DOCUMENT RESUME

ED 104 746 SO 008 211

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TITLE	Planning Student Flow with Linear Programming: A Tunisian Case Study.
INSTITUTION	California Univ., Berkeley. Program in International Education Finance.
SPONS AGENCY	Agency for International Development (Dept. of State), Washington, D.C.
PUB DATE	31 Jan 75
NOTE	22p.; Paper presented at the Annual Meeting of the American Educational Research Association (Washington, D. C., April 1975)
EDRS PRICE	MF-$0.76 HC-$1.58 PLUS POSTAGE
DESCRIPTORS	*Developing Nations; Educational Needs; Educational Opportunities; *Educational Planning; Higher Education; Human Capital; Labor Economics; Labor Market; *Linear Programing; *Manpower Needs; Models; Productivity; Resource Allocations; Secondary Education; *Teacher Supply and Demand
IDENTIFIERS	*Tunisia

ABSTRACT

A student flow model in linear programming format, designed to plan the movement of students into secondary and university programs in Tunisia, is described. The purpose of the plan is to determine a sufficient number of graduating students that would flow back into the system as teachers or move into the labor market to meet fixed manpower targets. The problem is to simultaneously consider a large number of programs and levels in the education system as well as to represent the production of teachers over a number of planning years. A linear programming model was chosen because of the wide availability of codes designed specifically for the optimization of linear problems. The modeling process illustrates the conceptual distinction between predictive and prescriptive planning. Predictive planning assumes externally controlled variables while prescriptive planning assumes that the government or planning body controls all the important variables. This model demonstrates that linear programming is suitable for both types of planning and that the ultimate determinants of the model are the manpower needs of the country. The structure of the model, its bounds and constraints with respect to pupil and teacher flow and graduation targets, and diagrams of the structure presented in the paper. (Author/JH)

ED104746

January 31, 1975

Planning Student Flow With Linear Programming:

A Tunisian Case Study

by

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The financial support of the Program in International Education Finance through a 211d grant from the United States Agency for International Development is gratefully acknowledged, as is the assistance of the Government of Tunisia. The information and conclusions remain solely the responsibility of the author.

Abstract

Planning Student Flow With Linear Programming:

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A student flow model in linear programming form for Tunisia was constructed to plan the movement of students into secondary and university programs while providing for sufficient graduating students to flow back as teachers and the others to move into the labor market to meet fixed manpower targets. The modeling process illustrated the conceptual distinction between predictive and prescriptive planning and demonstrated that linear programming is suitable for both. The feasible region of the model is small in an essentially predictive problem but much larger in a prescriptive one.

PLANNING STUDENT FLOW WITH LINEAR PROGRAMMING:
A TUNISIAN CASE STUDY

In recent years, mathematical models of education involving student flow have become popular as a means of either predicting or planning future student enrollments and hence future needs for teachers, buildings, and budget allocations for education. Some models are explicitly linked to the economy (Adelman, 1966) while others regard the economy as exogenous or ignore it altogether (Bowles, 1965; Min, 1971; Bezeau, 1974). Two recent reviews (McNamara, 1971; Johnstone, 1974) indicate the truly international scope of activity in this field. Bartholomew (1973) considers these models in the broader disciplinary context of stochastic processes.

This study involved a flow model cast in a linear programming format, designed to solve a specific Tunisian planning problem.

In the following paragraphs, the terms pupil and student are used interchangeably as are secondary school and high school.

THE TUNISIAN PROBLEM

The model was designed to solve a manpower planning problem posed by the Government of Tunisia. They had arrived at manpower targets expressed as university graduations based on manpower needs predicted by an input-output model of the economy. The number of future graduates depends on the number of available teachers, among other things. The number of teachers depends on previous graduations and secondary school enrollments. Thus the presence of this long-term feedback loop of students returning to the system at a lower level as teachers prevented representation of the situation by a transition model. The percentage of university graduates who become teachers is of a magnitude that cannot be ignored. Furthermore, a shortage of teachers can seriously constrain the number of students and often cannot be corrected quickly.

The need to simultaneously consider a large number of programs and levels in the educational system as well as the need to represent the production of teachers over a number of planning years indicated an approach that efficiently models and solves very large problems. Linear programming was chosen for this reason and because of the wide availability of computer codes designed specifically for the optimization of linear problems.

A somewhat arbitrary distinction must be made between those characteristics of the system that are subject to government manipulation and those that are determined by social demand or are otherwise not controllable by the government. School entrances, dropout rates, and continuation rates are largely determined by social demand. The government does have leverage in determining which streams students enter, particularly at the secondary level. There are three types of secondary school streams or tracks, vocational, technical, and academic. They lead to university entrance, not at all, in small percentages, and in large percentages in that order. University entrances can be controlled by rationing entrance to secondary streams and by the examinations at the end of secondary school.

Tunisia spends 8.1 percent of its GNP and 25% of its government budget on education (UNESCO, 1973). These figures are among the highest in the world, and they cannot easily be increased. Thus the country is faced with the now common problem of rationing a service that is priced well below its cost. Political, social, and moral constraints prevent arbitrary or capricious rationing.

STRUCTURE OF THE MODEL

The model considers pupils at all post-primary educational institutions except primary teacher training institutions. Teachers are considered explicitly only for the secondary level.

The linear program minimizes a weighted function of enrollment subject to socially determined minimum throughput and minimum requirement for graduates. Costs are determined but are not used as constraints. Most of the constraints are of the stock and flow type. Stocks of pupils are supplied by the primary schools. They flow through the post primary institutions and into the labor market or back into the secondary schools as teachers.

Student flow consists of three general types (Figure 1) which can be labeled promotees, dropouts, and repeaters. A fourth type, the transferee, can be included as a promotee or a repeater depending on the type of transfer. Figure 1 shows that students flow or move in discrete steps called grades over time measured in planning years or just years. A group in a given grade in a certain year is known as a cohort. Students who are promoted remain in the same cohort as it moves through the system. Students who repeat, represented by the downward pointing arrows in Figure 1, move back one cohort for each repetition.

Figure 1 also indicates the grade and time span of the model, 12 years (excluding the base year) and 16 grades.

The horizontal structure of the system, which has been simplified out of Figure 1, is detailed in Figure 2. The promotion and repetition rates are shown. The residual is the dropout rate. It is clear that the three choice points are the beginning of high school, the end of the common track, and the end of high school.

FIGURE 1

SIMPLIFIED GENERAL PUPIL FLOW OVER TIME AND GRADES

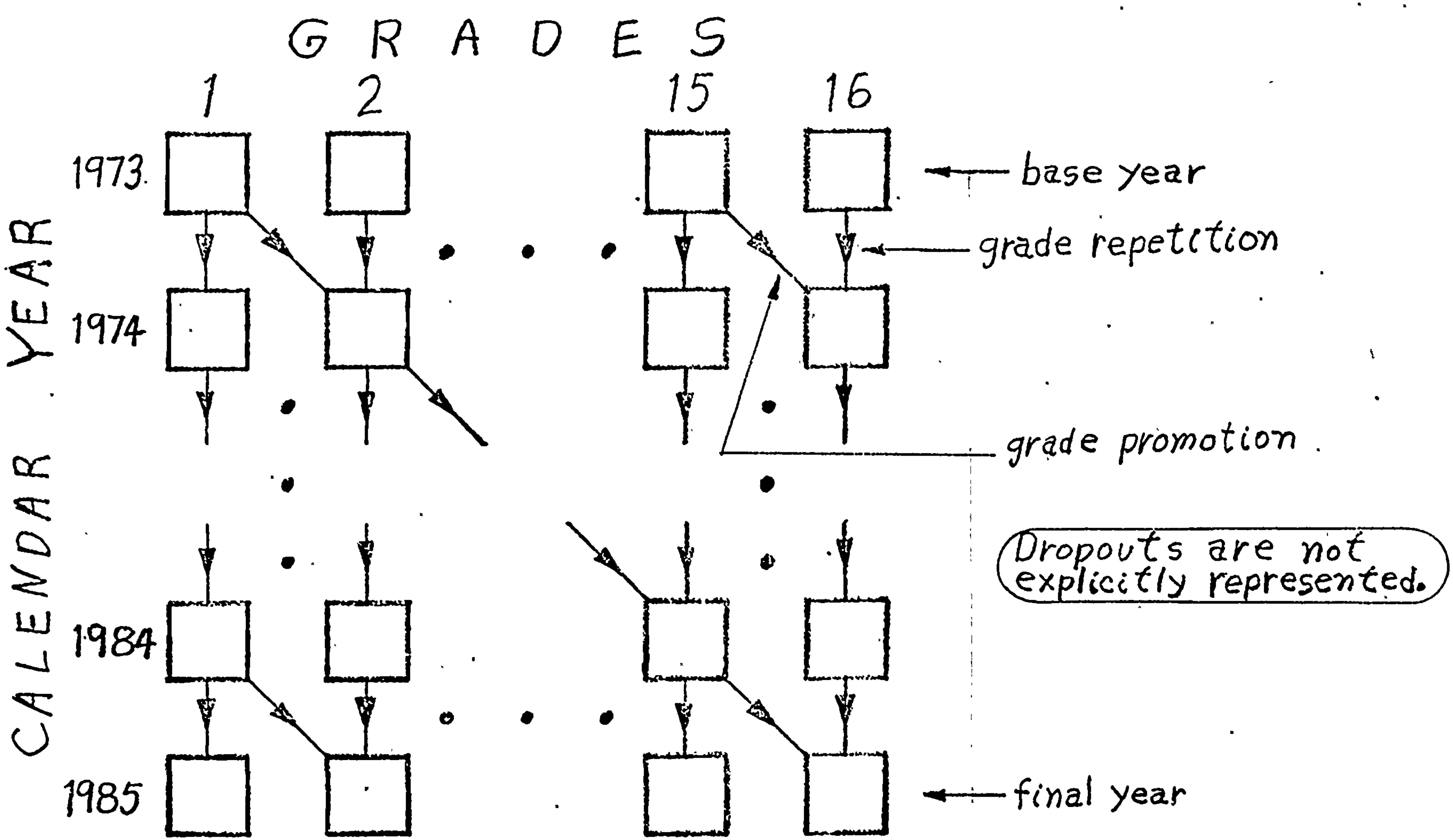

FIGURE 2
SINGLE YEAR FLOW OVER GRADES

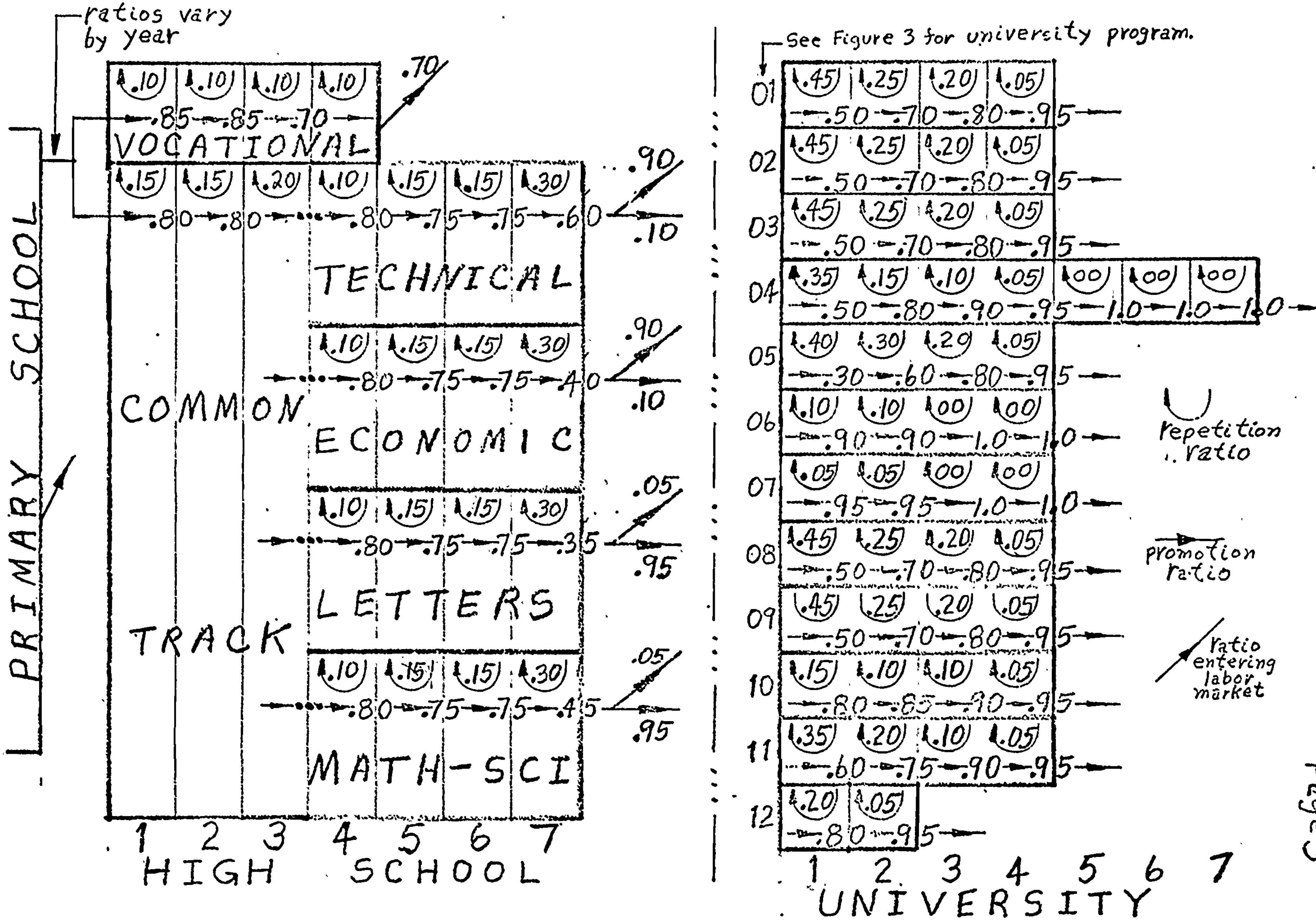

FIGURE 3

KEY TO UNIVERSITY PROGRAMS FOR FIGURE 2

Number	Program Description
01	letters and social sciences
02	pure science
03	law, economics and political science
04	medicine
05	theology
06	agriculture
07	commerce
08	journalism
09	education
10	technical education
11	engineering
12	public administration

The activities or variables represent students disaggregated by grade, year, and track or program. The extent of disaggregation is indicated in Figures 1 and 2.

CONSTRAINTS AND BOUNDS

A distinction needs to be made between constraint and bound. A bound is a special type of constraint that acts on only one variable, that is, it is a constraint that has only one nonzero coefficient other than the right hand side. A bound can always be treated as a constraint but the problem can be solved more efficiently if bounds are separated from other constraints.

The constraints of this model are entirely of the stock and flow type. The stock constraints are those of pupil supply and graduation targets. The flow constraints govern pupil flow from grade to grade over planning years. A distinction is made between flow within a program and flow between programs.

By convention, greater-than-or-equal-to constraints are called 'G' constraints and less-than-or-equal-to are called 'L' constraints.

Pupil Supply

Pupil supply occurs on the left side and the top of the general pupil flow diagram (Figure 1). The top is the supply of pupils for the entire system for the base year, 1973. In this model all 72 base-year variables were represented by fixed bounds. The left side is the entry supply for the lowest level of the system for the remaining 12 planning years. The supply comes from the last year of primary school since the primary level is exogenous. These were represented by equality constraints, reflecting the assumption that the proportion of primary school graduates entering secondary school is controllable or, at least, known.

Pupil Flow Within Programs

Pupil flow within levels involves promotion, repetition, and dropping out. It does not, in general, involve choice of program or institution. This more complex type of flow is treated in the next section.

The basic expression relates three variables: the enrollment in a given grade in a given year, the enrollment in the same grade in the previous year, and the enrollment in the prerequisite grade in the previous year. The first variable of the three receives the flow from the other two and, by convention, has a negative coefficient (Beale 1968, p. 79). In this model a coefficient of negative one was used. The second variable is the supply of promotees and the third is the supply of repeaters, both of which have positive coefficients. These coefficients, which range between zero and one, express the proportions of the total enrollment which can be promoted and repeated respectively.

These constraints were of the L type and can be stated as

$$C_P P + C_R R - E \leq 0$$

where the C's are coefficients and P, R, and E are promotees, repeaters, and enrollment respectively.

The use of an L type constraint is unusual since with the sign convention used a G constraint is necessary to ensure conservation of flow. It is clear that the constraints define a feasible region which is largely infeasible. The objective function is designed to force the optimal feasible solution to be truly feasible. This is discussed under "Nonconstraint Rows."

The purpose of formulating the flow constraints in this way was to model a system which is experiencing pressure to increase throughput. Pupils do not want to drop out of the system and grade repetition is considered undesirable by the government. In spite of this, repetition rates have been high in the past and the formulation used reflects, in part, a d[illegible] the government to reduce them. The problem is one of minimizing cost, hence enrollment, while catering to the throughput pressure. The flow constraints reflect the assumed minimum level of throughput determined by social pressure and government policy.

Pupil Flow Between Programs

There are two places within the system where branching occurs in the pupil flow. Here either the pupil or the system must choose an institution or a program. The first branching occurs between the common track at the end of the third year of high school and the four specialty tracks that follow (Figure 2). The second occurs at the end of high school, upon entrance to university.

The model was left free to optimize the flow to the four high school specialties, subject to conservation, dropout, and repetition. The ultimate determinates of the flow at this point are the manpower target constraints for future years.

The flow situation at the end of high school is much more complex. The 4 high school tracks flow into 12 university programs (Figure 4). Each track flows into at least two programs. Four of the 12 programs receive students from more than one track. Additional quantitative restrictions further complicate the situation.

The adequate representation of this situation required the definition of new variables termed double count variables, and some new constraints

FIGURE 4

UNIVERSITY PROGRAM DESTINATIONS
OF SECONDARY SCHOOL GRADUATES
BY TRACK

Secondary Track	University Program
technical	technical education engineering
economic	law, economics, and political science agriculture commerce public administration
letters	letters and social science law, economics, and political science theology journalism education
mathematics - science	pure science medicine agriculture technical education engineering

to help define their values. The original variables give stocks (enrollments) at the various levels in the system. The double count variables are for portions of enrollments in the first year of certain university programs. The portion is defined by the high school track which these students come from. Since the total enrollment is also a defined variable, the double count variables represent students who are already represented. They are special in this respect and their values must be excluded from certain statistics, as their name suggests.

Teacher Flow Constraints

The teacher flow constraints ensure that enrollments at the high school level do not exceed the amount for which teachers can be supplied. There are no teacher variables in this model. The teacher flow constraints act on the pupil variables under the assumption of fixed pupil/teacher ratios. The flow originates in the final grade of various university programs and goes into the secondary schools as teachers. The end of this flow is not explicitly represented, but the teacher supply can constrain the pupil flow.

For modeling purposes, teachers were divided into three groups defined by broad subject matter specialization. This determined the particular university programs they came from and the proportion of them required for each of the secondary school tracks. The three groups could be labeled arts, science, and economics. They were formed from a common sense aggregation of a more detailed breakdown. The economics group is small relative to the other two. One constraint was used for each subject matter group for each year except the last year.

The supply of teachers for a given year came from two sources, new teachers available from the universities and continuing teachers available from the previous year. The coefficients of the university graduate variables are the proportions that can be induced to go into teaching. The coefficient on the pupils of the previous year is the product of the teacher/pupil ratio, the teacher nonretirement rate, and the proportion of the total teachers from the subject matter group required for the given program (Figure 5).

The demand for these teachers is from the pupils of the current year. The coefficients for these variables are the negated product of the teacher/pupil ratio and the proportion mentioned above. The right-hand-side is zero.

Graduation Target Constraints

The graduation target constraints are G constraints which ensure that the number of university graduates in each program is adequate to meet the exogenously determined manpower requirements of the country. There is a constraint for each university program for each planning year. The right-hand-side is the exogenously determined requirement of graduates and all coefficients are equal to one.

NONCONSTRAINT ROWS

The term nonconstraint row indicates what these rows are not rather than what they are. In fact they serve a variety of purposes. Every mathematical program requires at least one to serve as the objective function. They are also used as change rows to parametrically program the objective function or a constraint. Finally, they may be used

FIGURE 5

HIGH SCHOOL TEACHING DESTINATIONS
OF UNIVERSITY GRADUATES

University Program and Group	High School Track	Proportion of Teachers in High School Program from Program Group
Arts Group	common	.68
01	letters	.80
05	math-science	.39
09	economics	.44
	technical	.26
	vocational	.23
Science Group	common	.32
02	letters	.20
10	math-science	.61
11	economics	.15
	technical	.74
	vocational	.77
Economics Group		
03	economics	.41

(See Figure 3 for key to university programs.)

simply to obtain additional information about the system. This program uses nonconstraint rows for the objective function and as information rows to obtain annual required recurrent expenditures. These two uses are discussed below.

The Objective Function

The objective function, which is minimized, is an arbitrarily weighted function of enrollments. The objective is to minimize enrollments differentially. Expensive enrollments that lead to future enrollments in the system are preferentially minimized over cheap enrollments that don't lead anywhere. This is similar to but not quite the same as minimizing cost. University enrollments received the highest weights followed by academic high school and vocational enrollments in that order.

It is clear that the manpower target constraints rather than the objective function determine the level of activity in the system. Excessively high manpower target levels could result in violation of conservation of flow without yielding an infeasible solution to the model. This seems unlikely and is easy to check, in any case.

Recurrent Expenditure Rows

Nonconstraint rows can be used to obtain additional information because of the way they are handled by the linear programming codes used by computers. They are carried along with the other rows in a computational sense, that is, the same values are reported by the program for all rows regardless of whether they have an upper limit, a lower limit or no limit. Since a nonconstraint row has no limit it cannot become a binding constraint and, therefore, its slack variable is always in the basis. If its right-hand-side is zero, as is usually the case, then the value of the structural or nonslack portion of the row is the negated value of its slack variable.

When the coefficients of a nonconstraint row are per pupil annual costs and the activities are enrollments, the value of the nonslack part of the row is equal to the total cost of schooling for those enrollments with non-zero coefficients. This can be read from the solution output. For this model cost rows were used for each program in each year.

THEORETICAL PERSPECTIVES

There are two basic philosophies of educational planning, neither of which is ever seen in pure form. These are the predictive and the prescriptive. The predictive philosophy assumes that important variables in the educational system are externally controlled by some outside force, such as social demand or local political bodies and that the job of the planner is to predict these variables so that correct decisions can be made concerning teacher training, school construction, etc. The prescriptive philosophy assumes that the government or the planning body controls all the important variables and the job of the planner is to calculate the correct values so they can be imposed on the system. Clearly, the observed mix of these two extreme assumptions depends on the prevailing political philosophy and the relative power of the groups involved in decision making. In practice, the planner must identify those variables that can merely be predicted and distinguish them from the variables that can be controlled. The resulting plan is very much affected by the choice of these two sets of variables.

At first glance, linear programming would appear to be more suited to prescriptive planning, since it sets variable values in such a way that some aim or objective is optimized. A closer look reveals its utility for both approaches. There are two phases to the LP solution process. The first phase finds a feasible solution, any feasible solution, and the second finds the optimal solution. If the student flow problem is purely predictive, that is, if the flow rates, pupil-teacher ratios and other important parameters are given, then the problem is one of finding a feasible solution, the only feasible solution. If such a model is solved with linear programming, the objective function is irrelevant. In a less extreme case of predictive planning, the feasible region may be very small. In contrast, if the important variables in the educational system are controllable then the feasible region is very large and the objective function assumes considerable importance, since it locates the optimal solution within the feasible region. Linear programming is therefore a useful solution procedure for any combinations of prediction and prescription.

If the only concern of a model is student flow, the use of a straightforward Markov transition approach is to be preferred over linear programming. The transition matrix can be applied sequentially to the enrollment vectors for each year. This greatly reduces the size of the main matrix representing the problem. The matrix of a multiyear planning problem represented in linear programming form will have Y^2 times as many elements as the Markov transition matrix where Y is the number of planning years. The transition matrix approach permits the calculation of teacher requirements and costs, but they cannot be used as constraints.

SUMMARY

Linear programming has proven to be a valid, useful, and convenient tool for large scale educational planning involving student flow modeling. It effectively solves the teacher feedback problem and permits the simultaneous solution of all parts of a model extending over a large number of planning years.

Any combination of predictive and prescriptive assumptions can be handled by LP. The essential distinction is in the size of the feasible region. An essentially predictive problem will have a small feasible region. The solution algorithm will spend most of its time searching for a feasible solution. An essentially prescriptive problem will have a large feasible region. The solution algorithm will spend most of its time optimizing the user-given objective function within the feasible region.

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