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ABSTRACT

Speed and accuracy in adding a column of one addend numbers directly down a column was compared to adding by looking for combinations which add to ten. Ninety-two fourth-grade subjects learned one of these two methods for one week and then learned the other method the following week. Using timed tests after each treatment, it was found that the Direct method was much faster (p less than .001) and just as accurate as the Tens method. This held for all ability levels. There was a slight preference for the Tens methods after training. The results suggest that a computational procedure which requires less decision making is more efficient. (Author)

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Abstract

Speed and accuracy in adding a column of one addend numbers directly down a column was compared to adding by looking for combinations which add to ten. Ninety-two fourth-grade subjects learned one of these two methods for one week and then learned the other method the following week. Using timed tests after each treatment, it was found that the Direct method was much faster ($p < .001$) and just as accurate as the Tens method. This held for all ability levels. There was a slight preference for the Tens method after training. The results suggest that a computational procedure which requires less decision making is more efficient.

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Teachers and students alike have debated the merits of column addition methods with some preferring to add a column directly, adding each number to the partial sum without looking for familiar combinations or deviating from a set procedure, while others feel they are faster and more accurate if they use combinations which add to ten. Other methods have been championed but most people add in one of these two ways. A person adding directly would add the numbers shown in Figure 1 by saying 6, 13, 17, 22, adding the numbers in the order they appear without analyzing the problem. In adding the numbers in Figure 1 using the Tens method, 6 and 4 are added first, then 7 and finally 5.

Insert Figure 1 about here

Preference for the Tens method where it exists may lie with the opportunity to avoid difficult facts such as 7 + 9 and 6 + 8. On this basis one would expect low ability children, the ones that often do not know their facts, to do better using the Tens method. But the Tens method is not without its critics. Some argue that the Tens method induces more errors becuase numbers are considered out of order, increasing the possibility of omitting a number from the sum. Probably the most overlooked aspect of the Tens method is the time consumed in determining how to proceed. When people estimate the time it takes them to add a column of numbers they do not usually consider the time spent initially in deciding how to begin and which combinations to use. This strategy usually results in time spent analyzing a column even if the Tens method does not apply.

It is interesting to consider the recommendations for teaching column addition available to teachers. Marks, Purdy,and Kinney (1965) recommend students be taught to add directly rather than by Tens, claiming that more errors are made using the Tens method. After interviewing 176 elementary school pupils, Lankford (1972) concluded that good computers

are more likely than are poor computers to add digits in order rather than jump around to find preferred combinations. Chesin and Quast (1970) surveyed college students enrolled in an elementary education course on methods used in column addition. They report three methods; adding down the column, adding up the column, and grouping addends to form partial sums of Ten. They found that 58% added down, 14% added up, and 28% added by tens. The authors hypothesized that more errors would be made adding with tens and probably for this reason recommend that column addition be taught by the adding down method. They suggested that studies be designed to compare the effectiveness of these methods. On the other hand, Riedesel (1967) suggests that the adding-to-make-tens method be taught as an alternate procedure, noting that children and adults use this method quite naturally. Collier and Lerch (1969) also recommend that the adding-by-tens method be taught because it makes mental calculation easier during column addition. They fail to consider speed or accuracy in their recommendation. Schminke, Maertens, and Arnold (1973) indicated that students will find their work easier if they look for tens. They further stated that looking for tens allows the students to "...overcome the drudgery of adding long columns of figures one addend at

a time, and enhances accuracy. "(p.225). In a book of math shortcuts, Locke (1972) directs students to look for Tens in adding, implying greater speed will result. Thus, it is clear that broad disagreement exists among mathematics educators on the best method of adding a column of figures: each method has its advocates with conflicting justifications clearly apparent.

PURPOSE

In an attempt to provide results which could guide those deciding how to perform column addition, a training study comparing the two methods was designed. The purpose of the study was to examine accuracy and speed measures after training on the Direct or Tens method. The following research questions were asked:

1. Do students trained on the Direct method have greater speed and/or accuracy in column addition?
2. Are there Aptitude-Treatment interactions?
3. How does the initial method used and preference after training relate to the criterion measures?
4. Do students that use their fingers to add score lower on the criterion measures?

METHOD

Sample While a wide age range of subjects should eventually be used in studying this problem, elementary school pupils were selected for this initial investigation. The subjects for this study were all fourth-grade pupils from a suburban area Wisconsin school. In the school, there were 92 Fourth-grade children grouped into four classes of 23 pupils each. Due to absenteeism, ten subjects did not complete the study and were dropped. Although the students were homogenously grouped for mathematics instruction, this study was conducted with students in their randomly-formed homeroom classes. The three aptitude levels were defined using the groupings determined by the teachers for mathematics instruction.

Description of instruments The pretest consisted of 50 single-digit column addition problems varying in the number of addends from two to seven. The first ten problems were two-addend "fact" problems while the remaining forty varied in the number of addends from three to seven. A table of random numbers was used to generate the addends to prevent any bias favoring one method over the other. Posttest 1 was a parallel form also constructed with a table of random numbers. Posttest 2 was identical to the pretest. Since there were eight worksheets and one test intervening, it is highly unlikely that any students recognized the identity.

Five minutes working time was allowed for each test. The five minute time was arrived at through pilot work with other fourth-grade pupils. The test length and time combination was effective in obtaining a spread of scores without topping out on the tests since less that three percent of the subjects finished any test.

Procedure From the four classes of students, two were selected to receive training or the Direct method first with the other two classes receiving training on the Tens method first. The treatments were reversed for the second phase of the study so that each group received training on each method during the two weeks of the study.

Initially, a five minute pretest of column addition was administered in a group setting. Immediately following the pretest, the teachers explained the two methods of adding using written protocols and asked the pupils to indicate which method they used on the pretest. Beginning the Monday after the pretest, subjects received five minutes explanation and ten minutes practice time using worksheets prepared by the experimenter. The teachers followed written protocols in presenting the method. While the instruction differed between treatment groups, all subjects used the same practice sheets. There were four days of training followed by a posttest

on Friday. On Monday following the Friday posttest, the treatments were reversed with those trained on the Direct method receiving training on Tens and those that had received training on Tens receiving training on the Direct method.

Following each posttest the pupils were asked several questions the first of which was, "Did you use the method you practiced this week in working the problems on this test?". If the student answered no to this his scores were dropped from the analysis on that posttest. Approximately 20% of those trained in the Direct method indicated they used Tens on the Posttest while less than 1% failed to use Tens when trained on that method. The number of subjects included in the analyses on the pretest, posttest 1 and posttest 2 are shown in Table 1.

Insert Table 1 about here

Data Analysis A number of alternatives were available for data analysis. A repeated measures design could have been used with main effects obtained by examining contrasts, but because of the counter balance design it was more direct to test main effects with analysis of variance on each posttest

separately. If the number of subjects per cell had been greater, it would have been possible to have performed a four-way analysis on method, ability, use of fingers, and sex. Since none of the two-way interactions were significant, little would have been learned by the four-way analysis. The two-way analysis selected is more conservative than either of the other method considered and since the results from it were conclusive no need was seen for other procedures.

Results

The main purpose of this study was to compare two methods of column addition. To accomplish this, six 2 (Treatment) x 3 (Ability) analyses of variance were performed on the number correct (C), number attempted (A), and percentage correct (C/A) for posttest 1 and posttest 2. The six analyses are shown in Table 2. As can be seen when the subjects received the Direct treatment the number correct and number attempted was significantly higher ($p < .001$) than when they received the Tens treatment. An examination of the group means

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Insert Table 2 about here

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presented in Table 3 shows that students using the Direct method were able to answer 15% more problems correct on posttest 1 than students using the Tens Method and 18% more

problems correct on posttest 2.

The results for number attempted parallels that for number correct as shown in Table 4.

Insert Table 3, 4, and 5 about here

As expected, there were highly significant ability differences in favor of higher ability students. This was the case for percentage correct as well as number correct and number attempted. There were no significant interactions.

As indicated previously, students' scores were not included in the analysis if they did not use the method trained on that week. Several points should be made about these subjects and their scores. First, there was no ability bias for those not following the method; there were about the same number of each ability group not following the method. Secondly, the mean number correct of the 12 subjects not using the Direct method on posttest 1 was 30.2 compared to 34.61 for those using the Direct method; for posttest 2 the comparison was 28.5 compared to 35.06. It should be kept in mind that all but two subjects used the Tens method when trained on it. The means of these subjects were nearly identical to the means of subjects in the Tens group, indicating consistency of performance with the Tens method. These data

further confirm the superiority of the Direct method. However, if the analyses are done with these scores included, the same treatment effects are present but are not as pronounced.

From Table 5 it can be seen that there were no differences in accuracy as measured by the percentage correct of those attempted. The subjects in each treatment group generally performed at the 90% level with the percentage slightly higher (92%) on posttest 2.

Initially, 47.5% of the subjects used the Direct method and 52.5% the Tens method. After the first training session the percentage preferring Direct had dropped to 42.5% and then to 41% after the second training session with the accompanying increases in the percentage preferring Tens. The shift in preference was unrelated to treatment condition with as many shifts to Tens by those subjects trained on Direct as trained on Tens. This shift to preference for Tens was made while the subjects were answering more problems correct with the Direct method. Subjects were given the results of posttest 1 prior to beginning the second week of the study on the alternate method, so they knew how well they performed with that method. While the number of subjects preferring Tens increased as the study progressed, the change in preference was nonsignificant ($x^2 = 1.25$).

During the pretest it was observed that many students either used their fingers, tapped the paper, or made marks

indicative of a "counting on" strategy of obtaining the sum of two numbers. As part of the posttest questionnaire, students were asked to indicate whether they used their fingers adding. A surprisingly high 46% of the sample answered yes to this question after each posttest. There was better than 88% agreement of those using their fingers on the two posttests. A 2 (Method) x 2 (Fingers) analysis of variance for each posttest revealed that those subjects using their fingers had significantly fewer problems correct and attempted fewer problems than those not using their fingers. For number correct on posttest 1, F=12.26, which is significant at the .001 level. For posttest 2, F=13.22, which is significant at the .001 level also. There was no interaction with method on either posttest. There was no difference in accuracy on either posttest.

As would be expected, more low ability subjects used their fingers than either of the other two ability groups. Specifically, 76% of the low ability subjects and 61% of the average ability group used their fingers, while only 14% of the high ability subjects indicated use of their fingers.

Analysis of the data for sex differences revealed that boys and girls were performing the same on all tasks.

Discussion

While much disagreement has existed on the best method of adding a column of numbers, this study demonstrates the advantage of the Direct method; it is 15-18% faster and just

as accurate. Not only did students trained on the Direct method answer more problems and have more correct than those trained on the Tens method, the same pattern of performance was observed in comparing students' scores adding Directly to their own scores adding by Tens. The number attempted and number correct followed the same pattern with the Direct trained subjects having higher means on both measures. This relationship resulted in the absence of any difference in percentage correct. Since both the number attempted and number correct were higher for the Direct trained subjects, no differences existed in the accuracy measure.

Because low ability students do not usually know all their addition facts it might be conjectured that they would score higher using the Tens method. The results of this study do not support such a conjecture. Students of all ability levels performed better with the Direct method as evidenced by the absence of any interaction of method and ability. The F values for interaction were nearly zero in every case.

The finding that students adding with their fingers are slower than those using their knowledge of facts is not surprising. But it is surprising that so many fourth grade pupils used their fingers when adding (46%). These students

were considerably above the national norm in mathematics on the Iowa Test of Basic Skills so one would expect them to know their facts as well as most fourth grade pupils. While further evidence is needed to compare the percentage using their fingers in other schools it is quite likely the percentage would be just as high. The absence of any method by use of fingers interaction together with an examination of the cell means shows that whether students use their fingers or not they still get more problems correct using the Direct method.

While the number of students using the Direct method was about the same as the number using the Tens method before exposure to the two methods, there was an increase in the number preferring Tens after the first week and again after the second week. There was just as much increase in Tens preference for those trained on Direct as trained on Tens after the first week. Since the number changing preference was nonsignificant, the increase in preference for Tens must be viewed cautiously. Related to preference is the number of subject following the training method on the posttests. Nearly all subjects used the Tens method when trained on it but more than 20% used the Tens method when trained on Direct. This can be interpreted as support for the strong appeal of the Tens method when it is known.

A survey of prospective elementary school teachers revealed that the 41% used Direct and 59% used Tens. This is the same percentage as shown by fourth grade pupils. Four reasons are put forth to explain the tendency to prefer tens; (1) there is a sense of speed when one finds one or more tens facts, (2) looking for tens breaks the monotony of adding one number after another, people like diversity and tend to avoid routine when they can. (3) those unsure of their facts feel they are fortunate when they can avoid 7 + 9 or 6 + 8, (4) the illusion of greater speed with tens results from considering time adding a column to begin when they start finding sums and not when they begin examining the problem.

A consistent criticism heard of the Tens method is that more errors will be introduced by jumping around looking for tens, with some numbers being added twice and others left out. This criticism does not appear valid judging by the results of this study. While the Tens method requires more time, users of it are just as accurate as users of the Direct method.

An interesting observation relates to the effect of extensive practice in column addition. At the end of the study each student had worked approximately 350 problems

yet except for treatment effects there was no appreciable improvement in performance. Evidently performance in column addition is at some base level and not easily changed. With more complex algorithms there is evidence that much improvement would be observed with this amount of practice.

Although more evidence is needed to generalize to other algorithms, one can infer from these data that computational procedure which stress understanding result in less computational facility. The advantage of algorithms lies in the routine established and the minimum amount of decision making or problem analysis required. While students may increase their understanding of addition, place value, and the number system when using a less routine procedure, this is at the expense of computational skill. That is not to say that understanding does not play a role in learning algorithms. The generalization stated above applies to the nature of the final algorithm to be used. The efficiency of the learning process is another issue.

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$$\begin{array}{r} 6 \\ 7 \\ 4 \\ +\underline{\ 5} \end{array}$$

Fig. 1

Table 1

Number of Subjects Scores Used
in the Analyses on each Test

	Treatment Group	
	Direct-Tens	Tens-Direct
Pretest	40	41
Posttest 1	28	40
Posttest 2	39	31

Table 2

Method X Ability Analyses of Variance

Variable	Source	df	MS	F	p
Posttest 1					
Number Correct	Method	1	259.64	8.61	.01
	Ability	2	528.48	16.30	.001
	Interaction	2	24.42	.75	NS
	Error	62	32.42		
Number Attempted	Method	1	548.78	14.81	.001
	Ability	2	277.00	7.48	.01
	Interaction	2	14.79	.40	NS
	Error	62	35.04		
Percentage Correct	Method	1	166.05	3.74	NS
	Ability	2	543.80	12.24	.001
	Interaction	2	31.31	.70	NS
	Error	62	44.45		
Posttest 2					
Number Correct	Treatment	1	496.81	12.09	.001
	Ability	2	930.09	22.63	.001
	Interaction	2	6.91	.17	NS
	Error	64	41.1		
Number Attempted	Treatment	1	362.44	8.49	.01
	Ability	2	543.77	12.74	.001
	Interaction	2	7.16	.17	NS
	Error	64	42.67		
Percentage Correct	Treatment	1	114.95	1.91	NS
	Ability	2	639.87	10.64	.01
	Interaction	2	2.79	.05	NS
	Error	64	60.13		

Table 3

Cell Means and Standard Deviations for the Number Correct

Method		Ability: Low	Average	High	Total Group
Pretest					
Direct-Tens	n	8	19	13	40
	$\bar{X}$	22.9	28.4	36.6	29.9
	s	4.5	7.9	8.5	9.0
Tens-Direct	n	13	15	13	41
	$\bar{X}$	20.8	28.2	35.0	28.0
	s	6.8	5.5	6.9	8.0
Posttest 1					
Direct	n	4	14	10	28
	$\bar{X}$	28.0	32.2	40.6	34.6
	s	5.9	7.6	5.9	9.3
Tens	n	12	15	13	40
	$\bar{X}$	24.6	29.3	33.9	29.7
	s	4.5	4.9	4.9	8.2
Posttest 2					
Tens	n	7	19	13	39
	$\bar{X}$	23.7	27.4	37.6	30.1
	s	6.3	7.7	5.9	8.7
Direct	n	10	11	10	31
	$\bar{X}$	30.6	32.7	42.1	35.1
	s	6.7	5.4	4.8	7.4

Table 4

Cell Means and Standard Deviations For the Number Attempted

Method		Low	Average	High	Total Group
			Pretest		
Direct-Tens	n	8	19	13	40
	$\bar{X}$	25.2	30.5	38.4	32.2
	s	6.2	7.6	8.1	8.7
Tens-Direct	n	13	15	13	41
	$\bar{X}$	26.5	31.5	37.6	31.9
	s	5.7	5.5	6.4	7.3
			Posttest 1		
Direct	n	4	14	10	28
	$\bar{X}$	36.0	36.3	43.4	38.8
	s	7.0	8.9	5.3	9.1
Tens	n	12	15	13	40
	$\bar{X}$	29.2	31.7	35.9	32.3
	s	4.9	4.5	5.1	7.0
			Posttest 2		
Tens	n	7	19	13	39
	$\bar{X}$	28.3	30.7	38.9	33.0
	s	6.8	8.3	5.8	8.3
Direct	n	10	11	10	31
	$\bar{X}$	34.4	35.2	42.6	37.3
	s	6.2	5.0	4.8	6.4

Note: Low, Average and High are columns under the heading "Ability".

Table 5

Cell Means and Standard Deviations For the Percentage Correct of those Attempted

Method		Ability: Low	Average	High	Total Group
Pretest					
	n	8	19	13	40
Direct-Tens	$\bar{X}$	91.2	92.2	94.9	92.9
	s	4.9	6.6	4.9	5.9
	n	13	15	13	41
Tens-Direct	$\bar{X}$	77.1	89.6	92.8	86.7
	s	16.9	7.6	6.1	9.1
Posttest 1					
	n	4	14	10	28
Direct	$\bar{X}$	78.8	89.1	93.3	89.1
	s	13.5	6.2	6.6	6.0
	n	12	15	13	40
Tens	$\bar{X}$	85.3	91.9	94.2	90.7
	s	9.6	4.1	2.3	12.8
Posttest 2					
	n	7	19	13	39
Tens	$\bar{X}$	84.4	89.6	96.5	91.0
	s	12.5	10.9	2.5	10.1
	n	10	11	10	31
Direct	$\bar{X}$	87.7	92.5	98.4	92.9
	s	6.6	5.4	.7	6.5