The thrust of this research exploration is aimed at one of the most pressing social problems facing America today: segregation in the public schools. The school system is only one of a complex set of systems, all interrelated, that comprise the entity that we call a city. The geography of the school system—the location of facilities and the location and movement of students to those facilities—is the only observable expression of a complex geo-political decision making system. Many considerations go into the making of essentially spatial decisions that are not primarily concerned with spatial efficiency or with the physical system’s day to day operation. It was for this reason that we decided to explore the nature of school location systems in the context of a hypothetical city as a means of minimizing nongeographical influences. We have borrowed the concept of spatial efficiency in school systems. Distance minimization is the comparative measure of the efficiency of alternate arrangements. In order to partially conform to conventional wisdom on urban spatial structure, we chose a polar grid system as a concession to the classical sector and ring theories. The execution of the problem was done using the Linear Programming Algorithm from the IBM Mathematical Programming Systems package on the Pennsylvania State University’s IBM/360 computer system. (Author/JM)
No. 9

City Space and Schools and Race: A Conceptual Safari into the Wilds of Urban Public Education and Its Geographic Role in Contemporary Urban Crises

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"... Contrast between different groups of people is what we recognize as culture. In our pluralistic society, we have strong commitments to maintain healthy sub-cultural contrasts. We also have strong commitments to maintain equality for all groups. However, when these cultural contrasts are the basis for discrimination in public services--schooling, occupations, housing--we have placed an undemocratic value on the differences."

Daniel Carson (1969, 161)

The thrust of this research exploration is aimed at one of the most pressing social problems facing America today; segregation in the public schools.

Let us acknowledge at the onset recognition that school segregation is only one symptom of the social disease we call racism, and that resolving the problems of school segregation is a symptomatic cure and does not directly attack the causes of racism. But if ever there is to be hope of finding a cure for our national cancer it must be in part in the education of our children. And how indeed are we to teach anti-racism within the walls of a segregated school.

Time is fleeting. The lag in the realization of social change to be derived from a re-education program is at least a full generation, and each step will be against the tide of cultural traditions of bigotry and racism. In the meantime the social climate gets continually worse.
Outmigration of whites from America's city centers has nearly quadrupled to an average rate of half a million a year between 1966 and 1968 (Carson, 1969, 160). In many major metropolitan areas Negro children dominate the public school population as white parents flee to the suburbs or place their children in private or parochial schools. The infamous "tipping point" concept observed in the housing and real estate literature may well apply to urban public schools. Not only individual schools but entire school systems may become segregated as a result of current social forces.

We are "headed for two societies--one black, one white--separate and unequal" (NACCD 1968). The trend must be checked, perhaps even reversed if our highly urbanized society as we know it is to survive.

The school system is only one of a complex set of systems, all interrelated, that comprise the entity we call a city. The geography of the school system--the location of facilities and the location and movement of students to those facilities--is the only observable expression of a complex geo-political decision making system. Many considerations go into the making of essentially spatial decisions that are not primarily concerned with spatial efficiency or with the physical system's day to day operation. Behind the location decision of every school facility in any real city exist enough non-geographical considerations to prevent an interpretation of spatial patterns that makes geographical sense. It was for this reason that we decided to explore the nature of school location systems in the context of a hypothetical city as a means of minimizing non-geographical influences.

We have borrowed from Yeates, as a point of departure, the concept of spatial efficiency in school systems (1968, 107). Distance mini-
zation is the comparative measure of the efficiency of alternate arrangements, and school integration is the dominant theme.

The Hypothetical City

In order to partially conform to conventional wisdom on urban spatial structure we chose a polar grid system as a concession to the classical sector and ring theories. The grid is the city, or perhaps more aptly the metropolitan area. It is assumed only that there exists only one school district covering the entire area. The number of sectors and rings was expanded until the number of areal units defined by their intersection was large enough to comprise an adequate data base; in this case there are 176 neighborhoods or tracts.

No specific population is considered, nor is there any particular type of school involved. We are considering only schools, and students and their mutual geographical needs. We arbitrarily designated five schools for our city and a total student population of 5000 to 7500. After numbering our grid areas from 1 to 176, a set of two digit random numbers was used to assign a student population of 1 to 99 for each of the tracts.

A twenty tract contiguous area was selected in which all school children were assumed to be black. The tracts selected were the most central five in each sector of the north-west quadrant of the city (Fig. 1). This area defines a ghetto in our city's residential space. The sectorial nature of the ghetto conforms to a degree to real world experience.

Keeping in mind Colin Clark's findings regarding the characteristics of population density, we multiplied our student population figures by a
density bias vector ranging from 1.0 to 0.5 to obtain a negative exponential function. In doing thus, higher population densities occur near the core of the city tapering off toward the edge. In addition, to concede to the popular notion of overcrowded ghettos, the black student populations were increased by an arbitrary 20%. The resultant student population totals 6628 with 1146 blacks (See reference numbers on Figure 1).

The next step was to locate the five schools within the hypothetical city. We felt it necessary to have at least one school in the ghetto with the remaining four evenly distributed radially throughout the city. Using only these guidelines random numbers were drawn to select the actual tract for location of the school facility (see Figure 1).

The Transportation Algorithm and Spatial Efficiency

Considering that our problem entails the movement of students to and from schools, the aggregate cost of such movement provides a means of comparative measure of the efficiency of spatial organization of our system. In lieu of cost data simple straight line distance will serve as a surrogate for transport cost. A homogeneous transport surface is assumed.

The objective of the algorithm is to move all students to and from school in such a way as to minimize aggregate transport cost. The objective can be expressed mathematically as:

\[
\sum_{i=1}^{n} \sum_{j=1}^{m} S_{ij} C_{ij} = \text{Minimum} \quad i = 1, 2, 3 \ldots n
\]

where: \( S_{ij} \) = Number of students to be moved from any ith tract to any jth school.

and: \( C_{ij} \) = The transport cost (distance) incurred in moving the students from the ith tract to the jth school.
In this particular problem the address integer \( j \) is any of the 5 schools serving the city, and the address integer \( i \) refers to any of the 176 tracts. Each source (tract) and destination (school) pair has associated with it a cost (distance) of movement which can be compared to all other transport costs. A summary of the necessary \( C_{ij} \)'s is provided in Table 1.

**TABLE 1**

*Mat of Data*

<table>
<thead>
<tr>
<th>Location of Students (Tracts)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>( j )</th>
<th>( m )</th>
<th>Number of Students at each Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( C_{11} )</td>
<td>( C_{12} )</td>
<td>( C_{13} )</td>
<td>( C_{ij} )</td>
<td>( C_{im} )</td>
<td>( a_1 )</td>
</tr>
<tr>
<td>2</td>
<td>( C_{21} )</td>
<td>( C_{22} )</td>
<td>( C_{23} )</td>
<td>( C_{2j} )</td>
<td>( C_{2m} )</td>
<td>( a_2 )</td>
</tr>
<tr>
<td>3</td>
<td>( C_{31} )</td>
<td>( C_{32} )</td>
<td>( C_{33} )</td>
<td>( C_{3j} )</td>
<td>( C_{3m} )</td>
<td>( a_3 )</td>
</tr>
<tr>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
</tr>
<tr>
<td>( i )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
</tr>
<tr>
<td>( n )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
</tr>
<tr>
<td>Number of Students in Each School</td>
<td>( b_1 )</td>
<td>( b_2 )</td>
<td>( \cdots )</td>
<td>( b_j )</td>
<td>( b_m )</td>
<td>Total Number of Students</td>
</tr>
</tbody>
</table>

---

*Note:* The table format is not perfectly aligned due to the page layout, but the content is clear and understandable.
Figure 1. Hypothetical City
The mxn matrix of transportation costs \((C_{ij})\) was calculated by measuring air distances from the center of each tract to the center of each of the five schools. Although air distances proved expedient in this case the model does not preclude the use of more complex forms of distance measurement. It has the advantage of being relatively bias free. The final matrix is of a 5 x 176 dimension and contains the results of 880 measurements.

The Solution

The solution to the problem as outlined above is to find a set of flows where:

\[
\sum_{i=1}^{176} \sum_{j=1}^{5} S_{ij}C_{ij} = \text{Minimum} \quad i = 1, 2, 3, \ldots 176 \quad (1-1)
\]

subject to the following constraints:

\[
\sum_{j=1}^{5} S_{ij} = a_i \quad i = 1, 2, 3, \ldots 176 \quad (1-2)
\]

\[
\sum_{i=1}^{176} S_{ij} = b_j \quad j = 1, 2, 3, \ldots 5 \quad (1-3)
\]

\[
\sum_{i=1}^{176} a_i = \sum_{j=1}^{5} b_j \quad (1-4)
\]

\(a_i\) = Number of students in the \(i\)th tract

\(b_j\) = Number of students in the \(j\)th school

The constraint (1-2) implies that the number of students assigned to one or more of the schools from each tract cannot exceed the number of students in that tract. Equally, constraint (1-3) implies that the total number of students assigned from the 176 tracts cannot exceed the defined capacity of a particular school. Finally, constraint (1-4)
implies that the total number of students available is exactly equal to the combined capacities of the schools. In this problem there is a total of 6628 students and the total capacity of the five schools is equally 6628 students.

The solution to the problem as structured will result in students being allocated to the various schools at the least aggregate cost to the system. From the solution to the problem, tracts can be grouped to show the student hinterland for each school. The solution to the problem is a normative one, and can be used as a base to which other solutions may be compared.

The execution of the problem was done using the Linear Programming Algorithm from the IBM Mathematical Programming Systems (MPS) package on the Pennsylvania State University's IBM/360 computer system.

Procedure

Consideration of spatial sensitivity in our hypothetical system as measured by change in the transport surface is variable along two dimensions. The first involves a racial constraint and compares a raceless free allocation system to a bi-racial system with varying mix conditions ranging from the totally segregated system to a completely integrated system. The second dimension involves variation in capacity at any given location, effectively eliminating the capacity constraint. This may be considered as an operational substitute for relocating schools, or it may be taken literally as an approach to reduce transport costs by adding facilities at some locations and abandoning facilities in others. Table 2 outlines the research strategy.
Table 2

<table>
<thead>
<tr>
<th>School Size</th>
<th>Free Allocation</th>
<th>Totally Segregated System</th>
<th>50/50 Plan</th>
<th>100% Integration System</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed and Equal</td>
<td>*</td>
<td>* Black Only</td>
<td>* Black Only</td>
<td>*</td>
</tr>
<tr>
<td>Variable and Open</td>
<td>*</td>
<td>* Black Only</td>
<td>* Black Only</td>
<td>*</td>
</tr>
</tbody>
</table>

The Map Series

Figure 2 illustrates the spatial consequences of a free allocation system. Each school is assigned a capacity of one fifth of the city's student population. School 1 is allocated a student body that is 68.3% Black which is actually a reflection of the degree of residential segregation. School 5 is composed of 18.2% Blacks which is near the city's average of 17.3% black students. The three remaining schools have no black students at all. The cost of moving the black student to school is computed to be 3841 cost units or about 10.6% of the system's aggregate transport cost.

In the completely segregated school system, the cost for black students remains about the same (3844). The 50/50 plan which keeps 50% of all Blacks at school 1 and equally redistributes the balance to
the other four schools experiences a cost increase for moving Blacks to 6917 cost units or 185% (Figure 3).

Stepping up the integration plan to achieve 100% integration involves a similar configuration but with somewhat larger flows (Figure 4). The cost of the plan is up to 9388 cost units; up 152% from the free allocation system. This figure represents 24.0% of the total system transportation cost.

Figure 5 shows the effect of the 100% integration plan on the city's spatial pattern. The assignments are color coded to emphasize the fragmentation that occurs on the ghetto side of the central city area. The yellow areas are allocated to the school located within the ghetto boundaries. In this plan the degree of integration, i.e., the percent of Blacks in each school is the same as the ratio for the city as a whole (17.3%).

In an effort to more clearly illustrate just what has occurred in Figures 2, 4, and 5, an additional map is offered to show changes in tract assignment as a result of the integration plan. Areas of similar color share similar assignments. The light lines show the link between tracts and the schools to which they are assigned. Only the colored areas have experienced change from one school to another as a result of the integration plan. Twenty-five tracts out of a total of 176 tracts for the entire city fall into this category, for a total change of 14.2%. The total cost for this plan comes to 39,775 cost units; up 10.9% over the free allocation system (Figure 6).

An alternative approach, as mentioned earlier, is to remove the capacity constraint from the individual schools and allow all students
Figure 4.

Allocation of Blacks with 100% Integration
to follow the path of least cost. Operationally, this was done by setting the capacity for all schools at some figure far above the potential allocation and providing a dummy source of students with a high transport cost associated with their movements. The real students file into the school which minimizes their transport cost and the dummy fills in the balance. When a solution has been reached, the dummy students and the cost accrued by their movement is subtracted from the results and optimal school capacities have been established.

Figure 7 shows the resultant spatial pattern from the procedure outlined above. In general, the more centrally located schools expanded their hinterlands at the expense of the more suburban schools. In all, 31 tracts were reassigned for a change of 17.6%. Aggregate transport cost was reduced to 32,967 cost units or about 9.0%.

Introduction of the 100% integration plan to this configuration triggers active changing. Figure 8 repeats the mapping procedure used in Figure 5. The price for a more efficiently tuned cost surface is reflected in the disruptive impact of the integration plan. The expansion of the fragmented area implies that a greater number of changes were required than in the fixed capacity series, to accommodate the integration plan. This observation is borne out by the pattern in Figure 9. Reassignment of tracts was far more active involving 30 tracts or about 17.8%. The heavy dark lines represent the original attendance area boundaries. Cost for this system is 37,813, still 9.5% below the fixed capacity system.
A cost summary of the systems considered appears in Table 3. The differences do not seem too significant in terms of the cost units we are dealing with in our problem, but in the context of a large metropolitan area it could amount to thousands of dollars.

Table 3

<table>
<thead>
<tr>
<th>System</th>
<th>Total Cost</th>
<th>Black</th>
<th>White</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free Allocation (fixed and equal capacity)</td>
<td>36,324</td>
<td>3,841</td>
<td>32,483</td>
</tr>
<tr>
<td>Free Allocation (variable capacity)</td>
<td>32,967</td>
<td>3,717</td>
<td>29,250</td>
</tr>
<tr>
<td>Segregated</td>
<td>3,844</td>
<td></td>
<td></td>
</tr>
<tr>
<td>50/50 Integration</td>
<td></td>
<td>6,917</td>
<td></td>
</tr>
<tr>
<td>100% Integration (fixed and equal capacity)</td>
<td>39,775</td>
<td>9,388</td>
<td>30,387</td>
</tr>
<tr>
<td>100% Integration (variable capacity)</td>
<td>37,813</td>
<td>9,388</td>
<td>28,425</td>
</tr>
</tbody>
</table>

The cost relationships can be conceptualized as in Figure 10. Since cost increases with increasing integration and decreases with varying capacity, it is implied that the costs of integration in our system could be at least partially offset by a varying capacity scheme.
Schematic presentation of cost associated with varying degrees of segregation and school capacity.

This degree of integration achieved per school facility is outlined in Table 4. Any of the four integration plans offers a feasible solution to the problem. If equality in quality of education is the prime objective, the fixed capacity-100% integration system offers the best possibilities even though the cost is greatest. Using the variable capacity scheme with the 50/50 plan will provide the most economical arrangement with a respectable balance of black and white students.

**Table 4**

<table>
<thead>
<tr>
<th>School</th>
<th>Free Allocation</th>
<th>50/50 Plan</th>
<th>100% Integration</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>68.3</td>
<td>43.2</td>
<td>17.3</td>
</tr>
<tr>
<td>2</td>
<td>0.0</td>
<td>10.8</td>
<td>17.3</td>
</tr>
<tr>
<td>Fixed &amp; Equal Capacity</td>
<td>3</td>
<td>0.0</td>
<td>10.8</td>
</tr>
<tr>
<td>Capacity</td>
<td>4</td>
<td>0.0</td>
<td>10.8</td>
</tr>
<tr>
<td>5</td>
<td>18.2</td>
<td><strong>23</strong>10.8</td>
<td>17.3</td>
</tr>
</tbody>
</table>
Table 4 (Cont.)

<table>
<thead>
<tr>
<th>School</th>
<th>Free Allocation</th>
<th>50/50 Plan</th>
<th>100% Integration</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>54.7</td>
<td>38.3</td>
<td>15.3</td>
</tr>
<tr>
<td>2</td>
<td>0.0</td>
<td>28.9</td>
<td>46.3</td>
</tr>
<tr>
<td>Variable Capacity</td>
<td>3</td>
<td>0.0</td>
<td>18.2</td>
</tr>
<tr>
<td>4</td>
<td>0.0</td>
<td>9.6</td>
<td>15.3</td>
</tr>
<tr>
<td>5</td>
<td>16.7</td>
<td>11.9</td>
<td>18.9</td>
</tr>
</tbody>
</table>

Summary

The notion of using a hypothetical city to analyse the nature of systems has a strong intuitive appeal. Extraneous influences that occur as unique features of real world cities are eliminated, thus removing the unobvious biases of data. The danger lies in introducing unwanted bias by means of the hypothetical structure. And, of course, there is the pitfall of inferring too much about the nature of the real world on the basis of the abstract model. We make no claim that our model city has counterparts anywhere in the real world. We do not feel it important that it should. It does serve to illustrate the complexities of urban school systems, and, we hope, gives insight to the spatial mechanics involved in a school integration scheme.

There are some obvious flaws in our construct. The density bias may have been too severe because of the large areal increase from core to the city's edge. The rigid boundary lines draw more attention to the graphics than is desirable. And the graphic representation of assignments does not always appear to be logical. But the thesis,
we believe, is defensible and serves our initial intent.

One further step that would greatly enrich the model would be the possibility of relocating school facilities in order to gain more efficiency in the transport surface. This could also be executed to maximize integration with a minimum of bussing. In addition ghetto expansion could be simulated with random numbers as a test of the systems flexibility.

We have dealt strictly with transport costs. But, of course, there are other costs to be considered in an integration scheme. The cost constraint of building, maintaining, or relocating physical facilities could be overwhelming. Location constraints imposed by physical barriers or availability of land could be a major factor. Or simply, economics of scale or the obtaining of extras (athletic facilities, etc.) could be used to complicate the scheme.

We have perhaps opened Pandora's Box rather than resolved any crucial questions. But, we believe the model has potential and would welcome suggestions as to its direction and development.
BIBLIOGRAPHY

