This study tested certain implied predictions regarding conceptual learning at each of four sequential levels of development: concrete level, identity level, classificatory level, and formal level. For this purpose, scaled batteries to assess the level of conceptual development of children, kindergarten through high school, were constructed and a cross-sectional/longitudinal study was begun in 1972-73. Four batteries are used in the study, one each for the following concepts: equilateral triangle, noun, tree, and cutting tool. Each battery has seven subtests, one for each of the four levels and one for each of three uses of concepts. The subtests can be scored to determine whether an individual has attained each level and each use. In 1972-73, to start the study, 50 boys and 50 girls of each grade group (kindergarten, third, sixth, and ninth) were tested. Based on preliminary results, five critical predictions were tested: (1) concepts are attained at four successively higher levels in an invariant sequence; (2) the level of concept attainment varies among children of the same age; (3) various concepts are attained by the same children at different rates; (4) concepts learned at the successively higher levels are used in understanding supraordinate-subordinate relationships; and (5) having the name of the concept and its attributes facilitates attainment of the concept and its uses. (CS)
Conceptual Development During the School Years

Herbert J. Klausmeier
V. A. C. Henmon Professor of Educational Psychology
University of Wisconsin--Madison

The research reported herein was supported by the Wisconsin Research and Development Center for Cognitive Learning, a research and development center supported in part by funds from the National Institute of Education, Department of Health, Education, and Welfare. The opinions expressed herein do not necessarily reflect the position or policy of the National Institute of Education and no official endorsement by that agency should be inferred.

Center Contract No. NE-C-00-3-0065
American psychology is changing rapidly, including the nature of experimentation and the substance of psychology itself. Piaget's (1970) genetic epistemology, Bruner's (Bruner, Olver, & Greenfield, 1966) instrumental conceptualism, the information theory of Newell and Simon (1972), Davis' (1973) creative problem solving, Gagné's (1970) cumulative learning, and Guilford's (1967) structure of intellect are representative of the trends in developmental psychology, learning, and psychological testing away from the study of muscle-twitch behaviorism toward complex learning phenomena, including concept learning, problem solving, and thinking generally. Despite these contributions to theory, there is a lesser contribution to education than might be expected (Rohwer, 1970; Klausmeier & Hooper, 1974). Also, the compartmentalization among the branches of psychology persists to the extent that even the leaders in one branch appear to be unaware of the methods and knowledge of the others (Neimark & Santa, 1975). An integration of the methods and knowledge of these areas is in order if they are to contribute as they might, both to theory formulation and to children's educational development through organized instruction during the school years.

In line with an integrative approach and dealing with only one area of human learning, an analytic descriptive model of conceptual learning and development was formulated (Klausmeier, 1971; Klausmeier, Chatala, & Frayer, 1974). It provides a theoretical framework for research on conceptual development.

---

and learning during the school years. According to this model, normally developing children and youth attain the same concept at four successively higher levels. Maturing individuals are able to progress from one level to the next as they are capable of the prerequisite mental operations of the particular level and if they have attained the concept at the prior level. In this way, the ability to learn a concept at each of the four successive levels is explained, first, in terms of the prerequisite mental operations and second, in terms of the external conditions that facilitate learning at the particular level. The initial manifestation of the mental operations at each level is presumed to be a product of both maturation and learning, or more broadly, development. The external conditions include instruction designed specifically for particular students at their particular levels of conceptual development.

As shown in Figure 1, a concept is attained at four successively higher levels. The four successive levels are concrete, identity, classificatory, and formal. Concepts once learned to a certain level may be used in various ways. Concepts acquired at only the concrete and identity levels may be used in solving simple problems which require only the relating of sensory perceptions. However, concepts acquired at the more mature classificatory and formal levels may be used in identifying newly encountered instances as examples and nonexamples of the concept. They also may be used in understanding taxonomic and hierarchical relationships of which the particular concept is a part, in understanding principles involving the concept, and in solving problems.

Acquiring the name of the concept and the names of the attributes may come at any of the four levels; however, having the name of the concept and the names of attributes is essential to attaining concepts at the formal level. Individuals
may acquire the name at about the same time they first attain the concept at lower levels but that this is not prerequisite.

The proposition that a concept is attained at the four successive levels applies to concepts that (a) have more than one example, (b) have observable examples or representations, and (c) are defined in terms of attributes. Not all concepts are of this kind. Some have only one example, e.g., the earth's moon. Some do not have observable examples, e.g., atom, eternity, soul. Still others are defined in terms of a single dimension; e.g., rough, thin, or in terms of a relationship, e.g., south, between, above. While not all four levels are applicable to these kinds of concepts, the identity level is applicable for those that have only one example; the classificatory level is for others, including those of one dimension or expressing a relationship; and the formal level is for others that have no observable, classifiable examples. With this introduction to levels and the kind of concepts under consideration, the mental operations pertaining to each level are explained.

Concrete Level

Attaining a concept at the concrete level is inferred when the individual recognizes an object that has been encountered on a prior occasion. The operations in attaining this level, as shown in Figure 2, are attending to an object, discriminating it from other objects, representing it internally as an image or trace, and maintaining the representation (remembering). The infant, for example, attends to a large red ball, discriminates it from other objects in the environment, represents the image of the ball internally and maintains the image (remembers), and recognizes the red ball when experienced later in the identical form as initially experienced. The name for the object concept may or may not be learned at this level of attainment.
Identity Level

Attainment of a concept at the identity level is inferred when the individual recognizes an object as the same one previously encountered when the object is observed from a different spatio-temporal perspective or sensed in a different modality, such as hearing or seeing. For example, the child's making the same response to the family poodle when seen from straight ahead, from the side, and from various angles is evidence of having attained the concept of poodle at the identity level. As shown in Figure 3, the operations of attending, discriminating, and remembering are involved in attainment at the identity level as they also are at the concrete level. However, whereas concept attainment at the concrete level involves only the discrimination of an object from other objects, attainment at the identity level involves both discriminating various forms of the same object from other objects and also generalizing the forms of the particular object as equivalent. Generalizing is the new operation postulated to emerge as a result of learning and maturation that makes attainment at the identity level possible.

Some psychologists such as Gagné (1970) treat concepts at the concrete and identity level as discriminations. Piaget (1970) refers to them as object concepts. The critical matter is not what they are called but to explain the internal and external conditions of concept learning.

Classificatory Level

As shown in Figure 4, the new operation at the classificatory level is generalizing that two or more things are alike in some way. The lowest level of attaining a concept at the classificatory level is inferred when the individual responds to at least two different examples of the same class of objects, events, or actions as equivalent. For example, when the child treats the family's toy poodle and the neighbor's miniature poodle as poodles,
the child has attained a concept of poodle at a beginning classificatory level. At this beginning level children seem to be able to classify, basing their classifications on some of the readily perceptible attributes of the concept; but they cannot give the basis of their classifications. It is not clear whether children use global properties or more discrete attributes of concepts at this lowest level of classifying. What is used probably varies according to children's learning styles (Kagan & Kogan, 1970) and also according to the nature of the particular concepts.

Individuals are still at the classificatory level when they can correctly classify a large number of instances as examples and others as nonexamples but they cannot define the word that stands for the concept and also cannot explain the basis of their classifying in terms of the defining attributes of the concept. At this higher phase in attaining concepts at the classificatory level, children seem to be able to discriminate some of the less obvious attributes of the concepts and to generalize correctly to a great variety of examples, some of which are very much like some nonexamples. Also, they seem to be able to make more explicit than they did earlier the basis of their classifying. In terms of Kofsky's (1966) analysis, what is included here from the beginning to the end of the classificatory level includes the following classificatory acquisition sequence of Inhelder and Piaget (1964): consistent sorting, exhaustive sorting, conservation of classes, knowledge of multiple-class membership, horizontal classification, and hierarchical classification.
Formal Level

Attainment of a concept at the formal level is inferred when the individual can give the name of the concept, can define the concept in terms of its defining attributes, can discriminate and name its defining attributes, and can evaluate actual or verbally described examples and nonexamples of the particular concept in terms of the presence or absence of the defining attributes. For example, maturing children demonstrate a concept of tree at the formal level if when shown some examples of trees, shrubs, and herbs they properly identify the trees and call them "trees"; give a societally accepted definition of tree; discriminate and name the defining attributes of tree; and evaluate how examples of trees differ from examples of shrubs and herbs in terms of the defining attributes. (Many college students cannot do all of these without further study, but some high school students who have studied biology can.) When individuals can do these things it may be inferred that they are also capable of performing the cognitive operations for the formal level which are now indicated.

As shown in Figure 5, persons may attain a concept at the formal level inductively or deductively. Whether persons use either of the inductive strategies or the deductive strategy depends on the kind of formal and informal instruction they experience, the kind of concept instances that they experience, their age, and other factors.

There are two patterns of inductive operations, as portrayed in Figure 6. One pattern involves formulating and evaluating hypotheses regarding the attributes of the concept and the other involves cognizing the attributes
that are common to the positive instances. The operations involved in the inductive hypothesis-testing strategy characterize individuals who cognize the information available to them from both examples and nonexamples of the concept. These individuals apparently reason like this: Instance 1 has land totally surrounded by water. It is a member of the class. Instance 2 has land but is only partially surrounded by water. It is not a member of the class. Therefore, lands totally surrounded by water belong to the class but lands only partially surrounded by water do not. Totally surrounded by water is one of the defining attributes of the concept. This individual has attained a partial but accurate concept of island based on experiences with only one positive and one negative instance.

A second way of inferring the concept inductively is by identifying the attributes that are common to the examples of the concept. The commonality approach is used more often than the hypothesizing approach by young children apparently because they are either incapable of getting information from nonexamples or they cannot carry out the hypothesizing and evaluating operations (Tagatz, 1967). Further, the commonality strategy is the only one possible when only positive instances of the concept are available to the individual. (Many textbooks give only one example with a verbal definition.)

As shown in Figure 7, learning a concept at the formal level deductively when given all the essential information entails assimilating this information by meaningful reception learning as described by Ausubel (1968),

---

3In explaining the formal level earlier, Klausmeier, Chatala, and Frayer (1974) subsumed the deductive operations under cognizing the common attributes.
remembering it, and then being able to use it later in identifying examples and nonexamples of the concept. Much concept learning at the formal level by upper elementary, high school, and college students follows this pattern. In their instruction, students are given the names of concepts and their attributes, verbal definitions, verbal examples, and verbal nonexamples, but no actual instances of the concepts.

Acquiring the Names of Concepts and Their Attributes

The importance of language in concept learning is widely acknowledged by American (Bruner, 1964) and Russian psychologists (Vygotsky, 1962). Having the labels of concepts enables the individual to think in symbols rather than in images. It also permits more mature individuals to attain some concepts, especially at the formal level, through language experiences in the absence of actual examples. By the present definition of the formal level the individual must know the defining attributes of the concept and must be able to communicate this knowledge. Verbalizing is normally used in this kind of communication. However, deaf individuals and others who lack speech may attain concepts at the formal level. They use other types of symbolic communication, for example, sign language. While speech, per se, is not necessary for the attainment of concepts at the formal level, some means for symbolizing and communicating the concept in the absence of examples is essential.

Methods for Measuring and Describing Conceptual Development

The preceding description of conceptual learning and development has been formulated over a period of years and is based on behavioral analyses
as described by Glaser and Resnick (1972) and a synthesis of many concept learning experiments carried out in laboratories and schools. However, certain implied predictions regarding conceptual development according to the four levels and the related uses have not been tested through research with school-age children. For this purpose scaled batteries to assess the level of conceptual development of children, kindergarten through high school, were constructed and a cross-sectional/longitudinal study was started in 1972-73.

Four batteries are used in the study, one each for the following concepts, equilateral triangle (Klausmeier, Ingison, Sipple, & Katzenmeyer, 1973a), noun (Klausmeier, Ingison, Sipple, & Katzenmeyer, 1973b), tree (Klausmeier, Marliave, Katzenmeyer, & Sipple, 1974), and cutting tool (Klausmeier, Bernard, Katzenmeyer, & Sipple, 1973). Each battery has seven subtests, one for each of the four levels and one for each of three uses of concepts. The subtests can be scored to determine whether an individual has attained each level and each use.

The design of the cross-sectional/longitudinal study as carried out in one school district is shown in Figure 8. (A partial replication is being carried out in another school district.) As shown in the design, provisions are made to determine the effects of repeated testing and of cohort groups. Major testing is provided for at one year intervals at four points in time. In 1972-73 to start the study, there were 50 boys and 50 girls of each grade group, kindergarten, third, sixth, and ninth. At the end of the study, the kindergarten children will be third-graders, the third-grade children sixth-graders, and so on. The fourth and final assessment is scheduled for completion during 1975-76.
Based on cross-sectional results of the first year that have already been reported (Klausmeier, Sipple, & Allen, 1974) and of the second year that have been analyzed, five critical predictions have been tested. The results which follow are stated in the form of principles of conceptual development rather than as statements of the prediction tested.

Principles of Conceptual Development

Principle 1. Concepts are attained at four successively higher levels in an invariant sequence.

There are five patterns of passing and failing to attain a concept at each of the four levels that are in line with the principle of an invariant sequence as stated earlier. The five patterns are to (a) fail all four levels (FFFF), (b) pass the concrete level and fail the next three levels (PFFF), (c) pass the concrete and identity levels but fail the next two levels (PPFF), (d) pass the first three levels but fail the formal level (PPPF), and finally, (e) pass all four levels (PPPP). There are 11 patterns not in line with principle. In these 11 patterns, the student fails an earlier level and then passes a later one, for example, fails concrete and passes identity or fails identity and passes classificatory. Figure 9 presents the supporting information for the three concepts for which validated assessment scales were available in the first year of the study. Ninety-two percent of all the children conformed to the five conforming pass-fail patterns for equilateral triangle, 94 percent for cutting tool, and 98 percent for noun. Data from 1973-74 presented in Table 1 indicate slightly higher percentages conforming, including for a fourth concept, tree. These very
high percentages conforming to the five pass-fail patterns provide one basis for the principle as stated.

Also in support of the principle proportion of each successively higher grade group attaining each successive level increased. This conclusion applies to all three concepts as may be inferred from Figures 10a, 10b, 10c, and 10d. Information in Figure 10a shows that the percentage of children attaining the concrete level of each of the three concepts increased as a function of the grade group. For example, about 28 percent of kindergarten children, 96 percent of the third-graders, 98 percent of the sixth-graders, and 99 percent of the ninth-graders fully attained the concept noun at the concrete level. This was the most difficult of the three concepts at the concrete level, particularly for the kindergarten children. One minor exception to the trend is noted. Slightly fewer sixth-graders than third-graders attained the concrete level of equilateral triangle.

Figure 10b shows a consistent developmental trend at the identity level of attainment, Figure 10c at the classificatory level, and Figure 10d at the formal level. The developmental trend is obvious at the formal level where kindergarten children attained equilateral triangle or noun at the formal level, and the spread among the successive grades was quite large and consistent for the three concepts.

Notice that the concepts are drawn from three different subject fields and that 70 percent or more of the kindergarten children attained the concrete and identity levels of equilateral triangle and cutting tool, the two concepts for which there are actual instances or readily constructed instances
in the immediate environment. This suggests that little instruction beyond the kindergarten might be needed at these two levels for this kind of concept. On the other hand, very few kindergarten children attained noun at these levels; further, attainment of noun at the formal level was very low by the sixth- and ninth-grade students. The form class, nominals, and the word for the class of things, noun, have been created by man. While the child early in life says words that are classed as nouns, the words themselves stand for many different kinds of things, ideas, events, etc. This apparently leads to the difficulty experienced by the children in identifying certain words as nouns and others as not nouns. It is probable that the ninth-graders who did not attain noun at the classificatory or the formal level will never do so without excellent instruction later in their school lives. However, such instruction could probably be arranged if a school so desired (Nelson & Klausmeier, 1974).

Principle 2. The level of concept attainment varies among children of the same age.

The variability in the levels of attainment is visualized in earlier Figure 10d. It is seen there that about 17 percent of the third-grade children attained equilateral triangle at the formal level; therefore 83 percent did not; similarly, about 93 percent of the ninth-grade students attained cutting tool at the formal level and about 7 percent did not. A similar pattern of variability is being found with all the concepts each year of the study. It is interesting to note, too, in Figure 10d that some third-graders attain a higher level than do some ninth-graders.
Principle 3. Various concepts are attained by the same children at different rates.

Reference back to Figures 10c and 10d shows that attainment of cutting tool at the classificatory and formal levels precedes attainment of equilateral triangle and noun. In fact, only 30 percent of the ninth-graders had attained noun at the formal level, whereas 90 percent had attained cutting tool at that level. In general, concepts for which there are few or no perceptible instances, e.g., noun, are attained to a certain level, e.g., formal, much later than are other concepts for which there are perceptible instances, e.g., cutting tool.

Principle 4. Concepts learned at successively higher levels are used more effectively in understanding supraordinate-subordinate relationships, in understanding principles, and in solving problems.

Table 2 shows the percent of children who attained each of the four levels as their highest performance and who then also passed the three uses. Without exception, a higher percent of the students who passed the formal level in comparison with those who passed only the classificatory level, performed each of the uses better. Further, the difference in the actual size of the percentages is large. For example, in connection with equilateral triangle the percentages of those who attained the formal level in comparison with those who attained only the classificatory level were as follows: 34 percent and 8 percent for understanding the supraordinate-subordinate relations, 43 percent and 5 percent for understanding the principles, and 34 percent and 3 percent for passing the problem-solving test. There were two minor exceptions to the second conclusion that children who attained a concept to only the concrete or identity level would be able to use the concept only in solving simple perceptual problems.
In line with the principle also, the proportion of children of successively higher-grade groups who mastered each concept use increased as shown in Figures 11a, 11b, and 11c. There was only one exception to this finding. Fourteen percent of the third-graders passed the supraordinate-subordinate test for equilateral triangle and only 12 percent of the sixth-graders did. Despite this one minor exception, the increase across grade levels on the various uses for every concept is marked. For example, with respect to understanding supraordinate-subordinate relations involving cutting tool the increase is as follows: kindergarten—8 percent; third grade—43 percent; sixth grade—66 percent; and ninth grade—82 percent.

It is worth emphasizing again that assisting students to attain concepts at the formal level is a worthy educational objective, because attainment at this level facilitates understanding relationships among concepts that comprise a taxonomy, understanding principles that involve the particular concept and other concepts, and in solving problems involving the particular concept and/or principles.

Principle 5. Having the names of the concept and its attributes facilitates attainment of the concept at the various levels and also the three uses of the concept.

Table 3 shows the high positive correlations between the vocabulary scores and the scores for the four levels, the three uses, and the combined levels and uses. Inasmuch as these positive relationships were found for concepts drawn from three different subject fields, similar results should be expected with other concepts.
A Note on Instructional Conditions to Facilitate Conceptual Development

Inasmuch as the internal conditions of concept learning vary according to the four levels, instructional conditions must also vary if the instruction is designed to utilize the available capabilities of the learner and be facilitative of learning. At the classificatory and formal levels, the following conditions have been identified as important in the design of instruction: the proper use of rational sets of examples and nonexamples (Klausmeier & Feldman, 1975, in press; Markle & Tiemann, 1970; and Tennyson, Woolley & Merrill, 1972), the proper matching of the examples and nonexamples (Tennyson, Woolley, & Merrill, 1972), the use of definitions (Feldman & Klausmeier, 1974), the use of cues (Frayer, 1970), and teaching a strategy (McMurray, Bernard & Klausmeier, 1974). It appears that knowledge about conceptual learning and instruction is sufficiently complete that instructional materials may be designed with precision to teach concepts.

Conjectures Regarding the Dimensions of Conceptual Learning and Development

If the longitudinal data support the cross-sectional data collected and analyzed thus far, what kind of view of development will be forthcoming? Will it correspond more closely to Piagetian stage theory (1970), to Bruner's instrumental conceptualism (Bruner, Olver, & Greenfield, 1966), or to Gagné's cumulative learning theory (Gagné, 1968, 1970)? Before this question can be answered it is essential to examine the possible attributes of stages of cognitive development.

According to Flavell (1971), stages in cognitive development could be conceived as analogous to metamorphosis in insects, for example, the butterfly: egg, larva, pupa, adult butterfly. This concept of stages is shown in
Figure 12. Let us think of all the operations, concepts, classification skills, and other cognitive items that comprise each stage as the totality of the stage and relate these items in their totality to the metamorphosis analogy.

Four conclusions follow regarding the cognitive items. First, the items interact with one another in specified ways during each stage in the course of being utilized by the individual; they are not isolated and unrelated. It is appropriate to describe them as organized cognitive structures of the individual in the same sense that there is a distinct and differentiated structure at the larva and the pupa stages of the butterfly. Second, the items, as organized into structures, are qualitatively different rather than just quantitatively different at each successive stage from those defining the previous stages of cognitive development. The items and structures are truly new and different structures rather than being improved versions of what had already been achieved. Third, each individual cognitive item functions at its asymptotic, mature level of proficiency as soon as it functions at all. For example, children in the preoperational stage this week are incapable of conservation. Next week they will be in the concrete operations stage and will conserve as well as they ever will during the stage of concrete operations. Fourth, all the items that define any particular stage make this abrupt, fully mature transition simultaneously. In line with these last two conclusions, therefore, the child abruptly enters the stage of concrete operations in full command of each operation, and is immediately capable of all the operations that define the stage.

Flavell (1971) does not accept the metamorphosis analogy to cognitive stages, but he does believe that stage is a useful theoretical construct in
the study of human cognitive development. Flavell's conception of stages is more like that shown schematically in Figure 13. The curves in Figure 13 imply a qualitative change in the maturing individual's repertoire of classification skills, concepts, principles, etc., at the beginning of each stage. But the items that define each stage mature gradually, rather than abruptly. The various items of any particular stage do not achieve their mature level until long after the end of the stage that marked their initial appearance.

Flavell interprets Piagetian stages as in Figure 13b and indicates that Piaget himself probably accepts 13a. It is probable that each of the three patterns is accepted by some Americans.

In connection with the rate at which various items or dimensions of a particular stage mature, Flavell depicts three possible patterns, as shown in Figures 14a, 14b, and 14c. Some items may emerge at about the same time but mature at different rates and draw farther apart (Figure 14a). Others may emerge at different points in time and reach their final level at the same time (Figure 14b). Still others may emerge at different points in time, mature at parallel rates, and therefore also reach full maturity at different times (Figure 14c). There is no empirical information to support any of these three patterns.

Bruner's (1966) treatment of enactive, ikonic, and symbolic representation indicates that these three means of representation emerge during the first years of life according to the order as given; however, rather than each means dropping out successively, it remains throughout life. Specifically related to classifying, Bruner and others (1966) identified the modes that persons of various ages, including 6 through 19, use in classifying things and events.
Figure 15 shows the percentages of groupings based on various modes. Six-year-olds classify according to perceptual properties more than do older children, but, equally important, all age groups thereafter still do some classifying on the basis of perceptual attributes or properties—colors, sizes, shapes, and places of things. After age 9, a sharp decrease occurs in arbitrary functional classification. However, total functional classification, which includes both intrinsic and arbitrary, increases to age 12, drops off slightly, and then increases to age 19. In other words, as arbitrary functional decreases, intrinsic functional increases, resulting in a total functional increase from about 49 percent of all responses at age 6 to 73 percent at age 19. Regardless of the specifics, these developmental curves and others identified by Bruner (Bruner, et al., 1966) do not support any of the possible stage patterns depicted by Flavell (Figure 12).

Gagné (1968) defined both development and learning as changes in capabilities, the difference between learning and development being the amount of time required to bring about the changes. The capabilities resulting from learning and development accrue incrementally and cumulatively, possibly additively, as successive capabilities are attained. Gagné apparently presumes that there are no particular points in life, corresponding to stage endings and beginnings, that determine when a person is ready to learn or to develop any particular capability. Rather, the present capabilities of individuals are the determinants of what they can start to learn or develop next.

I am uncertain yet about the dimensions of conceptual learning and development across the school years. However, our cross-sectional data, corresponding to Bruner's viewpoint, indicate that concepts are learned to
successively higher levels, starting at the concrete level in early childhood and continuing into adulthood to the formal level. Whereas Bruner specified three modes of acting upon the environment and representing experience as sequentially emerging prior to age 5, the new cognitive operations that are essential for each successive level of concept attainment appear to emerge with maturation and learning across a long time interval, until much later in life. The specific cognitive operations and levels referred to here are discriminating among objects—concrete level, generalizing that the same object is alike in some way—identity level, generalizing that different objects and events are alike in certain attributes—classificatory level, and hypothesizing and evaluating the attributes of concepts and evaluating examples and nonexamples on the basis of their defining attributes—formal level. Further, as noted earlier, there is much variability among concepts, for example cutting tool and noun, as to when in the life span of the same individual the particular operations on the particular contents emerge, that is, attributes that are readily perceptible, as is the case with tree and cutting tool, or attributes that are inferred by reasoning, as is the case with noun.

In closing it would seem that any theory of cognitive development that would be helpful in understanding children's conceptual development during the school years must take into account the measurement of cognitive development, the short-term learning conditions related to the level at which the student is regarding particular classes of concepts, and the long-term emergence of the qualitatively different cognitive operations that are essential for attaining concepts at each of the particular levels.
References


McMurray, N. E., Bernard, M. E., & Klausmeier, H. J. Lessons designed to teach fourth grade students the concept equilateral triangle at the formal level of attainment. Madison, Wis.: Wisconsin Research and Development Center for Cognitive Learning, Practical Paper No. 14, 1974.


Tennyson, R. D., Woolley, F. R., & Merrill, M. D. Exemplar and nonexemplar variables which produce correct concept classification behavior and specified classification errors. Journal of Educational Psychology, 1972, 63(2), 144-152.

Figure 1. Levels of concept attainment, extension and utilization.
Attending to perceptible features of an object

Discriminating the object from other objects

Remembering the discriminated object

Acquiring the concept name

Remembering the concept name

Figure 2. Cognitive operations in concept attainment at the concrete level.
Attending to perceptible features of an object
Discriminating the object from other objects
Remembering the discriminated object
Generalizing that two or more forms of the thing are the same object

Attending to perceptible features of an object
Discriminating the object from other objects
Remembering the discriminated object
Generalizing that two or more forms of the thing are the same object

Acquiring the concept name
Remembering the concept name

Figure 3. Cognitive operations in concept attainment at the identity level.
Attending to the less obvious attributes of examples of the class of objects

Discriminating the example from nonexamples

Remembering the discriminated example

Generalizing that two or more forms of the example are the same thing

Generalizing that two or more examples are equivalent and belong to the same class of things

Acquiring the concept name

Remembering the concept name

Figure 4. Cognitive operations in concept attainment at the classificatory level.
Figure 5. Kinds of operations and strategies of concept attainment at the formal level.
Figure 6. Cognitive operations and inductive strategies of concept attainment at the formal level.
Figure 7. Cognitive operations and deductive strategies of concept attainment at the formal level.
(Table entries are approximate mean ages)

<table>
<thead>
<tr>
<th>Cohort</th>
<th>1972-73</th>
<th>1973-74</th>
<th>1974-75</th>
<th>1975-76</th>
</tr>
</thead>
<tbody>
<tr>
<td>1967 6 (N=100)</td>
<td>7</td>
<td>8*</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>1968 [Kindergarten]</td>
<td>6(N=40)</td>
<td>7*</td>
<td>8* (Cohort effect group)</td>
<td></td>
</tr>
<tr>
<td>1969 7(N=40)</td>
<td>6*</td>
<td>7* (Cohort effect group)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1967 8*</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1967 6*</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1967 7(N=40)</td>
<td>9 (N=100)</td>
<td>10*</td>
<td>11*</td>
<td></td>
</tr>
<tr>
<td>1965 [Third Grade]</td>
<td>9(N=40)</td>
<td>10*</td>
<td>11*</td>
<td></td>
</tr>
<tr>
<td>1964 10(N=40)</td>
<td>9*</td>
<td>10*</td>
<td>11*</td>
<td></td>
</tr>
<tr>
<td>1966 11*</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1967 12(N=100)</td>
<td>13</td>
<td>14</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>1961 [Sixth Grade]</td>
<td>12(N=40)</td>
<td>13*</td>
<td>14*</td>
<td></td>
</tr>
<tr>
<td>1962 13(N=40)</td>
<td>12*</td>
<td>13*</td>
<td>14*</td>
<td></td>
</tr>
<tr>
<td>1963 14*</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1961 15(N=100)</td>
<td>16</td>
<td>17*</td>
<td>18*</td>
<td></td>
</tr>
<tr>
<td>1959 [Ninth Grade]</td>
<td>15(N=40)</td>
<td>16*</td>
<td>17*</td>
<td></td>
</tr>
<tr>
<td>1958 16(N=40)</td>
<td>15*</td>
<td>16*</td>
<td>17*</td>
<td></td>
</tr>
<tr>
<td>1960 17*</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1958 15*</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1958 18*</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 8. Sampling design for the longitudinal descriptive study.
*These groups will not be continued if cohort and practice effects are not found after the 1st year. If effects are found, decisions about continuing will be made after data are analyzed.
Table 1

Proportion of Total Subject Population Conforming to Predicted Pass-Fail Patterns of Attainment: Comparing the Four Concepts

<table>
<thead>
<tr>
<th>Pass-Fail Sequence</th>
<th>Equilateral Triangle (N=351)</th>
<th>Cutting Tool (N=349)</th>
<th>Noun (N=362)</th>
<th>Tree (N=354)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FFFF</td>
<td>.01</td>
<td>.00</td>
<td>.07</td>
<td>.00</td>
</tr>
<tr>
<td>PFFF</td>
<td>.01</td>
<td>.00</td>
<td>.12</td>
<td>.01</td>
</tr>
<tr>
<td>PPFF</td>
<td>.14</td>
<td>.06</td>
<td>.41</td>
<td>.15</td>
</tr>
<tr>
<td>PPPF</td>
<td>.56</td>
<td>.34</td>
<td>.32</td>
<td>.44</td>
</tr>
<tr>
<td>PPPPP</td>
<td>.25</td>
<td>.57</td>
<td>.08</td>
<td>.37</td>
</tr>
<tr>
<td>Total</td>
<td>.97</td>
<td>.97</td>
<td>1.00</td>
<td>.98</td>
</tr>
</tbody>
</table>

Source: Klausmeier, Sipple, and Allen (In press).
Figure 9. Percentage of Children Conforming to a Predicted Invariant Sequence in Attaining Concepts at Four Successive Levels (Source: Klausmeier, Sipple, & Allen, 1974.)
Figure 10a

Figure 10b

Figure 10c
Figures 10a, 10b, 10c, 10d

Percent of Each Grade Group Fully Attaining the Four Concept Levels of Equilateral Triangle, Cutting Tool, and Noun
(Source: Klausmeier, Sipple, & Allen, 1974.)
Table 2

Relationship Between Attainment Levels and Uses of Equilateral Triangle, Cutting Tool, and Noun

<table>
<thead>
<tr>
<th>Concept Uses</th>
<th>Concrete as Highest</th>
<th>Identity as Highest</th>
<th>Classificatory as Highest</th>
<th>Formal as Highest</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>S-S Pr P-S*</td>
<td>S-S Pr P-S</td>
<td>S-S Pr P-S</td>
<td>S-S Pr P-S</td>
</tr>
<tr>
<td>Equilateral Triangle</td>
<td>.00 .00 .00</td>
<td>.07 .00 .00</td>
<td>.08 .05 .03</td>
<td>.34 .43 .34</td>
</tr>
<tr>
<td>Cutting Tool</td>
<td>.00 .00 .00</td>
<td>.24 .03 .16</td>
<td>.27 .07 .20</td>
<td>.75 .48 .73</td>
</tr>
<tr>
<td>Noun</td>
<td>.00 .00 .00</td>
<td>.00 .01 .24</td>
<td>.02 .18 .55</td>
<td>.11 .86 .93</td>
</tr>
</tbody>
</table>

Source: Klausmeier, Sipple, & Allen, 1974
*S-S = Understanding Supraordinate-Subordinate Relations
Pr = Understanding Principles
P-S = Solving Problems
Figures 11a, 11b, 11c
Percent of Each Grade Group
Fully Attaining the Three Uses of Equilateral Triangle, Cutting Tool
and Noun
(Source: Klausmeier, Sipple, & Allen, 1974.)
Table 3
Correlations for Vocabulary Scores and Scores on Levels of Concept Attained, Uses of Attained Concepts, and Combined Levels and Uses: Equilateral Triangle, Cutting Tool, and Noun

<table>
<thead>
<tr>
<th>Concept</th>
<th>Four Concept Levels</th>
<th>Three Concept Uses</th>
<th>Combined Levels and Uses</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equilateral Triangle</td>
<td>.57</td>
<td>.56</td>
<td>.70</td>
</tr>
<tr>
<td>Cutting Tool</td>
<td>.51</td>
<td>.43</td>
<td>.52</td>
</tr>
<tr>
<td>Noun</td>
<td>.67</td>
<td>.75</td>
<td>.79</td>
</tr>
</tbody>
</table>

Source: Klausmeier, Sipple, & Allen, 1974
Figure 12. Metamorphosis analogy to stages of cognitive development. (Based on Flavell, 1971, p. 426.)

Figure 13a and 13b. Possible patterns of stages of cognitive development. (Based on Flavell, 1971, p. 426.)
Legend: $X_1$: Reading performance  
$X_2$: Mathematics performance

Figure 14. Possible patterns of relationships among dimensions of development. (Based on Flavell, 1971, p. 437.)
Figure 15. Percentages of students using different modes of classifying, or grouping. (Bruner, Olver, & Greenfield, 1966, p. 73)