Analyzed is the Piagetian concept of cognitive structure (concerning an individual's thought organization) and noted is the lack of precise psychological definition. Reviewed are deficiencies in Piaget's theories relating to developmental stages of the cognitive structure. Elements of the cognitive structure are defined; five basic assumptions are set forth (including the existence of unique and indeterminate cognitive structures for all living animals); and the definitions and assumptions are mathematically represented. Discussed are the relationships between cognitive structure and intelligence and advocated is the substitution of a cognitive structure index for the standard IQ measure. The relationship between cognitive structure and learning is said to be demonstrated by the need for a teacher to match the instructional material's structural complexity to the complexity level of the learner's cognitive structure. (CL)
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THE PROCESS OF COGNITIVE STRUCTURE

COMPLEXIFICATION

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The University of Minnesota Research, Development and Demonstration Center in Education of Handicapped Children has been established to concentrate on intervention strategies and materials which develop and improve language and communication skills in young handicapped children.

The long term objective of the Center is to improve the language and communication abilities of handicapped children by means of identification of linguistically and potentially linguistically handicapped children, development and evaluation of intervention strategies with young handicapped children and dissemination of findings and products of benefit to young handicapped children.
I. Introduction

To Jean Piaget (1963), thought is not only a psychological but also a biological phenomenon. Given that, thought must comply to the biological principles of organization and autoregulation. The principle of organization refers to the tendency of various parts of an organism to function together in a coordinated manner as an organized whole. In other words, according to this principle, the mechanisms and capabilities of an organism tend to form systems. The principle of autoregulation refers to the capability of an organism to interact with its environment in such a way as to maintain certain crucial organism processes. In other words, autoregulation relates to the capacity of an organism to regulate and preserve itself and its interactive capabilities with its environment. Autoregulation is closely related to organization as autoregulation functions only through an implicit organization (Piaget, 1971a).

From a Piagetian perspective, much of human development is the result of an on-going interaction between the process of autoregulation and the process of organization which are designated by the corresponding biological principles. An important product of that interaction is cognitive structure. The term "cognitive structure" has been primarily used by psychologists in...
the explication of the thinking capabilities of a live human. However, the term "cognitive structure" is also applicable to some non-human thinking organisms such as chimpanzees and orangutans; thus, the term may best be viewed as referring to organisms that live and think. Basically, a cognitive structure reflects the organization of thought of an organism for some interval of time and is a construct useful in many sciences that discuss the human mind and its constructions (e.g., sociology of knowledge, cognitive psychology).

Cognitive structures are psychological structures which relate to the realm of thought. Surprisingly, neither the term "cognitive structure" nor its parent term "psychological structure" or "structure" has a definition shared by psychologists. Flavell (1971) has commented on this lack of definitional consensus and offered a synthesis which describes structure in terms of a set of elements with an interconnecting organization acting in such a way that the organization is relatively stable and affords a wide range of capabilities. Furth (1969) described structure in terms of interconnections of the parts of a whole, and stated that the term is synonymous with organization, form, system, or coordination. Gardner (1964) defined cognitive structures as "enduring arrangements of cognitive processes that shape the expression of intentions under particular types of environmental conditions." Kagan (1970) talked about the role of attention in changing cognitive structure but failed to give a definition or reference as to meaning or formalization.

For Piaget (1970), structure relates neither to a whole nor to its constituent parts, but instead to the array of pertinent transformations.
that can be posited on the whole. Three explanatory concepts useful in his
description of structure are wholeness, transformation, and self-regulation.
The last concept has been discussed already, the concept of transformation
is self-evident, and the first concept refers to the complete integration
or structuration necessary for the existence of the whole. The unifying
element inherent in structure can be transformational, relational, or simply
definable in terms of some set of morphisms. However, totality or whole-
ness must be further explained at at least two levels: 1) in terms of its
essence, or its defining properties; 2) in terms of the mechanism of
structure formation, preformation, or interactive (dialectic) construction.
With regard to the first level, an approximation can be obtained through
formalization in terms of some model. However, a model only captures and
depicts the formal properties of the totality of the structure (Gödel, 1931).
As for the second level, further explanation of the totality of the structure
can occur with the solution of such questions as the following: 1) if the
structure is preformed, from what did it evolve?: 2) if the structure is
constructed, what were the materials and what was the process of construction?
Since the transformations defining structured wholes are simultaneously
organized and organizing in the realm of the cognitive (Piaget, 1971a), it
is difficult to state necessary prerequisites for a structure. In addition,
structures function in a closed manner in that they cannot transcend their
existence.

Other scientists such as linguists also use the term "structures". For example, linguistic structures for Noam Chomsky are transformational
and generative. Language, an outgrowth of cognition and its structures,
readily lends itself to structural analysis because of its symbolic nature. However, language is but a part of the total cognitive repertoire of an individual and thus linguistic structures can never exhaust the intelligibility of a cognitive structure.

A few general comments can be made about the term "cognitive structure". First, developmental psychologists with similar theoretical orientations posit similar but not equivalent definitions. Some developmental psychologists even contend to examine cognitive structures, when they do not even define what it is they are examining. Of the definitions cited, the one provided by Piaget is the most exacting, but even that definition is not well-defined and logically or mathematically precise. As for Chomsky and the other mathematical linguists who have made such progress in the structural analysis of language processes, thus far they have not extended their discussions on structure in the linguistic domain to the cognitive domain. One obvious immediate conclusion is that the structural analysis of cognition will be severely retarded as long as there remains no precise, meaningful, and generative definition of cognitive structure.

Definitional precision notwithstanding, Piaget has constructed a developmental theory of cognitive structures. In this theory, Piaget designates the three major qualitatively-distinct periods of cognitive development as sensorimotor, concrete, and formal (Piaget, 1950). As is the case with all organic development, cognitive development has three primary components: genomic preprogramming, environmental press, and all-prevading autoregulation. The order of development of the periods is contended to be universally invariant as is the order for the sequence of stages that constitute the periods.
The rule-systems underlying the structure at various periods have been defined with varying specificity. Inhelder and Piaget (1958) have formalized the period of formal operations; Apostel, Grize, Papert, and Piaget (1963) have attended to the filiation of concrete operational structures. However, the formalization is neither unified, nor complete, nor rigorously precise. Such an unfortunate state of affairs may be due to the imprecise definitions of cognitive structures. However, in some fields, with a paucity of assumptions, quite elegant results have been obtained. For example, ethology has some fine theoretical stances even with the few assumptions being considered (Ruwet, 1972). Also, Einstein (1917) constructed his theory of special relativity with precise definitions and from only two assumptions: 1) the constancy of the velocity of light, and 2) the principle of relativity of physical laws which states that physical laws retain their form in varying coordinate systems. Moreover, his approach was synthetic. Two opposing views were fused to form a consistent and richly explanatory theory. Thus, given a few all-encompassing, synthetic postulates, it might be possible to construct a richly informative structural theory of cognitive development.

Notwithstanding the lack of total unity of Piagetian cognitive theory, the model of the formal operational period itself has some weaknesses. N. Isaacs (1950) commented on the incompleteness of Piagetian logic and suggested that Piagetian logic be extended to account for multivalent or modal logical operations. The logic proper to scientific investigation, language, and concepts should also be included in a complete formulation of the formal cognitive period. In fact, N. Isaacs introduced the notion of "psycho-logic" as a separate discipline to consider issues such as those. Parsons (1960) commented on the lack of precision of Piagetian logic and
in fact questioned even the intents of Piagetian investigation of adolescent thought. Bruner (1960) criticized the gaps in explanation in Piagetian theory as regards the structures, mechanisms, and "strategies" inherent in an operational system. He also questioned the notion of equilibriums. Bart (1971) extended Piaget's model of formal operations to account for possible development of cognitive structures within the formal operational period.

The concrete operational period formalized by Piaget also has deficiencies. One primary weakness is the fact that Piaget provided eight mathematical structures labelled groupings I-VIII to depict concrete operations and ignored structural representations that may capture the totality and interconnectedness of the period (Flavell, 1963). Thus, one of the defining characteristics of the concrete cognitive period as represented by Piaget is its lack of complete integration and unity. As for the sensorimotor period of thought, Piaget (1971) mentions the existence of a logic proper to the period, but neither references nor defines the axioms proper to the logic.

Obviously, there is needed well-defined integrated conceptualizations of cognitive periods and their constituent cognitive structures. Cognitive structure is not static but is mutable, dynamic, and complexifiable. Also, one of the distinctive features of an organism is its autoregulatory character (Piaget, 1971). Thus, it seems that cognitive structure obtained at each period of thought should have a unity that allows it to exist, function, and develop as a part of its physical and social environment. Pursuant to this goal, the rule-systems obtained at the various periods should be formally and unambiguously articulated. Furthermore, the cognitive structure defined by any rule-system must account for humans rather than epistemic, platonic subjects. Thus, such a model must be compatible with idiosyncratic cognitive
developmental patterns determined by various genetic creodes (Flavell, 1971; Werner, 1957). Thus, there needs to be constructed a "periodic table" of cognitive structures in order for progress in the structural examination of cognitive development to be accelerated. This paper is intended to contribute to the structural analysis of cognitive structures as it provides a precise formulation of cognitive structures.

II. Elements of the Interpretive Framework

A Definitions.

A pragmatic first step in the precise conceptualization of cognitive structure and its related theory is the definition of such crucial terms as cognitive structure. The cognitive structure of an individual animal for any given time interval \((t_2, t_1)\) is the regulatory entity which controls all the cognitive processes of which the organism is capable. The cognitive structure may also be defined as a composition of its three constituent parts: elements, operations, and rules. Elements are the input and output of the cognitive structure and are all those entities that are attended to, thought of, and cognized; in other words, elements are defined as the objects and the products of the cognitive structure. Operations are the actions performed on the elements and are defined with the elements that they act on and the elements that they produce being designated. Rules are the relations among the operations which govern the order and forms of employment of the operations; for example, some rule might indicate that certain operations may be used after certain other operations and that other operations may not.
An example of an element is included in the following case. A person sees a glass and thinks about it. Thus, the glass is an element that can be subject to an operation. If the glass is displaced and is again focused on, an operation has occurred. The set of elements in a cognitive domain is the domain of the cognitive structure and the set of operations may be defined as a set of functions defined on the domain. The domain of a cognitive structure, though very large, does not necessarily include all sensory input; however, during development, the domain will most likely increase in size for any individual.

In considering cognitive structure from a developmental perspective, one crucial concern is the isolation and identification of certain levels of development. Various cognitive theorists, Bruner (1966) and Piaget (1971b) and others, speak of certain isolable levels, which can be referred to as periods: periods would be "long" intervals of time over which some order and unity is maintained within a cognitive structure. Within the identifiable periods, there are sub-divisions called stages. Within the interpretive framework, a period is designated by cognitive structures that have the same rules and a stage is designated by cognitive structures that have the same rules and similar operations with an operation 1 being similar to an operation 2 if 1 is embedded in 2 or 2 is embedded in 1. Intra-stage change occurs with changes in the domain and changes of addition or subtraction in the functional domains and ranges in the cognitive structure. Inter-stage intra-period change occurs with changes of addition or subtraction of operations in the cognitive structure. Inter-period change occurs with changes in the rules of the cognitive structure.
If one is dealing with a continuous phenomenon of development, the course of cognitive structures could be sub-divided ad infinitum, but periods and stages are sufficient at this time. In fact, within the interpretive framework, development of cognitive structure and the process of cognitive structure complexification is not viewed as continuous but rather as dense, discrete, and denumerable as the set of rational numbers.

Associated with period and stage are such adjectives as optimal, maximal, and possible rather than real or actual. Perhaps cognitive structure is best described as a noumenon, or thing-in-itself, rather than as a phenomenon which can be easily observed. If such is the case, a theoretical attack could indeed be fruitful. However, the question of what bounds apply to cognitive structure does exist and will be considered later in the paper.

B. Assumptions

Basically there are five assumptions in the interpretive framework.

Assumption 1. For each living animal, a cognitive structure exists.

The existence is determined or determinable through interaction. Interaction cannot be instantaneous and comply to physical laws: thus, the structure exists as a dynamic entity over a time interval rather than for a single point in time. The existence of the structure is purely functional, in terms of how it is interacting. If the contrary were true, i.e., the structure did not exist, the idea of explaining it or modelling it would be ludicrous.
Assumption 2: The existing cognitive structure for any living animal is unique.

This assumption asserts that there is one and only one regulatory structure for cognitive processes. Its range is all cognitive phenomena—all phenomena that occur because of a nervous system. The uniqueness is only for one time interval, because, at a later time, the cognitive structure might not be the same. If the structure were not unique, an adequate foundation would be much more difficult, for one would have to consider carefully which structure was being described and why.

Assumption 3: The cognitive structures for any living animal over time form an inclusion chain.

This assumption contends that for a time interval \((t_2, t_1)\) beyond a time interval \((t_1, t_0)\) with time being measured from conception of the animal, the cognitive structure at \((t_2, t_1)\) includes that of \((t_1, t_0)\).

The inclusion chain is not in terms of performance: the cognitive structure itself is monotonically increasing or at least not decreasing, and is not subject to motivational factors, fatigue, or the like. There are certain counterexamples: physical or chemical alteration of the structure through, for example, gunshot wounds or noxious chemicals. Aging, or at least senility, might also be considered a counter-example.

Assumption 4: The set of all cognitive structures manifested by an assemblage of \(N\) living animals forms a semi-lattice with a common infimum.

Genetic considerations notwithstanding, the cognitive structures would all be commonly zeroed at conception. In addition, assumption 4
asserts that for any two cognitive structures manifested in a group of animals, there exists a cognitive structure that is included in the two cognitive structures but not necessarily a cognitive structure that includes the two cognitive structures. Furthermore, if previous structures for each animal are also introduced into the semi-lattice, the only necessary inclusion is for the single animal. Of course, every cognitive structure would include the minimal point. Perhaps such a semi-lattice could be constructed for all living animals with cognitive structures, but, needless to say, its construction would be a non-trivial task. The semi-lattice posited does not constrain each animal to follow an invariant path, but allows for somewhat individualized or idiosyncratic development.

Assumption 5: The cognitive structure for any living animal is fundamentally uncertain and indeterminate.

The uncertainty refers to predictability, diagnosis, operationality, and explication of the structure with respect to present, future, and even past action of the structure. Probabilistic methods, group-theoretic methods, or even cybernetic methods might be useful in description, but the problem of cognitive structure cannot be solved exactly. Good examples of indeterminacy exist in law, chess games, and even mathematical discoveries or inventions (Hadamard, 1952).

C. Representation

The definitions and assumptions heretofore-cited bear elements that are susceptible to mathematical representation. One such element is cognitive structure. A cognitive structure C may be defined as an ordered
triple \((D, F, R)\) where \(D\) is the domain, \(F\) is the set of functions defined on \(D\), and \(R\) is the set of rules defined on \(F\).

Let \(\text{Dom}\) refer to domain and \(\text{Range}\) refer to range, then \(\text{Range} \ F \subset \text{Dom} \ F\) and \(F: (f | f: D \rightarrow D)\). Also, for living animal \(i\), there exists a cognitive structure \(C_i\) for animal \(i\). Let \(t_0, t_1, \text{and} t_2\) be three successive times, then \(C(t_1, t_0) \subset C(t_2, t_1)\) is a representation of assumption 3.

In addition to the five assumptions cited, two other widely held views may be heeded in any discussion on cognitive structure and its subsequent representation. One such view is the contention by Piaget (1950) that there is an invariant sequence of periods. Within the interpretive framework, that Piagetian contention is interpreted in terms of the statement that the \(R\)'s manifested by animals in their cognitive structures form an inclusion chain. Another view is that there are most likely alternative routes of psychological development (Langer, 1969; Waddington, 1957; Werner, 1957). That view is interpreted by stating that cognitive stages designated by cognitive structures do not form inclusion chains and thus the \(D\)'s and \(F\)'s in the cognitive structures of animals would not most likely form inclusion chains. Thus, \(C\)'s for each animal and \(R\)'s manifested in all \(C\)'s determine linear orders. \(C\)'s for the set of animals determine semi-partial orders, and \(D\)'s and \(F\)'s determine unknown orders.

III. Discussion

A formulation of cognitive structure has been provided which has emanated primarily from Piagetian theory. This formulation has also taken into consideration the suggestion of W. Kessen (1966) that in discussing the underpinnings of cognitive capabilities one should "talk in terms of 'operators' related by 'rules'." Within this framework, three levels of
cognitive structure complexification are posited: 1) changes in the domain constitute the most surface and immediate level; 2) changes in the function set constitute the intermediate level; 3) changes in the rule set constitute the deepest level of complexification.

One concern regarding developmental patterns considered by Flavell (1971) and in part by Van Den Daele (1969) that has partly been considered is discussion as to whether an additive model of substitution model of cognitive structure complexification is the most reasonable. It is posited that both models are operative if one considers cognitive structure from an ethological viewpoint with the ethological concept of threshold (Eibl-Eibesfeldt, 1970). A cognitive operation i which has a high threshold is less likely to be employed than a cognitive operation j which has a low threshold given that both operations can be used on the same cognitive element which is being thought of. What is posited is that over time the function (operation) set for a cognitive structure increases in a cumulative, additive manner as reflected in assumption 3 but, as new functions are added, those functions will tend to have low thresholds and other older functions that are more consolidated and definable on similar function domains may tend to develop high thresholds; the threshold changes in functions would mirror the substitution model of cognitive development. An example of this situation is that when an adult is asked a theoretical question he will tend to respond by using a cognitive operation proper to formal reasoning; however, with effort, he would respond with the use of a cognitive operation proper to preoperational reasoning and give a childlike response. One problem for cognitive researchers is the determination of methods to effect threshold change in cognitive operations.
Though there are many other topics that can be considered with respect to cognitive structure such as mechanisms determining rate of cognitive structure complexification, two topics that are of substantial importance are the relationships between cognitive structure and intelligence and between cognitive structure and learning. Presently, research on intelligence is dominated by the linear I.Q. model which has been criticized by various researchers (Bart, 1971; Scarr-Salapatek, 1971). Cognitive structure would be a fine candidate for a qualitative, richly informative replacement for I.Q. as an index of intelligence. The new formulation would not be a simple linear order, but instead would be partially-ordered perhaps as a set of triples. For example, (1,0,0) is less than (0,1,0), (0,0,1) and (1,0,0) which are less than (1,1,1). Thus, a lattice formulation provides some basis for lines of ordinal scales of intelligence. However, the determination of interval and ratio scales of intelligence from a cognitive structural framework is highly problematic as it requires the determination of metrics on cognitive structures.

If research on intelligence is instituted with cognitive structure being used as the index of intelligence, many experimental questions and methods of data analysis traditionally used with intelligence research will have to be reconsidered.

Another topic is the relationship between cognitive structure and learning. The structural complexity of that which is to be learned must be less than or equal to the complexity of the cognitive structure of the learner in order for learning to occur. In other words, if a teacher wanted to teach a pupil some body of subject matter, the assimilatory
capabilities of the student's cognitive structure could handle only a
structured body of material that can be included in his cognitive structure.
Furthermore, close to some transition, the structure of the subject matter
could somehow determine the route of the transition. (Piaget mentions a
certain compulsion or necessity associated with a structure in transition.)
On the other end, knowledge output would also have to be less than the
structure which generates it. Thus, the expulsion and ingestion of
information must proceed according to the structure of the organism.

Though an attempt at precise conceptualization of cognitive structure
has been provided, it is far from complete due to the extraordinary
complexity of cognitive structures. The charge thus is two-fold: 1) the
planning of experiments and the scrutiny of research to determine the
psychological validity of aspects of this interpretive framework of cog-
nitive structure; 2) further delineation, and articulation of the theory
of cognitive structure. These designated activities should contribute
substantially to the goal of synthesis of psychological formulations of
cognition.
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