Model identification of time-series data is essential to valid statistical tests of intervention effects. Model identification is, at best, inexact in the social and behavioral sciences where one is often confronted with small numbers of observations. These problems are discussed, and the results of independent identifications of 130 social and behavioral time-series by two judges are presented. The majority (75 percent) of the series were represented by one of four basic models: "white noise" (i.e., independent observations); first-order autoregressive; first-order moving averages model in the first difference; and "white noise" in the first difference. (Author)
Model Identification in Time-Series Analysis: Some Empirical Results
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The interrupted time-series experimental design has recently become
a useful methodology in the social sciences. Campbell and Stanley(1963)
sensitized social scientists to the many possibilities for the design but
much of the theoretical framework remained to be discovered prior to formal
application. Box and Tiao(1965) and Box and Jenkins(1970) developed the
basic statistical structure and Glass, Willson and Gottman(1972) developed
the statistical machinery to test for intervention effects in time-series
studies.

The "success" of the application of these statistical methods depends
on the proper identification of the underlying model assumed for the data.
Unfortunately, model identification remains a rather inexact pursuit.
The implications of incorrectly identifying a model are, in many cases, a
serious threat to the validity of the results. The purpose of this paper
is to present the results of the independent identification of some 130
time-series by two judges and to focus on some relevant aspects of the model
identification problem.

Most of the data were taken from research reports in social or behavioral
publications. The series are not random samples; they represent personal
choices of one judge accumulated over the last several years. However, the
series do reflect a variety of things observed (e.g., a person, a city, a
nation) and a range of applications: alpha brain waves, crime rates, exam-
ination scores, stock prices, word association test scores, students'
time spent studying, learning curves, etc. The length of the series varied
from less than 20 to over 200 time points.

Each judge identified the series by observing the correlogram or pattern
of autocorrelations and partial autocorrelations. The degree of differenc-
ing d necessary to produce stationarity was first chosen and then the ap-
propriate p and q were selected to complete the identification of the series
as an Auto-Regressive-Integrated-Moving-Average model of order p, d and
q (ARIMA(p,d,q)). Agreement was obtained on approximately 80% of the series
with much of the 20% disagreement due to inherent model ambiguity. Consensus
was reached with respect to the disagreements and the series were categorized
according to the value of p, d and q.

Approximately 75% of the non-seasonal series (18% of the series were
identified as seasonal) were covered by four models: The "white noise"
process ARIMA(0,0,0), 18%; the first-order autoregressive process ARIMA(1,0,0),
23%; "white noise" after first differencing ARIMA(0,1,0), 11%; the integrated
moving-averages process ARIMA(0,1,1), 23%. In no case was differencing
above the second order required to produce stationarity. The most often
encountered models were the first-order autoregressive (23%) and the first-
order integrated moving averages.

The process of identifying these series brought to light rather clearly
some of the problem areas regarding proper choice of p, d and q. The salient
components of model identification are the following.
Identifying $d$. This is a critical area since over-differencing a series introduces dependency and under-differencing fails to remove dependency. In the series identified about half were $d=0$, and half were $d=1$ with only 6 cases $d=2$.

Equivalence of Models. While unique model representation is desirable, there are inherent ambiguities among the various possibilities. For example, the ARMA$(1,0,1)$ with autoregressive parameter $\phi$ of one is formally equivalent to the ARIMA$(0,1,1)$. In general, the choice of representation depends on the values of the autoregressive and moving-average parameters $\phi$ and $\theta$.

Deterministic Drift. Series with a consistent rise or fall can be identified by inspecting the mean of the $d$th difference. A statistically significant non-zero mean implies the need for incorporation of a drift parameter into the model formulation. Alternatively, the raw series may be adjusted to some base rate to correct for the drift (e.g., traffic fatality data adjusted by constant miles driven).

Seasonal Model. Seasonal models are identifiable by large autocorrelations at lag $s$, the length of the cycle. The series may be identified as seasonal and represented as a complex multiplicative model (e.g., $(0,0,1) \times (0,1,1)$) or by adjusting the original series for cyclic fluctuations and then identifying the adjusted series.

Changes in the Model. Series in the social and behavioral science areas seem to be uniquely beset with problems of the intervention altering the fundamental nature of the series. For example, the Scandinavian countries' traffic series show marked discrepancies between the pre- and post-WWII periods. Proper identification of both pre and post-intervention series is essential for correct formulation into the statistical estimation phase of the intervention effect.

While these problems appear formidable they can, for the most part, be reduced in complexity since most of the series actually encountered in practice are rather simple in form.
REFERENCES


