This investigation tested the hypothesis that the probabilistic structure underlying psychotherapy interviews is Markovian. The "goodness of fit" of a first-order Markov chain model to actual therapy interviews was assessed using a $\chi^2$ test of homogeneity, and by generating by Monte Carlo methods empirical sampling distributions of selected characteristics of interaction processes against which the same characteristics in the actual interviews were compared. The model provided an adequate fit and should provide a useful tool for further investigations into the character and course of the therapy process. (Author)
PSYCHOTHERAPY AS STOCHASTIC PROCESS:
FITTING A MARKOV CHAIN MODEL TO INTERVIEWS OF
ELLIS AND ROGERS

James W. Lichtenberg
Student Counseling Bureau
University of Minnesota

Abstract

This investigation tested the hypothesis that the probabilistic structure underlying psychotherapy interviews is Markovian. The "goodness of fit" of a first-order Markov chain model to actual therapy interviews was assessed using a $\chi^2$ test of homogeneity, and by generating by Monte Carlo methods empirical sampling distributions of selected characteristics of interaction processes against which the same characteristics in the actual interviews were compared. The model provided an adequate fit and should provide a useful tool for further investigations into the character and course of the therapy process.
Introduction

The malleability of verbal (vocal) behavior has been extensively demonstrated across a large number of different verbal response classes and subject-experimenter parameters (Greenspoon, 1962; Holz & Azrin, 1966; Kanfer, 1968; Krasner, 1958; Salzinger, 1959; Williams, 1964). An extrapolation and application of these research findings has been to the area of psychotherapy, and in both "clinical analogues" and actual therapy settings, therapist manipulation of client verbal behavior has been demonstrated (Heller & Marlatt, 1969; Krasner, 1965; Salzinger, 1969; Strong, 1964; Truax, 1966). The literature also supports the notion of therapist responses serving as discriminative stimuli for client verbalizations (Auld & White, 1959; Barnabei, 1973; Frank & Sweetland, 1962). Not only do therapist responses result in an increase in the frequency of certain client responses, but particular client responses are more or less probable following certain therapist responses. The paradigm depicting this verbal situation is presented in Figure 1.
As presented in Figure 1, each therapist response (R) "occasions" or serves as a discriminative stimulus (S\textsuperscript{D}) for the next client response as well as a reinforcing stimulus (S\textsuperscript{R}) for the previous response.

A literature also exists on client influence on therapist behavior (Alexik & Carkhuff, 1967; Auld & White, 1959; Bandura, Lipsher & Miller, 1960; Carkhuff & Alexik, 1967; Friel, Kratochvil & Carkhuff, 1968; Heller, Meyers & Kline, 1963). In these studies the paradigm implied reverses itself.

Here the client's verbal responses serve as discriminative stimuli for the next therapist response and reinforce the previous response. The two paradigms are not in contradiction with each other, but taken alone they are incomplete. Both depict therapy as a sequence of discrete and unidirectional one-step contingencies of influence between therapist and client utterances. An alternative paradigm for the therapy process which is consistent with the literature cited above would acknowledge the mutual and interactive influence of both the therapist and client on each other's verbal behavior. Such an "interlocking paradigm" (Skinner, 1957; Strong, 1964) would depict each verbal response of therapist and client serving as a reinforcing stimulus (S\textsuperscript{R}) for the other's previous response and as a discriminative stimulus (S\textsuperscript{D}) occasioning the next response.
Within this paradigm the mutual and sequential dependencies of the verbal responses in the counseling interaction are incorporated.

The effect of any given $S^R$ on a previous response is to increase the probability of occurrence of another response of the same response class, and the effect of any given $S^D$ is an increase in the probability of occurrence of a response of the response class conditioned to it (Reynolds, 1968). The sequential interaction of the therapy process, therefore, can be viewed as a chain of probabilistic events—a stochastic process.

A finite first-order Markov chain model was proposed as a stochastic process model for psychotherapy. A Markov chain is a formal stochastic process model, and if appropriate to actual therapy interaction, it would provide a formal probabilistic description of the therapeutic process and offer a means of predicting the probable course and outcome of that interaction process.
Introduction to Markov Chains

In terms of the interlocking paradigm (Figure 3) the therapy process is viewed as a series of transitions from response to response—from $T \rightarrow C \rightarrow T$, etc. A transition is defined as a move between two events. The first event in the transition is the antecedent, and the second event is the consequent. In a sequence of transitions, each event, with the exception of the first and last responses in the sequence, serves a dual function as both antecedent and consequent.

An estimate of the probability of any given event (e.g. response, class) being followed by any other specified event may be determined by dividing the number of occurrences of a particular event-event transition by the number of times its antecedent occurs as the antecedent of any transition in the sequence.

These transition probabilities ($p_{ij}$) may be organized into a transition matrix ($P$) with rows ($i$) = antecedents; columns ($j$) = consequents. Figure 4 represents a transition matrix for a therapy interaction sequence in which four possible response classes have been distinguished for both therapist and client.

0 has been entered in the upper left and lower right quadrants of the matrix to indicate that $T_i - T_j$ and $C_i - C_j$ transitions cannot occur under the interlocking paradigm.
The matrix summarizes the interaction sequence in terms of the probabilities of transition among the various response classes. If it may be assumed, however, that

a) those transition probabilities are stationary, i.e. the matrix represents constant probabilities of transition within the sequence which do not depend on their place in the series of transitions, and

b) the outcome of any transition (consequent) is dependent only on its antecedent, i.e. if the antecedent is known, no additional information is provided in the prediction by knowing the path of events leading to the antecedent (first-order dependency),

then the interaction sequence represented by the matrix is a Markov chain (Kemeny & Snell, 1960).

The particular advantage of being able to justifiably employ the two Markov assumptions to the therapy process is significant. Not only does the P matrix provide information as to the likely consequent of any possible response at any time in the interaction; but it can be shown that by raising the P matrix to the nth power using the rules of matrix multiplication one derives a matrix of probabilities of the process being in each of the possible response classes after n transitions given the initial response class (Kemeny & Snell, 1960; Howard, 1971). In such an n-step matrix the rows (i) again correspond to the antecedent responses and the columns (j) correspond to the consequents, but the probabilities (P_{ij}) are those of being in each of the possible consequent states, given the antecedent, in n transitions. This allows one to predict not only
the immediate consequent of each T and C response in terms of the other's likely response, but the course and eventual outcome of the interaction sequence as well.

It was, therefore, the purpose of this study to validate those Markov assumptions against actual therapy interviews. To the degree that the therapist-client interaction satisfied the assumptions of transition probability stationarity and 1-step dependence among response events, the Markov chain model might be considered a valid descriptive and predictive model of the therapy interaction process.
Methodology

The Markov model was tested individually against transcriptions of 6 psychotherapy interviews: 4 conducted by Albert Ellis and 2 conducted by Carl Rogers. For each interview, therapist and client verbal responses, defined as everything spoken by one participant between any two consecutive responses of the other (this in accord with the interlocking paradigm), were categorized by response class. Response classes were defined in terms of speaker (T or C) and the mode of communication expressed by the speaker. Four modes of communication were identified:

a) **Personal**: personal, affective, self-disclosing statements which focus on and share personal reactions to things impinging upon the speaker.

b) **Descriptive**: descriptive, impersonal, non-affective statements which, though they may be about either the speaker himself or things outside himself, reflect an objectivity or distance about them.

c) **Cognitive**: cognitive or analytical statements displaying either overtly or covertly an integration or tying together of ideas, concepts or events.

d) **Directive**: directive, leading, structuring or otherwise imperative statements which either explicitly or implicitly direct the attention or behavior of the other person, or which imply what the other person should or should not be thinking or doing.
The response classification scheme resulted in eight mutually exclusive interaction response classes (speaker crossed with mode). An Introductory state (from which the interaction began) and a Terminal state (in which the interaction ceased) were also included. The Introductory state had as its consequent the first actual interaction response; the Terminal state had as its antecedent the final interaction response. The two additional categories were defined so to delimit the interaction sequence. The sequence always began at the Introductory state, to which it was impossible to return. The Terminal state always concluded the interaction sequence; once it had been entered, transition from it was impossible. The addition of the Introductory and Terminal states resulted in a 10 x 10 transition matrix for each of the 6 interviews.

Insert Figure 5 about here

Each interview transcript was classified by two independent judges previously trained to a level of inter-rater agreement of $\kappa = .80$ (Cohen, 1960; Tinsley & Weiss, in press) on counseling interview material similar to the actual interviews used in this study. Rater disagreements were settled by re-rating and, if necessary, negotiation. Transition probabilities for each interview were computed and organized into their own transition matrix.

Testing stationarity

Stationarity of the transition probabilities was tested using a $\chi^2$ test of homogeneity (Hertel, 1968; Suppes & Atkinson, 1960). Each
interview was divided in half and the frequencies of transitions in the first half were compared with the frequencies in the second to determine if the interview halves were significantly different ($p \leq .05$). Significantly large $\chi^2$ values indicated unequal frequencies across the interview and suggested nonstationary transition probabilities across the interview sequence.

**Testing dependency**

First-order dependency was tested for each of the six interviews in the following manner. For each of the interviews two "process characteristics" were computed: the mean distance (i.e. average number of transitions) between response classes, and the standard deviations of the distances between classes (Hertel, 1968). Using the Markov assumptions of transition probability stability and first-order dependence among responses, a population of 1000 Markov sequences was generated using Monte Carlo methods. The transition probabilities of the actual interview served as parameters for the generated population. For each of the 1000 generated Markov sequences, the two process characteristics were computed, resulting in an empirical sampling distribution ($N=1000$) for both characteristics and against which was tested the hypothesis that the interview had been generated by a first-order Markov process. If 97.5% of the empirical sampling distribution of a process characteristic fell above or below the characteristic value for the actual interview, the hypothesis that the interview had been generated by the proposed Markov process was rejected in favor of a model specifying the complete independence of the responses.
Summary of Results

Interview 1

One row of the transition matrix displayed a significant $\chi^2$ value, suggesting that row of transitions to be unstable. As a whole, however, the interview appeared to represent a stable process. Eighteen of the process characteristics were significant—7 among the 64 mean distances between response classes and 11 among the 64 standard deviations of those distances.

Interview 2

None of the rows of the transition matrix displayed a significant $\chi^2$ value, suggesting stability of the individual transitions in the interview and of the interview as a whole. None of the process characteristics were significant.

Interview 3

As in interview 2, none of the rows of the transition matrix displayed a significant $\chi^2$ value, again suggesting stability of the transitions and of the interview as a whole. Also as in interview 2, none of the process characteristics were significant.

Interview 4

None of the rows of the transition matrix displayed a significant $\chi^2$ value, suggesting stability of the transitions in the interview and of the interview as a whole. However, two of the process characteristics

...
were significant—one among the 64 mean distances between response classes and one among the 64 standard deviations of those distances.

**Interview 5**

None of the rows of the transition matrix displayed a significant $\chi^2$ value, suggesting stability of the transitions and of the interview as a whole. Three process characteristics were significant—one among the 81 mean distances between response classes and two among the 81 standard deviations of those distances.

**Interview 6**

One row of the transition matrix displayed a significant $\chi^2$ value, suggesting that row of transitions to be unstable. As a whole, however, the interview appeared to represent a stable process. Among the process characteristics, 21 were significant—one among the 81 mean distances between response classes and 12 among the standard deviations of those distances.
Discussion

Interpretation of the results is not as straightforward a task as is the case with classic randomized experiments and their analyses. While rejection of the statistical hypotheses suggests the failure of the Markov model to fit the therapist-client interaction, it does not rule out the possibility of some alternative Markov model (e.g. of second or third order) or of some other non-Markovian (albeit stochastic) model of the therapy process. At the same time, failure to reject the hypotheses does not conclusively prove the interviews to be Markovian, though it certainly offers support to this thesis. At this stage one must "interpret the retention of the null hypothesis" and do so without really knowing the power of the analysis to detect deviations from the hypothesized model. Until the state of the art advances, one must proceed cautiously in discussing the fit of the model to the data.

In light of the proposed interlocking paradigm for the therapy interaction process, the two Markov assumptions of stationarity of the transition probabilities and one-step dependence among responses do not seem unreasonable; it was for this reason that the particular Markov model was selected for testing and validation. However, in light of general theorizing about the counseling process, the assumptions may likely seem absurd. Therapeutic interaction is usually assumed to vary in style within the interview; indeed, therapy "stages" or process "phases" are commonly referenced in the literature (Bordin, 1955, 1974; Brammer & Shostrum, 1968; Cashdan, 1973). The assumption of probability stationarity suggests the converse. The responses made by both therapist
and client are generally assumed dependent upon the full course of the interview up to that point. The assumption of one-step dependency explicitly states that only the immediately preceding response need be considered in determining the next response.

Considering that approximately 98 percent of the tests on interview 1 were significant and that 26 percent were on interview 6, we feel that the appropriateness of the Markov model for these two interviews is at this point questionable. It appears to us that these percentages of significant tests are higher than one might reasonably expect given only chance deviation from the model and we must reject the Markov model on these two interviews.

However, the results of interviews 2, 3, 4 and 5 support the Markov model as a model for therapeutic interaction. The few significant tests for interviews 4 and 5 we believe can be understood in terms of the number expected to be significant due to chance deviation from the model. Interviews 2 and 3 displayed no significant deviations from the Markov model.

To be sure, even in those four interviews for which the Markov chain assumptions were satisfied, the Markov model cannot be assumed to account for all the variation in the therapy interaction process. But the model does suggest a lawfulness and possible probability structure for the process—and importantly, one which can permit the prediction of not only immediate, but also distant, consequences of interaction and interventions in that process.

The intent of this study was to explore the possibility and reasonability of employing a finite Markov chain model in the study of
the verbal interaction of the therapy process. More specific application of the model is yet to be conducted. But while the full potential of the model may yet to be realized, it is expected that the model will lead to the development of fruitful hypotheses relative to the therapy process.

Can the stationary transition probabilities distinguish among the various therapeutic orientations (Fiedler, 1951; Zimmer & Cowles, 1972)? Can therapeutic and non-therapeutic interaction be discriminated in terms of those probabilities (Reusch, 1972, 1973)? Can client response style as expressed in the transition probabilities provide clues as to problem diagnosis (e.g. Bales, 1970; Carson, 1969)? How long might the therapy process be expected to continue until termination? Assuming the inclusion of appropriate response categories, is the process more likely to end in success or failure? How might interventions be selected to achieve improved, if not optimized, interaction? Application of the model to the therapy process may answer some of these questions and will hopefully raise additional ones. To date such questions, if asked, have generally had to rely on answers derived from investigations employing a multitude of variables external to the therapy process (e.g. sex, age, educational level). A finite Markov chain model of the therapy process proposes a self-contained system in which prediction for course and outcome of the process are the result of the lawfulness and dynamics of the process itself.
References


Fiedler, F. E. Factor analysis of psychoanalytic, non-directive, and Adlerian therapeutic relationships. *Journal of Consulting Psychology,* 1951, 15, 32-38.


1. This paper was presented at the annual convention of the American Educational Research Association, Washington, D.C.: April, 1975.

The authors wish to acknowledge the American Academy of Psychotherapists who provided the interview transcripts used in this study, and the University of Minnesota Computer Center which partially supported this project. They also wish to express their appreciation to Jef Evans and David Stulhman who served as raters for the interviews.

This paper is based in part on the doctoral dissertation of the first author.
Figure 1. Paradigm in which the responses of the therapist (T:R) serve as discriminative stimuli ($S^D$) and reinforcing stimuli ($S^R$) for client responses (C:R).
Figure 2. Paradigm in which the responses of the client (C:R) serve as discriminative stimuli (S^D) and reinforcing stimuli (S^R) for therapist responses (T:R).
Figure 3. Interlocking paradigm in which the responses (R) of both therapist (T) and client (C) serve as discriminative stimuli ($S^D$) and reinforcing stimuli ($S^R$) for the other's responses.
Figure 4. An 8 x 8 therapy transition matrix in which 4 therapist and 4 client response classes have been distinguished.
<table>
<thead>
<tr>
<th>Antecedent</th>
<th>Term</th>
<th>$T_1$</th>
<th>$T_2$</th>
<th>$T_3$</th>
<th>$T_4$</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
<th>$C_4$</th>
<th>Intro</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_1$</td>
<td>1.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_2$</td>
<td>0.02</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.23</td>
<td>0.48</td>
</tr>
<tr>
<td>$T_3$</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.19</td>
<td>0.62</td>
<td>0.15</td>
<td>0.04</td>
<td></td>
</tr>
<tr>
<td>$T_4$</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.05</td>
<td>0.84</td>
<td>0</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>$C_1$</td>
<td>0</td>
<td>0.50</td>
<td>0.42</td>
<td>0.08</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$C_2$</td>
<td>0</td>
<td>0.03</td>
<td>0.45</td>
<td>0.33</td>
<td>0.19</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$C_3$</td>
<td>0</td>
<td>0.50</td>
<td>0.50</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$C_4$</td>
<td>0</td>
<td>0.66</td>
<td>0.17</td>
<td>0.17</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intro</td>
<td>0</td>
<td>1.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 5. A $10 \times 10$ therapy transition matrix employed in this study (Interview 6).