The technique originating from the study of ecological systems for the examination of dynamic educational systems over time is utilized. The technique is based on the theory of energy exchanges in interactions between variables. Using this technique, interactions among variables can be specified and simulated over time using an analog computer. Variables can then be examined under controlled conditions. The procedure was found to duplicate known findings from Atkinson's achievement motivation theory. Over long time periods, changes not hypothesized appear among variables which may have ramifications for empirical research procedures. (Author)
A PROCEDURE FOR THE INVESTIGATION OF HYPOTHEZIZED RELATIONSHIPS AMONG DYNAMIC VARIABLES OVER TIME

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INTRODUCTION

Studies in education of necessity deal with systems which are both dynamic and time dependent. Usually, data are collected at some specified time period and the system described as it appears at the time of the study. Although valuable information is gained, the dynamic nature of the system remains largely unexplored and the dynamics of the system are often ascertained from extrapolation from knowledge about the system at a particular time. A similar situation exists in fields of research other than the social sciences.

In an attempt to understand dynamic systems in ecology, Odum (1971) has developed a technique for the investigation of the systems encountered in that field. This technique allows for the specification of the mechanism by which components of the system interact. As well, relationships of the components to one another may be specified as linear, non-linear and may include two or more components. These relationships may be examined over time and the effects which components have on each other due to feedback can also be included and examined.

The system may be represented mathematically by differential equations and modeled, thus indicating the changes in the components over time as they vary in accordance with the relationships specified for the system. In ecological studies, the simulation not only indicates changes in the components of the system, but also functions as an empirical predictor. It is the purpose of this paper to indicate how the technique can be used in educational situations, to report initial attempts at the utilization of the method in educational situations, and to indicate studies which might enlarge upon initial studies in order to develop empirical models. Emphasis throughout the paper will be on adapting the technique as it exists in ecology to methods useful in educational modeling.
GENERAL PRINCIPLES

The energy systems approach to the study of dynamic systems (Odum, 1971) is based on energy and the laws of energetics. The advantages derived from using energy as a basis are several. First, the laws of energetics are well understood and precisely defined. Second, they are empirical and predictive. Third, there are no known exceptions to the basic laws of energetics. The use of energy as a basis for understanding psychological or other social science systems is not as rash as it may appear at first thought. All human functions are energy dependent, whether they be physical functioning or the complex types of mental processes operating by electrochemical means. Information itself is an energy form in that it is able to facilitate numerous interactions, in fact, the study of information in terms of energetic laws has met with success in bioenergetics and communications engineering (Gatlin, 1972).

There is perhaps a more basic tie between the laws of energetics as used in physical sciences and their use in the social science context. Energy is operationally defined—it is defined not by what it is but by what it does. Thus the laws of energetics pertain to what energy does and the rules by which it operates rather than defining energy itself. In a similar manner, psychological constructs are also operationally defined. Mental ability, for example, is defined by how the individual performs on inferential measures, or motivation is defined by its effect on performance. The definition becomes limited by the factors which influence each of the psychological constructs. Neither construct is normally defined by what it actually is—some combination of neural networks and electrochemical activity.
Since the laws of energetics determine the confines of the system within which energy operates, and as such are true for all forms of energy, they become principles of the system which explain and limit the feature under consideration. It may be that energetic laws stated in terms more applicable to psychological considerations can serve as the 'ground rules' for understanding how the components of a psychological system function. These principles then become the rules which govern the operation of any system and are not necessarily specific to energy considerations alone.

The principles underlying the procedure can then be restated as adaptations from Odum (1971) to make them more readily applicable to psychology or social science. In ecology, the system is described symbolically. This allows a great deal of information to be represented, as well as aiding in the development of the differential equations which represent the model. Therefore, the symbolism will be included, where appropriate, to indicate how the principles are incorporated into the symbolism representing the system. The general principles are presented in the following discussion. There has been no attempt to present the analog of the principle which exists in energetics, since this aspect cannot be dealt with adequately within the scope of this paper.

1. The components of the system, more commonly referred to as variables, are grouped into two categories. If the variable is one which is not a part of the system being considered but influences the behavior of the system from outside the system, it is termed a forcing function. For example, if the system being examined is one which involves some cognitive processes in a learning situation, the stimulus presented to the learner is presented from outside the system (the learner in this case). The forcing function provides impetus for the changes which
occur within the system but is not changed as variables within
the system change. As such it could be considered as the
independent variable in an experimental situation.

Variables in the system itself (termed 'variables' or 'state
variables') are components of the system which are quantifiable,
vary with time as a result of changes in the system, or at least
are free to do so, and which dissipate or depreciate with time.
They are a part of the dynamics of the system and as such could
be considered as the dependent variables in experimental situations.
In some instances, variables can be best understood by envisaging
them as storages (such as information or memory). The distinction
is thus made by whether the component changes due to the dynamic
nature of the system or whether it is outside the system and
therefore is not affected by changes which occur within the system
over time. Forcing functions need not be constant. However, the
variations in the forcing function are pre-determined and are not
affected by the changes produced on the system itself.

Symbolically, a forcing function is represented as shown in
Figure 1. Usually, the nature of the forcing function is indicated
inside the symbol, for example inscribing I=k would indicate the
forcing function represented some level of influence which was set
to a constant level.

Figure 1
5. State variables are represented symbolically as shown in Figure 2. Lines to and from the variable are joined to parts of the system to indicate what effect that variable has on other parts of the system (J1), what parts of the system affect the variable (J2), and indicate depreciation of the variable over time (J3), a concept to be discussed in principle #5.

Figure 2

2. Variables and/or forcing functions may operate in the system in such a fashion that the effect of the combination is additive. When this occurs, the combined effect can never be exactly equal to the sum of the effects of the combination, although it may be very nearly equal. Thus some of the effect is always lost when several effects are combined additively to produce a combined effect. For example, this principle states that if two stimuli are presented simultaneously, their combined additive effect is less than the sum of the two effects.

3. Variables and/or forcing functions may interact with one another in such a way that the result is not additive. Symbolically, this is shown in Figure 3.
A and B represent the effect of variables which interact, and C indicates the result of the interaction, that is, the combined effect of A and B. D indicates that some effect is once again lost during the interaction, as was the case when two variables combined additively.

Many types of interactions are possible (Odum, 1971) but only the most common will be discussed. Figure 4 shows a multiplicative interaction. In this case, the interaction of A and B is such that the resulting effect is given by C and D takes into account the fact that the resulting effect is less than the combination of the two interacting variables. If either A or B remain constant while the other varies over time, then the relationship is linear, if both A and B vary, then the resulting effect is curvilinear over time. Thus the relationships allow the system to adjust itself to linear or non-linear variations depending on whether the variables change or remain constant.
Another type of interaction representing inverse variation over time is shown in Figure 5. In this case, the combined effect of A and B is such that the result (C) is reduced when B increases and increased when B decreases.

D represents the loss of some of the effect when A and B interact. It is also possible to specify other types of interactions. For example A and B may interact in such a fashion that the relationship is best explained using the logarithm of A and B. In such a case, transformations can be accounted for before the interaction and interaction could still be direct or inverse, linear or non-linear. Conversions of the interactions to mathematical representations will be discussed later.
4. In any interaction, the sum of the effects of the interacting variables and/or forcing functions must be equal to the sum of the combined effect of the interaction and the losses which occur when the interaction takes place. This is a conservation principle which states that the combined effect of the interaction of any variables cannot exceed the sum of the effects of the variables which interact. Since some effect is always lost in interaction, the resulting effect is always less than the sum of the effects of the interacting variables.

5. Losses occur in various parts of the system as a result of interactions between components of the system and as a result of the depreciation of the variables. Previous principles indicate that no interactions are possible without the simultaneous occurrence of some loss in the resulting effect of the combined variables due to interaction. Ideally, this loss should be almost zero. However, the more difficult the interaction is to achieve, the more loss is associated with the interaction. Other considerations are also important however, and the modifications these make to the system will be discussed in principle #7. The other losses are peculiar to the variables in the system and are the result of the depreciation of the variable over time. This loss may vary in amount but in any case is always proportional to the magnitude of the variable at any particular time. This loss is shown by the symbol appearing in Figure 6.
6. The basic function of a system is to provide stability through organization and diversity. This organization is brought about through interactions of variables with each other in such a fashion that the influence of each variable on the others is optimum. Diversity insures that many types of interactions are possible so that the interactions necessary to benefit the system exist. Some examples may be of help here. A system in which information is organized benefits the system, whereas, less benefit or even detrimental effects are derived from the lack of organization of information in a system. Components of a system can only benefit the system as a whole when the components are operating in an organized fashion. Diversity provides for alternate ways of accomplishing those things beneficial to the system, for example, problem solving is enhanced when diversity of experience exists so that numerous alternatives are available to the problem solver.

7. Systems receive optimum benefit when interactions are such that the loss incurred during the interactions are 50 percent.
When losses at interaction are less than 50 percent, optimal interaction between variables does not occur. If losses are greater than 50 percent the interaction is too difficult to achieve and the resulting effect is less than the loss. The nature of this relationship is shown in Figure 7.

![Figure 7](image)

8. Variables which are the result of the interaction of other variables are said to be of higher quality than those which interacted. Quality in this case refers to the ability of variables of higher quality to affect more other variables to a greater degree than those of lower quality. For example, the interaction of concepts to give rise to principle, resulting in entities which can affect more other cognitive interactions than was the case for the concepts alone.

9. Because the system is concerned only with forcing functions, variables, and the nature of their interaction, variables which are not included in the system considerations are held constant.
this means that other variables may exist within a system being studied, but by not including them as variables they are not assigned a dynamic nature and are, therefore, being held constant.

10. In some instances, variables interact in the system only under certain circumstances. In this case, there is provided in the system some type of logic or switching devices which allow the system to adhere to boundary conditions.

REPRESENTATION OF A SYSTEM

Most of the symbols by which the system can be represented have already been indicated. These symbols are joined by lines to indicate how the variables and forcing functions interact. In the cases where variables interact only under certain conditions, symbols which indicate comparisons and switches must be included. Figure 8 shows how this can be symbolized in a system.

![Figure 8](image-url)
The magnitude of variable Q is compared with the value of the constant K, as shown by the comparator symbol. If Q is greater than K, the comparator opens switch S and allows variable Q to be affectual in some fashion. If K is greater than Q, the effect of Q is not realized in the system.

In order to facilitate adapting the mathematical representation of the model to the simulation procedure on an analog computer, the magnitude of all variables is expressed as some portion of the maximum value for that variable. Therefore, all variables have values between zero and one, but maximum values for variables given in raw score may be different. For example, variable A may have a maximum value of 100, variable B a maximum of 50. If both variables were assigned a value of 20 for simulation purposes, variable A would have a value of .2 and variable B would have a value of .8. This scaling process is necessary because analog computers operate on values between 0 and 1. The scaling process has other useful results—it indicates how close to maximum any variable is without having to keep in mind the maximum value which that variable has, since it is one for all variables regardless of the numerical raw score maximum.

The effect which variables have on each other and on the system are determined entirely by the interactions which take place among the variables. When the interaction shown in Figure 4 takes place the resulting effect, C, is given by \( C = kAB \) The constant, K, takes into account the losses incurred during the interaction and represented by D in the diagram. Similarly, when A and B interact as shown in Figure 5, the resulting effect, C, is given by \( C = kA(l - k_2B) \). In this case the
relationship is inverse, and because the maximum value which B can have is 1, the relationship $1 - k_2B$ expresses the inverse aspect of the relationship. When B is large, the expression $k_1(1 - k_2B)$ decreases and when B is small the reverse is true. $k_1$ and $k_2$ take into account the losses at interaction, represented by D.

![Diagram](image)

**Figure 9**

Figure 9 represents the relationships for some variable Q and its interaction with a variable outside the system (a forcing function). In this case, the interaction is such that the magnitude of Q affects how much the external variable will affect Q. If Q is large then the effect of the external variable on Q will increase. As well, Q is shown to have some effect on another system and also the normal depreciation of Q over time is shown.

The effect of Q on the interaction of Q and A is given mathematically as $k_2AQ$. The effect of the interaction on Q is given by $k_1AQ$. Since $k_1$ and $k_2$ are different, the effect of Q on the interaction, and the
effect of the interaction on Q are different. However, since the effect of Q on the interaction with A depends on the magnitude of both A and Q, the product of A and Q takes this into account. If either A or Q were zero then no interaction would occur. For the same reason the effect of the interaction of A and Q on Q is given by the product of A and Q. Differences in the two effects are accounted for by two different constants.

Since we do not know how Q is affecting the other system we can only indicate that the effect is determined by the magnitude of Q, or $k_3Q$. The depreciation of Q over time is known to be dependent on the magnitude of Q and is therefore $k_4Q$. The amount of depreciation of Q in some unit of time is given by $1/k_4$.

The dynamics of the system over time are described by the changes in the variables. In this simple system, only one variable exists in the system and therefore the system changes can be described mathematically by indicating what changes occur in this variable. This can be done by adding the expressions going into Q and subtracting the expressions going out of Q. This becomes the differential equation showing the change in Q and for this case is:

$$Q = k_1AQ - k_2AQ - k_3Q - k_4Q$$

When this equation is integrated over time it yields the change in the magnitude of Q, given some original condition of Q.

If the interaction symbol in Figure 9 were to have a negative sign in it, this would indicate a different relationship between A and Q. In this case, as Q increases the effect of A on Q decreases. The effect of Q on the interaction then becomes $k_2A(1-Q)$ and the effect of the
interaction on Q becomes $k_1A(1-Q)$. The differential equation then is:

$$Q = k_1A(1-Q) - k_2A(1-Q) - k_3Q - k_4Q$$

The final representation of the model is the analog computer version of the differential equations (one for each variable in the system). In this model, all of the effects can be manipulated by manipulating the constants ($k$'s) in the equations, and the change of Q over time plotted using the computer. As well, different initial conditions for A and Q can be used, and different periods of time chosen for simulation. The computer makes possible simultaneous manipulation of many relationships while presenting for inspection changes in variables in a manner which we can comprehend—one or a few at a time.

The crucial part of the modeling procedure is to determine what relationships exist among the variables in the systems being considered. Relationships determine the effect of the variables on each other and determine the changes in variables over time. The relationships, therefore, indicate what happens, the mechanism by which it happens, and the variables which are affected within the system, as well as including the effect of variables external to the system.

The highlights of the principles and procedures for an energy systems approach to modelling dynamic systems has been outlined. Since this is a fairly complex technique, more information may be necessary before the technique is used. However, by presenting the results from the use of the procedure in the following discussion, some of the procedure may become easier to understand. As well, presentation of the use of the technique will indicate the usefulness of the technique and also indicate what areas require further study.
MODELING A SYSTEM

The energy systems approach, as outlined, was used to model achievement motivation and some related sociological concepts (Loose, 1974). Since the focus of this paper is on the procedure, only the relationships represented in the model will be given without the more detailed justification from research literature.

Atkinson (Atkinson, 1958, 1965; Atkinson and Feather, 1966) formulated the theory of achievement motivation which has been a useful guide for research in areas concerned with motivation and achievement in academic situations. A review by Bower, Bayer, and Scheirer (1970) summarizes several hundred studies using this theory. As well many other investigations bespeak the utility of the theory. In summary, the relationships from the theory are:

1. The incentive to succeed decreases as the perceived probability of success increases. \( I_s = 1 - P_s \).

2. The incentive to avoid failure is equal in magnitude to the perceived probability of success. The negative value indicates avoidance of the task. \( I_f = -P_s \).

3. The perceived probability of failure decreases as the perceived probability of success increases. \( P_f = 1 - P_s \).

4. The tendency to succeed is greatest at intermediate probability of success and increases as motive to succeed increases. The relationship is multiplicative such that \( T_s = M_s \times P_s \times (1-P_s) \).

5. The tendency to avoid failure is greatest at intermediate probability of success and increases as the motive to avoid failure (anxiety) increases. The negative value of this relation indicates
avoidance. The relationship is multiplicative such that

\[ T_f = M_f \times (1 - P_s) \times (-P_s). \]

6. When the tendency to succeed is greater than the tendency to avoid failure, achievement-oriented activity is likely to be attempted. In this case, tasks are chosen for which the perceived probability of success is intermediate, that is, tasks which maximize the tendency to succeed.

7. When the tendency to avoid failure is greater than the tendency to succeed, achievement-oriented activity is likely to be avoided.

8. When the tendency to avoid failure is greater than the tendency to succeed, and there is a high level of extrinsic motivation, achievement-oriented activity is likely to be attempted. Tasks are chosen for which the perceived probability of success is very low or very high in order to minimize the tendency to avoid failure.

In addition, the concept of locus of control (Rotter, 1966) was found to be pertinent to achievement as well. Those persons identified as having internal locus of control believed that their actions are of some consequence in determining the outcome in a given situation. Persons having an external locus of control believed that the outcome is not affected by their actions but is only governed by fate or chance.

Findings of Clarke (1973), Hersh and Schiebe (1967), Crandall and Lacey (1972), Reid and Cohen (1973), and others indicate a tie between locus of control and achievement motivation theory. Also, findings from Joe (1971), McGhee and Crandall (1968), and Hersh and Schiebe (1967) indicate that social pressure has an affect on achievement as well. Thus the
following relationships were added to those from achievement motivation theory:

1. An increase in locus of control produces an increase in internality.

2. As locus of control increases, motive to avoid failure (anxiety) decreases.

3. As locus of control decreases, motive to succeed decreases.

4. As locus of control increases and retention increases, the perceived probability of success increases.

5. Social pressure is equivalent to extrinsic motivation in the relationships stated in the achievement motivation theory.

6. Social pressure will influence the individual to participate in achievement activity only if locus of control is low.

7. Success in achievement activity increases locus of control.

As an aid to explaining the energy symbol model, Figure 10 shows a schematic representation of the model. This schematic is meant to give a general explanation of how the model is conceptualized. The achievement motivation theory stated that the tendency to succeed ($T_s$) was the result of the multiplicative relationship of probability of success ($P_s$), incentive ($1 - P_s$), and motive to succeed ($M_s$). Likewise, the multiplication of probability of success, incentive, and motive to avoid failure ($M_f$) produced the tendency to avoid failure ($T_f$). These relationships are shown in the schematic beginning at $P_s(1 - P_s)$ and proceeding up and right for $M_f \times P_s(1 - P_s)$ and down and right for $M_f \times P_s(1 - P_s)$.

Certain decision processes then determine interaction with the learning situation. If ($T_s > T_f$) is true, then tendency to succeed is
dominant. The schematic indicates that the tendency to succeed interacts with I. I represents the intensity of the learning situation. Here intensity was defined as the combination of factors such as sequence, strength of stimulus, and task complexity which influence the valence of the learning situation presented. I was an attempt to represent those aspects of the learning situation which affect acquisition and performance, and not meant as a specific breakdown of particular aspects of presentation in learning situations. This aspect was considered in the model as an external force on the individual, although this is not shown in the schematic. Because it was an external force, it was only of interest in that an increase in intensity fosters greater interaction of the learner with the situation.

If \( T_s > T_f \) is false, that is, \( T_f > T_s \), then other decisions must be made. Locus of control (L) must be less than some stipulated threshold value which determines "low" locus of control. Social pressure (SP) must exceed the tendency to avoid failure. Social pressure, in this case, included extrinsic motivation referred to by Atkinson as well as peer group pressures and pressures arising out of social norms. When these conditions are satisfied, the individual interacts with the learning situation to a degree which is dependent on the tendency to avoid failure and the intensity of the learning situation.

The interaction of the individual with the learning situation produces achievement (A). In the model, achievement is the sum of the effort expended in the situation as well as successes and failures. Thus, a negative achievement can indicate avoidance. Retention (R) was
Figure 10: General Schematic of the Model
defined to be the accumulation of skills, knowledge and experience resulting from achievement activity. The schematic indicates an interaction between achievement and retention. This interaction is such that retained information interacts with achievement to produce more retained information. It also affects achievement in such a way as to change probability of success and to influence locus of control. Locus of control, in turn can modify motive to succeed, motive to avoid failure, and probability of success.

The precise nature of interactions is represented by the model in energy language symbols as shown in Figure 11. The definition of each of the variables is as described in achievement motivation theory and the general description of the model. The explanation of the energy model will be done for each variable to indicate how the total system is affected.

In moving from left to right in the model, quality increases. Further, small amounts of high quality can amplify or substantially increase the effect of lower quality variables. Thus, when retention interacts with locus of control to affect probability of success, small interactions can produce fairly large changes in probability of success. This role of quality manifests itself in the importance of feedback in systems, whereby feedback of high quality substantial changes.

Lines which join variables with comparators function only to show what quantities are being compared. The symbol representing the variable perceived probability of success ($P_s$) has two inputs and three outputs. Two outputs go to the negative multiplier. These two function to produce the product of probability of success and incentive, or $P_s(1-P_s)$. 
The other output represents the depreciation of $P_s$ over time, since no quantity can exist for a time period without some depreciation.

One input to $P_s$, which results from the interaction of $R$ with $A$ represents the increase in $P_s$ which occurs when the individual, through achievement activity, increases $R$, that is succeeds in learning. The relationship indicates that $P_s$ can increase, although at a slower and slower rate when large effort is put into achievement even though retention is not high, or may even be decreasing. The input to $P_s$ results from the interaction $R$ and $L$. Thus, if retention and locus of control increase, then probability of success also increases. Since the system is dynamic, changes in other variables may mean total input into $P_s$ is greater than the output so that the value of $P_s$ may increase. The reverse is equally possible.

There are three outputs from $M_s$ and one input. One output takes part in the interaction of $M_s$ with the result of the interaction of probability to succeed and incentive. The result of the interaction produces, in part, the changes in tendency to succeed. The output from $M_s$ into the multiplier which has an output from $L$ specifies the interaction of locus of control with motive to succeed. As $L$ increases, or $M_s$ increases, motive to succeed is enhanced. Should $L$ and $M_s$ decrease, input to $M_s$ would also decrease. The interaction thus gives rise to the input to $M_s$. The other output from $M_s$ is the depreciation of $M_s$ over time.

The input to $T_s$ results from the interaction of $P_s$, incentive, and $M_s$, as explained previously. One of the outputs is depreciation; the other is an interaction with the intensity of the learning situation.
Since the interaction is multiplicative, as $T_s$ increases achievement is increased. Decreasing either $T_s$ or $I$ results in a lower input to $A$, that is, less recognizable achievement.

Motive to avoid failure ($M_f$) has four outputs and one input. The input results from an interaction of level of $M_f$ with $A$ in a fashion which causes an increase in $M_f$ when $A$ decreases. Thus one of the outputs is a necessary part of this interaction. The output which interacts with the result of the interaction of probability and incentive produces $T_f$. Atkinson postulated that tendency to avoid failure depended on the interaction of probability to avoid failure, incentive to avoid failure and motive to avoid failure. However, since $P_f = 1 - P_s$ and $I_f = -P_s$, the magnitude of the value of $P_s \times I_s$ and $P_f \times I_f$ is the same. The negative value refers to the avoidance character of the product of $P_f$ and $I_f$. Since $T_s$ and $T_f$ are compared to see which is larger (discussed later) the model duplicates Atkinson's hypothesis. The drain on $M_f$ which is affected by $L$ indicates an interaction which is such that $M_f$ is drained more rapidly when $L$ is high. Thus when $L$ is large (internal) $M_f$ decreases. The remaining drain is depreciation. For purpose of the model it is shown but is not considered to be significant as compared to the effect of $L$. It is, therefore, assumed to be zero.

Input to $T_f$ results from the interaction of probability, incentive, and motive as discussed previously. One of the outputs is depreciation, the other is an interaction of $T_f$ with $I$ to produce some achievement flow. Inputs to $A$ have already been discussed in conjunction with $T_s$ and $T_f$, outputs to interact with $M_f$ discussed in conjunction with $M_f$, and the output to interact with $R$ discussed in conjunction with $P_s$. The remaining
output is depreciation of A.

The output from R to interact with A has been partially discussed in conjunction with $P_s$. However, the result of the interaction which produced an input to $P_s$ also produces an input to L and to A. Thus as A and R increase $P_f$ increases as well. The remaining output is the depreciation of R, commonly referred to as forgetting. All inputs and outputs for L have been discussed except for the output due to depreciation of L.

The remainder of the diagram specifies the conditions under which flows occur. The comparators connected to $P_s$ determine the interval for decisions which determine in part whether $T_s$ or $T_f$ will interact with I. If $P_s$ is greater than a lower bound specified and less than an upper bound, and if $T_s > T_f$, then $T_s$ will interact with I to produce A. If $P_s$ is less than the specified lower bound or greater than the specified upper bound, and if L is less than a specified value, and if $T_f$ is less than SP, then $T_f$ will interact with I to produce a A. The conditions for interaction are dictated by the factors which influence participation for individuals of various types, as discussed previously.

The dotted portion of the model is included to indicate that the variables represented in the model can also be connected to other processes not considered. Retention is an obvious variable to use to indicate this, but the same may be true for other variables in the model.

Mathematical Representation of the Model

Utilizing the energy diagram in Figure 11, differential equations which represent the model were derived. The change in any variable is the sum of inputs minus sum of outputs. The symbols used in the equations correspond to the symbol used for the variable in the diagram. The
equations are derived in accordance with the examples given previously and are as follows:

\[
\begin{align*}
\dot{P}_s &= k_{21}P_s + k_{22}R_A - k_{23}P_s (1-k_{18}P_s)M_s - k_{24}P_s (1-k_{18}P_s)M_f - k_{20}P_s \\
\dot{M}_s &= k_{27}L_M - k_{26}M_s - k_{28}P_s (1-k_{18}P_s)M_s - k_{67}L_M_s \\
\dot{I}_s &= k_{12}P_s (1-k_{18}P_s)M_s - k_{11}T_I - k_{13}T_s \\
\dot{M}_f &= k_{15}M_f (1-k_{50}A) - k_{16}P_s (1-k_{18}P_s)M_f - k_{17}L_M - k_{68}M_f (1-k_{50}A) - k_{99}M_f \\
\dot{T}_f &= k_{40}P_s (1-k_{18}P_s)M_f - k_{41}T_f I - k_{42}T_f \\
\dot{A} &= k_{35}T_s I + k_{36}T_f I - k_{39}A - k_{37}M_f (1-k_{50}A) - k_{38}R_A \\
\dot{R} &= k_{43}R_A - k_{44}R_A - k_{51}L_R - k_{52}R \\
\dot{L} &= k_{31}R_A - k_{32}L_R - k_{33}L_M - k_{30}L_M_f - k_{34}L
\end{align*}
\]

Table 2 includes a more detailed explanation of each of the terms. The numbers of the different constants correspond to the various pots on the analog computer representation. As already mentioned, the depreciation of \( M_f \) was considered negligible as compared to losses produced by the interaction of \( M_f \) with \( L \). It is, therefore, not included in the analog version of the model. The final representation is the analog representation of eight equations.

RESULTS OF MODELING PROCEDURE

The analog computer representation of the model was put on the computer. Using the computer, results from the modeling procedure were obtained in two phases. In the first phase, the inputs and outputs which were necessary to produce results in keeping with predictions made with achievement motivation theory and research findings were determined. The
second phase consisted of collecting data for selected variables in the mode to determine the behavior of the variables under different initial conditions.

**Determination of Energy Flows in the Model**

Since the study was conceived, in part, to investigate the usefulness of applying the energy systems approach to modeling an educational situation, the procedures used in determining the inputs and outputs necessary to duplicate theory were considered as data or results. Thus the determination of the relationships to be modeled, the energy diagram of the model, the analog schematic were procedures of a conceptual nature. The use of the analog computer to simulate the interaction of the relationships over time was a verification of the conceptual model, even in the initial stages of input/output determination and, therefore, provided data about verification procedures.

**Data from Initial Procedures**

In order to make an estimation of inputs and outputs in the model, it was first necessary to assign maximum values to the variables. The maximum values for $I$, $SP$, $M_s$, $M_f$, $L$, $A$, and $R$ were stipulated as 100, providing a range of 0 to 100. The range of $P_s$ was from 0 to 1.0. Since $T_s$ and $T_f$ are determined mathematically by the relationship given in the achievement motivation theory, the maximum for these variables was 25 when $M_s$ and $M_f$ were given maxima of 100.

It was then necessary to assign values to those variables which could serve as initial starting conditions in the model. Since two diverse conditions exist, that is, those for whom $M_s$ is high and $M_f$ is low, and those for whom $M_f$ is high and $M_s$ is low, two different sets of initial conditions were necessary to represent the two types of conditions.
The assigned initial conditions for the variables are shown in Table 1. These conditions were assumed as representative of the two conditions while at the same time attempting to change as few of the variables as possible. For $M_f > M_s$, $P_s$ was chosen as .1 rather than .9 on the assumption that non-achievers generally would be more likely to perceive most learning situations as difficult rather than easy.

<table>
<thead>
<tr>
<th>Variable</th>
<th>$M_s &gt; M_f$</th>
<th>$M_f &gt; M_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>R</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>I</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>L</td>
<td>75</td>
<td>3</td>
</tr>
<tr>
<td>$M_s$</td>
<td>75</td>
<td>10</td>
</tr>
<tr>
<td>$M_f$</td>
<td>10</td>
<td>75</td>
</tr>
<tr>
<td>$T_s$</td>
<td>18.75</td>
<td>.9</td>
</tr>
<tr>
<td>$T_f$</td>
<td>2.5</td>
<td>14.25</td>
</tr>
<tr>
<td>$P_s$</td>
<td>.5</td>
<td>.1</td>
</tr>
</tbody>
</table>

Before inputs, outputs and initial conditions could be used for simulations with the model, it was necessary to determine the constants which appear in all of the differential equations. A rough determination of the constants was attempted, since having no knowledge about the magnitude of the inputs and outputs would require an enormous number of
manipulations of constants in order to obtain a feasible solution. The estimation of the constants required an estimation of the inputs to and outputs from the variables.

Estimation procedures began with a decision to estimate inputs and outputs for the case where initial conditions were those which pertained to initial conditions for $M_s > M_f$. The input into A when $T_s$ interacts with I was assigned a value of 7 units. Then inputs to and outputs from each variable were estimated. Some procedures followed during estimation were to try to keep the sum of inputs to a variable minus the sum of outputs from a variable close to zero if the variable was considered to be relatively stable. If the variable was expected to increase, then those inputs which would cause the increase were made larger than they would be if no increase was expected. Likewise, if the variable was expected to decrease, the outputs were larger than would be the case if steady state was expected. In all cases, these were estimated so that the sum of inputs to the multiplier equalled the sum of the outputs from the multiplier (the outputs included the dissipation). A summary of the estimated inputs and outputs, the description of its origination and destination, and the number of the constant associated with it are shown in Table 2. Since conditions for $M_s > M_f$ were used as starting conditions these are shown on the table. However, the conditions relevant when $M_f > M_s$ required only a few changes and the changes are shown in parentheses for the appropriate variables. This makes it possible to show all of the constants for both conditions on one table.

The calculation of the value for each constant was done by using the initial conditions associated with the constant and the input or output produced. The initial conditions were the values assigned to the
<table>
<thead>
<tr>
<th>Constant or Pot #</th>
<th>Description</th>
<th>Estimated Amount</th>
<th>Calculated Value</th>
<th>Actual Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>00</td>
<td>I to $T_s xI$</td>
<td>50</td>
<td>.500</td>
<td>.500</td>
</tr>
<tr>
<td>02</td>
<td>$SP$</td>
<td>50</td>
<td>.500</td>
<td>.500</td>
</tr>
<tr>
<td>04</td>
<td>I.C. for $P_s$</td>
<td>.5</td>
<td>.500 (.100)</td>
<td>.500 (.100)</td>
</tr>
<tr>
<td>05</td>
<td>I.C. for $R$</td>
<td>50</td>
<td>.500</td>
<td>.500</td>
</tr>
<tr>
<td>07</td>
<td>I.C. for $L$</td>
<td>75 (3)</td>
<td>.500</td>
<td>.500</td>
</tr>
<tr>
<td>09</td>
<td>I.C. for $A$</td>
<td>50 (10)</td>
<td>.500 (.100)</td>
<td>.500 (.100)</td>
</tr>
<tr>
<td>10</td>
<td>I.C. for $T_s$</td>
<td>18.75 (.9)</td>
<td>.670 (.036)</td>
<td>.670 (.036)</td>
</tr>
<tr>
<td>11</td>
<td>from $T_s$ to $T_s xI$</td>
<td>2.0</td>
<td>.001</td>
<td>.001</td>
</tr>
<tr>
<td>* 12</td>
<td>from $P_s x (1-P_s) xM_s$ to $T_s$</td>
<td>2.02</td>
<td>.011</td>
<td>.040</td>
</tr>
<tr>
<td>13</td>
<td>depreciation of $T_s$</td>
<td>0</td>
<td>0</td>
<td>.1089</td>
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<tr>
<td>14</td>
<td>I.C. for $M_f$</td>
<td>10 (75)</td>
<td>.100 (.750)</td>
<td>.100 (.750)</td>
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<tr>
<td>* 15</td>
<td>into $M_f$ from $M_f x (1-A)$</td>
<td>1.5</td>
<td>.031</td>
<td>.023</td>
</tr>
<tr>
<td>* 16</td>
<td>from $M_f$ to $P_s x (1-P_s) xM_f$</td>
<td>2.0</td>
<td>.080</td>
<td>.107</td>
</tr>
<tr>
<td>17</td>
<td>from $M_f$ to $M_f xL$</td>
<td>0</td>
<td>0</td>
<td>.772</td>
</tr>
<tr>
<td>18</td>
<td>proportion of $P_s$ in $(1-k_{10} P_s)$</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>19</td>
<td>lower limit for $P_s$</td>
<td>.4</td>
<td>.400</td>
<td>.400</td>
</tr>
<tr>
<td>20</td>
<td>depreciation of $P_s$</td>
<td>0</td>
<td>0</td>
<td>.091</td>
</tr>
<tr>
<td>21</td>
<td>into $P_s$ from $I x R$</td>
<td>.1</td>
<td>.266</td>
<td>.220</td>
</tr>
<tr>
<td>22</td>
<td>into $P_s$ from $R x A$</td>
<td>.1</td>
<td>.400</td>
<td>.400</td>
</tr>
<tr>
<td>Constant or Pot #</td>
<td>Description</td>
<td>Estimated AMOUNT</td>
<td>Calculated Value</td>
<td>Actual Value</td>
</tr>
<tr>
<td>------------------</td>
<td>-------------</td>
<td>------------------</td>
<td>------------------</td>
<td>--------------</td>
</tr>
<tr>
<td>23</td>
<td>from $P_s$ to $P_s x (1-P_s) x M_s$</td>
<td>.05</td>
<td>.002</td>
<td>.027</td>
</tr>
<tr>
<td>24</td>
<td>from $P_s$ to $P_s x (1-P_s) x M_s$</td>
<td>.05</td>
<td>.200</td>
<td>.200</td>
</tr>
<tr>
<td>25</td>
<td>I.C. for $M_s$</td>
<td>75 (10)</td>
<td>.750 (.100)</td>
<td>.750 (.100)</td>
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<tr>
<td>26</td>
<td>depreciation of $M_s$</td>
<td>0</td>
<td>0</td>
<td>.009</td>
</tr>
<tr>
<td>27</td>
<td>into $M_s$ from $M_s x L$</td>
<td>2.5</td>
<td>.004</td>
<td>.011</td>
</tr>
<tr>
<td>28</td>
<td>from $M_s$ to $P_s x (1-P_s) x M_s$</td>
<td>2.0</td>
<td>.107</td>
<td>.107</td>
</tr>
<tr>
<td>29</td>
<td>I.C. for $T_f$</td>
<td>2.5 (14.25)</td>
<td>.100 (.570)</td>
<td>.100 (.570)</td>
</tr>
<tr>
<td>30</td>
<td>from $L$ to $M_f x L$</td>
<td>.5</td>
<td>.007</td>
<td>.007</td>
</tr>
<tr>
<td>31</td>
<td>into $L$ from $Rx_A$</td>
<td>2.0</td>
<td>.080</td>
<td>.143</td>
</tr>
<tr>
<td>32</td>
<td>from $L$ into $Rx_L$</td>
<td>.5</td>
<td>.013</td>
<td>.007</td>
</tr>
<tr>
<td>33</td>
<td>from $L$ into $L x M_s$</td>
<td>1.0</td>
<td>.018</td>
<td>.005</td>
</tr>
<tr>
<td>34</td>
<td>depreciation of $L$</td>
<td>0</td>
<td>0</td>
<td>.002</td>
</tr>
<tr>
<td>35</td>
<td>into $A$ from $T_f x L$</td>
<td>7.0</td>
<td>.075</td>
<td>.800</td>
</tr>
<tr>
<td>36</td>
<td>into $A$ from $T_f x L$</td>
<td>4.0</td>
<td>.320</td>
<td>.800</td>
</tr>
<tr>
<td>37</td>
<td>from $A$ into $M_f x (1-A)$</td>
<td>1.0</td>
<td>.204</td>
<td>.092</td>
</tr>
<tr>
<td>38</td>
<td>from $A$ to $Ax R$</td>
<td>4.0</td>
<td>.160</td>
<td>.382</td>
</tr>
<tr>
<td>39</td>
<td>depreciation of $A$</td>
<td>0</td>
<td>0</td>
<td>.057</td>
</tr>
<tr>
<td>40</td>
<td>into $T_f$ from $P_s x (1-P_s) x M_f$</td>
<td>2.02</td>
<td>.081</td>
<td>.017</td>
</tr>
<tr>
<td>41</td>
<td>from $T_f$ into $T_f x L$</td>
<td>2</td>
<td>.160</td>
<td>.001</td>
</tr>
<tr>
<td>42</td>
<td>depreciation of $T_f$</td>
<td>0</td>
<td>0</td>
<td>.085</td>
</tr>
<tr>
<td>43</td>
<td>into $R$ from $Rx_A$</td>
<td>2.0</td>
<td>.080</td>
<td>.210</td>
</tr>
</tbody>
</table>
Table 2 - continued

<table>
<thead>
<tr>
<th>Constant or Pot #</th>
<th>Flow Description</th>
<th>Estimated Flow</th>
<th>Calculated Value</th>
<th>Actual Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>44</td>
<td>from R to RxA</td>
<td>1.0</td>
<td>0.040</td>
<td>0.004</td>
</tr>
<tr>
<td>50</td>
<td>proportion of A in (1-k_80*A)</td>
<td>1.0</td>
<td>1.000</td>
<td>1.0</td>
</tr>
<tr>
<td>51</td>
<td>from R to LxR</td>
<td>1.0</td>
<td>0.027</td>
<td>0.013</td>
</tr>
<tr>
<td>* 52</td>
<td>depreciation of R</td>
<td>0</td>
<td>0</td>
<td>0.006</td>
</tr>
<tr>
<td>53</td>
<td>upper limit for P_s</td>
<td>0.6</td>
<td>0.600</td>
<td>0.600</td>
</tr>
<tr>
<td>54</td>
<td>limit for min L</td>
<td>30</td>
<td>0.300</td>
<td>0.300</td>
</tr>
<tr>
<td>65</td>
<td>Reduce 10 gain</td>
<td>0.1</td>
<td>0.100</td>
<td>0.100</td>
</tr>
<tr>
<td>66</td>
<td>Reduce 10 gain</td>
<td>0.1</td>
<td>0.100</td>
<td>0.100</td>
</tr>
<tr>
<td>* 67</td>
<td>from M_s to M_s*xL</td>
<td>2.0</td>
<td>0.004</td>
<td>0.005</td>
</tr>
<tr>
<td>68</td>
<td>from M_f to M_f*(1-A)</td>
<td>1.0</td>
<td>0.205</td>
<td>0.204</td>
</tr>
</tbody>
</table>

* indicates flow into 10 gain of amplifier
variables (shown in Table 2); the constant was the unknown; and the product of these was set equivalent to the value of the estimated input. For example, the input from the interaction of L and R into P_s had an estimated value of .1 and was represented in the differential equation for P_s by the term \( k_{21} LR \). The initial condition assigned to L was 75, and to R, 50. Therefore, \( k_{21} (75)(50) = .1 \), and the unscaled value for \( k_{21} \) is .0000266. In order to scale the constant to account for the difference in the maxima for L, R, and P_s, the following equation was used:

\[
K_{\text{scaled}} = \frac{K_{\text{unscaled}} \times \text{maximum scale value of output variable}}{\text{Maximum scaled value of input variable}}
\]

The source in this case is the point from which the input to P_s originates (the multiplier) and the receiver is P_s. The maximum value of the multiplier is \((\text{maximum L}) (\text{maximum R}) or 100 \times 100\). The maximum value of P_s is 1. Therefore,

\[
k_{21}(\text{scaled}) = \frac{.0000266 \times 100 \times 100}{1} = .266
\]

This procedure was repeated for all the constants to obtain an estimate of reasonable starting values for the constants. Scaled values for the estimated inputs and outputs are also shown in Table 2.

**Data from Analog Computer Procedures**

The scaled values for the constants were then used as starting values for the analog computer simulation. The constants were manipulated in order to obtain values for the variables over time which were compatible with achievement motivation theory and related research. The research in achievement motivation theory generally covered time periods
Figure 12: Simulation Using Basic Flows ($M_g > M_f$)
Figure 13: Simulation Using Basic Flows ($M_f > M_s$)
which are very short. Many different inputs and outputs were found which duplicated the theory over short time periods but which were not feasible over longer time periods. Non-feasible solutions were considered as those in which one or more variables behave in a manner which is not explainable, or not compatible with computer operation. For example, $P_s$ greater than 1 or less than 0 is infeasible because this situation is both theoretically and logically impossible. Also, values greater than plus or minus 1 overload the computer.

Decisions about which constant to manipulate were dependent on which variables were discrepant or overloading. The depreciation of each variable over time was initially set at zero for all of the variables. Therefore, it was necessary to utilize these outputs to achieve a solution. Since it was easier to work from the energy language diagram in order to determine which inputs or outputs were pertinent to a given discrepancy, these were labelled on the diagram with the numbers of the constants which controlled the flows, rather than working from the equations themselves.

The inputs and outputs were manipulated until a reasonable solution was reached which would duplicate theory and research for the case where $M_s > M_f$. Then the initial conditions of the variables were set for the case where $M_f > M_s$. Once again the inputs and outputs were manipulated until the results conformed to those expected with reference to theory and research. By refining inputs and outputs for the case where $M_s > M_f$ and for the case where $M_f > M_s$, the conditions which would provide solutions to both cases was finally achieved. This final set of constants, which were used in the second phase of the study, and which were felt to be
to be those which best produced adherence of the model to theory and research, are shown as actual constant values in Table 2. Values in parentheses are those values which pertained when $M_f > M_s$.

Figure 12 shows the results obtained when the flows shown in Table 2 were used for initial conditions where $M_s > M_f$. Figure 13 shows the results obtained when initial conditions for $M_f > M_s$ were used. In both cases the magnitude of the variables was plotted against time, although in these two figures the time interval represented in the graphs was short, compared to the time during which it was possible to simulate the interactions of the variables under the conditions. The magnitude scale ranges from 0 to 1. Thus the values represented on the graph are the portion of total of maximum value possible for each variable. Simulated time for both figures was 6.5 seconds for the total time scale. This time was important in comparing these plots to the other plots but it had no real meaning because the amount of actual time represented by 1 second of simulated time was not known. Some attempt was made to determine orders of magnitude for the time scale in later parts of the study.

The variables represented in Figure 12 behave as was predicted by the achievement motivation theory. Those for whom $M_s > M_f$, achieved only when the probability was intermediate. The probability of success increased and tendency to succeed functioned in a fashion which adhered to theory, that is, as probability increased the tendency to succeed decreased. Motive to succeed and motive to avoid failure were relatively stable. There was a slight increase in retained information, as would be expected as a result of achievement activity. Locus of control also showed some slight variation.
Figure 14: Simulation Using Basic Flows ($M_s > M_p$)
Figure 15: Simulation Using Basic Flows ($M_f > M_g$)
In Figure 13, in which $M_f > M_s$, $M_s$ and $M_f$ were also found to be relatively stable. In this case, social pressure was high enough to encourage entry into the achievement activity; hence, the initial avoidance of achievement, shown by the decrease in $A$ to a negative value, eventually gave rise to some achievement activity. The entry into achievement activity came at the point where $T_f$ was less than social pressure, as the relationships stipulated, and also was attempted when the probability of success was very low, as was predicted. $T_f$ decreased initially in order to satisfy the relationship between motive, probability and incentive. Its behavior was then in keeping with theory in that it decreased when the probability of success increased. Retention showed a leveling when achievement activity increased, indicating the effect of the activity on retained information.

Having determined the inputs and outputs which would result in a duplication of theory and previous research findings, the model was allowed to simulate the effect of allowing these relationships to interact over a much longer period of time. The results of this are shown in Figures 14 and 15. In Figure 14, $M_s > M_f$ and the initial conditions which pertained for Figure 12 were in effect. In Figure 15 $M_f > M_s$ and the initial conditions which pertained for Figure 13 were in effect. The total simulated time in Figures 14 and 15 was 100 seconds as compared to the 6.5 seconds in the previous two figures. Thus, Figures 12 and 13 are represented in .65 of the time scale used in Figures 14 and 15.

In Figure 14, $M_s$ and $M_f$ were still relatively stable in that they fluctuated about some central point. However, there is a trend toward lower levels for both motives. Analysis of $A$ and $P_s$ indicated that $A$
only increased when $P_s$ was intermediate, and that achievement activity increased the probability. $M_s$ and $T_s$ also functioned in a fashion such that $T_s$ increased when probability was intermediate and decreased when the probability increased. It was difficult to determine precisely whether or not $M_f$ $T_f$ functioned as expected in this case, because $M_f$ was so low that the result of the multiplicative relationship did not fluctuate significantly.

In Figure 15, $M_f$ and $T_f$ did function as predicted by achievement motivation theory for a short period of simulated time. $T_f$ decreased due to the low $P_s$, and the avoidance of achievement activity. $M_f$ remained high. However, when achievement activity was begun, and $P_s$ increased, $T_f$ and $M_f$ decreased. This was due to the effect of the other relationships which were expected to affect these variables over time. The maintenance of achievement activity was not possible because the motive to succeed was very low and as $T_f$ decreased below $M_s$, achievement activity could only be undertaken when $P_s$ was intermediate, something which did not occur.

Having duplicated the relationships which were derived from research and achievement motivation theory, four variables were chosen and manipulated in order to ascertain what would occur if these relationships were to pertain over a longer time period. The variables chosen for manipulation were $P_s$, $L$, $R$, and $I$. These were manipulated one at a time while all others were kept constant. The energy flows determined in the first part of the study were utilized and kept constant also. For the conditions where $M_s$ was high, manipulation of $P_s$ produced variations in the oscillations of $P_s$, $R$, $L$, and $A$. The minimum oscillations in these variables were produced when $P_s$ was .60, the maximum value considered in the model to be in the intermediate range of probability.
Initial R increase was greatest when $P_s$ was at its lower limit for the initial conditions of the variables in the model. Manipulations of the initial conditions for R produced changes in the variables which indicated that the greatest increase in R and $M_s$ was at low initial R. Intermediate values of R produced less increase in R and $M_s$ but more oscillations than did the high or low initial conditions of R.

When locus of control was manipulated, greatest increases in L were found when initial L was low. In this case, initial increase in A was also large. As L increased, initial increases in A were less than those at the low level of L, and the number of oscillations in the variables were less. Increasing I increased the number of oscillations and slightly decreased the magnitude of the fluctuation of the oscillating variables. When I was very low, a tendency toward normal depreciation of the variables was observed, since there was little to influence achievement activity.

Simulations done for the conditions when $M_f$ was high produced somewhat different results. The manipulation of $P_s$ indicated that A was maintained longer for more difficult tasks. R was also slightly higher for these tasks. However, $M_f$ remained high longer when the tasks were more difficult. Changing the initial conditions for R to a low initial value produced achievement over a longer period of time. $M_f$ also remained high longer. $P_s$, in this case, did not change much. For high intitial R, $M_f$ and A decreased rapidly and $P_s$ increased rapidly.

Manipulations of L were constrained by the value considered in the model as the value below which L would be considered as 'low'. When L remained below this, increasing L produced increases in $P_s$ and A and
decreases in $M_f$ as well as smaller initial gains in $R$. When $L$ was increased above the 'low' value, $A$ and $P_s$ increased much more rapidly, $M_f$ decreased rapidly and initial gains in $R$ were increased. The manipulation of $I$ produced some of the most dramatic changes. Increasing $I$ brought increases in $A$, $P_s$, $R$, $L$, and $T_s$, and decreases in $M_f$ and $T_f$. When the level of $I$ was very high, the changes in the variables were such that $T_s$ exceeded $T_f$, $P_s$ became intermediate, and the oscillations typical of the high $M_s$ conditions replaced the general decline conditions common to the high $M_f$ conditions.

The model was found to duplicate theory and research well over the short time period as well as being able to function over longer periods of time to produce reasonable results in light of empirical research. Some combinations of initial conditions were found which resulted in infeasible solutions. These instances will be discussed here since they may be a limitation of the model. There is great latitude in the choice of initial conditions at which the simulation begins. This range of conditions available may make it possible to combine conditions in the model which are not in this combination in individuals. For example, the upper limit for $L$ in individuals with high $M_s$ was .78 or 78 percent of the total range of $L$ possible. It may be that an individual with high $M_s$ and with a locus of control that is higher than .78 is a non-existent type of individual. There is some justification for such an argument. An $L$ of 1 or almost 1 would mean, theoretically, that the individual was sure that he could control the outcome in all situations. Since achievement-oriented individuals choose intermediate probabilities, there is already good evidence to indicate that the idea of almost absolute control of situation outcome is not part of the high $M_s$ individual's characteristics. Discrepancies in the model may also exist because the
relationships are incomplete. This does not mean that the theory is incorrect, or the model simulating the theory is incorrect. The achievement motivation theory was derived from using college students for much of the original experimentation. Almost all of the studies which were reported in Chapter II used randomization of subjects to groups, or there was a range of individuals with high $M_s$ in the sample. Thus, the few individuals who were at the extremes of the sample are somewhat neglected when means and standard deviations were used in analyses. The theory, then, becomes based on a normal distribution, and focuses on the mean. The model too focuses on the 'average individuals' by adjustment of the initial conditions and there is no provision in the model for taking into account other features of ability which are operative and not represented well by the variable 'R'. Some variables which serve as limitations to relationships may also exist for conditions when values are at the extreme range of possibility, but these do not function in this model.

**Implications**

The implications of the information acquired in the study are twofold in nature. There are immediate implications for further study in attempting to determine the validity of the model and the changes in variables over time which the simulations produced. The other realm of implications applies should the outcomes indicated by the model be found to be true after empirical testing. These implications are for experimental design and for the procedures with which we currently look at variables in education.
Immediate Implications

A series of empirical investigations are envisaged to determine the validity of the model. Determining the time scale for the model using the time taken for the normal depreciation of retained information for a given situation would allow the determination of the constant for depreciation and could be applied to other variables. Knowledge of this nature would indicate not only the time scale of the model but also the time lag for the variables used in the model, specifically A and R.

With prior knowledge of the time scale, it could be empirically determined whether or not variables such as A and R do fluctuate about some median level for those persons for whom $M_s$ is dominant. The investigations would also indicate whether or not there is a time lag as indicated in the simulations.

The simulations indicated that $M_f$ was not stable, but rather that it was a function of A and R. Studies in which $M_f$ was measured at intervals during an achievement activity would indicate whether or not $M_f$ is stable. These experiments would need to utilize those individuals for whom $M_f$ is dominant and $L$ is low in order to provide information about the validity of the model. Another type of empirical investigation which would indicate model validity would be to determine whether or not intense, structured learning experiences do produce a change from the dominance of $M_f$ to conditions in which $M_s$ is dominant.
Other Implications

Should the model prove valid, there are implications for research methodology which are of importance. Validity of the model would suggest that investigations into the nature of retention of learned information must take into account the time lag which exists between maximum storage in different variables. Thus, experimental procedures which utilize procedures in which R is measured at a time when A is at a peak do not provide true information on R because there is the time lag between A and R. Knowledge of the time scale indicated by the simulations would provide a key to the length of time which might enable the measurement of maximum retention after achievement has reached a maximum.

Some of the simulations resulted in more oscillations which were less severe than were found in other simulations. These fluctuations may be the indicators of the conditions which provide optimal utilization of energy and thus also indicate when conditions have been achieved which provide maximum benefit to the system. These conditions may be indicated by many small oscillations rather than fewer large ones. In the learning situation, maximum benefit might be referred to as greatest retention in the smallest time period.

In order to put empirical data from educational systems into the model and thus provide empirical indications of the results of specified
interactions, it is necessary to find some means for measuring inputs and outputs between variables in some fashion. Should some technique be found, this would not only be another way of providing some validity for the model, but it would also be a powerful predictor as a research tool, predicting both changes and expected magnitude of the change in a given period of time. This aspect of the procedure is currently being examined. The use of causal models (Harriott and Muse, 1973; Madaus, Woods, and Nutall, 1973; Anderson and Evans, 1974) may provide the link necessary.

Prior to the establishment of the validity of the predictions made by simulations with the model, it still remains that the model was able to duplicate research results well in theoretical mode rather than as an empirical device. It, therefore, suggests examination as a tool whereby theoretical constructs and relationships can be manipulated in order to see what changes occur and whether or not these changes are reasonable. In this respect alone, it has value as a means for manipulating a number of relationships simultaneously when it becomes impossible for these manipulations to be done mentally.

The existence of relationships which indicated that the principle governing maximum benefit to the system is operating in educational situations may also indicate that other such relationships exist. This is of importance in that many predictions in educational situations are done on the assumption that relationships are linear. The simulations using the model indicated that non-linear relationships and the maximum concept of benefit to the system are useful in the consideration of successful systems and may be of utility as theoretical constructs in education.
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