With reference to data from a Swedish investigation into learning difficulties in mathematics, an analysis is presented of models and taxonomies applied to special education problems. The author suggests that such developmental cognitive theories as those of J. Bruner, J. Piaget, and R. Gagne are suitable as a starting point for a taxonomy in mathematics for low achievers but contain inconsistencies. Proposed is a three dimensional model which relates the child's stage of development, instructional objectives and activities, and mathematics content categories.
Magne, O. Educational technology in special education. Didakometri
(Malmö, Sweden: School of Education), No. 43, 1974.

With reference to data from a Swedish investigation into learning difficulties in mathematics, an analysis is presented of problems in educational technology in special education. It seems necessary to take as a point of departure the set of broad aims accepted by the government. In the following goal-analysis, the teacher ought to capitalize on the existing aptitudes of pupils in special education, but also attempt to improve their aptitudes.

When we confront some of the usual taxonomies with findings in research on the attainment of low-achievers, a number of inconsistencies seem to arise. This result indicates that we ought to construct alternative taxonomies. The author suggests that the developmental cognitive theories of Bruner, Piaget, Vygotsky etc., are suitable as a starting-point for a taxonomy in mathematics for low-achievers. A taxonomic model has been developed from these assumptions.

Keywords: Educational technology, mathematics, special education, taxonomy.
A Swedish project on individually given remedial teaching in mathematics

During the years 1963-70 a Swedish project was carried out on individually given remedial teaching for children with learning difficulties, mainly in mathematics (dyscalculia). The Swedish National Board of Education sponsored the project and gave considerable economic aid to local authorities who arranged this form of special education. I myself was responsible for the planning and evaluation of the project.

The experiences of the project have been summarized in several reports, both on the particular topic of how low-achievers learn mathematics, and in various other more general respects, such as the efficacy of special education. In a book in Swedish (Learning Difficulties in Mathematics), published half a year ago, I have also described a rationale of the remedial teaching in mathematics clinics and in ordinary classes (Magne, 1973).

Background of children with learning difficulties in mathematics

There are, compared with writing and reading, very few studies on the educational background and conditions of children with learning difficulties in mathematics. In addition to the investigations I have conducted, I would like to mention one by Ross in the United States (1964), the ones by Schönell (1957) and Lytton (1961 and 1967) from English schools and Weinschenk's (1970) from Germany. I can also mention the 35th Yearbook of the National Council of Teachers of Mathematics, called The Slow Learner in Mathematics.

If I exclude some specific works from neurophysiology, I should like to summarize these studies in the following way.

1. Low-achievers in mathematics have, frequently but not always, an inadequate learning aptitude. One author has said: "Intelligence appears to play by far the largest part in mathematical attainments of every kind" (Barakat, 1950).
In some countries, but not in Sweden, it has been shown that parents of low-achieving children tend to belong to the lowest socio-economic classes.

Low-achievers regularly tend to display symptoms of emotional disorders, or a disturbed working disposition, involving lack of interest, home or school maladjustment, short attention span, lack of persistence, distractivity, limited initiative, insecurity, or in some cases anxiety.

Abnormal physical or social conditions have been found, but seem to be of less importance than intellectual handicaps or emotional disturbances.

Among the low-achievers there are a number of under-achievers, that is children with an adequate learning aptitude. Under-achievers in mathematics are few compared with the number of under-achievers in reading and writing. The proportion is less than five percent of any age-group.

Under-achievers display the same symptoms as low-achievers.

There are certain findings indicating that under-achievement in mathematics may be caused by hereditary conditions.

Thus arithmetic low-achievement and under-achievement appear as complex and multi-factorial disabilities. I would like to stress this observation, since we have found it important when constructing programs for students with learning difficulties in mathematics. No simple taxonomic technique seemed to be valid.

In addition, I think we have to accept that most students with learning handicaps have a similar complex and multi-factorial background. Therefore, when they come to the particular form of special education, which has been found to suit them, children ought to be treated with respect not to one handicap, but to several.

A few words on theories

We now come to the question of theories for special education. Several conflicting background theories have been suggested, although none of these is particularly well-adapted to the treatment of those handicapped in this way. In the Swedish government report on the inner organization of the school (SIA, 1974), several general theories on educational diagnosis are discussed. These are called
the medical model
the psychometric model
the administrative model
the sociological model and
the pedagogical model.

I think we must confess that in many cases we have developed remedial teaching techniques without first having carefully considered the specific problems involved. That is to say, we have developed a remedy without sufficient knowledge of the disease. Or, we have taken steps to solve a problem which nobody has distinctly formulated. We have followed our hearts rather than our heads. This may stem from the fact that in many respects we lack the necessary research background to be able to describe a comprehensive theory for coping with all types of special education or treatments of personality handicaps. Perhaps we only have a theory for treating a small number of handicaps, and only one theory for each case.

The assumption we did adopt for our mathematics project in Sweden was based on findings in developmental cognitive studies, e.g. by Bruner, Piaget and Vygotsky. We know, for instance, that it is extremely difficult to raise a low-achiever or under-achiever to the level of his or her peers, but that this may be achieved, if the training is systematic and intensive. This assumption, which we called the intensity approach, is based upon researches by several child psychologists. I will specially mention the learning theory of Piaget, who has advanced what has been called a conflict learning model. Similar learning theories, suitable for remedial education, have been suggested by Freire (1970) and Witty (1949). Lack of time prevents me from going into further details to describe these theories now.

For the effective treatment of low-achievers or under-achievers in mathematics, we found it necessary temporarily to intensify the special education in a way which can not be done in ordinary classes.

We also found it necessary to give priority to higher cognitive processes such as discrimination, concept formation, problem solving, and structuring. Simple respondent or operant learning - or simple labelling - did not seem to be effective according to our studies. Finally, there were very strong indications that the remedial training we were engaged in failed to give results, unless we paid particular attention to the childrens' individual needs, attitudes, feelings, motives, interests and so on.

This theory may be adapted to other types of special education. But
apparently other, and partly conflicting, theories have been suggested or used for other handicaps (cf. for instance Johnson & Myklebust, 1967 and Kylen, 1974). I think, therefore, that for the time being, we shall have to apply different theories to the treatment of varying types of special education. One comprehensive theory does not seem to be possible today.

According to this theory, the systematic treatment of a child with learning difficulties in mathematics may be described in the following ten points:

1. Diagnosis
2. Teaching with increased teacher resources
3. Treatment of the child's mathematics difficulties on an individual basis
4. Simultaneous treatment of the child's other behavior problems
5. Adaptation of content, instructional aids, and strategies to the behavior level of the child
6. Foundation of learning in the child's needs and interests
7. Personal involvement by child through activity
8. Formation of experiences, often as re-schooling
9. Development of concepts and structures and attitudes
10. Consolidating of skills after formation of concepts and structures

The utility of educational technology in special education

We all accept that an efficient learning situation is one in which well-defined goals are established. From this point of view it is useful and often necessary to state general and afterwards also behavioral objectives. A systematic treatment of this kind is often called educational technology.

Sometimes, however, the use of educational technology in schools is limited to lists of intellectual behavioral objectives. This is no sensible way to apply research findings to school learning practices. Before we go to specific aims, it seems essential to discuss a set of broad aims. It is, for instance, important to remember not only to capitalize on the existing aptitudes of the students in special education, but also to attempt to improve their aptitudes.

Cronbach (1967) has suggested three ways of adapting patterns of education to individual differences as indicated in the third column of table 1.
Table 1. Patterns of educational adaptation to individual differences.

<table>
<thead>
<tr>
<th>Educational goals</th>
<th>Instructional treatment</th>
<th>Possible modifications to meet individual needs</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Fixed</td>
<td>Fixed</td>
<td>1a. Alter duration of schooling by sequential selection.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1b. Train to criterion on any skill or topic, hence alter duration of instruction.</td>
</tr>
<tr>
<td>(2) Options</td>
<td>Fixed within an option</td>
<td>2a. Determine for each student his prospective adult role and provide a curriculum preparing for that role.</td>
</tr>
<tr>
<td>(3) Fixed within a course or program</td>
<td>Alternatives provided</td>
<td>3a. Provide remedial adjuncts to fixed &quot;main track&quot; instruction.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3b. Teach different pupils by different methods.</td>
</tr>
</tbody>
</table>

Addition by the author:

(4) Options within a course | Alternatives provided | 4a. Provide individual curricula, individual help, and/or different methods. |

For special education the third pattern "Fixed goals, treatment options" seems to be the most interesting one. This pattern means according to Cronbach "adaptation by erasing individual differences". In this case we accept a set of goals within a given course of program, but there are alternative instructional treatments.

I would like to comment on Cronbach's table that it is possible to make further additions or amendments, e.g. the one suggested as a fourth pattern. In some Swedish schools this pattern is being implemented since a year ago.

When we continue to the question of the utility of behavioral objectives, we meet great difficulties, because of the extremely complex symptom picture in special education. It is not helpful and, in fact often harmful, just to list a number of so-called "cognitive objectives". (Bloom, 1956; Gagné, 1970). Let us, for instance, analyze the "mastery curriculum" by Bloom and associates (1971). This list of objectives consists mainly of low-level objectives in the cognitive domain. Higher-level objectives in this domain and essentially all objectives in the psycho-motor or affective domains are lacking.

One further remark may be added. According to the following line of argument it is wrong to base mathematics learning on the usual taxonomies (e.g. Bloom's or Gagné's) when we consider low-achievers or under-achievers. Among other things, I will suggest that in special education.
other categories of learning are more important than the ones included in these taxonomies.

Let us have a look at one of the more comprehensive tables of specifications. This one has been provided by Wilson (1971).

Table 2. Two-dimensional schema of objectives (Wilson, 1971).

<table>
<thead>
<tr>
<th>CONTENT</th>
<th>BEHAVIORS</th>
<th>INTEREST AND ATTITUDES</th>
<th>APPRECIATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number Systems</td>
<td>Computation</td>
<td>Comprehension</td>
<td>Application</td>
</tr>
<tr>
<td>Algebra</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Geometry</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The essential idea in this table of specifications is that behavioral objectives of mathematical instruction can be classified in two ways:

1. by categories of mathematical content and
2. by levels of behavior.

It can be instructive to test the validity of a table such as Wilson's with a simple case study. I will give the following example of what some authors would call a simple labelling behavior without understanding.

Example: A boy of 9 tries to solve an addition problem 46 + 13 as follows.

He uses finger counting,

Step 1: 46 + 13
Step 2: 46
Answer: 46 + 13 = 59 (Correct?)

Which category of the table does this attempt represent? Computation? Comprehension? Application? Analysis?

I would like to say: ALL! The outcome of this attempt, together with other items, may be interpreted in the following way, using the vocabulary in Wilson's table.

This boy was not able to add correctly, although the answer happens to be correct. We may call his performance an outcome of a cognitive disposition acquired through misinterpretations of conceptual patterns, and this mental disposition yielded wrong results in addition. The obvious interpretation is that his performance is the consequence of higher processes, but not of simple labelling. He failed in his computation because
he is wrong in his comprehension, application, and analysis of addition
corcepts.

Therefore, I can not accept the view, represented in some hierarchical
taxonomies (such as those of Bloom, Gagné - and Wilson). Perhaps com-
putation should be considered the highest level and analysis the lowest one.
Application seems to be so closely connected to every other cognitive
trait or situation, whether its cognitive level is high or low, that I will put
the question of whether application can be looked upon as an independent
category. In the same way analysis and comprehension seem to be involved
in every learning, problem-solving or thinking situation, irrespective of
its complexity.

For the conditions we studied in our mathematics project, we came to
the conclusion that we preferred a taxonomy, or classification, organized
by levels of cognitive complexity. We tried to formulate a taxonomy which
should have theories of Bartlett, Bruner, and Piaget as a starting-point.
These theories recognize children's stages of cognitive development as
well as their strategies of problem-solving.

Thus, it is my opinion that children's mathematical experience should
match their stages of development in mathematical ideas. There are two
sides to the problem: one is to be able to recognize the performance level
a particular child has reached with respect to a certain area of experience,
and the other is to know what kind of activity corresponds to this stage of
development. It is essential that both these factors are known, and only
then can "matching" be done.

This interpretation of our mathematics project is summarized in
table 3. I have also given an example of a possible case of the achieve-
ment of a non-treated low-achiever. I will not go into detail now, but I
would like to stress that a taxonomy of this type seems to be much better
suited to the data from this project than the conventional taxonomies.

This taxonomic model can be developed to include further character-
istics of mathematics instruction, e.g., the mathematical content, the in-
structional aids, and evaluation techniques. The model will then be multi-
dimensional, of course. If this is done, Cronbach's teaching patterns will
be covered.

It has been shown by Weaver (1970) that in contemporary mathematics
programs different and partly conflicting types of content characteristics
can be identified. This may also be the case for other school subjects.
Such types of characteristics of mathematics programmes embrace for
instance
Table 3. General objectives of mathematics instruction for low-achievers and under-achievers (Magne, 1974).

<table>
<thead>
<tr>
<th>The Objectives and Activities Corresponding to the Stage of Development</th>
<th>The Child’s Stage of Development</th>
</tr>
</thead>
<tbody>
<tr>
<td>Needs, Attitudes, Interests</td>
<td>Intuitive Perception and Thinking</td>
</tr>
<tr>
<td>Observation, Exploration, Ordering of Observations</td>
<td>Possible Achievement of a Non-treated Low-achiever</td>
</tr>
<tr>
<td>Basic Concepts</td>
<td></td>
</tr>
<tr>
<td>Patterns</td>
<td></td>
</tr>
<tr>
<td>Computational Skills</td>
<td></td>
</tr>
<tr>
<td>Communication</td>
<td></td>
</tr>
<tr>
<td>Discrimination Between Methods or Models</td>
<td></td>
</tr>
<tr>
<td>Critical and Creative Interpretation</td>
<td></td>
</tr>
</tbody>
</table>

If we consider only the usual conventions, the domain of school mathematics could be represented by the general objectives according to Table 3 with the added dimension of mathematics content. This is made explicit in the following figure, where the content categories are taken from Weaver.

It seems possible that this taxonomy or similar taxonomies are suitable for other forms of remedial teaching. Finally, it is my belief that a taxonomy of this type better fulfills our aims to find suitable objectives for children learning mathematics in ordinary classes.
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