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ABSTRACT

This report begins with a brief historical sketch of the origins of the mathematics curriculum and the responsiveness of mathematics curriculum to the demands of society. The current North Carolina mathematics curriculum is then described and evaluated. A "strands" approach to the development of curriculum and a framework for planning are then proposed. This framework is based on consideration of courses as student-centered (e.g., applied mathematics, consumer mathematics), subject-centered (e.g., algebra I, geometry) and mixed (e.g., business mathematics, applied geometry). The rationale for these strands is provided, and sequences of courses which students might elect are diagrammed. Individual courses, including a remedial clinic, are then described. Descriptions include sample materials where available, discussion of objectives and topics to be covered, and an overview of special issues related to each course. (SD)

THE SECONDARY SCHOOL MATHEMATICS CURRICULUM

A Report to the Division of Mathematics,
North Carolina State Dept. of Public Instruction
and the
State Advisory Council on Secondary School Mathematics

November, 1974

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FOREWORD

Prior to the modern mathematics era, a series of commissions and committees made insightful and comprehensive suggestions and recommendations for the improvement of secondary mathematics education. The implementation of these recommendations was usually exceedingly slow. The modern mathematics movement differed in approach to curriculum revision. Writing groups were formed to produce experimental mathematics textbooks that would serve as prototypes for commercial textbooks. Often, members of the writing groups themselves produced commercial renderings of their experimental textbooks. The rapid implementation and wide adoption of the "new math" is usually attributed to the production of sample text materials that could be wholly transplanted into mathematics classrooms all over the country.

The innovation that permitted a nationwide insertion of new math reforms into the mathematics curriculum also served to discourage local initiative in curriculum planning. Little thought needed to be exerted at the local level. Curriculum planning could begin and end with the selection and teaching of a modern mathematics textbook. Few accommodations and adaptations were made to meet unique local needs. The involvement of teachers in the process of thinking through the purpose and content of the mathematics curriculum was minimal. Many were cheated of the valuable experiences derived from participating in curriculum evaluation workshops, cooperatively developing new experimental courses, and writing local curriculum guides that articulated the mathematics program from grades K through 12. Mathematics programs implemented by their developers will be successful, while mathematics programs imposed upon teachers who have had no involvement in their development are too often misapplied.

What is the purpose of a State guide for secondary school mathematics? It is not to prescribe a uniform statewide mathematics curriculum. It should be a guide, a tool, an aide to encourage and support the all-important involvement of mathematics teachers in local planning and experimentation. A guide does not fill in all the details so that no additional work is necessary at the local level. Rather, it provides a general orientation and framework that must be "fleshed out."

The authors have assumed that a State Guide for Secondary School Mathematics will consist of three parts:

Part I: The Mathematics Curriculum (What mathematics should be taught and to whom and when?)

Part II: Learning and Teaching Mathematics (How should students be taught the mathematics selected?)

Part III: Evaluating Mathematics Instruction (How can outcomes of mathematics instruction be assessed?)

The focus of this report is the Mathematics Curriculum, Part I of three parts. This report envisions Part I of the Guide as providing these basic aids to the design of effective local mathematics programs:

- Provide an examination of historical forces that have shaped the structure of the mathematics curriculum.
- Identify deficiencies and shortcomings in many existing curricula that should be avoided in future curricula revisions.
- Propose a pattern of mathematics offerings that is flexible and diverse enough to accommodate many local innovations and interpretations.
- Identify and describe a number of courses for which state adoptions of textbooks can be made and so that local units can have the economic flexibility to offer a variety of mathematics courses.
- Clarify the existing confusion and misconceptions surrounding consumer math, general math, business math, vocational math, and technical math. The differences among these student-centered courses should be delineated as well as the all-important separation of these courses from remedial arithmetic.
- Create a new perspective, "a different set of eyes," through which curriculum revision can be viewed and healthy debate stimulated.

ORIGINS OF THE SECONDARY SCHOOL MATHEMATICS CURRICULUM

The Mathematics Curriculum in Transition

"A new period in mathematics education is beginning."

The mathematics curriculum of the secondary school is in transition. That period of reform and revolution known as "modern mathematics" is now coming to a close. A new period in mathematics education is beginning. Poised on the threshold of the next era, we are like the two-faced god, Janus, of Roman mythology. One face turns backward to study the past, and the other face looks forward toward developing the courses and curricula that will meet the needs of the future. Thus, before projecting the mathematics content needed for the near future, we need to examine some of the forces that shaped the high school mathematics curriculum into its present-day mold.

The Modern Mathematics Movement

"...this period of curricular reform initiated comprehensive and far-reaching changes..."

The most recent force to mold the curriculum was the new or modern mathematics movement. Lasting almost 20 years, this period of curricular reform initiated comprehensive and far-reaching changes backed by a great deal of funding, missionary zeal, and extensive publicity. Any attempt to define modern mathematics must fail because of the diversity among the modern programs and the large number of changes and innovations that they brought to the mathematics curriculum at both the elementary and secondary school levels. In addition, modern mathematics programs did not remain static during this period. Present-day modern mathematics programs as implemented in the schools often bear little resemblance to the original experimental programs and the visions of their authors and developers.

"A great many of the innovations of the modern mathematics period will be incorporated and continued on into future eras."

What is the legacy of the modern mathematics movement? Pedagogically, it revived interest in discovery teaching, laboratory methods of instruction, and concern for teaching the "why" of mathematical algorithms and procedures. In pursuit of teaching understanding, many simple and elegant ways of explaining difficult concepts were developed that are now included routinely in mathematics textbooks. Mathematically, the structure of the discipline was emphasized and used to derive and justify the new results. Many topics such as sets, number bases, symbolic logic, non-metric geometry, probability, inequalities, metric postulates for geometry, and concepts from number theory and abstract algebra were included in the high school curriculum. Topics like solid geometry, trigonometry, analytic geometry, algebraic proofs, integers, variables, and solutions of open sentences were "pushed down" to a lower grade placement and oftentimes consolidated in other courses. This is only an incomplete recounting of the major changes. Many, many more minor changes were also incorporated into the curriculum during this period. Changes as numerous and extensive as these do not merely disappear. A great many of the innovations of the modern mathematics period will be incorporated and continued into future eras.

"Instructional time diverted to studying the new topics is bought at the expense of time spent on mastery of fundamental operations."

With perfect hindsight, it can be seen that some aspects of modern mathematics were unsuccessful and ill-conceived. Modern mathematics had its critics from the beginning, and now more people are willing to lay new mathematics to rest and get back to the "basics." Criticism has been directed toward the worthlessness of new topics such as number bases, set notation, numeral-number distinctions, number properties, and symbolic logic. Modern mathematics programs have been accused of replacing intuition with formalism and symbolism - of substituting rote memorization of rules for rote memorization of field properties and formal proofs. The lack of attention to applications and uses of mathematics, the absence of motivation of topics, and the disregard of a generic development of mathematics in new programs are decried. However, the most severe criticism has centered upon the decline in students' computational skills in arithmetic and manipulative facility in algebra and trigonometry. Test results seem to verify this criticism. Instructional time diverted to studying the new topics is bought at the expense of time spent on mastery of fundamental operations.

"...new math is dead because it is obsolete."

Is new math dead? The answer is "yes;" new math and its era are past. After nearly 20 years, modern mathematics is no longer "modern" or "new;" it is "standard" and "traditional." The term is dead, and so is the thrust of that curriculum. New mathematics was a curriculum attuned to the needs and goals of the 1960's. The 60's was a turbulent decade featuring a prestige-ridden space race, social upheavals, demonstrations for causes, and educational systems in open international competition. A "revolution" in school mathematics was consistent with the mood of the times. But the modern mathematics programs congruent with the society of the 60's, no longer suffice for the 1980's. The reform period known as modern mathematics has ended for the same reason all reform movements end; new times and social climate demand new responses. In short, new math is dead because it is obsolete.

"...modern mathematics was most successful during the decade of the 1960's..."

Was new math a failure? An enterprise as complex and diverse as modern mathematics cannot be judged a success or failure as a whole. Rather, it is composed of many successes and failures. Many of these successes and failures already have been enumerated and have entered the total wisdom concerning mathematics education. It now appears that modern mathematics was most successful during the decade of the 1960's when it was most attuned to the interests and aspirations of students, teachers, parents, and society in general. At that time, new topics, understanding of algorithms, and mathematics for mathematics' sake were more valued than computational skills and applications. Students displayed great interest in mathematics and generally were considered well prepared upon college entrance. Computational skills remained at acceptable levels while understanding of concepts showed marked improvement in comparisions of modern programs with traditional programs. During this period, the evidence suggested more successes than failures for new mathematics.

Today, computational skills and utilitarian values are considered important. Thus, at the time when modern mathematics programs are fully implemented throughout grades K-12, the goals of mathematics education and society have shifted away from those of modern mathematics. Hence, the curriculum that now is being criticized as producing more failures than successes is an out-moded one.

"Now a new curriculum must be designed."

Now a new curriculum must be designed. Not some traditional curriculum transplanted from the 1940's or 1950's, for that will not be in harmony with the 1980's. Rather, we must follow the lead of the mathematicians and mathematics educators who initiated the modern mathematics movement. We must use vision and experiment and try new approaches in the mathematics curricula. We must discard outmoded ideas and obsolete conceptions of mathematics instruction. To develop a new curriculum, we must probe further back in the history of the secondary school mathematics curriculum to identify the forces that have shaped the curriculum.

College Entrance Requirements

"The goal, content, grade placement, and emphasis of the courses in the high school mathematics curriculum have been almost totally dominated by college entrance requirements."

Modern mathematics is only the latest in a long list of issues, innovations, and movements that have influenced the content of school mathematics. A thorough study of the movements and counter movements in curricular reform may be found in the Thirty-Second Yearbook of the National Council of Teachers of Mathematics, entitled "A History of Mathematics Education in the United States and Canada." Some of the movements and issues of certain periods have had profound effects while other periods and other suggested reforms have had little or no effect. There is one factor that above all others has continuously exerted the most influence on the content of high school mathematics--college entrance requirements. The goals, content, grade placement, and emphasis of the courses in the high school mathematics curriculum have been almost totally dominated by college entrance requirements.

High schools emerged before and during the Civil War and by 1875 they were well established in the northern part of the country. These schools replaced the academies whose number declined after 1850. (2:27) Of these early high schools Willoughby states:

"...when more mathematics was added to the curriculum, it was because college entrance requirements had increased."

"Curriculum in secondary schools had been determined largely by college entrance requirements; because secondary schools were neither free nor compulsory until late in the nineteenth century, most of the pupils in these schools were preparing for college. Thus, when more mathematics was added to the curriculum, it was because college entrance requirements had been increased... It is interesting to note that the order in which the three subjects, arithmetic, algebra, and geometry, are presently taught in this country is the same order in which Harvard and other colleges began requiring them in the eighteenth and nineteenth centuries."[1]

Yale in 1745, followed by Princeton in 1760 and Harvard in 1807, made arithmetic an entrance requirement and hence no longer a college course. (2:18) Harvard required algebra for entrance in 1820, with Yale and Princeton following in 1847 and 1848 respectively. (2:27) By the middle of the nineteenth century, arithmetic and algebra were commonly taught subjects in the academies and high schools since many colleges required them for entrance. Geometry was first required for entrance to Harvard in 1844 and by 1875 was commonly

[1]Willoughby. Contemporary Teaching of Secondary School Mathematics. p. 4.

taught in secondary schools. (4:47) The high schools of this period served a small number of students, most of whom were preparing for college. The mathematics curriculum was molded by entrance requirements and as a result was quite utilitarian and vocational for the intended audience.

By the end of the 19th century, a chaotic situation had arisen. The entrance requirements being established for college admission varied widely from one college to another. Secondary schools often had to design courses just to meet the requirements of particular colleges. A person had to select a college early in order to determine the preparatory work he should follow in high school to qualify him for admission to that college. (4:47) In this period, there was no standardization as to what constituted arithmetic, algebra, or geometry, no uniformity as to the grade placement and amount of time devoted to mathematics each week. (2:163-166)

"...a 1890's committee seeking to establish uniformity in college preparatory schools serving 7% of the total school-age population set the framework for today's mathematics curriculum..."

In 1892 the Committee on Secondary School Studies (Committee of Ten) was appointed to bring agreement and uniformity to the problem of high school-college articulation. The mathematics subcommittee recommended that the study of arithmetic be completed by the end of the eighth year. They suggested the topics in arithmetic that should be included or excluded. Algebra through quadratic equations was to be studied for five periods a week and placed at the 9th grade level. The study of geometry, including solid geometry, and more algebra was recommended for the 10th and 11th years. The study of trigonometry and more advanced topics in algebra was suggested for boys going on to scientific schools. (1:9) The committee advised that all students should take the same mathematics although some may not take more mathematics after algebra in the ninth grade. In mathematics courses all students, college and non-college bound students should be treated alike, i.e. as college-bound students. Thus, an 1890's committee seeking to establish uniformity in college preparatory schools serving seven per cent of the total school-age population set the framework for today's mathematics curriculum in comprehensive high schools serving 7 % of the total school-age population. (2:162)

"...no other force did move to mold the current conception of the mathematics curriculum than college entrance requirements and the mathematics subcommittee of the Committee of Ten."

Of course, there have been many movements, reforms, and revisions in the mathematics curriculum in its progression from the 1890's to the present day. The rise of junior high schools prompted some enrichment of the arithmetic of the 7th and 8th grades. Concepts of algebra and informal geometry entered the junior high curriculum to form an integrated mathematics course as a replacement for arithmetic. Severe criticism of the sterility of college preparatory mathematics during the depression forced experimentation with courses that could be shown to have value in the life of a citizen. In this way general mathematics evolved as an alternative for algebra in the 9th grade. The modern mathematics movement introduced or "pushed down" content that was usually taught at higher levels. Despite all the movements and reforms, no other force did more to mold the current conception of the mathematics curriculum than college entrance requirements and the recommendations of the mathematics subcommittee of the Committee of Ten.

Many current practices and attitudes can be traced back to the antecedents of the mathematics curriculum and the Committee of Ten.

- College entrance requirements determine the mathematics that is selected to be taught in secondary schools.
- College preparatory mathematics is narrowly focused toward providing a direct track to calculus.
- The amount of materials to be mastered in each grade increases after each reform movement so more content gets pushed down.
- Mathematics at the high school level is equated with college preparatory mathematics.
- Students must choose between college preparatory mathematics courses or virtually no mathematics courses.
- Students must fit to the mathematics content offered in a course instead of providing some courses where the mathematics content is selected that will serve the students.
- An attitude that beyond the eighth year, mathematics is only for the able students and remedial arithmetic and computation meets the needs of less able students.

•"Can these attitudes be continued, or has the time come for the democratization of mathematics?"

Can these attitudes and practices be continued or has the time come for the democratization of mathematics? Should the mathematics curriculum be broadened to provide a full sequence of elective courses for non-college bound students as well as college-bound students?

Must only a narrow slice of mathematics be selected for inclusion in high school courses or can all of mathematics be sampled for topics that will help students to understand the increasing role of mathematics in our society? Can non-academic mathematics courses be developed that teachers will want to teach and that will attract students on an elective basis rather than as a captive audience? These are some of the questions that must be faced in the next era of mathematics education.

A Fable

"What if high schools had evolved as institutions almost solely for non-college bound students preparing for vocations and the responsibilities of a citizen?"

What if high schools had evolved as institutions almost solely for non-college bound students preparing for vocations and the responsibilities of a citizen? Suppose that later on these high schools added a college preparatory track as they began to enroll students who went on to college. What kind of mathematics curriculum would have been proposed by an 1890 committee, and what kind of curriculum would we inherit today? One could imagine that the mathematics curriculum coming down to us would prescribe such courses as Commercial Arithmetic, Measurement (direct and indirect including experimentation, gathering and organizing data, and drawing conclusions), Plane and Spherical Trigonometry (emphasis upon computation and mensuration), Surveying, Navigation, and Applications of Mathematics to Vocations (including special computational devices and tricks of the trade). A modern mathematics movement along the way no doubt would have added probability and statistics to the commercial arithmetic course, as well as the applications course. Flow charting to teach algorithmic problem solving would have been added to the measurement course. Uses of new computing devices, particularly computers, would have been included in the Applications to Vocations course.

"...many of the college-bound students would be reluctant learners in the traditional vocational curriculum since it would not serve their needs."

What about the students in this hypothesized high school who are preparing for college? All students who were able would no doubt be encouraged to follow the vocations and life preparation courses even if they were not sure whether they were non-college bound. Otherwise, they might close doors to their chances to enter life and a vocation. Of course, many of the college-bound students would be reluctant learners in the traditional vocational curriculum since it would not serve their needs. Thus, as soon as the math requirements for their graduation are met, they should b· discouraged from taking any more mathematics. If necessary to get college-bound students through math, special courses called Algebra and Geometry could be designed and taught for them. However, these courses would be definitely low prestige courses to be avoided by good mathematics teachers. Students in the life and vocations track would be discouraged from taking them.

NEED FOR CHANGES IN THE EXISTING CURRICULUM

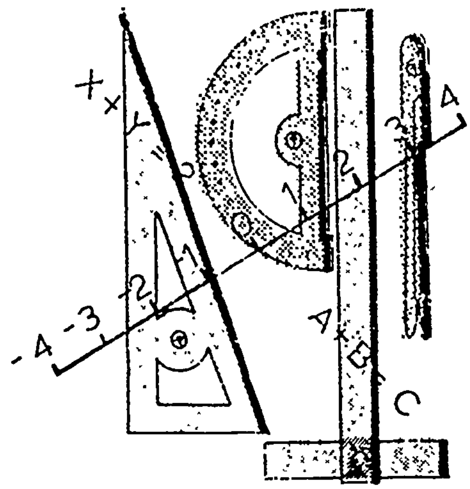
The Existing Curriculum

"...two [sequences] are traditional college preparatory routes with the only choice being the year in which the sequence is begun."

Shown in Figure 1 on the following page is the mathematics curriculum proposed in the last State Curriculum Guide. Three sequences are shown, of which two are traditional college preparatory routes with the only choice being the year in which the sequence is begun. Fundamental Mathematics 3 (general mathematics) is sequenced as a pre-algebra course to mark time for less able students to get ready to be funneled into Algebra I and the college preparatory track in the 10th year.

"...the third sequence is a shunt to eject as quickly as possible from further mathematics courses the students who do not fit into college preparatory mathematics."

The legacy of the Committee of Ten is evident in the accompanying description of algebra, geometry, and advanced mathematics as "higher" mathematics, not "different" mathematics. Consumer mathematics and business mathematics are recommended as special courses for those who cannot be successful in higher mathematics. Clearly, the general mathematics-consumer mathematics sequence presents no real mathematics program. Rather, the third sequence is a shunt to eject as quickly as possible from mathematics courses those students who do not fit into college preparatory mathematics. This curriculum design equates high school mathematics with college preparatory mathematics. The choice given students is following the "high road" of algebra, geometry, etc., or the "low road" of general mathematics, consumer mathematics,



The Mathematics Program, Grades 7-12

Sequences

Grade 7: 201 Fundamental Mathematics 1

↓

Grade 8: 202 Fundamental Mathematics 2

↙ ↘

203 Fundamental Mathematics 3

- 210 Algebra 1 → 220 Geometry → 230 Algebra 2
- 691.2 Business Mathematics (See Publication No. 368)
- 241 Consumer Mathematics

210 Algebra 1

↓

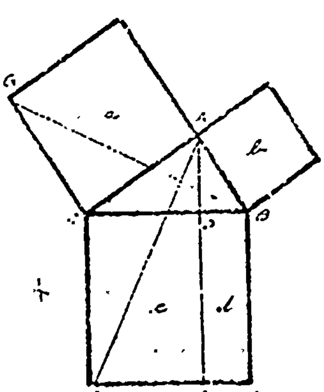
220 Geometry

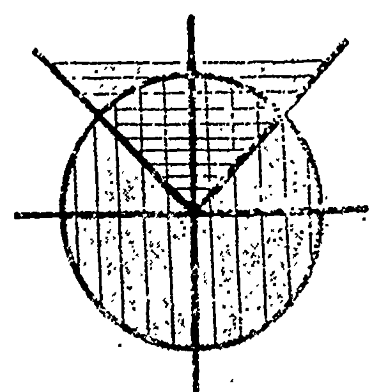
↓

230 Algebra 2

↓

240 Advanced Mathematics





and fast exit from further mathematics courses. No wonder non-college preparatory courses are held in low esteem and avoided by students and teachers alike. Everyone knows that these courses represent not a path of choice and prestige but a path of rejection and coercion.

Recent Innovations in the Existing Curriculum

"It is unfair to present the ... sequence in the last State Guide as the existing curriculum."

It is unfair to present the more than ten-year-old sequence in the last State Guide as the existing curriculum. There have been adjustments and additional courses that have revised the pattern shown in Figure 1. Figure 2 shows some of the possible patterns of course offerings that might be found in some schools (6). Shown in Figure 2 are some new courses such as Algebra I spread over two years (Algebra I-A and I-B or Introductory Algebra A and Introductory Algebra B), Remedial or Laboratory Math, and Vocational Math. Other new courses that are not shown are Pre-algebra, Algebra II with Computing, and The Man-Made World (ECCP,7). These new additions to the curriculum deserve scrutiny and thorough appraisal.

"Pre-algebra and Algebra I spread over two years is worthy of Committee of Ten thinking."

The invention of courses like Pre-algebra and Algebra I spread over two years is worthy of Committee of Ten thinking. These courses evolved because the content of Algebra I is considered to represent such a desirable mathematics experience that all students regardless of need should be streamed through it. Since all students do not show an inclination or aptitude for this essential mathema-

Grade	Course Sequences				
7th	Math 7	Math 7	Math 7	Math 7	*Math 7, Math 8
8th	Math 8	Math 8	Math 8	Math 8	*Math 8, Algebra I
9th	Remedial or Laboratory Math	General Math**	Introductory Algebra A	Algebra I	Geometry
10th	Applied or Vocational Math	Algebra I	Introductory Algebra B	Geometry	Algebra II
11th		Consumer Math	Geometry	Algebra II	Advanced Math
12th			Algebra II	Advanced Math	Calculus or Linear Algebra and Probability and Statistics

*Three years of mathematics can be taught in grades 7 and 8 to students of high mathematical ability provided there is careful planning in the determination of course content.

**In addition to the flexibility of course offerings suggested by the sequences listed in the five columns, note that the arrows emanating from General Math suggest additional possibilities. Thus, the columns are not intended to impose rigid restraints on the choices open to any student.

FIGURE 2. Examples of Course Options in Grades 7-12

tics experience, special courses are designed to mold students into the right shape for Algebra I. Pre-algebra and Algebra IA-IB are the "shoe horns" that are supposed to force one shoe size on all feet.

"Pre-algebra and Algebra IA-IB have produced some undesirable side effects."

Pre-algebra and Algebra IA-IB have produced some undesirable side effects. Students completing Algebra IA-IB are sometimes considered terminal students in the college preparatory track and are discouraged from going into Geometry. The student is the victim of a hoax; for having been lured and "shoe horned" into the college preparatory track, the student is then dropped. To provide credit for two years spent in a one-year course, the student is often given one credit for Algebra I and one credit for General Mathematics. This practice further deprecates General Mathematics and shows the contempt in which it is held. It shows the General Mathematics student that the content he is studying has little value since it counts for only one-half the value of Algebra I.

"No course can be justified on the basis of its need for another course."

The title, "Pre-algebra" (like precalculus) implies a serious pitfall often encountered in justifying a mathematics course. Pre-algebra implies that the goal of the whole course is "getting the student ready for algebra." Thus, the justification of each topic becomes, "You need to learn this so that you can use it in Algebra." Obviously, a student who does not follow Pre-algebra with Algebra I has wasted his time. No course can be justified on the basis of its need for another course. Every mathematics course must stand on its own merits as worthy of learning and enjoyable to study even if no more mathematics courses are taken. Delayed gratification or "pie in the sky by and by" will not serve as a sound basis for a year's work in mathematics.

"Students who have been unsuccessful in mastering computational skills for nine years are unlikely to be "saved" in a senior high school arithmetic course."

Lack of computational skills exhibited by some students has prompted the formation of basic mathematics courses (Remedial or Laboratory Math). Basic mathematics is a euphemism for remedial arithmetic. While absence of arithmetic skills is unfortunate, the assumption that a senior high school mathematics faculty can right the "wrongs" of elementary and junior high school mathematics instruction is optimistic and arrogant. (Senior high teachers can at most review and maintain developed skills, but they are largely unsuccessful in developing skills that are not present.) Students who have been unsuccessful in mastering computational skills for nine years are unlikely to be "saved" in a senior high school arithmetic course. Students deficient in fundamental operations will not become proficient through one more dose of the same old treatment. Moreover, senior high school teachers are usually unfamiliar with the developmental approaches and concrete activities necessary to put meaning into arithmetic instruction. The approach of devising one more course to teach computation at the senior high school level contains the seeds of failure.

"The preceding observations do not recognize many of the good elements of the existing curriculum,,,"

The foregoing comments on the existing curriculum have been negative and cynical. The preceding observations do not recognize many of the good elements of the existing curriculum or the large number of students who are very successful products of it. The focus of the remarks has been to present a new conception of the mathematics curriculum and pave the way for a curricula of alternatives and choices.

A CURRICULUM OF CHOICES

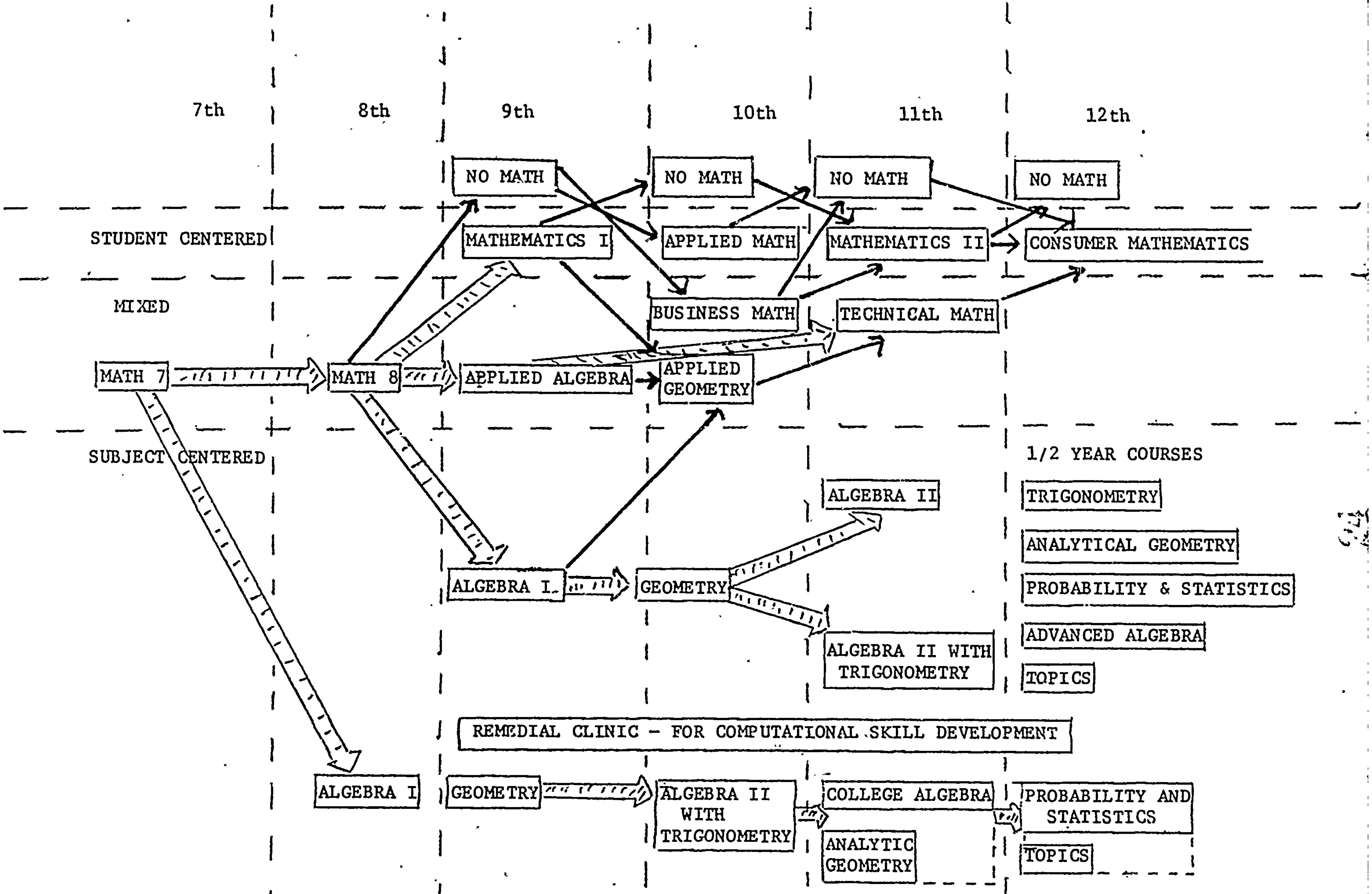
Shown in Figure 3 (next page) is a proposed framework for planning secondary school mathematics programs. Beneath the myriad of choices is the basic change of providing a full four-year sequence of mathematics courses that are alternatives to but co-equal in mathematical value with the traditional Algebra I, Geometry, Algebra II sequence. The course offerings described are divided into three basic strands: subject-centered courses, student-centered courses, and mixed courses.

Description of Strands

Courses such as Algebra I, Algebra II, Geometry, Trigonometry, and Arithmetic are examples of subject-centered mathematics. These courses have a fixed core of content that is basically the same from class to class, school to school, and state to state. Students must fit the content rather than the content fit the student. A teacher asked to teach Algebra I will know what to teach without knowing whether he is teaching 8th graders, 9th graders, adults, monkeys, horses, or an empty classroom.

Student-centered courses are designed to fit the content to the student. There are no imperatives or fixed subject matter that must be covered. Subject matter may be drawn from any part of mathematics and its fringes or related fields. A student-centered course cannot be fully prescribed until the students and their input are considered. Mathematics courses included in this category are Consumer Mathematics, General Mathematics, Vocational or Career Mathematics, and Advanced General Mathematics.

Mixed courses obviously contain elements of both subject-centered and student-centered courses. A common core of fixed content is pre-



Double arrows show prerequisites.

Single arrows show possible paths, not prerequisites.

FIGURE 3

scribed for all students, but there is an opportunity for choice and selection to meet student interests and needs.

Rationale for the Student-Centered Strand

It should be emphasized that the courses in the student-centered strand are very different from each other and from courses in the other strands. In particular, these are NOT courses in remedial arithmetic. The only computation taught in these courses is for maintenance of existing skills, not compensatory instruction for lack of skills. There would be no more time spent on arithmetic instruction in the student-centered courses than would be the case in courses in Algebra II or Geometry. Any remedial instruction in arithmetic required by students must be provided in a novel, non-traditional way. Students who after eight years have not mastered computational skills have problems so serious that they cannot be solved in the first-aid approach of a remedial course. They need the specialized intensive care provided by an individualized diagnostic-prescriptive approach in a remedial clinic. In summary, the student-centered courses are not arithmetic courses but mathematics content chosen and organized around a central theme.

A further distinction should be made between student-centered courses and the other strands. Unit planning and unit teaching will need to be undertaken, rather than chapter, section, or lesson planning and teaching. Thus, teaching a student-centered mathematics course will resemble more the methods used to teach science and social studies than the methods of teaching usually used in college preparatory mathematics.

MATH 7

Sample Material:

1. Kane, Robert et. al. Mathematics Target System Cincinnati, Ohio: American Book Company.

Additional enrichment:

2. Johnson et. al. Activities in Mathematics, First Course, Scott Foresman, 1971.

Patterns
Numbers
Measurement
Probability

Additional review material:

3. Stein, Edwin. Refresher Arithmetic. Boston: Allyn and Bacon, Inc. (Parts I and II)

or

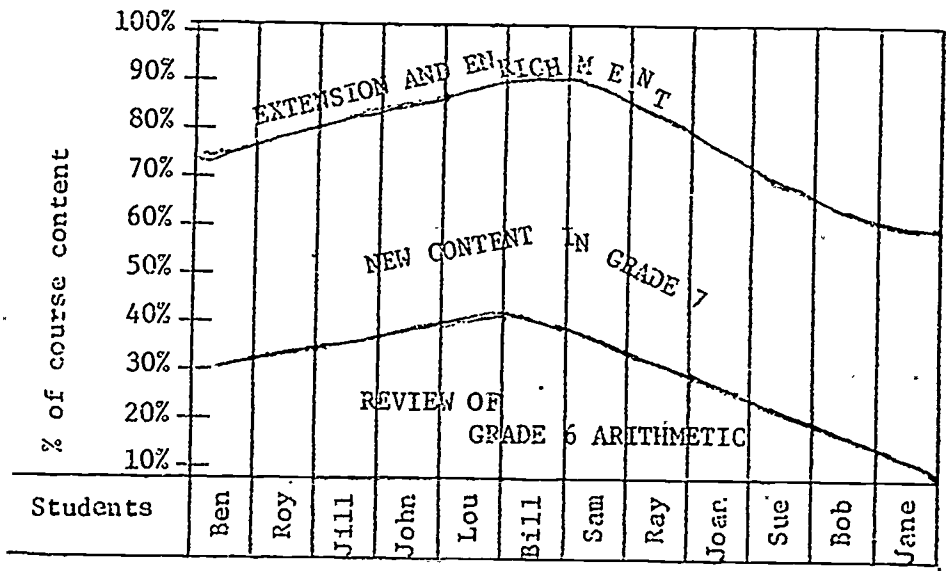
4. Proctor, Charles. SRA Computational Skills Development Kit, Chicago, 1965.

In recent years, several seventh-grade mathematics courses could be found in the same school. The mathematics books state adopted for grade seven vary in their coverage, sophistication, and reading level. Students are usually grouped according to ability. More able students get an enriched course from the more sophisticated book. Average students use a less sophisticated book. Low achievers are grouped together and study from special materials. Thus, the practice of ability grouping and selecting math materials for each type of student has produced tracking in both seventh and eighth grade mathematics.

Ability grouping and tracking are now under a severe attack which they likely will not survive. Schools will be under pressure to group students heterogeneously. Without ability grouping, schools will not be offering distinctly separate courses in seventh and eighth grades and hence will not be using different books in different sections of the same math course. The past practice of adopting seventh and eighth grade books at varying levels will no longer serve the needs of the schools.

In an effort to anticipate and meet future constraints on the schools, one common course in mathematics is recommended for all students in the 7th grade. Basically, the same mathematics material would be used in all 7th grade classes. Changes would have to be made both in the selection of these materials and in teaching methodology to enable teachers to cope with herero-geneously grouped classes.

The Math 7 course for heterogeneously grouped classes consists of three concurrent strands of content shown in the diagrams. One strand is review



and remedial instruction in the mathematics the student has already encountered in the elementary school. The second strand is the essential new material in mathematics to be developed in grade 7. (The emphasis is upon the word essential.) The third strand involves the extension of new concepts and enrichment activities. From the diagram it can be seen that exposure to the core or essential content would be the same for all students. Time devoted to review and corrective instruction in arithmetic will vary with the individual. Students who have mastered computational skills would be permitted more time for enrichment and extension of concepts. Thus, although Math 7 is described as one common course for all students, it can be observed from the diagram that Math 7 is just the opposite - a different course for each student determined by the students needs.

The Math 7 course as shown in the diagram would allow teachers to meet individual needs of students within a heterogeneous setting. Efforts to meet individual needs in the past have resulted in the use of individualized systems of instruction. Examples are IPI and IMS at the elementary level and the construction of Learning Activity Packages (LAPS) and self-paced programs at the secondary level. Students work through material independently, at their own pace, and usually in isolation from their peers. Math 7 is <u>not</u> intended to be taught under an individualized system, self-paced, or LAPS approach.

The approach in Math 7 should be called <u>personalized instruction</u> to distinguish it from individualized systems of instruction or independent, self-paced instruction. In a personalized system of instruction, students study from regular textbooks and supplementary materials. They are taught as whole classes, in groups, and individually. In a personalized system of instruction, more emphasis is placed upon individualizing the content to be studied, rather than the pacing or time permitted for learning. The teacher is the most central component of a personalized system of instruction, rather than an adjunct or resource person as in an individualized system of instruction.

Materials for a personalized system of instruction must be comprehensive and flexible. Every seventh-grade classroom will need more than a textbook. The teacher must be provided with placement tests, diagnostic tests, or individualized skill development kits, a textbook for main content and extensions, workbooks, spirit masters, reteaching materials, mastery tests, enrichment materials, prepared laboratory activities, accompanying laboratory materials and classroom library books. A teacher in a personalized system of instruction has too much to do in planning instruction for students that she has no time to develop materials, tests, enrichment activities, work sheets, practice material, and laboratory activities. These materials must be immediately at hand or this kind of instruction will fail. No longer can a textbook be provided with the expectation that the teacher will provide the rest. In sum, teaching in heterogeneous classes brings added demands upon the teacher's time and creativity. To meet these demands, teachers must be provided with all the prepared materials that the teachers themselves were formerly expected to produce in ability-grouped classes. It can not be stated too strongly - state adoptions for Math 7 and 8 must be for a total package of materials and not just textbooks.

Of course, materials alone do not make a personalized system of instruction succeed or fail (quite unlike an individualized system). The teacher's ability to implement, coordinate, and monitor instruction is the central ingredient. To do this effectively, the teachers must organize. Usually, the simplest organization is the best. The diagram below shows a sample plan for a week's work in Math 7.

MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY
Core Content (group-based instruction)	Review of Arithmetic (Individualized Instruction Skills Kit)	Core Content (group-based instruction)	Review of Arithmetic (Skills Kit) (Low Achievers) Enrichment (for able students)	Core Content (Workbooks for most) Enrichment for some.

On Monday and Wednesday, new content is developed using instruction with the whole class together. On Friday, the class is divided into groups. Most students practice and review core content in workbooks, spirit masters, practice tests or laboratory activities. Some more able students may study some extensions of the core content in the textbook. Tuesdays and Thursdays are devoted to self-paced individualized instruction. All students work on review and maintenance of computational skills on Tuesday. On Thursday, some students work on computational skills while others work on enrichment and extensions. In this way the class is never totally individualized or totally on group-based instruction.

Incorporating a planned program of individualized instruction on sixth grade mathematics in Math 7 means that less time can be spent on developing new material. Therefore, the essential new content to be introduced must be pared to a minimum. The core strand should probably represent no more than 40% of the total course content. The remaining 60% of the courses content would be split between sixth grade review and extensions and enrichment. Some of the topics, that could be considered in the core strand might be:

- extensions of algorithms to work with larger numbers. (i.e. 3-place division)
- operations with decimals, estimating, reasonableness of answers; multiplying and dividing with powers of ten.
- relating fractions and decimals, learning decimal and fraction equivalents; repeating decimals.
- metric geometry - emphasis on SI. Some work with customary system.
- extending operations with fractions and mixed numerals; multiplication and division.
- ratio, proportion, and percent.
- interpreting and constructing graphs.

In addition to enrichment in the form of games, activities, readings, the enrichment strand could contain content like the following:

- exponents and scientific notation

- sets
- properties of operations
- non-metric geometry; subsets of lines
- factoring, primes, and divisibility rules
- using g.c.f. and ℓ.c.m. in reducing and adding fractions and mixed numerals.
- modular arithmetic
- Roman numerals and non-decimal numeration systems
- non-repeating and non-terminating decimals.
- square roots and cube roots
- graphing on a coordinate lattice
- congruence, similarity, and constructions
- integers and rational numbers.

MATH 8

Sample Material:

1. Kane, Robert, et al. Mathematics Target System. Cincinnati, Ohio. American Book Company

Additional Enrichment

2. Johnson, et al. Activities in Mathematics: Second Course Scott Foresman 1971.

graphs
statistics
proportions
geometry

Additional Review Material

3. Stein, Edwin. Refresher Arithmetic. Boston: Allyn and Bacon Inc. Parts I and II

or

4. Proctor, Charles. SRA Computational Skills Development Kit Chicago 1965.

The discussion and recommendations made for MATH 7 apply equally to MATH 8. In MATH 8 the review strand would include review and corrective instruction on the core content of MATH 7.

The core content of MATH 8 should include:

- integers and rational numbers
- introduction to irrational numbers through roots and non-repeating, non-ending decimals.
- solving equations involving proportions
- mensuration formulas for solids
- exponents and scientific notation.

The extension and enrichment strand could include:

- numerical trigonometry
- number theory
- statistics

- probability
- constructions
- equations and inequalities
- graphing ordered pairs of rational numbers
- transformational geometry
- precision, accuracy, and significant digits.

MATHEMATICS I

Sample Material:

1. Dodes, Irving Allen. Mathematics - A Liberal Arts Approach - A Textbook in General Mathematics. New York: Hayden Book Co., Inc., 1964.

In the preface to his book, Dodes writes:

"How does this book differ from other books in general mathematics? There is really no similarity. This book emphasizes the strength and power of mathematics, not the practice and details of manipulation. A non-artist can enjoy a painting by Picasso, and a non-mathematician can enjoy the kind of poetry which is called mathematics. If you wish to teach arithmetic, whether of the consumer or commercial type, this is not the book to use. It does not explain how to write a check or how to fill out an income tax blank."

Dodes' description is not mere idle talk. His book is genuinely different and in the way he describes. In exactly the same way, Mathematics I is genuinely different from what is being presently passed off as general mathematics at the ninth grade level. The present day course in general mathematics has gotten so far afield from its original intent that we find it necessary to invent the term "Mathematics I" to distinguish this rediscovered course in general mathematics from today's counterfeit versions. Mathematics I is not a course in arithmetic or diluted algebra and geometry with the reasoning removed.

Mathematics I is organized in what can best be described as a liberal arts approach to mathematics. It is a course about mathematics - an appreciation course. Its primary purpose is to get students to like it - who may think they don't like mathematics. Therefore, its success must be judged on the basis of positive attitudes and the stimulation of aesthetic values, as much as on factual information learned.

The content must incorporate topics that show mathematics as an interesting human activity containing ideas that have fascinated, entertained, and captivated people for centuries. It would contain familiar mathematics topics with non-standard treatments as well as topics on the fringes of mathematics that are little seen in the high school curriculum. No content concerning business appliaations, consumer uses, or vocational examples would enter in this course. Rather than emphasizing "work ethic" mathematics (i.e. mathematica as a tool in life and the world of work), it would emphasize "leisure ethic" mathematics as well as a reasoning and intellectual stimulation. Many standard mathematical topics could be treated in their relationship to hobbies, recreations, and avocations.

A course such as this would be non-traditional in other ways. Students would be expected to use the library in the same way it is expected in any literary course. Much of the classwork may involve individuals or groups performing laboratory activities, preparing presentations, or constructing projects. Students would be assigned to follow up aspects of topics in depth using encyclopedias and other sources. Biographies and other works would be used to develop plays to depict contributions of great mathematicians. (Archimedes discovering the principle of displacement in the bathtub would no doubt be the most interesting role.)

Much of the students' work in the class might consist of written or oral reports, construction of physical models, making of cassette tapes, videotapes, or slide shows, staging of plays or debates, role-playing incidents, production of bulletin boards or other teaching materials, demonstrations, laboratory activities, and peer teaching. Students could be evaluated on their strengths rather than their weaknesses. One student can be graded on his ability to teach the class a card trick and the understanding of its mathematical basis. Another student may be graded on an oral report describing the history of women in mathematics. A third may be graded on the construction of a special slide rule, or physical model of a mathematical phenomena. This would allow students who normally are unsuccessful in a paper-pencil mode to find alternative ways of exhibiting achievement and self-expression. Many students who have limited ability to express themselves in language and symbolism can often show excellent work in other less school-like and more life-like modes of exhibiting achievement.

Students in Mathematics I are learning a great many important aspects about mathematics that other students in algebra will not learn. (It would be desirable for algebra students to learn them, but there is not time if they are to meet the vocational goals of preparing for college.) It is good for the students in Mathematics I to learn mathematics that is not known to algebra students. The course does not get the image of being remedial or lower-level. Students in Mathematics I can boost their academic self-image and self-esteem in knowing that they are learning mathematics that the supposedly "smart, superior algebra kids" do not know.

Presenting a list of topics for Mathematics I presents a danger. Teachers may feel that the list represents a syllabus showing content that should be covered. This is contrary to the philosophy of a student-centered course as being flexible. There is no pressure to finish a textbook, cover all topics, or follow the sequence of topics shown. Teachers should feel free to supplement, substitute, or rearrange content as desirable within the constraints of the course theme. The interests of the class should be the major factor in determining the selection and length of any particular unit.

APPLIED ALGEBRA

Sample Material: None. Select from and build upon topics and ideas from:

1. Stein, Edwin. Algebra in Easy Steps. Princeton, N. J.: D. Van Nostrand Co., Inc., 1961.

2. Herrick, Hanzl-Hendrick and Adams. Essential Skills in Algebra. New York: Harcourt, Brace and World, Inc., 1962.

3. Proctor, et. al. SRA Algebra Skills Kit. Chicago, Ill.: Science Research Associates.

Although the present course in Algebra I is unsuitable for many students, there are some students who need and can profit from the study of algebraic ideas and problem-solving in practical situations. Applied Algebra is a course to meet these needs. It is built around the concept of motivating the study of algebra. Students will not have to ask, "What good is this?" or "What is it used for?" The uses of algebra will be as constantly before the student as the mechanics of algebra. Thinking in a concrete way will be more emphasized than the abstract, deductive mathematics of Algebra I.

The content of Applied Algebra would consist of four main strands: 1) the mechanics of manipulating algebraic entities, 2) the direct application of each newly developed technique to show its usefulness, 3) the development of formula reasoning and functional dependence, and 4) the formulation of algebraic models to solve problems.

The first strand contains the basic algebraic facts that will be presented without proofs. The algebraic content is presented as generalized arithmetic rather than as an abstract deductive system (i.e., a ring of polynomials over the field of real numbers). Topics to be covered would be:

- review of integers
- building up expressions using variables
- fundamental operations with polynomials
- special products and factoring in simple situations
- fundamental operations with rational expressions using simple expressions
- solving fractional equations involving simple fractions
- fundamental operations with powers and radicals
- solving radical equations
- solving quadratic equations

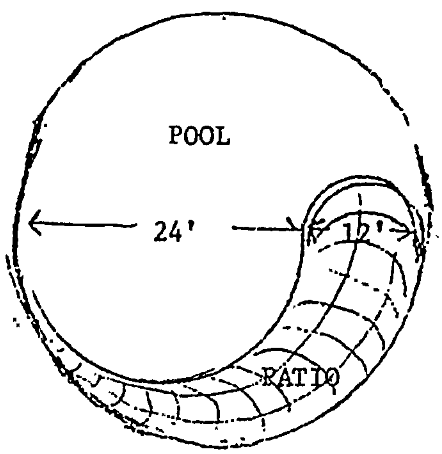
The second strand would emphasize or highlight the use of each of the manipulative skills listed above. Situations and problems already formulated would be used to motivate the need for studying a topic. That is, the student would be shown what they will be able to do as a result of mastering each new aspeet of the topic. After the topic is developed, it will be practiced in applied situations where its use can be shown. The applications of strand 2 are much more low level than those of strand 4. In strand 2, students substitute into already derived formulas or relationships and interpret the results within the setting of the p roblem. Some of the word problems in present textbooks dealing with physical situations that only call for direct translation with little real formulation would fall in this category.

Strand 3 emphasizes algebra as a subject that extends our power to reason. In this phase, solving, manipulating, and, more importantly, interpreting literal equations would be stressed. Functional relationships would be stressed as well as the ideas of variation.

The fourth strand involves identifying relationships in problems that can be represented algebraically or graphically. The stress in this strand is the formulation of a problem into mathemtical terms. Students would generate formulas or graphical means to represent relationships in sets of orderred pairs, relationships in word problems, physical principles or relationships shown in geometry situations.

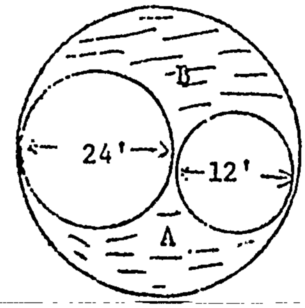
An example of modeling in a geometric situation is as follows:

A pool-patio combination is being constructed using the design shown The area of the pool and the area of the patio must be found. Find the area of each.



Solution: To formulate the problem mathematically, we must make some simplifying assumptions.

1. The patio-pool combination forms a circle of diameter 36'.
2. The curves are arcs of circles with diameters of 24' and 12' as shown in the accompanying sketch.
3. The area of region A = the area of region B.



With these assumptions and the area formula for a circular region, algebraic expressions for the area of the pool and the patio can be written and solved.

This example shows the role of assumptions in formulating a mathematical model to solve problems, a much neglected area in mathematics education.

Applied Algebra would not cover as broad a range of topics in algebra as Algebra I nor to the same depth. Rather, it would concentrate on the mastery of a narrow band of essential content. The remaining time would be spent on motivation of the study of algebra, applications of the subject, and how it is used to model and solve realistic and worthwhile problems.

It is intended that this course would contain enough mastery of algebraic content to serve the needs of students who must have algebra for college admission or job qualifications. In this sense, it is a college preparatory track course and not a terminal course in mathematics. Students who are more mature in their development and need little motivation other than intellectual stimulation should take Algebra I.

APPLIED GEOMETRY

Sample materials: None

Applied geometry is a course built around the concept of showing the uses and usefulness of geometry and geometric thinking. The course itself would be organized around four main strands: 1) geometric facts, 2) meaningful applications of geometric facts in practical problems, 3) logical reasoning and deduction, and 4) problem-solving using geometric modeling.

The content of geometry would be transmitted as basic facts without formal proofs. Theorems would be presented as statements concerning relationships in the physical world rather than propositions that can be logically derived from a set of assumptions called axioms. Instead of proofs, there would be verifications such as measuring to justify that an angle inscribed in a circle is one-half its intercepted arc. Or, pouring sand to show the volume of a right circular cone is one-third the volume of a right circular cylinder with the same base and height.

As theorems are proved, applications of them to the direct solutions of problems would be given. Applications would include descriptive drawings, the science of perspective and other draftsmen and trades orientations of geometry. Drawings with ruler and protractor would be utilized as well as constructions with compass and straightedge.

Logical reasoning would be introduced through the use of flow charts. Students would learn to analyze problems and reduce them to a step-by-step algorithmic procedure. From this background, deduction would be introduced as well as some geometric proofs in flow chart form. A more general approach to deductive proof would follow, much like the work undertaken by Harold Fawcett. His work in the 1930's at the Ohio State Laboratory School is described in the 13th Yearbook of the National Council of Teachers of Mathematics entitled, "The Nature of Proof." The emphasis of the logic reasoning strand would not focus entirely upon proofs of geometric theorems but on teaching for understanding of and transfer of deductive thinking and logical reasoning to other areas and activities.

The fourth strand would emphasize solving problems using geometric content as mathematical models. The stress would be upon problem formulation, i.e., abstracting from physical situations, relationships that can be represented as relationships among points, lines, circles, triangles, and angles. The students would then solve the mathematical formulation and translate the mathematical solution back to the physical settings. Problems would include construction problems and locus problems. Another example would be drawing the pattern for covering a Christmas ball where 5 congruent lunes are to cover a ball with a 5-inch diameter. Another example would be to find the inside diameter of a pipe that must slip over a triangular rod.

A part of the course would have the spirit of a geometric drawing course for a draftsman or a skilled machinist. Included also would be the notion of proof and deduction in a broader sense and problem formulation skills associated with mathematical modeling.

It is intended that this course would contain enough geometric content to serve the needs of students who must have geometry for college admission or job qualifications. In this sense, it is a college preparatory track course and not a terminal course in mathematics. Students who are more mature in their development of abstract thinking should take the regular geometry course which is less oriented toward concrete thinking.

MATHEMATICS II

Sample material (selections from):

1. Dodes, Irving Allen. Mathematics - A Liberal Arts Approach - A Textbook in General Mathematics, New York: Hayden Book Company, Inc., 1964.

2. Harnadek, Anita. Mathematical Reasoning. Troy, Mich.: Midwest Publications Co., Inc., 1969.

3. Jacobs, Harold. Mathematics - A Human Endeavor - A Textbook for Those Who Think They Do Not Like the Subject, W. H. Freeman, 1970.

4. Triola, Maria. Mathematics and the Modern World. Menlo Park, Ca.: Cummings Publishing Co., 1973. (Mathematics in the social sciences, psychology, art, and music; capsule history of mathematics)

5. Johnson et. al. Applications in Mathematics, Course A, Chicago: Scott Foresman Co., 1972. (Sampling and statistics bookley and prediction and probability booklet.)

6. Johnson et. al. Applications in Mathematics, Course B, Chicago: Scott Foresman Co., 1974. (Computing devices booklet and decision-making booklet.)

Mathematics II is a continuation of Mathematics I in theme and spirit. The expected level of achievement would be higher in Mathematics II than Mathematics I. However, the emphasis upon variety of modes of instruction and evaluation would be continued and heightened by utilizing the higher level of maturity and independence of the students.

Some of the topics in Mathematics II can reflect more modern mathematics. Thus, topics such as computing, flow charting, intuitive topology, Boolean algebra and switching circuits, analyzing simple games mathematically, symbolic logic and truth tables, famous and unsolved problems.

Presenting a list of topics for Mathematics II presents a danger. Teachers may feel that the list represents a syllabus showing content that should be covered. This is contrary to the philosophy of a student-centered course as being flexible. There is no pressure to finish a textbook, cover all topics or follow the sequence of topics shown. Teachers should feel free to supplement or substitute or to rearrange content as desired within the constraints of the course theme. The interests of the class should be the major factor in determining the selection and length of any particular unit.

CONSUMER MATHEMATICS

Sample Material

1. Lankford, Francis and Coe, William. Consumer Mathematics. New York: Harcourt, Brace, Javanovich, Inc. 1971.

2. Mathematics Education Staff. Consumer Mathematics - Teaching Units. North Carolina State Department of Public Instruction. 1969.

Like general mathematics, the content and purpose of consumer mathematics is frequently misunderstood. Consumer mathematics is often taught like it is a course in arithmetic or consumer applications of arithmetic. Teaching arithmetic or its social applications is not the primary purpose or function of consumer mathematics. Consumer mathematics is a problem-solving course. More than any other course in the mathematics curriculum, consumer mathematics should teach students processes and procedures for solving problems - real problems, not the exercises called problems found in other mathematics courses.

The main purpose of consumer mathematics is to help students learn to make informed decisions, particularly, but not exclusively, those where mathematics forms part of the input used to arrive at a final decision. The point to be stressed is that mathematics provides only one of several inputs needed for making a wise consumer decision. Real problems have many more facets than just those of cost or financial gain. Matters of taste, life-style, service, religious convictions, aesthetic appeal, and humanistic values are all considerations that cannot be ignored. A narrow course in arithmetic can be harmful in that it can distort the importance of the mathematical input in consumer decisions and ignore many other more important components. A narrow mathematical approach encourages a "right answer" mentality whereas in consumer decisions there are no right or wrong answers - only selections among alternatives.

Is consumer mathematics (a better name would be consumer education) a course that should be taught by mathematics teachers or are they too ill-prepared to teach it? The answer is yes they should teach it. One, a mathematics teacher is a person as well as a mathematics teacher. Surely their education and life experiences prepare them as well as any teacher to teach this course. (If not, then it is questionable whether they are fit to teach children in any other mathematics course.) Secondly, there is a significant mathematics component to be considered in any consumer decision. The mathematics teachers are best suited to handle this aspect provided they teach understanding of quantitative concepts and mathematical reasoning instead of computation and remedial arithmetic. Thirdly, mathematics is thought by many to teach people to think quantitatively and logically. Thus, mathematics teachers should be extremely well-equipped to teach a course in thinking and problem-solving.

If there is no fixed content in consumer mathematics and if arithmetic plays a very minor role, what is the content of consumer mathematics? The core of consumer mathematics is the processes it teaches. All aspects of processes of decision making should be taught and illustrated again and again in each new unit. Some of the processes to be taught are:

- anticipating and identifying problems that will require decisions
- generating a plan to attack the problem
- gathering data and information
- standard tools, references, and techniques for becoming more informed
- organizing data
- drawing inferences and conclusions
- searching for hidden motives, fraudulent intent and misrepresentations
- developing alternatives and working hypotheses
- analyzing alternatives with reasonable objectivity
- arriving at decisions and justifying them
- taking appropriate action

- reflecting upon decisions and their wisdom after they have been accomplished

Clearly, this is not a simplistic, "right answer" approach.

The classroom teaching of consumer mathematics should reflect this philosophy. Since the processes are the important variants, the particular content used as a vehicle to teach the processes is of secondary importance. Topics such as house-buying, family budgeting, furniture-buying, investments, property taxes, and insurance should receive less emphasis since students have difficulty relating to these topics. The goal of learning decision- making processes can be more easily reached with topics such as renting a room or apartment, buying a motorcycle or 10-speed bike, income taxes on an after-school job, buying stereos and radios; avoiding rip-offs and other topics more attuned to the student's daily-life activities.

The methods of instruction must be varied also. Unit planning and teaching should be the rule with each unit designed to showcase all the steps in making an informed decision. A project method could be used where students identify the problems they wish to work on, gather data from their community, do their own research in standard references or tables and present a written or oral project report. They would conduct interviews and possibly devise their experiments, laboratory tests, or comparison of features of service and price. Class time would often be devoted to discussion of events of current interest and debates on consumer issues. Students should be taught to consult and utilize information in references such as Consumer Reports. Students would be given specific and frequent work on following directions, reading instruction booklets, filling in forms, or carrying out directions shown in diagrams.

In summary, consumer mathematics does not teach arithmetic or about how to save money. It teaches problem-solving skills for real problems and informed decision-making.

TECHNICAL MATHEMATICS

Sample Materials

1. Levine. Vocational and Technical Mathematics in Action.

2. Practical Problems in Math for

   Auto Technicians
   Carpenters
   Electricians
   Machinists
   Masons
   Printing Trades
   Sheet Metal Technicians

   Albany, N.Y. Delmar Publishers

3. Mathematics for Machine Technology
   Mathematics for Plumbers and Pipefitters
   Mathematics for Sheet Metal Fabrications

   Albany, N.Y. Delmar Publishers

4. Skelly, Esther G. Medications and Mathematics for the Nurse. Albany, N.Y. Delmar Publishers, 1972.

5. McMackin, Frank, et al. The Mathematics of the Shop. Albany, N.Y. Delmar Publishers, 1970.

Technical mathematics is more of a subject-centered course than a student-centered course. It contains a fixed core of content that would be expected to be mastered.

Algebra, geometry and trigonometry provide the needed background for students who wish to become scientists, engineers, and mathematicians. Business mathematics provides the necessary background for students who will enter the business world or business colleges. Technical mathematics provides the needed background for students who wish to enter skilled crafts and trades. These students need the mathematics that will enable them to pass employment and apprenticeship tests or to enter a technical or vocational curriculum for technicians and skilled workers.

A shortage of craftsmen, technicians, and skilled workers continues in our society at a time when we have an oversupply of both unskilled labor and college-trained specialists. The high school curriculum is in a unique position to offer a program that will both meet the interests of many students and also prepare these students for entry into the skilled trade and service areas. Mathematics has a prominent role to play in this program. Mathematics courses that have been offered have not provided enough mathematics to allow students to exit high school with enough mathematics to enter skilled areas. This oversight must be rectified.

The fixed portion of the content of Technical Mathematics should draw upon Algebra, Geometry, Graphing, Measurement, Ratio and Proportion, and Numerical Trigonometry. The applications of these mathematical topics to particular vocations should be determined by local needs. These should be in cooperation with local technical and trade schools as well as local industries to determine the skilled trades of the area and the mathematical requirements for entry. Sample employment and apprenticeship tests should be used which simulate the type of situation that a prospective employee might experience. The sample should not be confined to just shop or industrial vocations. Vocations should be included that have been traditionally female as well as those which have been traditionally male.

REMEDIAL CLINIC

Sample materials - None.

After eight years of instruction in arithmetic, there are often some students whose computational skill and comprehension of measurement fall below the level of functional competence. Students who lack mastery at this stage are suffering from more than poor teaching at previous grade levels. These students have learning disabilities that cannot be overcome by placement in an arithmetic course, no matter how well it is taught. These students need expert care on an intensive individualized basis.

Remedial Clinic is not a course, it is a location in the school staffed by a teacher with special qualifications. Students in the school who are performing well below some minimum level of functioning in mathematics are identified and then scheduled into the clinic, one or two at a time, for one hour a day. The students may be taken out of their study hall or they may be assigned to remedial clinic instead of their math class or some other class. Students would remain in remedial clinic until some minimal level of competence is achieved At that time the clinic would discharge them. Thus, the more the student progresses, the sooner he is released to do something he would rather do.

The teacher in the remedial clinic would need special qualifications. He must know and be able to identify learning disabilities in mathematics whether of cognitive, affective, emotional, or social origin. He must be familiar with diagnostic-prescriptive teaching techniques and implement them in an intensive way. Through testing, observations, and interviews, profiles and protocols would be constructed for each student. The teacher would then administer the prescribed treatment and monitor progress. The remedial clinic teacher must be trained to administer diagnostic tools such as aptitude batteries, achievement tests, spatial relations tests, reading and comprehension inventories, projective techniques, attitude scales, modality of thought scales, manual skill tests, and many other types of instruments that can produce measures to construct a student's profile.

Along with mastery of diagnostic tools, the remedial teacher needs special training to identify and effectively use a variety of remedies. Computational skills may be taught using novel algorithms such as the Trackenburg System or byusing crutches such as nomographs, slide rules, tables, and other novel computing devices. Some students may learn through a laboratory mode, using manipulative materials while others may require a traditional role memorization mode of instruction. Still others may only achieve the ability to use a calculator and may never learn the algorithms. Clearly no one mode or approach will serve all these students.

Because of the special qualifications needed by the teacher and the expense, the remedial clinic may serve more than one discipline. The remedial teacher may diagnose and prescribe for both math and reading. A mathematics or English teacher might then follow up by overseeing the administration of the treatment. In this way, the special diagnostic ability of the remedial teacher can be combined with the knowledge of a subject matter teacher.

APPLIED MATHEMATICS

Sample material

1. Lyng, Merwin, Meconi, L.J., and Zwich, Earl J. Career Mathematics Industry and the Trades, Boston: Houghton Mifflin Company, 1974.

2. Johnson, et al, Applications in Mathematics, Course A and Course B. Scott Foresman, 1972, 1974.

3. Gold, Marvin and Robert E. Carlberg. Modern Applied Mathematics. Houghton Mifflin Company, 1971.

Applied mathematics is a course that shows the diverse uses of mathematical content and mathematical modes of thought. Applied mathematics is not to be confused with Technical Mathematics. Technical Mathematics, Business Mathematics, Algebra, Geometry, etc. are courses to provide the particular mathematical content needed to enter a chosen vocation or profession. Those courses have some fixed core content that must be mastered. Applied mathematics is a course for general education, i.e., to prepare for a full life as an informed citizen. It is an appreciation-of-mathematics course, stressing the utilitarian values of mathematics in a society. Saying this in another sense, Mathematics I and II present the subject from the "historical Greek" perspective of the discipline and Applied Mathematics presents it from a "historical Roman" perspective.

Although one of the sample books listed is a trades math book, applied math is not thought to be a trades math course. Applications to mathematics in the trades is only one aspect of the course. Applications of mathematics in the sciences, sports, social science and other areas would also be discussed.

Perhaps no existing textbook would fulfill the diversity of applications envisioned. Selections would be made from many sources. However, the two books by Johnson, et al listed above most nearly capture the spirit of showing the wide uses of mathematics in a general education or appreciation-type setting.

Examples of some interesting topics that may not usually be covered would be:

surveying

navigation

map-making involving projections.

Rationale for the Subject Matter Strand

These comments should in no way be considered a criticism of present-day mathematics teachers. On the whole, teachers have done an excellent job of teaching what they have been requested to teach. The problem is that we have requested them to teach entirely too much in too short a time. Consequently, they have been forced to choose between teaching too much too quickly and choosing the most important topics from important topics.

Separation of Algebra II and Trigonometry

Until recently trigonometry was considered an advanced course at the secondary level and the first course (either separately or with college algebra) at many colleges and universities. With the expectation that students be able to begin their college course work with calculus, trigonometry became required at the secondary level and (in most college scientific-mathematical curricula) non-credit or remedial at the post secondary level. With the advent of "pre-calculus" courses, trigonometry was pushed even further down in the curriculum until it achieved its present state of a group of topics scattered throughout several years of mathematics. The results have not been exceptionally rewarding. Not only have the number of students enrolled in the remedial courses increased, but an informal survey revealed that faculties of the college mathematics teachers feel that competency in trigonometry has declined (among those not taking the remedial courses in college). A corresponding decline in ability to operate with algebraic expressions also has been noted. Spending three semesters in studying algebra and trigonometry appears to be the best solution. This would allow more time to study the existing topics and provide time for topics not now covered.

Advanced Algebra

Under the existing program, little time is available to such topics as mathematical induction, series and sequences, elementary matrix and determinant theory, solution of higher ($n>2$) degree polynomial equations and other topics formerly covered in college algebra. Understanding of many of these topics is assumed both by the authors of the college texts and the college professors who teach the courses. A one-semester course based upon these expectations would serve the dual purposes of preparing and motivating students.

Analytical Geometry

Twenty years ago, college students typically took a course in analytical geometry as a prerequisite or co-requisite for calculus, to increase the time available for "higher level" mathematics, calculus and analytic geometry were integrated into a three-semester sequence of courses, with the result that a minimum of analytic geometry was taught. The claim that the "coordinate geometry" sections of plane geometry and Algebra II provide the necessary background in analytic geometry is suspect. In many cases, these are the topics sacrificed to teach other topics in the limited time. Many calculus students have difficulty in visualizing two-dimensional problems--three dimensions are almost too much to ask. Curve sketching ability is almost non-existent if one asks for other than the circle and parabola. A one-semester course in analytic geometry which builds upon the geometry, algebra, and trigonometry taken earlier would provide a sound basis for the calculus.

Building upon the earlier mathematics courses, analytical geometry has the potential to be the final course in a high school which successfully integrates the material previously studied. Whether or not the student plans to study calculus is immaterial; the content of analytical geometry warrants adequate emphasis in the mathematics curriculum.

Probability and Statistics

Recent applications of mathematics to the social and biological sciences as well as the "hard" sciences have increasingly involved probability and statistics. Understanding predictions of voting patterns, the validity of opinion polls and the merchant's application of opinion questionnaires to decision making requires an understanding of probability and statistics. A one-semester course in probability and statistics is suggested. It is suggested that such a course could be constituted so that a second year of algebra would not be a prerequisite and students could enter with basic algebraic concepts.

Topics in Mathematics

A one-semester course of the "liberal arts math" approach could be offered to students whose preparation is not necessarily for the calculus. Such courses should be developed at the local level and geared to local interests and strengths. Topics such as computer programming could also be offered under this umbrella. The rigor and sophistication could vary so that students interested in mathematics but not deeply prepared could profit from a one-year sequence in Probability and Statistics followed by other topics in mathematics.

Calculus in the High School?

Analysis and/or calculus are conspicuous by their absence in this report. The writers feel that a thorough understanding of algebra, geometry, trigonometry, and analytic geometry provides the best background for further study of the calculus.

DISCUSSION OF SPECIFIC COURSES

Subject-Centered Strand

9th	10th	11th	12th (half-year courses)
Algebra I	Geometry	Algebra II and Trigonometry	Trigonometry
			Analytic Geometry
			Probability & Statistics
			Advanced Algebra
			Topics

Students taking Algebra II and wishing to prepare for calculus should take Trigonometry and Analytic Geometry. Probability and Statistics and Advanced Algebra could be taken either at the same time or given as fifth-year options.

Students taking Algebra II and Trigonometry should take at least two of the half-year courses with Advanced Algebra and Analytical Geometry being the preferred sequence. If possible, "doubling up" should be permitted so that Probability and Statistics and/or Topics also could be taken.

Secondary students not preparing for calculus but wishing to study additional mathematics should have an adequate background to take Probability and Statistics and Topics after acquiring basic algebraic and geometric concepts.

Descriptions and Changes

The courses implied by existing textbook adoptions appear adequate for Algebra I and Geometry. In both cases, it is recommended that less time be given to right angle trigonometry and less time to proof of algebraic properties. If a school must offer a combined two-semester course in Algebra II and Trigonometry, the existing textbooks probably are adequate.

The recommended course, Algebra II (without trigonometry), probably could be taught from existing texts. A syllabus which indicated those topics to be included and those to be omitted should be prepared. This would require omission of trigonometric functions and consequently the polar form of complex numbers and real functions. Complex numbers would then be introduced as ordered pairs, or preferably, algebraic expressions of the form $a + bi$; a and b real numbers and $i^2 = {}^-1$.

It is suggested that a separate text for trigonometry be adopted. One sequence of topics follows.

Trigonometry and Elementary Functions: (e.g., Garland and Nichols, Modern Trigonometry, Holt, Rinehart and Winston, 1968) - half-year course.

I Introduction. Review of Pertinent Algebra; Sets, Inequalities, Geometry Concepts; the Circle, Triangle Function.

II The Unit Circle

or

IIa The Wrapping Function

III Trigonometry Functions

IV Equations and Identities (including Sums and Differences, Multiples and Solving Identities

V Inverse Trigonometry Functions

VI Polar Form of Complex Numbers, DeMoivres' Theorem

VII Solving Triangles (Law of Sines, Cosines, etc.)

VIII Introduction to Vectors

Similarly, a separate text for Analytic Geometry is a necessity. One suggested outline is as follows:

Analytic Geometry: (A very good outline is found on pages 121-130 of Mathematics in Florida Secondary Schools, State Department of Education, 1964) - half-year course.

I Basic Concepts Coordinate Systems; Distances

II The Straight Line - Slope, Equation, Families of Lines, Locus of Points

III Conic Sections

IV Rapid Sketching of Simple Curves - Algebraic, Transcendental

V Polar Coordinates

VI Parametric Equations

VII 3-D Analytic Geometry (as time permits)

Probability and Statistics: (e.g. Willoughby, Steven S., Probability and Statistics, Silver Burdett, Morristown, N. J., 1968 and/or Moesteller et al, Statistics by Example, Addison Wesley, Reading, Mass., 1973.)

Two different approaches to Statistics and Probability are represented above. Willoughby's book presupposed only a rudimentary knowledge of sets and basic algebra and develops the usual sequence of topics from a non-rigorous, intuitive basis. The chapter headings indicate the scope of the course:

I A Fast Look at Probability

II How to Count Efficiently (Permutations, Combinations)

III Probability

IV Organizing and Reporting Data

V Theoretical Distributions

VI Inferential Statistics

VII Game Theory

The four-part series, "Statistics by Example," attempts to motivate the study by exploring "real life" problems. Subtitles and prerequisite mathematics of the four pamphlets are indicated below.

I Exploring Data - Arithmetic, Rates, Percentages

II Weighing Chances - Notion of Probability, Elementary Algebra

III Detecting Patterns - Elementary Probability, Intermediate Algebra

IV Finding Models - Elementary Probability, Intermediate Algebra

The authors of this series state in their preface: "We feel that there is a need to explain real life problems, because the student is unlikely to have had experience with any kind of statistics beyond taking averages and the teacher with any other than the theoretical side of statistics, if that." Consequently, there is more explanatory text than is found in the usual mathematics book, and the extra-mathematical aspects of each problem are considered as well as the mathematical ones. (Another aspect of this text is that, for the highly motivated student, self study is possible.)

Both of the preceding texts enable students to investigate probability, descriptive statistics, and inferential statistics from a fairly rigorous standpoint without requiring calculus.

Advanced Algebra: Topics to be chosen from the following list:

Polynomial functions of degree m, $m \geq 2$

Synthetic Division

Bounds of Real Roots

Descartes' Rule of Signs

Matrices, Determinants

Cramer's Rule

Mathematical Induction

Sequences and Series

Rational Roots Theorems

Systems of Equations in Two or More Variables

Combinations, Permutations

Topics in Mathematics: Possible topics include:

Number Theory

Computer Programming

Boolean Algebra

Symbolic Logic

Linear Programming

Recreational Mathematics Topics

Additional Geometry (e.g., Construction Theory, Vector Geometry, Projective Geometry, Finite Geometry)

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