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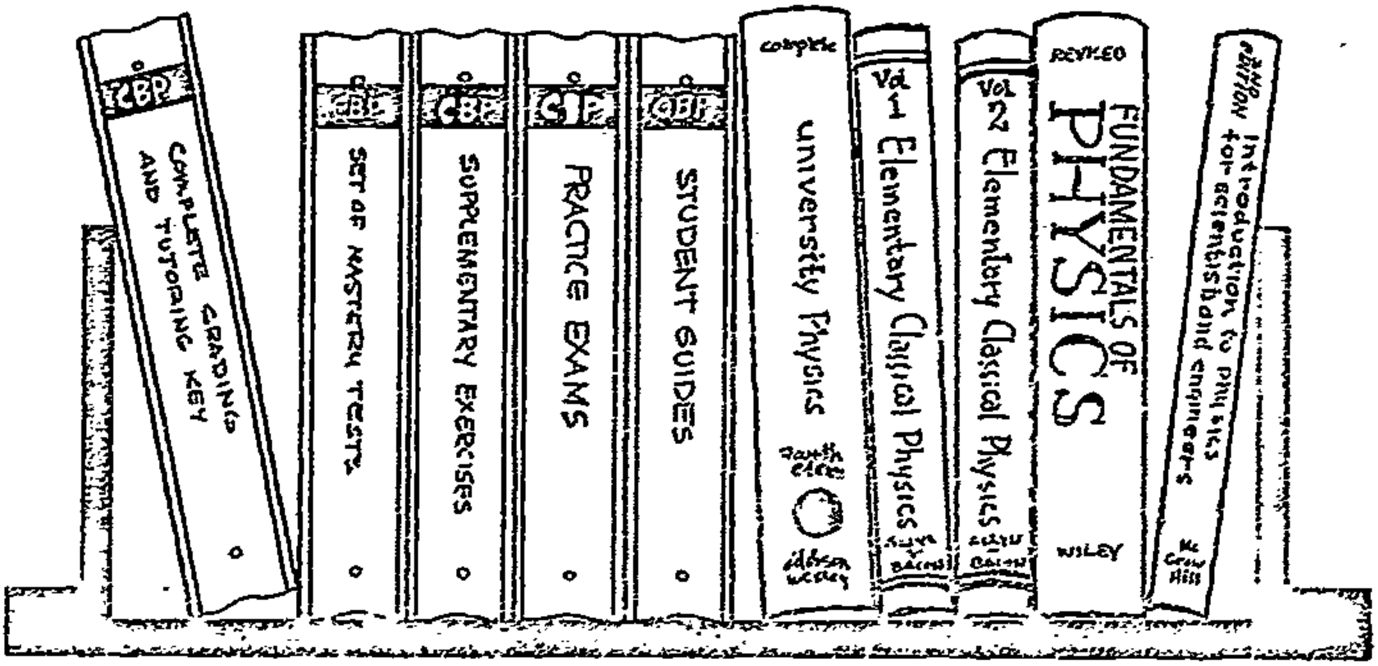
ABSTRACT

This is part of a series of 42 Calculus Based Physics (CBP) modules totaling about 1,000 pages. The modules include study guides, practice tests, and mastery tests for a full-year individualized course in calculus-based physics based on the Personalized System of Instruction (PSI). The units are not intended to be used without outside materials; references to specific sections in four elementary physics textbooks appear in the modules. Specific modules included in this document are: Module 6--Work and Energy and Module 7--Applications of Newton's Laws. (CP)

STUDY MODULES FOR CALCULUS-BASED GENERAL PHYSICS*



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Comments

These modules were prepared by fifteen college physics professors for use in self-paced, mastery-oriented, student-tutored, calculus-based general physics courses. This style of teaching offers students a personalized system of instruction (PSI), in which they increase their knowledge of physics and experience a positive learning environment. We hope our efforts in preparing these modules will enable you to try and enjoy teaching physics using PSI.

Robert G. Fuller
Director
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These modules were prepared by the module authors at a College Faculty Workshop held at the University of Colorado - Boulder, from June 23 to July 11, 1975.

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COMMENT TO USERS

In the upper right-hand corner of each Mastery Test you will find the "pass" and "recycle" terms and a row of numbers "1 2 3 ..." to facilitate the grading of the tests. We intend that you indicate the weakness of a student who is asked to recycle on the test by putting a circle around the number of the learning objective that the student did not satisfy. This procedure will enable you easily to identify the learning objectives that are causing your students difficulty.

ERRATA

In Work and Energy, p. 2, Objective 3 should read

3. Work-energy theorem - Relate the work done on a particle to the change in its kinetic energy, and solve problems of particle motion in one dimension using this relationship.

Work and Energy: p. 7, last line, "$d = K_f/F$..." P. 9, G(4)(d) "...10^3t^2 J."

On p. 10, the solution to Problem H, part (b) should be

$$v(t) = \sqrt{225 + (2)(7.2 \times 10^4)t/(1.5 \times 10^3)} \ldots .$$

In the Grading Key to Mastery Test C, in the second Eq. (2) of Solution 6:

$$F = \left(\frac{1}{2}\right) \frac{3.00M_T}{(100 + 3.00t)^{1/2}} \ldots .$$

We shall correct these and any other errors brought to our attention when the CBP Modules are reprinted. We would be happy to receive your suggestions or any corrections that you discover necessary in using the modules.

COMMENT TO USERS

It is conventional practice to provide several review modules per semester or quarter, as confidence builders, learning opportunities, and to consolidate what has been learned. You the instructor should write these modules yourself, in terms of the particular weaknesses and needs of your students. Thus, we have not supplied review modules as such with the CBP Modules. However, fifteen sample review tests were written during the Workshop and are available for your use as guides. Please send $1.00 to CBP Modules, Behlen Lab of Physics, University of Nebraska - Lincoln, Nebraska 68588.

FINIS

This printing has completed the initial CBP project. We hope that you are finding the materials helpful in your teaching. Revision of the modules is being planned for the Summer of 1976. We therefore solicit your comments, suggestions, and/or corrections for the revised edition. Please write or call

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WORK AND ENERGY

INTRODUCTION

Energy is much in the news lately. The term "energy" usually refers to the inherent ability of a material system, such as a person, a flashlight battery, or rocket fuel, to bring about changes in its environment or in itself. Some common sources of energy are the fuel used to heat hot water, the gasoline that propels a car, the dammed water that drives the turbine in a hydroelectric plant, and the spinning yo-yo that can climb up its own string. Inanimate energy sources are of central importance in raising the standard of living of mankind above the subsistence level.

The physicist distinguishes among several types of energy, including kinetic energy (associated with a flying arrow or other moving object), elastic energy (associated with stretched or compressed strings), chemical energy (associated with fuel-oxygen systems or a storage battery), thermal energy (associated with the sun and other objects that are hotter than their surroundings), and nuclear energy. Applications of the energy concept in the science of mechanics, which you are studying now, usually concentrate on kinetic energy, potential energy (to be introduced in the module Conservation of Energy), and work (the transfer of energy by the action of a force). Sometimes the phrase "mechanical energy" is used to refer to the forms of energy of importance in mechanics.

When you get on your bicycle, you have undoubtedly noticed that it takes a good deal of effort to get yourself moving rapidly. If you exert yourself very strenuously, you can reach a given speed after a short distance; or you can take it easy and pedal over a longer distance to reach the same speed. In some sense it always takes the same amount of "work" to reach a given speed - either a large exertion for a short distance or a small exertion for a long distance. You may also have noticed that if you are carrying a passenger on your bike, then it takes more "work" to reach the same speed.

It turns out that these intuitive relationships among the "work" done on a system, its mass, and changes in its speed can be sharpened into a precise statement, called the work-energy theorem. (One caution, though: the technical definition of work needed for this precise statement is different from its everyday usage and physiologic meanings; e.g., you do no work on a heavy box by merely holding it still.) As you will begin to see in the present module, this relationship between work and mechanical energy gives you a new and powerful tool for the solution of many problems, a tool that is often easier to use than a direct application of Newton's second law.

PREREQUISITES

Before you begin this module, you should be able to:	Location of Prerequisite Content
*Calculate the dot (scalar) product of two vectors, given rectangular or polar descriptions (needed for Objectives 1, 3, 4 of this module)	Vector Multiplication Module
*Distinguish between displacement and position, speed and velocity (needed for Objectives 1, 3, 4 of this module)	Rectilinear Motion Module
*Given position or velocity as functions of time, find the velocity or position (needed for Objective 4 of this module)	Rectilinear Motion Module
*Construct free-body diagrams (needed for Objectives 1, 3, 4 of this module)	Newton's Laws Module
*Apply Newton's second law of motion (needed for Objectives 1, 3, 4 of this module)	Newton's Laws Module
*Calculate the definite integral of polynomial functions (needed for Objectives 1, 3 of this module)	Calculus Review
*Find the derivatives of polynomial and rational functions of one variable (needed for Objectives 1, 3, 4 of this module)	Calculus Review

LEARNING OBJECTIVES

After you have mastered the content of this module you will be able to:

1. Work - Define the work done by a force and the work done on a particle; calculate the work done by a constant or variable force oriented parallel or obliquely to the displacement of the particle.

2. Kinetic energy - Define and calculate the kinetic energy of a particle or system of several particles, given their masses and velocities.

3. Work-energy principle - Relate the work done on a particle to change its kinetic energy, and solve problems of particle motion in one dimension using this relationship.

4. Power - Define power and apply the relationships of power to work, force, velocity, and kinetic energy in connection with the motion of a particle in one dimension.

GENERAL COMMENTS

Your text deals exclusively with the energy concept in the context of Newtonian mechanics. Actually, energy is much broader than that, and your understanding will be helped if you are aware of other scientific uses of the energy concept.* The text's definitions of work and kinetic energy are chosen to be useful in mechanical theory, and therefore may sound very abstract to you.

*For instance, see Introductory Physics: A Model Approach [by Robert Karplus (Benjamin, New York, 1969)], Chapters 4, 9, 11, and 14, especially if you have not had physics in high school.

TEXT: Frederick J. Bueche, Introduction to Physics for Scientists and Engineers (McGraw-Hill, New York, 1975), second edition

SUGGESTED STUDY PROCEDURE

After reading the General Comments, please read each of the comments below along with the text section primarily dealing with the objective as keyed in the table. Your text readings are from Chapter 8. Follow the order of the objectives and pay special attention to the examples in the text and to the additional Problems with Solutions in the study guide. After concluding the reading and examples, work out the assigned problems later in the study guide. Hints for solving these, if needed, are given at the end. Finally, check your learning by taking the Practice Test.

In relation to Objective 1, note that the physicist's technical definition of work is introduced gradually by proceeding from a simple case (constant force) in Section 8.1 to the general case (variable force) in Eq. (8.3). The integral in Eq. (8.3) is called a line integral because the integration extends along the line (path) of the particle's motion. In this course you will only be asked to evaluate the work for particle motion along a straight line. (Skip Section 8.2 for now.) Note that the vector force in Eq. (8.3) may vary in magnitude and in direction; what matters is the component along the path $d\vec{s}$. (Note: we shall use $d\vec{l}$ for line integrals because $d\vec{s}$ is more commonly reserved for areas.)

BUECHE

Objective Number	Readings	Problems with Solutions: Study Guide	Problems with Solutions: Text*	Assigned Problems: Study Guide	Additional Problems (Chapter 8)
1	Secs. 8.1, 8.3	A, B, C	Illus. 8.3, 8.4	I, J	1 to 4
2	Sec. 8.4	D	Illus. 8.5	K	
3	Sec. 8.4	E, F	Illus. 8.5	L, M	5 to 8, 14
4	Sec. 8.2	G, H	Illus. 8.1, 8.2	N, O	Quest. 7 Probs. 9 to 14

*Illus. = Illustration(s). Quest. = Question(s).

For Objective 2 don't be put off by the reference to impulse at the beginning of Section 8.4. The same result follows directly from Newton's law as stated in Eq. (5.1) in Section 5.1. You may be surprised by the integral over the variable v on page 116. The purely mathematical manipulation is an application of the abstract definition of the definite integral to the sum that has to be evaluated. The kinetic energy $K = (\frac{1}{2})mv^2$ is defined in the text after Eq. (8.5). It is a scalar quantity that may be found for a system of several particles by merely adding the kinetic energies of the individual particles.

Objective 3 is treated only briefly on p. 117; you will gain your understanding of the work-energy theorem by working on problems or by looking at another text.

For Objective 4 go back to Section 8.2. Equation (8.2c) shows that the power P is constant if both force and velocity are constant, and this applies in Illustration 8.1. When a car or other vehicle has an engine that delivers constant power and the velocity changes, then so does the force, as in Problem H. If the force is constant and leads to an acceleration of the moving object, then the power must vary according to Eq. (8.2c) as in Problem G.

TEXT: David Halliday and Robert Resnick, Fundamentals of Physics (Wiley, New York, 1970; revised printing, 1974)

SUGGESTED STUDY PROCEDURE

After reading the General Comments, please read each of the comments below along with the text section in Chapter 6 primarily dealing with the objective as keyed in the table. Pay special attention to the examples in the text and to the additional solved problems in the study guide. After concluding the reading and examples, work out the assigned problems later in this study guide. Hints for solving these, if needed, are given at the end. Finally, check your learning by taking the Practice Test.

In relation to Objective 1, note that the definition of work is gradually introduced in Sections 6-2 to 6-4, proceeding from the constant-force case in Eq. (6-2) to the variable-force case in Eq. (6.10). Equation (6-9) apparently has an infinitesimal left-hand side and finite right-hand side.

HALLIDAY AND RESNICK

Objective Number	Readings	Problems with Solutions: Study Guide	Problems with Solutions: Text (Examples)	Assigned Problems: Study Guide	Additional Problems*
1	Secs. 6-2 to 6-4	A, B, C	1, 2	I, J	Quest. 1 to 5, 9, Probs. 1 to 10
2	Sec. 6-5	D		K	Quest. 7, Probs. 13, 15
3	Secs. 6-5, 6-6	E, F	3, 4	L, M	Quest. 6, Probs. 11, 12, 14, 16 to 24
4	Sec. 6-7	G, H	5	N, O	Probs. 25 to 36

*Quest. = Question(s).

It is a mixture of two equations:

$$\Delta W = \vec{F}\cdot\Delta\vec{r} = F\cos\phi\,\Delta r \text{(finite } \Delta r)$$

and (1)

$$dW = \vec{F}\cdot d\vec{r} = F\cos\phi\,dr \text{ (infinitesmal } dr).$$

The latter is applicable in the limit as Δr approaches zero. By integrating both sides you obtain Eq. (6-10). As explained in the Introduction, the work concept is especially useful when forces in a problem vary as functions of position, because the integration over position in Eq. (6-10) can easily take this variation into account. The integral in Eq. (6-10) is called a line integral because the integration extends along the line (path) of the particle's motion. In this course you will only be asked to evaluate the work for motion along a straight line.

A very important point to remember in applications of Eqs. (6-2) (constant force) or (6-10) (variable force) is that the force may be the resultant force acting on a particle, or it may be a particular force of interaction of the particle with another object (pull of a rope, friction, force of gravity). The two very good examples in Section 6-2 both happen to have zero resultant force (zero acceleration), and that will not always be true (Examples 3 and 4).

Objective 2 is satisfied by Eq. (6-12) and the last paragraph of Section 6-5. Note that kinetic energy $K = (\frac{1}{2})mv^2$ is a scalar quantity, not a vector, so that the relative directions of the velocities of many particles in a system are immaterial as far as the total kinetic energy is concerned.

Objective 3 is stated in Eq. (6-14) and illustrated by Examples 3 and 4. Applications of the work-energy theorem are very diverse and can involve the use of Newton's laws to help find the forces that must be known if the work is to be calculated. Here you must again remember to distinguish between the resultant force on a moving body (this accelerates the body and affects its kinetic energy) and the individual forces of interaction with various separate objects that may do positive or negative work on the moving body. Free-body diagrams will help you keep these in mind.

Objective 4 is treated in Section 6-7, and one example with a constant force is given. If the force is not constant, Eq. (6-15) applies. Together with Eq. (1) in this study guide, you can show that the first equation in Example 5 is true in general:

$$P = dW/dt = (\vec{F}\cdot d\vec{r})/dt = \vec{F}\cdot(d\vec{r}/dt) = \vec{F}\cdot\vec{v}. \qquad (2)$$

If a car or other vehicle has an engine that delivers constant power (constant P), and the velocity changes, then the resultant force $\vec{F}$ acting on it must vary; if the resultant force is constant and the velocity varies, then the power P must vary. Keep all the possibilities in mind, and look closely at the study guide Problems G and H keyed to this objective.

TEXT: Francis Weston Sears and Mark W. Zemansky, University Physics (Addison-Wesley, Reading, Mass., 1970), fourth edition

SUGGESTED STUDY PROCEDURE

After reading the General Comments, please read each of the comments below along with the text section in Chapter 7 primarily dealing with that objective as keyed in the table. Pay special attention to the examples in the text and to the additional solved problems in the study guide. After concluding the reading and examples, work out the assigned problems given later in this study guide. Hints for solving these, if needed, are given at the end. Finally, check your learning by taking the Practice Test.

We suggest that you begin reading Section 7-2, which deals with Objective 1. Note that the physicist's technical definition of work is contrasted carefully with the everyday notion. We consider the expression "work done by force" to be better physics idiom than "the work of a force" used to introduce Eq. (7-3). The integral in Eq. (7-3) is called a line integral because the integration extends along the (possibly) curved line (path) of the particle's motion. In this course you will only be asked to evaluate the work for motion along a straight line.

SEARS AND ZEMANSKY

Objective Number	Readings	Problems with Solutions: Study Guide	Problems with Solutions: Text	Assigned Problems: Study Guide	Additional Problems
1	Sec. 7-2	A, B, C	7-1, 7-2	I, J	7-1 to 7-6, 7-15, 7-21
2	Secs. 7-1, 7-3	D	Example (Sec. 7-3)	K	7-7 to 7-9
3	Sec. 7-1	E, F		L, M	7-16 to 7-19(a), (b), (d)
4	Secs. 7-9, 7-10	G, H	Example (Sec. 7-9)	N, O	7-38 to 7-45(a), (c), 7-46, 7-47

For Objective 2, return to Section 7-1, where the kinetic-energy formula is derived. You may have been surprised to see the integral over the variable v on p. 94. This purely mathematical manipulation applies the definition of the definite integral as the limit of a certain sum, provided that the limits of integration correspond on the two sides of the equation, as pointed out in the text just before Eq. (7-1). Continue with Section 7-3. For a system consisting of several particles, the total kinetic energy is the sum of the individual particles' kinetic energies.

TEXT: Francis Weston Sears and Mark W. Zemansky, University Physics (Addison-Wesley, Reading, Mass., 1970), fourth edition

SUGGESTED STUDY PROCEDURE

After reading the General Comments, please read each of the comments below along with the text section in Chapter 7 primarily dealing with that objective as keyed in the table. Pay special attention to the examples in the text and to the additional solved problems in the study guide. After concluding the reading and examples, work out the assigned problems given later in this study guide. Hints for solving these, if needed, are given at the end. Finally, check your learning by taking the Practice Test.

We suggest that you begin reading Section 7-2, which deals with Objective 1. Note that the physicist's technical definition of work is contrasted carefully with the everyday notion. We consider the expression "work done by force" to be better physics idiom than "the work of a force" used to introduce Eq. (7-3). The integral in Eq. (7-3) is called a line integral because the integration extends along the (possibly) curved line (path) of the particle's motion. In this course you will only be asked to evaluate the work for motion along a straight line.

SEARS AND ZEMANSKY

Objective Number	Readings	Problems with Solutions: Study Guide	Problems with Solutions: Text	Assigned Problems: Study Guide	Additional Problems
1	Sec. 7-2	A, B, C	7-1, 7-2	I, J	7-1 to 7-6, 7-15, 7-21
2	Secs. 7-1, 7-3	D	Example (Sec. 7-3)	K	7-7 to 7-9
3	Sec. 7-1	E, F		L, M	7-16 to 7-19(a), (b), (d)
4	Secs. 7-9, 7-10	G, H	Example (Sec. 7-9)	N, O	7-38 to 7-45(a), (c), 7-46, 7-47

For Objective 2, return to Section 7-1, where the kinetic-energy formula is derived. You may have been surprised to see the integral over the variable v on p. 94. This purely mathematical manipulation applies the definition of the definite integral as the limit of a certain sum, provided that the limits of integration correspond on the two sides of the equation, as pointed out in the text just before Eq. (7-1). Continue with Section 7-3. For a system consisting of several particles, the total kinetic energy is the sum of the individual particles' kinetic energies.

Objective 3 requires you to put together Eqs. (7-2), (7-3), and the unnumbered equation in the box just before Eq. (7-2):

$$\int_{\vec{s}_1}^{\vec{s}_2} \vec{F}\cdot d\vec{s} = \frac{1}{2} mv_2^2 - \frac{1}{2} mv_1^2. \qquad (1)$$

For Objective 4, turn ahead to Sections 7-9 and 7-10. The important result is presented in Eq. (7-17). The power P is, of course, constant when both vector force and velocity are constant. When a car or other vehicle accelerates with an engine that delivers constant power, however, then the velocity changes and so does the force (Problem H). If the force is constant and leads to acceleration of the moving object, then the power must vary according to Eq. (7-17) (Problem G).

In connection with all objectives, note how free-body diagrams are used in Figures 7-3 and 7-4.

TEXT: Richard T. Weidner and Robert L. Sells, Elementary Classical Physics (Allyn and Bacon, Boston, 1973), second edition, Vol. 1

SUGGESTED STUDY PROCEDURE

After reading the General Comments, please read each of the comments below along with the text section in Chapter 9 primarily dealing with the objective as keyed in the table. Pay special attention to the examples in the text and to the additional Problems with Solutions in the study guide. After concluding the reading and examples, work out the assigned problems later in this study guide. Hints for solving these, if needed, are given at the end. Finally check your learning by taking the Practice Test.

We suggest that you begin by reading Section 9-1, which deals with Objective 2 by means of an abstract mathematical manipulation. For a system consisting of several particles, the total kinetic energy is the sum of the individual particles' kinetic energies.

WEIDNER AND SELLS

Objective Number	Readings	Problems with Solutions		Assigned Problems	Additional Problems
		Study Guide	Text (Examples)	Study Guide	
1	Secs. 9-2, 9-3, 9-5	A, B, C	9-1(a), 9-2	I, J	9-2, 9-5, 9-6, 9-9, 9-10, 9-13 to 9-16(a), 9-17, 9-20
2	Sec. 9-1	D		K	
3	Secs. 9-3, 9-4, 9-5	E, F	9-1(b), 9-3, 9-4	L, M	9-3, 9-4, 9-7, 9-8, 9-11, 9-26
4	Sec. 9-6	G, H		N, O	9-22 to 9-25

To deal with Objectives 1 and 3, continue your reading in Sections 9-2 and 9-3.* Note that one is looking for changes in speed and kinetic energy, hence one selects the component of force parallel to the direction of motion for the left-hand side of Eq. (9-4), which is called the work. Section 9-2 considers only the change in kinetic energy due to a force acting on a particle over an infinitesimal displacement $d\vec{r}$, a limitation not clearly visible from the right-hand sides of Eqs. (9-4) and (9-6).

*Seven lines from the bottom on p. 134, the first equation should read $v_{f1} = v_{i2}$, not $v_{fi} = v_{i2}$.

The limitation of infinitesimal displacements is removed in Section 9-3 where there is a careful derivation of the work:

$$W_{i\to f} = \int_i^f \vec{F}\cdot d\vec{r} \qquad (1)$$

done on a particle by the resultant force $\vec{F}$. Two very useful theorems follow, concerning the work done by each individual force of interaction ($\vec{F}_1$, $\vec{F}_2$, ...) and the work done by each rectangular component (F_x, F_y, F_z) of the force.

Regarding the statement after Eq. (9-7a), consider this remark: The relation between work and kinetic energy holds regardless of the path followed by the particle, but the numerical values of the work done and kinetic energy change will usually differ for different paths. Study the informative examples at the end of the section carefully. Sections 9-4 and 9-5 add more details concerning Objective 3. You may skip the first subsection "Work Energy and Impulse Momentum" in Section 9-4.

For Objective 4, turn to Section 9-6. The important result is presented in Eq. 9-13, which is illustrated by Problems G and H in this study guide. The power P, according to the equation, is constant when both vector force and velocity are constant. When a car or other vehicle is accelerated by an engine that delivers constant power, however, the velocity changes and so does the force. If the force is constant and leads to acceleration of the moving object, then the power must vary according to Eq. (9-13).

In connection with all the objectives, note how free-body diagrams are used in Figures 9-4, 9-5, 9-6, and 9-8.

PROBLEM SET WITH SOLUTIONS

A(1). The forces $\vec{F}_1$, $\vec{F}_2$, and $\vec{F}_3$ act on a particle of mass M. The forces vary with the particle's position.

(a) State the work done by force $\vec{F}_2$ as the particle moves along curve C from position $\vec{r}_1$ to position $\vec{r}_2$.

(b) State the total work done on the particle when it moves subject to the three forces as described above.

Solution

$$\text{(a)}\quad W_a = \int_{\vec{r}_1\ (\text{along C})}^{\vec{r}_2} \vec{F}_2 \cdot d\vec{r}$$

$$\text{(b)}\quad W_b = \int_{\vec{r}_1\ (\text{along C})}^{\vec{r}_2} (\vec{F}_1 + \vec{F}_2 + \vec{F}_3) \cdot d\vec{r}.$$

B(1). A particle moves along the y axis from y_1 to y_2 according to the force law given below. Find the work done on the particle and by the particle in each case. The constant y_0 equals 3 m.

(a) $F_y(y) = [(y/y_0)^2 - 16]$ N: $y_1 = -y_0$, $y_2 = 2y_0$;

(b) $F_y(y) = [(y/y_0)^3 - 3(y/y_0)^2 + 6(y/y_0) + 3]$ N: $y_1 = 0$, $y_2 = 2y_0$.

Solution

$$W = \int_{y_1}^{y_2} F(y)\, dy \text{ is work done on particle.}$$

$-W$ is work done by particle.

$$\text{(a)}\quad W = \int_{-y_0}^{2y_0} \left[\left(\frac{y}{y_0}\right)^2 - 16\right] dy = \left[\frac{1}{3}\left(\frac{y}{y_0}\right)^3 - 16\left(\frac{y}{y_0}\right)\right] y_0 \Bigg|_{-y_0}^{2y_0}.$$

$$W = [\tfrac{1}{3}(8) - 16(2)]y_0 - (\tfrac{1}{3} + 16)y_0 = -45y_0 = -135 \text{ J} \quad (-W) = +135 \text{ J}.$$

Since W is negative and $(-W)$ is positive, work is done by the particle.

(b) $W = \int_0^{2y_0} [(\frac{y}{y_0})^3 - 3(\frac{y}{y_0})^2 + 6(\frac{y}{y_0}) + 3]\, dy = [\frac{1}{4}(2)^4 - (2)^3 + 3(2)^2 + 3(2)]y_0.$

$W = (4 - 8 + 12 + 6)y_0 = 14y_0 = 42$ J $\quad (-W) = -42$ J.

Since W is positive, work is done <u>on</u> the particle.

(<u>Note</u>: The integration may be carried out with the help of the substitution $y/y_0 = x$, $dy = y_0\, dx$.)

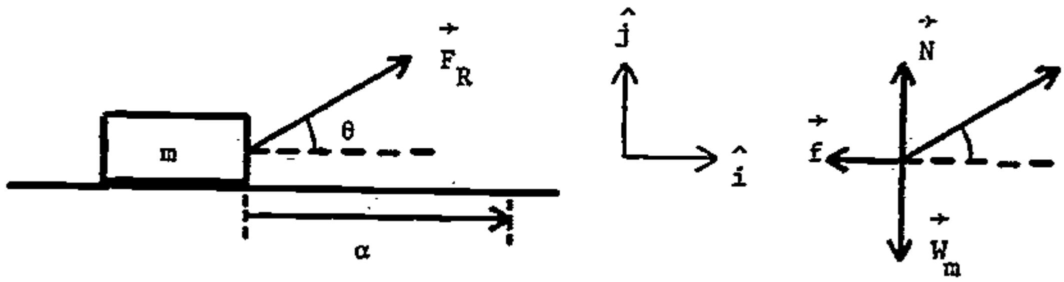
C(1). A block of mass $m = 3.00$ kg is drawn at constant speed a distance $d = 4.0$ m along a horizontal floor by a rope exerting a constant force of magnitude $F_R = 8.0$ N making an angle $\theta = 15.0°$ with the horizontal. Sketch a free-body diagram for the block, then compute:

(a) the work done on the block by the resultant force;

(b) the work done by the rope on the block;

(c) the work done by friction on the block.

<u>Solution</u>



Oata: $m = 3.00$ kg $\qquad \vec{F}_R = 8.0$ N (rope)

$\theta = 15.0 \qquad \vec{F}_R = F_{Rx}\hat{i} + F_{Ry}\hat{j}$

$d = 4.0$ m $\qquad \vec{f} = -f\hat{i}$ (friction)

$\Delta\vec{r} = d\hat{i}$

(a) Since the block was pulled at constant speed, $\vec{F} = \vec{F}_R + \vec{f} + \vec{W}_m + \vec{N} = 0$.

Total work $W = \int \vec{F}\cdot d\vec{r} = 0$.

(b) The work done by the rope:

$$W_R = \int \vec{F}_R\cdot d\vec{r} = F_R \cos\theta\, d$$
$$= (8.00)(0.966)(4.0)$$
$$= 30.9 \text{ J}.$$

(c) The work done by friction $W_f = \int \vec{f}\cdot d\vec{r} = -fd = -(7.74)(4.0) = 30.9$ J.
(From the x component of $\vec{F} = 0$, we find $F_{Rx} - f = 0$, $f = F_{Rx} = 7.73$ N.)
Comparing the answers of (a), (b), and (c), we find $W = W_R + W_f$ checks.

D(2). A 1000-kg sportscar moving at 30.0 m/s and 5000-kg truck moving at 12.0 m/s approach an intersection at right angles. What is the total kinetic energy of the two-vehicle system?

Solution

The directions of motion are irrelevant, since kinetic energy is a scalar and depends only on v^2. Call the sportscar #1, the truck #2:

$$K = K_1 + K_2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2}(1000)(30.0)^2 + \frac{1}{2}(5000)(12.0)^2$$
$$= (500)(900) + (2500)(144) = 4.5 \times 10^5 + 3.6 \times 10^5 = 8.1 \times 10^5 \text{ J}.$$

E(3). A 2000-kg car travels at 20.0 m/s on a level road. The brakes are applied long enough to do 1.20×10^5 J of work.

(a) What is the final speed of the car?

(b) What further distance is required to stop the car completely if the brakes are applied again and the constant decelerating force on the car is 4.0×10^3 N? (Use two methods and compare the results.)

(c) What ultimately happens to the car's initial kinetic energy?

Solution

(a) The kinetic energy of the car (mass M = 2000 kg) is converted to thermal energy of the brake drums and linings by friction. The work done $W = 1.20 \times 10^5$ J reduces the car's kinetic energy from the initial K_i to the final $K_f = K_i - W$. The speed drops from the initial $v_i = 20.0$ m/s to the final $v_f = ?$, where

$$K_i = \tfrac{1}{2}Mv_i^2, \quad K_f = \tfrac{1}{2}Mv_f^2,$$

$$v_f = \sqrt{2K_f/M} = \sqrt{2(K_i - W)/M} = \sqrt{v_i^2 - 2W/M}$$

$$= \sqrt{400 - \frac{2(1.2 \times 10^5)}{2 \times 10^3}} = \sqrt{400 - 120} = \sqrt{280} = 16.7 \text{ m/s}.$$

(b) With applied force $F = 4.0 \times 10^3$ N, the car moves a distance d, so work in the amount $W' = Fd = K_f$ is done to stop the car.

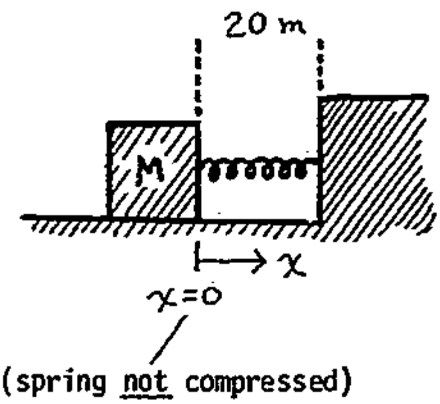
$$d = K_f/K = \tfrac{1}{2}Mv_f^2/F = \frac{1}{2} \times \frac{2 \times 10^3 \times 280}{4.0 \times 10^3} = 70 \text{ m}.$$

Find deceleration a, time to stop t, and distance moved d at the average speed $v_{av} = (1/2)v_f$. Use Newton's second law:

$$a = F/M = -\frac{4000}{2000} = -2\ m/s^2;\quad t = vf/a = \frac{16.7}{2.0} = 8.35\ s;$$

$$d = v_{av}t = \tfrac{1}{2}(16.7)(8.35) = 70\ m.$$

(c) Thermal energy in brakes - see part (a).



F(3). The block of mass M in the diagram rests on a frictionless surface. The spring exerts a force $F_x = -kx$ on the block, where k = 0.75 N/m and positive x is measured to the right. The spring is <u>not</u> fastened to the block.

(a) How much work must be done on this system by an external agent to move the block slowly from x = 0 to x = 0.80 m?

(b) If the block is now released, how much work will the spring do on the block?

(c) If the block has a mass M = 0.330 kg, what is its final speed?

<u>Solution</u>

(a) The force $F'_x = -F_x = kx$ must be exerted on the block to move it slowly, compressing the spring. Hence the work W is

$$W = \int_0^{0.80} F'_x\,dx,$$

$$W = \int_0^{0.80} kx\,dx = \frac{1}{2}kx^2\Big|_0^{0.80} = \frac{1}{2}(0.75)\,[(0.80)^2 - (0)^2],$$

$$W = 0.24\ J.$$

Since W is positive, work is done by the external agent.

(b) Assuming the spring has zero mass, the force it exerts on the block is F_x, equal in magnitude to the force F' exerted by the agent in (a). Hence the work it will do on the block is W = 0.24 J, which will slide away when it passes x = 0. The block acquires kinetic energy in this process.

(c) Kinetic energy:

$$K = \frac{1}{2}Mv^2 = W = 0.24 \text{ J}.$$

$$\text{Final Speed:} \quad v = \sqrt{\frac{2K}{M}} = \sqrt{\frac{(2)(0.24)}{(0.33)}} = \sqrt{1.44} = 1.2 \text{ m/s}.$$

G(4). A 1800-kg elevator starts from rest and is pulled upward at a constant acceleration of 2.50 m/s.

(a) Draw a free-body diagram for the elevator.

(b) Find the upward speed of the elevator as function of time after it started.

(c) Find the power required to raise the elevator as function of time.

(d) Find the kinetic energy of the elevator as function of time.

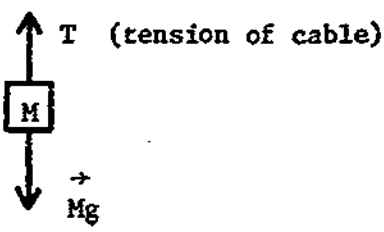
(e) Evaluate the power and kinetic energy numerically for the time when the elevator has risen 20.0 m.

(f) Describe the relation between the power in (c) and the kinetic energy in (d).

Solution

(a) Newton's second law:

$$\vec{F} = \vec{T} - M\vec{g} = M\vec{a}.$$



(b) $v(t) = at$ (since a is constant)

$= 2.5t$ m/s.

(c) $P(t) = Tv(t) = M(g+a)at$

$= (1800)(9.8+2.50)2.5t = 5.5 \times 10^4 t$ W.

(d) $K = (1/2)Mv^2 = (1/2)Ma^2t^2 = 5.6 \times 10^4 t^2$ J.

(e) Height $h = (1/2)at^2$, $h = 20.0$ m, $a = 2.50$ m/s,

$$t = \sqrt{2h/a} = \sqrt{40.0/2.5} = \sqrt{16.0} = 4.0 \text{ s}.$$

$$P = M(g+a)at \doteq (1800)(9.8 + 2.50)(2.50)4 = 2.2 \times 10^5 \text{ W}.$$

$$K = (1/2)Ma^2t^2 = (1/2)(1800)(2.5)^2(4)^2 = 0.90 \times 10^5 \text{ J}.$$

(f) Because of the gravitational force, not all the power is available to increase K, but some is used to raise the elevator.

H(4). The following questions deal with a sports car of mass 1500 kg (including the driver and a companion). Its maximum power is 7.2×10^4 W:

(a) How long would it take the car to accelerate from 15.0 m/s to 30.0 m/s on a straight, level road, if the engine delivered maximum power continuously?

(b) What is the speed $v(t)$ of the car as function of time after the accelerator is "floored" (i.e., maximum power P, initial speed $v_0 = 15$ m/s)?

(c) What is the accelerating force $\vec{F}(t)$ exerted on the car in (b)?

Solution

(a) Call speeds $v_0 = 15.0$ m/s and $v_1 = 30.0$ m/s. Gain in kinetic energy $K_1 - K_0$ comes from power delivered up to time t. Maximum power is $P = 7.2 \times 10^4$ W.

$$K_1 - K_0 = Pt; \quad t = (K_1 - K_0)/P = \frac{1}{2}M(v_1^2 - v_0^2)/P;$$

$$t = (1/2)(1500)(30^2 - 15^2)/7.2 \times 10^4 = 7.0 \text{ s}.$$

(b) $v(t)$ can be found from $K(t)$ (kinetic energy):

$$\tfrac{1}{2}Mv^2 = K(t) = K_0 + Pt; \quad v(t) = \sqrt{2(K_0 + Pt)/M} = \sqrt{v_0^2 + 2Pt/M},$$

$$v(t) = \sqrt{225 + (2)(7.2 \times 10^4)/(1.5 \times 10^3)t} = \sqrt{225 + 96t}$$

$$= (15\sqrt{1 + 0.43t}) \text{ m/s}.$$

(c) $$P = Fv \quad \text{or} \quad F = \frac{P}{v} = \frac{7.2 \times 10^4}{15\sqrt{1+0.43t}}$$

$$= \left(\frac{4.8 \times 10^3}{\sqrt{1 + 0.43\,t}}\right).$$

N directed along the road.

Problems

I(1). A particle subject to several forces moves at constant speed along the y axis. One force is $\vec{F} = (ax + b)\hat{i} + (cy^2 + d)\hat{j}$, where $a = 10.0$ N/m, $b = 5.0$ N, $c = 9.0$ N/m^2, and $d = 26.0$ N. Find the amount of work done by the force $\vec{F}$ on the particle as it moves from $y = 2.00$ m to $y = 4.0$ m.

J(1). A 3.5-kg block attached to a rope is allowed to slide down a plane inclined at 30° to the horizontal. The coefficient of friction is 0.40 and the tension applied to the rope is 8.0 N, in the direction parallel to the incline.

(a) How much work is done by the rope's tension on the block while it slides a distance of 2.00 m along the plane?

(b) How much work is done on the block by the force of gravity while it slides a distance of 2.00 m along the plane?

(c) Compare the results of (a) and (b), and comment on the difference.

K(2). A girl on a motor scooter chasing a horse has one-fourth the kinetic energy of the horse, which has 10 times her mass. She speeds up by 1.00 m/s and then has one-third the horse's kinetic energy. What were the original speeds of (a) the girl (on the motor scooter) and (b) the horse?

L(3). A 7.0-kg block slides down a rough plane inclined at 60° to the horizontal. After sliding a distance of 15.0 m from its starting point, its speed is 12.0 m/s. The coefficient of friction is 1.00.

(a) Draw a free-body diagram for the block.

(b) Evaluate the work done by each of the forces acting on the block.

(c) Calculate the initial velocity of the block.

M(3). A block of mass 1.80 kg slides on a horizontal frictionless table. It has the initial speed 5.0 m/s and is aimed end-on at the free end of a spring capable of exerting the force $-kx$ when it is deformed by the distance x (positive for stretching, negative for compression) from its unextended length L. The constant $k = 180$ N/m. The other end of the spring is firmly nailed to the table. The spring has mass m_s.

(a) How much work must the block do on the spring in order to come to rest?

(b) How far will the spring be compressed when the work in (a) is done on it?

(c) How does the mass of the spring affect the answers to (a) and (b)?

N(4). A 5000-kg truck has an engine that can deliver a maximum of power of 5.0×10^5 W. The driver steps on the gas when the truck is moving at 15.0 m/s and keeps the accelerator floored (i.e., maximum power output - assume it is constant) for 5.0 s.

(a) What will be the truck's speed at the end of the 5.0 s?

(b) What will be the speed of the truck as function of time after the accelerator is floored?

(c) (<u>Optional</u> difficult problem) How far does the truck travel during the 5.0 s while the accelerator is floored?

O(4). A satellite rocket of 5.0×10^4 kg mass acquires a speed of 2.0×10^3 m/s in one minute after launching. The rocket motor functions at constant thrust (constant force), and the rocket's mass change in one minute is negligible.

(a) What is the average power developed by the rocket during the first minute of flight?

(b) What instantaneous power is developed by the rocket as function of time after launch?

(c) Compare the instantaneous power after one minute of flight with the average power in (a) and comment on the relationship.

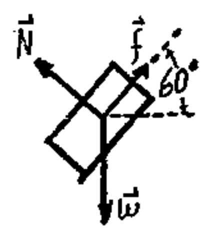
Solutions

I(1). The other forces and the motion of the particle can be ignored; only F_y contributes to the work, which is calculated from an integral along the y axis. Answer: $W = 220$ J.

J(1). The friction and motion of the block are not needed to find the work required. Applying the definition of work to the tension and the force of gravity will give the answers directly, since the block's displacement is stated. Answers: (a) $W = -16.0$ J; (b) $W = +34$ J.

K(2). Introduce as variables the girl's and horse's masses and speeds. Use the information given to set up three equations. The mass of the motor scooter does not enter and one mass cancels. Answers: (a) $v = 6.5$ m/s; (b) $v = 4.1$ m/s.

L(3). There are three forces: gravity, friction, and the normal force due to the inclined plane. Since the displacement is given, the net force, acceleration, and speed do not need to be calculated. Answers: (b) 0, 890 J, -520 J; (c) $v_i = 6.0$ m/s.



M(3). To come to rest, the block has to lose its kinetic energy. Since the spring is at rest at the end, when the block is at rest, any work done to give the spring kinetic energy will be returned. Hence the mass of the spring does not affect the result. Answers: (a) 22.5 J; (b) 0.50 m.

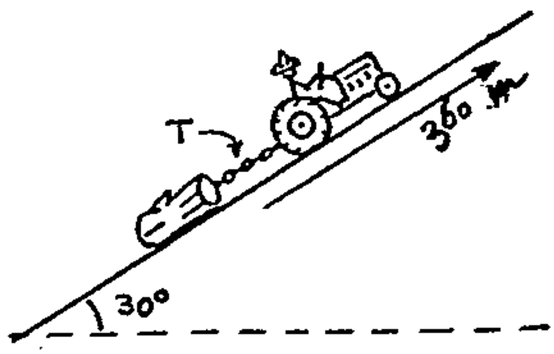
N(4). At constant power, the kinetic energy increases at a uniform rate. The speed can be found from the uniform rate of kinetic energy increase. Answers: (a) 35 m/s; (b) $(225 + 200\,t)^{1/2}$; (c) 130 m.

O(4). The average power can be found from the total kinetic-energy change during the time interval. The instantaneous power must be found from the force and instantaneous velocity. Answers:

(a) 1.7×10^9 W; (b) 5.6×10^5 t.

PRACTICE TEST

1. A tractor pulls a log of mass 200 kg up a 30° slope at constant speed of 1.20 m/s. The log is moved a distance of 360 m.



   (a) Draw a free-body diagram for the log.

   (b) Introduce symbols to state an expression for the work done by the force of gravity on the log during the operation.

   (c) Evalute the work stated in (b) numerically and give an interpretation of the sign of the result.

   (d) What is the power supplied by the tractor, assuming that the log moves without friction.

2. A brick with mass 4.0 kg is pushed along a horizontal wooden plank with the initial speed of 1.50 m/s. The coefficient of friction with the plank is 0.250.

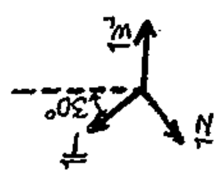
   (a) What is the brick's initial kinetic energy?

   (b) How far along the plank will the brick slide before coming to rest?

   (c) What happens to the initial energy of the brick?

Practice Test Answers

1. (a)



   (b) $W = \int_{0}^{360} \vec{W}_L \cdot \hat{i}\, dx$.

   (c) -3.5×10^5 J.

   (d) 1180 W.

2. (a) 4.5 J.

   (b) 0.46 m.

   (c) Heat up the brick and plank.

Date____________________

WORK AND ENERGY

pass recycle

Mastery Test Form A

1 2 3 4 5

Name__________________________ Tutor__________________________

Dennis, an impish 30.0-kg boy, runs down a grassy slope (30° from the horizontal) at 3.00 m/s. Suddenly he falls down and continues to slide on the seat of his pants (coefficient of friction 0.8).

1. Make a sketch and free-body diagram for Dennis while he is sliding.
2. How far does he slide before coming to a stop? Use the work-energy theorem.
3. How much work is done by the force of gravity on the boy? Set up the work integral carefully and be prepared to explain the sign of this work.
4. What is the power developed by the force of friction when his pants first make contact with the ground?
5. Describe qualitatively how this power varies during his slide.

Date______________________

WORK AND ENERGY

pass recycle

Mastery Test Form B

1 2 3 4 5

Name______________________ Tutor______________________

A block of mass 4.0 kg is drawn across a rough horizontal floor by a force of 12.0 N applied at an angle of 37° with the horizontal. The coefficient of friction is 0.200. The block starts from rest and is moved a total of 7.0 m before the force ceases to act suddenly.

1. Draw a sketch of the phenomenon and a free-body diagram for the block.
2. Find the work done on the block by the applied force and by friction after setting up the work integral.
3. What is the kinetic energy at the moment the force ceases acting?
4. What is the power expended by the applied force at the moment before it ceases acting?
5. State the work-energy theorem (as applied to a particle) in the form of an equation and briefly identify the meaning of each symbol used.

Date________________

WORK AND ENERGY

pass recycle

Mastery Test Form C

1 2 3 4 5 6

Name__________________________ Tutor__________________________

A 5.0-kg piece of ice can slide in a 2.00 m long frictionless chute, at an angle of 60° with the horizontal.

1. Draw a sketch of the situation, and a free-body diagram of the ice while a person is pushing it up the chute at constant speed.
2. How much work is done on the ice by the force of gravity when it is pushed from the bottom to the top of the chute? Set up and evaluate the work integral.
3. With what kinetic energy will the ice arrive at the bottom of the chute if it starts at the top with a speed of 2.0 m/s and slides down freely? Apply the work-energy theorem.

A 20 000-hp (1.5×10^7 W) railroad locomotive accelerates a 10 000-ton (1.00×10^7 kg) train from a speed of 10.0 m/s to 30.0 m/s at full power along a straight track.

4. How long a time interval is required for this process, neglecting friction?
5. Find the kinetic energy and the speed of the train as functions of time during the interval.
6. Find the force accelerating the train as function of time during the interval.

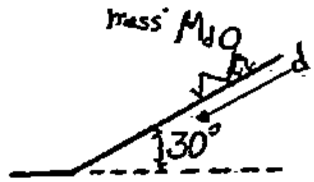
MASTERY TEST GRADING KEY - Form A

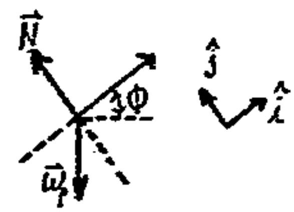
What To Look For	Solutions

1. Three forces $\vec{N}$, $\vec{f}$, $\vec{W}_D$: (Normal, friction, weight).

   Angle of slope.

   Equations (1) to (4) not required -- prepares for later parts.





1. Resultant force:

$$\vec{F} = \vec{N} + \vec{f} + \vec{W}_D = F_x\hat{i} + F_y\hat{j}.$$

$$F_x = M_D a_x \quad F_y = 0 \qquad (1)$$

$$F_x = f - W_D \sin\phi F_y = N - W_D \cos\phi, \qquad (2)$$

$$\vec{f} = \mu\vec{N}, \quad W_D = M_D g, \qquad (3)$$

$$\text{Data: } M_D = 30.0\text{ kg}, \quad \phi = 30°, \qquad (4)$$

$$v_i = -3.00\text{ m/s}, \quad g = 9.8\text{ m/s}^2.$$

2. Setting up W and K for theorem; correct solution of resultant F; correct use of resultant, not merely friction, in work-energy theorem.

2. Initial kinetic energy K_i is dissipated through friction, but gravitational force tends to maintain it. K is lost owing to (-) work done by net force:

$$W = K_f - K_i = -K_i \quad \text{since } K_f = 0,$$

$$W = \int \vec{F}\cdot d\vec{r} = -F_x d; \quad K_i = \tfrac{1}{2}M_D v_i^2,$$

$$F_x = \mu N - M_D g \sin\phi = M_D g(\mu\cos\phi - \sin\phi).$$

(<u>Note</u>: $F_y = 0$, $N = W_D \cos\phi = M_D g \cos\phi$.)

$$-F_x d = -K_i, \quad d = \frac{K_i}{F_x} = \frac{\tfrac{1}{2}M_D v_i^2}{M_D g(\mu\cos\phi - \sin\phi)},$$

$$d = \frac{v_i^2}{2g(\mu \cos \phi - \sin \phi)}$$

$$= \frac{(3.00)^2}{(2 \times 9.8)(0.8 \times 0.866 - 0.50)}$$

Two significant figures OK.

$d = 2.4$ m.

3. Definition of work

3. $W_{gravity} = \int \vec{F}_D \cdot d\vec{r}$, $d\vec{r} = dx\hat{i} + 0\hat{j}$,

$\vec{F}_D = -M_D g(\hat{i} \sin \phi + \hat{j} \cos \phi)$.

This can be obtained as indicated, or from $dW = F\, dr \cos \theta$, where this angle θ is 90° more than ϕ in the problem.

Limits of $\int$: $x = 0$ to $x = -d$,

$\vec{F}_D \cdot d\vec{r} = -M_D g \sin \phi \, dx$,

$$W_{gravity} = \int_0^{-d} (-M_D g \sin \phi \, dx)$$

$= M_D g \sin \phi d$

$= (30.0)(9.8)(0.50)(2.4)$

$= 350$ J (Positive).

Sign is +, two significant figures.

The force of gravity does work on Dennis, thus decreasing the rate of kinetic energy loss.

4. Definition $P = (dw/dt)f$ is less convenient here, but OK.
(-) sign since f, v opposite.
N comes from Problem 2.
Two significant figures.

4. $P = \vec{f} \cdot \vec{v}$ ($\vec{f}, \vec{v}$ oppositely directed)

$= -\mu N v_i$

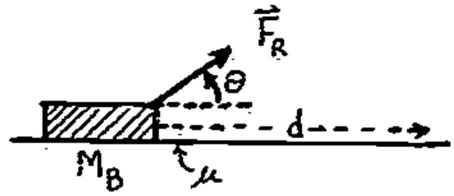
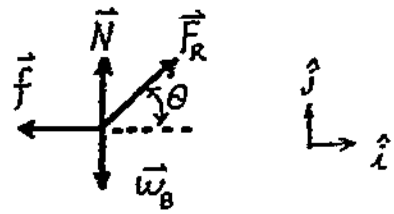
$= -\mu W_D \cos \phi v_i$

$= -(0.8)(30.0)(9.8)(0.866)(3.00)$

$= 610$ W

5. The power decreases as speed decreases; f stays constant until he stops.

MASTERY TEST GRADING KEY - Form B

What To Look For	Solutions
1. Four forces: weight $\vec{W}_B$; normal $\vec{N}$, rope $\vec{F}_R$; friction $\vec{f}$. Note angle of rope.	1.   Data: $M_B = 4.0$ kg $\mu = 0.200$ $f = \mu N$; $F_R = 12.0$ N $d = 7.0$ m $\theta = 37°$; $g = 9.8\ m/s^2$ $W_B = M_B g$
2. Definitions of W; note sign (+) and angle. Note sign (-) only. Use correct f.	2. $W_R = \int \vec{F}_R \cdot d\vec{r} = \int_0^{7.0} F_R \cos\theta\, dx = F_R \cos\theta d = (12.0)(0.8)(7.0) = 67$ J. $W_F = \int \vec{f} \cdot d\vec{r} = \int_0^{7.0} (-\mu N)\, dx = -\mu[W_B - F_R \sin\theta]\, d = -(0.20)[(4.0)(9.8) - (12)(0.6)]\, 7.0 = -45$ J.
3. Use this, not laws of motion (latter OK but tell student). More significant figures useful for subtraction, but not required.	3. Use the work-energy theorem: $W = K_f - K_i = W_R + W_f$, $K_i = 0$, $K_f = W = 22$ J. $W_R + W_f = 67.2 - 44.8 = 22.4$ J (from Problem 2).

4. Recognize formula for instantaneous power, NOT average $P_{av} = W/(\text{time for movement})$.

   Use kinetic energy to find v.

4. $P = \vec{F}_R \cdot \vec{v}_f = F_R \cos \theta v_f$

   $= (12.0)(0.08)(3.35) = 32$ W.

   $W = K_f = \frac{1}{2}M_B v_f^2$

   $v_f = \sqrt{2W/M_B} = \sqrt{(2)(22)/4.0} = 3.3$ m/s.

5. Resultant force should be identified. Integral optional.

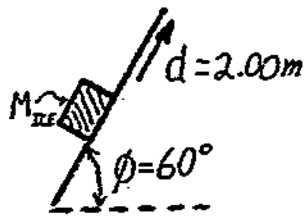
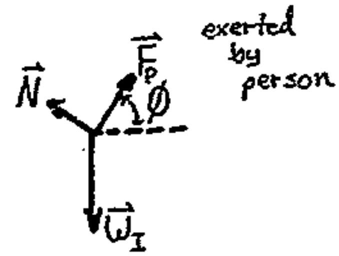
   Initial and final K should be identified in relation to work. Ask for oral explanation if the written answer is incomplete.

5. $W = K_f - K_i$

W	=	K_f	-	K_i
work done by resultant force on particle		final		initial
		kinetic energy		

   $= \int \vec{F} \cdot d\vec{r}$.

MASTERY TEST GRADING KEY - Form C

What To Look For	Solutions
1. Note three forces: gravity, person, normal. Angle of slope. $W_I = M_I g$.	1.  
2. General integral. Note angle ϕ, is usually θ in definition minus 90° or $\theta = \phi + 90°$.	2. $W_g = \int \vec{W}_I \cdot d\vec{r} = -W_i d \sin \phi$ $= -M_I g d \sin \phi = -(5.0)(9.8)(2.00)(0.866)$, $W_g = -85$ J. (Ask Student to interpret sign).
3. Source of W.	3. $W = K_f - K_i$. Work done during slide is done by force of gravity. Since motion is down, $W = -W_g$ from Problem 2. $K_i = \frac{1}{2} M_I v_i^2$, $K_f = W + K_i = 85 + (\frac{1}{2})(5.0)(2.0)^2$ $= 85 + 10 = 95$ J.
4. Apply work-energy theorem and definition of power (for constant P).	4. Symbols: M_T = mass of train, i = initial, F = resultant force, P = power, t = time, f = final Speed is related to kinetic energy. Use $W = K_f - K_i = P\,\Delta t$ since P is constant. $K_i = \frac{1}{2} M_t v_i^2$, $K_f = \frac{1}{2} M_T v_f^2$, $\Delta t = (K_f - K_i)/P = \frac{1}{2} M_T (v_f^2 - v_i^2)/P$ $\Delta t = (\frac{1}{2})(10^7) \frac{30^2 - 10^2}{1.5 \times 10^6} = \frac{800}{3} = 270$ s.

5. Introduce time variable. Use initial speed. Solve for $v(t)$.

5. $K(t) = K_i + Pt = \frac{1}{2}M_T v_i^2 + Pt$

$= (\frac{1}{2})(10^7)(10)^2 = 1.5 \times 10^7 t$

$= 10^7(50 + 1.5t)$ J (t in seconds).

$K(t) = \frac{1}{2}M_T v(t)^2,$

$v(t) = \sqrt{2K(t)/M_T} = \sqrt{100 + 3.0t}$ m/s .

6. Two methods possible. Use $v(t)$ from Problem 5 for both. Get same result. F NOT constant; a NOT constant $\neq (v_f - v_i)/t$.

6. $P = Fv, \quad F = P/v,$ (1)

$F = M_T a = M_T dv/dt.$ (2)

OR

$$F = \frac{P}{v} = \frac{1.5 \times 10^7}{(100+3.00t)^{\frac{1}{2}}}\ \text{N} \quad \text{(t in seconds)} \qquad (1)$$

$$F = M_T a$$

$$= M_T \frac{d}{dt}\sqrt{100+3.00t} = \frac{1}{2}\,\frac{30M_T}{(100+3.00t)^{\frac{1}{2}}}$$

$$= \frac{1.5 \times 10^7}{(100+3.00t)^{\frac{1}{2}}}\ \text{N}. \qquad (2)$$

Module ___

APPLICATIONS OF NEWTON'S LAWS

INTRODUCTION

Perhaps at some time you have had occasion to swing a massive object at the end of a rope. Maybe you have watched a parent swing a child around by his outstretched arms or have been fortunate enough to watch an athlete throw the hammer. But all of you have heard or watched an automatic washer go through a spin-dry cycle. How was this spinning drum with holes in its periphery able to speed up the "drying" process? The clothes were too large to pass through the holes in the drum and were "held" in a circular path but the water droplets were small enough to pass through the holes. We all know what happens when the stone is no longer restrained in the revolving slingshot. The water droplets fly through the holes in a straight-line path and are then disposed of.

In this module you will explore the nature of the forces responsible for this circular motion. You will also look into the motion of several bodies connected together such as a plow to a horse, a train to a locomotive, or a barge to a tugboat.

PREREQUISITES

Before you begin this module, you should be able to:	Location of Prerequisite Content
*Identify forces acting on an object and draw a free-body diagram of the object (needed for Objectives 1 and 3 of this module)	Newton's Laws Module
*Determine acceleration for uniform circular motion (needed for Objective 2 of this module)	Planar Motion Module
*Apply Newton's second and third laws to single-body problems (needed for Objectives 2, 4, 5 of this module)	Newton's Laws Module

LEARNING OBJECTIVES

After you have mastered the content of this module you will be able to:

1. Centripetal force - For a particle undergoing circular motion, draw a free-body diagram and identify the interactions responsible for the centripetal force; these forces may be gravitational forces or contact force exerted by another body.

2. Uniform circular motion - Search out the necessary conditions concerning radius, speed, and forces to solve problems by applying Newton's second law to a particle undergoing circular motion.

3. Free-body diagram - For a system of two or three interacting bodies, (a) identify the forces of interaction; and (b) draw a free-body diagram (using a particle representation) for each body.

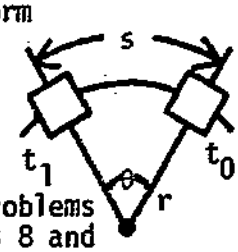
4. Newton's third law - Apply Newton's third law to determine action-reaction force pairs between the bodies of a two- or three-body system.

5. Two- or three-body motion - Use Newton's second law to solve problems relating the motion of several bodies comprising a system and the external and internal forces acting where (a) the acceleration is uniform; or (b) the motion is uniform circular.

TEXT: Frederick J. Bueche, Introduction to Physics for Scientists and Engineers (McGraw-Hill, New York, 1975), second edition

SUGGESTED STUDY PROCEDURE

Read Section 10.7 in Chapter 10, relative to Objectives 1 and 2. The ω that appears in Eq. (10.13) is called angular speed and is related to the radius of motion r and the linear speed v of the body by $\omega = v/r$. For uniform circular motion $\omega = \theta/t$, where θ is the angle subtended in a time t.



$\omega = \theta/t$, $v = s/t$, where the arc distance $s = r\theta$. $t = t_1 - t_0$.

For Objectives 1 and 2, read General Comments 1 and 2. Then study Problems A, B, and C in this Study Guide. Solve Problems J and K and Problems 8 and 20 in Chapter 10 of your text.

Read Chapter 5, Section 5.5 of the text for Objectives 3, 4, and 5; carefully study the illustrations in this section. For Objective 5, read General Comment 3 below. Study Problems D through I in this Study Guide before working Problems L through Q. To check mastery, try the Practice Test.

BUECHE

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text*	Study Guide	Text	
1	Sec. 10.7, General Comment 1	A, B				
2	Sec. 10.7, General Comment 2	A, B, C	Illus. 10.3, 10.4	J, K	Chap. 10, Probs. 8, 20	Chap. 10, Probs. 9, 11, 18
3	Sec. 5.5	D, E, F, G, H, I				
4		D, F				
5	Sec. 5.5	D, E, F, G, H, I	Illus. 5.5, 5.6	L, M, N, O, P, Q		Chap. 5, Probs. 15, 17, 19-22, 24-26

*Illus. = Illustration.

TEXT: David Halliday and Robert Resnick, Fundamentals of Physics (Wiley, New York, 1970; revised printing, 1974)

SUGGESTED STUDY PROCEDURE

Read Section 5-13 in Chapter 5 of your text in relation to Objectives 1 and 2. Go over the three excellent examples in this section. In Example 9 remark the method for changing v to revolutions per second. This latter unit is a measure of angular speed and designated ω. (Note this usage in Problem A.) Next read General Comments 1 and 2. Study Problems A, B, and C before solving Problems J and K of this module and Problems 67 and 77 in Chapter 5 (pp. 94a, 94b).

Go over Examples 1 and 2 in Section 5-6 in relation to Objective 4 and Examples 5 and 6 in Section 5-11 in relation to Objective 5. Also read General Comment 3. Before solving Problems L through Q, study Problems D through I.

Try the Practice Test before attempting a Mastery Test.

HALLIDAY AND RESNICK

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text	Study Guide	Text (Chap. 5)	(Chap. 5)
1	Sec. 5-13, General Comment 1	A, B				
2	Sec. 5-13, General Comment 2	A, B, C	Ex. 9, 10, 11	J, K	67, 77	68, 70-72, 74
3		D, E, F, G, H, I				
4	Sec. 5-6	D, F	Ex. 1, 2			
5	Sec. 5-11, General Comment 3	D, E, F, G, H, I	Ex. 5, 6	L, M, N, O, P, Q		47, 58-61, 64, 65

*Ex. = Example(s).

TEXT: Francis Weston Sears and Mark W. Zemansky, University Physics (Addison-Wesley, Reading, Mass., 1970), fourth edition

SUGGESTED STUDY PROCEDURE

Your readings will be from Chapters 2, 5, and 6 of the text. Read Sections 6-7 and 6-8 in relation to Objectives 1 and 2. Go through the four worked examples in these sections. Note in Example 1 of Section 6-7 how the speed of the body is changed from revolutions per second (in Problem A we designate this as ω) to centimeters per second. Study the excellent multiflash photographs in Figure 6-16 (p. 85). For Objectives 1 and 2, read General Comments 1 and 2. Then study Problems A, B, and C. Solve Problems J and K in this module and Problems 6-32 and 6-35 of your text.

Read Section 2-5 of Chapter 2 before studying Example 1 in Section 2-6 and Example 4 in Section 5-6 (p. 64) for Objective 4. Then study Example 5 in Section 2-6 (p. 22), Example 5 in Section 5-6 (pp. 64, 65), and General Comment 3 for Objective 5. Study Problems D through I before solving Problems L through Q. Then try the Practice Test.

SEARS AND ZEMANSKY

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text	Study Guide	Text	
1	Sec. 6-7, General Comment 1	A, B				
2	Secs. 6-7, 6-8, General Comment 2	A, B, C	Ex. 1, 2, 3, (Sec. 6-7) Ex. (Sec. 6-8)	J, K	6-32, 6-35	6-29, 6-30, 6-31, 6-33, 6-36, 6-39
3		D, E, F, G, H, I				
4	Secs. 2-5, 2-6, 5-6	D, F	Ex. 1, 2 (Sec. 2-6), Ex. 4 (Sec. 5-6)			
5	Secs. 2-6, 5-6, General Comment 3	D, E, F, G, H, I	Ex. 5 (Sec. 2-6) Ex. 5 (Sec. 5-6)	L, M, N, O, P, Q		5-26 to 5-28, 5-30 to 5-32, 5-34 to 5-36

*Ex. = Example(s).

TEXT: Richard T. Weidner and Robert L. Sells, Elementary Classical Physics (Allyn and Bacon, Boston, 1973), second edition, Vol. 1

SUGGESTED STUDY PROCEDURE

Read Chapter 8, Sections 8-5 and 8-6 of the text and study Examples 8-9 and 8-10 carefully in relation to Objectives 1 and 2. Then read General Comments 1 and 2. Study Problems A, B, and C in this study guide before solving Problems J and K and Problems 8-22 and 8-27 in your text.

Study Example 8-6 (p. 112) in relation to Objectives 3 and 5. For Objective 5, read General Comment 3 before studying Problems D through I. Solve Problems L through Q before you try the Practice Test.

WEIDNER AND SELLS

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text	Study Guide	Text	
1	Sec. 8-5, General Comment 1	A, B				
2	Secs. 8-5, 8-6, General Comment 2	A, B, C	Ex.* 8-9, 8-10	J, K	8-22, 8-27	8-18, 8-24, 8-25 (hard)
3		D, E, F, G, H, I				
4		D, F				
5	Sec. 8-2, General Comment 3	D, E, F, G, H, I	Ex. 8-6	L, M, N, O, P, Q		8-2, 8-13 to 8-15, 8-19

*Ex. = Example(s).

GENERAL COMMENTS

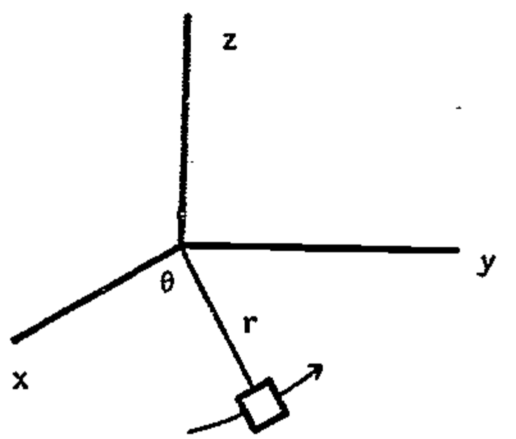
1. Centripetal Force

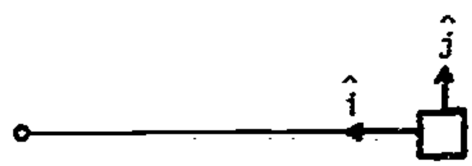
You should not be misled into thinking that a centripetal force is different in nature from other forces. Centripetal forces may be pushes or pulls exerted by strings or rods. They may arise from gravity or friction, or they may be the resultant force arising from a combination of causes.

For a particle undergoing uniform circular motion, the centripetal force is merely the name given the resultant force on the particle. For nonuniform circular motion, the centripetal force is the component of the resultant force directed toward the center of the circular path.

In working uniform-circular-motion problems, you may wish to "freeze" the motion of the body (take a snapshot of it) at a given time and set up a rectangular coordinate system at the body with one axis pointing toward the center of motion. You may also wish to use what are called cylindrical coordinates. This is comparable to using polar coordinates in the xy plane and the z axis or (r,ϕ,z). Hence you can place your origin at the center of motion and resolve the forces into their r and z components in order to apply Newton's second law in these directions:

$$\Sigma F_z = ma, \quad \Sigma F_r = mv^2/r.$$





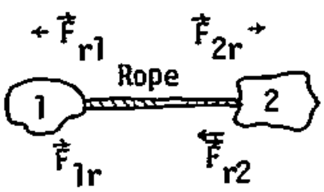
2. Noninertial Frame

You should notice that we are working in an inertial reference frame again (a frame in which Newton's first law holds). You must not be trapped into thinking that you can use the standard approach to problems (using Newton's laws) in a noninertial reference frame. Take, for an example, a very smooth glass block resting on a highly polished merry-go-round surface (there is still some friction). Assume you are on the merry-go-round as it begins to turn. What happens to the block? The block slides off the merry-go-round. Why? According to Newton's first law the block should remain at rest unless acted upon by an unbalanced force. But what was the interaction that gave rise to this force? Only the observer on the ground (inertial system) can interrupt the situation. The block was initially at rest, and if the surfaces were frictionless, according to Newton's first law it would remain at rest. The merry-go-round would simply rotate under the block. But because there is friction and now relative motion between the surfaces the block will slide off. If the frictional force is great enough the block will rotate with the merry-go-round.

Therefore, this module will deal only with inertial coordinate systems.

3. Massless Ropes and Frictionless Pulleys

Most of the cords, ropes, and cables you will meet in problems in this module have masses so small they can be neglected. Such a "massless" cord, rope, or cable exerts the same magnitude of pull on each object to which it is attached at either end. Do you see why this is so? Consider the rope in the diagram at the right, connecting objects 1 and 2. Newton's third law guarantees that



$$\vec{F}_{1r} = \vec{F}_{r1} \quad \text{and} \quad \vec{F}_{2r} = \vec{F}_{r2}, \tag{1}$$

where $\vec{F}_{1r}$ is the force that object 1 exerts to the left on the rope, $\vec{F}_{r1}$ is the force that the rope exerts to the right on object 1, etc. Applying Newton's second law to the rope yields

$$F_{2r} - F_{1r} = M_r a_r = 0 \tag{2}$$

because the rope is massless ($M_r = 0$). Combining these three equations gives us

$$F_{r1} = F_{2r}, \tag{3}$$

as claimed. When the rope passes over a pulley (which will also be "massless" in this module, as well as frictionless), it is only necessary to imagine that the rope is divided into very short segments where it wraps around the pulley. Since the frictionless pulley exerts no force on such a segment along its line of motion, Eq. (3) holds again for each of these segments. But then, with Newton's third law guaranteeing equality of the magnitude of the force from one segment to the next, this means that Eq. (3) must hold also over the whole length of the rope. Thus, for a "massless" rope we need talk about only one magnitude of force; this is usually called the "tension in the rope."

PROBLEM SET WITH SOLUTIONS

A(1). A rubber eraser of mass 0.040 kg is placed on a horizontal metallic disk of radius 0.200 m that is rotating with a uniform speed. The coefficient of friction between the eraser and the disk is 0.50.

(a) Draw a free-body diagram for the eraser.

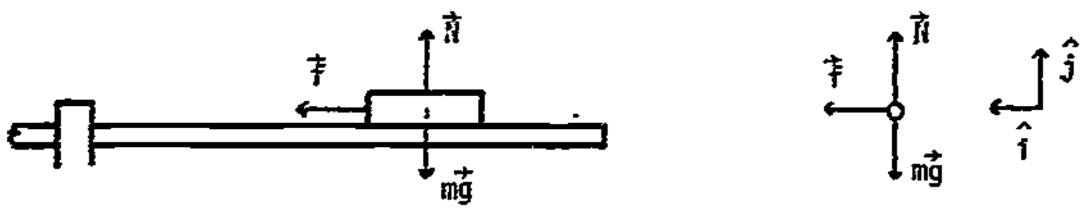
(b) Identify the interaction(s) responsible for the centripetal force.

(c) The eraser is positioned 0.050 m from the center of the disk. Find the minimum number of revolutions per minute (r/min) required so that the eraser slips on the disk.

(d) If the disk is spinning at 70 r/min describe the region of the disk where the eraser can be placed so that it will ride on the disk without slipping. Why is there no slipping in this region?

Solution

(a)



(b) Only the frictional force is responsible for the centripetal force. $\vec{N}$ and $m\vec{g}$ have zero components toward the center.

(c) Applying Newton's second law, with the radius of the disk represented by R:

$$\Sigma F_x = ma_x, \qquad f = mv^2/R,$$

$$\Sigma F_y = ma_y, \qquad N - mg = 0.$$

But $f = \mu N = \mu mg$ just when slipping is about to begin. Thus

$$\mu mg = mv^2/R \text{ or } v^2 = R\mu g;$$

and

$$v = [R\mu g]^{1/2} = [(0.050\ \text{m})(0.50)(9.8\ \text{m/s}^2)]^{1/2}$$
$$= 0.49\ \text{m/s}.$$

The circumference of the path is $2\pi R$ or one revolution:

$$\omega = 0.49 \text{ m/s } (\text{r/min})\left(\frac{1}{2\pi(0.050)}\right)\left(\frac{\text{s}}{\text{min}}\right)\left(\frac{60}{1}\right) = 94 \text{ r/min}.$$

(d) $\omega = 70$ r/min or $v = 70 \left(\frac{\text{r}}{\text{min}}\right)\left(\frac{\text{min}}{\text{s}}\right)\left(\frac{1}{60}\right)\left(\frac{1}{\text{r}}\right)\left(\frac{2\pi R(\text{meters})}{1}\right)$,

$v = 7.33R$ Hz.

The centripetal force required for this speed is mv^2/R or

$$m(7.33)^2R \text{ Hz}^2 = 2.15R \text{ kg/s}^2.$$

Friction is responsible for the centripetal force, so $f = 2.15R$ kg/s^2, but

$$f \leq \mu mg = (0.50)(0.40 \text{ kg})(9.8 \text{ m/s}^2) = 0.20 \text{ N}.$$

Thus, for maximum frictional force:

$$R = \frac{\mu mg}{2.15 \text{ kg/s}} = \frac{0.20 \text{ kg m/s}^2}{2.15 \text{ kg/s}^2} = 0.093 \text{ m}.$$

If R is less than this value, then $f < 0.20$ N. Hence for $R \leq 0.093$ m slipping will not occur.

B(2). A banked circular highway curve is designed for frictionless traffic moving at 60 km/h. The radius of the curve is 200 m. A car is moving along the highway at 80 km/h on a stormy day.

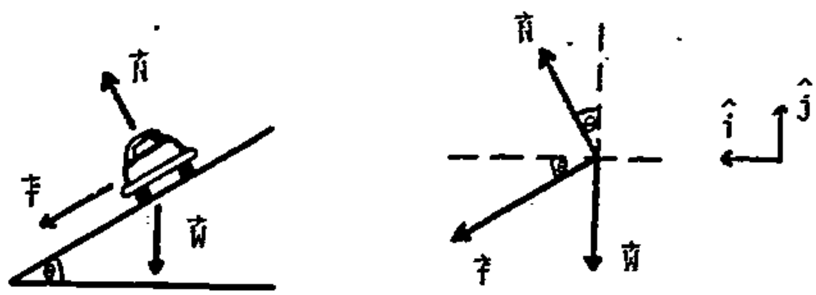
(a) Draw a free-body diagram of the car negotiating the curve.

(b) Identify the interactions responsible for the centripetal force.

(c) What is the minimum coefficient of friction between tires and road that will allow the car to negotiate the turn without sliding off the road?

Solution

(a) Free-body diagram is shown in the figure below.



(b) The component of the resultant force that is directed toward the center of the uniform circular motion is the "centripetal" force, or the force holding the body in a circular path. $\vec{W}$ has zero component in this direction; $\vec{f}$ has a component of magnitude f cos θ in this direction; $\vec{N}$ has a component of magnitude N sin θ in this direction. Therefore, $\vec{f}$ and $\vec{N}$ are responsible for the centripetal force. The centripetal force is given by

$$f \cos \theta + N \sin \theta = mv^2/R. \qquad (4)$$

(c) The highway is banked for a car moving at 60 km/h or 16.7 m/s, which means that the car need not depend upon friction but only upon the normal force to provide the centripetal force. Thus,

$$N \sin \theta = mv^2R \qquad (5)$$

where we have set f = 0 in Eq. (4). But since in the $\hat{j}$ direction there is zero acceleration,

$$N \cos \theta - f \sin \theta - mg = 0. \qquad (6)$$

Setting f = 0, we have

$$N \cos \theta = mg. \qquad (7)$$

Dividing Eq. (5) by Eq. (7), we obtain

$$\frac{N \sin \theta}{N \cos \theta} = \frac{mv^2/R}{mg} \quad \text{or} \quad \tan \theta = \frac{v^2}{Rg},$$

$$\tan \theta = \frac{(16.7 \text{ m/s})^2}{(200 \text{ m})(9.8 \text{ m/s}^2)} = 0.142,$$

$$\theta = 8.1°.$$

Now including friction when the car is traveling at v = 80 km/h or 22.2 m/s, from Eq. (4) we find

$$N \sin \theta + f \cos \theta = mv^2/R,$$

and from Eq. (6):

$$N \cos \theta - f \sin \theta = mg.$$

Since $\vec{f} = \mu\vec{N}$, we have

$$N \sin \theta + \mu N \cos \theta = mv^2/R,$$

$$N \cos \theta - \mu N \sin \theta = mg.$$

Dividing these two equations, we get

$$\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta} = \frac{mv^2}{Rg} = K.$$

Solving for μ:

$$\mu = \frac{K\cos\theta - \sin\theta}{\cos\theta + K\sin\theta}, \quad K = \frac{(22.2\ \mathrm{m/s})^2}{(200\ \mathrm{m})(9.8\ \mathrm{m/s^2})} = 0.250,$$

$$\mu = \frac{(0.250)(0.990) - (0.141)}{(0.990) + (0.250)(0.141)} = 0.100.$$

C(2). A passenger rides around in a Ferris wheel of radius 20.0 m that makes one revolution every 10 seconds. The mass of the passenger is 75 kg.

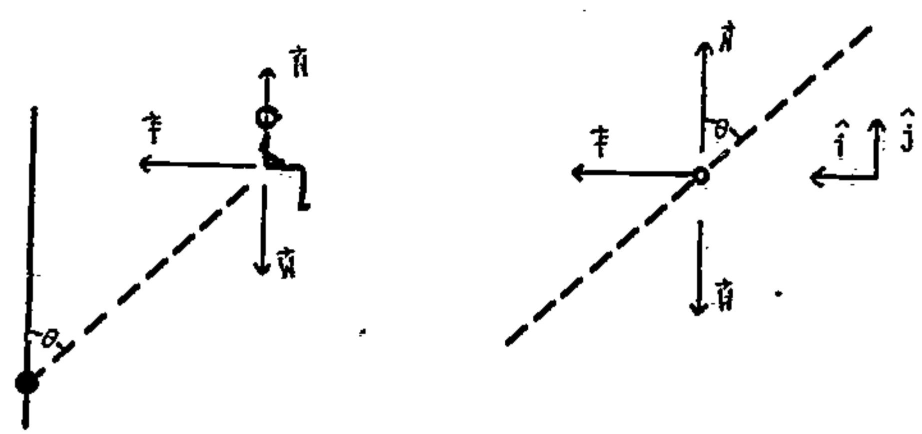
(a) Draw a free-body diagram of the passenger at an arbitrary point along the circle.

(b) What interaction(s) is responsible for the centripetal force?

(c) What force does the passenger exert on the seat when he is at the bottom of the circle?

Solution

(a)



(b) Two interactions are responsible for the centripetal force - the force the seat exerts on the posterior of the passenger ($\vec{N} + \vec{f}$) and the weight of the passenger ($\vec{W}$) ($\vec{N}$ = normal force, $\vec{f}$ = frictional force). The centripetal force is given by

$$F_c = W\cos\theta + f\sin\theta - N\cos\theta.$$

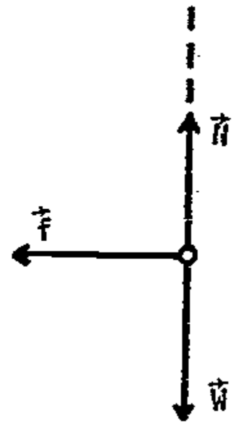
(c) The speed of the Ferris Wheel is $v = (2\pi R)/10$ s, where R is the radius of the Ferris Wheel and $2\pi R$ the circumference of the circular path of the passenger.

$v = 12.6$ m/s.

The centripetal acceleration is

$a = v^2/R = 7.9$ m/s^2.

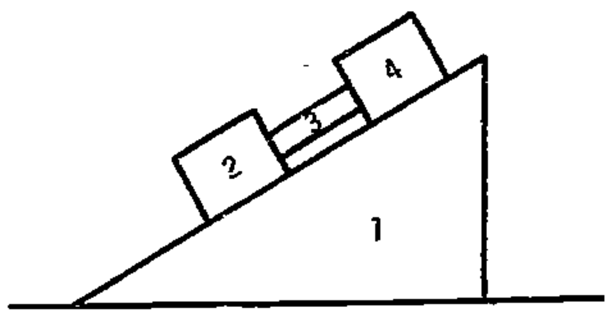
At the bottom of the circle, the passenger's acceleration is directed upward. Therefore the net upward force on the rider is



$F = ma = (75\text{ kg})(7.9\text{ m/s}^2) = 592$ N.

The upward force exerted by the seat must have a greater magnitude than the man's weight [$W = (75\text{ kg})(9.8\text{ m/s}^2) = 735$ N] by 592 N. Therefore the seat must push up with a total force of 592 N + 735 N = 1327 N on the passenger, and the passenger exerts an equal but opposite force on the seat.

D(3). An inclined plane of mass m_1 slides to the right on a horizontal surface. The coefficient of friction between these two surfaces is μ_1. Blocks 2 (mass m_2) and 4 (mass m_4) are connected by a rod (massless) and slide down the incline. The coefficient of friction between the incline and the two blocks is μ.

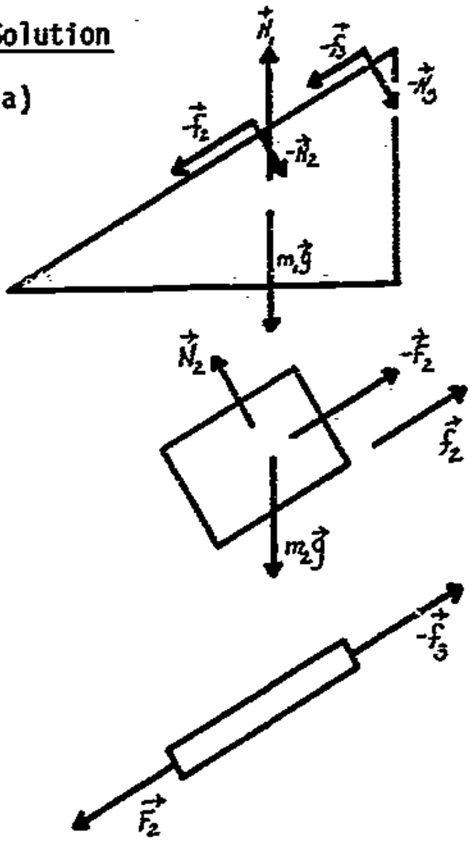


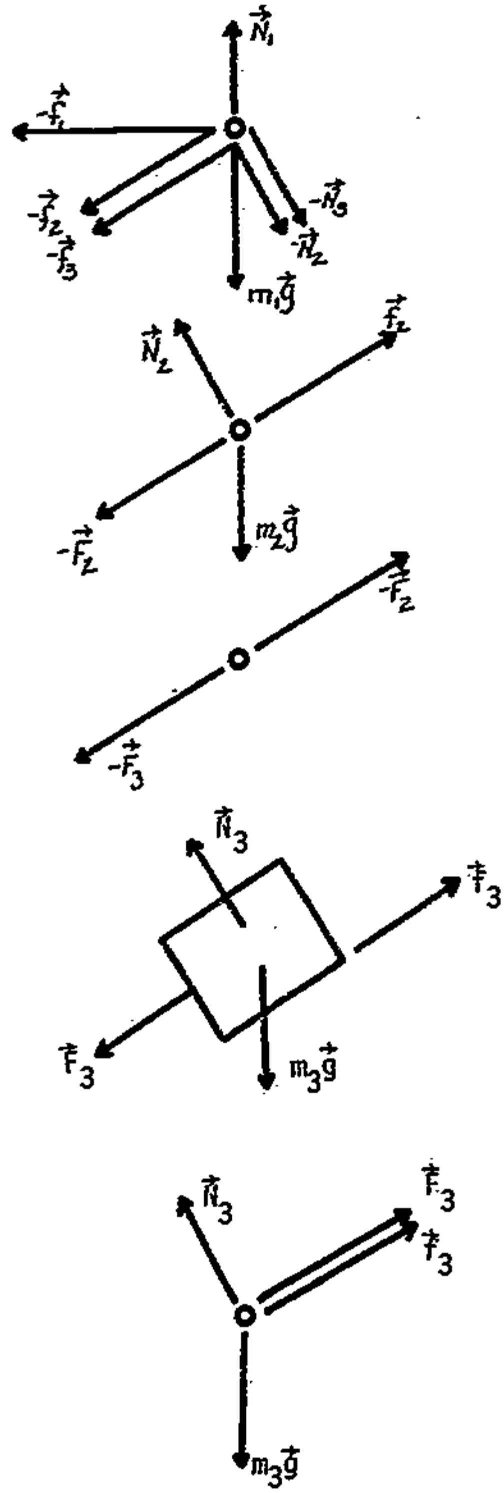
(a) Draw a free-body diagram of each body.

(b) Using Newton's third law, identify all action-reaction pairs among these four bodies.

Solution

(a)





(b) $\vec{f}_2$ = frictional force #1 exerts on #2
$-\vec{f}_2$ = frictional force #2 exerts on #1

$\vec{N}_2$ = normal force #1 exerts on #2
$-\vec{N}_2$ = normal force #2 exerts on #1

$\vec{f}_3$ = frictional force #1 exerts on #3
$-\vec{f}_3$ = frictional force #3 exerts on #1

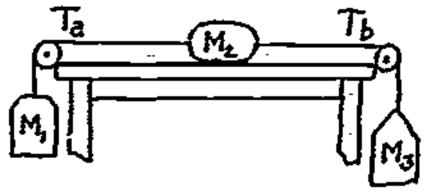
$\vec{N}_3$ = normal force #1 exerts on #3
$-\vec{N}_3$ = normal force #3 exerts on #1

$\vec{F}_2$ = force #2 exerts on rod
$-\vec{F}_2$ = force rod exerts on #2

$\vec{F}_3$ = force rod exerts on #3
$-\vec{F}_3$ = force #3 exerts on rod

See General Comment 3 to see how $\vec{F}_2$ and $\vec{F}_3$ are related.

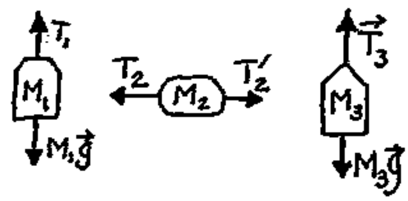
E(3). In the figure at the right the table top is frictionless; the pulleys are frictionless and massless. Find the tensions in the cords, and the acceleration of the system, assuming M_1, M_2, and M_3 are known.



Solution

When you move on from considering the motion of a single object to finding the motion of two or three objects, the chief new ingredient, you will find, is the solution of a set of two or three simultaneous equations.

The free-body diagrams are shown at the right: $|\vec{T}_1| = |\vec{T}_2| = T_a$ and $|\vec{T}_2'| = |\vec{T}_3| = T_b$. For M_1 we shall take the x axis pointing vertically upward; for M_2, horizontally to the right; and for M_3, vertically downward. (Of course, other choices are possible at this point.) The interesting components of Newton's second law for each of the blocks are then



$$M_1a_x = T_a - M_1g, \quad M_2a_x = T_b - T_a, \quad \text{and} \quad M_3a_x = M_3g - T_b. \tag{8}$$

(Note that the above choice of axes has made $a_{1x} = a_{2x} = a_{3x}$; their common value has been denoted simply by a_x in the above equations.)

We now have a set of three equations involving the three unknowns a_x, T_a, and T_b. Fundamentally, the approach to solving such a set is to combine them so as to obtain a (necessarily smaller) set in which at least one of the unknowns does not occur. When such a procedure is carried out repeatedly, we eventually end with just one equation that involves only one of the unknowns. With the equations above, one might start by "solving" the first for T_a:

$$T_a = M_1a_x + M_1g, \tag{9}$$

and substituting this into the second equation:

$$M_2a_x = T_b - M_1a_x - M_1g. \tag{10}$$

This equation along with the third of Eqs. (8) now gives us our first new set. Note that this new set consists of two equations in two unknowns; T_a has been eliminated from this set. Proceeding again, we solve the third of Eqs. (8) for T_b and substitute the result into Eq. (10). This yields the one equation

$$M_2a_x = M_3g - M_3a_x - M_1a_x - M_1g, \tag{11}$$

which can finally be solved to obtain a_x in terms of known quantities:

$$a_x = (M_3 - M_1)g/(M_1 + M_2 + M_3). \tag{12}$$

It is now a straightforward matter to substitute this into Eq. (9) to obtain T_a, and into Eq. (10) or the third of Eqs. (8) to obtain T_b:

$$T_a = \frac{M_1(M_2 + 2M_3)g}{M_1 + M_2 + M_3} \quad \text{and} \quad T_b = \frac{M_3(2M_1 + M_2)g}{M_1 + M_2 + M_3}. \tag{13}$$

Often, one can find variations of the above solve-and-substitute procedure that are quicker. For example, simply adding all three of Eqs. (8) yields

$$M_1a_x + M_2a_x + M_3a_x = -M_1g + M_3g, \tag{14}$$

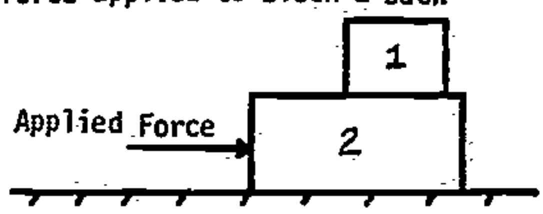
which can be solved for a_x, to obtain Eq. (12).

F(4). Block 1 of mass 1.00 kg rests on block 2 of mass 4.0 kg, which rests on a frictionless table. The coefficients of friction between the two blocks are $\mu_s = \mu_k = 0.40$. If block 2 is given a hard enough push, block 1 will slide off. We want to apply the largest possible force to block 2 such that block 1 will not slip.

(a) Draw free-body diagrams for both blocks, showing and labeling all forces.

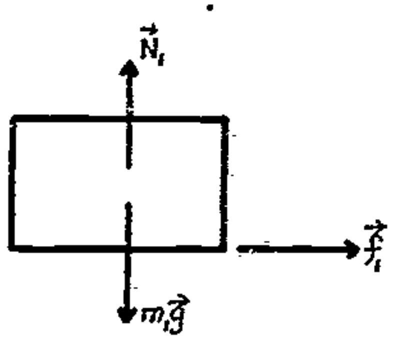
(b) Using Newton's third law, identify the action-reaction forces between the two blocks.

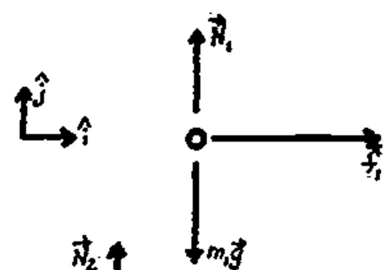
(c) Determine this largest possible force applied to block 2 such that block 1 will not slip.

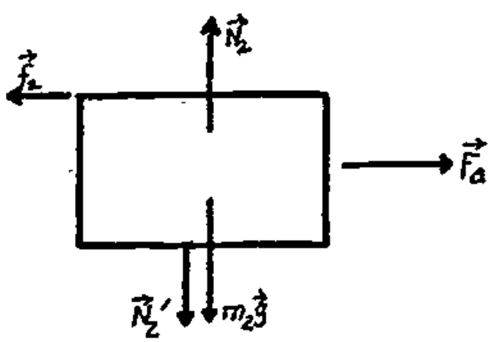


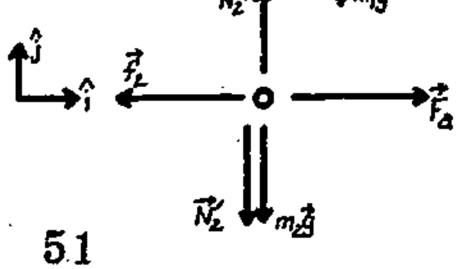
Solution

(a)









(b) $\vec{f}_1$ is the frictional force that block 2 exerts on block 1.
$\vec{f}_2$ is the frictional force that block 1 exerts on block 2.
$\vec{f}_1 = -\vec{f}_2$, $|\vec{f}_1| = |-\vec{f}_2| = f$ (action-reaction pair).
$\vec{N}_1$ is the normal force that block 2 exerts on block 1.
$\vec{N}_2'$ is the normal force that block 1 exerts on block 2.
$\vec{N}_1 = -\vec{N}_2'$, $|\vec{N}_1| = |-\vec{N}_2'| = N$ (action-reaction pair).

(c) Apply Newton's second law to each of the blocks, i.e., $\Sigma F_x = ma_x$ and $\Sigma F_y = ma_y$. For block 1,

$$f = m_1 a_{1x} \tag{15}$$

and

$$N - m_1 g = 0. \tag{16}$$

For block 2,

$$Fa - f = m_2 a_{2x}, \tag{17}$$

$$N_2 - N - m_2 g = 0 \quad \text{or} \quad N_2 = N + m_2 g = (m_1 + m_2)g. \tag{18}$$

Now

$$f \le \mu N = \mu m_1 g. \tag{19}$$

This normal force ($\vec{N}$) is that action-reaction force between the blocks. If block 1 is to stay stationary on block 2, they must have the same acceleration:

$$a_{1x} = a_{2x} = a. \tag{20}$$

From Eqs. (15) and (19),

$$a = f/m_1 \le \mu(m_1/m_1)g = \mu g. \tag{21}$$

Substituting Eq. (15) into Eq. (17) we obtain

$$Fa = m_1 a_{1x} + m_2 a_{2x}.$$

Using Eq. (20), we find

$$Fa = (m_1 + m_2)a,$$

and using Eq. (21), our result is

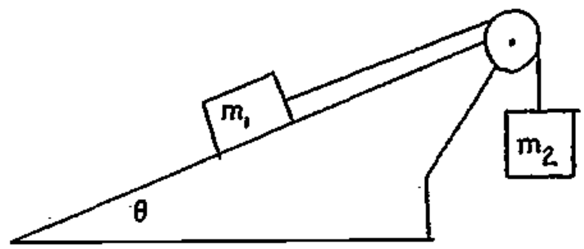
$$Fa \le (m_1 + m_2)\mu g$$
$$\le (1.00\text{ kg} + 4.0\text{ kg})(0.40)(9.8\text{ m/s}^2)$$
$$\le 20\text{ N}.$$

G(5). A block of mass m_1 on a smooth inclined plane of angle θ is connected by a cord over a small frictionless pulley to a second block of mass m_2 hanging vertically.

(a) Draw a free-body diagram of each block.

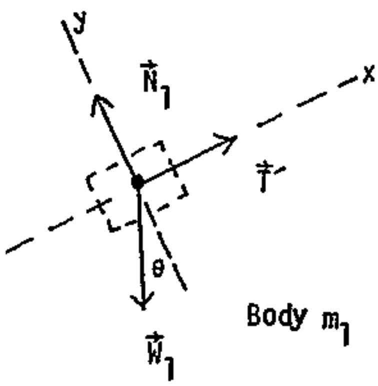
(b) What is the acceleration of each block?

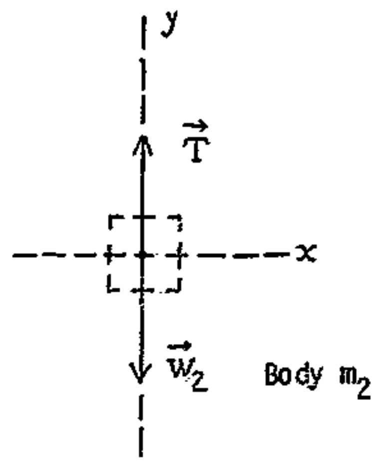
(c) What is the tension in the cord?



Solution

(a)





(b) Consider the rope and the pulley to be massless. Block 2 exerts a downward force $\vec{T}_2$ on the rope, and the rope exerts an upward force $\vec{T}$ (tension) on block 2. $\vec{T}_2 = -\vec{T}$ (by Newton's third law). Block 1 exerts a force $\vec{T}_1$ toward the -x direction on the rope, and the rope exerts a force $\vec{T}'$ on the block. $\vec{T}_1 = -\vec{T}'$ (by Newton's third law). Since the rope and pulley are massless, from the discussion in General Comment 3, $|\vec{T}'| = |\vec{T}| = T$.

For m_1 the coordinate system is oriented such that the x axis lies along the incline with x increasing toward the right. This is done for convenience because two of the forces lie along this direction. Applying Newton's second law in component form to m_1:

$$\Sigma F_x = ma_x: \quad T - W_1 \sin\theta = m_1 a_1, \tag{22}$$

$$\Sigma F_y = ma_y: \quad N_1 - W_1 \cos\theta = m_1(0). \tag{23}$$

For m_2 the coordinate system is oriented in the conventional way with the y axis vertical. Applying Newton's second law in component form to m_2:

$$\Sigma F_y = ma_y: \quad T - W_2 = m_2 a_2. \tag{24}$$

Now m_1 must have the same magnitude of acceleration as m_2 because of the "nonstretchable" connecting rope. Thus $a_1 = a = -a_2$ (when m_1 moves right, a_1 is positive while m_2 goes down, making a_2 negative). Equation (22) becomes

$$T - W_1 \sin\theta = m_1 a \quad \text{or} \quad T = m_1 g \sin\theta + m_1 a.$$

Equation (23) becomes

$$N_1 - W_1 \cos\theta = 0, \qquad N_1 = m_1 g \cos\theta.$$

Equation (24) becomes

$$T - W_2 = -m_2 a, \qquad T = m_2 g - m_2 a.$$

Combining Eqs. (22) and (24) we obtain

$$m_1 g \sin\theta + m_1 a = m_2 g - m_2 a$$

or

$$a = g(m_2 - m_1 \sin\theta)/(m_1 + m_2).$$

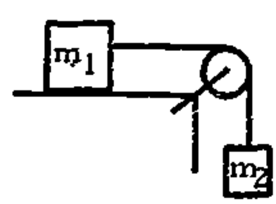
Let's check the units: $\sin\theta$ is unitless so the units of mass cancel, leaving those of acceleration.

Limiting case: If $\theta = 0$, we would have the case:

$$a = gm_2/(m_1 + m_2);$$

if $m_2 = 0$, $a = 0$;

if $m_1 = 0$, $a = g$.



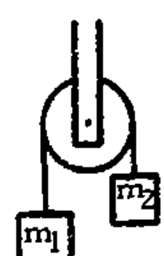
Is this reasonable?

If $\theta = 90°$, we would have the case:

$$a = [(m_2 - m_1)/(m_1 + m_2)]g;$$

if $m_1 = m_2$, $a = 0$;

if $m_1 = 0$, $a = g$.



Is this reasonable?

(c) From Eq. 24,

$$T = m_2(g - a) = m_2 g - \frac{g(m_2 - m_1 \sin\theta)}{m_1 + m_2} = \frac{m_1 m_2 g(1 + \sin\theta)}{m_1 + m_2}.$$

H(5). A 3.6-kg block and a 7.2-kg block connected together by a string slide down a 30° inclined plane. The coefficient of friction between the 3.6-kg block and the plane is 0.100; between the 7.2-kg block and the plane it is 0.200.

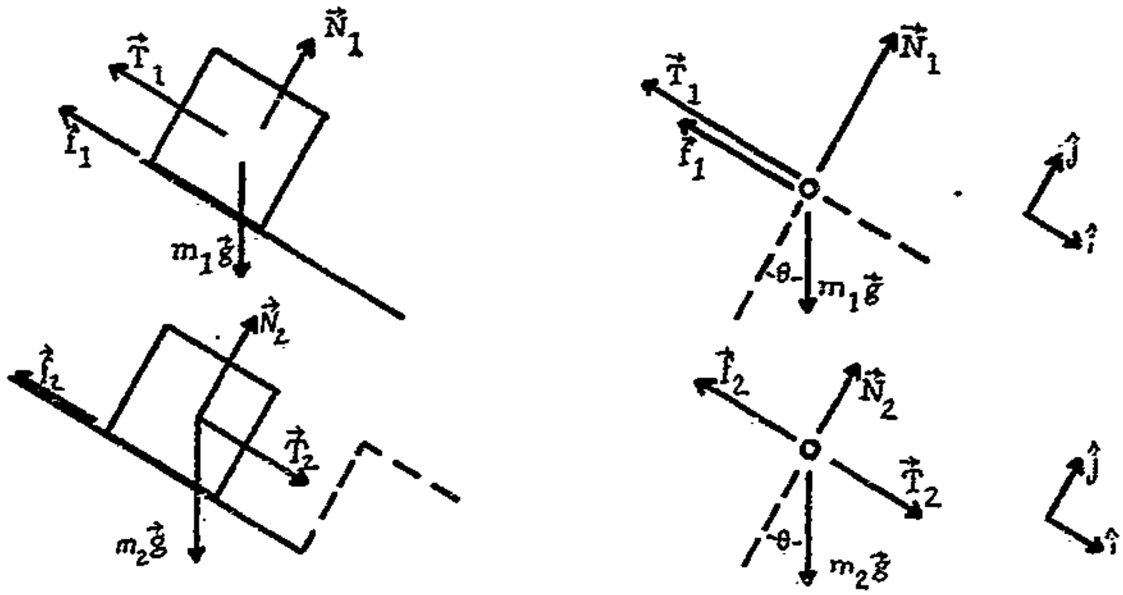
(a) Draw a free-body diagram of each block.

(b) Find the acceleration of the blocks.

(c) Find the tension in the string assuming that the 3.6-kg block leads.

(d) Describe the motion if the blocks are reversed.

Solution



(b) First we must determine the acceleration of each block as though they were not hooked together. If block 2 were accelerating faster than 1, the blocks would be in physical contact. Assume each block acts independently of the other, i.e., $T_1 = T_2 = 0$. Applying Newton's second law for block 1, we find

$$\Sigma F_x = ma_x: \quad m_1 g \sin\theta - f_1 = m_1 a_1,$$

$$\Sigma F_y = ma_y: \quad N_1 - m_1 g \cos\theta = 0.$$

But since $f_1 = \mu_1 N_1$,

$$m_1 g \sin\theta - \mu_1 m_1 g \cos\theta = m_1 a_1$$

or

$$a_1 = g(\sin\theta - \mu_1 \cos\theta) = (9.8\ \mathrm{m/s^2})[0.50 - (0.100)(0.87)],$$

$$a_1 = 4.1\ \mathrm{m/s^2}.$$

For block 2, we find

$$\Sigma F_x = ma_x:\ \ m_2 g \sin\theta - f_2 = m_2 a,$$

$$\Sigma F_y = ma_y:\ \ N_2 - m_2 g \cos\theta = 0.$$

But since $f_2 = \mu_2 N_2$,

$$m_2 g \sin\theta - \mu_2 m_2 g \cos\theta = m_2 a_2,$$

$$a_2 = g(\sin\theta - \mu_2 \cos\theta) = 3.20\ \mathrm{m/s^2}.$$

Therefore, there <u>is</u> a tension in the rope. Apply Newton's second law again: for block 1,

$$\Sigma F_x = ma_x:\ \ m_1 g \sin\theta - f_1 - T_1 = m_1 a_1,$$

$$\Sigma F_y = ma_y:\ \ N_1 - m_1 g \cos\theta = 0,$$

or

$$m_1 g \sin\theta - \mu_1 m_1 g \cos\theta - T_1 = m_1 a_1. \tag{25}$$

For block 2,

$$\Sigma F_x = ma_x:\ \ m_2 g \sin\theta + T_2 - f_2 = m_2 a_2,$$

$$\Sigma F_y = ma_y:\ \ N_2 - m_2 g \cos\theta = 0,$$

or

$$m_2 g \sin\theta + T_2 - \mu_2 m_2 g \cos\theta = m_2 a_2. \tag{26}$$

By General Comment 3, $\vec{T}_1 = \vec{T}_2 = \vec{T}$:

$$a_1 = a_2 = a.$$

Adding Eqs. (25) and (26), we find

$$m_1 g \sin\theta - \mu_1 m_1 g \cos\theta + m_2 g \sin\theta - \mu_2 m_2 g \cos\theta = (m_1 + m_2)a,$$

$$a = \frac{m_1 g(\sin\theta - \mu_1 \cos\theta) + m_2 g(\sin\theta - \mu_2 \cos\theta)}{m_1 + m_2}, \qquad (27)$$

$$a = \frac{(3.6\text{ kg})(9.8\text{ m/s}^2)[(0.50)-(0.100)(0.87)]+(7.2\text{ kg})(9.8\text{ m/s}^2)[(0.50)-(0.200)(0.87)]}{10.8\text{ kg}},$$

$$a = 3.5\text{ m/s}^2.$$

(c) From Eq. (26), we find

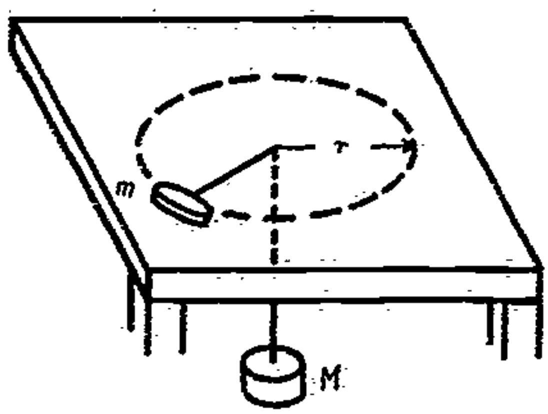
$$T = m_2 a + \mu_2 m_2 g \cos\theta - m_2 g \sin\theta,$$

$$T = (7.2\text{ kg})(3.5\text{ m/s}^2)+(0.200)(7.2\text{ kg})(9.8\text{ m/s}^2)(0.87)-(7.2\text{ kg})(9.8\text{ m/s}^2)(0.50),$$

$$T = 2.20\text{ N}.$$

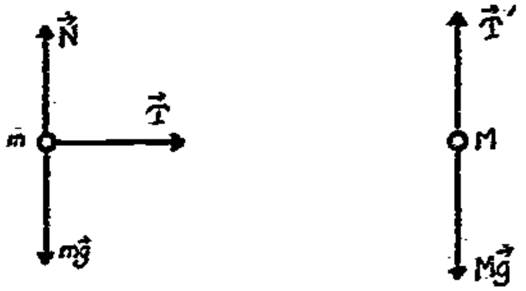
(d) Now the blocks are in contact and in our free-body diagram $\vec{T}_1$ will point down the plane, and $\vec{T}_2$ will point up the plane. $\vec{T}_1 = \vec{T}_2$ by Newton's third law and $\vec{a}_1 = \vec{a}_2 = \vec{a}$. Equations (25) and (26) will be the same except the signs of T_1 and T_2 will be changed; but when we add the equations T_1 and T_2 will cancel leaving the same equation for a or Eq. (27).

I(5). A mass on a frictionless table is attached to a hanging mass M by a cord through a hole in the table. Find the condition (v and r) with which m must spin for M to stay at rest. Draw free-body diagrams for both objects.



Solution

Draw a free-body diagram for both objects. (r = radius of circular path for the object of mass m. v is uniform speed of the object of mass m.) The object of mass m is undergoing uniform circular motion, where $\vec{T}$ provides the centripetal force. Hence $T = mv^2/r$. The



connecting cord is massless, and the hole just changes the direction of the force that the object of mass M exerts on the object of mass m via the rope $|\vec{T}'| = |\vec{T}|$. For the object of mass M, applying Newton's second law we find $T' - Mg = 0$, so $T' = T = Mg = mv^2/r$ or

$$v^2/r = Mg/m.$$

Problems

J(2). An old street car rounds a corner on unbanked tracks. The radius of the tracks is 9.1 m and the car's speed is 4.5 m/s. The loosely hanging hand straps swing to one side.

(a) Draw a free-body diagram of a strap.

(b) What interaction(s) are responsible for the centripetal force on the strap?

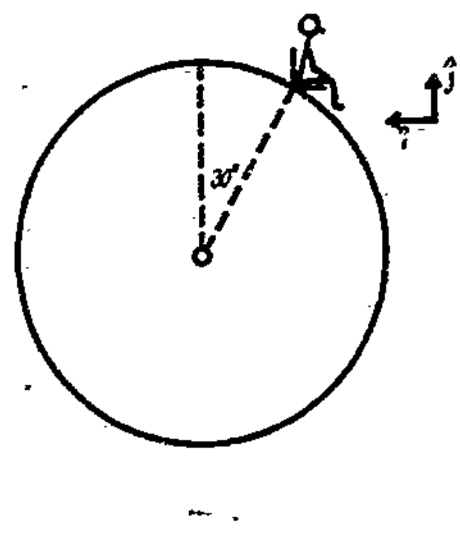
(c) What angle will the strap make with the vertical?

K(2). A boy of 30.0 kg rides on a Ferris wheel that has a radius of 4.0 m and makes one revolution every 8.0 s.

(a) Draw a free-body diagram for the forces on the boy when the boy is in the position shown.

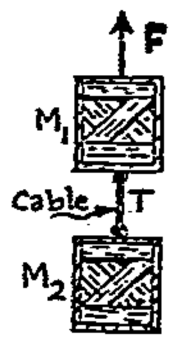
(b) What interaction(s) are responsible for the centripetal force on the boy?

(c) What force does the seat of the Ferris wheel exert on the boy in the position shown?



STUDY GUIDE: Applications of Newton's Laws

L(5). In the figure at the right, the crane exerts a constant upward force on the upper crate. The crates have equal mass: $M_1 = M_2 = M$.



(a) Draw a free-body diagram for each crate.

(b) Write Newton's second law in component form for each crate.

(c) Find the acceleration of the crates and the tension $\vec{T}$ in the cable.

(d) Given that F = 1100 N and M = 50 kg, how long does it take to lift the crates 15.0 m, starting from rest?

M(5). Exactly at 5:00:00 p.m., a Rock Island switch engine proceeds to push two boxcars across the Vine Street crossing. The engine exerts a force $\vec{F}_1$ on boxcar 1; the two boxcars experience frictional forces $\vec{f}_1$ and $\vec{f}_2$, respectively. Assume all these forces are constant.

(a) Draw a free-body diagram for each of the boxcars.

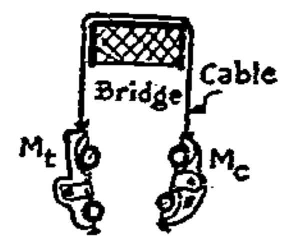
(b) Write Newton's second law in component form for each of them.

(c) Find the acceleration of the boxcars, and the force that the first exerts on the second.

(d) You are waiting to cross on your bicycle, with the front of boxcar 2 directly in front of you, when this train starts off from rest. How much longer will you have to wait? $F_1 = 7.0 \times 10^4$ N, $f_1 = 1.20 \times 10^4$ N, $f_2 = 1.00 \times 10^4$ N, and $M_1 = M_2 = 2.00 \times 10^4$ kg.

N(5). A tow-truck is pulling a car, by means of a long cable, across an icy bridge in the wintertime. Though proceeding very slowly, the truck slides off on one side; but fortunately for the driver, the car simultaneously slides off on the other side. The mass of the car, M_c, exactly equals that of the truck plus driver, so that in a little while the system is hanging at rest, as shown. However, the driver tires of waiting to be rescued, and finally manages to climb out onto the bridge; this unbalances the system by his weight, $M_d\vec{g}$, and the car starts to descend. The bridge is so slippery that the friction between the cable and the bridge can be neglected.

(a) Construct free-body diagrams, showing and identifying all forces acting, for both the car and the truck.

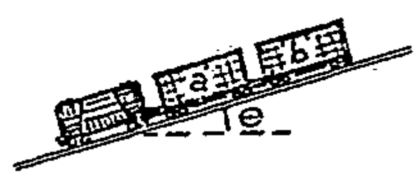


(b) Write Newton's second law in component form for each vehicle.

(c) Calculate the acceleration of the system, and the tension in the cable.

(d) After the car has fallen 2.00 m, the truck strikes the bridge. How fast is it moving then? The masses of the car and truck are $M_c = 1050$ kg and $M_t = 950$ kg.

O(5). A switch engine is pushing two boxcars up an incline, with an acceleration of 1.00 m/s^2 along the incline. Use free-body diagrams and Newton's second law to find the forces $\vec{F}_{ea}$, which the engine exerts on boxcar a, and $\vec{F}_{ab}$, which boxcar a exerts on boxcar b. As your final step, insert the numerical values $\theta = 6.0°$, $M_a = 1.00 \times 10^4$ kg, $M_b = 2.00 \times 10^4$ kg, and (for the frictional forces) $f_a = f_b = 1.00 \times 10^4$ N.

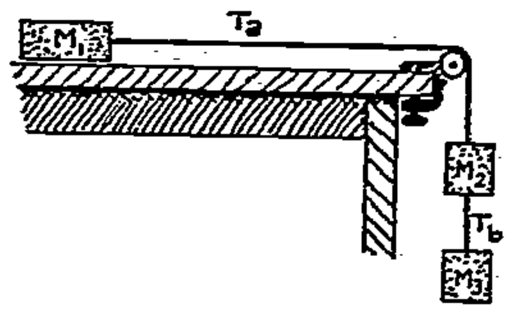


P(5). In the figure at the right, the table top and the pulley are frictionless; the pulley and the cords are massless.

(a) Draw free-body diagrams for all the blocks.

(b) Use Newton's second law to find the acceleration of the system and the tension $\vec{T}_b$.

(c) Given that $M_1 = M_2 = M_3$, how far does M_3 fall in 0.300 s? How fast is it going then?

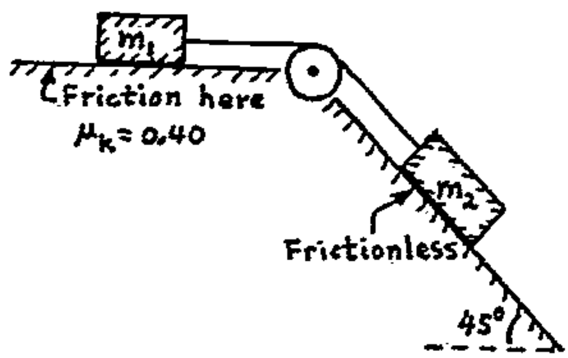


Q(5). Two blocks are connected by a massless string and placed on horizontal and inclined surfaces as indicated in the figure at the right.

(a) Draw free-body diagrams for the two blocks, showing all the forces acting on each block.

(b) Find the magnitude of the acceleration of the system given m_1 = 10.0 kg and m_2 = 20.0 kg.

(c) Find the tension $\vec{T}$ in the string.



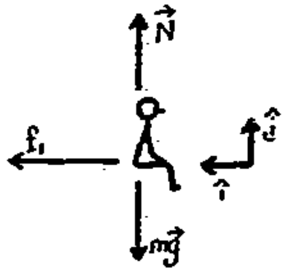
Solutions

J(2). (a)

(b) The horizontal component of the force that the ceiling exerts on the strap.

(c) 13°.

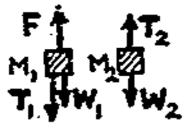
K(2). (a)



(b) The components of the frictional force, normal force, and weight along a radius of the wheel to the position of the boy.

(c) $\vec{f} = 37\hat{i}$ N, $\vec{N} = 230\hat{j}$ N, $\vec{S} = \vec{f} + \vec{N} = (37\hat{i} + 230\hat{j})$ N.

L(5). (a)



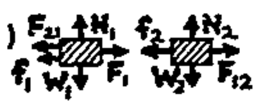
$|\vec{T}_1| = |\vec{T}_2| = T$.

(b) $Ma_{1y} = F - W_1 - T$, $Ma_{1x} = 0$; and $Ma_{2y} = T - W_2$, $Ma_{2x} = 0$, where $a_{1y} = a_{2y}$;

(c) $a_y = (F - W_1 - W_2)/2M = F/2M - g$, $T = (1/2)F$;

(d) $t = 5.0$ s.

M(5). (a)



(b) $M_1a_x = F_1 - F_{21} - f_1$, $M_1a_y = 0$; and $M_2a_x = F_{12} - f_2$, $M_2a_y = 0$;

(c) $a_x = \dfrac{F_1 - f_1 - f_2}{M_1 + M_2}$,

$$F_{12} = \frac{M_2(F_1 - f_1) + M_1 f_2}{M_1 + M_2};$$

(d) 10 s.

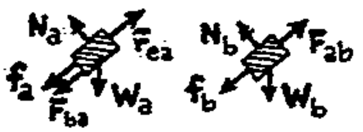
N(5). (a)

(b) $M_ta_{ty} = T - M_tg$, $M_ta_{tx} = 0$, and $M_ca_{cy} = T - M_cg$, $M_ca_{cx} = 0$, with $a_{cy} = -a_{ty}$;

(c) $a_{ty} = \dfrac{M_c - M_t}{M_c + M_t}g$, $T = \dfrac{2M_tM_c}{M_t + M_c}g$;

(d) 1.4 m/s.

O(5).



With the x axis upward along the incline, $M_a a_x = F_{ea} - F_{ba} - f_a - W_a \sin\theta$, and $M_b a_x = F_{ab} - f_b - W_b \sin\theta$, where $F_{ba} = F_{ab}$. So $F_{ab} = M_b a_x + f_b + W_b \sin\theta$
$= M_b(a_x + g \sin\theta) + f_b = 5.0 \times 10^4$ N,
and $F_{ea} = M_a a_x + f_a + W_a \sin\theta + F_{ab}$
$= 8.1 \times 10^4$ N.

P(5). (a)

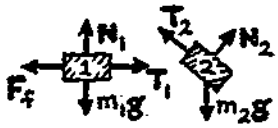
$|\vec{T}_1| = |\vec{T}_2| = T_a$, and $|\vec{T}_2'| = |\vec{T}_3| = T_b$.

(b) $a_x = \dfrac{M_2 + M_3}{M_1 + M_2 + M_3} g$ (down);

$T_b = \dfrac{M_1 M_3 g}{M_1 + M_2 + M_3}$;

(c) $3gs^2/100 = 0.29$ m; $gs/5 = 2.0$ m/s.

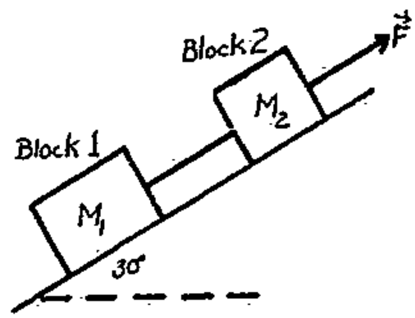
Q(5). (a)



(b) 3.3 or 3.4 m/s^2;
(c) 72 N.

PRACTICE TEST

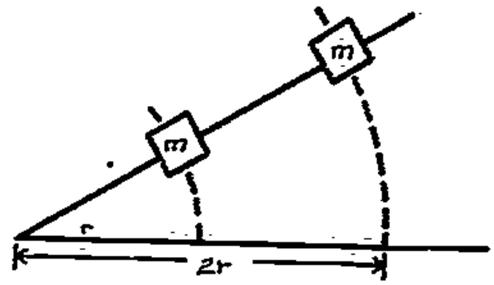
1. A constant force $\vec{F}$ pulls on block 2, so that both objects accelerate upward along the incline. The coefficient of friction between each block and the sloping surface is μ_k.



   (a) Draw a free-body diagram for block 1, showing all the forces acting on it. Identify the agent responsible for each of these forces.
   (b) In a separate diagram, do the same for block 2.
   (c) Set up a suitable coordinate system, and write Newton's second law in component form for block 1. In your final equations evaluate any gravitational and/or frictional forces in terms of g and/or μ_k.
   (d) Do the same for block 2.

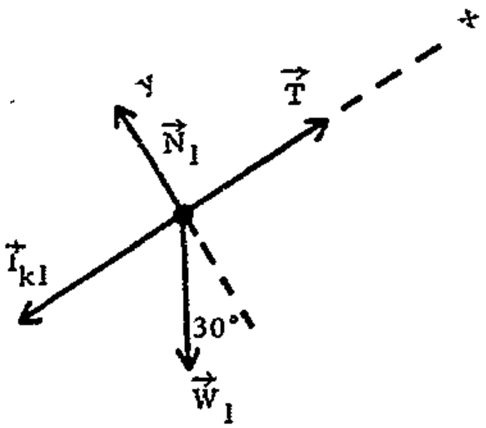
(e) If F = 200 N, M_1 = 16.0 kg, M_2 = 4.0 kg, μ_k = 0.50, determine the acceleration of the blocks.

2. Two small blocks of equal mass are attached to two strings of equal length and are set into rotation on a horizontal frictionless plane with uniform circular motion. Find the ratio of the tension in the inner string to that in the outer string.



Practice Test Answers

1. (a)



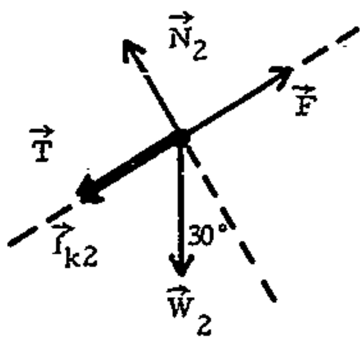
$\vec{N}_1$ is the normal force (⊥ component of plane on block);

$\vec{f}_{k1}$ is the frictional force (∥ component of plane on block);

$\vec{W}_1$ is the weight of block or attraction of earth for block;

$\vec{T}$ is the tension in rope-pull of rope on block.

(b)



$\vec{N}_2$ is the normal force;
$\vec{f}_{k2}$ is the frictional force;
$\vec{W}_2$ is the weight;
$\vec{T}$ is the tension in rope;
$\vec{F}$ is the force pulling agent exerts on block.

(c) $a_x = (T/M_1) - \mu_k g \cos 30° - g \sin 30°$.

(d) $a_x = [(F - T)/M_2] - \mu_k g \cos 30° - g \sin 30°$.

(e) $a = [F/(M_1 + M_2)] - \mu_k g \cos 30° - g \sin 30°$, $a = 0.9\ m/s^2$ upward along the plane.

2. $T_1/T_o = 3/2$.

APPLICATIONS OF NEWTON'S LAWS
Mastery Test Form A

Date ______________

pass recycle

1 2 3 4 5

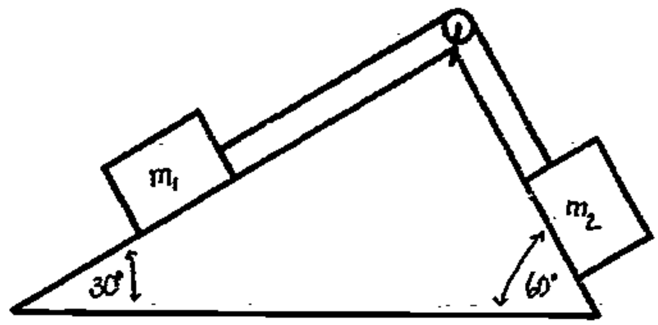
Name ______________________ Tutor ______________________

1. Two bodies of equal mass (4.0 kg) are placed on inclined planes and connected by a rope that passes over a frictionless pulley as shown. All surfaces are frictionless and the rope has negligible mass.

   (a) Make a free-body diagram for each body.

   (b) Describe any action-reaction forces between the two bodies.

   (c) Find the acceleration of the system and the tension in the rope.



2. A 40-kg child stands in place on a merry-go-round. He is 3.00 m from the center of rotation, and the merry-go-round makes one turn every 7.0 s.

   (a) Draw a free-body diagram of the child.

   (b) What interaction(s) is/are responsible for the centripetal force?

   (c) Determine the magnitude of the total force that the merry-go-round exerts on the child.

APPLICATIONS OF NEWTON'S LAWS
Mastery Test Form B

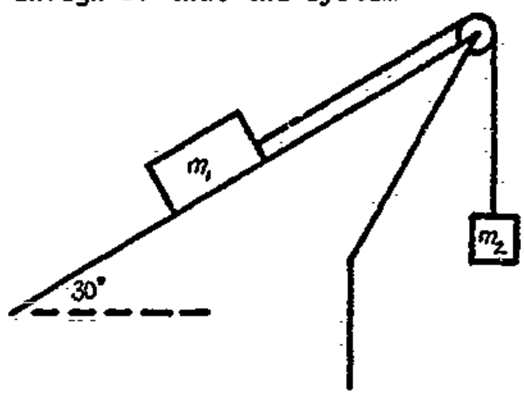
Date ______________

pass recycle

1 2 3 4 5

Name ______________________ Tutor ______________________

1. Each block in the figure has a mass of 5.0 kg. The pulley is frictionless, but the inclined surface has a coefficient of sliding friction $\mu = 0.200$. The static friction is small enough so that the system moves.



   (a) Draw free-body diagrams for both blocks.

   (b) Describe any action-reaction forces between the two bodies.

   (c) Find the acceleration of the blocks.

2. A youngster is flying a 0.50-kg model airplane at the end of a 6.0-m guide control. The control makes an angle θ upward with respect to the horizontal, while the airplane flies in a circular path at a uniform speed of 9.5 m/s. Assume an aerodynamic lift force in an upward vertical direction.

   (a) Sketch the situation and draw a free-body diagram for the airplane.

   (b) What is the force or forces responsible for the centripetal force?

   (c) Express the tension in the guide control as a function of the angle θ of the guide with the horizontal.

   (d) For $\theta = 30^{\circ}$, determine the vertical aerodynamic lift force on the airplane.

Date ______________

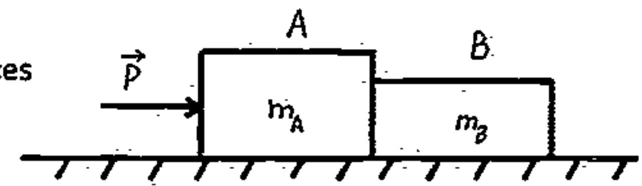
pass recycle

1 2 3 4 5

APPLICATIONS OF NEWTON'S LAWS
Mastery Test Form C

Name ______________________ Tutor ______________________

1. A piston exerts a constant horizontal force $\vec{P}$ on block A so that both blocks accelerate toward the right. The coefficient of kinetic friction between each block and the horizontal surface is μ.



   (a) Draw a free-body diagram for both bodies.

   (b) Describe any action-reaction forces between the blocks.

   (c) If $P = 100$ N, $m_A = 1.50$ kg, $m_B = 0.50$ kg, and $\mu = 0.50$, determine the acceleration of the blocks.

   (d) Determine the force that block A exerts on block B.

2. A car drives at 40 m/s around a circular track of radius 200 m. The track is correctly banked so the car doesn't depend on friction as it rounds the corner.

   (a) Draw a free-body diagram of the car.

   (b) What force (or forces) is responsible for the centripetal force?

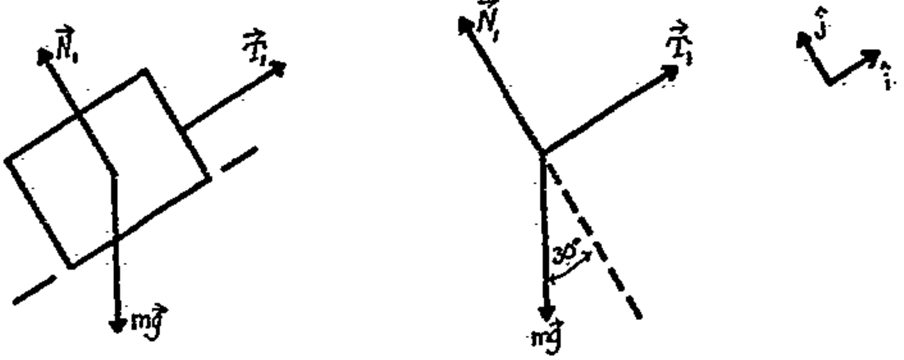
   (c) Determine the angle of the bank (i.e., the angle of incline the road makes with the horizontal).

MASTERY TEST GRADING KEY - Form A

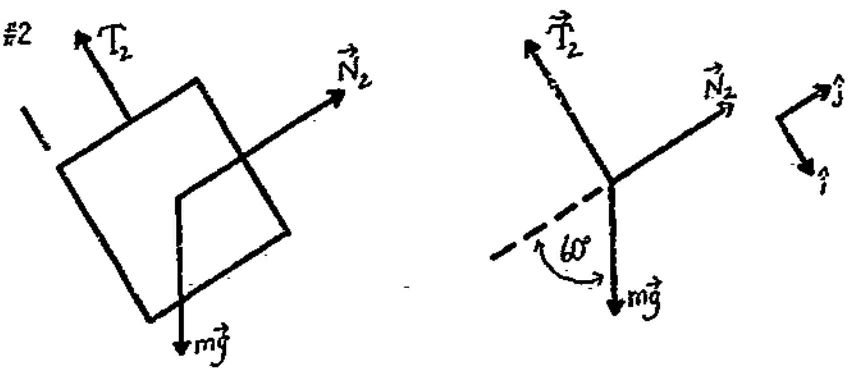
What to Look For	Solutions

1.(a)

Body #1



Body #2



(b) Ask what the rope and pulley do.

(b) None: The action-reaction pairs are between the blocks and rope. The rope transfers a force from one block to the other while the pulley changes the direction of this force.

(c) Watch for the orientation of the coordinate systems. Assume body 2 moves down the 60° incline and body 1 moves up the 30° incline.

(c) Taking the orientation of our coordinate systems as shown in part (a), we apply Newton's second law:

Body 1,

$\Sigma F_x = ma_x$: $T_1 - mg \sin 30^\circ = ma_1$, (1)

$\Sigma F_y = ma_y$: $N_1 - mg \cos 30^\circ = 0$,

Body 2,

$\Sigma F_x = ma_x$: $mg \sin 60^\circ - T_2 = ma_2$, (2)

$\Sigma F_y = ma_y$: $N_2 - mg \sin 60^\circ = 0$.

$T_1 = T_2 = T$

3. See General Comments;

$a_1 = a_2 = a.$

Two bodies move as a system.

Adding Eqs. (1) and (2), we get

$mg \sin 60^0 - mg \sin 30^0 = 2ma,$

$a = (g/2)(\sin 60^0 - \sin 30^0),$

$a = (\frac{9.8 \ m/s^2}{2})(0.87 - 0.50)$

$a = 1.8 \ m/s^2.$

To solve for T, we can go back to either Eq. (1) or (2).

From Eq. (1),

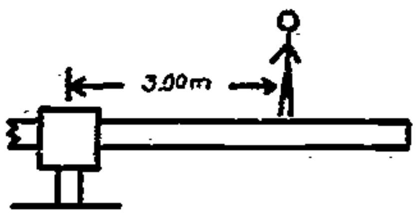
$T = mg \sin 30^0 + ma$

$= m(g \sin 30^0 + a)$

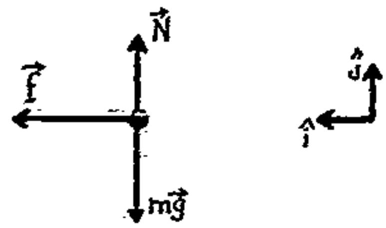
$= (4.0 \ kg)[(9.8 \ m/s^2)(0.50) + 1.8 \ m/s^2]$

$T = 26 \ N.$

2.(a)



2.(a)



(b) The frictional force between the child's feet and the merry-go-round is responsible for the centripetal force.

(c) The merry-go-round exerts a normal force $\vec{N}$ and a frictional force $\vec{f}$ on the child, so we need both $\vec{f}$ and $\vec{N}$.

(c) Applying Newton's second law:

$\Sigma F_x = ma_x: \quad f = mv^2/r,$

$\Sigma F_y = ma_y: \quad N - mg = 0.$

$v = \frac{2\pi r}{8.0 \ s} = \frac{2\pi(3.00 \ m)}{8.0 \ s},$

$f = \frac{(40 \ kg)4\pi^2(3.00 \ m)^2}{(8.0 \ s)^2(3.00 \ m)},$

The magnitude of the total force exerted by the merry-go-round is given by $F_T = (N^2 + f^2)^{1/2}$.

$f = \frac{4\pi^2(40 \ kg)(3.00 \ m)}{(8.0)^2 s^2} = 74 \ N,$

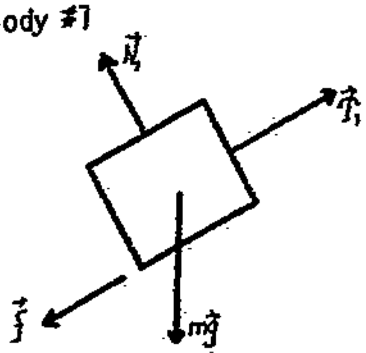
$N = mg = (40 \ kg)(9.8 \ m/s^2) = 390 \ N,$

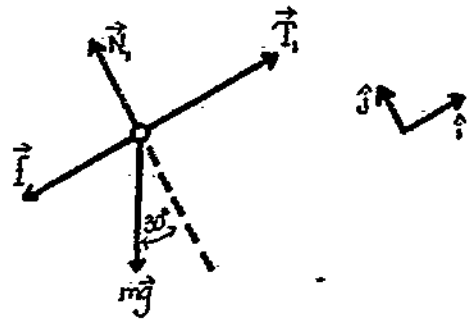
$F_T = [(390 \ N)^2 + (74 \ N)^2]^{1/2} = 400 \ N.$

MASTERY TEST GRADING KEY - Form B

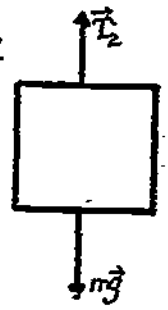
What to Look For	Solutions

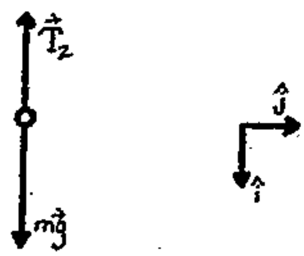
1.(a) Body #1





Body #2





(b) See General Comment 3.

(b) None: The action-reaction pairs are between the blocks and the rope, but not between the blocks.

(c) Assume block 2 goes vertically down while block 1 slides up the plane.

(c) Applying Newton's second law to body 1:

$$\Sigma F_x = ma_x: \quad T_1 - f - mg \sin 30^0 = ma_1,$$

$$\Sigma F_y = ma_y: \quad N_1 - mg \cos 30^0 = 0. \quad (1)$$

For body 2: $\Sigma F_x = ma_x$: $mg - T_2 = ma_2$. (2)

For body 1:

$f = \mu N_1 = \mu mg(\cos 30^0)$.

Substituting this into Eq. (1),

$\bar{T}_1 = \bar{T}_2 = \bar{T}$.

$$T_1 - \mu mg(\cos 30^0) - mg(\sin 30^0) = ma_1. (3)$$

See General Comment 3.

Add Eqs. (3) and (1).

$a_1 = a_2 = a$.

$mg - \mu mg \cos 30^\circ - mg \sin 30^\circ = 2ma,$

The two bodies move at the same rate.

$a = g/2[1 - \mu \cos 30^\circ - \sin 30^\circ]$

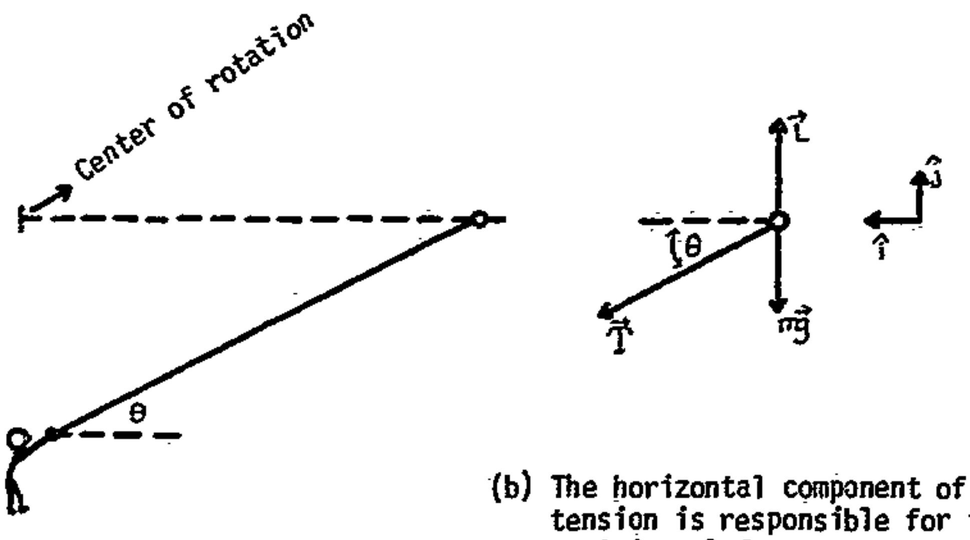
$a = (9.8\ m/s^2/2)[1 - (0.200)(0.87) - 0.50],$

$a = 1.6\ m/s^2.$

What would have resulted if we had assumed block 1 was moving down the plane?

The frictional force on block 1 would change direction and the calculated acceleration would be negative, showing we had chosen the incorrect situation.

2.(a)



(b) The horizontal component of the tension is responsible for the centripetal force.

(c) Note the radius of the circular path of the airplane is R cos θ.

(c) Applying Newton's second law to the airplane, we find

$$\Sigma F_x = ma_x: \quad T \cos \theta = mv^2/r, \qquad (4)$$

$$\Sigma F_y = ma_y: \quad L - T \sin \theta - mg = 0 \qquad (5)$$

From Eq. (4):

$$T \cos \theta = mv^2/R \cos \theta,$$

$$T = mv^2/R(\cos\theta)^2$$

$$T = \frac{(0.50\ \mathrm{kg})(9.5\ \mathrm{m/s})^2}{(6.0\ \mathrm{m})\cos^2\theta}$$

$$T = (7.5/\cos^2\theta)\ \mathrm{N}.$$

(d) From Eq. (5):

$$L = T\sin\theta + mg,$$

$$L = \frac{7.5}{(\cos 30^\circ)^2}\sin 30^\circ$$

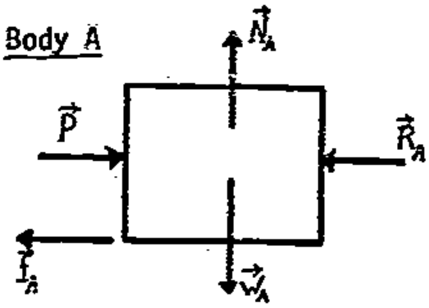
$$+ (0.50\ \mathrm{kg})(9.8\ \mathrm{m/s^2}) = 9.8\ \mathrm{N}.$$

MASTERY TEST GRADING KEY - Form C

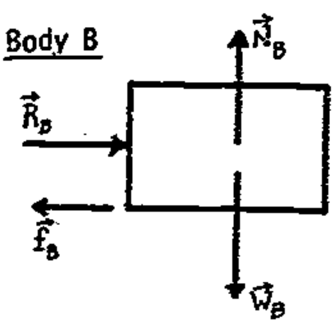
What to Look For	Solutions

1. (a)

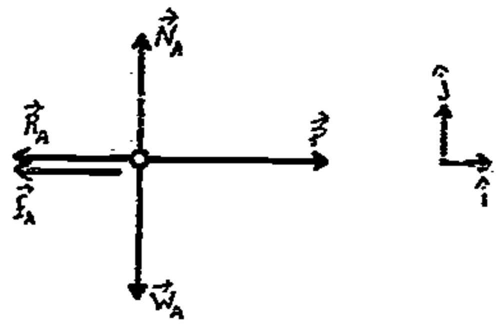
Body A

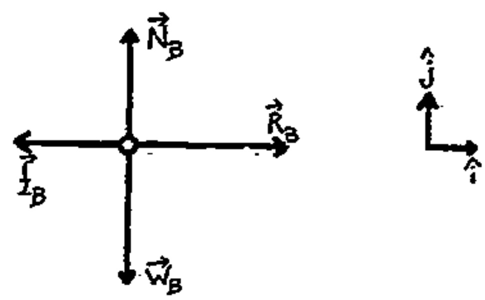


Body B



1. (a)





(b) R_A and R_B are an action-reaction pair. R_A is the force that block B exerts on block A. R_B is the force that block A exerts on block B. $R_A = -R_B$.

(c) Applying Newton's second law to block A, we find

$$\Sigma F_x = ma_x: \quad P - f_A - R_A = m_A a_{Ax},$$

$$\Sigma F_y = ma_y: \quad N_A - W_A = 0, \qquad (1)$$

but

$$f_A = \mu N_A = \mu W_A = \mu m_A g.$$

Substituting this into Eq. (1),

$$P - \mu m_A g - R_A = m_A a_{Ax}. \qquad (2)$$

Block B,

$$\Sigma F_x = ma_x: \quad R_B - f_B = m_B a_{Bx},$$

$$\Sigma F_y = ma_y: \quad N_B - W_B = 0, \qquad (3)$$

$$f_B = \mu N_B = \mu m_B g, \quad R_B - \mu m_B g = m_B a_{Bx}. \qquad (4)$$

$R_A = R_B$ as discussed in part (b).

$a_{Ax} = a_{Bx} = a.$

Add Eqs. (2) and (4):

$$P - \mu m_A g - \mu m_B g = (m_A + m_B)a,$$

$$a = \frac{P - \mu g(m_A + m_B)}{m_A + m_B} = \frac{P}{m_A + m_B} - \mu g,$$

$$a = \frac{100\ \text{N}}{2.00\ \text{kg}} - (0.50)(9.8\ \text{m/s}^2) = 45\ \text{m/s}^2.$$

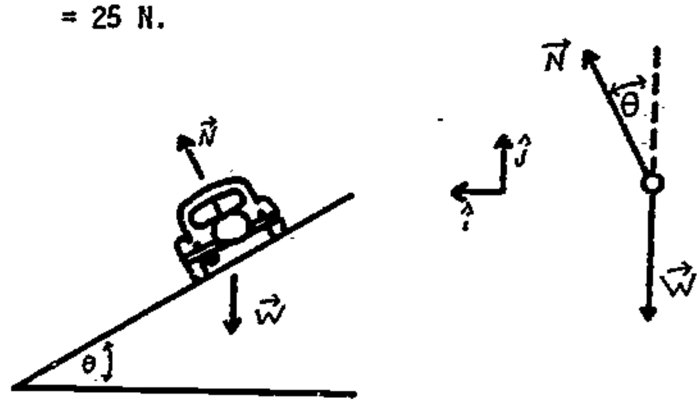
(d) From Eq. (4),

$$R_B = \mu m_B g + m_B a$$

$$= (0.50)(0.50\ \text{kg})(9.8\ \text{m/s}^2) + (0.50\ \text{kg})(45\ \text{m/s}^2)$$

$$= 25\ \text{N}.$$

2. (a)



(b) The component of the normal force in the horizontal direction is the centripetal force.

(c) Apply Newton's second law,

$$N \sin\theta = mv^2/r,$$

$$N \cos\theta - mg = 0.$$

Divide these two equations:

$$\frac{N \sin\theta}{N \cos\theta} = \frac{mv^2/r}{mg},$$

$$\tan\theta = v^2/rg$$

$$\theta = \arc\tan\ (v^2/rg)$$

$$= \text{arc tan}\ \left(\frac{(40\ m/s)^2}{(200\ m)(9.8\ m/s^2)}\right)$$

$$= \text{arc tan}\ (0.82).$$

$$\theta = 39°.$$