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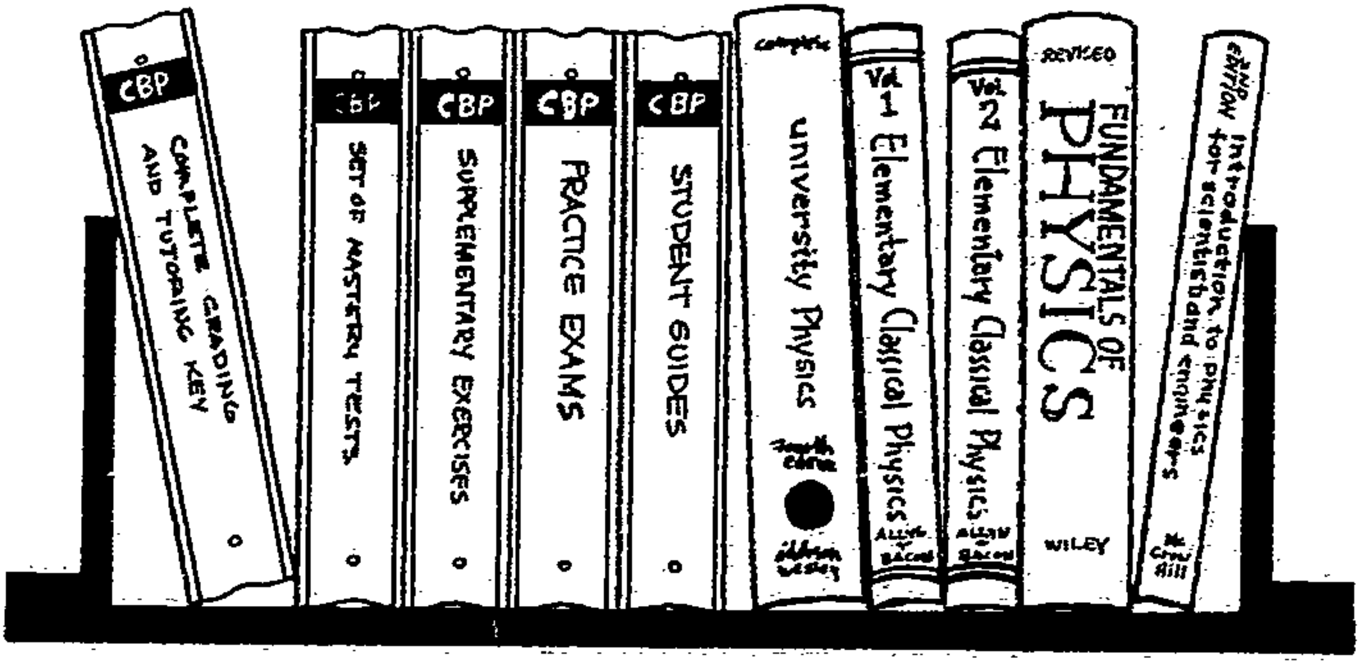
ABSTRACT

This is part of a series of 42 Calculus Based Physics (CBP) modules totaling about 1,000 pages. The modules include study guides, practice tests, and mastery tests for a full-year individualized course in calculus-based physics based on the Personalized System of Instruction (PSI). The units are not intended to be used without outside materials; references to specific sections in four elementary physics textbooks appear in the modules. Specific modules included in this document are: Module 1--Dimensions and Vector Addition and Module 2--Rectilinear Motion, plus a trigonometry and calculus review. (CP)

STUDY MODULES FOR CALCULUS-BASED GENERAL PHYSICS*



CBP Workshop
Behlen Laboratory of Physics
University of Nebraska
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*Supported by The National Science Foundation

Comments

These modules were prepared by fifteen college physics professors for use in self-paced, mastery-oriented, student-tutored, calculus-based general physics courses. This style of teaching offers students a personalized system of instruction (PSI), in which they increase their knowledge of physics and experience a positive learning environment. We hope our efforts in preparing these modules will enable you to try and enjoy teaching physics using PSI.

Robert G. Fuller
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These modules were prepared by the module authors at a College Faculty Workshop held at the University of Colorado - Boulder, from June 23 to July 11, 1975.

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WHY

TRY

PSI?

The personalized system of instruction (PSI)[1-3] has been used in physics courses in a number of colleges and universities for several years. Studies have shown that in some cases the students in a PSI physics course learn physics better than students in a lecture course. In almost all cases, a majority of the students prefer the PSI course to the lecture course, yet they report working harder in the PSI course. Why is it that this system of instruction is not more widely used? At least one reason is the large effort required of the PSI instructor to develop the study modules for his course. We have written these modules to enable you to use PSI in your calculus-based physics course. With these modules we believe the effort required to use PSI will be greatly reduced. We hope you will give PSI a try.

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[1] For study modules on the use of these CBP Modules, see Appendix A.

[2] J. Gilmore Sherman, (ed.) Personalized System of Instruction, 41 Germinal Papers, A Selection of Readings on the Keller Plan, (W. A. Benjamin, 1974)

[3] Fred S. Keller and J. Gilmore Sherman The Keller Plan Handbook, Essays on a Personalized System of Instruction, (W. A. Benjamin, 1974)

COMMENT TO USERS

In the upper right-hand corner of each Mastery Test you will find the "pass" and "recycle" terms and a row of numbers "1 2 3 ..." to facilitate the grading of the tests. We intend that you indicate the weakness of a student who is asked to recycle on the test by putting a circle around the number of the learning objective that the student did not satisfy. This procedure will enable you easily to identify the learning objectives that are causing your students difficulty.

ERRATA

Our apologies to Dr. Van Bluemel for misspelling his name in the list of CBP Module Authors! This will be corrected when reprinted.

ERRATA

Calculus Review: p. 15, first three entries in the Table of Derivatives should read

x^{-3} . . . $-3/x^4$ $\quad$ x^{-2} . . . $-2/x^3$ $\quad$ x^{-1} . . . $-1/x^2$.

P. 8, second to last line ..."...= 8/2 = 4." P. 9, line 7, "I = $(1/3)2^3 - (1/3)0$..."

COMMENT TO USERS

It is conventional practice to provide several review modules per semester or quarter, as confidence builders, learning opportunities, and to consolidate what has been learned. You the instructor should write these modules yourself, in terms of the particular weaknesses and needs of your students. Thus, we have not supplied review modules as such with the CBP Modules. However, fifteen sample review tests were written during the Workshop and are available for your use as guides. Please send $1.00 to CBP Modules, Behlen Lab of Physics, University of Nebraska - Lincoln, Nebraska 68588.

FINIS

This printing has completed the initial CBP project. We hope that you are finding the materials helpful in your teaching. Revision of the modules is being planned for the Summer of 1976. We therefore solicit your comments, suggestions, and/or corrections for the revised edition. Please write or call

CBP WORKSHOP
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NOTE TO THE INSTRUCTOR

You need to make several specific decisions as you prepare for the student use of these materials.

1. You must select a textbook. These materials are not intended to be used independent of other reading and study materials. In fact, these modules have been keyed to four different calculus-based general physics textbooks.[1-4] If you wish your students to read only one of these books, you may remove the pages in each module that refer the students to the other textbooks. We intend that these materials can be easily adaptable to any other general physics textbook that you choose, but you will have to provide the suggested study procedures for such a text.

2. You should select a sequence in which you wish your students to study these modules and your selected text. We have labeled these modules by their physics content and provide a flow chart for you on pp. 2 and 3 of Preparation of Your Orientation Module. You may wish to give them numbers to suggest a learning sequence for your students. In general, your students will find a sequence that follows your selected textbook preferable to other sequences.

3. You may need to prepare an orientation module for your students. We have provided a study guide to assist you in the preparation of an orientation module. (See the next page.) If you have never taught using the self-paced, mastery-oriented, student-tutored style of instruction, known as the personalized system of instruction (PSI) or Keller Plan, you will find it useful to read the references given on the previous page.

4. You may wish to provide additional learning activities for your students. We have found that film loops and audio tapes can be very useful to students. We have provided in our modules some references to film loops. You can provide audio-tape instructions about problem solving as you wish. The task of working a problem while listening to suggestions from you on a cassette tape can be a good learning experience for your students.

These modules, each of which includes a student study guide with worked problems and a practice test, as well as equivalent mastery tests and grading keys, provide the basic written ingredients for a PSI calculus-based general physics course. You can also use these materials to improve a traditional lecture-recitation course, through the use of enriched homework assignments, repeatable testing, or in other ways.

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[1] Frederick J. Bueche, Introduction to Physics for Scientists and Engineers (McGraw-Hill, New York, 1975), second edition.

[2] David Halliday and Robert Resnick, Fundamentals of Physics (Wiley, New York, 1970; revised printing, 1974).

[3] Francis Weston Sears and Mark W. Zemansky, University Physics (Addison-Wesley, Reading, Mass., 1970), fourth edition.

[4] Richard T. Weidner and Robert L. Sells, Elementary Classical Physics (Allyn and Bacon, Boston, 1973), second edition, Vols. 1 and 2.

PREPARATION OF YOUR ORIENTATION MODULE

INTRODUCTION

Perhaps you have walked into a large shopping center and wandered around looking for the items you came to purchase. The feelings of lostness you had may be similar to the feelings of students as they wander into a self-paced, mastery-oriented, student-tutored physics course. As the instructor, you need to provide them with the assurance that all is well because you know where everything is (if you don't, please read Appendix A). You also need to provide your students with a road map and catalog for your physics course. This combination road map-catalog is often called the orientation module. (If you want your orientation module to match the CBP modules have it typed on an IBM Selectric 12 pitch typewriter with letter gothic type.)

PREREQUISITES

Before you begin this module you should be familiar with the personalized system of instruction (See Appendix A) and the content structure of the physics textbook you have chosen for student use.

LEARNING OBJECTIVES

After you have mastered the content of this module, you will be able to write an orientation module for your course which contains:

1. A description of the organization of the physics modules and the format of each module.
2. A suggested procedure for student progress in completing the modules.
3. An identification of the resources for learning and of their availability.
4. A statement of the requirements and responsibilities of students.
5. An explanation of the grading policy.

GENERAL COMMENTS

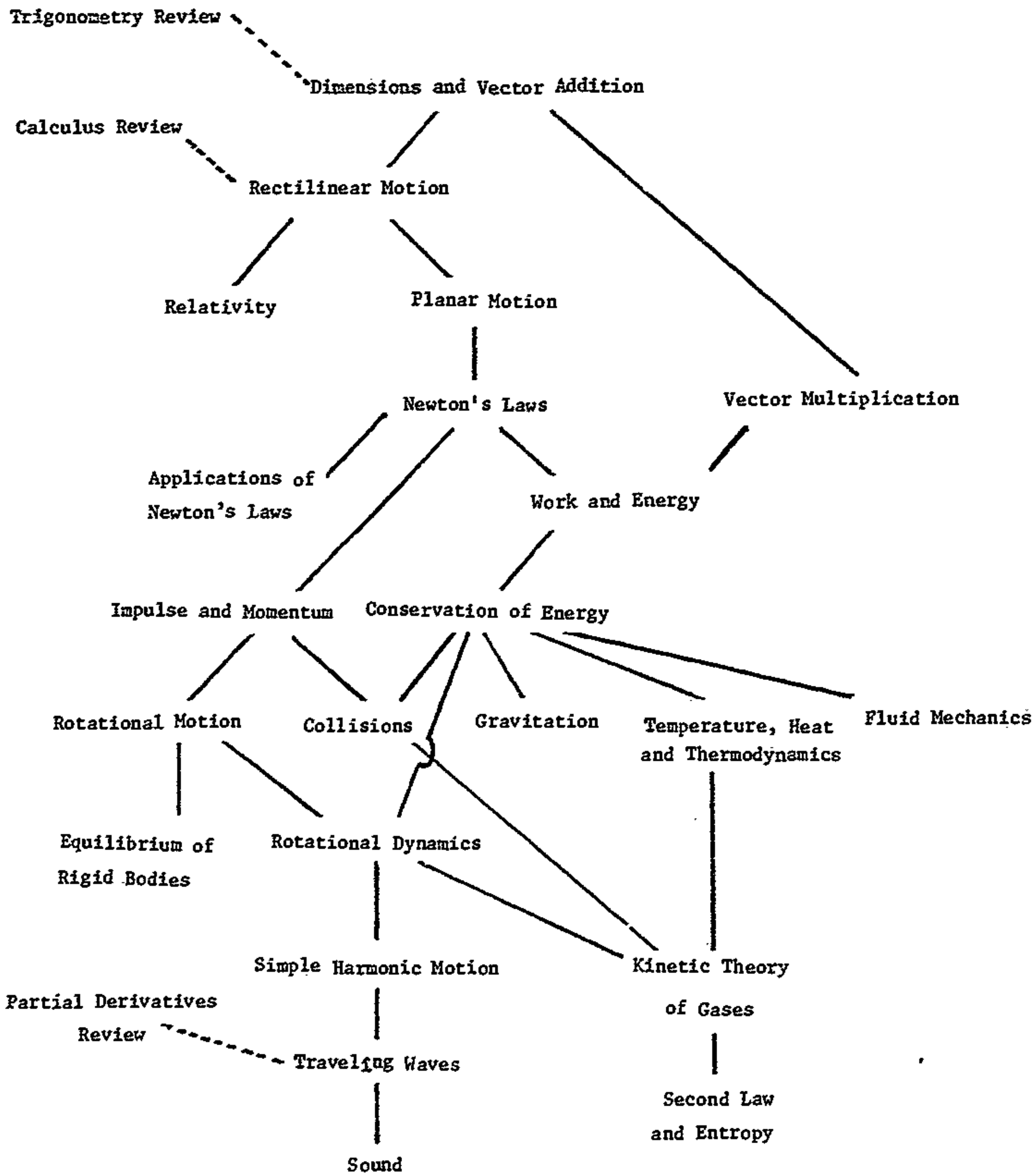
The orientation module you must prepare serves as the introduction of your students to your PSI physics course. For many students this may be their first experience in a course that requires so much active involvement from them. The self-pacing feature of PSI will demand that each student take responsibility for the effective use of his study time. The repeated testing enables students to perfect their problem solving skills. Your orientation module is to encourage your students to develop these personal attributes.

SUGGESTED STUDY PROCEDURE

To complete Objective 1, you must become familiar with the structure of the physics textbook you have chosen. You need to know what the prerequisites are for the various textbook chapters. The CBP study modules we have prepared are a compromise between the sequence of physics topics in four textbooks. You may wish to rearrange the sequence in which you use these modules or you may change the sequence of topics in the textbook. Examine the following prerequisite charts for the CBP modules.

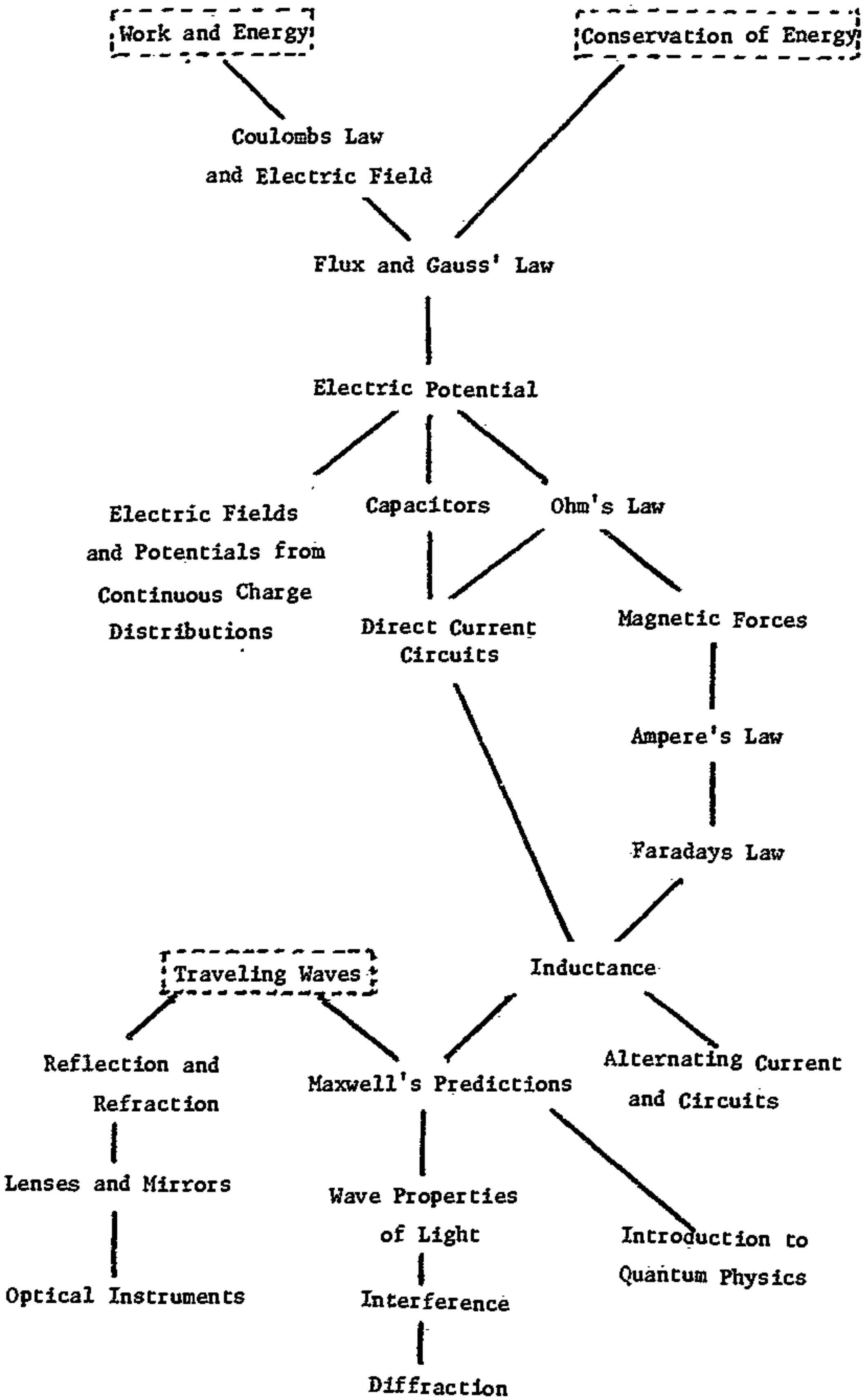
Mechanics, Sound, Heat

CBP Content and Prerequisite Description



Electricity, Magnetism, Light

CBP Content and Prerequisite Description



SUGGESTED STUDY PROCEDURE (Continued)

For example, you wish your students to study the CBP module on Work and Energy, they should previously completed the modules on Newton's Laws, Vector Multiplication, Planar Motion, Rectilinear Motion, and Dimensions and Vector Addition. The Mathematics skills required are treated in the trigonometry review and the calculus review modules. You will notice that the CBP modules permit some branching, e.g. after the student has completed the Newton's Laws module, he can do any one of three modules, Applications of Newton's Laws, Work and Energy, or Impulse and Momentum.

Decide how you want your students to proceed through the materials. You can allow branching if you wish. Fill out the sequence chart below.

Module Sequence Number	Textbook Chapter(s)	CBP Module
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		
16		
17		
18		
19		
20		

Examine the format of several of the study modules. You will notice that each module contains sections labeled: introduction, prerequisites, learning objectives, general comments, four separate suggested study procedures (one for each textbook), problem set with solutions, practice test (with answers), at least three forms of mastery tests and grading keys. You need to remove the mastery tests and grading keys from each module for use during the usual class testing times. You need to remove the suggested study procedures that do not relate to your textbook. Then you have a study guide in the format that you will give your students. Prepare a short description of the study guide to put in your orientation module.

To complete Objective 2 you need to write a suggested procedure for your students to progress through your course. Below is an example.

"How the course will be run. Since you are to do the work, I shall not lecture but provide you with reading assignments, study hints, and exercises. You work where and when you wish, at your own rate within the ten-week limit of the course. I have divided the subject matter into fifteen modules, nine basic ones required for a C grade, and six advanced ones leading to B or A. Thus, each basic module covers about a week's work at a basic level."

"Prepare yourself through study at home. Whenever you feel you have mastered the material in a module, you present yourself for a test during class hours. If you are successful, you go on to another module; if not, you go back for more work on the module on which you were tested."

"DO NOT GO ON TO THE NEXT MODULE UNTIL YOU HAVE PASSED THE TEST ON THE PREVIOUS ONE: IT IS MORE IMPORTANT TO KNOW A FEW THINGS WELL THAN TO KNOW A LOT BADLY."

"If you do not understand something, use the class time to get tutorial help. If you wish to attend lectures, you can go to those given for Section 1 or 2."

(Karplus, UC-B)

An additional example is found as Exhibit A, page 9.

For Objective 3, you need to identify the resources available to assist your students in their learning of physics. An example of resources for students use is shown below.

"The materials you should have for this course are:

A. Elementary Classical Physics, Vol. 1, by R. T. Weidner and R. L. Sells, hereafter called "W & S".

B. Study Guide in Physics, Vol. 1, Mechanics, by Victor Namias, hereafter called "Namias".

C. A three-ring binder in which to keep:
Information and notices concerning this course
Study guides to the units as they are issued to you
Your notes on the reading
Your solutions to the problems
Your progress chart
Other miscellaneous material, such as Physics 111 quizzes and solutions, etc."

(Baker, RU)

Additional examples of student resources are found as Exhibit B, pages 10 and 11.

For Objective 4, you may wish to offer a general statement of the responsibilities of the students, provide a list of student responsibilities, or to prescribe proper problem solving performance. Examples are shown as Exhibit C on pages 11 - 13.

A progress chart or student record sheet can be helpful as a guide for students. They can then guage their pace according to a long term performance goal. An example is shown as Exhibit D on page 13.

For Objective 5, finally, your orientation module provides the written grading contract with the student. In the orthodox Keller Plan course the grade is determined by the number of modules that a student masters.

"2. Grading in Physics 111

The final grade in Physics 111 will depend on the number of units completed by the end of the semester. The relation between semester grade and unit completed is given on the progress chart that each student receives."

(Baker, RU)

By now, the Keller Plan has been adapted to fit a wide variety of grading systems: a final examination may be used for part of the grade.

"Grading: The term grade will be determined largely by how many units are completed by the end of the semester (Friday of Review Week). The number of units completed out of 24 will count 90% of the term grade. The comprehensive final exam may be taken at the option of the student and will count an additional 10%. The A range is 90-100%, B is 80-90%, C is 70-80%, D is 60-70% and F is less than 60%. Completing less than 14 units will result in a term grade of F."

(Fuller, UN)

Two levels of mastery may be required.

"The two examinations on each unit are designed to be of increasing difficulty, which reflect the increasing mastery of the subject area by the student. (The examinations are not intended to be a test of a high level of intelligence or sophisticated physical understanding, just mastery).

Exam P	Average Mastery of unit	(To pass the course this exam must be passed in all units.)
Exam S	Complete Mastery	

"Course grading will go as follows:

A - all 25 examinations passed

B - 20 examinations passed, 15P + 5S

C - 15 examinations passed, 15P

No D grades will be assigned except by failure of the final exam. To pass the course, the first "P" unit examination is each of the 15 subject areas must be passed."

(Snow, UMR)

A small bonus may be given for keeping ahead of schedule.

"Grading policy. Your final grade will be based on your work, as follows:

For each module completed "pass"	1 point
For each module completed "partial"	.7 Points
For each module completed "pass" ahead of "slow pace" (Modules 1-9 only)	.12 points bonus
Final exam	0 to 5 points
Maximum possible	21
For grade of A	19
For grade of B	15
For grade of C	12
For grade of D	9
For grade of F	8 or less"

(Karplus, UC-B)

PROBLEM SET WITH SOLUTIONS (The number in parenthesis is the number of the learning objective that is covered by the question.)

A(1) Are the CBP modules in the same sequence as your textbook?

SOLUTION: Check the CBP prerequisite diagrams. In particular, notice the location of the module - Equilibrium of Rigid Bodies. In many texts equilibrium is treated much earlier in the book than as shown in CBP. Plan your strategy to avoid difficulty with this problem and look for other possible sequence troubles.

B(2) Are you going to regularize student progress by the use of deadlines?

SOLUTION: The original Keller article (see Reference in Appendix A) suggested unlimited time and minimal punishment for procrastination. Many physics teachers find that suggestion incompatible with their institution's rules or with their own teaching style. Deadlines can subvert the intentions of a PSI course and need to be carefully considered. A PSI physics course can teach physics and help students learn about the logical consequences of irresponsible behavior.

C(3) What additional learning resources are you going to make available to your students?

SOLUTION: The use of audio tapes or video tapes to augment the textbook reading and problem assignments has been successfully used in a number of colleges. Talking a student through physics problems, by means of an audio casette tape can be very effective. You can prepare your own audio tapes quite easily and add another dimension to your interaction with your students.

D(4,5) Have you described clearly what you are requiring of your students?

SOLUTION: Just as the learning objectives in each module show what physics content must be learned by the student, so your orientation module shows what the students must do to pass mastery tests and how their grade is to be determined. An essential ingredient in a successful PSI course is an open grading contract. A student must be able to decide what tasks he must do to obtain the grade he desires.

PRACTICE TEST

Write your orientation module and try it on your students. (We are interested in how you use the CBP modules. Please send us a copy of your orientation module - CBP Workshop, Behlen Laboratory of Physics, University of Nebraska, Lincoln, NE 68508)

MASTERY TEST

Write a short test over your orientation module and check it before a student is permitted to take the first mastery test. It is a good way to reward a student for reading your orientation material.

Exhibit A - Suggested Procedure

The Teaching Method

"Unlike courses which have a rigid pace locked to a series of lectures, and which too often leave most of a class either bored or confused, this course allows you to proceed at a pace of your own choice. You will not be held back by slower students, or forced to go ahead when you are not ready. Some of you will finish the course in 5 or 6 weeks, but fast may not be best."

"The material of this quarter is divided into 14 units, each of which covers somewhat less than one chapter of the text (Halliday and Resnick). You have enrolled in sections of about 10 students led by a tutor whose job is to help you learn the material of each unit in sequence. For each unit we have written a sheet which spells out objectives of the unit, indicates relevant sections of the text and in some cases contains supplementary material, and includes a set of problems to help you learn and to test your understanding."

"After you have completed the objectives of a unit by self study, working with other students, help from your tutor, or any other method which works, go to your tutor. The tutor will provide you with a written quiz which you should be able to work in about a half hour by yourself (no book) in the quiz room. Immediately after taking the quiz, you and the tutor go over the quiz in order to decide how well you understand the material. If your quiz is correct, or if you understand things well enough to find and correct your errors on the spot without help, you pass on to the next unit. If you cannot do the quiz by yourself, the tutor will help you find your difficulties and suggest some review work which should enable you to pass the next quiz. We do not care how many quizzes you take to pass a unit; the important thing is the number of units passed. Most students pass most units on the first or second try, but once in a while someone needs three or four quizzes to understand a unit. You can't lose; a passed quiz advances you in the course and the others get you some individual tutoring where you need it most. The important thing is to avoid putting off a quiz; you should take at least two or three quizzes per week to finish the course."

"The results will surprise you. You will learn a remarkable amount of science very well, well enough to discuss it with your tutor and with other students. You will acquire the ideas needed to develop positions of your own on scientific aspects of some problems of public concern, and you will get some experience with the most powerful decision making tool we know; the making of a mathematical model which approximates the real world and can be used to make predictions regarding real situations."

(Swanson, UC-SD)

Exhibit B - Learning Resources

"Learning Materials You are required to purchase these items (approximate costs):

1. Complete set of Physics 5A Study Guides $ 2.50
(purchase syllabus cards from Physics Department, Room 154 LeConte)

2. Resnick and Halliday, PHYSICS I or PHYSICS I & II 11.50 (20.50)
(1966 Edition) PHYSICS I is used in 5A and 5B, PHYSICS II in 5C and 5D. One part of the two-volume edition is easier to carry around with you.

The following are recommended--you may use them in the Physics library or Moffitt on 2-hour reserve, buy them for yourself, or share them with friends:

3. Young, FUNDAMENTALS OF MECHANICS AND HEAT (1974) 12.00
(referred to for additional reading and text for some advanced modules; more readable and insightful than #2, but fewer worked-out examples)

4. Karplus, INTRODUCTORY PHYSICS, A MODEL APPROACH 14.00
(qualitative background material, especially if your high school physics course was weak)

5. Taylor, PROGRAMMED STUDY AID FOR INTRODUCTORY PHYSICS I 4.00
(step-by-step worked out physics problems, keyed to Resnick and Halliday; very good if you have difficulty)

6. Feynmann, THE FEYNMANN LECTURES ON PHYSICS
(excellent advanced supplementary reading)

7. Helmholz and Moyer, BERKELEY PHYSICS COURSE (rev.)
Sears and Zemanski, UNIVERSITY PHYSICS, Vol. I
(alternate references, in case you get bogged down)

The following are provided for your use and are NOT for sale:

1. Worked-out solutions to Exercises in all modules
(in classroom, Physics Library, and T.A. Office)

2. Audiotape supplements to some modules
(in Physics Library, cassette players available there also)"

(Karplus, UC-B)

"MINI-LECTURES AND DISCUSSIONS
ON T.V. TAPE"

"The tapes listed below are available for your viewing in the library. Just go past the Audio-visual desk and ask the T.V. secretary for the number and title you want. Additional titles will be posted on the bulletin board in room 202B as they become available.

1. Problem 4, Page 708, Cooper. Describes how to use vector methods to solve projectile problems.

2. Problem 5, page 708, Cooper. Shows how to use vectors to analyze motion.

3. Circular Motion. Why is Acceleration Towards the Center of the Circle? Page 41, Cooper. Explains the rather abstract material in Cooper. This tape shows the difference between velocity and speed..."

(Sperry, CWSC)

<u>Exhibit C</u> - Student Responsibilities

"1.4 Class time will be used for studying, test taking, asking the instructor and/or tutors and/or fellow students questions about those things that you don't understand. You may also meet the tutors, the instructor, and fellow students outside regular class time in the Keller Plan Study Hall - Room SC3-207."

"1.5 Thus the pattern for this semester should be as follows: Study the material indicated in your study guide, work some of the suggested problems, and answer some of the suggested questions. When you feel that you have mastered the material you will then take a test. When you pass the test, you will go on to another unit. If you do not pass, you will study the suggested material again, and take another test when you feel ready, and so forth."

(Munsee, SCU-LB)

"<u>Your responsibilities</u>. Your principal responsibility, of course, is learning about Newtonian mechanics. To make this possible for all students in a class using the PSI format, I have found certain specific requirements very helpful:

1. Honor system: all tests must be treated confidentially. You should never take a test paper from the classroom, communicate the content of a test to another student (though we encourage you to help fellow students master the material in other ways), or have the assistance of other students, the text, or notes during any of the tests except for Module 4 and the final exam, as described there.

Exhibit C - Student Responsibilities - continued

2. Attendance: You are required to attend one weekly conference with your tutor during a mutually determined class period to discuss your progress, until you have completed the nine basic modules. Beyond this, attend classes as needed for your progress, to take tests, get help, or confer with other students.

3. The final examination is required. It will take place Tuesday, March 8, 1:30 - 4:30.

4. Your Progress Sheet must be kept up-to-date by bringing it to class so completed modules can be recorded.

5. Bulletin Board: consult the bulletin boards in Room 60 at least once a week to get information concerning any special situations, changes in procedure, corrections to the study guides, or other matters that you need to know.

6. Errors: If you find mistakes in the study guides, especially in Exercise answers or solutions, tell your tutor immediately so they can be checked and announced on the bulletin board for the benefit of other students.

7. Use your initiative: there will be no planned group activities in Room 60 after the first meeting. We will respond to your questions and suggestions. If you procrastinate, we recommend you change to Section 1 or 2."

(Karplus, UC-B)

"PROBLEM SOLVING PERFORMANCE PRESCRIPTIONS FOR KELLER PLAN TESTS

THUS SPAKE ZARATHUSTRA...in order to receive a "pass" mark on the KELLER PLAN EXAMINATIONS a student must satisfy the following prescription for problem solving and provide the information noted.

1. SKETCH your interpretation of a word problem in the form of a figure and define all symbols that you employ.

2. Define a coordinate system and sign convention appropriate for the problem.

3. Write and identify by name, if appropriate, any physical and mathematical laws or theorems that you employ in the solution.

4. If you simply employ a formula derived by mathematically manipulating physical laws, specify any approximations involved in the derivation or any physical conditions that must be satisfied in order to use the formula.

5. Be specific in distinguishing between vector quantities, components of vectors, magnitudes of vectors, and scalar quantities.

Exhibit C - Student Responsibilities - (continued)

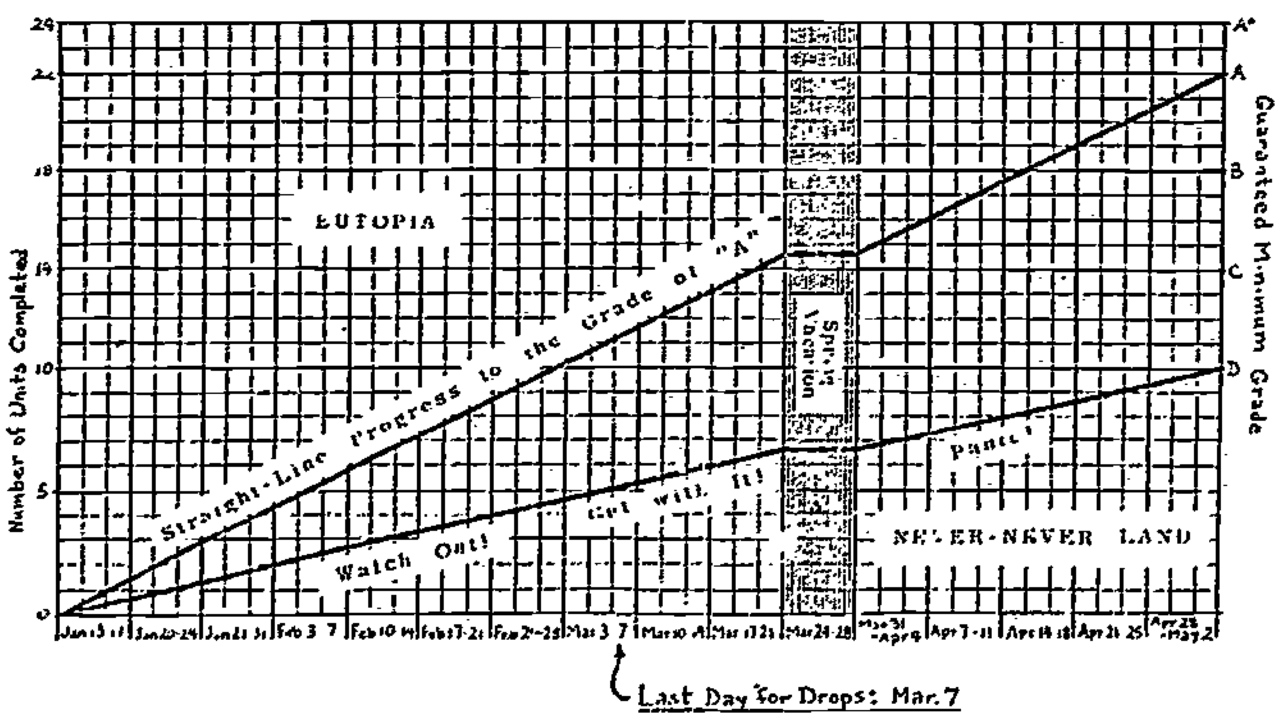
6. When you reach that point in your solution where you substitute numerical quantities for the symbols in your equations, include the units associated with each numerical value and carry these units or the resultant set remaining after appropriate cancellations, until you arrive at a solution. NUMERICAL ANSWERS WITHOUT UNITS OR WITH INCORRECT UNITS HAVE NO MEANING AND DO NOT CONSTITUTE A SOLUTION. ZERO CREDIT.

7. Write legibly and proceed in an orderly manner toward your solution. You are not under a time constraint, so the opportunity for an orderly presentation is available to you. USE IT."

(Snow, UMR)

Exhibit D - Progress Chart

☆ PROGRESS CHART ☆ KELLER-PLAN PHYSICS ☆



(Joseph, UN-L)

Module____

STUDY GUIDE

DIMENSIONS AND VECTOR ADDITION

INTRODUCTION

If someone asked you how to get to the top of Pike's Peak from Boulder, Colorado, you might say something such as "Eighty miles south and two miles straight up!" This is a quantity which must have both a magnitude (80 miles, 2 miles) and a direction (south, up) specified.

There are other quantities similar to this, such as the velocity of an automobile or the pull (force) of a horse on a wagon. These quantities and many others can be represented by vectors. In a diagram we represent a vector by an arrow; the length and direction of the arrow indicate the size and direction of the quantity being represented.

Vectors turn out to be extremely useful in formulating the concepts of physics, and they help to find numerical answers to problems. The study of vectors (vector analysis) also can be beautiful and rewarding in its own right.

Vectors can be added, subtracted, and multiplied; the chief objective of this module is to enable you to calculate the sum and difference of vectors expressed in various ways. A second objective is to review the basic metric units in which physical quantities are expressed and to enable you to use these units to avoid errors in equations and numerical results.

PREREQUISITES

Before you begin this module, you should be able to:	Location of Prerequisite Content
*Write the SI units, with proper prefixes, for mass, length, and time (for Objective 1 of this module)	
*Use rectangular and polar coordinates in two dimensions to plot the location of a point (for Objective 3 of this module)	
*Locate a point in space using rectangular coordinates in three dimensions (for Objectives 3 and 4 of this module)	
*Obtain values of unknown side(s) or angle(s) of a given triangle, and find values for sin θ and cos θ for θ larger than 90° (for Objective 4 of this module)	Trigonometry Review

LEARNING OBJECTIVES

After you have mastered the content of this module you will be able to:

1. Dimension and units - Given an equation and the metric units of all quantities, determine whether or not the equation is dimensionally consistent.

2. Vectors and scalars - Distinguish whether a given item is a vector, a scalar, or neither.

3. Vectors, polar and component form - Transform a given two-dimensional vector from polar form (magnitude and direction) to rectangular form (components and unit vectors) and vice versa.

4. Vector addition and subtraction - Given two three-dimensional vectors in rectangular form, calculate the sum and difference.

Note: If you think that you are already competent on these four objectives, you can skip directly to the Practice Test, and you may wish to try the Mastery Test as soon as possible. The first module has purposely been made somewhat less demanding than later modules, and you may have already encountered this material. The advantage of this method of instruction is that you do not have to spend any more time studying than necessary for you to attain mastery; on the other hand, taking a mastery test before you are reasonably sure that you can pass is wasteful of both your time and the instructor's time.

TEXT: Frederick J. Bueche, Introduction to Physics for Scientists and Engineers (McGraw-Hill, New York, 1975), second edition

SUGGESTED STUDY PROCEDURE

Read all of Chapters 1 and 2 (pp. 1-18), and study Sections 2.2 to 2.5 carefully. You may also find it helpful to read through the material on accuracy and significant figures on p. 12 and on mathematics in Appendix 3, p. 830.

After you have completed the text, look over the Problem Set (below), and read through the General Comments carefully. Then work Problems A through G in the Problem Set, checking your answers with those provided. If you have trouble with one or more of these problems, reread the text and the General Comments carefully, and you may wish to work the appropriate additional problems from the text (see Table below). When you feel prepared, try the Practice Test, which has problems similar to the Mastery Test.

We shall now make specific comments concerning the material treated in Sections 2.2 to 2.5 in Bueche. Bueche shows clearly in Section 2.2 how to add vectors by transforming them to component form and then adding the x and y components separately by means of a table. You should study Illustrations 2.1 and 2.2 carefully.

BUECHE

Objective Number	Readings	Problems with Solutions: Study Guide	Problems with Solutions: Text	Assigned Problems: Study Guide	Additional Problems
1	Secs. 1.6, 1.7, General Comment 1	A		E	Chap. 1, Problems 10, 11
2	Sec. 2.1	B			Chap. 2, Question 1
3	Secs. 2.2, 2.4	C		F	Chap. 2, Problems 1, 2, 3, 5
4	Secs. 2.2, 2.3, 2.5, General Comment 2	D	Illus. 2.1-2.4	G	Chap. 2, Problems 7, 9, 18, 19, 21

The transformation equations for vectors from Section 2.2 are summarized below. You should be able to look at each equation, to state the meaning of each symbol, and to imagine the transformation involved. It is generally not helpful to memorize the equations uncritically.

Polar form to rectangular components:

$$x = R\cos\theta; \qquad y = R\sin\theta.$$

Rectangular components to polar form:

$$R^2 = x^2 + y^2; \qquad \tan\theta = y/x.$$

Section 2.5 describes a geometrical method to add vectors and introduces the idea of vector subtraction. It is important to realize that vectors can in fact be added by means of a diagram drawn carefully with ruler and protractor, and in some cases this is a faster and easier method than that of using trigonometry and the components.

Bueche does not give an illustration for vector subtraction; thus we supply one here:

Example

If $\vec{A} = 4\hat{i} + 6\hat{j} - 12\hat{k}$ and $\vec{B} = 8\hat{i} - 3\hat{j} + 2\hat{k}$, find $\vec{B} - \vec{A}$.

Solution

$$\vec{B} + (-\vec{A}) = (8\hat{i} - 3\hat{j} + 2\hat{k}) + (-4\hat{i} - 6\hat{j} + 12\hat{k}),$$

$$\vec{B} - \vec{A} = 4\hat{i} - 9\hat{j} + 14\hat{k}.$$

TEXT: David Halliday and Robert Resnick, Fundamentals of Physics (Wiley, New York, 1970; revised printing, 1974)

SUGGESTED STUDY PROCEDURE

You should read Chapter 1, Sections 1-1 through 1-5 (pp. 1-8) for general information. Study Chapter 2, Sections 2-1, 2-2, 2-3 (pp. 11-18), and 3-9 (pp. 35-36) carefully.

After completing the text, read General Comments 1 and 2, and study the Problem Set below. Make sure to work out your own answers for Problems E, F, and G. If you have trouble, re-read the appropriate sections of the text and General Comments; if necessary, work the additional problems listed in the Table. When you feel prepared, try the Practice Test, which has problems similar to the Mastery Test.

Halliday and Resnick

Objective Number	Readings	Problems with Solutions	Assigned Problems	Additional Problems
		Study Guide	Study Guide	
1	Sec. 3-9, General Comment 1	A	E	
2	Sec. 2-1	B		Chap. 2, Question 5
3	Sec. 2-3	C	F	Chap. 2, Problems 10, 11, 12
4	Secs. 2-2, 2-3, General Comment 2	D	G	Chap. 2, Problems 5, 8, 15, 25

TEXT: Francis Weston Sears and Mark W. Zemansky, University Physics (Addison-Wesley, Reading, Mass., 1970), fourth edition

SUGGESTED STUDY PROCEDURE

You should read all of Chapter 1, with careful study of Sections 1-3, 1-5, 1-6, 1-7, 1-8, and 1-9. After you have finished Chapter 1, you should read the General Comments below.

After completing the reading, work through the Problem Set and check your answers with those provided. If you have trouble with one or more of the problems, re-read the appropriate sections of the text or the General Comments, and you may find it helpful to work some of the additional problems listed in the Table below.

Sears and Zemansky

Objective Number	Readings	Problems with Solutions	Assigned Problems	Additional Problems
		Study Guide	Study Guide	
1	Secs. 1-2, 1-3, General Comment 1	A	E	
2	Secs. 1-4, 1-5	B		
3	Sec. 1-7	C	F	1-8, 1-9
4	Secs. 1-6, 1-8, 1-9, General Comment 2	D	G	1-2 (not easy), 1-5, 1-10, 1-11, 1-13, 1-15

Section 1-3 of Chapter 1 contains a good example, similar to the one presented below in General Comments, of how to check through the dimensions of an equation and a numerical result. The units chosen are not from the metric system that we use in this study guide, but the principle is the same.

Figure 1-5(b) has the label for $\vec{B}$ in the wrong place; it should be to the right of the triangle.

Section 1-B employs the notation ΣF_x and ΣF_y (p. 9). The Greek letter Σ (sigma) is used in this way to refer to "the sum of." Thus in Section 1-B, ΣF_x is shorthand for $F_{1x} + F_{2x} + F_{3x}$, and ΣF_y is shorthand for $F_{1y} + F_{2y} + F_{3y}$.

In Section 1-9, Sears and Zemansky introduce the idea of vector subtraction; since they have not provided an example, we furnish one, as follows:

Example

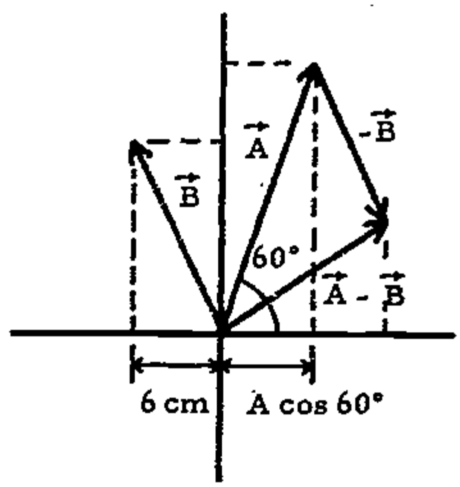
Suppose $\vec{A}$ has a magnitude of 14 cm and direction 60° above the positive x axis. Suppose $\vec{B}$ has $B_x = -6.0$ cm and $B_y = 8.0$ cm. Find $\vec{A} - \vec{B}$ in terms of rectangular components.

Solution

Vector	x component	y component
$\vec{A}$	$14 \cos(60°) = 7.0$ cm	$14 \sin(60°) = 12.0$ cm
$\vec{B}$	-6.0 cm	8.0 cm
$\vec{A} - \vec{B}$	$A_x - B_x = 13.0$ cm	$A_y - B_y = 4.0$ cm

$(\vec{A} - \vec{B})_x = 13.0$ cm

$(\vec{A} - \vec{B})_y = 4.0$ cm



Sears and Zemansky do not describe unit vectors. Therefore, you should carefully read General Comment 2 below.

TEXT: Richard T. Weidner and Robert L. Sells, Elementary Classical Physics (Allyn and Bacon, Boston, 1973), second edition, Vol. 1

SUGGESTED STUDY PROCEDURE

You should read all of Chapter 1 (pp. 1-5) for general background. Study Sections 2-1 through 2-5 and 2-7 carefully. After you have read the text you should read the General Comments below and work through Problems A through G in the Problem Set. If you have trouble with the problems, re-read the appropriate sections of the General Comments and the text, or work several of the relevant additional problems listed in the Table below. When you are ready, try the Practice Test.

Weidner and Sells

Objective Number	Readings	Problems with Solutions	Assigned Problems	Additional Problems
		Study Guide	Study Guide	
1	Sec. 1-4, General Comment 1	A	E	
2	Sec. 2-1	B		
3	Secs. 2-3, 2-7	C	F	2-2, 2-6
4	Secs. 2-2, 2-4, 2-5, 2-7, General Comment 2	D	G	2-7, 2-9, 2-11

Note: In Section 2-3, in the third line below Figure 2-10, the words "left" and "right" in the parentheses should be interchanged to read "(right or left, up or down)."

In Section 2-4, Figure 2-12, note that the component vectors $\vec{A}_x$ and $\vec{B}_x$ add vectorially to give the component vector $\vec{R}_x$. If we represent these vectors with numbers, A_x is positive, B_x is negative, and R_x is the algebraic sum of A_x and B_x. Thus, write $R_x = A_x + B_x$ in the last line of the caption to Figure 2-12.

GENERAL COMMENTS

There are two general comments in this module:

1. Use of units for checking equations and answers; and
2. Vectors in three dimensions - unit vectors and rectangular components.

These comments are intended to supplement your textbook but may overlap somewhat.

1. Use of Units for Checking Equations and Answers. Note: This topic is also treated in Halliday and Resnick, Section 3-9, pp. 35-36 [D. Halliday and R. Resnick, Fundamentals of Physics (Wiley, New York, 1970; revised printing, 1974)].

There is a simple and convenient way to check for errors in your equations and answers to problems. The check consists of determining whether the physical units are correct; in an equation, both sides should be expressed in the same units, and a numerical answer should come out to have the units appropriate to the quantity being sought. For example, suppose you were to derive the following equation:

$$v_0 = \left(\frac{x}{t}\right) - \frac{1}{2}at, \quad \text{where } [v_0] = m/s, \quad [x] = m, \quad [t] = s, \quad \text{and} \quad [a] = m/s^2.$$

(Note: We shall use the square brackets to indicate that the symbol within the brackets is expressed in the units to the right of the equals sign. We shall also use the following abbreviations for the basic SI units: meter, m; second, s; kilogram, kg.) You can check the equation as follows:

$$v_0 = \left(\frac{x}{t}\right) - \frac{1}{2}at,$$

$$m/s = (m/s) - (m/s^2)(\cancel{s}) = (m/s) - (m/s).$$

Thus both sides of the equation are expressed in the same units, and the equation is said to be dimensionally consistent. (Notice that the numerical factor 1/2 does not have units.) Of course, you are not assured that the equation is correct; this method will not catch mistakes in signs or in numerical factors. For example, the equation could be

$$v_0 = 5(x/t) + 10at,$$

and it would still be dimensionally consistent. However, the method does catch a surprisingly large number of errors. It is also worth remembering that the trigonometric, exponential, and logarithmic functions are all dimensionless.

Suppose that you proceed to put numbers into an equation in order to calculate a result. You can also quickly evaluate the correctness of the units of the answer. For example, using the above equation, with $x = 16$ m, $t = 2$ s, and $a = 10\ \text{m/s}^2$,

$$v_0 = 16\ \text{m}/2\ \text{s} - \frac{1}{2}(10\ \text{m/s}^2)(2\ \cancel{\text{s}})$$

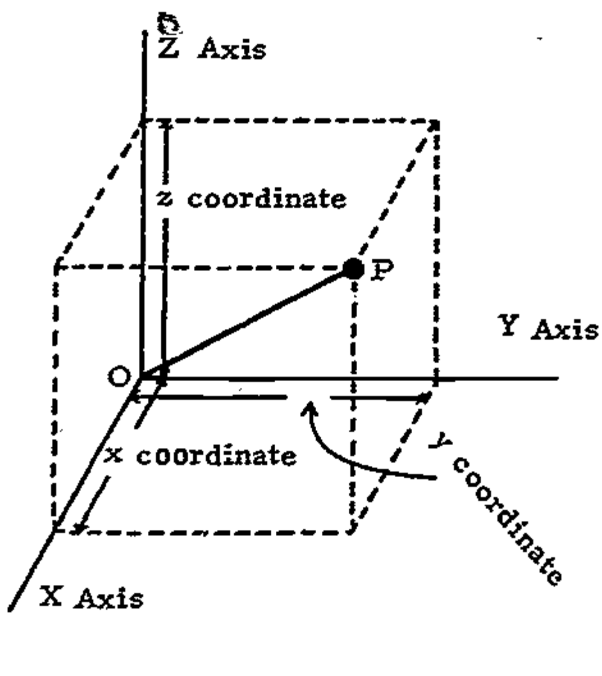
(where the slash indicates how we have canceled the units in the second term on the right) and

$$v_0 = 8\ \text{m/s} - 10\ \text{m/s} = -2\ \text{m/s}.$$

The answer is in proper units for velocity, and this increases our confidence that this answer is correct.

Note: Another way to identify possible errors in a numerical result involves estimating whether the magnitude of the answer is in the proper "ballpark," but this technique will be described and examples given in subsequent modules, for example, in Collisions.

2. Vectors in Three Dimensions - Unit Vectors and Rectangular Components. The text describes how to add and subtract two or more vectors that are confined to a plane. It is also possible to add vectors in three-dimensional space using a three-dimensional system of coordinates. We shall employ rectangular coordinates (see diagram) in which each point in space (P) is located by means of its distance from three mutually perpendicular axes, the x-, y-, and z-axes.



As in two dimensions, a vector can be resolved into components along the coordinate axes. We shall find it useful to employ the concept of unit vector, which is defined to be a vector of unit length with no dimensions.

We shall use the notation $\hat{i}$, $\hat{j}$, and $\hat{k}$ to refer to the unit vectors oriented along the positive x-, y-, and z-axes, respectively (see Fig. A).

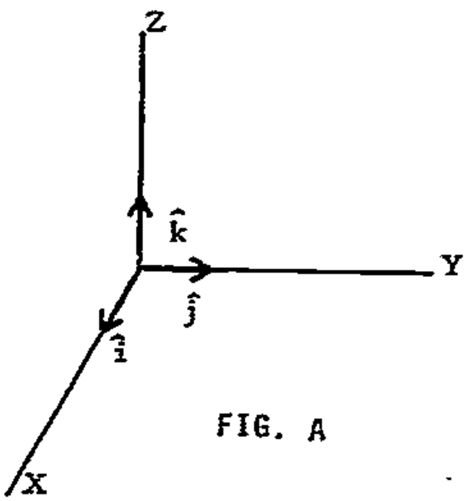


FIG. A

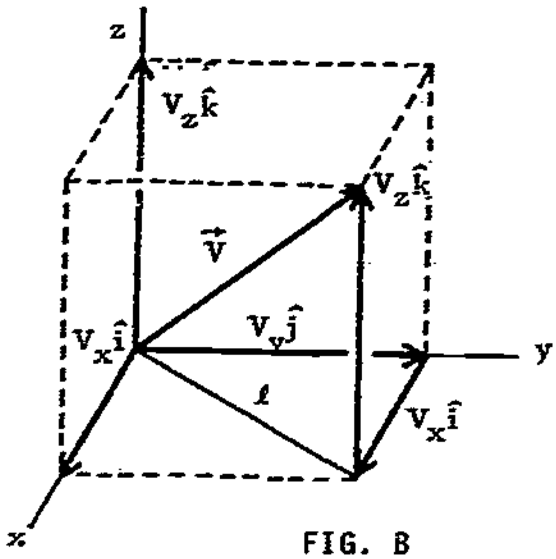


FIG. B

You may have found that your text was not sufficiently explicit about the resolution of vectors that do not lie in one of the coordinate planes. Perhaps Figure B will help to clarify this. If you have a vector $\vec{V}$ and need to find its components, your first step is to construct a rectangular box as shown by the dashed lines. The lengths of the sides of the box then give you the components V_x, V_y, and V_z. In other words, $\vec{V} = V_x\hat{i} + V_y\hat{j} + V_z\hat{k}$, and you can see that this is simply restating the laws of vector addition by noting how, starting at the origin, one adds $V_y\hat{j}$ plus $V_x\hat{i}$ plus $V_z\hat{k}$ to equal $\vec{V}$. You should note that $V_x\hat{i}$ and $V_z\hat{k}$ have each been drawn at two different locations. It is also important to be careful of signs; if, say, the vector $\vec{V}$ pointed more to the left so that the box cut the y axis to the left of the origin, then V_y would be negative.

One can also see from the drawing, by use of the Pythagorean theorem, that $\ell^2 = V_y^2 + V_x^2$. Furthermore, $V^2 = \ell^2 + V_z^2$; therefore, the length or magnitude of the vector $\vec{V}$ is given by

$$|\vec{V}| = \sqrt{V_x^2 + V_y^2 + V_z^2}\,.$$

(*Note*: We shall use an absolute value sign, $|\ |$, to indicate the magnitude of a vector.)

PROBLEM SET WITH SOLUTIONS

A(1). Given that velocity (v) has units of length/time, that acceleration (a) has units of length/time^2, and that distance (d) has units of length, evaluate whether or not the following equation is dimensionally consistent:

$$v^2 = 2ad^2.$$

Solution

Units of v^2: $(m/s)^2 = m^2/s^2$. Units of $2ad^2$: $(m/s^2)(m)^2 = m^3/s^2$. The units of v^2 and $2ad^2$ are not the same; therefore the equation is not dimensionally consistent and cannot be correct.

B(2). State whether each of the following items could be represented by a vector, a scalar, or neither:

(a) The moon itself,

(b) The mass of the moon,

(c) The velocity of the moon.

Solution

(a) Neither,

(b) A scalar; one need specify only a single number,

(c) A vector, one must specify both magnitude (speed) and direction.

C(3). (a) If $\vec{A}$ has x component $A_x = 12$ m, and y component $A_y = -18$ m, express $\vec{A}$ in polar form (A, θ). (Use a trigonometric table if necessary.)

(b) If $\vec{A}$ has magnitude $|\vec{A}| = 16$ mm and $\theta = 72°$, express $\vec{A}$ in terms of the unit vectors $\hat{i}$ and $\hat{j}$.

Solution

(a) $|\vec{A}| = \sqrt{(12)^2 + (-18)^2}$ m $= \sqrt{144 + 324}$ m $= \sqrt{468}$ m $= 22$ m.

$\tan\theta = -18/12 = -1.5$; using a table for $\tan\phi = +1.5$, we find $\phi = 56°$; but we want $\theta = 360° - \phi = 304°$, since a negative y component and a positive x component places the vector $\vec{A}$ in the fourth quadrant.

(b) $(4.9\hat{i} + 15.2\hat{j})$ mm.

D(4). Given $\vec{A} = 4\hat{i} - 8\hat{j} - 6\hat{k}$ and $\vec{B} = -2\hat{i} - 3\hat{j} + 5\hat{k}$, calculate the following:
(a) $\vec{A} + \vec{B}$.
(b) $\vec{A} - 2\vec{B}$.

Solution

(a) $\vec{A} + \vec{B} = (4 - 2)\hat{i} + [(-8) + (-3)]\hat{j} + [(-6) + 5]\hat{k} = 2\hat{i} - 11\hat{j} - \hat{k}$.
(b) $\vec{A} - 2\vec{B} = (4\hat{i} - 8\hat{j} - 6\hat{k}) - (-4\hat{i} - 6\hat{j} + 10\hat{k}) = (4 + 4)\hat{i} + (-8 + 6)\hat{j} + [(-6) - (+10)]\hat{k} = 8\hat{i} - 2\hat{j} - 16\hat{k}$.

Problems

E(1). Given that distance (d) has units of length, that acceleration (a) has units of length/time^2, and that t represents time, determine whether or not the following equation is dimensionally consistent.

$$d = 4at^2.$$

F(3). (a) If $\vec{A}$ has magnitude 0.15 m and direction 260° with respect to the positive x-axis, find A_x and A_y.
(b) If $\vec{A} = (8.4\hat{i} + 2.8\hat{j})$ cm/s, express $\vec{A}$ in polar form.

G(4). Given $\vec{D} = \hat{i} + 3\hat{j} - 2\hat{k}$ and $\vec{E} = 4\hat{i} - 5\hat{j} + \hat{k}$, calculate the following:
(a) $3\vec{D} + \vec{E}$;
(b) $4\vec{E} - 2\vec{D}$. (Note order!)

Solutions

E(1). The equation is dimensionally correct.

F(3). (a) $A_x = -0.026$ m, $A_y = -0.15$ m. (Note: Because the original magnitude had only two significant figures, we have rounded off the answer for $|\vec{A}|$ to two figures.)
(b) $|\vec{A}| = 8.9$ cm/s; $\theta = 18°$.

G(4). (a) $7\hat{i} + 4\hat{j} - 5\hat{k}$;
(b) $14\hat{i} - 26\hat{j} + 8\hat{k}$.

PRACTICE TEST

1. Given the equation $y = (v_0 \sin \theta_0)t - (1/2)gt^2$, and $[y] = m$, $[v_0] = m/s$, $[t] = s$, $[g] = m/s^2$ determine whether or not the equation is dimensionally consistent.

2. State whether each of the following items can be represented as a vector, a scalar, or neither:
   (a) the United States,
   (b) the number of eggs in a carton,
   (c) the velocity of the wind at a particular point.

3. (a) If $A_x = -8.0$ cm and $A_y = 16$ cm, express $\vec{A}$ in polar form (magnitude, direction). (Use a trigonometric table if necessary.)
   (b) If $\vec{A} = (26\ m, 195°)$, express $\vec{A}$ in terms of $\hat{i}$ and $\hat{j}$.

4. Given $\vec{A} = -13\hat{i} + 4\hat{j} + 4\hat{k}$ and $\vec{B} = +6\hat{i} - 9\hat{k}$, calculate the following quantities:
   (a) $\vec{A} + \vec{B}$,
   (b) $\vec{A} - \vec{B}$.

Practice Test Answers

1. Yes, the equation is dimensionally consistent.
2. (a) Neither; (b) scalar; (c) vector.
3. (a) $|\vec{A}| = 18$ m, $\theta = 120°$. (Note θ was rounded off to two significant figures. With three figures, $\theta = 117°$.)
   (b) $\vec{A} = (-25\hat{i} - 6.7\hat{j})$ m.
4. (a) $\vec{A} + \vec{B} = -7\hat{i} + 4\hat{j} - 5\hat{k}$.
   (b) $\vec{A} - \vec{B} = -19\hat{i} + 4\hat{j} + 13\hat{k}$.

Date ____________

DIMENSIONS AND VECTOR ADDITION

Mastery Test Form A

pass recycle
1 2 3 4

Name ____________________ Tutor ____________________

1. Given the equation, $E = (1/2)mv^2 + (1/2)kx^2$, and

   $[E] = kg\ m^2/s^2$,

   $[m] = kg$,

   $[v] = m/s$,

   $[k] = kg/s^2$,

   $[x] = m$,

   evaluate whether or not the equation is dimensionally consistent.

2. State whether each of the following items can be represented as a vector, a scalar, or neither.

   (a) Jack's age;

   (b) the color of Jack's eyes;

   (c) the length of Jack's shirt sleeve.

3. (a) If $\vec{A} = (4.0\hat{i} - 3.0\hat{j})$ m, express $\vec{A}$ in polar form (magnitude, direction).

   (b) If $\vec{B} = (15\ m/s, 130°)$ express $\vec{B}$ in terms of $\hat{i}$ and $\hat{j}$. (<u>Note</u>: You may need to use a trigonometric table.)

4. If $\vec{D} = (-7.4\hat{i} + 3.1\hat{j} + 1.8\hat{k})$ m/s and

   $\vec{C} = (-4.3\hat{i} + 2.0\hat{j})$ m/s, calculate the following:

   (a) $\vec{D} + \vec{C}$,

   (b) $\vec{D} - 3\vec{C}$.

Date________

DIMENSIONS AND VECTOR ADDITION

Mastery Test Form B

pass		recycle	
1	2	3	4

Name______________________ Tutor______________________

1. Given the equation $I = (mv^2) + r^2p$, and

   $[I] = kg\ m^2/s$,

   $[m] = kg$,

   $[v] = m/s$,

   $[r] = m$,

   $[p] = kg\ m/s$,

   evaluate whether or not the equation is dimensionally consistent.

2. State whether each of the following items can be represented as a vector, a scalar, or neither:

   (a) the direction from Chicago to New York,

   (b) the position of New York with respect to Chicago,

   (c) the distance from New York to Chicago.

3. (a) If $\vec{A}$ has components $A_x = -6.0$ m/s, $A_y = -8.0$ m/s, express $\vec{A}$ in polar form (magnitude, direction).

   (b) If $\vec{B} = (5.0\ m, 210°)$, express $\vec{B}$ in terms of $\hat{i}$ and $\hat{j}$. (<u>Note</u>: You may need to use a trigonometric table.)

4. Given $\vec{A} = 5\hat{i} + 4\hat{j} - 6\hat{k}$ and $\vec{B} = -2\hat{i} + 2\hat{j} + 3\hat{k}$, calculate the following:

   (a) $\vec{A} + 2\vec{B}$,

   (b) $\vec{A} - \vec{B}$.

Date____________

OIMENSIONS ANO VECTOR AOOITION

pass recycle

MasteryTest Form C

1 2 3 4

Name____________________ Tutor____________________

1. Given the equation, $a = k^2 A \cos(\omega t) - k^2 xB \sin(kx)$, and

   $[a] = m/s^2$,
   $[\omega] = 1/s$,
   $[A] = m$,
   $[k] = 1/m$,
   $[x] = m$,
   $[B] = m$,
   $[t] = s$,

   evaluate whether or not the equation is dimensionally consistent.

2. State whether each of the following items can be represented by a vector, a scalar, or neither:
   (a) the volume of a tank,
   (b) the shape of a tank,
   (c) the location of a tank in a storage area.

3. (a) If $\vec{A} = (-1.0\hat{i} + 2.7\hat{j})$ m/s^2, express $\vec{A}$ in polar form (magnitude, direction).

   (b) If $\vec{B} = (7.0$ m/s^2, 170°), express $\vec{B}$ in terms of $\hat{i}$ and $\hat{j}$. (Note: You may need to use a trigonometric table.)

4. If $\vec{C} = 6.3\hat{i} + 4.0\hat{j} - 3.8\hat{k}$ and $\vec{D} = -4.6\hat{j} + 5.0\hat{k}$, calculate the following quantities:
   (a) $\vec{C} + \vec{D}$,
   (b) $\vec{C} - \vec{D}$.

MASTERY TEST GRADING KEY - Form A

What To Look For	Solutions
1. Make sure the student did not just put a guess, but that the units for both sides of the equation are worked out.	1. $\text{kg m}^2/\text{s}^2 = (\frac{1}{2})\text{kg m}^2/\text{s}^2 + (\frac{1}{2})\text{kg m}^2/\text{s}^2$. The equation _is_ dimensionally consistent.
2. Student recognizes differences among vectors, scalars, and other quantities, and correct answers.	2. (a) scalar, (b) neither, (c) scalar.
3. (a) Check that significant figures and units are correct. If not included or incorrect, point this out to student. (b) Same as (a); answer should be expressed in terms of unit vectors, but B_x and B_y are acceptable.	3. (a) $A = \sqrt{(4.0)^2 + (-3.0)^2} = 5.0$ m. $\tan\theta = \frac{-3.0}{4.0} = -0.75, \quad \theta = 323°$. (b) $B_x = B\cos\theta = 15 \times -0.643 = -9.6$, $B_y = B\sin\theta = 15 \times 0.766 = 11.5$, $\vec{B} = (-9.6\hat{i} + 11.5\hat{j})$ m/s.
4. (a) and (b) Check that significant figures and units are correct. If incorrect or not included, point this out to student.	4. (a) $\vec{D} + \vec{C} = (-11.7\hat{i} + 5.1\hat{j} + 1.8\hat{k})$ m/s. (b) $\vec{D} - 3\vec{C} = (+5.5\hat{i} - 2.9\hat{j} + 1.8\hat{k})$ m/s.

MASTERY TEST GRADING KEY - Form B

What To Look For	Solutions
1. Make sure the student did not just put a guess, but that the units for both sides of the equation are worked out.	1. $kg\ m^2/s = kg\ m^2/s^2 + kg\ m^3/s$. The equation is not dimensionally consistent.
2. Correct answers.	2. (a) neither, (b) vector, (c) scalar.
3. (a) Check that significant figures and units are correct. If not included or incorrect, point this out to student.	3. (a) $A = \sqrt{(-6.0)^2 + (-8.0)^2} = 10.0$ m/s. $\tan\theta = \frac{-8.0}{-6.0} = 1.33$, $\theta = 233°$.
(b) Same as (a); answer should be expressed in terms of unit vectors, but B_x and B_y are acceptable.	(b) $B_x = 5\cos\theta = 5 \times -0.866 = -4.3$. $B_y = 5\sin\theta = 5 \times -0.5 = -2.5$. $\vec{B} = (-4.3\hat{i} - 2.5\hat{j})$ m.
4. (a) and (b) Check that significant figures are correct.	4. (a) $\vec{A} + 2\vec{B} = \hat{i} + 8\hat{j}$. (b) $\vec{A} - \vec{B} = 7\hat{i} + 2\hat{j} - 9\hat{k}$.

MASTERY TEST GRADING KEY - Form C

What To Look For	Solutions
1. Make sure the student did not just put a guess, but that the units for both sides of the equation are worked out.	1. $m/s^2 = 1/m$. The equation is <u>not</u> dimensionally consistent.
2. Correct answers.	2. (a) scalar, (b) neither, (c) vector.
3. (a) Check that significant figures and units are correct. If not included or incorrect, point this out to student.	3. (a) $A = \sqrt{(-1.0)^2 + (2.7)^2} = 2.9\ m/s^2$. $\tan\theta = \frac{2.7}{-1.0} = -2.7$, $\theta = 110°$.
(b) Same as (a); answer should be expressed in terms of unit vectors, but B_x and B_y are acceptable.	(b) $B_x = 7\cos\theta = 7 \times -0.985 = -6.9$, $B_y = 7\sin\theta = 7 \times 0.174 = 1.2$, $\vec{B} = (-6.9\hat{i} + 1.2\hat{j})\ m/s^2$.
4. (a) and (b) Check that significant figures and units are correct. If incorrect or not included, point this out to student.	4. (a) $\vec{C} + \vec{D} = 6.3\hat{i} - 0.6\hat{j} + 1.2\hat{k}$. (b) $\vec{C} - \vec{D} = 6.3\hat{i} + 8.6\hat{j} - 8.8\hat{k}$.

TRIGONOMETRY

This module begins with a self-check test. If you can correctly answer 90% of these test items, you do not need to study this module. (A table of trigonometric functions is given on p. 8 of this module.) The answers to these questions are given at the end of the module, i.e., immediately preceding the table.

SELF-CHECK TEST

1. Without the use of tables, convert degrees to radians and radians to degrees:

   (a) $30° =$ _______rad; (c) $225° =$ __________rad;

   (b) $(3/4)\pi$ rad = _____°; (d) $(5/3)\pi$ rad = ______°.

2. (a) What is the maximum value for the sine of an angle, cosine of an angle, and tangent of an angle? Give at least one angle that has the maximum value for the named function.

   (b) Which angle in Problem 1 has the largest value for the sine, for the cosine, and for the tangent, respectively?

3. One acute angle of a right triangle is 37°. The length of the side opposite the angle is 12.0 cm.

   (a) What are the ratios of the lengths of the sides of this triangle?

   (b) For this triangle, find the lengths of the other two sides of the triangle. (Show your work!)

4. One acute angle of a right triangle is 40°. The length of the hypotenuse is 12.0 cm. Find the lengths of the other two sides.

5. In a right triangle the hypotenuse is $2\sqrt{3}$ and one side is 3.

   (a) Find the missing side.

   (b) What are the angles?

6. A surveyor wishes to determine the distance between two points A and B, but he cannot make a direct measurement because a river intervenes. He steps off at a 90° angle to AB a line AC, which he measures to be 264 m. He measures an angle with his transit at point C to point B. Angle BCA is measured to be 62°. With this information, calculate AB.

7. Show that, for any angle θ,

   $\sin^2\theta + \cos^2\theta = 1$.

RIGHT TRIANGLE

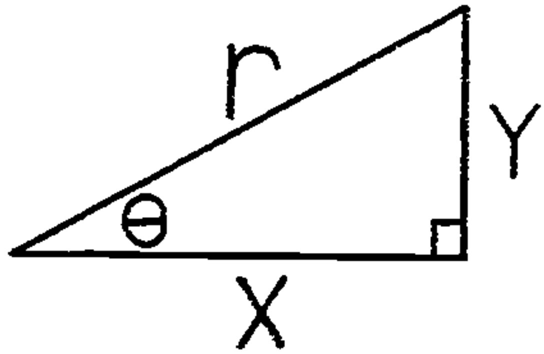
Many of the applications of physics will require you to have a thorough knowledge of the basic properties of right triangles, i.e., triangles that have one angle equal to 90°.

The trigonometric functions are defined with respect to a right triangle as follows:

$\sin\theta = y/r$

$\cos\theta = x/r$

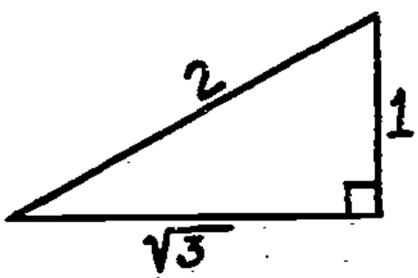
$\tan\theta = y/x$

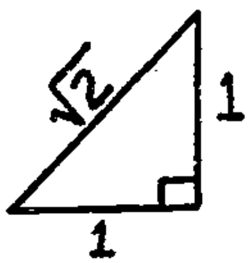


The values of the trigonometric sine, cosine, and tangent functions for a given θ can be determined from a table such as in the appendix to your text or the last page of this module. You can also get the values by use of most slide rules ("S" and "T" scales) and many electronic calculators.

The 30°-60°-90° and 45°-45°-90° Triangles

It is also useful to remember the values of the functions for $\theta = 30°$, 45°, and 60° by means of the triangles below.





These triangles are right triangles, and you should check that the sides indeed satisfy the Pythagorean theorem. Note also that in any right triangle the longest side is the hypotenuse.

Using the basic definitions and the above triangles, one finds

$$\sin 30° = 1/2 = 0.500,$$
$$\sin 45° = 1/\sqrt{2} = \sqrt{2}/2 = 1.414/2 = 0.707\ldots,$$
$$\sin 60° = \sqrt{3}/2 = 1.732/2 = 0.866,$$

$$\cos 30° = \sqrt{3}/2 = 1.732/2 = 0.866,$$
$$\cos 45° = 1/\sqrt{2} = \sqrt{2}/2 = 0.707,$$
$$\cos 60° = 1/2 = 0.500,$$

$$\tan 30° = 1/\sqrt{3} = \sqrt{3}/3 = 1.732/3 = 0.577,$$
$$\tan 45° = 1/1 = 1.000,$$
$$\tan 60° = \sqrt{3} = 1.732.$$

The 3-4-5 Triangle

The 3-4-5 triangle (since the sides are in the ratio of 3 : 4 : 5) is known as the 37°-53°-90° triangle:

$$\sin 37° = 0.6, \qquad \sin 53° = 0.8,$$
$$\cos 37° = 0.8, \qquad \cos 53° = 0.6,$$
$$\tan 37° = 0.75, \qquad \tan 53° = 1.33.$$

You should memorize these three special triangles so that you can compute the values of sine, cosine, and tangent for the angles involved. A

ANGLES > 90°

When calculating the products of vectors (see Dimensions and Vector Addition module), it is often necessary to determine the sine and cosine of angles greater than 90°, whereas most trig tables list values only for angles less than or equal to 90°. Two alternative ways of remembering the necessary relationships are as follows:

Method I Recall the definitions of sine and cosine for general angles:

$$\sin\theta = y/r \quad \text{and} \quad \cos\theta = x/r,$$

where x and y are the horizontal and vertical projections, respectively, of the radial distance r, as shown in the figures below for angles in the various quandrants.

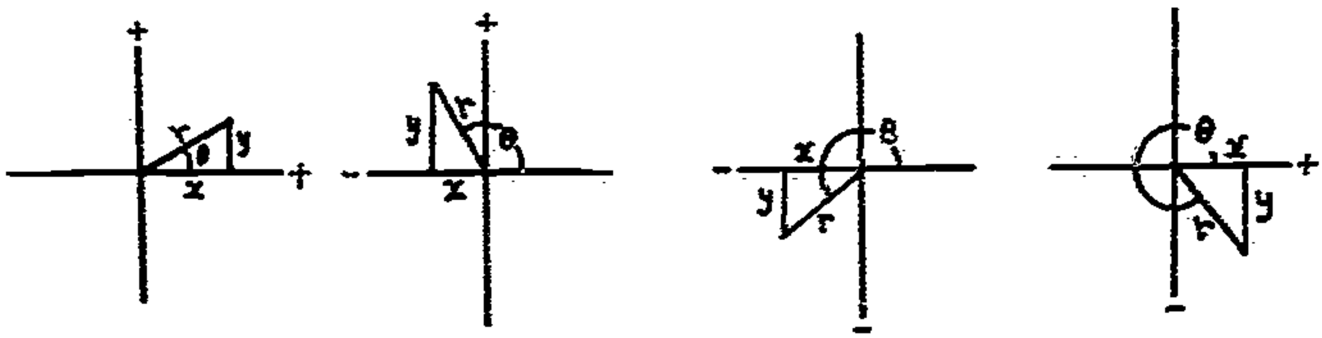


Figure 1

<u>Method II</u> Recall the graphs of sin θ and cos θ:

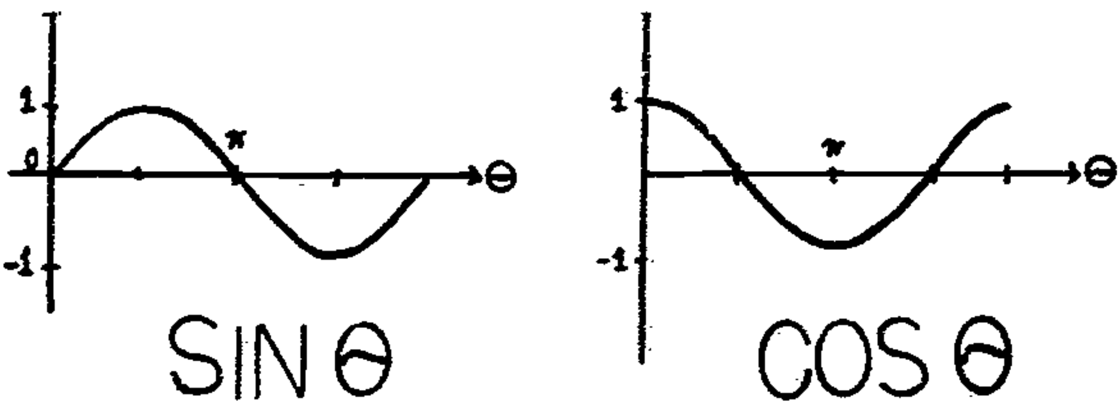


Figure 2

<u>Example</u>

Find sine θ and cos θ where θ = 150° (= 180° - 30°).

<u>Solution I</u>: Comparison of Figures 1(a) and 1(b) together with the definitions of sine and cosine, shows that sin 150° = sin 30°, cos 150° = -cos 30°. Then sin 30° and cos 30° can be looked up in a table or on a slide rule (or, for this example, easily computed).

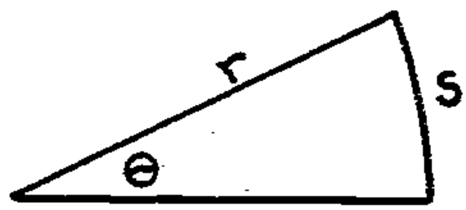
<u>Solution II</u>: Inspection of Figure 2(a) shows that sin 150° = sin 30°; inspection of Figure 2(b) shows that cos 150° = -cos 30°. The sine and cosine of 30° are determined as in Solution I.

RADIAN MEASURE

Many of the problems of planar and rotational motion and waves will depend upon your knowledge of radian measure of angles. Let us therefore take a look at radian measure. The number of radians in an angle at the center of a circle is equal to the (arc subtended by the angle) divided by the radius, or

$$\text{angle in radians} = \frac{\text{arc length}}{\text{radius}},$$

$$\theta = \frac{s}{r}.$$



To travel completely around a circle with a radius of 1.00 m, you will go in an arc of length 6.28 m (2π rad). But once around the circle is equal to 360°.

Derive a formula to convert back and forth between angular measurements in degrees and in radians. Use your formula to convert 45° to radian measure.

USEFUL TRIGONOMETRIC IDENTITIES

In the solution of many problems in physics you may need to use a trigonometric identity. Listed below are some of the most useful ones:

$$\sin^2\theta + \cos^2\theta = 1, \tag{1}$$

$$\sin(A \pm B) = (\sin A)(\cos B) \pm (\sin B)(\cos A), \tag{2}$$

$$\cos(A \pm B) = (\cos A)(\cos B) \mp (\sin A)(\sin B). \tag{3}$$

You can develop the relationships for the sine and cosine of 2A by letting A equal B in Eqs. (2) and (3).

PRACTICE TEST

1. Convert the following angles to radian measures and give their sine, cosine, and tangent values:

   (a) 60°;

   (b) 53°;

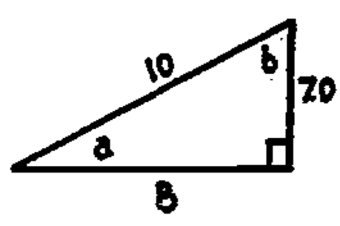
   (c) 37°.

2. Find the unknowns of triangle A:

   B = ______________.

   a = ______________.

   b = ______________.

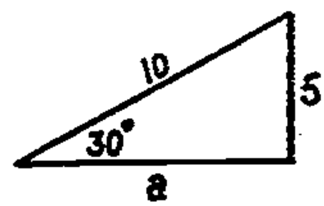


Triangle A

3. One acute angle of a right triangle is 20°. The length of the hypotenuse is 6.0 in. Use trigonometry to calculate the lengths of the two sides.

4. In a 45°-45°-90° right triangle, what is the ratio of the hypotenuse to the sides?

5. State what a in the triangle is equal to without using a trigonometry table or the Pythagorean theorem.



6. A car was traveling exactly northeast. If it went a total distance of 42.4 km, how far north had it actually gone?

Practice Test Answers

1.

θ (radians)	$\sin\theta$	$\cos\theta$	$\tan\theta$
(a) $\pi/3$	0.87	0.50	1.70
(b) $53\pi/180 = 0.925$	0.80	0.60	1.30
(c) $37\pi/180 = 0.646$	0.60	0.80	0.75

2. B = 7.0. a = 44°. b = 46°.

3. 2.05 in., 5.6 in.

4. Ratio of hypotenuse to each side = $\sqrt{2} : 1$.

5. $a = 5\sqrt{3}$.

6. 30 km.

If you have had difficulties with the Practice Test, please work through this Review Module once more, with the assistance of any general mathematics book including a chapter on trigonometry.

SELF-CHECK TEST ANSWERS

1. (a) $\pi/6$ rad; (b) 135°; (c) $5\pi/4$ rad; (d) 300°.

2. (a) $\sin\theta = 1$, $\theta = \pi/2$ or 90°; $\cos\theta = 1$, $\theta = 0$ or 0°;
   $\tan\theta \to \infty$, $\theta \to \pi/2$ or 90°.
   (b) $\sin 3\pi/4$ maximum, cos 30° maximum, tan 225° maximum.

3. (a) The sides are in the ratio of 3 : 4 : 5. (b) 20 cm, 16 cm.

4. 7.7 cm, 9.2 cm.

5. (a) $\sqrt{3}$, (b) 30°, 60°, 90°; one side = (1/2) hypotenuse.

6. 497 m.

7. $x/r = \cos\theta$, $y/r = \sin\theta$, $x^2 + y^2 = r^2$;

or

$$x^2/r^2 + y^2/r^2 = 1.$$

Thus, $\cos^2\theta + \sin^2\theta = 1$.

NATURAL TRIGONOMETRIC FUNCTIONS

Angle: Degree	Angle: Radian	Sine	Cosine	Tangent	Angle: Degree	Angle: Radian	Sine	Cosine	Tangent
0°	0.000	0.000	1.000	0.000					
1°	0.017	0.017	1.000	0.017	46°	0.803	0.719	0.695	1.036
2°	0.035	0.035	0.999	0.035	47°	0.820	0.731	0.682	1.072
3°	0.052	0.052	0.999	0.052	48°	0.838	0.743	0.669	1.111
4°	0.070	0.070	0.998	0.070	49°	0.855	0.755	0.656	1.150
5°	0.087	0.087	0.996	0.087	50°	0.873	0.766	0.643	1.192
6°	0.105	0.105	0.995	0.105	51°	0.890	0.777	0.629	1.235
7°	0.122	0.122	0.993	0.123	52°	0.908	0.788	0.616	1.280
8°	0.140	0.139	0.990	0.141	53°	0.925	0.799	0.602	1.327
9°	0.157	0.156	0.988	0.158	54°	0.942	0.809	0.588	1.376
10°	0.175	0.174	0.985	0.176	55°	0.960	0.819	0.574	1.428
11°	0.192	0.191	0.982	0.194	56°	0.977	0.829	0.559	1.483
12°	0.209	0.208	0.978	0.213	57°	0.995	0.839	0.545	1.540
13°	0.227	0.225	0.974	0.231	58°	1.012	0.848	0.530	1.600
14°	0.244	0.242	0.970	0.249	59°	1.030	0.857	0.515	1.664
15°	0.262	0.259	0.966	0.268	60°	1.047	0.866	0.500	1.732
16°	0.279	0.276	0.961	0.287	61°	1.065	0.875	0.485	1.804
17°	0.297	0.292	0.956	0.306	62°	1.082	0.883	0.469	1.881
18°	0.314	0.309	0.951	0.325	63°	1.100	0.891	0.454	1.963
19°	0.332	0.326	0.946	0.344	64°	1.117	0.899	0.438	2.050
20°	0.349	0.342	0.940	0.364	65°	1.134	0.906	0.423	2.145
21°	0.367	0.358	0.934	0.384	66°	1.152	0.914	0.407	2.246
22°	0.384	0.375	0.927	0.404	67°	1.169	0.921	0.391	2.356
23°	0.401	0.391	0.921	0.424	68°	1.187	0.927	0.375	2.475
24°	0.419	0.407	0.914	0.445	69°	1.204	0.934	0.358	2.605
25°	0.436	0.423	0.906	0.466	70°	1.222	0.940	0.342	2.748
26°	0.454	0.438	0.899	0.488	71°	1.239	0.946	0.326	2.904
27°	0.471	0.454	0.891	0.510	72°	1.257	0.951	0.309	3.078
28°	0.489	0.469	0.883	0.532	73°	1.274	0.956	0.292	3.271
29°	0.506	0.485	0.875	0.554	74°	1.292	0.961	0.276	3.487
30°	0.524	0.500	0.866	0.577	75°	1.309	0.966	0.259	3.732
31°	0.541	0.515	0.857	0.601	76°	1.326	0.970	0.242	4.011
32°	0.559	0.530	0.848	0.625	77°	1.344	0.974	0.225	4.332
33°	0.576	0.545	0.839	0.649	78°	1.361	0.978	0.208	4.705
34°	0.593	0.559	0.829	0.675	79°	1.379	0.982	0.191	5.145
35°	0.611	0.574	0.819	0.700	80°	1.396	0.985	0.174	5.671
36°	0.628	0.588	0.809	0.727	81°	1.414	0.988	0.156	6.314
37°	0.646	0.602	0.799	0.754	82°	1.431	0.990	0.139	7.115
38°	0.663	0.616	0.788	0.781	83°	1.449	0.993	0.122	8.144
39°	0.681	0.629	0.777	0.810	84°	1.466	0.995	0.105	9.514
40°	0.698	0.643	0.766	0.839	85°	1.484	0.996	0.087	11.43
41°	0.716	0.656	0.755	0.869	86°	1.501	0.998	0.070	14.30
42°	0.733	0.669	0.743	0.900	87°	1.518	0.999	0.052	19.08
43°	0.750	0.682	0.731	0.933	88°	1.536	0.999	0.035	28.64
44°	0.768	0.695	0.719	0.966	89°	1.553	1.000	0.017	57.29
45°	0.785	0.707	0.707	1.000	90°	1.571	1.000	0.000	

Module________

STUDY GUIDE 1

RECTILINEAR MOTION

INTRODUCTION

How long does it take you to go home? This depends on how far you are from home (displacement), how fast (velocity) you can travel, and how often you must start and stop (acceleration).

This module treats kinematics, which is the part of physics concerned with the description of the motion of a body. The body may be an automobile, a baseball, a raindrop, a flower in the wind, or a running horse. The change in position of a body can be described in terms of the vector quantities: displacement, velocity, and acceleration. Calculus can be used to define the relationships among these quantities. It is therefore essential to know some basic techniques of calculus to understand the content of this module.

The applications in this module only consider motion in one dimension. A later module will treat the more general case of motion in two or three dimensions, but the fundamental concepts will be essentially the same.

PREREQUISITES

Before you begin this module, you should be able to:	Location of Prerequisite Content
*Distinguish between vector and scalar quantities (needed for Objectives 1 and 2 of this module)	Dimensions and Vector Addition Module
*Differentiate and integrate simple polynomials (needed for Objectives 3 to 5 of this module)	Calculus Review
*Differentiate the sine and cosine functions (needed for Objectives 4 and 5 of this module)	Calculus Review Trigonometry Review

LEARNING OBJECTIVES

After you have mastered the content of this module you will be able to:

1. Displacement, velocity, acceleration - Write the mathematical definitions of displacement, instantaneous velocity, and acceleration, and define all terms.

2. Average-instantaneous, position-displacement, velocity-speed - Distinguish between average and instantaneous values of velocity and acceleration, and distinguish between position and displacement and between velocity and speed.

3. Graphical - (a) Given a graph of position as a function of time, for one-dimensional motion, determine either average or instantaneous velocity. (b) Given a graph of velocity as a function of time, determine acceleration and displacement.

4. Analytical - (a) Given a mathematical expression for position as a function of time, for one-dimensional motion, determine an equation for velocity as a function of time. (b) Given a mathematical expression for velocity as a function of time, determine the equations for acceleration and displacement as a function of time.

5. Constant acceleration problems - In the case of one-dimensional motion of a body with constant acceleration, determine the displacement, velocity, and/or acceleration of the body; e.g., a body falling freely near the surface of the earth.

GENERAL COMMENTS

Physics is an area of human knowledge based on accumulative learning. The concepts of rectilinear motion are the foundation for the study of physics of moving bodies. Mastery of the objectives of this module is essential to a successful understanding and completion of subsequent modules.

Rectilinear motion is the motion of a single particle along a straight line. Thus, we may suspend our use of vector notation in working problems, since in one dimension velocity and acceleration may be considered as their respective x components and treated algebraically. The vector notation will be kept when we are speaking rigorously of definitions, but otherwise the symbol a, for example, will represent both the vector acceleration and the absolute value of the acceleration, which is scalar. In later modules we shall have to be more particular and specify by subscripts, a_x, a_y, or a_z, for example, to which scalar component of the vector $\vec{a}$ we are referring. In one dimension, vector addition (subtraction) reduces and is equivalent to algebraic addition (subtraction), noting that careful observance must be made of signs.

ADDITIONAL LEARNING MATERIALS

There is a series of film loops available from John Wiley and Sons, Inc., that may assist in mastering Objectives 1, 2, and 3:
Velocity from Position,
Position from Velocity, I,
Position from Velocity, II,
One-Dimensional Motion.

TEXT: Frederick J. Bueche, Introduction to Physics for Scientists and Engineers (McGraw-Hill, New York, 1975), second edition

SUGGESTED STUDY PROCEDURE

Read Chapter 4, Sections 4.1 to 4.4, 4.6, 4.7, and work at least Problems A through I of this module before attempting the Practice Test. Projectile Motion (Sec. 4.8) will be covered in a later module.

It is essential to work the problems related to Objective 4 (Analytical) because the book does not treat this subject in the reading. You may wish to refer to F. W. Sears and M. W. Zemansky [University Physics (Addison-Wesley, Reading, Mass., 1970) fourth edition], Sections 4-5 and 4-6.

Illustrations 4.4 and 4.5 are the most important part of the reading for Objective 5.

BUECHE

Objective Number	Readings	Problems with Solutions	Assigned Problems	
		Study Guide	Study Guide	Text
1	Secs. 4.1, 4.2	A		
2	Secs. 4.1, 4.2	B		
3	Secs. 4.1, 4.2, 4.4	C	F	Problem 10
4	Not covered in this text (see Sears and Zemansky, Secs. 4-5, 4-6)	D, G	H	Problems 27, 28
5	Secs. 4.4, 4.6, 4.7	E	I	Problems 13, 15

TEXT: David Halliday and Robert Resnick, Fundamentals of Physics (Wiley, New York, 1970; revised printing, 1974)

SUGGESTED STUDY PROCEDURE

Read Chapter 3 and work at least Problems A through I before attempting the Practice Test.

It is essential to work the problems related to Objective 4 (Analytical) because the discussion in the book is limited to the one example for constant acceleration. You will be required to differentiate and integrate functions as shown in Problems D, G of this module and Problems 4 and 5 of the text.

Examples 7 and 8 on pp. 37, 38 are the most important part of the reading for Objective 5.

HALLIDAY AND RESNICK

Objective Number	Readings	Problems with Solutions	Assigned Problems	
		Study Guide	Study Guide	Text
1	Secs. 3-3, 3-4, 3-6	A		
2	Secs. 3-3, 3-4, 3-6	B		
3	Secs. 3-2, 3-5, 3-6, 3-8	C	F	10, 11, 12
4	Sec. 3-8 (in particular, Example 4)	D, G	H	4, 13
5	Secs. 3-8, 3-10	E	I	15, 16, 21, 30, 34

TEXT: Francis Weston Sears and Mark W. Zemansky, University Physics (Addison-Wesley, Reading, Mass., 1970), fourth edition

SUGGESTED STUDY PROCEDURE

Read Chapter 4 and work at least Problems A through I before attempting the Practice Test. The section on Velocity Components (4-9) will be treated in a later module.

Examples 1 and 2 on pp. 48 and 49 are the most important part of the reading for Objective 5.

SEARS AND ZEMANSKY

Objective Number	Readings	Problems with Solutions	Assigned Problems	
		Study Guide	Study Guide	Text
1	Secs. 4-2 through 4-4	A		
2	Secs. 4-2 through 4-4	B		
3	Secs. 4-3, 4-5, 4-6	C	F	4-23, 4-24
4	Secs. 4-5, 4-6, 4-8	D, G	H	4-28, 4-29
5	Sec. 4-7	E	I	4-20, 4-25, 4-26

TEXT: Richard T. Weidner and Robert L. Sells, Elementary Classical Physics (Allyn and Bacon, Boston, 1973), second edition, Vol. 1

SUGGESTED STUDY PROCEDURE

Read Chapter 3 and work at least Problems A through I before attempting the Practice Test.

It is essential to work the problems related to Objective 4 (Analytical) because the discussion in the book is limited to the one example for constant acceleration. You will be required to differentiate and integrate functions as shown in Problems D and G of this module and Problems 3-12 and 3-13 of the text.

Examples 3-3, 3-4, and 3-5 on pp. 28-32 are the most important part of the reading for Objective 5.

WEIDNER AND SELLS

Objective Number	Readings	Problems with Solutions	Assigned Problems	
		Study Guide	Study Guide	Text
1	Secs. 3-2 through 3-4	A		
2	Secs. 3-2 through 3-4	B		
3	Secs. 3-2 through 3-5	C	F	3-1, 3-2
4	Sec. 3-5 [Middle of p. 27 to Eq. (3-7) on p. 28]	D, G	H	3-12, 3-13
5	Secs. 3-5, 3-6	E	I	3-20, 3-23, 3-24, 3-32

PROBLEM SET WITH SOLUTIONS

A(1). Write the mathematical definitions of displacement, instantaneous velocity, and acceleration, and define all terms.

Solution

Displacement:

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1,$$

where $\vec{r}_2$ is position at point 2 and $\vec{r}_1$ is position at point 1.

Instantaneous Velocity:

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}.$$

Instantaneous Acceleration:

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}.$$

B(2). A woman swims the length of a 75-m pool and back again (one lap) in 60 s with a constant speed. (a) What is her displacement at the end of one lap? (b) What is her average velocity for one lap? (c) What is her average speed for one lap?

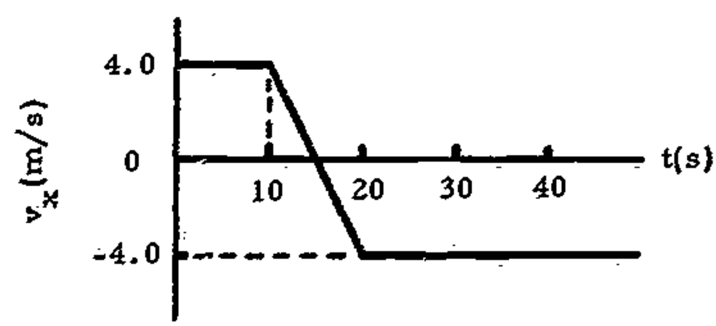
Solution

(a) $\vec{r}_2 = \vec{r}_1$; therefore, $\Delta\vec{r} = 0$.

(b) Since $\Delta\vec{r} = 0$, $\vec{v}_{av} = 0$.

(c) $v_{av} = 150\ \text{m}/60\ \text{s} = 2.5\ \text{m/s}$.

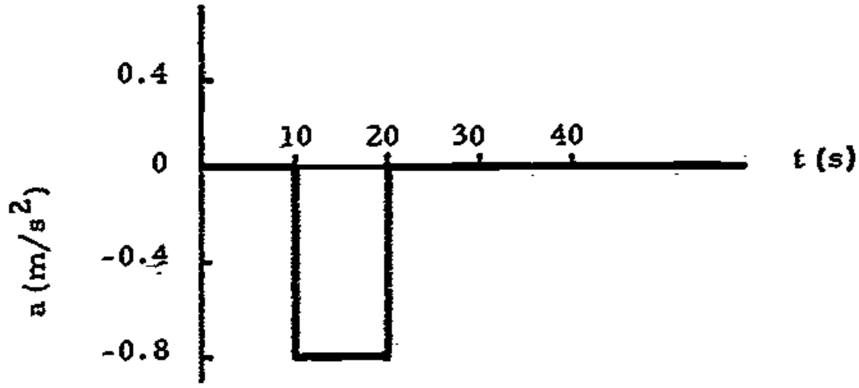
C(3). A particle's velocity is shown in the graph below. At $t = 0$, its displacement is $x = 0$.
(a) Sketch the acceleration vs. time graph corresponding to this velocity vs. time graph.



Solution

Acceleration is the slope of the v vs. t graph. For the first 10 s the acceleration is zero (v = 0). The acceleration is also 0 for t = 20 s. For the time between 10 and 20 s the slope is a constant. Therefore

$$a = a_{av} = \frac{v}{t} = \frac{v_2 - v_1}{t} = \frac{(-4.0) - (4.0)\ \mathrm{m/s}}{10\ \mathrm{s}} = -0.80\ \mathrm{m/s^2}.$$



C(3). (b) Sketch the graph of position (x) vs. time (t) corresponding to the velocity vs. time graph.

Solution

At t = 0, x = 0. The position at any time is the area under the v vs. t graph up to that time.

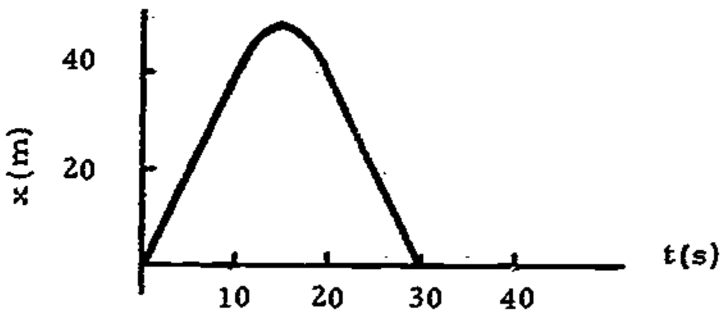
At t = 5 s; x = 4 m/s × 5 s = 20 m.

At t = 10 s; x = 4 m/s × 10 s = 40 m.

At t = 15 s; x = 40 m + $(\frac{1}{2})$4 m/s × 5 s = 50 m.

At t = 20 s; x = 50 m + $(\frac{1}{2}(-4)$ m/s × 5 s = 40 m.

At t = 30 s; x = 40 m + (-4) m/s × 10 s = 0.



Note: The curve is not linear between t = 10 and 20 s. In fact, it is a parabola, which will be shown later.

C(3). (c) Determine the average acceleration between $t = 0$ and 20 s.

Solution

$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{-4 - (+4)\ m/s}{20\ s} = \frac{-8\ m/s}{20\ s} = -0.4\ m/s^2.$$

D(4). The vertical position of a body under constant acceleration is given by

$$y = y_0 + v_{y0}t + \frac{1}{2}at^2,$$

where t = time,

y = vertical position at time t,

v_{y0} = initial velocity in vertical direction,

a = acceleration in vertical direction,

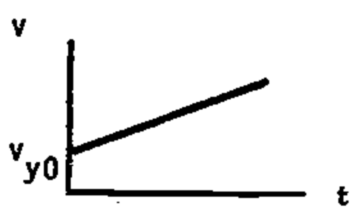
y_0 = initial position.

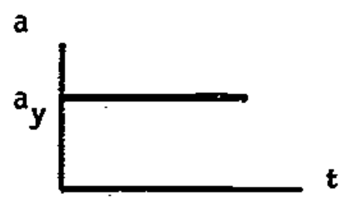
By the use of calculus, find the velocity and acceleration of the body as a function of time. Also construct a graph of velocity vs. time and acceleration vs. time for this case.

Solution

$$v_y = \frac{dy}{dt} = \frac{d}{dt}(y_0 + v_{y0}t + \frac{1}{2}at^2) = v_{y0} + at$$

$$a_y = \frac{dv_y}{dt} = \frac{d}{dt}(v_{y0} + at) = a.$$





E(5). An object is thrown vertically upward in a uniform gravitational field that produces an acceleration of $a = (-g = -9.8\ m/s^2)$. It has a speed of 9.8 m/s when it has reached one-half its maximum height.

(a) How high does it rise?

(b) What is its speed 1 s after it is thrown?

(c) What is its acceleration when it reaches its maximum height?

Solution

This problem requires the solution of the folowing equation:

(a) $v^2 - v_0^2 = 2ay$ (constant acceleration).

When at one-half height, $y_{1/2} = y_{max}/2$; and since $v = 0$ at $y = y_{max}$,

$$-v_0^2 = -2gy_{max} \text{ or } v_0^2 = 2gy_{max}.$$

Using $v^2 - v_0^2 = 2ay$ (at $y = y_{max}/2$) gives

$$v^2 - 2gy_{max} = -2gy,$$

and using $y = y_{1/2} = y_{max}/2$ gives

$$v^2 = 2gy_{max} - gy_{max} = gy_{max}.$$

Solving for

$$y_{max} = \frac{v^2}{g} = \frac{(9.8\ m/s)^2}{9.8\ m/s^2} = 9.8\ m.$$

Also:

$$v_0 = \sqrt{2gy_{max}} = \sqrt{2(9.8\ m/s)^2} = 13.9\ m/s.$$

(b) $v - v_0 = at$; $v = -gt + v_0 = -9.8 + 13.9 = 4.1\ m/s$.

(c) $a = -9.8\ m/s^2$ (acceleration is constant).

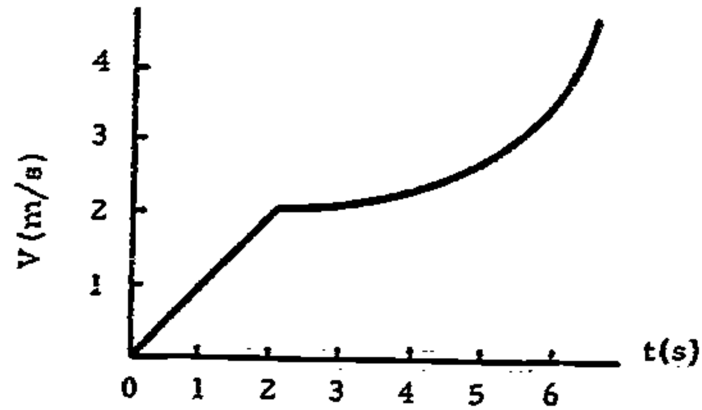
Problems

F(3). The velocity vs. time graph of a particle is shown in the figure below.

(a) During what time interval is the particle traveling with constant acceleration?

(b) Calculate graphically the displacement of the particle during the first 3 s.

(c) Estimate the particle's acceleration at t = 5 s.



G(4). A particle moves along the x axis in such a way that its x coordinate is given by $x = 3 + 20t - t^2$. x is in meters; t is in seconds. Find:

(a) Its velocity at $t = 4$ s.

(b) Its position at $t = 4$ s.

(c) Its acceleration at $t = 4$ s.

H(4). Let $x = A \cos \omega t$ where A and ω are constants. Find the expressions for velocity as a function of time and acceleration as a function of time.

I(5). A rocket is fired vertically and ascends with a constant vertical acceleration of $+20\ m/s^2$ for 80 s. Its fuel is then all used, and it continues upward with an acceleration of $-9.8\ m/s^2$. Air resistance can be neglected.

(a) What is its altitutde 80 s after launching?

(b) How long does it take to reach its maximum altitude?

(c) What is this maximum altitude?

Solutions

F(3). (a) Acceleration is constant for $t = 0$ to 2 s.

(b) 4 m (area under curve).

(c) $0.6\ m/s^2$ (slope at $t = 5$ s).

G(4). (a) $v = \frac{dx}{dt} = \frac{d}{dt}(3 + 20t - t^2) = 20 - 2t$ at $t = 4$ s, $v = 12$ m/s.

(b) $x = 3 + 20t - t^2$ at $t = 4$ s, $x = 67$ m.

(c) $a = \frac{dv}{dt} = \frac{d}{dt}(20 - 2t) = -2$ m/s^2.

H(4). $v = \frac{dx}{dt} = \frac{d}{dt}(A \cos \omega t) = A \frac{d}{dt}(\cos \omega t) = -A\omega \sin \omega t.$

$a = \frac{dv}{dt} = \frac{d}{dt}(-A\omega \sin \omega t) = -A\omega \frac{d}{dt} \sin \omega t = -A\omega^2 \cos \omega t.$

I(5). (This problem must be treated as two separate constant-acceleration problems.)
(a) 6.4×10^4 m. (b) 2.4×10^2 s. (c) 1.9×10^5 m.

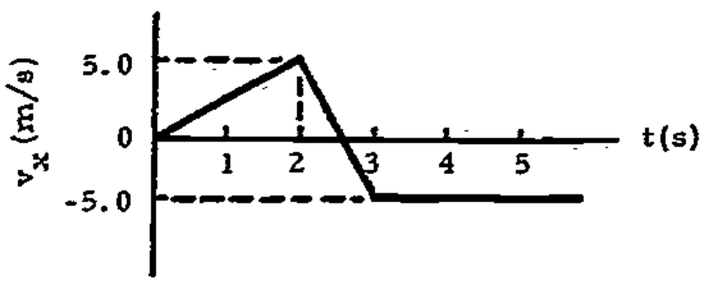
PRACTICE TEST

1. Write the mathematical definitions of displacement, instantaneous velocity, and acceleration.

2. A particle's velocity is shown on the graph. At $t = 0$, the displacement is $x = 0$.

   (a) Sketch the displacement and acceleration as a function of the time.

   (b) Determine the average acceleration during the time interval $t = 0$ to 2.5 s.



3. The position of a particle as a function of time is given by $y = \alpha t^3 - \beta t$, where α and β are constants. Find the expressions for velocity as a function of time and acceleration as a function of time.

4. A ball is thrown vertically upward from the ground with an initial speed of 24.5 m/s, under the influence of gravity ($g = 9.8$ m/s^2).

   (a) How long does the ball take to reach its highest point?

   (b) How high does the ball rise?

   (c) What is its velocity and acceleration at its maximum height?

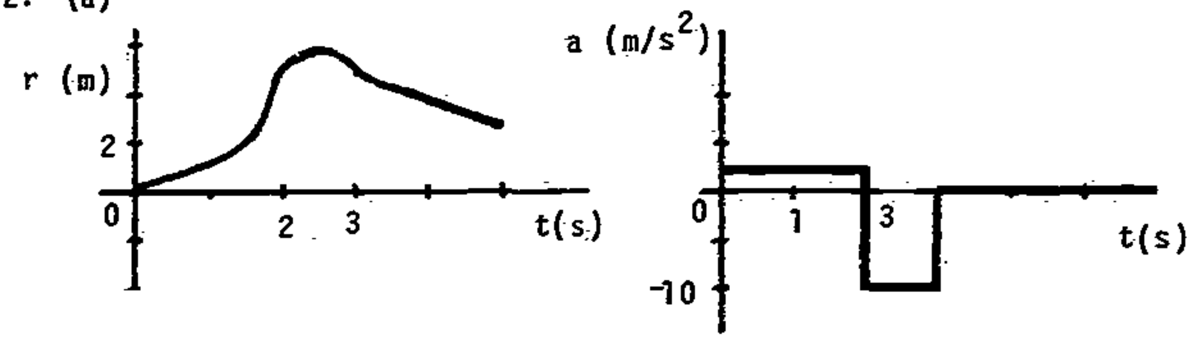
Answers to Practice Test

1. $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$.

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}.$$

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}.$$

2. (a)



(b) $\vec{a}_{av} = (\vec{v}_2 - \vec{v}_1)/(t_2 - t_1) = 0/2.5 = 0.$

3. $v = dy/dt = 3\alpha t^2 - \beta$, $a = dv/dt = 6\alpha t$.

4. (a) At the highest point, $v = 0$. Therefore we can use the equation,

$v(t) = 0 = v_0 + at$ (and v_0 = initial velocity = 24.5 m/s),

$t = -v_0/a = (-24.5 \text{ m/s})/(-9.8 \text{ m/s}^2) = 2.5$ s.

(b) $x(t) = x_0 + v_0 t + (1/2)at^2$

$= 0 + (24.5 \text{ m/s})(2.5 \text{ s}) + (1/2)(-9.8 \text{ m/s}^2)(2.5 \text{ s})^2$

$= 61.2 \text{ m} - 30.6 \text{ m} = 30.6$ m.

(c) $v_{max} = v_0 + at = 24.5 \text{ m/s} + (-9.8 \text{ m/s}^2)(2.5 \text{ s})$

$= 24.5 - 24.5 = 0$ (which we already knew from the fact that at the maximum point the ball stops rising and begins falling.

$a = -g = \text{const} = -9.8 \text{ m/s}^2$.

Date________________

RECTILINEAR MOTION

Mastery Test Form A

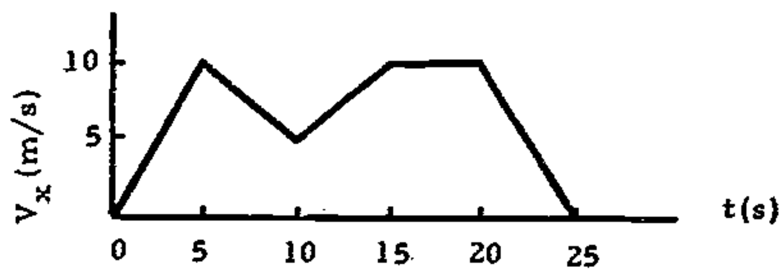
pass recycle

1 2 3 4

Name________________________________ Tutor____________________

1. Write the mathematical definitions of displacement, instantaneous velocity, and acceleration.

2. The graph below shows the straight-line velocity of an object as a function of time. What is the average acceleration during the time interval t = 5 to 15 s ? Plot a graph of the acceleration as a function of time. Indicate numerical values on the axes as accurately as freehand will permit.



3. The speed of a particle along the x axis is given as

$$v_x = \alpha t^2 \qquad (\alpha = \text{constant}).$$

(a) What is the position of the particle as a function of time if $x = 0$ at $t = 0$?

(b) What is the acceleration of the particle as a function of time?

4. On the moon, the acceleration due to gravity is 1/6 as large as on the earth. An object is given an initial upward velocity of 98 m/s at the surface of the moon.

(a) How long will it take for the object to reach maximum height?

(b) How high above the surface of the moon will the object rise?

Date________________

RECTILINEAR MOTION

Mastery Test Form B

pass recycle

1 2 3 4

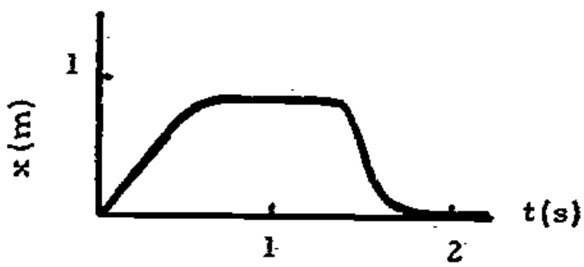
Name____________________ Tutor________________

1. Write the mathematical definitions of displacement, instantaneous velocity, and acceleration.

2. A graph of position (x) vs. time (t) for a particle is shown below.

   (a) Locate all regions where the instantaneous speed is zero.

   (b) What is the average speed for the time interval $t = 0$ to $t = 2$ s?



3. The position of a particle as a function of time is given by

   $$x = A \sin \omega t,$$

   where A and ω are constants. Find expressions for velocity and acceleration as a function of time.

4. A jet plane that is landing touches the ground at a speed of 100 m/s. It decelerates uniformly, coming to a stop after 40 s.

   (a) What is the plane's acceleration?

   (b) How far down the runway does the plane move before stopping?

Date________________

RECTILINEAR MOTION

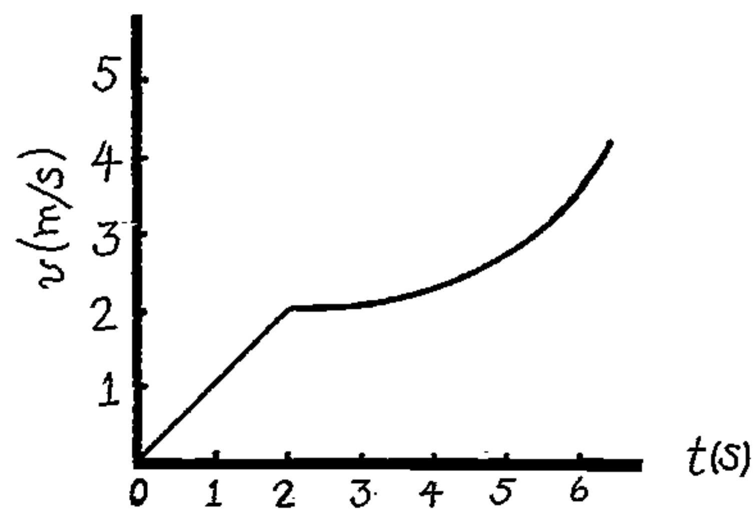
Mastery Test Form C

pass recycle
1 2 3 4

Name________________________ Tutor________________________

1. Write the mathematical definitions of displacement, instantaneous velocity, and acceleration.

2. The velocity vs. time graph of a particle is shown in the following figure:



(a) During what portion of the motion is the particle traveling with constant acceleration?

(b) Estimate the displacement of the particle during the first 3 s.

(c) Estimate the particle's acceleration at t = 5 s.

3. A particle moves along the x axis according to the equation

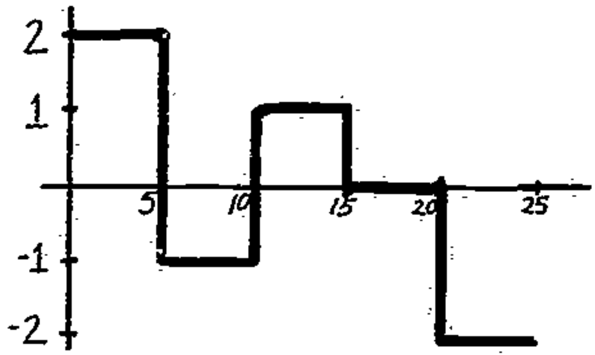
$$x = 5t + t^2,$$

where x has units of meters and t has units of seconds. Calculate the instantaneous velocity and acceleration at t = 2 s.

4. A ball thrown vertically upwards returns to the starting point in 4 s. Find its initial speed.

MASTERY TEST GRADING KEY - Form A

What To Look For	Solutions
1. $\vec{r}$, $\vec{v}$, and $\vec{a}$ are written as vectors. $\lim_{\Delta t \to 0}$ and/or d/dt is used in definitions of $\vec{v}$ and $\vec{a}$.	1. (a) Displacement: $\vec{r} = \vec{r}_2 - \vec{r}_1$. (b) Instantaneous velocity: $\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$. (c) Instantaneous acceleration: $\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$.
2. (a) Average does not equal instantaneous. (b) Units for acceleration.	2. (a) $a_{av} = \frac{dv}{dt} = \frac{(10 - 10)\ m/s}{10\ s} = 0\ m/s^2$.
3. (a) Integration to obtain x from v_x and application of condition x = 0 at t = 0 to find integration constant equal to zero. (b) Differentiation to obtain a_x from v_x.	3. (a) $v_x = \alpha t^2 = dx/dt$, $\int \frac{dx}{dt} dt = \int \alpha t^2\, dt = \alpha \int t^2\, dt$, $x = \alpha t^3/3$. (b) $a_x = \frac{dv_x}{dt} = \frac{d}{dt}(\alpha t^2)$, $a_x = 2\alpha t$.



4. (a) $v_f = 0$ when $y = y_{max}$. Check units.

4. (a)

$$v_f - v_i = at,$$

$$t = (v_f - v_i)/a.$$

Since $a = (-98/6)$ m/s^2,

$v_i = 98$ m/s, and

$v_f = 0$ at $y = y_{max}$,

$$t = \frac{(0 - 98)\ \text{m/s}}{-9.8/6\ \text{m/s}}$$

$$t = 60\ \text{s}.$$

(b)

$$v_f^2 - v_i^2 = 2ay,$$

$$y = \frac{v_f^2 - v_i^2}{2a}$$

$$= \frac{0 - (98\ \text{m/s}^2)^2}{-2(9.8/6)\ \text{m/s}^2} = 2940\ \text{m}.$$

MASTERY TEST GRADING KEY - Form B

What To Look For

1. $\vec{r}$, $\vec{v}$, and $\vec{a}$ are written as vectors. $\lim_{\Delta t \to 0}$ and/or d/dt is used in definitions of $\vec{v}$ and $\vec{a}$.

2. (a) Slope of x vs. t graph.
   (b) Average velocity is zero because total displacement is zero.

3. Differentiation of this function will be useful in the module on simple harmonic motion.

4. Since acceleration is constant, instantaneous acceleration is equal to average acceleration. Check units.

Solutions

1. (a) Displacement: $\vec{r} = \vec{r}_2 - \vec{r}_1$.
   (b) Instantaneous velocity:
   $$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}.$$
   (c) Instantaneous acceleration:
   $$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}.$$

2. (a) Instantaneous speed is zero in the interval $1/2 \leq t \leq 3/2$ s.
   (b) Average speed:
   $$v = \frac{\text{total distance traveled}}{\text{total time}},$$
   $$v = 2 \text{ m}/2 \text{ s} = 1 \text{ m/s}.$$

3. $x = A \sin \omega t$, $\quad v = \frac{dx}{dt} = A\omega \cos \omega t$,
   $$a = \frac{dv}{dt} = -A\omega^2 \sin \omega t.$$

4. (a) $a = \frac{v_f - v_i}{t}$, $\quad t = 40$ s,
   $v_i = 100$ m/s,
   $v_f = 0$,
   $$a = (0 - 100 \text{ m/s})/40 \text{ s},$$
   $$a = -2.5 \text{ m/s}.$$
   (b)
   $$y = v_i t + \frac{1}{2} a t^2,$$
   $$y = (100)(40) + \frac{1}{2}(-2.5)(40)^2,$$
   $$y = 4000 - 2000 = 2000 \text{ m}.$$

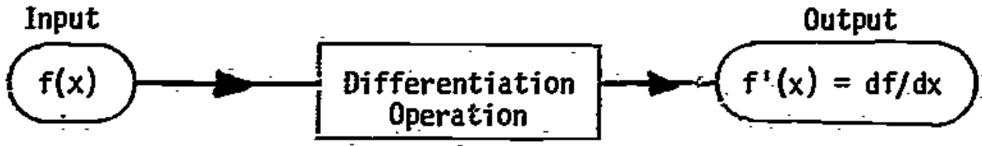
MASTERY TEST GRADING KEY - Form C

What To Look For	Solutions
1. $\vec{r}$, $\vec{v}$, and $\vec{a}$ are written as vectors. $\lim_{\Delta t \to 0}$ and/or d/dt is used in definitions of $\vec{v}$ and $\vec{a}$.	1. (a) Displacement: $\vec{r} = \vec{r}_2 - \vec{r}_1$. (b) Instantaneous velocity: $\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$. (c) Instantaneous acceleration: $\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$.
2. (a) Acceleration is slope of v vs. t graph. (b) Displacement is area under v vs. t graph, and the answer is approximately 4 m. (c) Slope of graph. Check units.	2. (a) Instantaneous acceleration for $0 \le t \le 2$ s. (b) Displacement graphically - area under curve: $x = \frac{1}{2}(2\text{ m/s} \times 2\text{ s}) + (2\text{ m/s} \times 1\text{ s})$ $= 4$ m. (c) Acceleration is slope of v vs. t: $a = \frac{1}{2}\text{ m/s}^2$.
3. Check differentiation and note that acceleration is constant.	3. $x = 5t + t^2$ $v = dx/dt = 5 + 2t$; at $t = 2$ s, $v = 9$ m/s, $a = \frac{dv}{dt} = \frac{d}{dt}(5 + 2t) = 2\text{ m/s}^2$.
4. If student uses $t_{up} = t_{down}$, let him/her justify statement.	4. $y = y_0 + v_0 t + \frac{1}{2}at^2$, $y = y_0 = 0$, $0 = v_0 + \frac{1}{2}at$, $v_0 = -\frac{at}{2}$, $a = -9.8\text{ m/s}^2$, $t = 4$ s, $v_0 = -\frac{(-9.8)4}{2}$, $v_0 = 19.6$ m/s

CALCULUS

DIFFERENTIATION

Differentiation of a function, say f(x), is a mathematical operation which yields a second function called the derivative of f [symbolized by f'(x) or dy/dx]. This procedure is represented in the diagram, which shows



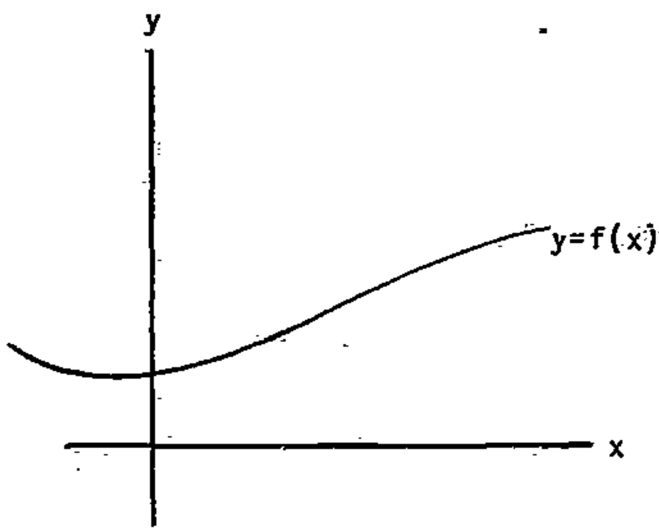
the function f(x) being input to an "analytical machine" that manufactures as output the derivative of f. A detailed mathematical prescription for the differentiation operation is nontrivial. It usually involves a quarter or semester course, which requires time, attention, and effort on the student's part.

Our goal here is to provide you with graphical and intuitive understandings of what information the derivative supplies; and tabular means for determining derivatives.

Graphical Interpretation of the Derivative

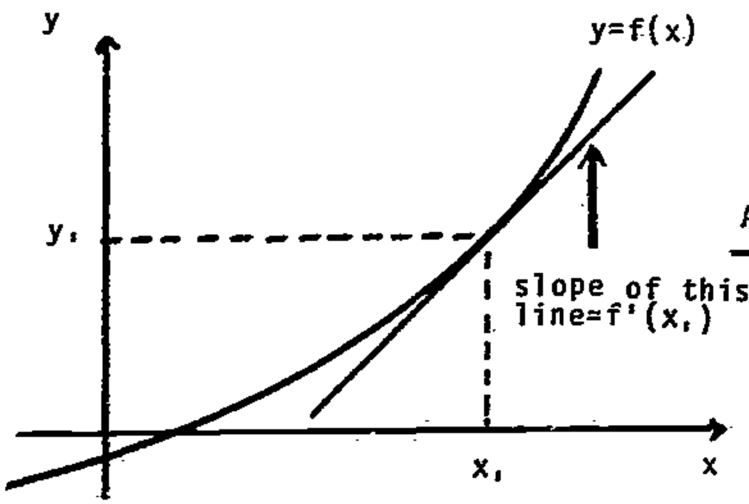
A graphical understanding of the derivative will prove useful time and again in your study of physics. So, let's get at it.

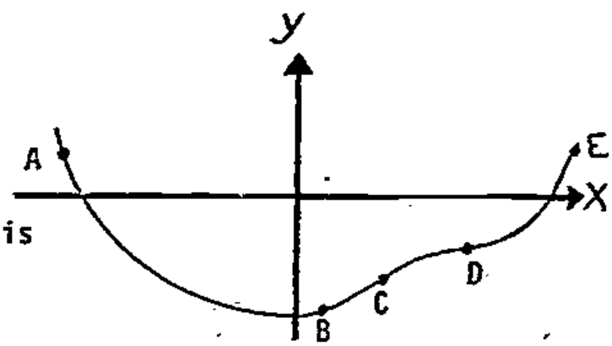
A function f(x) is graphed in the xy plane by letting y = f(x), i.e. for each value of x (for which f is defined) there corresponds one value of y obtained from the "rule" y = f(x). This number pair (x,y) plots as a point in the plane. As x changes, this point sweeps out a curve. Such a curve might look like the one shown in the figure on the right.



Question:	What information does the derivative $f'(x)$ provide about this graph?
Answer:	Let x_1 be a specific value of x. The value of the derivative for $x = x_1$ is denoted by $f'(x_1)$ or $(dy/dx)x_1$ and is numerically equal to the slope of the line tangent to the curve $y = f(x)$ at the point (x_1,y_1) where $y_1 = f(x_1)$.

Read this interpretation carefully while studying Figure 1.





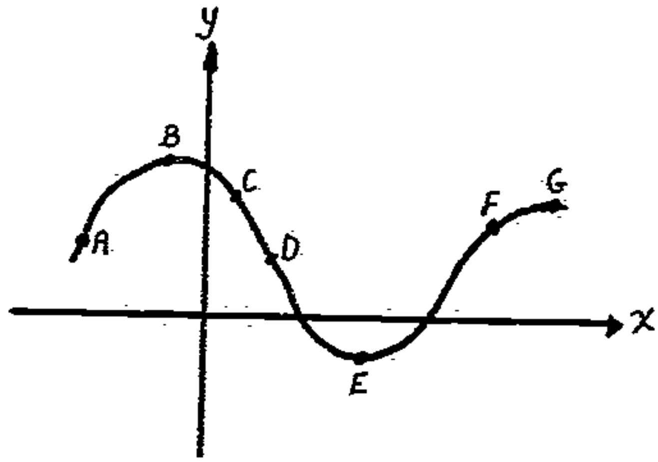
Example 1: In Figure 2, which point or points is:

	Answer
$f' = 0$?	B,D.
$f' > 0$?	C,E.
$f' < 0$?	A.
$\|f'\|$ the greatest?	E.

Each of the answers in Example 1 is obtained by looking at the graph and ascertaining the needed information about the slope of the curve (actually of the tangent line). For example, point B has been drawn at the lowest point on the curve. A tangent line at B is horizontal and therefore has a zero slope. Since the tangent line at A slopes downward (i.e., has a negative slope) $f'< 0$ at A.

Exercise A: By inspecting the graph decide whether to insert >, =, or < in each of the blanks.

(a) f'_A _____ 0	(i) f'_B _____ f'_E
(b) f'_B _____ 0	(j) f'_E _____ f'_F
(c) f'_C _____ 0	(k) f'_D _____ f'_E
(d) f'_D _____ 0	(l) f'_A _____ f'_F
(e) f'_E _____ 0	(m) f'_C _____ f'_D
(f) f'_F _____ 0	(n) f'_D _____ f'_F
(g) f'_G _____ 0	(o) $\lvert f'_D\rvert$ _____ $\lvert f'_F\rvert$
(h) f'_A _____ f'_B	(p) $\lvert f'_D\rvert$ _____ $\lvert f'_B\rvert$

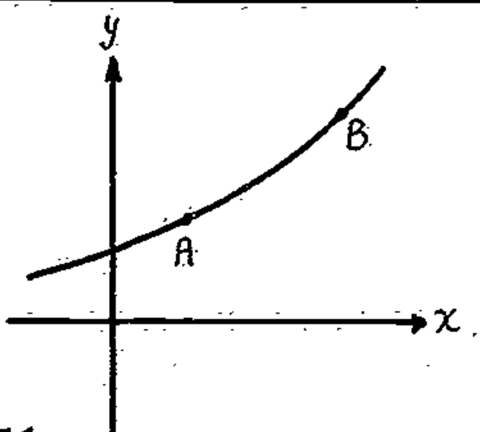


Exercise B: With reference to the graph of Exercise A, complete the table with appropriate words and symbols for the sentence:

At point _____, f is _______ and f' _____ 0.

Point	Increasing, decreasing, not changing	<, =, >
A		
B	not changing	
C		
D		
E		
F		>
G		

Consider the graph. At both points A and B, f is increasing and f' is positive. But more can be said. At B the rate of increase of f is greater than A. This statement is said in derivative language by saying that the derivative is greater at B than at A. We can summarize these remarks with



$$f'_B > f'_A > 0.$$

f increasing more rapidly at B than at A.

f increasing at A.

Clearly f' (or dy/dx) is related to the rate of change of f, and this rate changes from point to point if the slope of $y = f(x)$ is changing as x changes. The following statement (definition) relates (defines) instantaneous rate of change of a function and its derivative.

> If $y = f(x)$, the instantaneous rate of change of y per unit change in x at x_1 is defined to be $f'(x_1)$.

This statement is often abbreviated to say "$f'(x_1)$ is the rate of change of f with respect to x at x_1."

<u>Determining f' by Using the Table</u>

Suppose $f(x) = x^2$. What is $f'(x)$? If you don't happen to know already, then two options are offered here.

<u>Option 1</u>. The formal definition of the derivative gives

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}.$$

Evaluate it. OR

<u>Option 2</u>. Turn to p. 15 of this module entitled "Table of Derivatives."

Locate x^2 in the f column and read 2x from the f' column.

Answer: $f'(x) = 2x$.

Now Option 1 is the way you do it when you really want to dig mathematics - and you have a month or two to learn about limits, etc. Guess what we have in mind here?

Let's do another. Let $f(x) = x^9$. Look under the f column. What now? x^9 does not appear as an entry. Oh yes, it does. It appears as x^p. The table gives $f'(x) = px^{p-1}$, or since $p = 9$ in this case, $f'(x) = 9x^8$. Furthermore, $f'(1) = 9 \times 1^8 = 9$. Thus the slope of (the tangent line to) $y = x^9$ at (1,1) is 9; and at $x = 1$ the rate of change of y with respect to x is 9.

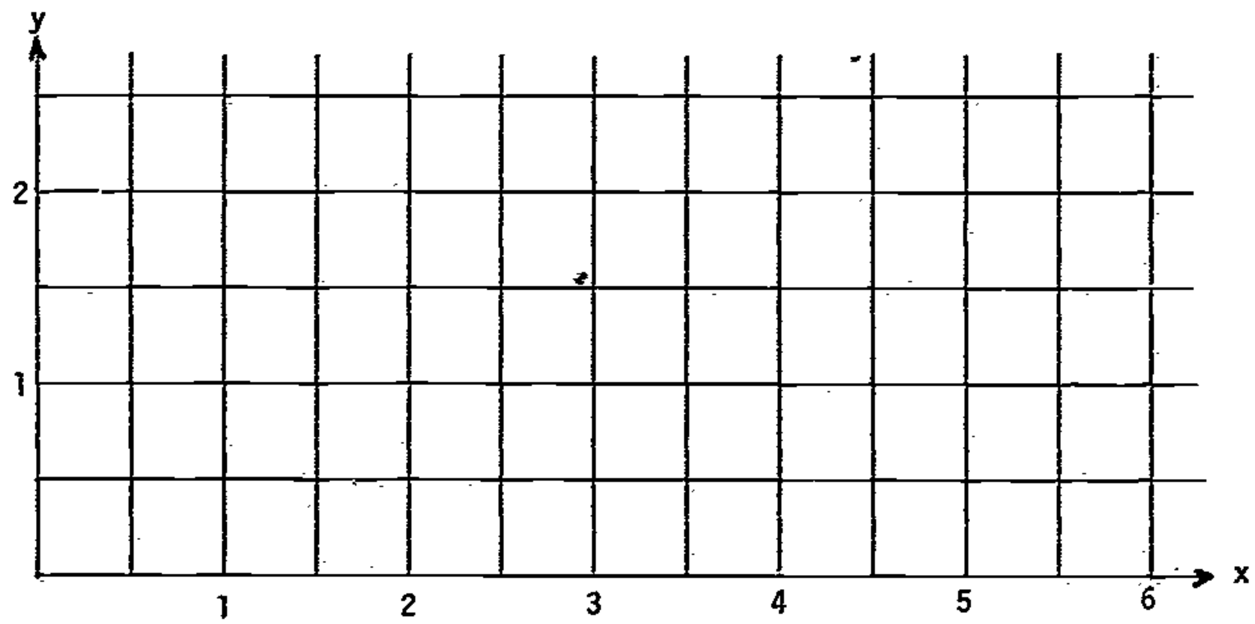
<u>Exercise C:</u> Let $f(x) = \sqrt{x} = x^{1/2}$.

(a) $f'(x) =$ ________________. (b) $f'(1) =$ ________________.

(c) Rate of change of x with respect to x at $x = 1$ is ________________.

(d) Graph f for $0 \leq x \leq 6$. Construct the tangent line to f at (1,1).
Slope = ______________. Does this agree with part (b)?

x	0.00	0.50	1.00	1.50	2.00	2.50	3.00	4.0	5.0	6.0
$\sqrt{x}$	0.00	0.71	1.00	1.22	1.41	1.58	1.73	2.00	2.24	2.45



Exercise D: Complete the table.

$f(x)$	$f'(x)$	x_1	$f'(x_1)$
x^2		-1	
x^{-4}		2	
6		1	
$\sin x$		$\pi/2$	
$\cos x$		$\pi/2$	
$\tan x$		$\pi/4$	
$\sin(2x)$		$\pi/2$	
$\cos(\pi x)$		1/2	
e^x		0	
e^{-2x}		1	
$\sqrt{x^2+16}$		3	
$1/\sqrt{x^2+9}$		4	

A Few Important Properties of Derivatives

$$\frac{d}{dx}(kf) = k\,\frac{df}{dx} \qquad \text{(k is a constant).} \tag{P1}$$

$$\frac{d}{dx}(f+g) = \frac{df}{dx} + \frac{dg}{dx}\,. \tag{P2}$$

$$\frac{d}{dx}(fg) = \frac{df}{dx}\,g + f\,\frac{dg}{dx}\,. \tag{P3}$$

$$\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{(df/dx)g - f(dg/dx)}{g^2}\,. \tag{P4}$$

Example 2

(a) $\frac{d}{dx}(6x^2) \overset{*}{=} 6\,\frac{d}{dx}(x^2) = 6(2x) = 12x.$ (*P1)

(b) $\frac{d}{dx}(x^2 + \sin x) \overset{*}{=} \frac{d}{dx}(x^2) + \frac{d}{dx}(\sin x) = 2x + \cos x.$ (*P2)

(c) $\frac{d}{dx}(e^x \cos x) \overset{*}{=} \frac{d}{dx}(e^x) \cos x + e^x \frac{d}{dx}(\cos x) = e^x \cos x - e^x \sin x.$ (*P3)

(d) $\frac{d}{dx}\left(\frac{e^x}{\ln x}\right) \overset{*}{=} \frac{\ln x(de^x/dx) - e^x(d/dx)(\ln x)}{(\ln x)^2} = \frac{e^x \ln x - e^x/x}{(\ln x)^2}.$ (*P4)

Exercise E:

(a) $f(x) = 2x^3 - \sin x$: $f'(x) =$ ______

(b) $g(x) = 3x^2e^{-x}$: $g'(x) =$ ______

(c) $h(x) = e^x/(x + a)$: $h'(x) =$ ______

(d) $F(x) = x \sin(\pi x)$: $F'(1) =$ ______

(e) $G(x) = e^{4x} \tan(\pi x)$: $G'(1/4) =$ ______

(f) $H(x) = e^x\sqrt{x + 9}$: $H'(0) =$ ______

THE DEFINITE INTEGRAL

The definite integral is a mathematical operation that requires as input a function, say $f(x)$, and two numbers a and b, which are the coordinates of the end points of an interval on the x axis, i.e., $a \le x \le b$. Given this input the definite integral of f on $[a,b]$ yields as output one number, say I. This is symbolized in the figure. Just as with the derivative, a careful prescription of this operation is nontrivial. Again we shall first offer you some graphical understanding as to the meaning of the number I and then give you a procedure (using integral tables) to determine I.

Graphical Interpretation of the Integral

> The definite integral of a function $f(x)$ on the interval $a \leq x \leq b$ is numerically equal to the area enclosed by the curve $y = f(x)$, the x axis, the line $x = a$, and the line $x = b$.

The shaded area in Figure A is the area determined by the definite integral of f on a,b.

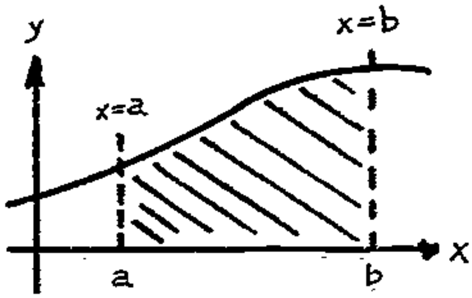


Figure A

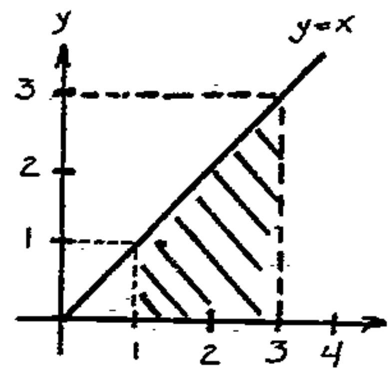


Figure B.

Determination of I

Suppose $f(x) = x$, $a = 1$, and $b = 3$. The area to be found is shown in Figure B. Of course, it's very easy to do so since the figure whose area we seek is a trapezoid. In fact,

$$\text{Area} = (\text{Average height}) \times (\text{base}) = 2 \times 2 = 4.$$

This simple calculation is possible because $y = x$ graphs as a straight line. Almost any other function would not be so trivial.

Here's how you do the integral with the "Table of Integrals" provided in this module (p. 15). Look under the function column f and find x. Read off the antiderivative (indefinite integral) for x, namely, $(1/2)x^2$. Evaluate this function at the upper limit b (= 3 in this case) and subtract the value of this function at the lower limit a (= 1). The result of this calculation is the value of the definite integral. Thus we have

$$I = \frac{1}{2}(3^2) - \frac{1}{2}(1^2) = \frac{9}{2} - \frac{1}{2} = \frac{8}{4} = 4,$$

in exact agreement with our previous calculation.

Let's try another. Let $f(x) = x^2$ from 0 to 2. The area to be determined is shaded in Figure C. From the table the antiderivative for x^2 is listed as $(1/3)x^3$. The desired area is

$$I = \frac{1}{2}2^3 - \frac{1}{3} = \frac{8}{3}.$$

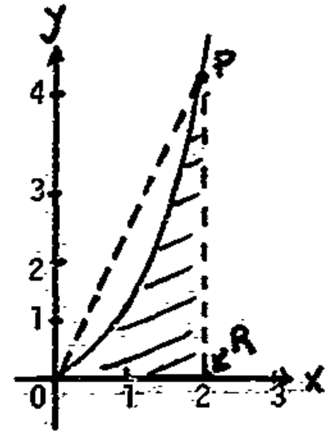
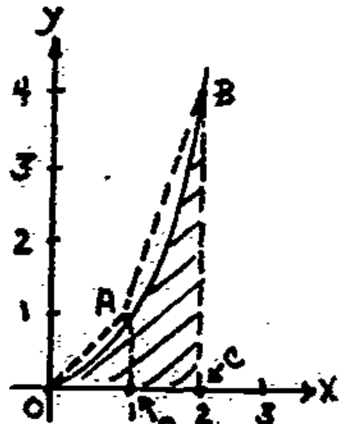


Figure C

Does this look reasonable? The shaded area is less than that of the triangle OPR whose area is (1/2) (base) (height) or (1/2) (2) (4) = 4, which exceeds 8/3 as expected.

Figure D



We can make a better estimate for the desired area by using the area of the triangle OAD and the trapezoid ABCD (See Figure D). The result for this improved <u>over</u>estimate is

$$\frac{1}{2}(1)\ (1) + \frac{1}{2}(1+4)\ (1) = \frac{1}{2} + \frac{5}{2} = 3,$$

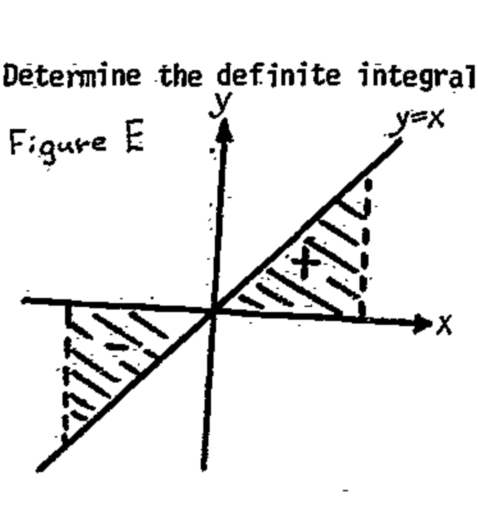
which is indeed less than 4 and 12.5% greater than 8/3, the result from the integral.

<u>Exercise F:</u> Continue this estimating process one more time by dividing the x axis equally four times and calculating the area of the four figures. <u>Answer:</u> $2\frac{3}{4}$.

Essentially, what the integral does is continue this process of dividing the interval [0,2] into increasingly larger number of subintervals. In fact, the integral is the limit of the approximate areas as the number of subintervals approaches infinity. In this sense then, the definite integral can be thought of somewhat intuitively as the sum of many (infinitely many, in fact) terms each of which approaches zero as the total number approaches infinity. We avoid the difficulty of actually doing this by using the table of antiderivatives.

<u>Exercise G:</u> Let $f(x) = x$, $a = -2$, $b = 2$. Determine the definite integral of f on [a,b]. <u>Answer:</u> 0.

Figure E



If you did Exercise G correctly, the result is zero. But how can this be? Here's how. Look at Figure E. Areas below the x axis are treated by the integral as negative. As you can see, the two area in the figure exactly cancel, leaving a net area of 0.

Exercise H:

(a) $f(x) = \cos x$, $a = 0$, $b = \pi/2$, $I =$ ________________.

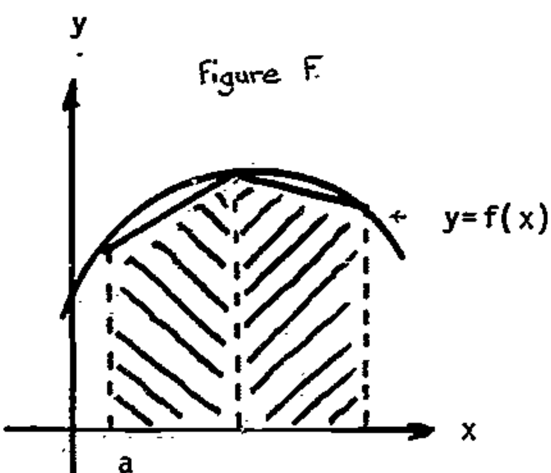
(b) Sketch a graph of cos x and estimate the area determined here by partitioning the interval $[0,\pi/2]$ into two and then three equal parts. Does your result for part (a) appear reasonable?

(c) $f(x) = \cos x$, $a = 0$, $b = \pi$, $I =$ ________________.

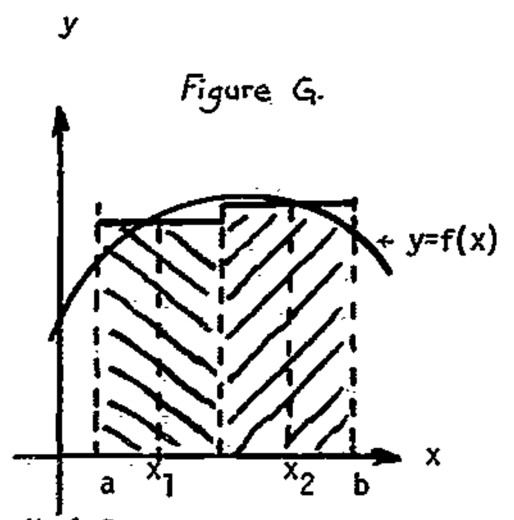
(d) Explain your result for part (c).

Estimating the Value of a Definite Integral

You may find yourself needing to evaluate a definite integral of a function that is not in your table. You have already seen one way to estimate the integral by partitioning the integration interval a,b into N equal subintervals and approximating the area by a set of N trapezoids. This technique is depicted graphically in Figure F below for N = 2.



N=2 Trapezoid Approximation



N=2 Rectangle Approximation

A second technique is shown in Figure G. Here the function is approximated by N rectangles. The height of each rectangle is the value of the function at the midpoint of the corresponding subinterval. From the figure you can see that the N = 2 approximation for I is then

$$I = f(x_1)\,\Delta x + f(x_2)\,\Delta x = \sum_{k=1}^{2} f(x_k)\,\Delta x,$$

where x_k is the value of x at the center of the k-th subinterval. For an N partition then,

$$I = \sum_{k=1}^{N} f(x_k)\,\Delta x.$$

As N increases the rectangular approximation gets closer and closer to the area being sought. In fact, the definite integral can be defined by

$$I = \lim_{N\to\infty} \sum_{k=1}^{N} f(x_k)\,\Delta x.$$

This suggests the usual notation for a definite integral, which is

$$\int_a^b f(x)\,dx.$$

The symbol $\int$ is called an integral sign. It symbolizes both the summation sign Σ and the limiting process. The values of a and b are included so as to indicate the end points of the integration interval. Finally, dx symbolizes the Δx in the sum. We shall use this notation for the integral. To be sure you understand, here is the appropriate way to write the integrals with the results we have obtained so far.

Function	Interval	Integral		Value
x	$[1,3]$	$\int_1^3 x\,dx$	=	4
x^2	$[0,2]$	$\int_0^2 x^2\,dx$	=	8/3
x	$[-2,2]$	$\int_{-2}^2 x\,dx$	=	0
$\cos x$	$[0,\pi/2]$	$\int_0^{\pi/2} \cos x\,dx$	=	1
$\cos x$	$[0,\pi]$	$\int_0^{\pi} \cos x\,dx$	=	0

One more comment before you go to work: Most integral tables will indicate the antiderivative for $f(x)$ by $\int f(x)\,dx$, i.e., the definite integral without limits. For example,

$$\int x\,dx = \frac{1}{2}x^2,$$

so that

$$\int_1^3 x\,dx = \frac{1}{2}(3)^2 - \frac{1}{2}(1)^2 = \frac{9}{2} - \frac{1}{2} = 4,$$

as we found earlier.

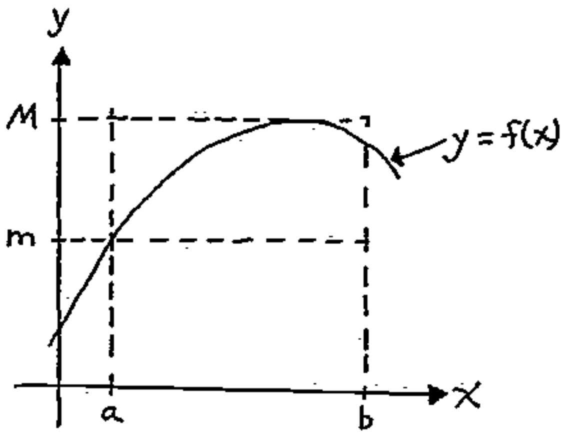


Figure H

One More Comment on Estimating Integrals

Consider the definite integral suggested by Figure H. A valuable technique for getting bounds on a definite integral is suggested. Let m be the minimum value of f on [a,b] and M the maximum value of f on [a,b]. Consider the rectangle of height m and base (b - a). Its area is necessarily less than the area determined by

$$\int_a^b f(x)\,dx.$$

Similarily the area M(b - a) is greater than that of the integral. Thus,

$$m(b - a) \leq \int_a^b f(x)\,dx \leq M(b - a).$$

Note: The equality signs are included in this equality to take care of the case where f is constant on the interval. Then $m = M = f(x)$ and

$$m(b - a) = \int_a^b f(x)\,dx = M(b - a).$$

Exercise I: Evaluate the following definite integrals. Also determine m and M and determine bounds on the integral. [You may need to sketch a graph of f(x) to determine m and M].

Integral		$m(b-a)$	$M(b-a)$
$\int_1^2 x^2\,dx =$		$1(1) = 1$	
$\int_{-1}^2 x\,dx =$			
$\int_0^{\pi/4} \cos x\,dx =$			
$\int_0^1 e^x\,dx =$			
$\int_0^{16} \sqrt{x+9}\,dx =$			
$\int_0^3 \frac{x\,dx}{\sqrt{x^2+16}} =$			
$\int_0^2 \frac{dx}{x+2} =$			
$\int_{-1}^1 e^{-x}\,dx =$		$e^{-1}(2) \approx 0.3$	$e(2) \approx 5.4$

Remember in each case you should check to see that

$$m(b-a) \le \int_a^b f(x)\,dx \le M(b-a).$$

Two More Important Properties of the Integral

The following properties of integrals are frequently required:

$$\int_a^b k\,f(x)\,dx = k\int_a^b f(x)\,dx \quad (k \text{ is a constant}), \tag{P1}$$

$$\int_a^b [f(x) + g(x)]dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx. \qquad \text{(P2)}$$

Example 3

(a) $\int_0^3 4x\,dx = 4\int_0^3 x\,dx$ [by (P1)]

$= 4\,(3^2/2 - 0^2/2)$

$= 18.$

(b) $\int_0^1 (x + x^2)\,dx = \int_0^1 x\,dx + \int_0^1 x^2\,dx$ [by (P2)]

$= (\frac{1^2}{2} - \frac{0^2}{2}) + (\frac{1^3}{3} - \frac{0^3}{3})$

$= \frac{1}{2} + \frac{1}{3} = \frac{5}{6}.$

Exercise J:

(a) $\int_1^2 (x^2 - x)\,dx =$ ____________________.

(b) $\int_0^{\pi/2} (\sin x + \cos x)\,dx =$ ____________________.

(c) $\int_0^3 (x^2 + e^{-x})\,dx =$ ____________________.

(d) $\int_0^1 (x + \sin \pi x)\,dx =$ ____________________.

TABLE OF DERIVATIVES

f(function)	f'(derivative of function)
.	.
.	.
.	.
x^{-3}	$-3/x^2$
x^{-2}	$-2/x$
x^{-1}	ln x
constant	0
x	1
x^2	$2x$
x^3	$3x^2$
.	.
.	.
.	.
x^p	px^{p-1}

TABLE OF INTEGRALS

f(function)	∫f(antiderivative)
.	.
.	.
.	.
x^{-3}	$-x^{-2}/2$
x^{-2}	$-x^{-1}$
x^{-1}	ln x
constant (c)	cx
x	$x^2/2$
x^2	$x^3/3$
x^3	$x^4/4$
.	.
.	.
.	.
x^p	$x^{p+1}/(p+1)$