}{8} + \frac{9}{8}\right)$$

$$= mv^2\left(\frac{1}{8} + \frac{9}{8}\right) = mv^2\left(\frac{10}{8}\right),$$

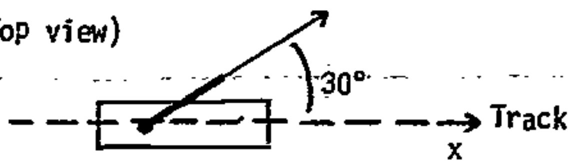
$K_i \neq K_f$.

Thus the collision is not elastic. Also, since v_1 has a j component it cannot be $\perp$ to v_2; therefore collision is inelastic.

MASTERY TEST GRADING KEY - Form B

What To Look For	Solutions
1.(a) $\Delta K = 0$;	1.(a) Elastic collision - a collision in which kinetic energy is conserved.
(b) $\Delta K \neq 0$;	(b) Inelastic collision - a collision in which kinetic energy is not conserved.
(c) Objects stick together after collision.	(c) Perfectly inelastic collision - a collision in which the colliding objects stick together after the collision.
(d) $\Delta\vec{P} = 0$ if $\Sigma\vec{F}_{ext} = 0$.	(d) Conservation of linear momentum: If the sum of the external forces acting on a system is zero, then the total linear momentum of the system remains constant.
2.(a) $\vec{p}$ conserved only in direction defined by track.	2.(a) Momentum not conserved in direction $\perp$ to track.

(Top view)



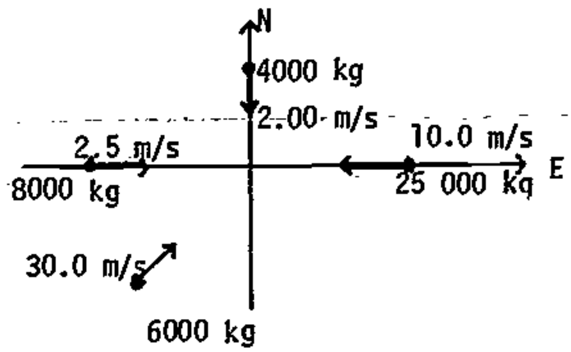
What To Look For	Solutions
(b) $\vec{p}$ used is total $\vec{p}$ times cos 30°. Answer dimensionally correct? Units correct? Answer reasonable?	$P_x = 0 = (P_x)_{proj} + (P_x)_{car}$ $= [(100)(600)\cos 30.0°]\ \text{m/s} + (10\,000)v,$ $v = -[(100)(600)\cos 30.0°/10\,000]\ \text{m/s}$ $= -(6\cos 30.0°)\ \text{m/s} = -5.2\ \text{m/s}.$

MASTERY TEST GRADING KEY - Form C

What To Look For	Solutions
1.(a) $\Delta K = 0$.	1.(a) Elastic collision - a collision in which kinetic energy is conserved.
(b) $\Delta K \neq 0$.	(b) Inelastic collision - a collision in which kinetic energy is not conserved.
(c) Objects stick together after collision	(c) Perfectly inelastic collision - a collision in which the colliding objects stick together after the collision.
(d) $\Delta\vec{P} = 0$ if $\Sigma\vec{F}_{ext} = 0$.	(d) Conservation of linear momentum: if the sum of the external forces acting on a system is zero, then the total linear momentum of the system remains constant.

2.(a) Make sure diagram is clear!

2.(a)



(b) $\Delta\vec{P} = 0$. Vector nature of $\vec{p}$. Answer dimensionally correct? Units correct? Answer reasonable?

(b) Momentum is conserved, junk pile moves with $\vec{v}_{c.m.}$

$$\vec{v}_{c.m.} = \frac{\vec{P}}{M} = \left(\frac{(6000)(15.0v_2\hat{i} + 15.0v_2\hat{j}) + (25\,000)(-10\hat{i})}{6000 + 25000 + 4000 + 8000} + \frac{(4000)(-20\hat{j}) + (8000)(25\hat{i})}{6000 + 25000 + 4000 + 8000}\right) \text{ m/s}$$

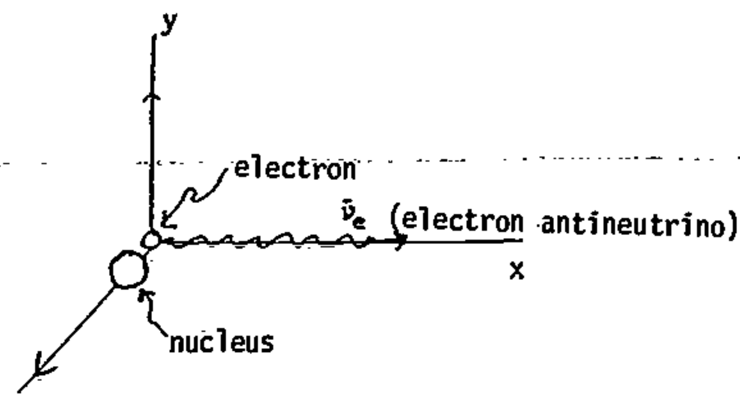
$$= (77\hat{i} + 47\hat{j})/(43) \text{ m/s} = (1.8\hat{i} + 1.1\hat{j}) \text{ m/s}.$$

(c) recognize perfectly inelastic collision

(c) No - perfectly inelastic.

MASTERY TEST GRADING KEY - Form D

What To Look For	Solutions
1.(a) $\Delta K = 0$.	1.(a) Elastic collision - a collision in which kinetic energy is conserved.
(b) $\Delta K \neq 0$.	(b) Inelastic collision - a collision in which kinetic energy is not conserved.
(c) Objects stick together after collision.	(c) Perfectly inelastic collision - a collision in which the colliding objects stick together after the collision.
(d) $\Delta\vec{P} = 0$ if $\Sigma\vec{F}_{ext} = 0$.	(d) Conservation of linear momentum: If the sum of the external forces acting on a system is zero, then the total linear momentum of the system remains constant.
2. $\Delta\vec{P} = 0$. Vector nature of $\vec{p}$. Answer dimensionally correct? Units correct? Answer reasonable?	2.



(a) $\vec{p}_f = 0$,

$\vec{p}_{nuc} = -(0.64\hat{i} + 1.20\hat{j}) \times 10^{-22}$ kg m/s.

(b) $K = \dfrac{p^2}{2m} = \dfrac{1.81 \times 10^{-44}}{2(5.8 \times 10^{-26})}$ kg$\cdot$m^2/s^2,

$K = 1.6 \times 10^{-19}$ J.

MASTERY TEST GRADING KEY - Form E

What To Look For	Solutions
1.(a) $\Delta K = 0$.	1.(a) Elastic collision - a collision in which kinetic energy is conserved.
(b) $\Delta K \neq 0$.	(b) Inelastic collision - a collision in which kinetic energy is not conserved.
(c) Objects stick together after collision.	(c) Perfectly inelastic collision - a collision in which the colliding objects stick together after the collision.
(d) $\Delta\vec{p} = 0$ if $\Sigma\vec{F}_{ext} = 0$.	(d) Conservation of linear momentum: If the sum of the <u>external</u> forces acting on a system is zero, then the total linear momentum of the system remains constant.
2. $\Delta\vec{p} = 0$. Vector nature of $\vec{p}$? Answer dimensionally correct? Units correct? Answer reasonable?	2. $\xrightarrow{+}$

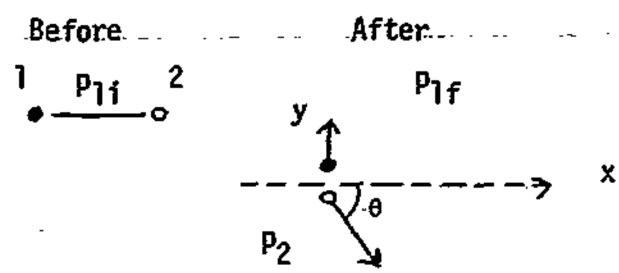
$$\Delta\vec{p} = 0. \qquad p_i = m_1 v_{1i} - m_2 v_{2i} = p_f = (m_i + m_2)v_f.$$

$$m_2 = \frac{m_i(v_{1i} - v_f)}{(v_{2i} + v_f)} = 10.0\text{ kg}\,\frac{(50 - 20.0)}{(30 + 20.0)} = 6.0\text{ kg}.$$

Collision is inelastic since objects stick together; therefore it is not elastic!

MASTERY TEST GRADING KEY - Form F

What To Look For	Solutions
1.(a) $\Delta K = 0$.	1.(a) Elastic collision - a collision in which kinetic energy is conserved.
(b) $\Delta K \neq 0$.	(b) Inelastic collision - a collision in which kinetic energy is not conserved.
(c) Objects stick together after collision.	(c) Perfectly inelastic collision - a collision in which the colliding objects stick together after the collision.
(d) $\Delta\vec{P} = 0$ if $\Sigma\vec{F}_{ext} = 0$.	(d) Conservation of linear momentum: If the sum of the <u>external</u> forces acting on a system is zero, then the total linear momentum of the system remains constant.
2. $\Delta\vec{P} = 0$. Vector nature of $\vec{p}$? Answer dimensionally correct? Units correct? Answer reasonable?	2.



$$\vec{p}_2 = \vec{p}_{1i} - \vec{p}_{1f},$$

$$\vec{p}_{1i} = m_1 v_{1i}\hat{i}, \qquad \frac{p_{1i}^2}{2m_1} = \frac{p_{1f}^2}{2m_1} + \frac{p_2^{\,2}}{2m_2},$$

$$\vec{p}_{1f} = m_1 v_{1f}\hat{j}, \qquad \frac{m_1^2 v_{1i}^2}{2m_1} = \frac{m_1^2 v_{1f}^2}{2m_1} + \frac{m_1^2}{2m_2}(v_{1i}^2 + v_{1f}^2),$$

$$\vec{p}_2 = m_1(v_{1i}\hat{i} - v_{1f}\hat{j}), \qquad v_{1i}^2 = v_{1f}^2 + (m_1/m_2)(v_{1i}^2 + v_{1f}^2),$$

$$v_{1f}^2 = v_{1i}^2(m_2 - m_1)/(m_2 + m_1),$$

$$v_{1f} = v_{1i}\sqrt{(m_2 - m_1)/(m_2 + m_1)},$$

$$\tan\theta = \sqrt{(m_2 - m_1)/(m_2 + m_1)} = \sqrt{(2/4)} = \sqrt{2}/2.$$

$$\theta = \tan^{-1}(\sqrt{2}/2).$$

See diagram for definition of θ.

Module ____ 1

STUDY GUIDE

EQUILIBRIUM OF RIGID BODIES

INTRODUCTION

Most of the objects that one sees are in a state of equilibrium, that is, at rest or in a state of uniform motion. Many man-made structures are designed to achieve and sustain a state of equilibrium, and this, in turn, sets requirements to the materials (their sizes and shapes) that can be used. This module will give you some practice in analyzing the forces that result in equilibrium. From this analysis, if you are given the values of an appropriate set of forces you can find the remaining ones. On the other hand, in designing a stable system you can find the requirements of materials and dimensions that will ensure equilibrium.

PREREQUISITES

Before you begin this module, you should be able to:	Location of Prerequisite Content
*Apply Newton's second law for the solution of problems involving frictional forces (needed for Objectives 2 and 3 of this module)	Newtons Laws Module
*Locate the center of mass of a uniform rigid body (needed for Objective 1 of this module)	Impulse and Momentum Module
*Define and use the definition of torque (needed for Objectives 1 and 3 of this module)	Rotational Motion Module

LEARNING OBJECTIVES

After you have mastered the content of this module, you will be able to:

1. Equilibrium - Define the following terms and describe the application of each to a physical object or system: first condition of equilibrium (translational); second condition of equilibrium (rotational); center of gravity.

2. Translational equilibrium - Analyze translational equilibrium problems by identifying all forces, making a free-body diagram, and applying the first condition of equilibrium to solve for the unknown parameters. These problems may involve weight acting at the center of gravity, tensions in ropes or wires, compressional forces on rods or hinges, and frictional forces.

3. Rotational equilibrium - Analyze problems involving both the first and second conditions of equilibrium. These problems will involve torques as well as the forces referred to in Objective 2.

GENERAL COMMENTS

The solutions to the problems of this module are rather formal; that is, all problem solutions follow a regular procedure, which if done carefully will almost always produce the desired result. By learning the general procedures and practicing on a few examples you should find no difficulty in solving any problem in this module.

Since your text may not delineate these steps clearly, they are summarized here:

1. Draw an imaginary boundary separating the system under consideration from its surroundings.
2. Draw vectors representing magnitude, direction, and point of application of all external forces to the system. (This is a free-body diagram.)
3. Choose a convenient reference frame, resolve all of the external forces along these axes, and then apply the first condition of equilibrium.
4. Choose a convenient axis, evaluate all of the external torques around it, and apply the second condition of equilibrium.

The resulting simultaneous equations can then be solved for the desired quantities.

TEXT: Frederick J. Bueche, Introduction to Physics for Scientists and Engineers (McGraw-Hill, New York, 1975), second edition

SUGGESTED STUDY PROCEDURE

Study the text Sections 3.1 through 3.4 in Chapter 3 and Section 11.9 in Chapter 11. Next study the Problem Set in this module, and work text problems, remembering to use the rules given in General Comments, until you are satisfied that you have met Objectives 1 to 3.

Try the Practice Test before attempting a Mastery Test.

BUECHE

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text*	Study Guide	Text (Chap. 3)	(Chap. 3)
1	Secs. 3.1, 3.2, 3.4, 11.9					
2	Sec. 3.1	A, B	Illus. 3.1 to 3.3	E	3-5, 8, 10	1, 2, 6, 7, 9
3	Sec. 3.2	C, D	Illus. 3.4 to 3.7	F	18-21	22-25

*Illus. = Illustration(s).

STUDY GUIDE: Equilibrium of Rigid Bodies 3(HR 1)

TEXT: David Halliday and Robert Resnick, Fundamentals of Physics (Wiley, New York, 1970; revised printing, 1974)

SUGGESTED STUDY PROCEDURE

Study text Sections 12-1 through 12-3 in Chapter 12. Then study the Problem Set in this module, remembering to use the rules given in General Comments. Work text problems until you are satisfied that you have met Objectives 1 to 3.

Try the Practice Test before attempting a Mastery Test.

HALLIDAY AND RESNICK

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text*	Study Guide	Text (Chap. 12)	(Chap. 12)
1	Secs. 12-1, 12-2					
2	Sec. 12-3	A, B	Chap. 5, Ex. 3, 4	E	7, 15, 22, 26	
3	Sec. 12-3	C, D	Chap. 12, Ex. 1, 2	F	6, 10, 12, 20, 29	8, 9, 11, 13, 14, 16-21, 27-29

*Ex. = Example.

TEXT: Francis Weston Sears and Mark W. Zemansky, University Physics (Addison-Wesley, Reading, Mass., 1970), fourth edition

SUGGESTED STUDY PROCEDURE

Study the text Sections 2-1 through 2-7 of Chapter 2 and 3-1 through 3-4 of Chapter 3. Note that most of Chapter 2 was covered in your study of Newton's laws. Review it now from the viewpoint of statics.

Study the Problem Set in this module; work Problems E and F, remembering to use the rules of General Comments. Work text problems until you are satisfied that you have met Objectives 1 to 3.

Try the Practice Test before attempting a Mastery Test.

SEARS AND ZEMANSKY

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text*	Study Guide	Text	
1	Secs. 2-2, 2-6, 3-2, 3-4					
2	Secs. 2-6, 2-7	A, B	Sec. 2-6, Ex. 2 to 5; Sec. 2-7, Ex. 1, 3, 4	E	2-9, 2-10, 2-18, 2-21	2-3 to 2-8, 2-11, 2-12, 2-14, 2-16, 2-19, 2-20, 2-22 to 2-25
3	Secs. 3-2, 3-3	C, D	Sec. 3-2, Ex. 1 to 3	F	3-4, 3-12, 3-17, 3-19	3-3, 3-6 to 3-11, 3-13, 3-15, 3-16, 3-18, 3-20

*Ex. = Example(s).

TEXT: Richard T. Weidner and Robert L. Sells, Elementary Classical Physics (Allyn and Bacon, Boston, 1973), second edition, Vol. 1

SUGGESTED STUDY PROCEDURE

Study the text Sections 12-5 and 12-6 in Chapter 12. Also review Sections 8-1 and 8-2 of Chapter 8 where equilibrium is treated as a special case of Newton's laws.

Study the Problem Set and work Problems E and F. Work text problems, remembering to use the rules from General Comments, until you are satisfied that you have met Objectives 1 to 3.

Try the Practice Test before attempting a Mastery Test.

WEIDNER AND SELLS

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text*	Study Guide	Text	
1	Secs. 12-5, 12-6					
2	Sec. 12-6	A, B	Ex. 8-2	E	8-1, 8-2, 8-4, 8-12	
3	Sec. 12-6	C, D	Ex. 12-10	F	12-31, 12-32, 12-34, 12-35	12-29, 12-30, 12-33, 12-37

*Ex. = Example(s).

PROBLEM SET WITH SOLUTIONS

A(2). A block of mass 12.0 kg slides at constant velocity when pushed by a force of 50 N applied at 30.0° as shown in Figure 1. Find the coefficient of sliding friction between the block and floor.

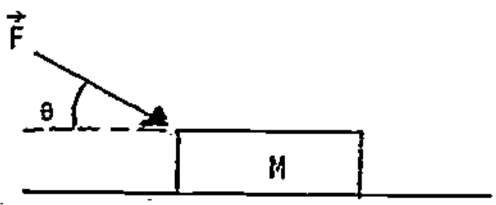


Figure 1

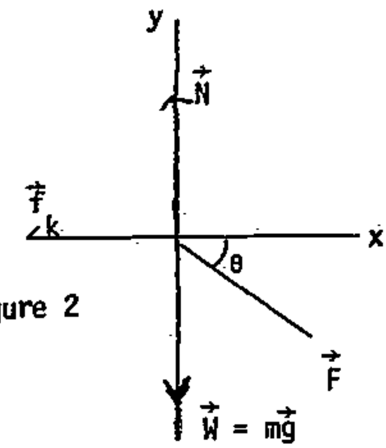


Figure 2

Solution

The expression "at constant velocity" suggests that this is a translational equilibrium problem. First isolate the system, which in this case is just the block M. Then identify all forces and draw a free-body diagram showing all forces. See Figure 2: $\vec{W}$ is the weight of the block, $\vec{N}$ is the normal force of the floor on the block, $\vec{F}$ is the applied force, and $\vec{f}_k$ is the kinetic frictional force for which we assume $\vec{f}_k = \mu_k\vec{N}$. Since nothing is said in the problem about the points of application of forces we assume we shall not need to apply the second condition of equilibrium. Thus, applying the first condition of equilibrium:

$$\Sigma F_y = N - W - F\sin\theta = 0, \qquad \Sigma F_x = F\cos\theta - f_k = 0,$$

and also $f_k = \mu_k N$. Note that in this case the normal force $\vec{N}$ is not simply the weight. We can combine the last two equations to eliminate the frictional force:

$$F\cos\theta - \mu_k N = 0.$$

This equation can then be combined with the first equation to eliminate the normal force N and we then solve for μ_k:

$$N = (F\cos\theta)/\mu_k, \quad (F\cos\theta)/\mu_k - W - F\sin\theta = 0, \quad \mu_k = (F\cos\theta)/(W + F\sin\theta).$$

This result is certainly plausible since it is a dimensionless ratio of forces that is expected, and if we let $\theta = 0$ it reduces to the defining equation for the coefficient of friction (with $W = N$).

Substituting in the numbers from the problem we get the answer

$$\mu_k = (50)(\cos 30.0°)/[(12.0)(9.8) + 50\sin 30.0°] = 0.300.$$

B(2). A weight W is hung from the middle of a tight clothesline 20.0 m long (see Fig. 3). The weight causes the center to drop 0.50 m below the horizontal (assume the line stretches slightly). Find the tension in the line in terms of W.

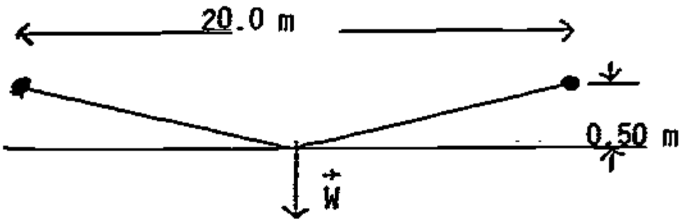


Figure 3

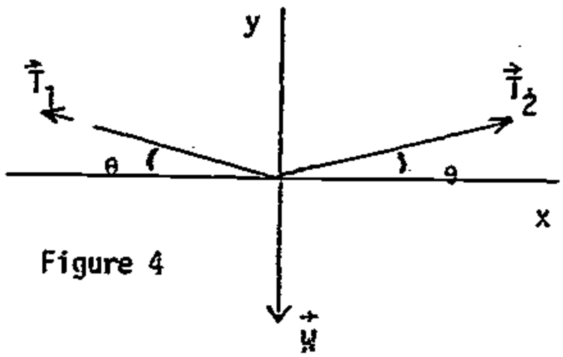


Figure 4

Solution

We assume a first condition of equilibrium and isolate the point of application of W on the line as the object in equilibrium, resulting in Figure 4. "Clothesline" implies flexibility, which implies that T must be along the "line," and hence the force diagram will have the same angles as the line itself. Thus

$$\tan\theta = 0.50/10.0 = 0.050.$$

$$\Sigma F_x = T_2 \cos\theta - T_1 \cos\theta = 0,$$

hence,

$$T_1 = T_2, \quad \Sigma F_y = 2T_1 \sin\theta - W = 0, \quad T_1 = W/(2\sin\theta) = 10.0W.$$

C(3). A uniform beam 5.0 m long with mass 10.0 kg is hinged at a wall. The outer end is supported by a guy wire making an angle of 30.0° with the horizontal beam, and an object of mass 20.0 kg is hung on the beam at a point 4.5 m from the wall. See Figure 5. Find the tension in the guy wire and the vertical and horizontal components of the force on the beam hinge by the wall.

Figure 5

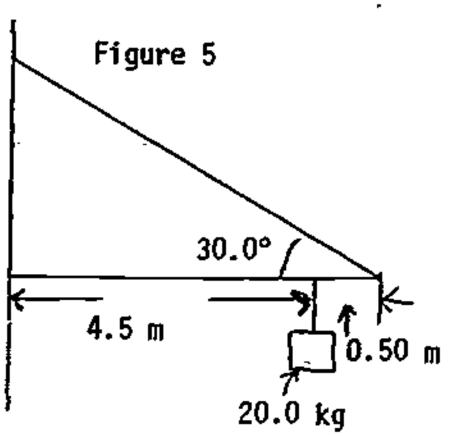
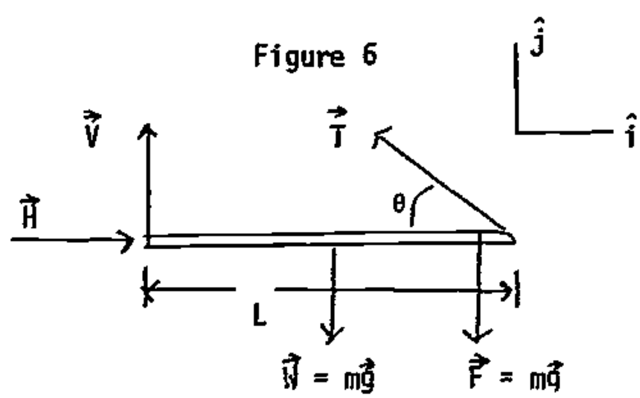


Figure 6



Solution

For the free-body diagram, see Figure 6. The first condition of equilibrium results in the following equations:

$$\Sigma F_y = V + T \sin \theta - W - F = 0, \quad \Sigma F_x = H - T \cos \theta = 0.$$

The second condition of equilibrium applied to an axis perpendicular to the paper at the hinge gives

$$\Sigma\tau = T(\sin \theta) L - (W)(L/2) - F(0.90)L = 0.$$

Solution of this last equation for T using the values of L, W, F, and θ from the problem gives T = 450 N. The first two equations can then be solved to give

$$H = 390 \text{ N}, \quad V = 69 \text{ N}.$$

D(3). Physics teachers can be very devious. One such teacher drilled several holes in an otherwise good wooden meterstick and filled the holes with lead. He then gave the meterstick, a knife edge, and a hanging 0.100-kg mass to a student and asked him to find the mass and center of gravity of the modified meterstick. The student found that the stick balanced at 0.58 m with the hanging mass at 0.66 m and also balanced at 0.73 m with the hanging mass at 0.93 m. Find the mass and the center of gravity.

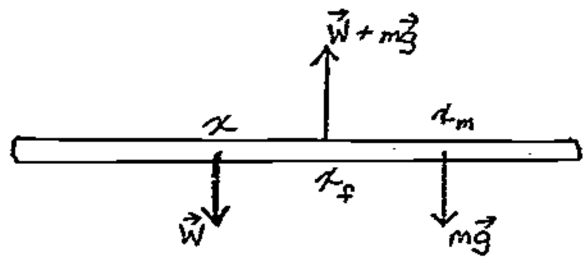


Figure 7

Solution

Assume the meterstick weight W to act at X and write two equations for $\Sigma\tau = 0$ around the 0.0 end. The upward force at the knife edge can be found from the first condition of equilibrium:

$$[W + (0.100)(9.8)](0.58) - WX - (0.100)(9.8)(0.66) = 0,$$

$$[W + (0.100)(9.8)](0.73) - WX - (0.100)(9.8)(0.93) = 0.$$

Simultaneous solution of these equations gives

$$X = 0.48 \text{ m}, \quad W = 0.78 \text{ N}, \quad m = 0.080 \text{ kg}.$$

Problems

E(2). A 2.50-kg mass is hung from the ceiling by a long rope. It is pulled to the side by a horizontal force of 10.0 N. Find the tension in the long rope and the angle it will make with the vertical.

F(3). A uniform ladder 10.0 m long, with a mass of 20.0 kg, rests against a smooth wall and on a very rough floor, with the base of the ladder 6.0 m from the wall. An 80-kg man has climbed the ladder to 8.0 m along the ladder from the bottom. Find the horizontal force that the floor must supply to keep the ladder from slipping.

Solutions

E(2). Draw a free-body diagram of the forces on the 2.50-kg mass, apply the first condition of equilibrium to the forces, and solve for the desired quantities: $T = 26.0$ N, $\theta = 22.0°$.

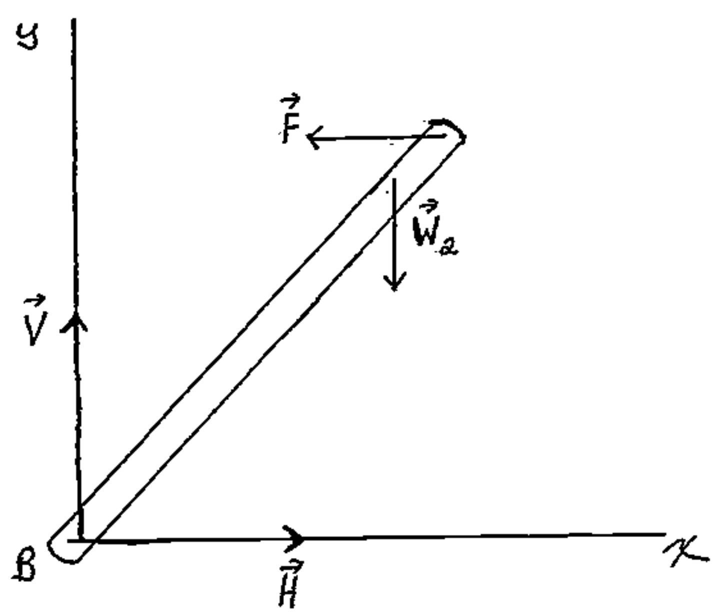
F(3). Draw a free-body diagram showing all forces on the ladder (Fig. 8). Note that "smooth wall" implies no friction, which implies that F is perpendicular to the wall. Notice the general similarity of this diagram to the free-body diagram of Problem C (except for orientation, direction, and magnitude of forces) and solve in a similar fashion.

$$\Sigma F_y = V - w - W_2 = 0 \text{ (this equation is not necessary)},$$

$$\Sigma F_x = H - F = 0, \quad \Sigma\tau_B = 10.0F \sin\theta - 5.0W \cos\theta - (8.0)(W_2) \cos\theta = 0.$$

The result is $H = 540$ N.

Figure 8



PRACTICE TEST

1. Describe how to find the center of gravity of an irregularly shaped planar object.

2. An object with a weight of 15.0 N is hung from two points as shown in Figure 9. Draw a free-body diagram and find the tension in the two ropes A and B.

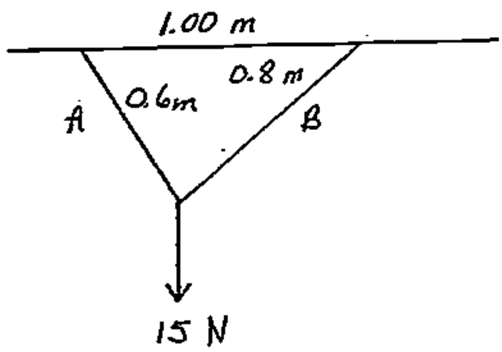


Figure 9

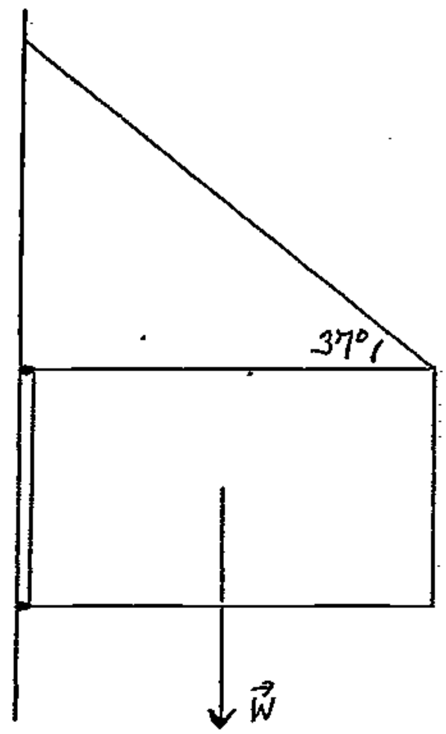


Figure 10

3. A uniform gate of weight W is hung from two hinges as shown in Figure 10, but to relieve the strain on the hinges a guy wire is installed as shown and tightened until there is no horizontal force on the lower hinge. Draw a free-body diagram and find the horizontal force on the top hinge, in terms of W.

Practice Test Answers

1. Assuming a plane object, suspend the object from a point on an edge and mark the vertical line (plumb line); then suspend the object from a different point and again mark the vertical line. The center of gravity is at the intersection of these lines.

2. Choose the point of application of the 15.0-N force as the point in equilibrium and draw a free-body diagram (Fig. 11). Apply the first condition of equilibrium:

$$\Sigma F_x = T_B \cos 37° - T_A \cos 53° = 0, \qquad \Sigma F_y = T_A \sin 53° + T_B \sin 37° - 15 = 0.$$

Simultaneous solution of these equations gives us

$$T_B = 9.0\ \text{N}, \qquad T_A = 12.0\ \text{N}.$$

3. Draw the free-body diagram as in Figure 12. Then

$$\Sigma F_x = H - T \cos 37° = 0, \qquad \Sigma F_y = V_1 + V_2 + T \sin 37° - W = 0,$$

$\Sigma\tau = 0 = LT \sin 37° - W(L/2) = 0$ (taking torques about the top hinge),

$$T = \frac{(1/2)W}{\sin 37°} = 0.83\ W.$$

From the first equation

$$H = T \cos 37° = 0.83W \cos 37° = 0.67W.$$

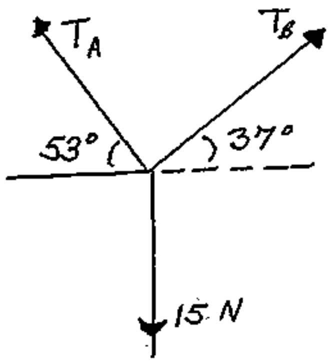


Figure 11

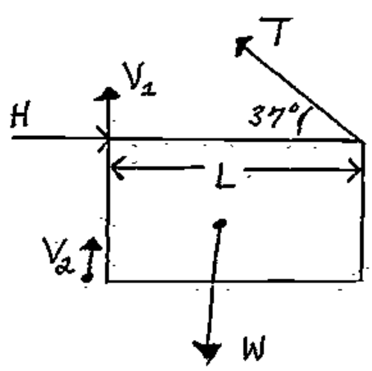


Figure 12

Date ____________________

EQUILIBRIUM OF RIGID BODIES

pass recycle

Mastery Test Form A

1 2 3

Name ______________________________ Tutor ______________________________

1. Describe what is meant by the "center of gravity" of an object.

2. A block of mass 6.0 kg is pulled up an incline at a constant velocity as shown in Figure 1. If the coefficient of sliding friction is 0.300, how large a force F is required?

3. A weight of 2.00×10^4 N is supported by a beam-and-guy-wire arrangement as shown in Figure 2. The beam is uniform and has a mass of 400 kg. Draw a free-body diagram and find the vertical and horizontal components of the forces on the hinge at A.

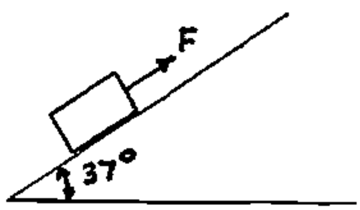


Figure 1

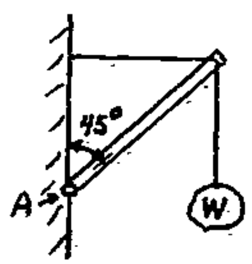


Figure 2

Date________________

EQUILIBRIUM OF RIGID BODIES

pass recycle

Mastery Test Form B

1 2 3

Name____________________ Tutor____________________

1. State the first condition of equilibrium (translational) and describe how it differs from the second condition of equilibrium (rotational).

2. Two weights of 17.0 N each are suspended by cords 2.00 m long and attached to the ceiling at points 4.0 m apart. The weights are then connected with a cord that is 2.00 m long as shown in Figure 1. Draw a free-body diagram and find the tension in the connecting cord.

3. A playground seesaw, made from a plank 4.0 m long with a uniformly distributed mass of 50 kg, is to be used by two children with masses 60 kg and 40 kg. If the children are to sit at the ends of the plank, where should the fulcrum be placed for a balance?

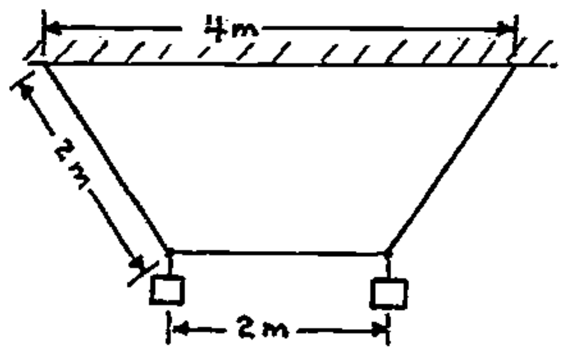


Figure 1

Date________________

EQUILIBRIUM OF RIGID BODIES

pass recycle

Mastery Test Form C

1 2 3

Name____________________ Tutor____________________

1. State the second condition of equilibrium (rotational), and describe how it differs from the first condition of equilibrium (translational).

2. A rope of tensile strength 4500 N is tied to two sturdy posts 40 m apart. The rope is broken by applying a transverse force at the center. If the rope broke when the center was displaced by 1.00 m, how large was the transverse force?

3. A nonuniform rod of length L is attached to a wall by a hinge A as shown in Figure 1. The lower end rests on a frictionless floor. The center of gravity is (1/3)L from the lower end. Draw a free-body diagram, and find the force on the hinge.

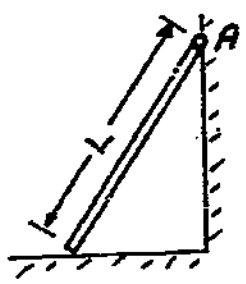


Figure 1

MASTERY TEST GRADING KEY - Form A

What To Look For	Solutions
1. (a) Be sure the student has made the concise statement asked for in the question. (b) Ask the student for the meaning of one or two key words in his answer.	1. Center of gravity - That point about which the resultant of the gravitational torques on all the particles composing the rigid body is zero.
2. (a) Check for a free-body diagram showing only directions and magnitudes of all external forces (not wrong, but explain to student). (b) Check for a correct statement of $\Sigma F_x = 0$ and $\Sigma F_y = 0$. (c) Be sure that he has obtained the correct answer, including magnitude, direction, and units.	2. 49 N up the incline.
3. (a) Check for a free-body diagram showing only directions, magnitudes, and points of application of all external forces. (b) Check for a correct statement of $\Sigma F_x = 0$, $\Sigma F_y = 0$, and $\Sigma\tau = 0$. (Be sure an explicit choice of axis has been made for calculation of torques.) (c) Be sure student has obtained the requested answer, including magnitude, direction, and units.	3. $\vec{V} = 2.40 \times 10^4$ N up, $\vec{H} = 2.20 \times 10^4$ N to right.

MASTERY TEST GRADING KEY - Form B

What To Look For	Solutions
1. (a) Be sure that the student has made the concise statement asked for in the question. (b) Ask the student for the meaning of one or two key words in his answer.	1. First condition of equilibrium (translational) - The vector sum of the forces acting on the object equals zero, i.e., the object has a constant velocity.
2. (a) Check for a free-body diagram showing only directions and magnitudes of all external forces. (b) Check for a correct statement of $F_x = 0$ and $F_y = 0$. (c) Be sure that student has obtained the correct answer including magnitude, direction and units.	2. 9.8 N along horizontal cord.
3. (a) Check for a free-body diagram showing only directions and magnitude, and points of application of all external forces (if missing, not wrong). (b) Check for a correct statement of $\Sigma F_x = 0$; $\Sigma F_y = 0$; $\Sigma \tau = 0$. (Be sure an explicit choice of axis has been made for calculation of torques.) (c) Be sure that student has obtained the requested answer including magnitude, direction, and units.	3. 1.70 m from the 60-kg child.

MASTERY TEST GRADING KEY - Form C

What To Look For	Solutions
1. (a) Be sure that the student has made the concise statement asked for in the question. (b) Ask the student for the meaning of one or two key words in his answer.	1. Second condition of equilibrium (rotational) - The sum of the torques at a chosen pivot point equals zero, i.e., the object will not rotate.
2. (a) Check for a free-body diagram showing only directions and magnitudes of external forces (not wrong if missing). (b) Check for a correct statement of $\Sigma F_x = 0$ and $\Sigma F_y = 0$. (c) Be sure that student has obtained the requested answer, including magnitude and units.	2. 400 N.
3. (a) Check for a free-body diagram showing only directions, magnitudes, and points of application of all external forces. (b) Check for a correct statement of $\Sigma F_x = 0$; $\Sigma F_y = 0$; $\Sigma \tau = 0$. (Be sure an explicit choice of axis has been made for calculation of torques.) (c) Be sure that student has obtained the requested answer including magnitude, and direction.	3. $\vec{F} = W/3$ upwards with no horizontal component.

Module ____

STUDY GUIDE

ROTATIONAL DYNAMICS

INTRODUCTION

A diver, in making several turns in the air, grabs his knees to achieve a high rate of rotation, and a skater does much the same thing when she goes into a spin with arms and legs extended but brings them in close to her body for the extremely rapid part of this motion. This module considers the physics describing these motions, and those of other rotational systems - starting or stopping a record turntable (or a washing-machine tub), unwinding of winch cord as a bucket is dropped into a well, etc.

PREREQUISITES

Before you begin this module, you should be able to:	Location of Prerequisite Content
*State Newton's second law for linear motion (needed for Objective 2 of this module)	Newton's Laws Module
*Analyze and solve problems involving conservation of energy (needed for Objective 2 of this module)	Conservation of Energy Module
*Relate work and energy (needed for Objective 3 of this module)	Work and Energy Module
*Define the center of mass (needed for Objective 1 of this module)	Impulse and Momentum Module
*Define angular momentum and relate angular momentum to torque for a point-mass particle (needed for Objective 4 of this module)	Rotational Motion Module
*Relate angle, angular velocity, and angular acceleration (needed for Objectives 2 and 3 of this module)	Rotational Motion Module

LEARNING OBJECTIVES

After you have mastered the content of this module, you will be able to:

1. Moment of inertia - (a) Apply the definition of moment of inertia, and the parallel-axis theorem where needed, to calculation of moments of inertia of simple extended bodies (not requiring integration), or demonstrate general understanding of the concept of moment of inertia by ranking several regular and irregular bodies according to their moments of inertia.

   (b) Write down the moments of inertia of a circular hoop (identical to that for an oil drum without ends), a circular disk (identical to that for a solid cylinder), and a long thin rod about an axis through the center of mass, and perpendicular to the plane of hoop or disk, or perpendicular to the length of the rod.

2. Rotation about fixed axis - In cases of rotation about a fixed axis, solve problems using Newton's second law of motion for rotation, or by using conservation of energy.

3. System of objects - For a system of objects rotating about a fixed axis where some of the following quantities are given, find others: moment of inertia, angular momentum, angular velocity, rotational kinetic energy, work, and power.

4. Conservation of angular momentum - For a system of objects rotating about a fixed axis, solve problems where angular momentum is conserved about some axis, but where angular velocity changes because the system changes size or shape; be able to recognize those groups of objects for which angular momentum will be conserved about a given axis.

GENERAL COMMENTS

You have seen conservation of energy in an earlier module (Conservation of Energy), but in this module it is broadened to include cases that have kinetic energy of rotation. Systems to which we may apply conservation of energy are those where only conservative forces act (gravity, spring, etc.) or else those where "other" forces are not doing any work on the system. Forces acting at the fixed axis of rotation of some body, for instance, are not acting through any distance, and thus do no work. When you get to Problems C and H, you will see that they each involve a body rotating about a fixed axis; thus energy conservation applies to their motions.

When you look for conservation of angular momentum, you must find a system of objects on which all torques that come from forces acting THROUGH the "plastic bag" around the system add vectorially to zero. You should study Problem E, which looks carefully at this situation.

TEXT: Frederick J. Bueche, Introduction to Physics for Scientists and Engineers (McGraw-Hill, New York, 1975), second edition

SUGGESTED STUDY PROCEDURE

Read Chapter 11, Sections 11.3 to 11.7, for Objectives 1. In particular, study Figure 11.11. Study Problems A and B in the Problem Set, and work Problems F and G. For Objectives 2 and 3, read the text Chapter 12, Sections 12.1 through 12.8. Study Problems B, C, and D and work Problem H. Sections 12.6 to 12.8 also apply to Objective 4. Study Problem E and work Problems I, J, and K. If you need more practice, try the Additional Problems in the text.

Take the Practice Test before trying a Mastery Test.

BUECHE

Objective Number	Readings	Problems with Solutions: Study Guide	Problems with Solutions: Text*	Assigned Problems: Study Guide	Additional Problems
1	Secs. 11.3 to 11.7	A, B		F, G	Chap. 11, Probs. 5, 6, 7
2	Secs. 12.2, 12.8	B, C, E	Illus. 12.8	H, K	Chap. 11, Probs. 12, 13, 14; Chap. 12, Prob. 18
3	Secs. 12.1 to 12.3, 12.7	C, D, E,		H, J, K	Chap. 12, Probs. 1, 3, 4
4	Secs. 12.6 to 12.8	E		I, J	Chap. 12, Probs. 2, 16, 12

*Illus. = Illustration(s).

TEXT: David Halliday and Robert Resnick, Fundamentals of Physics (Wiley, New York, 1970; revised printing, 1974)

SUGGESTED STUDY PROCEDURE

Read Chapter 11, Section 11-4 for background, and then Sections 11-5 through 11-7. Example 5 can be omitted, since this module does not require the calculation of rope tensions. Note also that the text has very few energy-conservation examples, which are called for in Objective 2. To prepare for Objective 1, study Problems A and B, then work Problems F and G. For Objectives 2 through 4, study Problems C, D, and E before working Problems F through K. If you need more practice, try the Additional Problems in the text.

Take the Practice Test before attempting a Mastery Test.

HALLIDAY AND RESNICK

Objective Number	Readings	Problems with Solutions	Assigned Problems	Additional Problems
		Study Guide	Study Guide	(Chap. 11)
1	Sec. 11-5	A, B	F, G	Quest.* 4, 6; Probs. 15, 19(b)
2	Sec. 11-6	B, C, E	H, K	17, 18, 23, 46
3	Sec. 11-6	C, D, E	H, J, K	25, 27, 31, 32
4	Sec. 11-7	E	I, J	35, 37, 39, 41, 43, 45

*Quest. = Question(s).

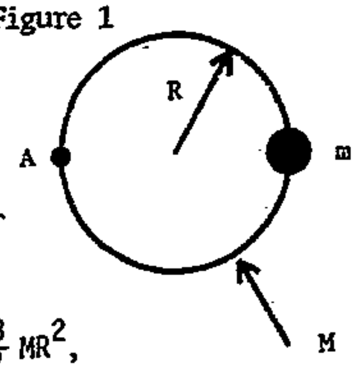
TEXT: Francis Weston Sears and Mark W. Zemansky, University Physics (Addison-Wesley, Reading, Mass., 1970), fourth edition

SUGGESTED STUDY PROCEDURE

Read Chapter 9, Sections 9-6 and 9-7 for Objectives 1 and 2. You may skim Section 9-7, but should study Section 9-6 more carefully, noting in particular Figure 9-13 on p. 137. Now read Sections 9-8 and 9-9 for Objectives 3 and 4. Study Problems A through E before working Problems F through K. Work some of the Additional Problems from the text if you are unsure of your mastery, before taking the Practice Test. Then try a Mastery Test.

Your text does not cover the parallel-axis theorem, which is called for in Objective 1. Briefly, the theorem states that if you know the moment of inertia I_A about axis A through the center of mass, you can calculate the moment of inertia about an axis B that is parallel to axis A and distant from it by D, using the formula $I_B = I_A + MD^2$, where M is the mass of the object. A text where this is proved and discussed is Fundamentals of Physics,* Chapter 11, Section 11-5. Note that Problem A is a good example of how to use this theorem, as is the following problem.

Figure 1



Problem: Calculate the moment of inertia about axis A of a cylinder (mass M) with a small mass m attached as shown in Figure 1.

Solution: Moments of inertia are additive:

$I_{MA} + I_{mA} = I_A$, $\quad I_m A + m(2R)^2$, and since I for cylinder is the same as that for a disk, $\quad I_{M(\text{about c.m.})} = MR^2/2$.

By the parallel-axis theorem, $\quad I_{MA} = I_M + MR^2$, $\quad I_{MA} = \frac{3}{2} MR^2$,

and the total moment of inertia is $I = (3/2)MR^2 + 4mR^2$.

SEARS AND ZEMANSKY

Objective Number	Readings	Problems with Solutions Study Guide	Assigned Problems Study Guide	Additional Problems
1	Sec. 9-7	A, 8	F, G	9-15, 9-14(a), (b), 9-16, 9-18
2	Sec. 9-6	8, C, E	H, K	9-23, 9-24(a), 9-26(b), 9-34, 9-38, 9-39(a)
3	Secs. 9-8, 9-9	C, D, E	H, J, K	9-24(b), 9-29(b), (c), (d), 9-31, 9-35, 9-37, 9-39(b), (c), (d), 9-41, 9-43, 9-49
4	Sec. 9-9	E	I, J	9-41, 9-43, 9-49, 9-51

*David Halliday and Robert Resnick (Wiley, New York, 1970; revised printing, 1974).

TEXT: Richard T. Weidner and Robert L. Sells, Elementary Classical Physics (Allyn and Bacon, Boston, 1973), second edition, Vol. 1

SUGGESTED STUDY PROCEDURE

Read Chapter 11, Sections 11-5 through 11-7 and Chapter 12, Sections 12-2 through 12-5. Study Figure 12-1. Note that Examples 12-5 to 12-8 involve more complex situations then called for in the objectives - either due to a moving axis of rotation or because Newton's second law for linear motion is used together with the second law for rotation.

WEIDNER AND SELLS

Objective Number	Readings	Problems with Solutions Study Guide	Assigned Problems Study Guide	Additional Problems
1	Secs. 12-2, 12-7	A, B	F, G	12-27(a)
2	Secs. 12-3, 12-4	B, C, E	H, K	12-14, 12-17, 12-26, 12-27(b), (c), 12-28
3	Secs. 12-3, 12-4	C, D, E	H, J, K	12-19, 12-27(e), (f)
4	Sec. 11-7	E	I, J	11-14, 11-18, 11-19, 11-23

PROBLEM SET WITH SOLUTIONS

A(1). Calculate the moment of inertia of four equal masses M at the corners of a square of side L as shown in Figure 2, with respect to axis A and also with respect to axis B, both axes perpendicular to the paper.

Solution

Axis A is at the center of mass:

$$I_A = \sum_{i=1}^{i=4} M_i r_{iA}^2 = M(r_{1A}^2 + r_{2A}^2 + r_{3A}^2 + r_{4A}^2),$$

where r_{1A} is the distance from axis A to first mass; r_{2A} is the distance from axis A to second mass, etc. Convince yourself that $r_{1A} = r_{2A} = r_{3A} = r_{4A} = L(\sqrt{2}/2)$, and $I_A = 2ML^2$.

To calculate I about axis B, we can use the parallel-axis theorem, or we can calculate

$$I = \sum_i M_i r_{iB}^2 = M(r_{1B}^2 + r_{2B}^2 + r_{3B}^3 + r_{4B}^2).$$

Using this last method first, we find that $r_{1B} = 0$, $r_{2B} = L$, $r_{3B} = L\sqrt{2}$, $r_{4B} = L$, and we get $I_B = M(0^2 + L^2 + 2L^2 + L^2) = 4ML^2$. Now, using the parallel-axis theorem, we have

$$I_B = I_A + (\text{total mass})(\text{distance between axes A and B})^2,$$

$$I = 2ML^2 + (4M)[L(\sqrt{2}/2)]^2 = 4ML^2,$$

and both methods agree.

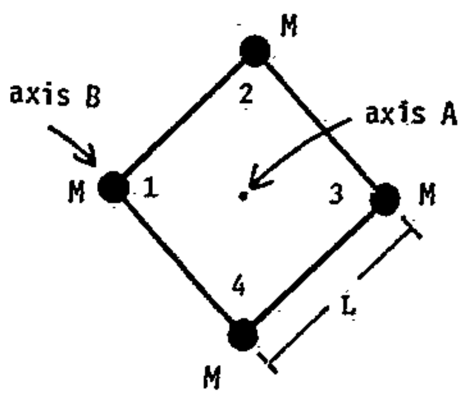


Figure 2

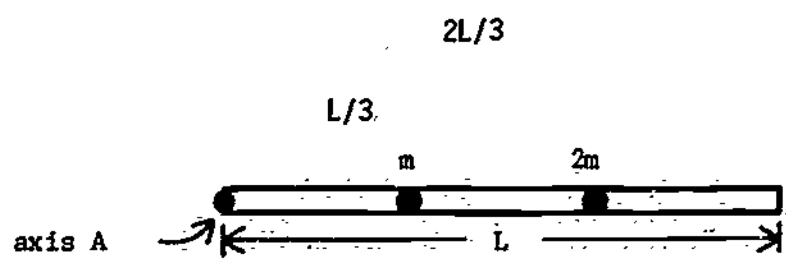


Figure 3

B(1,2). Consider a rod of mass M and length L, frictionlessly pivoted about axis A (out of the page) with masses m and 2m at L/3 and 2L/3 as shown in Figure 3. The moment of inertia of this rod about A is

$$I = ML^2/3 + mL^2 \quad \text{(The student should work this out!).}$$

Calculate the angular acceleration of the rod and masses if just released under gravity.

Solution

Calculate total torques about A:

$$\tau_1 = mg(L/3), \quad \tau_2 = (2m)g(2L/3), \quad \tau_3 = (MgL)/2,$$

$$\Sigma\tau_{ext} = +(MgL)/2 + (5/3)mgL = I_\alpha = (ML^2/3 + mL^2)\alpha \quad \text{(into the paper),}$$

thus

$$a = \frac{(5/3)mgL + MgL/2}{ML^2/3 + mL^2} = \frac{g}{L}\frac{5/3 + M/2m}{1 + M/3m} \quad \text{(into the paper).}$$

C(2,3). Calculate the angular velocity of the rod in Problem B when it reaches a vertical position, after having rotated through 90°.

Solution

We apply conservation of energy: $K_i + U_i = K_f + U_f$. We take a PE reference height equal to 0 where the rod starts, and we note that the center of mass of the rod will have a height at the end of $-L/2$, m will have a height $-L/3$, and 2m will have height $-2L/3$.

$$K_i = 0, \quad U_i = 0, \quad K_f = I\omega^2/2,$$

$$U_f = Mg(-L/2) + mg(-L/3) + 2m(-2L/3),$$

thus

$$\omega^2 = \frac{2}{I}(gL)\left(\frac{M}{2} + \frac{5m}{3}\right) = \frac{2gL}{L^2}\left(\frac{M/2 + (5/3)m}{m + M/3}\right),$$

$$\omega = \text{angular velocity} = \left(\frac{2g}{L}\right)^{1/2}\left(\frac{M/2 + (5/3)m}{m + M/3}\right)^{1/2} \quad \text{(into the paper).}$$

D(3,4). A disk of mass M and radius R rotates frictionlessly with angular speed ω_0 about an axis through its center. At its center is a cricket of mass m. Since the disk is isolated, no torques exist external to the (M + m) system.

(a) Calculate the final angular velocity ω_f, after the cricket jumps from the center to a point R/2 from the center as shown in Figure 4, where he holds fast, rotating with the disk.
(b) Calculate KE before and after. Why is it not the same?

Solution

(a) Angular momentum is conserved, thus

$$L = I_0\omega_0 = I\omega_f, \qquad I_0 = MR^2/2, \qquad I = MR^2/2 + m(R/2)^2$$

and

$$\omega_f = \omega_0 \left(\frac{I_0}{I}\right) = \omega_0\left(\frac{MR^2}{MR^2 + m(R^2/2)}\right) = \frac{\omega_0}{1 + m/2M} \quad \text{(in same direction as } \omega_0\text{)}.$$

(b) $K = I\omega^2/2$, $L = I\omega = I_0\omega_0$, thus

$$K_f = L^2/2I, \qquad K_i = L^2/2I_0$$

The L is the same, but I's are different. Thus $K_f < K_i$. We may regard the cricket's landing as an inelastic collision with the disk, in which some energy was converted into heat.

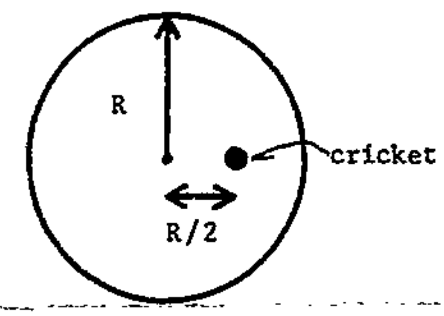


Figure 4

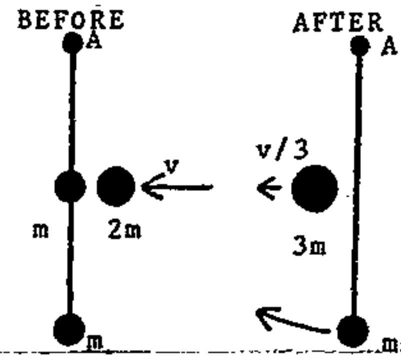


Figure 5

E(2,3,4). A massless rod of length 2L is free to rotate about axis A as shown in Figure 5, with two masses m attached. Along comes mass 2m and "captures" one of the masses as shown. Find the height to which the remaining mass will rise.

Solution

First we notice that angular momentum about A is conserved for the system of (rod + all three masses) before and after the collision. This is because the torques that act on the system (rod + three masses) just before and after the collision are zero about A. There are four forces external to the system: gravity acting on the three masses and the force of the support on the rod at A. Every one of these forces exerts zero torque about point A, because all forces pass through A. Notice that once the collision is over and the (rod +

mass) starts to rotate, there will be a torque from the gravity on the mass, and angular momentum of (rod + mass) will not be conserved after the collision. Conserving angular momentum about A we get

$$2mvL = (3mv/3)L + I\omega, \quad I\omega = mvL = \omega[m(2L)^2], \quad \omega = v/4L.$$

Energy is not conserved in this collision (inelastic collision), as the student may check ($mv^2 \neq mv^2/6 + mv^2/8$). Mechanical energy <u>will</u> be conserved, though, for the (rod + mass) system after the collision. No further collisions are involved, gravity is a conservative force, and the force on the rod at the pivot does no work:

$$K_i + U_i = K_f + U_f, \quad I\omega^2/2 + 0 = 0 + mgh,$$

$$h = \left(\frac{m(4L^2)}{2}\right)\left(\frac{v}{4L}\right)^2\left(\frac{1}{mg}\right) = \frac{v^2}{8g}.$$

<u>Problems</u>

F(1). Calculate the moment of inertia of three masses at the corners of an equilateral triangle of side L about axes A and B, perpendicular to the page, see Figure 6.

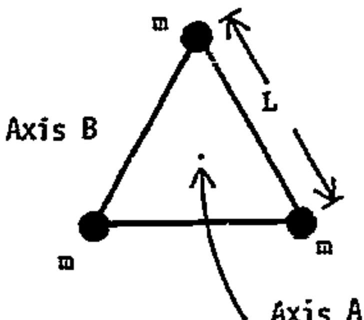


Figure 6

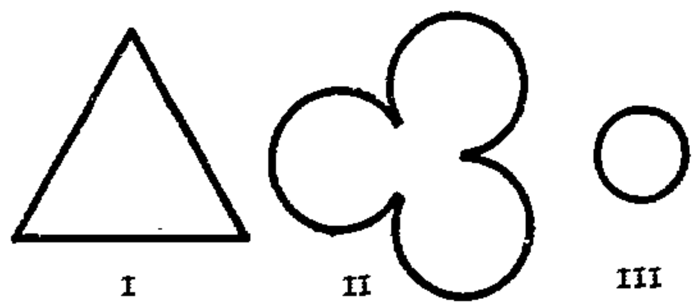


Figure 7

G(1). Put the masses of Figure 7 in order of decreasing moments of inertia about an axis perpendicular to the paper through the center of mass. All systems (I, II and III) have the same mass and uniform density.

H(2,3). Calculate the angular velocity of the disk in Figure 8 in position 2 released from the position 1.

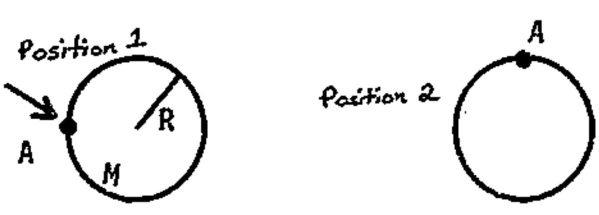


Figure 8

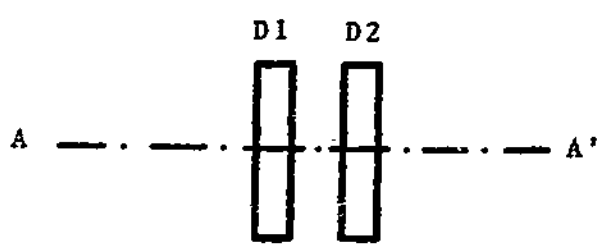


Figure 9

I(4). Consider disk D1 in Figure 9 of mass M and radius R rotating with angular velocity ω_0 about axis AA′, while disk D2 is not in motion. (Its mass is 2M and its radius is R.) Suppose that disks D1 and D2 suddenly become attached without external intervention. Calculate the final common angular velocity of the combined disks.

J(3,4). Suppose two 60-kg skaters are going in a circle, with angular speed 1.00 rad/s, each holding on to a 2.00-m (massless) broom handle. Combined, they are able to "power" themselves at the rate of 400 W. How long will it take them to pull on the broom and get within 1.00 m of each other?

K(2,3). Block A (mass = 10.0 kg) rests on a frictionless surface in Figure 10. The pulley is a disk of mass 20.0 kg, radius 0.50 m, and is free to rotate without friction about an axis at its center. A massless rope that is tied from block A passes without slipping over the pulley, and is tied to block B (mass = 30.0 kg). This system is released from rest. Find the angular velocity of the pulley disk when the block A has moved 4.0 m. Hint: The new element in this problem is the mixing of angular motion of the disk with the linear motion of the blocks. The velocity of block A must also be the velocity of the rope. The rope must have the same speed as the tangential velocity of the pulley where they meet (no slipping); this gives a relation between ω of the pulley and V of the block ($\omega = V/R$). Use energy conservation and solve for ω, after eliminating V.

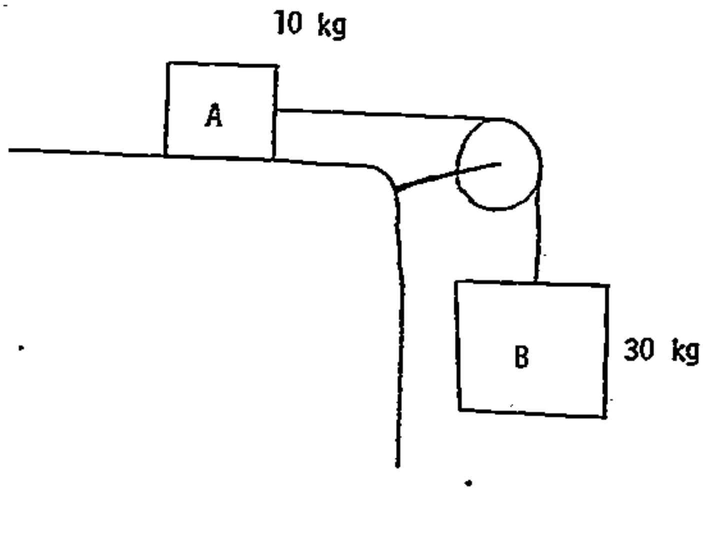


Figure 10

Solutions

F(1). $I_A = ML^2$; $I_B = 2ML^2$.

G(1). In each case, we estimate the position of the center of mass, and judge in which case there is more mass "away from the center": for which one is $\Sigma_i M_i r_i^2$ the largest. In Figure 7, Case II clearly has most mass furthest from the center of mass, and Case III has the least. Therefore, $I_{II} > I_I > I_{III}$.

H(2,3). $\omega = \sqrt{(4/3)(g/R)}$ (into the paper).

I(4). $\omega_f = \omega_0/3$.

J(3,4). Angular momentum is conserved:

$$I_0\omega_0 = I_1\omega_1 = L, \qquad I_0 = 2(60\ \text{kg})(1\ \text{m})^2, \qquad I_1 = 2(60\ \text{kg})(1/2\ \text{m})^2,$$

$$\text{Work needed} = \Delta KE = \frac{I_1\omega_1^2}{2} - \frac{I_0\omega_0^2}{2} = \frac{L^2}{2I_1} - \frac{L^2}{2I_0} = (KE)_0\left(\frac{I_0}{I_1} - 1\right) = \frac{I_0\omega_0^2}{2}\left(\frac{I_0}{I_1} - 1\right)$$

$$= \frac{60\ \text{kg} \times 2 \times 1.00\ \text{m}^2}{2} \times (1.00\ \text{rad/s})^2\left(\frac{r_0^2}{r_1^2} - 1\right) = (60\ \text{J})\left(\frac{1^2}{(1/2)^2} - 1\right)$$

$$= 180\ \text{J}.$$

$$t = \frac{\text{work}}{\text{power}} = \frac{180\ \text{J}}{400\ \text{W}} = \frac{1.80}{4}\ \text{s} = 0.45\ \text{s}.$$

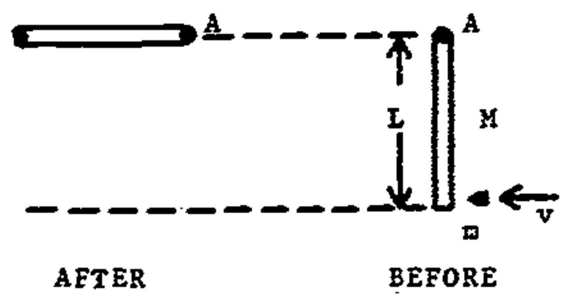
K(2,3). 13.7 rad/s.

PRACTICE TEST

1. A rod of mass M and length L is suspended to rotate freely about axis A coming out of the page. It is struck by a bullet of mass m and speed v, which becomes imbedded in the rod after impact. The rod-plus-bullet has rotated but is brought to rest after 90° of rotation by gravity as shown in Figure 11. (a) What is the angular velocity of the rod-plus-bullet just after the bullet is imbedded? (b) What value must the bullet speed have for this to happen?

2. A uniform disk of radius 0.200 m and mass 5.0 kg is rotating at an angular speed of 2.50 rad/s about a fixed axis through its center. It is brought to rest in 5.0 s by a uniform torque. Find the value of this torque.

Figure 11



Practice Test Answers

1. (a) Conserve angular momentum about axis A: $mvL = I\omega = (ML^2/3 + mL^2)\omega$.

$$\omega = \frac{mvL}{ML^2/3 + mL^2} = \frac{v}{L}\left(\frac{1}{1 + M/3m}\right).$$

(b) Conserve energy after collision:

$$mgh + mg\frac{h}{2} = \frac{I\omega^2}{2} = \left(\frac{ML^2/3 + mL^2}{2}\right)\left[\frac{v}{L}\left(\frac{1}{1 + M/3m}\right)\right]^2,$$

$$v^2 = 2gh(1 + M/2m)(1 + M/3m), \quad v = \sqrt{2gh(1 + M/2m)(1 + M/3m)}.$$

2. $\omega - \omega_0 = \alpha t, \quad \alpha = \dfrac{2.50\ \text{rad/s}}{5.0\ \text{s}} = 0.50\ \text{rad/s}^2,$

$$I = mr^2/2 = (5.0\ \text{kg})(0.200\ \text{m})^2/2 = 0.100\ \text{kg m}^2.$$

$$\tau = I\alpha = (0.100\ \text{kg m}^2)(0.50\ \text{rad/s}^2) = 0.050\ \text{N m}.$$

Date ____________________

ROTATIONAL DYNAMICS

pass recycle

Mastery Test Form A

1 2 3 4

Name ______________________________ Tutor ______________________________

1. A thin cord is wrapped around a disk of mass 10.0 kg and radius 0.100 m, which is free to rotate about a fixed axis at its center. See Figure 1. The end of this cord is tied to a 10.0-kg mass that is released from rest and travels 0.50 m vertically. At this point, what is the angular velocity of the disk?

2. A merry-go-round of radius 4.0 m and mass 220 kg in the shape of a disk is rotating at 1.00 rad/s. A 70-kg man standing next to the merry-go-round runs directly toward the center at 3.00 m/s, jumps and lands on the edge, turning with the merry-go-round after he lands. What is the angular speed of the merry-go-round after this maneuver?

3. In Figure 2 there are three objects of equal mass and uniform density. Number them 1, 2, 3, assigning 1 to the object with the largest moment of inertia, 2 to the middle, and 3 to the object with the smallest moment of inertia about an axis through the center of mass and perpendicular to the paper.

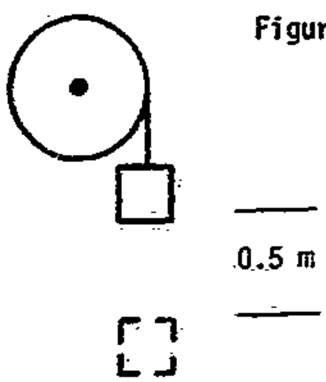


Figure 1

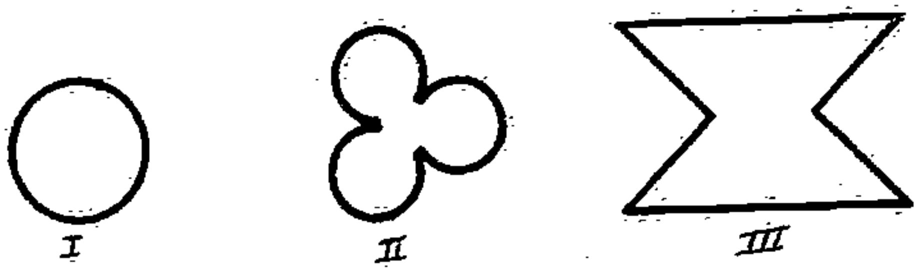


Figure 2

ROTATIONAL DYNAMICS

Mastery Test   Form B

Name_______________

Date_______________

pass   recycle
1   2   3   4

Tutor_______________

1. (a) While running through the forest at 12.0 m/s chased by a rampaging rhinocerous, Tarzan grabs on to a tree limb dangling by a strong bark strip at its upper end (see Fig. 1). If the limb has a mass of 50 kg, and a length of 5.0 m, what is the angular velocity just after Tarzan grabs on? Treat the limb as a uniform rod. Tarzan's mass is 80 kg.

   (b) How high does Tarzan swing if the bark holds?

2. In Figure 2 there are three objects of <u>equal mass</u> and uniform density. Number them 1, 2, 3, assigning 1 to the object with the largest moment of inertia, 2 to the middle, and 3 to the object with the smallest moment of inertia about an axis through the center of mass and perpendicular to the paper.

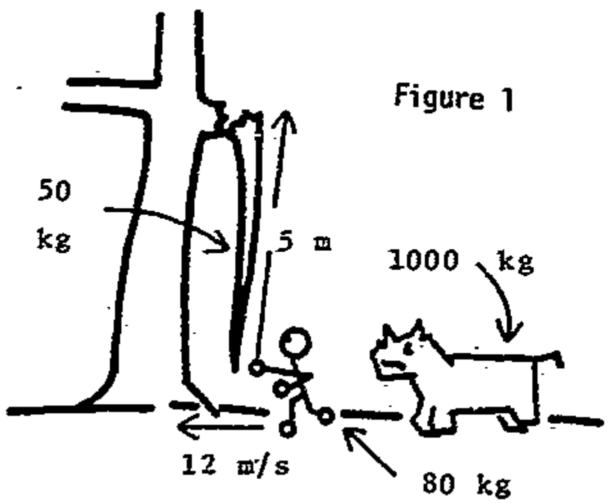
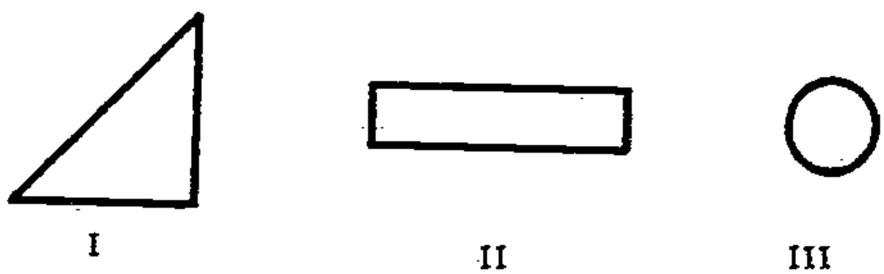


Figure 1

Figure 2



ROTATIONAL DYNAMICS

Mastery Test   Form C

Date____________________

pass   recycle

1   2   3   4

Name__________________________   Tutor____________________

1. Two uniform bars (one bar having mass M and the other having mass 2M) of length L are located side by side and each is pivoted about one end, as shown in Figure 1. The two are initially at rest in a horizontal orientation and released in such a way that they reach the vertical simultaneously.

   (a) Find the angular velocity of each bar immediately before impact.

   (b) Find the angular momentum of each bar immediately before impact.

   (c) After impact, the 2M bar rebounds at one-third of its preimpact angular velocity. What is the angular velocity of the M bar immediately after impact?

   (d) Compare the kinetic energy of the system immediately before collision with the kinetic energy immediately after collision.

2. In Figure 2 there are three objects of equal mass and uniform density. Number them 1, 2, 3, assigning 1 to the object with the largest moment of inertia, 2 to the middle, and 3 to the object with the smallest moment of inertia about an axis through the center of mass and perpendicular to the paper.

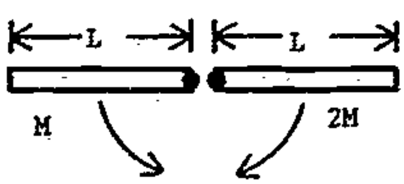
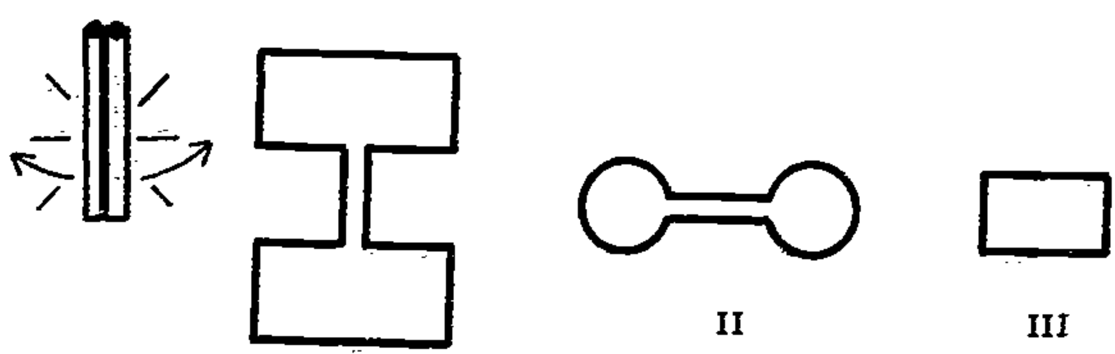


Figure 1

Figure 2



I

ROTATIONAL DYNAMICS

Date____________________

pass recycle

Mastery Test Form D

1 2 3 4

Name____________________________

Tutor____________________

1. An 80-kg astronaut is in a rotating simulator (mass of 40 kg concentrated at outer rim) with a frictionless center axle. The simulator is rotating at 1.00 rad/s. See Figure 1.

   (a) An alarm bell rings, and the astronaut is told he has 80 s to reach the escape door at the center of the simulator. If his power output is 200 W, will he escape in time? Start by calculating initial and final moments of inertia.

   (b) The motor used to start the simulator spinning delivers a constant torque of 100 N m. What is the angular acceleration? With the astronaut inside, at the outer edge, how long does it take to reach the speed above?

2. In Figure 2 there are three objects of <u>equal mass</u> and uniform density. Number them 1, 2, 3, assigning 1 to the object with the largest moment of inertia, 2 to the middle, and 3 to the object with the smallest moment of inertia about an axis through the center of mass and perpendicular to the paper.

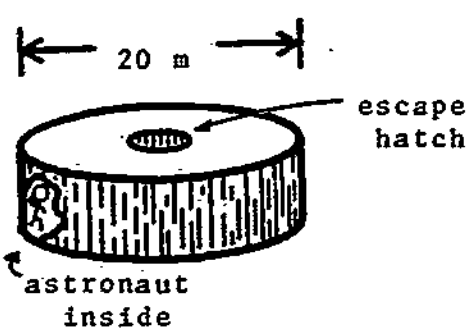
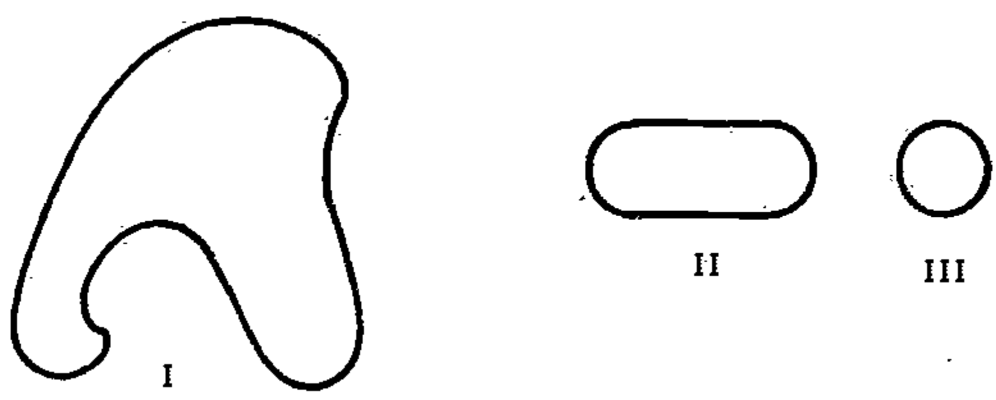
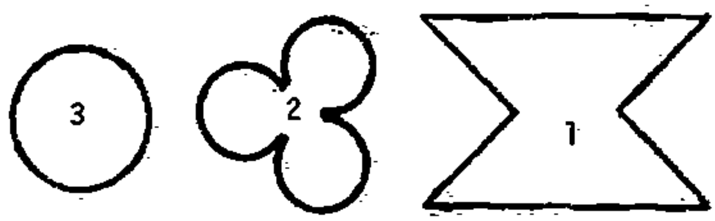


Figure 1

Figure 2



MASTERY TEST GRADING KEY - Form A

What To Look For	Solutions
	1. $(KE + PE)_{start} = (KE + PE)_{end}$. Take reference for gravitational PE at final position of falling mass: $0 + mgh = mv^2/2 + I\omega^2/2 + 0$, $I = mR^2/2$, $\omega = v/R$, $mgh = (3/4)\ MR^2\omega^2$, $\omega = (\frac{4}{3}\frac{gh}{R^2})^{1/2} = (\frac{4}{3}\frac{(9.8\ m/s^2)(0.50\ m)}{(0.100\ m)^2})^{1/2}$ $= 25.6$ rad/s (into the paper).
2. Note that radial velocity of 3.00 m/s has no effect on the problem.	2. $I_{empty} = MR^2/2$, $I_{with\ man} = MR^2/2 + MR^2$. Conserve angular momentum: $\omega_0 R^2/2 = \omega_{final}(MR^2/2 + mR^2)$, $\omega_f = \omega_0(\frac{1}{1 + 2m/M}) = \frac{220}{360} = 0.61$ rad/s.
3. Object III is clearly the greatest. Ask student to explain reasoning if II and I are interchanged.	3.  I II III

MASTERY TEST GRADING KEY - Form B

What To Look For	Solutions
	1. (a) Conserve angular momentum about point where tree limb swings for system of (Tarzan + limb): M_T = Tarzan mass, M_L = mass of limb, L = length of limb.

$$M_T V_T L = I\omega,\quad I = M_L(L^2/3) + M_T L^2,$$

$$\omega = \frac{M_T V_T L}{M_L L^2/3 + M_T L^2} = \frac{V_T}{L}\left(\frac{1}{1 + M_L/3M_T}\right) = 2.00 \text{ rad/s}.$$

(b) Tarzan gains height h and tree limb center of mass gains $h/2$. Conservation of energy gives us

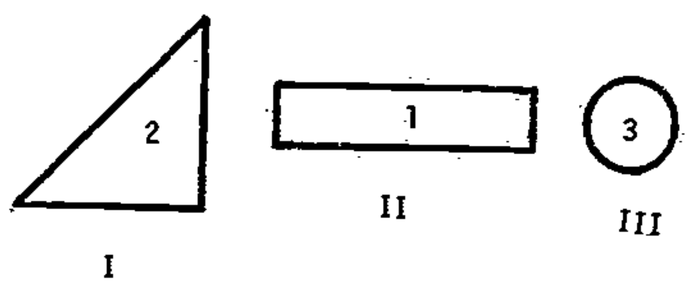
$$M_T g h + M_L g\frac{h}{2} = \frac{I}{2}\left(\frac{M_T V_T L}{I}\right)^2,$$

$$h = \left(\frac{V_T^2}{2g}\right)\frac{1}{(1 + M_L/3M_T)(1 + M_L/2M_T)}$$

$$h = \frac{(12.0\text{ m/s})^2}{2(9.8\text{ m/s}^2)(1 + 5.0/24)(1 + 5.0/16.0)} = 4.6\text{ m}.$$

2. Object III is clearly the smallest. Ask student to explain if Objects I and II are interchanged.

2.



MASTERY TEST GRADING KEY - Form C

What To Look For	Solutions
1. Ask student about $\vec{\omega}$, $\vec{\ell}$ directions, if he doesn't show them.	1. (a) $\frac{I\omega^2}{2} = mg(\frac{L}{2}) = (\frac{ML^2}{3})(\frac{\omega^2}{2})$, $\omega_1^2 = 3(\frac{g}{L})$ for each bar.

ω_1 into paper for 2M bar, out of paper for M bar.

(b) Angular momentum = $I\omega$. M bar has angular momentum $(ML^2/3)(3g/L)^{1/2}$ out of the paper and 2M bar has twice this, into the paper.

(c) $\ell_{before} = (\frac{mL^2}{3})(\frac{3g}{L})^{1/2}$,

$\ell_{after} = -(\frac{2}{3})(\frac{ML^2}{3})(\frac{3g}{L})^{1/2} + \ell_{bar}$

$\ell_{before} = \ell_{after}$, thus $\ell_{bar} = (\frac{5}{3})(\frac{ML^2}{3})(\frac{3g}{L})^{1/2}$

$= I_M\omega_M$;

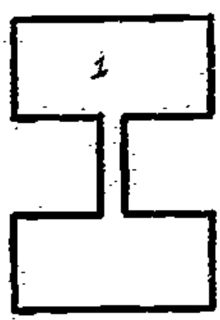
$\omega_M = (5/3)\omega_1$ into the paper.

$KE_{before} = \frac{I_1\omega_1^2}{2} + \frac{2I_1\omega_1^2}{2} = \frac{3I_1\omega_1^2}{2}$,

$KE_{after} = \frac{1}{2}[I_1(\frac{5}{3}\omega_1)^2 + 2I_1(\frac{\omega_1}{3})^2] = \frac{27}{18} I_1\omega_1^2$,

Since the kinetic energies are the same, the collision is elastic.

2. I = 1, II = 2, III = 3.



I

II

III

MASTERY TEST GRADING KEY - Form D

What To Look For	Solutions

1. (a) $I_0 = (10 \times 10^2 + 80 \times 10^2)$ kg m^2 = 12 000 kg m^2,

I_1 = 4000 kg m^2.

Angular-momentum conservation gives $I_1\omega_1 = I_0\omega_0 = L$.

ω_0 = 1.00 rad/s, L = 12 000 kg^2 m/s.

$$KE_{final} - KE_{initial} = \frac{L^2}{2I_1} - \frac{L^2}{2I_0} = \frac{L^2}{2I_0}\left(\frac{I_0}{I_1} - 1\right).$$

$$= KE_{initial}\left(\frac{I_0}{I_1} - 1\right).$$

Work needed = ΔKE = (6000 J)(12 000/4000 - 1).

To expend 12 000 J in 80 s we need

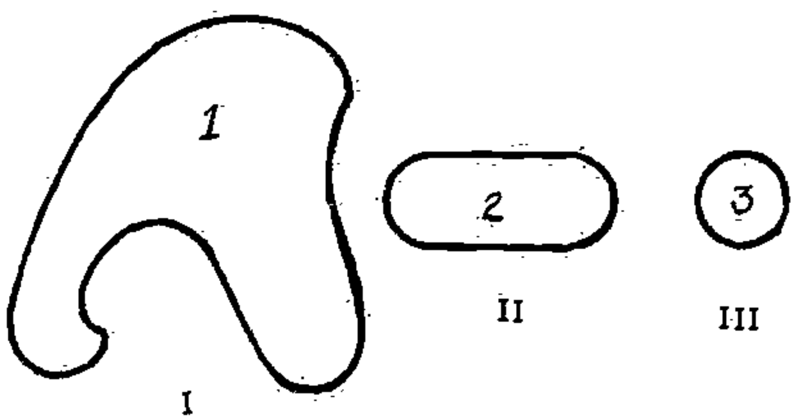
12 000 J/80 s = 150 W, thus astronaut will escape.

At 200 W he escapes in 60 s.

(b) $$\alpha = \frac{\tau}{I} = \frac{100 \text{ N m}}{(80 \text{ kg} + 40 \text{ kg})(10.0 \text{ m})^2} = \frac{1}{120} \text{ rad/s}^2,$$

$$\tau = \frac{\omega}{\alpha} = \frac{1.00 \text{ rad/s}}{1/120 \text{ rad/s}^2} = 120 \text{ s}.$$

2. I = 1, II = 2, III = 3.



Module ____

STUDY GUIDE

FLUID MECHANICS

INTRODUCTION

An invigorating shower in the morning is usually a pleasant experience except for the pesky shower curtain slapping your legs and allowing water to run on the floor. You would think that the downward stream of water would be enough to keep the curtain back even without water striking the curtain. But not so: fast-moving fluids (water spray causing a downdraft of air) contain a low-pressure region. Thus the pressure outside the shower is greater than the pressure inside - with the result that the curtain is blown in and flops against your legs.

More technical applications of fluid mechanics include airplane flight, streamlining of boats and cars, blood circulation, water towers, and weather forecasting. Even such mundane phenomena as the pressure of your water faucet and "curves" thrown by pitchers in baseball illustrate the ideas of fluid mechanics.

In this module conservation of energy will be recast into a form that is more suitable for application to fluids. No new fundamental physical laws will be introduced. The concepts of energy, work, and the conservation of matter will be used to study fluids at rest and in motion (statics and dynamics).

PREREQUISITES

Before you begin this module, you should be able to:	Location of Prerequisite Content
*Relate the resultant force on a particle to all forces of interaction of that particle and distinguish between weight and mass (needed for Objectives 1 and 2 of this module)	Newton's Laws Module
*Relate the work done on a particle to its change in kinetic energy (needed for Objective 3 of this module)	Work and Energy Module
*Define potential energy and apply the law of conservation of energy to mechanical systems (needed for Objective 3 of this module)	Conservation of Energy Module

LEARNING OBJECTIVES

After you have mastered the content of this module, you will be able to:

1. Pressure - State Pascal's principle and use it to solve problems involving the absolute or relative pressure, density, relative density, or depth of a fluid at rest.

2. Buoyancy - State Archimedes' principle and use it to find the pressure, density, relative density (specific gravity), or amount submerged for stationary objects floating in fluids.

3. Fluid flow - Describe various types of fluid flow: steady versus nonsteady, rotational versus irrotational, compressible versus incompressible, or viscous versus nonviscous, and give an example of each.

4. Hydrodynamics - State the equation of continuity and Bernoulli's principle, and use them to solve problems concerning the pressure, velocity, or height above some arbitrary reference level of incompressible, steady, nonviscous, irrotational fluid flow.

TEXT: Frederick J. Bueche, Introduction to Physics for Scientists and Engineers (McGraw-Hill, New York, 1975), second edition

SUGGESTED STUDY PROCEDURE

The reading includes all of Chapter 14 except Sections 14.3 and 14.4. Study the Problems with Solutions carefully, and solve the Assigned Problems before attempting the Practice Test. Begin by reading the material relative to Objective 1, that is, General Comment 1 and Sections 14.1, 14.2, and 14.5.

To supplement the text, read General Comment 2, where the characteristics of fluid flow are described more clearly. Pay particular attention to the treatment of fluid particles as in Figures 14.7, 14.10, and 14.12.

BUECHE

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text (Illus.*)	Study Guide	Text (Chap. 14)	(Chap. 14)
1	General Comment 1, Secs. 14.1, 14.2, 14.5	A, B	14.2	G, H	10	8, 9, 11
2	Sec. 14.6	C, D	14.3	I, J	18	15, 17, 19, 21
3	Sec. 14.7, General Comment 2					
4	Secs. 14.8 to 14.11, General Comment 3	E, F		K, L	26	23, 24, 25, 27

*Illus. = Illustration(s).

TEXT: David Halliday and Robert Resnick, Fundamentals of Physics (Wiley, New York, 1970; revised printing, 1974)

SUGGESTED STUDY PROCEDURE

The reading includes all of Chapter 15. Study the Problems with Solutions carefully, and solve the Assigned Problems before attempting the Practice Test. Begin by reading the material relative to Objective 1, that is, General Comment 1 and Sections 15-1 through 15-6.

In Sections 15-2, 15-3, and 15-4 the concepts of pressure and density, Pascal's principle, and Archimedes' principle are explained. Before these two principles are introduced, there is a treatment of the variation of pressure in a fluid at rest, which provides a theoretical basis for them. Equation (15-4) is a special case of the pressure variation in a moving fluid that you will encounter in Section 15-8 (Bernoulli's principle).

Pay particular attention to the treatment of fluid particles as in Figures 15-2, 15-11, and 15-13. See Section 15-7, Eqs. (15-5) and (15-6). The phrase "along any tube of flow" must be understood to follow these equations.

In Section 15-8, Bernoulli's equation has been derived for a pipeline, but can equally well be applied along any streamline. The necessity of keeping to a streamline was forgotten in the subsequent diagram of a Pitot tube (Fig. 15-15);

HALLIDAY AND RESNICK

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text (Chap. 15)	Study Guide	Text (Chap. 15)	(Chap. 15)
1	General Comment 1, Secs. 15-1 to 15-6	A, B	Ex.* 1,	G, H	8	7, 12
2	Sec. 15-4	C, D	Ex. 2	I, J	19	13, 15, 17, 20
3	Sec. 15-6, General Comment 2					
4	Secs. 15-7 to 15-9, General Comment 3	E, F	Sec. 15-9, 1, 2, 3	K, L	29	26, 30, 31, 39, 42

*Ex. - Example(s).

point b should be at the tip of the Pitot tube, so that the same streamline passes through points a and b. If we could neglect this condition, then we could move point a just inside the Pitot tube, where the velocity is zero - and thereupon find no pressure difference at all!

In Section 15-9, 3: Dynamic Lift, what Bernoulli's equation really does is to relate the pressure to the speed along each of two streamlines, one of which goes through point 1 (above the ball or wing), while the other goes through point 2 (below). But this makes no difference to the arithmetic, since p and v become the same for all streamlines if we follow them back sufficiently far in front of the ball or wing. It's important to note that v must be measured in the reference frame in which the streamlines are at rest. (Remember that the pipeline was at rest in the derivation of Bernoulli's equation.) Had we measured v in the rest frame of the ambient air, the ball would seem to be pulled down! Finally, note these are examples - don't try to remember the resultant equations!

TEXT: Francis Weston Sears and Mark W. Zemansky, University Physics (Addison-Wesley, Reading, Mass., 1970), fourth edition

SUGGESTED STUDY PROCEDURE

The reading includes all of Chapter 12 except Section 12-7 and all of Sections 14-1 to 14-4 of Chapter 14, except for the rocket example on p. 201 in Section 14-4. Study the Problems with Solutions carefully, and solve the Assigned Problems before attempting the Practice Test. Begin by reading the material relative to Objective 1, that is, General Comment 1 and Sections 12-1 through 12-5.

In Sections 12-1 to 12-6, the concepts of pressure and density, Pascal's principle, and Archimedes' principle are explained. Before these two principles are introduced, there is a treatment of the variation of pressure in a fluid at rest that provides a theoretical basis for them. Equation (12-4) is a special case of the pressure variation in a moving fluid you will encounter in Section 14-3 (Bernoulli's principle). Pay particular attention to the treatment of fluid particles as in Figures 12-1, 14-3, and 14-4. In Section 14-4, 5 and 6, what Bernoulli's equation really does for us is to relate the pressure to the speed along each of two streamlines, one of which goes through point 1 (above the ball or wing), while the other goes through point 2 (below). But this makes no difference to the arithmetic, since p and v become the same for all streamlines if we follow them back sufficiently far in front of the ball or wing. It's important to note that v must be measured in the reference frame in which the streamlines are at rest. (Remember that the pipeline was at rest in the derivation of Bernoulli's equation.) Had we measured v in the rest frame of the ambient air, the ball would seem to be pulled down! Finally, note that these are examples - don't try to remember the resultant equations!

SEARS AND ZEMANSKY

Objective Number	Readings	Problems with Solutions: Study Guide	Problems with Solutions: Text	Assigned Problems: Study Guide	Assigned Problems: Text	Additional Problems
1	Secs. 12-1 to 12-5, General Comment 1	A, B	Ex.* (Sec. 12-4)	G, H	12-5	12-1 to 12-4, 12-6
2	Sec. 12-6	C, D	Ex. (Sec. 12-6)	I, J	12-13	12-11, 12-19, 12-21, 12-33
3	Sec. 14-1, General Comment 2					
4	Secs. 14-1 to 14-4, General Comment 3	E, F	Sec. 14-4, 1 to 6	K, L	14-19	14-9, 14-11, 14-17, 14-21

*Ex. = Example(s).

TEXT: Richard T. Weidner and Robert L. Sells, Elementary Classical Physics (Allyn and Bacon, Boston, 1973), second edition, Vol. 1

SUGGESTED STUDY PROCEDURE

The reading includes all of Chapter 15 except Sections 15-1 and 15-2. Study the Problems with Solutions carefully, and solve the Assigned Problems before attempting the Practice Test. Note that the text covers the material in reverse order from this study guide. You may want to skim through the sections of the text and then decide whether to follow the table in order as suggested below or start with Objectives 3 and 4 and do Objectives 1 and 2 last. Either way is good.

Pay particular attention to the treatment of fluid particles as in Figures 15-5, 15-7, and 15-8. There will not be any problems like Example 15-4 on the Mastery Tests, where an integral of dF must be done. However, note that at the end of the example the difference in forces acting on the dam is not $(1/2)gLH^2$. What should it be? (Try dimensional analysis.)

WEIDNER AND SELLS

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text	Study Guide	Text	
1	Secs. 15-3, 15-7, General Comment 1	A, B	Ex.* 15-1	G, H	15-14	
2	Sec. 15-8	C, D	Ex. 15-5	I, J	15-17	15-15, 15-9 to 15-21
3	Sec. 15-4, General Comment 2					
4	Secs. 15-5, 15-6, General Comment 3	E, F	Ex. 15-2, 15-3	K. L	15-9	15-10 to 15-12

*Ex. = Example(s).

GENERAL COMMENTS

1. Pascal's Law

A fluid is a substance that does not maintain its shape against external distorting forces, or, more simply, a fluid is something that can flow. In fluid mechanics the basic laws of particle physics apply, but special formulations are needed. Instead of forces we often speak of pressure (force per unit area, in newtons per meter squared or pascals), instead of mass we often use density (mass per unit volume, in kilograms per meter cubed). Keep this in mind when reading your text and doing the problems.

With pressure we sometimes use gauge pressure, i.e., pressure above atmospheric pressure. For example, if the gauge pressure in your car tires is 25.0 lb, this means that the pressure is 25.0 $lb/in.^2$ above the atmospheric pressure of 14.7 $lb/in.^2$. The absolute pressure is 39.7 $lb/in.^2$ in your tires, but the pressure difference between the inside and outside of the tire is 25.0 $lb/in.^2$. Density is often stated relative to water, thus it is called specific gravity. For instance, the specific gravity of mercury is 13.6. This means that the density of mercury is 13.6 times that of water, or 13.6 g/cm^3.

Pascal's law may be stated as follows: "A pressure change in one portion of an incompressible liquid in a closed container is transmitted to all parts of the liquid." The liquid must remain at rest for this to be true. For a definition of incompressible, see General Comment 2. The hydraulic brake system of a car is a good example of Pascal's principle. The small force of the brake pedal on the small piston of the master cylinder produces a pressure in the brake fluid, which produces a large force on the correspondingly large piston at the wheel.

2. Characteristics of Fluid Flow

As is customary in physics, fluid motion will be idealized in order to obtain workable problems. You will recall that "frictionless" surfaces and bearings were much used in your studies of particle motion; for fluid motion, the corresponding adjectives are "steady, irrotational, incompressible, and nonviscous" - truly a frightening list! Yet the resulting idealized fluid motion is close enough to reality to have practical applications. The following summary defines these terms and gives an example of them.

Steady versus nonsteady flow: Fluid motion is steady if the velocity $\vec{v}$ of all particles passing any given point does not change with time. An example of steady flow is a gently flowing stream or flow of fluid in a pipe. Nonsteady motion is more common: the chaotic, turbulent motion of rapids, or the pounding waves at the seashore.

Incompressible versus compressible flow: Even though gases are highly compressible, sometimes the changes in density are unimportant. For example, the air around airplane wings at low speeds is nearly incompressible, and the mathematical analysis is much simpler. Liquids can usually be considered as incompressible, i.e., they do not change density.

Irrotational versus rotational flow: If a fluid element at a point has a net angular velocity about that point, the fluid flow is rotational. It is probably best illustrated by a small paddle wheel. In Figure 1 the paddle wheel does not turn, thus the motion is irrotational (as in a straight pipe). In Figure 2 the paddle does spin, thus the motion is rotational (as in a whirlpool).

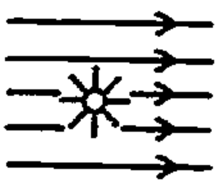


Figure 1

Figure 2

Nonviscous versus viscous flow: Viscosity is the internal friction of a fluid between individual layers of the fluid and between a layer of the fluid and the constraining tube through which it flows. Viscosity introduces retarding forces between layers of fluid in relative motion and dissipates mechanical energy. Air has very little viscosity, whereas thick syrup is very viscous. Nonviscous fluids have no velocity discontinuities in the fluid, but a complete discontinuity between the liquid and the wall of the pipe.

Streamlines: In steady flow a streamline is the path that a particle traces out as time passes. At each point the streamline (or path) is tangent to the velocity $\vec{v}$ of the particle. Thus the streamlines indicate the direction of the fluid's velocity at each point. A high velocity may be indicated by the streamlines being close together. Thus the direction of streamlines and the number of them per unit area give the direction and speed of the fluid's flow.

3. Hydrodynamics

All applications of Bernoulli's equation and the equation of continuity in this module concern nonviscous flow, in which mechanical energy of the fluid is conserved. Notice that the fluid pressure, which has the dimensions of force per area or energy per volume, plays the role of a potential energy per unit volume in Bernoulli's equation. That's a helpful idea when solving problems.

Note how the derivations of pressure dependence on depth in a fluid, the equation of continuity, and Bernoulli's equation employ either the technique of considering a small bit of fluid as though it were a particle or otherwise artificially isolating a region of fluid for purposes of the analysis. This is also a useful technique for solving problems, and I recommend that you keep it in mind to guide your work and to help you check your results in terms of your knowledge of particle mechanics.

PROBLEM SET WITH SOLUTIONS

A(1). Two hemispheres are put together as shown in Figure 3 and then evacuated to 0.100 atmospheric pressure. (1 atm = 1.013×10^5 N/m^2 or Pa.*)

(a) What is the minimum radius of the hemispheres if a force of 1.00×10^5 N cannot pull them apart?

(b) If the hemispheres were moved to the Moon (no atmosphere) what force would be required to hold them together?

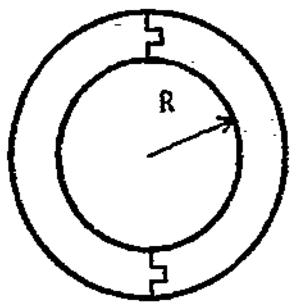


Figure 3

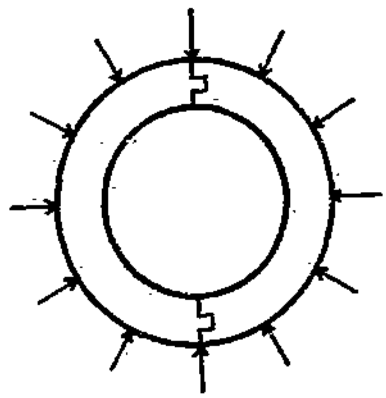


Figure 4

Solution

(a) See Figure 4. The pressure difference between the inside and the outside is (1.00 - 0.100) atm = 0.90 atm = 9.1×10^4 Pa. The atmospheric force on each element of area is directed radially inward. Considering one hemisphere only, we see that the vertical components cancel and the horizontal components add together to yield a total force equal to the pressure difference (Δp) times the area (A) of a circle of radius R. Therefore if we pull on the hemispheres with a total force of $F = \Delta pA$ they will just come apart:

$$F = \Delta pA = \Delta p\pi R^2,$$

$$R = (F/\pi\Delta p)^{1/2} = [10^5/\pi(9.1 \times 10^4)]^{1/2} = 0.59 \text{ m}.$$

(b) In empty space, the smallest external force, now directed inward, that will hold the hemispheres together is $F = \Delta pA = (P_{At}/10)A = 1.100 \times 10^4$ N.

B(1). A simple, uniform U-tube, open on both sides, contains mercury as in Figure 5. If water is poured into the left side until the column of water is 28.0 cm high, how high on the right does the mercury rise above its initial level? The density of mercury is 1.40×10^4 kg/m^3. Water and mercury do not mix (ρ for water equals 10^3 kg/m^3).

*The pascal (Pa) is the SI unit for pressure used in this module. 1 Pa = 1 N/m^2.

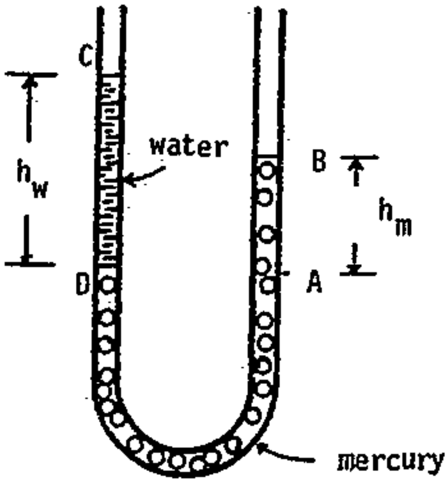


Figure 5

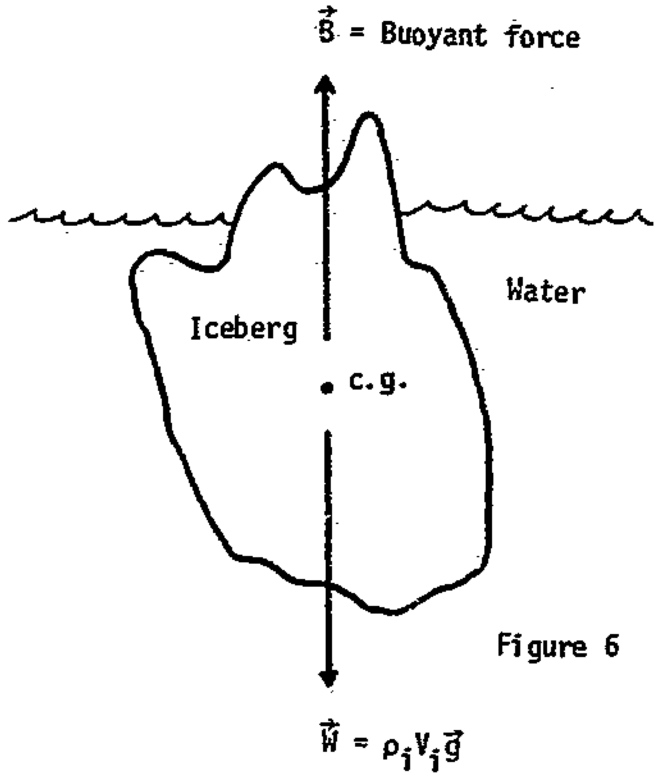


Figure 6

Solution

Consider first the mercury below D and A in the tube. If the pressure at D and A were not equal, the water would flow. Since it does not, we conclude that the pressure at D equals the pressure at A (Pascal's principle). The pressure at a height h below the surface of a liquid is ρgh, thus $P_D = \rho_w g h_w$ and $P_A = \rho_m g h_m$. Equating these, we find

$$\rho_w h_w = \rho_m h_m, \qquad h_m = (\rho_w/\rho_m)h_w = [10^3/(1.40 \times 10^4)]28.0 \text{ cm} = 2.00 \text{ cm}.$$

Before the water was added, D and B were at the same height. The left side fell by the same amount the right side rose up, thus the right side has risen 2.00 cm/2 = 1.00 cm above its original position.

C(2). The density of ice is 92% that of fresh water. What fractional volume of an iceberg on a lake floats on top of the water?

Solution

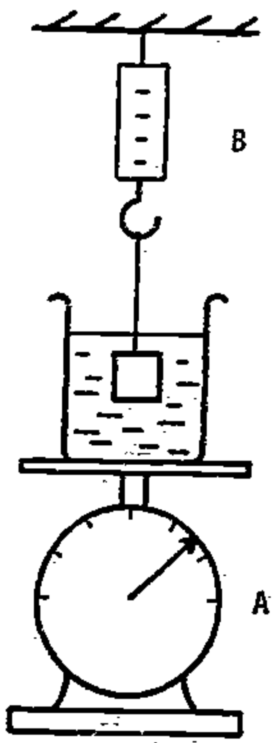
The weight of the ice is $\vec{W} = \rho_i V_i \vec{g}$, where ρ_i is the density and V_i is the volume of the ice. The weight of the volume V_w of water displaced is the buoyant force, $\vec{B} = \rho_w V_w \vec{g}$. Since the iceberg is in equilibrium,

$$|\vec{B}| = |\vec{W}|, \qquad \rho_i V_i g = \rho_w V_w g, \quad \text{and} \quad V_w/V_i = \rho_i/\rho_w = 0.92\rho_w/\rho_w = 92\%.$$

The volume of the submerged portion of iceberg is V_w = 92% of the total volume. So only 8.0% of the iceberg is exposed.

D(2). A beaker partly filled with water has a mass of 300 g (see Fig. 7). A piece of metal with density 3.00 g/cm^3 and volume 100 cm^3 is suspended by a spring scale B, so that the metal piece is submerged in water but does not rest on the bottom. What scale readings on A and B will be observed with the metal piece suspended in the water?

Figure 7



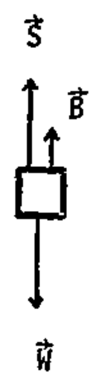


Figure 8

Solution

A free-body diagram of the metal piece is shown in Figure 8. $\vec{S}$ is the tension from the spring scale; $\vec{B}$ is the buoyant force from the displaced water; and $\vec{W}$ is the weight from gravity. Since the metal is in equilibrium,

$$\vec{W} = \vec{B} + \vec{S}.$$

By Newton's third law the buoyant force of the water on the metal is equal and opposite in direction to the force exerted on the water by the metal. This force on the water is the extra amount that the beaker and water weigh owing to the metal. The buoyant force is equal to the weight of the water displaced.

Since the volume of the piece of metal is 100 cm^3, the weight of the water displaced is (0.100 kg)(9.8 m/s^2) = 0.98 N. The weight of the metal piece is

$$mg = \rho V g = (3000\ \text{kg/m}^3)(10^{-4}\ \text{m}^3)9.8 = 2.94\ \text{N} = \vec{W},$$

$$\vec{S} = \vec{W} - \vec{B} = 2.94\ \text{N} - 0.98\ \text{N} = 1.96\ \text{N},$$

and scale B reads 200 g. The beaker now weighs its original weight plus an amount equal to the buoyant force or $(0.300\ kg)(9.8\ m/s^2) + 0.98\ N = 3.94\ N$, and scale A reads 400 g.

E(3,4). A water pipe having a 2.00-cm inside diameter carries water into the basement of a house at a velocity of 1.00 m/s at a pressure of 2.00×10^5 Pa. If the pipe tapers to 1.00 cm inside diameter and rises to the second floor 10.0 m above the input point, what are the (a) velocity and (b) water pressure there?

Solution

(a) By the equation of continuity $A_1v_1 = A_2v_2$,

$$v_2 = A_1v_1/A_2 = \pi R_1^2v_1/\pi R_2^2, \quad (1)$$

$$R_1 = 2R_2, \text{ thus } v_2 = (2R_2)^2v_1/R_2^2 = 4v_1 = 4(1.00\ m/s) = 4.0\ m/s.$$

(b) Bernoulli's equation is

$$p_1 + \rho gh_1 + (1/2)\rho v_1^2 = p_2 + \rho gh_2 + (1/2)\rho v_2^2. \quad (2)$$

Since $h_2 - h_1 = 10.0$ m, $p_1 = 2.00 \times 10^5$ Pa, $\rho = 1000\ kg/m^3$, $v_1 = 1.00$ m/s, and $v_2 = 4.0$ m/s, we can solve for p_2:

$$p_2 = p_1 - \rho g(h_2 - h_1) - (1/2)\rho(v_2^2 - v_1^2)$$

$$= 2.00 \times 10^5 - 1000(9.8)10.0 - (1/2)(1000)[(4.0)^2 - (1.00)^2]$$

$$= (2.00 \times 10^5) - (0.98 \times 10^5) - (0.075 \times 10^5) = 9.5 \times 10^4\ Pa.$$

Note the use of the two relationships (1) and (2) in combination.

F(3,4). Each wing of an airplane has an area of $10.0\ m^2$. At a certain air speed in level flight air flows over the upper wing surface at 50 m/s and over the lower wing surface at 40 m/s. The density of air is $1.29\ kg/m^3$. Assume that lift effects associated with the fuselage and tail are negligible. What is the weight of the plane?

Solution

The region above the wing is one of higher velocity and lower pressure than that below the wing. It is this pressure difference that gives rise to the lift on the wing. On streamlines connecting the upper and lower wing surfaces and a point far from the wing, Bernoulli's equation tells us that

$$p_u + \rho gh_u + (1/2)\rho v_u^2 = p_L + gh_L + (1/2)v_L^2.$$

The difference in height will contribute a very small amount to the pressure difference. (Try it, assuming a difference of, say, 1.00 m.)

$$p_L - p_u = (1/2)\rho(v_u^2 - v_L^2).$$

The total force for each wing is $F = \Delta pA$. The weight of the plane in level flight is then equal to the total force upon both wings:

$2\Delta pA = 2(1/2)\rho(v_u^2 - v_2^2)A$, weight $= (1.29)(50^2 - 40^2)10.0 = 1.16 \times 10^4$ N

(1180 kg mass).

Problems

G(1). In a car brake system the small piston at the brake pedal (or master cylinder) has a cross-sectional area of 2.00 cm^2, and the wheel cyclinder has a cross-sectional area of 40 cm^2. What is the force applied at the wheel cylinder if a force of 400 N is applied at the brake pedal?

H(1). A tube 0.0100 m^2 in cross-sectional area is attached to the top of a container 0.100 m high and of cross-sectional area 1.00 m^2 as in Figure 9. Water is poured into the system filling it to a depth of 1.00 m above the bottom of the vessel.
(a) What is the force exerted against the bottom of the vessel by the water (excluding atmospheric pressure)?
(b) What is the weight of the water in the system?
(c) Explain why (a) and (b) are not equal.

I(2). An iron casting weighs 1000 N in air and 600 N in water. What is the volume of the cavities in the casting? The density of iron is 7.8 times that of water. (Hint: find the volume of the iron in the casting and the volume of the displaced water.)

J(2). What is the minimum mass of wood (density = 0.80 times the density of water) necessary to support a 70-kg man standing on a block of wood floating on water?

Figure 9

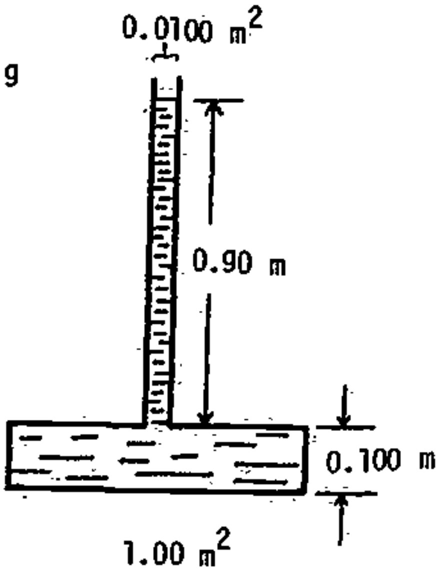
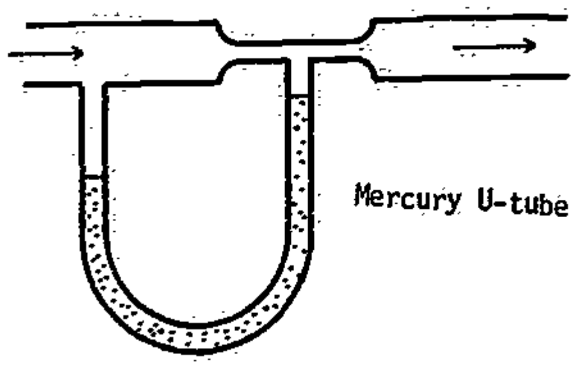


Figure 10



K(3,4). A tank is filled with water to a depth H. A hole is made in one of the vertical walls at a depth h below the water surface.
(a) Apply Bernoulli's equation to a streamline connecting the water surface and the hole to find the horizontal speed of water coming out of the hole. (The area of the hole is much less than the cross-sectional area of the tank.)
(b) If the hole has a bent pipe attached to it so that the water shoots directly upward, how high will the water rise?
(c) If the water is flowing horizontally out of the hole, how far from the foot of the tank will the stream strike the floor? (The bottom of the tank is at the same level as the floor.)

L(3,4). The section of pipe shown in Figure 10 has a cross-sectional area of $30.0\ \mathrm{cm}^2$ at the wider portion and $10.0\ \mathrm{cm}^2$ at the constriction. $3.00\ \ell$ of water is discharged from the pipe in 1.00 s.
(a) Find the velocities at the wide and the narrow portions.
(b) Find the pressure difference between these portions.
(c) Find the difference in height between the mercury columns in the U-tube (density of mercury = $1.40 \times 10^4\ \mathrm{kg/m^3}$).

Solutions

G(1). $|\vec{F}| = 8.0 \times 10^3$ N.

H(1). (a) $F = 9.8 \times 10^3$ N. (b) $W = 1.07 \times 10^3$ N. (c) There is also an upward force on the top of the 1.00-m^2 container.

I(2). Volume of cavities = $2.8 \times 10^{-2}\ \mathrm{m}^3$.

J(2). $m_{\mathrm{wood}} = 280$ kg.

K(3,4). (a) $v = \sqrt{2gh}$. (b) To the top of the water surface in the tank if viscous energy losses are neglected. (c) Horizontal distance = $2\sqrt{h(H - h)}$.

L(3,4). (a) 3.00 m/s; 1.00 m/s. (b) 4.0×10^3 Pa. (c) 2.90 cm.

PRACTICE TEST

1. The large piston of a hydraulic automobile lift is 40 cm in diameter. (a) What pressure is required to lift a car weighing 1000 kg? (b) If the fluid is forced into the main chamber by a small piston with an area of 5.0 cm^2, what force must be applied to this small piston to lift the 1000-kg car?

2. A 1.00-kg block of iron is floating on mercury as in Figure 11. How much lead would have to be placed on top of the iron in order that the iron might float barely below the surface as pictured? (The density of mercury is 13.6 g/cm^3; the density of iron is 7.9 g/cm^3; and the density of lead is 11.3 g/cm^3.)

3. Briefly describe what is meant by steady versus nonsteady flow; illustrate each with an example.

4. The water level in a tank of large cross-sectional area on the top of a building is 40 m above the ground. The tank is open to the atmosphere, and supplies water to the various apartments through pipes of 10.0 cm^2 cross-sectional area. Each faucet through which the water emerges has an orifice of 2.00 cm^2 effective area. (a) What is the gauge pressure in a water pipe at ground level when the faucet is closed? Recall that gauges read the pressure in excess of atmospheric pressure. (b) How long will it take to fill a 20.0-ℓ pail in an apartment 30.0 m above the ground? (c) With what speed is the water flowing in the pipe (of cross-sectional area 10.0 cm^2) leading to this faucet?

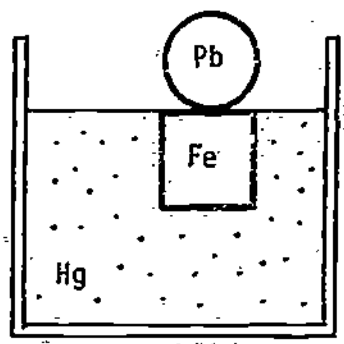


Figure 11

Practice Test Answers

1. (a) 7.8×10^4 Pa.
   (b) 39 N.
2. 720 g (Pb).
3. In steady flow the fluid velocity $\vec{v}$ at every point is constant in time. For examples, steady flow: gentle flowing stream; nonsteady flow: seaside surf.
4. (a) 3.9×10^5 Pa.
   (b) 7.1 s.
   (c) 2.80 m/s.

FLUID MECHANICS Date ______________

Mastery Test Form A

pass recycle

1 2 3 4

Name______________ ________ Tutor______________

1. The expansion tank of a household hot-water-heating system is open to air, and the water level is 10.0 m above a pressure gauge attached to the furnace.
(a) What is the gauge pressure at the furnace? Recall that gauges read the pressure in excess of atmospheric pressure. The density of water is 1000 kg/m^3.
(b) If the gauge is removed and the hole is filled with a plug having an area of 1.00 cm^2, what is the resultant force on the plug?

2. A 300 000-kg iceberg is floating on salty water ($\rho_{salt\ water}$ = 1030 kg/m^3). This iceberg has a relative density (or specific gravity) of 0.92.
(a) What volume of the iceberg is below the surface of the water?
(b) What volume of it is above the water surface?

3. Briefly describe what is meant by viscous versus nonviscous fluid flow and illustrate each with an example.

4. Point A in Figure 1 is 20.0 m above ground level; points B and C are 3.00 m above ground level. The cross-sectional areas at points B and C are 0.200 m^2 and 0.100 m^2, respectively. The surface area at point A is much larger than that at B or C.
(a) Compute the discharge rate in cubic meters per second.
(b) Find the gauge pressure at point B.

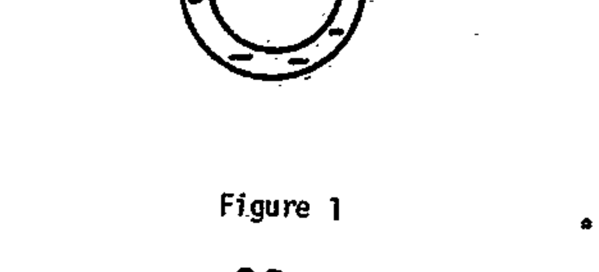


Figure 1

FLUID MECHANICS

Date ____________________

Mastery Test   Form B

pass   recycle

1   2   3   4

Name ________________________   Tutor ________________________

1. In a hydraulic press, the small piston has an area of 2.00 cm^2, and the large piston an area of 72 cm^2. What weight can be lifted if a force of 5.0 N is applied to the small piston?

2. A raft is made from four logs, held together by light, thin ropes. Each log has a diameter of 40 cm and a length of 2.00 m. The raft floats in fresh water (ρ = 1000 kg/m^3) with the logs exactly half submerged. When the River Riders climb on board the raft floats with the logs just barely submerged. If there are six River Riders, what is the average weight of each?

3. Briefly describe what is meant by rotational versus irrotational flow and give an example of each.

4. (a) You are designing an airplane to have a lift of 1.00×10^3 N per square meter of wing area. Make the usual assumptions of smooth flow, negligible viscosity, etc. If the air flows under the wing at 100 m/s, how fast must it flow over the top surface? (At the specified altitude, ρ_{air} = 1.25 kg/m^3.)

   (b) A fluid of density 0.60 g/cm^3 flows through a horizontal pipe whose area at the inlet is 25.0 cm^2, inlet speed is 2.00 cm/s, and outlet speed is 5.0 cm/s. Find the area of the outlet.

FLUID MECHANICS Date ________________

Mastery Test Form C

pass recycle

1 2 3 4

Name ______________________ Tutor ________________

1. A simple U-tube manometer contains water as in Figure 1. If 2.50 cm of oil having a relative density (specific gravity) of 0.80 is poured into the tube on the right, how high does the water in the left arm rise above its initial level?

2. A swimmer (volume $\equiv V_s = 0.070\ m^3$) is floating with just her nose and chin (volume $\equiv V_{nc} = 140\ cm^3$) out of the water. The water in the swimming pool has a density $\rho_w = 1000\ kg/m^3$. What is her specific gravity? Express your answer first in terms of the symbols V_s, V_c, and/or ρ_w and then numerically.

3. Briefly describe what is meant by compressible versus incompressible flow, and illustrate each with an example.

4. Oil of specific gravity of 0.80 flows through a pipe as shown in Figure 2, emerging from the lower end of the pipe into the atmosphere.

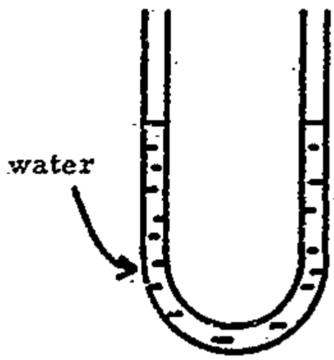
   $A_1 = 8.0\ cm^2$, $A_2 = 2.00\ cm^2$,

   $V_1 = 6.0\ cm/s$, $\Delta h = 50\ cm$,

   $p_2 = 1.01 \times 10^5\ Pa$.

   (a) Find the speed at which the oil emerges.
   (b) Find the absolute pressure at point 1 in the pipe.

Figure 1



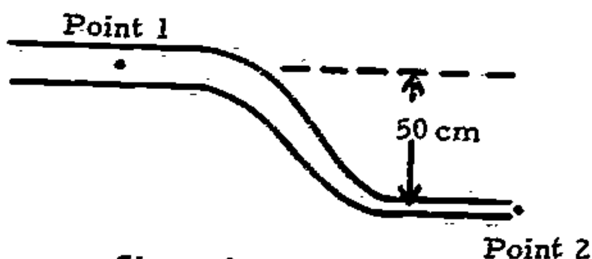


Figure 2

MASTERY TEST GRADING KEY - Form A

1. What To Look For:

(a) What is Pascal's principle? What does it apply to here? (p_0 = atmospheric pressure is transmitted from top to furnace.) (b) Why should we use gauge pressure?

Solution:

(a) $p = p_0 + gh$, gauge pressure $= p - p_0 = \rho gh = (1000)(9.8)(10.0)$

$= 9.8 \times 10^4$ Pa.

(b) $p = F/A$, $F = pA = (9.8 \times 10^4 \text{ Pa})(10^{-4} \text{ m}^2) = 9.8$ N.

2. What To Look For: (a) What is ρ_{ice}? $\rho_{ice} = 0.92\rho_{water} = 0.92(1000) = 920 \text{ kg/m}^3$.

Solution: (a) Weight of iceberg $= W_i = \rho_i V_i g$,

Buoyant force $= \vec{B} = \rho_w V_w \vec{g}$, where V_w = volume of water displaced = volume of ice below surface.

$B = W$, $\rho_i V_i g = \rho_w V_w g$, $\frac{V_w}{V_i} = \frac{\rho_i}{\rho_w} = \frac{920 \text{ kg/m}^3}{1030 \text{ kg/m}^3} = 0.89$.

Thus 89% of iceberg is below water.

$\rho_{ice} = \frac{m}{V}$; Volume $= \frac{m}{\rho_{ice}} = \frac{300\,000 \text{ kg}}{920 \text{ kg/m}^3}$, $V_i = 326 \text{ m}^3$,

$V_{below\ surface} = 0.89 \times 326 = 290 \text{ m}^3$.

(b) $V_{above\ surface} = 0.110 \times 326 = 36 \text{ m}^3$.

3. Solution: Viscous flow: friction between layers of the fluid, e.g., thick syrup. Nonviscous flow: no friction between layers, e.g., air.

4. What To Look For: Does $V_A = 0$ sound reasonable?

Solution: (a) Apply Bernoulli's theorem for points A and C along a streamline:

$$p_A + \rho g h_A + (1/2)\rho v_A^2 = p_c + \rho g h_c + (1/2)\rho v_C^2.$$

Now $p_A = p_C$ = atmospheric pressure, $h_A - h_C = \Delta h = 17.0$ m. By the equation of continuity $A_A v_A = A_C v_C$:

$$A_A >> A_C, \quad v_A = (A_C/A_A)v_C \approx 0, \quad \rho g(h_A - h_C) = (1/2)\rho v_C^2,$$

$v_C = \sqrt{2g\Delta h} = 2(9.8)(17.0) = 18.3$ m/s.

Discharge rate = $Av = (0.100\ m^2)(18.3\ m/s) = 1.83\ m^3/s$.

What To Look For: (b) Could also use points B and A if they find v_B first. Did they use the equation of continuity to find v_B?

Solution: (b) For a streamline between B and C, Bernoulli's equation states

$p_B + \rho g h_B + (1/2)\rho v_B^2 = p_C + \rho g h_C + (1/2)\rho v_C^2, \quad h_B = h_C.$

$A_B v_B = A_C v_C$, so $v_B = (A_C/A_B)v_C = 9.1$ m/s.

$p_B = p_C + (1/2)\rho(v_C^2 - v_B^2) = p_C +$ gauge pressure,

gauge pressure = $(1/2)(1000)(18.3^2 - 9.12^2) = 1.26 \times 10^5$ Pa.

MASTERY TEST GRADING KEY - Form B

1. What To Look For: What is Pascal's principle?

   Solution: Pascal's principle says that the pressure will be the same everywhere, thus on the small piston

   $p = F/A_s = (5.00\ N)/(2.00 \times 10^{-4}\ m^2) = 2.50 \times 10^4\ N.$

   On the large piston

   $F = pA_r = (2.5 \times 10^4)(72 \times 10^{-4}) = 180\ N.$

   Thus a weight of 180 N (mass = 18.4 kg) can be lifted.

2. What To Look For: What is Archimedes' principle? Did they multiply mass of water by g to obtain weight?

   Solution: The volume of the logs is $4(\pi r^2)\ell$. With the riders on,

   weight of logs + riders = buoyant force = weight of displaced water = $\rho_w 4(\pi r^2)\ell g$,

   weight of logs = weight of displaced water with no riders = $(1/2)\rho_w(4\pi r^2)\ell g$.

   Thus the weight of one rider is one-sixth the weight of all riders.

   $$W_{rider} = \frac{1}{6}[\rho_w 4\pi r^2\ell - \frac{1}{2}\rho_w 4\pi r^2\ell]g = \frac{\rho_w \pi r^2 \ell g}{3} = \frac{(1000)\pi(0.200)^2 2(9.8)}{3}$$
   $$= 820\ N \quad (\text{mass} = 84\ kg).$$

3. Solution: Irrotational flow: a fluid element at a point has zero net angular velocity about that point (e.g., flow in a straight pipe). Rotational flow: a fluid element has a net angular velocity about a point (e.g., whirlpools).

4. What To Look For: (a) Why is $h_1 \approx h_2$? ($\rho g \Delta h$ is very small if $\Delta h \approx 1.00$ m.)

   Solution: (a) Apply Bernoulli's equation:

   $p_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2 = p_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2$, $\quad p_1 - p_2 = 1.00 \times 10^3$ Pa

   (1 = below, 2 = above).

   $h_1 \approx h_2$, therefore $\quad (1/2)\rho v_2^2 = p_1 - p_2 + (1/2)\rho v_1^2$,

   $v_2 = [2(p_1 - p_2)/\rho + v_1^2]^{1/2} = [(2.00 \times 10^3)/1.25 + 100^2]^{1/2} = 108$ m/s.

   (b) Equation of continuity:

   $A_0 v_0 = A_i v_i$, $\quad A_0 = A_i\left(\frac{v_i}{v_0}\right) = 25.0\ cm^2\left[\frac{2.00\ cm/s}{5.6\ cm/s}\right] = 10.0\ cm^2.$

MASTERY TEST GRADING KEY - Form C

What To Look For: What is Pascal's principle?

Solution: See Figure 15. Points A and B are at the same pressure (Pascal's principle).

$p_A = \rho_{oil} g h_{oil}$, $p_B = \rho_w g h_w$, h_w = height of water above B, $p_A = p_B$.

Therefore,

$\rho_o g h_o = \rho_w g h_w$, $h_w = (\rho_o/\rho_w)h_o$, $\rho_o/\rho_w = 0.80$.

Thus, $h_w = 0.80(2.50) = 2.00$ cm. If the difference in height of the water is 2.00 cm, one side moved down by 1.00 cm, and the other side moved up by 1.00 cm.

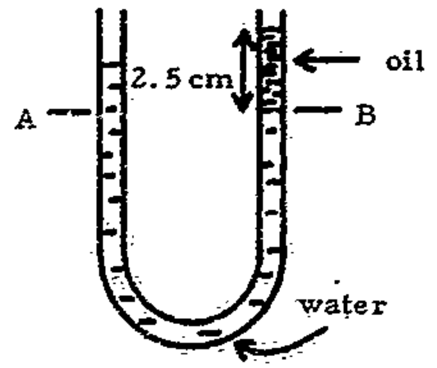


Figure 15

2. What To Look For: What is Archimedes' principle? (buoyant force = weight of displaced water.) Can you draw a free-body diagram of swimmer?

   Solution: Volume of swimmer is $V_s = m_s/\rho_s$.

   Weight of swimmer = $m_s g = \rho_s V_s g$, weight of displaced water = $\rho_w(V_s - V_{nc})g$.

   Weight of swimmer = weight of displaced water, $\rho_s V_s g = \rho_w(V_s - V_c)g$,

   ρ_s/ρ_w = specific gravity = $(V_s - V_c)/V_s = 1 - V_{nc}/V_s = 1 - (1.40 \times 10^{-4})/0.070 = 0.998$.

3. Solution: Incompressible: the fluid cannot change density under ordinary pressures of a few atmospheres (e.g., water). Compressible: the fluid easily changes density under pressure of a few atmospheres (e.g., a gas like air).

4. What To Look For: (a) What is the discharge rate at point 2? (A_2v_2.) (b) Can you show that the units in every term are N/m^2 (Pa)? What is the gauge pressure at point 1? (-3.9×10^3 Pa.)

Solution: (a) From the equation of continuity:

$A_1v_1 = A_2v_2$, $v_2 = (A_1/A_2)v_2 = (8/2)(6)$ cm/s = 24.0 cm/s.

(b) Apply Bernoulli's equation to 1 and 2:

$$p_1 + \rho g h_1 + (1/2)\rho v_1^2 = p_2 + \rho g h_2 + (1/2)\rho v_2^2,$$

$$p_1 = p_2 + \rho g(h_2 - h_1) + (1/2)\rho v_2^2 - v_1^2),$$

$h_2 - h_1 = -0.50$ m, $\rho = 0.80\rho_w \doteq 800$ kg/m^3,

$V_1 = 0.060$ m/s, $V_2 = 0.240$ m/s,

$$p_1 = 1.01 \times 10^5 + 800(9.8)(-0.50) + (1/2)(800)[(0.240) - (0.060)] = 9.7 \times 10^4 \text{ Pa}.$$