This theoretical paper discusses (1) the meaning of the concept of structure, how the concept of structure is used by psychologists, and how the concept can be defined, and (2) the role of the concept "structure" in cognitive development with specific reference to Piaget's vision of intellectual development. The meanings of the term structure in structural psychology, in gestalt psychology and in the structuralist movement are described, and it is emphasized that the concept of structure is a purely logical notion. Adjectives (like grammatical) serve only to modify rather than define the generic concept. Treating structure in this way implies that structural isomorphism has useful implications for work in new areas. Such analytical uses of structure are contrasted with Piaget's conception of cognitive structure which, it is alleged, has been used as grounds for his stages. The possible fallacy of assuming a relation between cognitive structures and cognitive stages in Piagetian theory is explained. Predictions concerning timing and sequence of stages based on this assumption are contrasted with Piaget's findings. (GO)
THE CONCEPT OF STRUCTURE IN COGNITIVE-DEVELOPMENTAL THEORY

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In his Introduction to Mathematical Philosophy, Russell observed that there has been a great deal of speculation in traditional philosophy which might have been avoided if the importance of structure, and the difficulty of getting behind it, had been realised. For example, it is often said that space and time are subjective, but are caused by things in themselves which must have differences intersel corresponding with the differences in the phenomena to which they give rise. Where such hypotheses are made, it is generally supposed that we can know very little about the objective counterparts. In actual fact, however, if the hypotheses as stated were correct, the objective counterparts would form a world having the same structure as the phenomenal world ... In short, every proposition having a communicable significance must be true of both worlds or of neither: the only difference must lie in just that essence of individuality which always eludes words and baffles description, but which, for that very reason, is irrelevant science [p. 61].

I take it as axiomatic that Russell's claim about the importance of structure in so-called traditional philosophy applies also to the infant science of cognitive development. That is, I am prepared to maintain that it is probably more important to ascertain how the organization of knowledge changes during the course of development than it is to ascertain either the explicit things we come to know or how we come to know them. I would even maintain that the structure of our knowledge probably is a more legitimate question for science to investigate than the "what" and "how" of our knowledge. Certainly
there can be no doubt that the structure question, in view of its comparative insensitivity to thorny philosophical questions of meaning, is much easier for science to address. This is not to say that a thoroughgoing emphasis on the ontogeny of knowledge structures is without cost. Concreteness, i.e., the close connection between a descriptive construct and some quantifiable variable, invariably is sacrificed and with it goes considerable predictive precision. However that may be, most of the questions which we presently need to have answered about cognitive development appear to be structural ones.

Persuant to the views just expressed, I should like to deal with two structure-related matters in this paper. First, I shall consider the concept of structure itself and the question of how this concept is to be defined. Second, I shall consider how, generally speaking, the concept of structure enters into cognitive developmental theory. In the brief space and time available, it is obvious that we shall not be able to examine more than few points relating to each of these matters. Since some selectivity, therefore, is inescapable, I have chosen to focus on points which are discrepant with what I take to be majority practice either in psychology generally or in cognitive-developmental psychology particularly.

The Concept of Structure

Historically, psychologists have operated with a concept of structure which may be charitably termed vague and intuitive. Titchener, who coined the terms "structural psychology" and "functional psychology," told us in 1898 that the aim of structural psychology is to discover
what the various forms of behavior are "as such" and that the aim of functional psychology is to discover the specific purposes that various forms of behavior fulfill in the existence of the organism. It is clear that Tichener regarded the task of structural psychology as more basic than the apparently teleological aim of functional psychology. It also is clear that Tichener intended to promote within psychology more or less the same distinction between structural disciplines and functional disciplines which was then extant in biology (e.g., anatomy vs. physiology). As to the meaning of the concept of structure and how we are to know a psychological structure when we see one, Tichener was silent—perhaps prudently so. The second great structural movement within psychology, gestalt psychology, was not any more precise about the meaning of the structure concept than Tichener had been. However, where Tichener had been silent, the original gestalters engaged in a brand of obscurantism of which Hegel no doubt would have been proud. Whatever their other merits may be, mysterious phrases such as "the whole is greater than the sum of its parts" do not prove definitionally helpful vis-à-vis the structure concept. At best, we derive from gestalt psychology a vague feeling that psychological structures are concerned with such things as "pattern," "form," "configuration," etc. But these are merely synonyms whose own meanings are less than clear.

Psychologists certainly are not the only group of scientists who waffle on the meaning of structure. Indeed, vague and intuitive conceptions of structure are the rule in all of psychology's sister biological sciences. However, definitional imprecision has not proved
very troublesome to other biological sciences because their structures are impersonal for the most part. That is, the biologist can make direct appeals to public experiential data, especially those concerned with vision and hearing, and to public testimony which the psychologist cannot use. Thus, if I ask the anatomist whose office is down the hall from mine about the sorts of structures he studies, I imagine that he will give me a tour of his quite remarkable collection of skeletons. Similarly, if I ask my wife, an audiologist, about the sorts of structures that she studies, she no doubt will show me a transverse-sectional diagram of the outer, middle, and inner ears. If I am especially obstinate, she may even do some dissecting for me. It is clear then that my anatomist colleague and my wife are confident that they know what a biological structure is. Of course, we know that they are somewhat deluded: pointing to something and saying "that is a stomach" or "that is a kidney" is not exactly a definition. But that is not the point. The point is that they have the advantage of being able to show me something palpable when I ask, "What is a structure?" and I cannot do the same in return because I, as a psychologist, deal in data which are essentially personal and nonpublic. However, if I were possessed of a general definition of the concept of structure, which both the anatomist and my wife can understand, then I might be able to generate some intelligible psychological illustrations.

Before proceeding to formulate a general definition of structure, we should at least note in passing the so-called "structuralist movement" which has been going on in the social sciences in recent years.
and concerning which our two discussants have written with considerable eloquence. The structuralist movement merits no more than passing mention in the present context because it has, in my view, done more to cloud the meaning of structure in the minds of psychologists than to clarify it. With the possible exception of Piaget, the members of this movement do not seem overly concerned with exhibiting a precise formulation of their key construct. Instead, they appear content to identify structure and (what amounts to the same thing) structural analysis with certain controversial tenets of the personal metaphysics of the movement's leading members. Among the more notable tenets are holism, anti-determinism, anti-reductionism, emergence, Hegelian dialectics, and the obscure middle ground between mechanism and vitalism that von Bertalanffy advocated. [For an overview of these tenets and an analysis of their role in the structuralist movement, the listener is directed to a paper by Looft and Svoboda in Dr. Riegel's book Issues in Developmental and Historical Structuralism.] Whatever other functions these obscure and, in the main, anti-scientific maxims may serve, their vagarity alone suffices to preclude their use in a formal definition.

The one fact which cannot be overlooked about structure, and overlooking it is but a step quickly taken, is that structure is a purely logical notion. That is, it is a concept which properly belongs to logic and not to science. If we commit the fallacy of confusing structure with its representations in various sciences (e.g., "grammatical" structure, "skeletal" structure, "mathematical" structure), then we shall not be able to formulate a satisfactory definition of the
generic concept. It should be obvious to common sense that the adjectives attached to "structure" in different disciplines serve only to modify and not define the generic concept. [Recall here the opening quotation.] If we try to define structure by listing the attributes of mathematical structures, physical structures, biological structures, etc. (as Piaget, for example, has attempted to do in his little book Structuralism), then we would have at best sufficient but not necessary conditions for structure.

In Principia, Whitehead and Russell divided logic into three branches: the study of statements about (unquantified) statements, statements about relations, and statements about classes. "Structure" may be reduced to two and only two primitive terms from the latter two branches, viz. relation and element. Every structure must have both relations and elements, and neither can be reduced to the other. Neither a collection of elements by itself nor a relation by itself constitutes a structure. [In psychology, our more atomistic schools are apt to forget the former point and the current structuralist movement is apt to forget the latter point.] When we analyze the structure of any domain, the first step is to posit certain undefined elementary phenomena and the second is to ascertain how they are related to each other. Any structure may contain more than one relation and several structures may be posited for a single domain of study.

As I have observed in other papers (Brainerd, 1973a, 1975), structural analysis has always been a sequential process in which investigators proceed through a series of successive approximations to the
"true" structure of the phenomena which they study. In physics, for example, the study of atomic structure proceeded from billiard balls of the nineteenth century to the nuclei and electrons of the Rutherford-Bohr model to the protons, neutrons and electrons of early quantum theory to the energy concentrations in space-time of modern quantum theory. Similarly, the study of heredity has proceeded from the cell to the nucleus to the chromosomes to the molecular constituents of the chromosomes. In short, the history of structural analysis in both the physical and biological sciences may be described as a successive penetration to ever deeper levels of analysis. Generally speaking, each subsequent step in a structural analysis takes the undefined elements of the previous step as its domain and then exhibits their structure. There is an inescapable methodological inference which is justified by the sequential character of structural analysis: As a matter of principle, it is incorrect to view any structure as the ultimate or basic structure of some domain. The elements of a structure can, at any moment, be analyzed into more basic elements and relations. This inference would seem to be important from the perspective of the aforementioned structuralist movement, because the inference quite obviously does not square with tenets such as holism and anti-reductionism.

From the working scientist's standpoint, the most important aspect of structural analysis is structural isomorphism. Obviously, one can group various domains of study in terms of their structural similarity. In mathematics, for example, certain number systems (e.g., integers, rationals, reals) under certain operations are known to
share the well-known group structure. The property of structural isomorphism allows us to establish the basic underlying identity of domains which bear no apparent resemblance to each other. A city and a map of a city provide a classic illustration. So far a common sense is concerned, the *prima facie* resemblance between a map of New York and the city itself is quite minimal. We would deem it extremely unlikely that a savage, a child, or anyone otherwise unfamiliar with maps would discover the connection. The *prima facie* dissimilarity between New York and a map notwithstanding, a visitor to the city who wishes to attend the opera does not need to wander about the city searching for Lincoln Center; he can consult his map. He knows that the basic elements of New York, the streets and buildings, are isomorphic with certain names on the map and he knows that the relations "to the west of" and "to the north of" which obtain between the city's elements are isomorphic with the relations "to the left of" and "above" which obtain between the names on the map. To get to the opera, our visitor need only translate the place names and relations of the map into the places and relations, respectively, of the city.

The map analogy brings me to the final point I should like to make about the general concept of structure. Whenever structures from two or more domains are isomorphic, all statements about any one of the domains whose truth or falsity depends solely on structure have counterparts in the other domains. Moreover, each of these counterparts is true if and only if the corresponding statement in the first domain is true and false if and only if the corresponding statement in the first domain is false. The significance of this consequence
of structural isomorphism is very great. Among other things, it means that, like our visitor to New York, the working scientist can know a very great deal about a new area of investigation before he conducts a single experiment. If he knows or has good reason to suspect that the new area is structurally isomorphic with some other well-studied area, then he knows too that the hard-won truths of the latter have counterparts in the former.

Cognitive Structure and Piaget's Theory

In the time remaining, let us turn to the role of structure in the study of cognitive development. In particular, let us consider how the concept of cognitive structure enters into Piaget's global vision of intellectual development. Since one of our other symposium participants, Dr. Fischer, has dealt with Piaget's theory in some detail, I shall, to avoid redundancy, mention only one fact about Piaget's cognitive structures which is of general significance from the perspective of developmental theory. This fact is intended to suggest some notable differences between Piaget's vaguely typological conception of cognitive structure and the analytic uses of structure mentioned above.

Piaget's structures are grounds for his stages. Explicitly, he uses putative changes in intellectual organization which occur during development to justify the contention that cognitive development is stage-like (cf. also Brainerd, 1974a; Pinard & Laurendeau, 1969):

Since the time of G. Stanley Hall, developmental theorists have been divided on two great questions. One question, heredity-environment, is primarily empirical with philosophical overtones. The other question,
continuity-discontinuity, is primarily philosophical with empirical overtones. The latter question may be summarized roughly as follows. For the sake of perspicuity, developmental psychologists slice the stream of behavioral development up into manageable segments (e.g., infancy, early childhood, middle childhood, adolescence, adulthood). But is it also possible that there are some schemes for slicing up behavioral development which are nonarbitrary? The so-called stage hypothesis specifies that there are, in fact, cases in which the slicing results in segments that are real and measurable entities rather than mere conventions (Brainerd, 1974b, 1974c). According to this hypothesis, the various segments posited in some theories comprise natural behavioral groupings.

Piaget subscribes to the stage hypothesis as just formulated (cf. especially Piaget, 1960, 1971). To him, the global stages which Dr. Fischer has reviewed are more than convenient descriptive headings under which to group a potpourri of logical and scientific reasoning skills. These stages are, in Piaget's (1973, p. 49) own words, "distinct natural breaks" in the developmental continuum which, because they are natural rather than conventional, can be measured. But how are we to validate these stages empirically? As I have observed elsewhere (Brainerd, 1974d), those who have sought to defend the stage hypothesis in the past have come up against a dilemma, viz. how much change is required in quantitative parameters (i.e., those whose estimates are allowed to take real numbers as values) before we can declare that a qualitative change or "discontinuity" has been observed? Piaget has tried to avoid the "How much 'more' is 'different'?" dilemma by substituting cognitive structures for quantifiable behavioral
parameters. The reader is asked to take it on faith that differences between structural models borrowed from abstract algebra automatically are qualitative and, hence, that the "How much 'more' is 'different'?" question does not arise with Piaget's stages. According to this view, "different structure" automatically entails "different stage." That is, if a structural model validated for one age range is different than a structural model validated for some subsequent age range, then there is a qualitative change or "natural break" between the two groups.

Piaget's line of reasoning vis-à-vis cognitive structures and cognitive stages seems dubious for at least two reasons. First and most important, waving a structural algebraic wand at an essentially quantitative data base does not magically transform continuity into discontinuity. Algebraic structures only model the data base. Moreover, it is not at all clear, at least not to me nor to any mathematician of my acquaintance, that the differences which obtain between various algebraic structures are even qualitative. For example, consider the groupement and the group, the structures which Piaget employs for the concrete-operational and formal-operational stages, respectively. Ignoring all the lofty claims and obscure suggestions in Piaget's two logic books, precisely what are the formal differences between these two structures? Actually, they differ only on a single postulate: there is only one identity element in the generic group but there can be more than one in a groupement. When viewed in this way, the difference between these structures does not look very much like and ipso facto qualitative one. In fact, the difference looks downright quantitative. Generally speaking, the differences which obtain
between common algebraic structures of the genre that Piaget has favored (e.g., fields, groups, lattices, rings) are neither obviously nor intuitively qualitative differences. Quite to the contrary, these differences usually amount to small adjustments in a single postulate. Thus, the "How much 'more' is 'different'?" question is still with us: Just how many changes must be made in how many postulates before two structures are qualitatively different?

Although I would maintain that what I have just said is obvious enough on logical grounds, it must be admitted that many developmental investigators accept the assumption that "different structure" means "different stage" as a working hypothesis. I shall not venture to put forth a definitive explanation of this disconcerting phenomenon. However, I shall risk a clinical impression gleaned from my correspondence and conversation with other investigators. Developmental researchers of my acquaintance are almost universally unfamiliar with the higher algebra of structure. For the most part, their mathematical training begins and ends with the mathematics of number—especially statistics. Moreover, they are understandably reticent about delving into a new branch of mathematics which may prove to have no concrete payoff for their own research. Therefore, some are willing to accept on faith the conclusion that the higher algebra of structure treats only of or primarily of qualitative differences. This attitude has, in my view, been the source of much misunderstanding.

The second problem with Piaget's assumption that "different structure" means "different stage" is purely empirical. It turns out that certain predictions which seem to follow from this assumption do not
square with known empirical fact. Before briefly mentioning the predictions and data, however, it is worth noting that even if the two were in line, the problem just discussed would remain.

In the literature, it has long been acknowledged that the structure-stage connection entails that the major cognitive skills associated with each stage, for which some structure constitutes a formal model, must be acquired in strict synchrony (e.g., Pinard & Laurendeau, 1969). That is, the order in which these skills are acquired always is idiosyncratic to individual children. Recently, it has also been shown on psychometric grounds that most of the behaviors modeled by a given structure must be observed to emerge fairly early during the age range assigned to the stage defined by this structure (Brainerd, 1974b). If this condition is not met, then it turns out that stages cannot be discriminated statistically.

Developmental studies of Piaget's stage-related reasoning skills have failed to confirm either of the preceding predictions. Concerning the synchrony prediction, nonidiosyncratic asynchronous acquisition of these skills has proved to be the rule rather than the exception. With the concrete-operational stage, for example, we know that children acquire many of the relational skills modeled by the groupement structure (e.g., transitivity, seriation, ordinal number) before they make much progress with the classificatory skills modeled by the same structure (e.g., class inclusion, matrix classification, cardinal number). [For reviews of asynchronies associated with the concrete-operational stage, cf. Brainerd (1973b, 1974a, 1974e) and Brainerd and Hooper (1974).] Concerning the abrupt emergence prediction, we
also know that stage-related reasoning skills are not acquired in this way. As a rule, these skills emerge gradually in discernible invariant sequences during the entire age range assigned to a given stage. Indeed, it is not at all uncommon for a reasoning skill modeled by some given structure to emerge during the age range assigned to some different structure. Class inclusion, for example, is supposed to be a concrete-operational skill. However, our most recent evidence (Brainerd & Kaszor, 1974; Hooper et al., 1974) indicates that class inclusion, when it is understood at all, is not grasped until sometime during the age range assigned to formal operations.

Since my time has expired, I shall not attempt to document my claims about what the literature shows any further. I shall simply conclude by observing that the data are quite extensive, that they have been discussed in review papers, and that interested listeners are directed to the appropriate reviews.
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