DOCUMENT RESUME

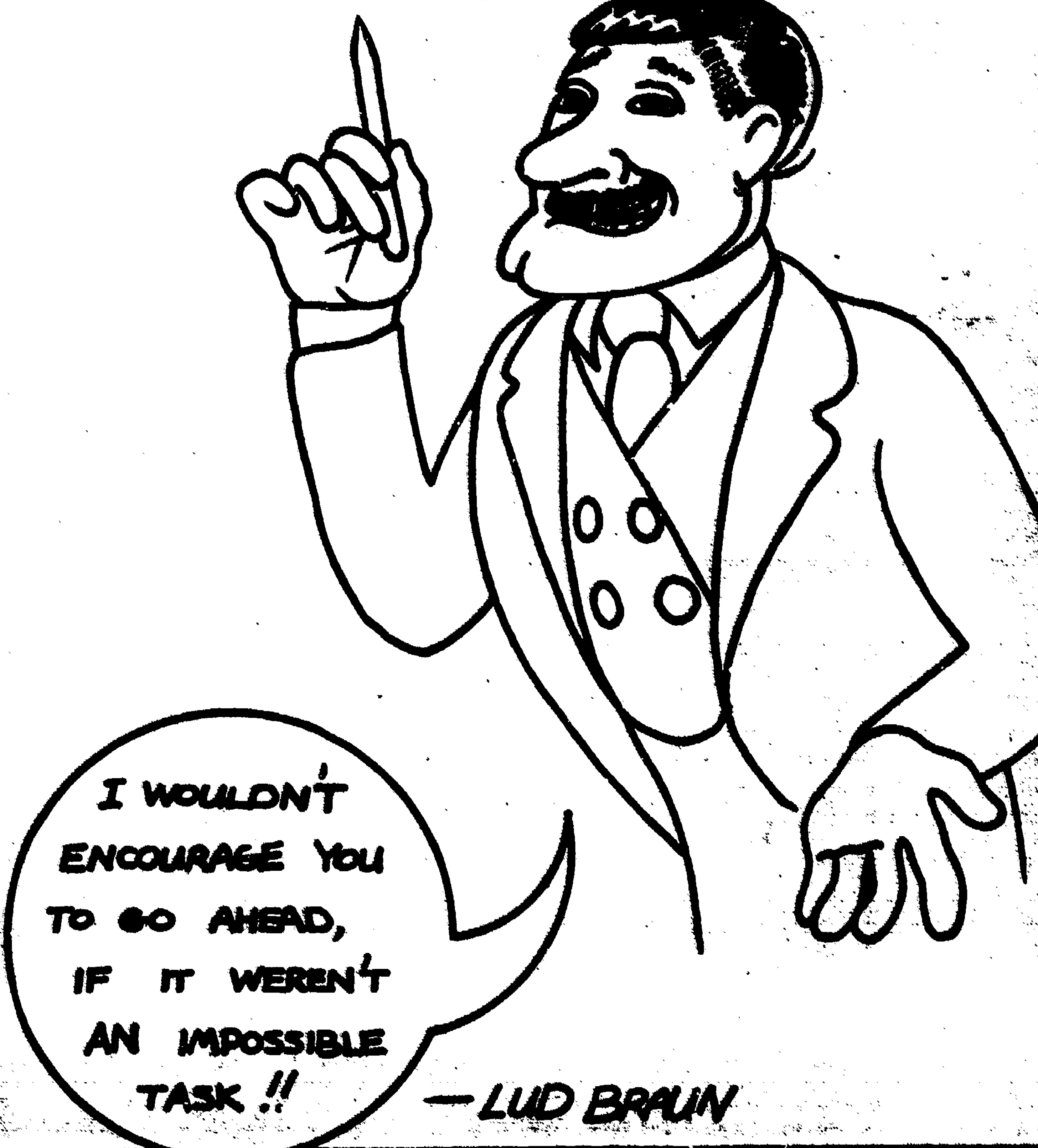
ED 103 273 SE 018 779

AUTHOR	Hoffer, Alan R.
TITLE	Conference on Mathematics Resource Materials (Eugene, Oregon, June 9-12, 1974).
INSTITUTION	Oregon Univ., Eugene. Dept. of Mathematics.
SPONS AGENCY	National Science Foundation, Washington, D.C. Div. of Pre-College Education in Science.
PUB DATE	Sep 74
NOTE	106p.
EDRS PRICE	MF-$0.76 HC-$5.70 PLUS POSTAGE
DESCRIPTORS	*Conference Reports; *Curriculum; Effective Teaching; Elementary School Mathematics; Elementary Secondary Education; Evaluation; Instruction; Instructional Materials; Mathematical Applications; *Mathematics Education; Mathematics Materials; *Middle Schools; *Resource Materials; Secondary School Mathematics
IDENTIFIERS	*National Science Foundation; NSF

ABSTRACT

This is a report of an advisory conference of a National Science Foundation project. The project was initiated to provide a guide to available instructional materials and resources for teachers who want to expand their knowledge of subject matter, teacher methods, or the interplay between mathematics and other disciplines. The main thrust of the project has been to develop materials for teachers of mathematically uninvolved students, with emphasis on performance levels equivalent to grades six through nine. The purpose of this conference was to receive reactions and recommendations from a broad spectrum of people, including directors and staff members of previous curriculum projects, supervisory level personnel from various cities, practicing teachers, mathematicians, scientists, and educators. Conference plans included five initial working groups on: content, applications, diagnosis and evaluation, didactics, and teacher utilization. This report includes an initial position paper on each topic, along with subsequent recommendations. Also included are recommendations made by secondary working groups established during the conference on the topics: organization and staffing policy, format and style, problem solving, calculators, and computers. (This conference report does not include, however, the actual guide to instructional materials and resources.) (Author/CR)

CONFERENCE ON MATHEMATICS RESOURCE MATERIALS



CONFERENCE ON MATHEMATICS RESOURCE MATERIALS

EUGENE, OREGON

JUNE 9-12, 1974

A REPORT BY

Alan R. Hoffer, Conference Director
Department of Mathematics
University of Oregon
Eugene, Oregon 97403

Supported by a grant from the National Science Foundation,
Materials and Instruction Development Section,
Division of Pre-College Education in Science

CONTENTS

I. PURPOSE and PLAN of CONFERENCE, a letter to L. O. Binder....................iii

II. PROJECT DESCRIPTION by Alan Hoffer and Ted Nelson.................... 1

III. INITIAL WORKING GROUPS

A. CONTENT

1. Remarks about Appropriate Content for the Mathematics Resource Books, a position paper by Robert B. Davis........ 13

2. Working Group Recommendations.............................. 27

B. APPLICATIONS

1. Models, Problems, and Applications of Mathematics, a position paper by Maynard Thompson....................... 31

2. Working Group Recommendations.............................. 41

C. DIAGNOSIS AND EVALUATION

1. Developing Teacher Competencies for Evaluating Mathematical Learning, a position paper by Larry L. Hatfield............ 45

2. Working Group Recommendations.............................. 60

D. DIDACTICS

1. Didactics, a position paper by William M. Fitzgerald....... 65

2. Working Group Recommendations.............................. 71

E. TEACHER UTILIZATION

1. Teacher Utilization, a position paper by Albert P. Shulte.. 74

2. Working Group Recommendations.............................. 83

IV. SECONDARY WORKING GROUP RECOMMENDATIONS

A. PROJECT DEVELOPMENT AND STAFFING POLICY......................... 86

B. FORMAT and STYLE... 88

C. PROBLEM-SOLVING.. 90

D. CALCULATORS.. 93

E. COMPUTERS... 95

V. LIST OF PARTICIPANTS.. 97

UNIVERSITY OF OREGON

September 9, 1974

L.O. Binder, Head
Materials and Instruction Development Section
Education Directorate
Division of Pre-College Education in Science
National Science Foundation
Washington DC 20550

Dear Larry:

This is a report of the advisory conference of the National Science Foundation Project: Topical Resource Books for Middle School Mathematics Teachers (NSF #GW-7910). The conference was held on June 9 - 12, 1974 at the Eugene Hotel in Eugene, Oregon. A description of the project plan is included in the enclosed materials.

The purpose of the conference was to receive reactions and recommendations from a broad spectrum of people who have experience and insight into the problems of teaching school mathematics. As it is reflected by the enclosed list of participants, the conference was attended by directors and others involved in previous curriculum projects, as well as representatives at the supervisory level from various cities, practicing teachers, mathematicians, scientists and educators.

The plan for the conference focused on five initial working groups on the topics: Content, Applications, Diagnosis and Evaluation, Didactics and Teacher Utilization. Five people were invited to write position papers - one for each of the initial working groups. These papers along with the Project Description were sent to each participant prior to the conference. The working groups reacted to these papers and made recommendations to the project staff. These papers and recommendations are included in the enclosed materials.

The sequence of activities at the conference consisted of a global discussion of the conference goals and position papers on the evening of June 9, 1974. On Monday, June 10 there was a report by the director on the project plan and progress to date. Then the five initial working groups met to discuss the position papers and topics of concern to their groups. That evening there were reports by the five working groups on the status of their deliberations. These reports then received reactions by members of the other

working groups. Tuesday morning the initial working groups reconvened to conclude their deliberations and make recommendations. The entire group came together before lunch to discuss these recommendations and to reorganize into five secondary working groups on topics which the participants thought would be of interest to the project staff. In the afternoon Professor Wayne Wicklegren gave a presentation on problem-solving, a topic which should pervade all of the proposed resources. Professor Wicklegren has developed his ideas in a book entitled How to Solve Problems. Following this the secondary working groups met and discussed the topics: Organization and Staffing Policy, Format and Style, Problem-Solving, Calculators, and Computers. Recommendations from these groups are enclosed. Wednesday morning there was a global discussion and concluding remarks. The conference terminated at noon on Wednesday, but some participants stayed on for further discussions.

I am most enthusiastic when I say that the conference was an extremely worthwhile endeavor. Every participant came here with concerns about the plan for the project. They openly engaged in discussion and made recommendations which have been helpful to me and will, I am certain, continue to be helpful to the project staff. Special recognition should be given to the five participants who wrote position papers for the initial working groups: Dr. Robert Davis, Professor Maynard Thompson, Professor Larry Hatfield, Professor William Fitzgerald and Dr. Albert Shulte. Their excellent papers were well received by all members at the conference.

It was valuable, I believe, that all participants were candid in the expression of their knowledge and experience. Theories by college personnel were challenged by practicing teachers and supervisors. Current school practices were, in turn, challenged by scientists and educators, and there were suggestions for the inclusion of heretofore nonmathematical considerations into mathematics curricula.

In summary the advisory conference by its recommendations approved of the intent and plan of the project and gave further direction to the project staff.

Respectfully submitted,

Alan R. Hoffer

Alan R. Hoffer
Project Director

ARH:km

cc: Lauren L. Woodby, Program Manager
Materials and Instruction Development Section

PROJECT DESCRIPTION

TOPICAL RESOURCE BOOKS   MATHEMATICS TEACHERS

By ALAN HOFFER and TED NELSON
Department of Mathematics
The University of Oregon
Eugene, Oregon

I. The Problem

A secondary school teacher of mathematics meets classes 25 to 30 hours per week and works with approximately 150 students. Within this assignment the teacher regularly collects and corrects homework papers, grades weekly tests for each student, tutors students individually before school, during the lunch period and after school, confers with parents and counselors, and sponsors student groups. The teacher is often expected to supervise the school grounds and be present during assemblies, athletic events, open house demonstrations, and meetings of the parent-teacher organization. A beginning teacher may supplement a low salary with a part-time job and male teachers are often asked to assist in coaching athletic teams.

The total demands upon a mathematics teacher make the job appear nearly an impossible task. As a result the teacher rarely has the time to extend his or her mathematical knowledge or to gain familiarity with new teaching techniques and innovative materials. During the summer months a teacher, if not working at a summer job, may take college courses to apply toward advance certification or higher placement on a salary schedule. The courses may be in advanced mathematics or educational theory, lacking synthesis, they often have little direct application to the teacher's school assignment. Inservice programs at colleges and universities have overall made a worthwhile

contribution. However, colleges simply do not have the resources to re-train or up-date all of the secondary teachers in the country much less induce them to return to campus. It is noteworthy that the National Council of Teachers of Mathematics places at least one annual meeting within 500 miles of each teacher in the country. These meetings have presented some excellent lectures, discussions and workshops. The displays at the larger meetings by commercial publishers, funded curriculum projects and the NCTM have demonstrated an abundance of new material. In spite of an increasing membership, a large percentage of mathematics teachers do not belong to the NCTM and many teachers have never attended an NCTM meeting to avail themselves of a two-day inservice experience.

In the important middle school years there are additional problems. Here we find many teachers with a very weak mathematics background - there still seems to be the belief on the part of some administrators that anyone can teach middle school mathematics. Many of these people became middle school mathematics teachers when there was a shortage of teachers and they now have tenure in those positions. On the other hand we have middle school teachers with adequate secondary mathematics credential but little preparation to diversify their teaching methods for students in these important transitional years. They learned their mathematics from textbooks and lectures and they teach middle school mathematics via textbooks and lectures. As a consequence, and because of a lack of awareness of alternatives, teaching in most mathematics classrooms is dictated by a linearly ordered textbook. Fast learners become disinterested while the slow learners are frustrated. Efforts to effectively individualize learning are severely restricted and often degenerate to a 9000 minute contest with programmed materials. Problem solving and creativity are usually non-existent as are delightful mathematical ideas and instructive applications. Students are placed into the traditional college bound (calculus preparatory) sequence or in remedial courses while innovative materials, capable of producing alternative mathematics courses, go unused.

We have then, on one hand, a harassed teacher lacking the confidence, time and local resources for self-improvement and, on the other hand, an

ocean of new materials and pedagogy which are available and desparately in need of synthesis. To one not well versed with the recent materials it may be difficult to convey in this paper the quantity and, in many instances, the excellent quality of the available resources. Mention is made later of an intensive eight-week workshop which was held during the summer of 1973 at the University of Oregon for master secondary school mathematics teachers. The workshop was held in the Mathematics Resource Center of the University of Oregon. The Resource Center is stocked with several thousand dollars worth of a variety of instructional materials all of which were published since these teachers completed their pre-service training. There are commercial instructional materials for metrication, laboratory activities, simulation activities, comprehensive problem solving situations, games and puzzles. The NCTM has several excellent publications, for instance, the series Experiences in Mathematical Discovery. Ideas for curriculum improvement are contained in some government-funded projects such as COLAMDA, CSMP, the Madison Project, MAGIC, the Oakland County Mathematics Project, SMSG, UICSM, UMM (Teachers College, Columbia University), and WYMOLAMP. From England we have the School Mathematics Project, the Kent Maths Project and possible extensions of the Nuffield Project materials. Applications from the sciences are available in the projects ESS, IEP, IIS, IPS, SAPA, SCIS and extensions of the USMES materials.

The problem of concern in this project is that teachers largely lack awareness of available instructional materials and resources and ways to translate these materials to use with their students. In addition, there are mathematics teachers nation-wide who have not been reached by inservice programs and who continue to be deficient either in subject matter knowledge, didactics or the interplay between mathematics and other disciplines.

II. The Need for Organized Resource Materials

The promise that new curriculum projects and new textbooks would solve the problems of teaching mathematics has apparently not been fulfilled. Four recent government sponsored conferences (at Cape Ann, Estes Park, Orono and Snowmass) have addressed themselves, either directly or indirectly, to

the problem of mathematics education. A perusal of the conference summaries seems to indicate an urgent need to help teachers expand their notions regarding teaching activities and content in order to more effectively deal with contemporary issues and developments in mathematics education. The recommendations of the conferences seem to fall equally into three areas: new emphasis in content and applications, didactics, and problem solving.

1. New Emphasis in Content and Applications

The new emphasis most frequently mentioned is a shift toward mathematics for general education, for the training of citizens who may successfully participate in society and, possibly, a shift away from a primary focus on the training of college bound students who may use mathematics as a post secondary school studies. There is a call for increased use of estimation, approximation, statistics, probability, calculators and computers, and practical uses of mathematics in society, science and technology.

2. Didactics

The strong emphasis on didactics in these reports is apparently a consequence of the lack of attention given to this area by curriculum reforms in the last two decades. It was recommended in these conferences, as well as in the recent recommendations by the CUPM, that mathematics teachers must have more knowledge about how children learn mathematics, about the effective use of laboratory activities and manipulatives, and various classroom teaching techniques.

3. Problem Solving

Although the ideas of problem identification and problem solving strategies might be listed under didactics, the urgency to develop this area - stressed in all of the conferences but most eloquently in the Estes Park Conference - deserves a category of its own. The comprehensive problem-solving approach suggested in the Estes Park report must be part of each mathematics teacher.

Are teachers prepared to accept these recommendations? Indeed, a more poignant question is, are teachers willing to accept these recommendations? Curriculum revision in the schools has hardly called upon the experiences and recommendations of school teachers. In many ways teachers have been

entirely left out of the decision making process. They have been led to believe that they have little if anything to say about what mathematics should be taught and to whom. Each new textbook series is imposed upon the teacher and students and there develops a contest to "cover the book," in some form or other. Each new textbook adoption leaves unchanged, and indeed further regresses, the basic attitudes and pedagogy of the teacher. It is evident that there will be no curriculum change, in practice, unless and until teachers change their attitudes and teaching techniques. There is evidence even today of teachers who teach rote techniques (as they may have learned them) in spite of the fact that their textbooks develop the ideas quite differently.

During the summer 1973 the project directors conducted an intensive eight-week Leadership Workshop for Mathematics Teachers of Educationally Uninvolved Students sponsored by the Oregon Mathematics Education Council. The participants were selected for their strong mathematical backgrounds, experience and leadership potential. The major effort for the summer was to help the teachers gain familiarity with new instructional materials. We soon realized what an enormous effort it was to collect, organize and synthesize even a small part of the available instructional materials. Even more surprising was the total lack of familiarity that these teachers had with the existing materials.

In discussions with middle school and secondary school teachers throughout the country it becomes apparent that teachers are asking for help - but the kind of help they can readily accept - not just another textbook change or new curriculum project. Most teachers have never heard of the Madison Project, for example, and many of those who have do not know how to use the materials in their program. Many teachers will claim that there are no materials for "low achievers" or uninvolved students and they are quite surprised to learn about the COLAMDA Project, Stretchers and Shrinkers, or Experiences in Mathematical Discovery, for example. Several teachers have contacted the participants in the 1973 Leadership Workshop asking for materials which may have come out of the workshop. Many of the workshop participants have made efforts to adapt and organize materials from the

workshop, and they have found this to be so useful that other teachers in their schools have asked for copies.

Indeed there seems to be a demand for organized resource materials - to help teachers effectively help themselves and their students.

III. Project Description

The project is concerned with the preparation of resource books for middle and secondary school mathematics teachers with emphasis on performance levels equivalent to grades six through nine. The project staff will primarily consist of four experienced school teachers working full time along with the project director (A. Hoffer) and assistant director (T. Nelson). Work on the project will take place in conjunction with the Mathematics Resource Center of the University of Oregon. The main thrust, at least initially, will be to develop materials for teachers of students who are mathematically uninvolved (in the broadest use of the term). This does not mean that the materials are limited to low ability students nor does it exclude the possible use of books at the high school level. As the project develops resource books may be generated which act as catalysts for alternative and possibly new mathematics courses at the junior and senior high school levels.

It is anticipated that the teacher-oriented resource books may be used for:

1. Inservice.

   There will be an analysis of the mathematical content to provide the teacher with a deeper understanding of the topic of the book and possible ways to modify the available material and extend or relate the topic to other areas. There will be a discussion of didactics including teaching strategies, motivational methods, learning difficulties and learning theories - with appropriate reference and emphasis on the affective consequences of the learning environment.

2. Resources.

   This will consist of a survey and synthesis of materials and activities

which pertain to the topic of the book including student worksheets, project assignments, and appropriate developmental and enrichment activities (manipulatives, games and simulation activities). Some pages may be reproduced for immediate class use, and there will be an annotated bibliography. Resources will also include skill development programs, diagnostic techniques, and evaluation procedures.

3. Problem Solving.

This will include the use of language and logic in understanding, reformulating and solving problems pertaining to the topic of the book. The interdisciplinary aspect of problem solving will be emphasized, and applications of the topic will be made to social and physical sciences as well as to consumer and vocational questions.

For easy teacher reference the topics and titles of the resource books will be compatible with the topics in current mathematics curriculum projects, state and local syllabi and textbooks. Each book will develop the topic from first principles to more involved concepts and applications. Emphasis will be placed on the use of logic, language and flowcharts in communicating, interpreting, organizing and applying mathematical concepts; on methods and activities in building skills and diagnosing weaknesses; on comprehensive problem solving situations and activities; on uses of mathematics in careers, consumer problems, society, science and technology; and on applications of computers.

The books will contain examples of and suggestions for the use of measuring instruments, drawing and constructing instruments, slide-rules, hand and desk calculators, computers, manipulatives, games and puzzles. There will be descriptions of various manipulatives - their uses, sources for purchase and ways to produce them from inexpensive materials. The manipulatives may be used to explore concepts, introduce topics, individualize, motivate drill and practice for problem solving. Some of the manipulatives which will be incorporated into the resource books are the abacus, attribute blocks, centimeter rods, chip trading, D-stix, dice, dominoes, fraction bars, geoboards, geoblocks, inch blocks, mini-computer, mirrorcards, multi-base

arithmetic blocks, mira and tangrams.

Topics for the resource books will be finalized in the early stages of the project following the recommendations of an advisory conference. The following is a workable list which proved to be useful for many of the participants in the University of Oregon Leadership Workshop.

RESOURCE BOOK TOPICS

1. Number Sense and Arithmetic Skills

   Counting, symbo , numerals, numeration, place value, powers of 10, expanded notation, large numbers, estimation, approximation, fractions, decimals, small numbers, operations, number properties, skill building, mental arithmetic, standard and non-standard algorithms, flow charts.

2. Measurement and the Metric System

   Linear measure, additiona and subtraction of lengths, perimeter, area, surface area, volume, weight, density, capacity, time, coinage, different units of measure, metric units, comparison between units, practical arithmetic, practical geometry, error, precision, accuracy.

3. Mathematical Sentences and Systems

   Integers, rationals, real numbers, exponents, roots and radicals, operations, postulates, clock arithmetic, polynomials, matrices, open sentences, linear equations and inequalities, systems of equations, polynomial equations.

4. Geometry

   Shapes (planar and solid), form and patterns, congruences, parallels, similarity, transformations (isometries, symmetry, enlargements, distortion), tessellations, topology, regions, polyhedra, deltahedra.

5. Relations and Graphs

   Pictographs, bar graphs, circle graphs, binary relations, networks, graphs of relations, functions, formulas, sequences, coordinates, coordinate graphs, interpretation of graphs, extrapclation, interpolation.

6. Number Patterns and Theory

Factors, multiples, divisors, primes, factorization, GCD, LCM, triangular numbers, magic squares, Pascal's triangle, Fibonacci numbers, sequences.

7. Ratio, Proportion and Scaling

Percent, interest, shares, weights and measures, balancing, growth rates, inverse rates, scale drawings, models, diagrams, descriptive geometry, map construction and reading, mean proportion, arithmetic and geometric mean, similarity, trigonometric ratios, surveying, mass points, music.

8. Statistics and Information Organization

Data analysis, tables, graphs, averages, sampling, randomization, measures of dispersion, formal frequency correlation, hypothesis testing, estimation, classification, forecasting, decision making, interaction with computers, misuses of statistics.

9. Probability and Expectation

Counting principles, permutations, combinations, experiments, trials, sample space, probability, models, independence, dependence, expectation, prediction.

10. Mathematics in Science and Society

Examples follow of some of the projects which may appear in this book and to some extent in the previous books.

a. Science. Weather forecasting; energy consumption; pendulums; falling bodies; rate of growth in humans; muscle fatigue; fastest living creatures (with respect to size); transpiration; migration of animals.

b. Social Science. Map making and reading; testing common generalizations; what's in a newspaper?; how many people elect a president?; polling; search strategies; decision making.

c. Occupations. Reading meters; painting a house; laying a sidewalk; bricklaying; building a structure; designing containers; packaging; measuring material; advertising; subdividing land into lots; data cards (what are the holes for?).

d. Miscellaneous. How do you ration gas?; kite designing; designing a geodesic dome; design fire evacuation for school (home); what is the difference between different speeds on a bicycle?; planning a trip; time zones around the world; what does a bicycle cost?; a car?; planning a parking lot; what is the value of household goods?; how

much time does an average person watch TV?; how much advertising on TV?

e. Games, Puzzles, Sports. Game strategy and flowcharts; how are league standings determined?; designing tournaments; determining averages; how does a billiard ball bounce?; scoring games; how far does a football tackle run during a game?; a basketball player?; puzzles.

It is envisioned that the books will be sectioned in the following manner with subsections tabbed for easy identification. There will be cross referencing between books and between subsections of each book as appropriate.

Section I. Historical Background and Biographical Sketches.
This will contain biographical sketches of mathematicians, scientists, environmentalists and other users of mathematics. There will be descriptions of historical developments of the topic; for example, in the Measurement and Metric System book, there will be a history of various units of measure, including the difficulties countries have had in converting to the nearly 200-year-old metric system, and the possible measurement techniques used in early times by Archimedes and others.

Section II. Content for Teachers.
This will contain an analysis of the mathematical content to provide teachers with a deeper understanding of the topic of the book and possible ways to extend and adapt the topic of the book to other learning situations. For example, the Geometry book will include an overview and use of transformations, models of spherical geometry, and a discussion of a descriptive drawing with relation to perspectivity and projective geometry.

Section III. Didactics.
This section will present ideas on how children learn mathematics with specific examples relating to the topic of the book. It will include topics from current learning theories involving the affective domain (self-concept, interest, motivation, attitudes), as well as the cognitive domain. There may be a discussion of cognitive styles, problem-solving strategies, and classroom techniques. It will also include current ideas developed by Dienes, Davis, (discovery and paradigms), Piaget (stages of learning), and others concerned with mathematics education.

Section IV. Classroom Materials.

This will be the main portion of the book. Each of the subtopics will be organized, indexed and developed to assist the teacher as a self-contained unit or as a supplement to an existing course. There will be suggestions for starting points to introduce a concept or idea; for example, area may be introduced via tessellations, tangrams, or geoboards. There will be follow-up activities and suggestions for open-ended interdisciplinary projects. There will be reproducible worksheets and activity sheets for students; suggested laboratory activities, games and manipulatives; simulation activities as well as activities and materials to promote skill building.

Section V. Diagnosis and Evaluation.

Suggested diagnostic materials will include interview techniques and written tests to determine, a priori, the students' knowledge of concepts. There will be activity tests and assignments which indicate whether or not the students can perform a task associated with each topical book; for example, given a metric ruler, have the students determine the area of the classroom, school yard, or athletic field. Evaluating students progress in learning is equally important and suggested evaluation techniques and materials for each of the topics will be provided; for example, evaluation techniques can be included in classroom activities, so the student doesn't realize he is being evaluated - as opposed to written tests on each unit.

Section VI. Applications and Problem Solving.

Each book will have a bank of applications from which teachers may select. This section will also contain reading selections from outside of mathematics which utilize the students' knowledge of the topic (reading from environmental areas, astronomy, biology, social science, etc.). References will also be made to pertinent projects in Book 10, Mathematics in Science and Society. There will be suggestions for comprehensive problem-solving activities and ways to encourage the students to identify, pursue, and solve problems of their own.

Section VII. Annotated Bibliography.
Each book will refer the teacher to a wide variety of different books, activities, government projects, commercial products, and articles which pertain to the subject under discussion. The references will contain descriptions of the substance of the reference and suggestions for ways to adopt and adapt the materials referred to.

It is naive to think that the creation of resource books will instantaneously change the classroom performance of teachers nationwide. Teacher utilization will continue to be a problem, and one which appropriately will be considered more fully in the implementation stages of the project. We will anticipate that the resource books once developed will encourage teachers to make the first step away from a highly structured textbook. With these books available teachers may gain a deeper understanding of mathematics, an awareness of varied instructional techniques and resources, familiarity with uses of mathematics in science and society, and just as importantly, an opportunity to become more involved in the decision making process for curriculum revision at the local level.

REMARKS ABOUT APPROPRIATE CONTENT

FOR THE MATHEMATICS RESOURCE BOOKS

By ROBERT B. DAVIS
Curriculum Laboratory
University of Illinois
Urbana, Illinois

I. These remarks are essentially addenda to the paper Topical Resource Books for Mathematics Teachers - Project Description by Hoffer and Nelson. The content (and book format) described in the Hoffer and Nelson paper is so complete and so representative that--in my personal view--it could well stand alone. My present remarks are essentially footnotes that should not distract anyone's attention from the Hoffer-Nelson outline.

II. It may be worth listing the background from which I am speaking:

A. The ability of elementary school children to handle "abstract pattern" mathematics.

Many of the new math projects have proved that children in grades 4, 5, and 6 have a very great--and virtually unused--ability to handle mathematical ideas of a certain kind, provided the foundation is always generalizing from instances.

Thus, William Johntz's Project SEED has dealt with exponents, going from specific examples: $(2 \times 2) \times (2 \times 2 \times 2) = 2 \times 2 \times 2 \times 2 \times 2$ to notations such as:

$$2^2 \times 2^3 = 2^5$$
$$3^2 \times 3^3 = 3^5$$
$$2^2 \times 2^5 = 2^7$$

and hence to:

$$2^a \times 2^b = 2^{a+b}$$

David Page's Illinois Arithmetic Project worked up to matters such as

$$[a] + [b] \leq [a + b] ,$$

having started with examples like

$$[2\ 3/5] = 2$$
$$[2\ 6/5] = 2$$

and so on.

The Madison Project dealt with identities, such as the following special case of the binomial theorem:

$$(\square + \triangle) \times (\square + \triangle) = (\square \times \square) + \left[(\triangle + \triangle) \times \square\right] + (\triangle \times \triangle)$$

having, again, started with specific instances.

Children in grades 2 through 5 find this interesting, attractive, exciting, and easy. Notice, though, that it is a definite kind of mathematics. It is not mathematics that depends upon complex verbal statements, nor does it require an elaborate running up and down the ladders of abstraction from symbol to meaning, then to the meaning of the meaning, etc. For want of a better word, I shall call this pattern-based mathematics. Note, however, that the patterns that are involved do make sense--they are not the gratuitous arbitrary patterns of common IQ-test questions.

Despite the fact that a number of separate projects have demonstrated clearly the success that this kind of mathematics can enjoy with children in grades 2 through 5, our usual school curriculum makes no significant use of such mathematics at these grades.

This is relevant to our present concern for grades 6 through 9, because of the following remark:

B. The Junior-High Slump. Most of these same projects (Page's, The Madison Project, SCIS, etc.) have found that children in grades 2 through 5 are better at this kind of mathematics than are children in grades 7 and 8.

Consequently, my personal guess is that a very strong national curriculum for the United States would assign a large role to science and to algebra in (say) grades 3 through 6, or thereabouts, but would probably slack off

for grades 7 and 8. By grade 9 one would again resume the serious pursuit of mathematics and science.

Notice that the traditional curriculum did in fact approximate this pattern, as John Mayor recognized by his remark: "Between sixth grade arithmetic and ninth grade algebra there comes a pause in the child's occupation that is known as junior high mathematics." Most of us who have worked with new math projects began, at least, with the expectation that the greatest improvements could be made at the junior high level, because of the base from which we were starting. By now, however, I am more inclined to think that the weak traditional program for grades 7 and 8 may well have been a response to the poor performance that is characteristic of 7th and 8th graders; if this is so, then these are the grades at which the least improvement may be possible.

C. Short-Term Goals and Long-Term Goals.

The situation described in remarks A and B, above, present us with a dilemma, and one that we shall encounter elsewhere in this paper: On the one hand, we want the books to be helpful to teachers now, to be relevant to their situation now. And what is their situation now? Their students come from relatively weak elementary school programs that have consisted mainly of rote arithmetic, which the children in most cases fail to understand (cf., e.g., Erlwanger, 1973). Whatever improvements a junior high teacher can make in this situation is probably at the junior high level (unless he can work with elementary school teachers or children, which is sometimes possible). So this is where the short-term thrust has to be (even though, in the long run, grades 7 and 8 may be a poor place for a sustained major thrust).

The dilemma arises from this conflict between short and long-term goals. In pursuing sensible short-term objectives, we don't want to destroy the possibility of working toward important long-term gains. If the junior high slump really does exist (and if, as some suspect, it is related to hormonal changes, and therefore not easily altered by pedagogy or curriculum), then the junior high years are NOT the best place to make our most serious thrusts. The big push should perhaps come in grades 2 through 6 (or so), possibly followed by a relative relaxation in grades 7 and 8. There is

enough data supporting the existence of a "junior-high slump" that we should be mindful of it as we plan these resource books--and, I suspect, teachers themselves should keep it in mind as they read the books. Thus teachers may need a short-term strategy--do as much as they can in grades 7 and 8--and may need to be planning a long-term strategy: develop ways to work with teachers and children in grades 2 through 6, and plan for <u>less</u> academic stress in grades 7 and 8. Of course, the data aren't 100% clear yet, and maybe some other developmental pattern will emerge as we study children in this age range more carefully, but there is enough evidence so that we should keep the junior-high slump in mind whenever we deal with possible modifications of the school curriculum.

D. <u>Gradual Cumulative Development vs. Segmentation</u>.

Another curriculum dilemma faces us: Shall we build toward a curriculum that develops increasing skill and knowledge, or shall we seek a curriculum that is segmented or modular, consisting of a virtually arbitrary sequence of largely independent units?

The argument for the segmented curriculum rests mainly on the fact that, since the units are essentially independent, the beginning of each unit can be a new starting point. Children do not become "lost" because of absences, moving from one school to another, or failing to master previous units. Given the typical junior high school situation, this can be a very big advantage.

But, as Robert Heinlein says, "There ain't no such thing as a free lunch." The advantages of independent units are bought at a price, namely, failure to develop any significant skills, abilities, or understandings. By contrast, a curriculum that builds systematically on the study of <u>functions</u> and on <u>analytic geometry</u> can build up to a high degree of proficiency at least as early as grades 8 or 9. (One such curriculum was developed by The Madison Project; the progress of students as they move through a <u>five-year</u> program, beginning in grade 3, has been recorded on film.) But, of course, this continuity and developmental build-up is also bought at a price: a student can fall off the train as it speeds along.

I personally am deeply distrustful of the segmented units approach <u>as it usually operates in actual practice</u>.

It can easily become an excuse for minimal progress, even a substitute for progress. Let me illustrate by citing some remarks made by Karen Fry, who is in charge of the physical education program at University High School, and who works in the University of Illinois's physical education department. She says that the University often sees students who have gone through 12 years of state-required physical education course, but are unable to throw a ball, unable to run, etc. How is this possible? Because of the segmented nature of the school P.E. program. Often schools stay with one activity--volley ball, say--about as long as it easily holds a child's interest. As soon as interest begins to flag, they move on to a new activity.

It is a hard job to take a boy who is over-anxious, who becomes too tense when he tries to throw a ball, and to help him relax, to give him the technical skill to throw well. In typical school settings this is too hard a job to pursue. Hence, instead of tackling this difficult task of individualized personal coaching, schools more often jump from one activity to the next. Most children enjoy this, some find it valuable, and the University of Illinois finds students who, after 12 years, still can't throw a ball.

There are complex questions involved here, but this is the kind of situation that makes me personally distrust sharply segmented, non-sequential curricula.

The resolution of the dilemma is probably our standard one: let's play for short-term relevance by being sufficiently segmented, but let's at the same time begin to mount a thrust toward a long-term implementation of a curriculum that builds cumulatively toward significant student growth over the years. The <u>students</u> can take this, and thrive on it, provided we can solve the school problems of providing such a program.

And what should the underlying theme be for a continuously building cumulative curriculum? Kenneth Appel of the University of Illinois's Math Dept. suggests <u>analytic geometry</u>, with a sizeable role for <u>applications</u>. This strikes me as a wise choice, since the study of functions, and the study of algebraic structures and techniques can easily be fitted into this main theme. I would propose that, for grades 2-12, there be four main (and simultaneous) themes: analytic geometry, arithmetic, computers, and applications. But of course I am speaking of a long-term goal toward which we

should try to converge, building in also appropriate short-term goals that face up to present realities.

(An alternative proposal, made independently by David Page and by Seymour Papert, is mentioned below.)

E. Summary. Thus, by personal experience and prejudice, I personally have these biases:

1. Toward seeking a far stronger math program in grades 2 through 6, and perhaps relaxing our program in grades 7 and 8;
2. Toward building a nation-wide curriculum that seeks considerable cumulative growth for individual students, instead of a sequence of largely independent units that do not add up to cumulative growth;
3. Recognition that, in advocating this, we are adding further stress on schools, giving them a harder job, but one that I think they should face up to;
4. Reconciling what is likely to be feasible in the short-run with what is likely to be most advantageous in the long-run by asking teachers to pursue a dual agenda: implement this now, but build toward that later on. (I claim we often do this, as when a medical student works at a part-time job to pay his way through medical school; the part-time job deals with immediate needs, while he is obviously building toward a different future.)
5. For the long-range cumulative curriculum, using analytic geometry, arithmetic, computers, and applications as our continuing central theme, for grades 2 through 12.

III. There is another dilemma that needs separate mention. Do we build toward maximum individual growth, with a generally "academic" bent, or do we build primarily so as to include all students, with great diversity (and lower expectations) in our offerings?

This seems to me to be the great unsolved problem in American education. By trying to educate nearly all Americans in nearly the same way, the United States has undertaken a task that is probably unique in human history--at

least when one makes due allowance for the complex culture we are dealing with. There are more traps here than may at first meet the eye.

I don't know the answer to this dilemma, but I do want to suggest that the idea of reaching for a "popular culture"--perhaps via diversity--that will be immediately acceptable to everyone may not be the best solution. Perhaps this "pop culture" will be unrewarding for everyone. We might, after all, be better off to maintain a smaller central core of basic knowledge--however unpopular that may seem--and try to get all Americans, or as many as possible, to master this basic core. (Obviously, it is this second road I am advocating when I urge analytic geometry, etc., as the theme for a cumulative curriculum.) Some relatively few students may require a quite different education--but maybe we make a mistake when we try to homogenize analytic geometry and major appliance repair.*

What I am trying to argue against is a mathematics curriculum that is "watered down" until every student can pretend to learn it. The pressure to move ever further toward this "common denominator" curriculum is immense--but the path of wisdom may be to resist it, nonetheless. There really ain't no such thing as a free lunch, and the "common denominator" curriculum can cost us plenty--perhaps even our sense of high standards and high achievement, which can damage our entire culture.

IV. Arithmetic.

Studies by Stan Erlwanger, Jack Easley, and others at the University of Illinois demonstrate clearly that students in grades 6, 7, 8, and 9 do not understand arithmetic. Obviously, there is a range of student competence--but far more students are deficient than would ordinarily be expected. (Notice that, in Section II, I am not suggesting that arithmetic precede analytic geometry; I urge that they be simultaneous in grades 7 through 12. Much analytic geometry is easy for children, and much of arithmetic is very difficult.)

In the case of arithmetic, "curriculum content" tends to merge with cognitive psychology and with diagnosis and evaluation. Erlwanger's case

*In rereading this section, I realize that I have not said this correctly; obviously, there may be many candidates for the "major appliance repair" route. But we can still try to avoid "watered-down" courses that won't be beneficial for anyone.

studies show just what this can mean.

Yet most junior and senior high teachers have thought little about arithmetic, about children's misconceptions concerning arithmetic, about how to identify these misconceptions, and about possible forms of remediation.

This needs to become a larger part of the secondary school curriculum.

V. Assembling Materials from Various Projects.

One proposal by Hoffer and Nelson deserves underscoring: their plan to assemble materials and results from all relevant projects, including David Page's Arithmetic Project, The Madison Project, ESS, SCIS, USMES, The Nuffield Project, SMSG, UICSM, The School Mathematics Project, etc., is excellent. To my knowledge this has never previously been done. There is a great wealth of excellent material available, and it needs to be brought together to form the foundation for future work on the mathematics curriculum.

VI. "Children's Literature."

David Page and Seymour Papert have independently developed and advocated special mathematical content designed for children.

Page speaks of this as "intermediate inventions," and offers as examples his "number line jumps" (where "standstill points" are a child's version of topological fixed-point theorems), his "maneuvers on lattices," and his "hidden numbers."

Papert speaks of "children's literature," and points out that in music one has a special literature prepared specifically for children; similarly with written literature. But where is the Dr. Seuss of mathematics? Where are the Bartok, Shostakovitch and Bach "Pieces for Children?"

Papert himself has made two. He has created a computer programming language (called LOGO) that is designed specifically to allow children to program a computer, and he has created his "Turtle Lab" array of peripherals to allow the computer to control equipment that may interest children: puppets, wheeled vehicles, TV pictures, music boxes, etc.

He has also developed a new geometry for children. Children orient in relation to themselves, and they jump back and forth from global to local properties. Papert's "turtle geometry" does the same, is immediately accessible to children, yet has the power of a legitimate branch of adult mathematics

(indeed, worthwhile "adult" theorems have been proved within turtle geometry, including problems about planetary motion).

Other computer programming languages designed specifically for children include <u>Picto</u>, developed by Paul Tenczar; <u>PAL</u>, now being developed by Hanna Kammerahl; and <u>DENKEN</u>, now being developed by [redacted], who is himself a student in high school.

Why, as Papert says, must we limit ourselves to taking pieces of adult mathematics off the shelf and offering them to children?

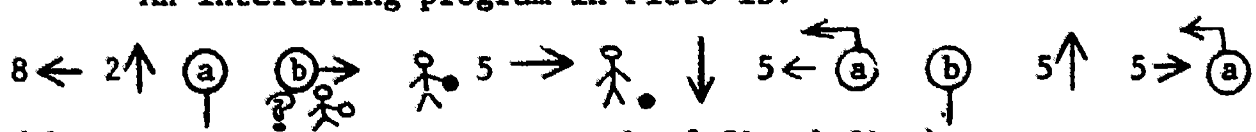
In Papert's LOGO, a square could be made by the program:

```
TO    SQUARE
10    FORWARD  10
20    RIGHT  90
30    SQUARE
END
```

and a circle could be made by the program:

```
TO    CIRCLE
10    FORWARD  1
20    RIGHT  1
30    CIRCLE
END
```

An interesting program in Picto is:



(this causes a man to move a stack of five balls.)

VII. <u>Tasks and Classroom Organization.</u>

An obvious (but often overlooked) point: many different social organizations of the classroom are possible. Which ones are best depends upon the specific tasks you have in mind.

Consider the case of chemistry: if we want to <u>demonstrate</u> something, the large lecture hall presentation may be best; if we want to <u>discuss</u> something (or even <u>debate</u> something), a smaller seminar group may be better; when <u>students</u> are to <u>carry out experiments</u>, an individualized arrangement in a properly equipped laboratory may be called for; if we want <u>a teacher to answer questions</u>, a smaller seminar group may again be the best setting.

A parallel situation exists in the case of mathematics, but we often forget it. We may try to do everything in a recitation format, or we may try to do everything in a math lab. This often makes us reject some content as unfeasible, when it is unfeasible only because it is being attempted in an inappropriate social setting.

VIII. <u>The Hoffer - Nelson Topics</u>.

Let me go through the topics as listed by Hoffer and Nelson, adding a few remarks:

A. 1. Number Sense and Arithmetic Skills.

Counting, symbols, numerals, numeration, place value, powers of 10. expanded notation, large numbers, estimation, approximation, fractions, decimals, small numbers, operations, number properties, skill building, mental arithmetic, standard and non-standard algorithms, flow charts.

2. Comment. It may be worth adding also some discussion on the computational role of <u>non-standard numerals</u>. For example, if we are given the problem

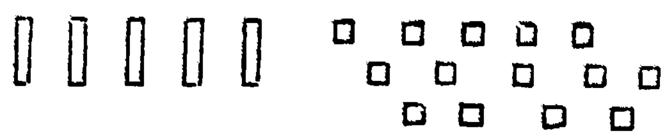
$$\begin{array}{r} 64 \\ \underline{-28} \end{array}$$

which has been stated in standard numerals, we may rewrite it in non-standard numerals

$$\begin{array}{r} {}^{5}\cancel{6}\,{}^{1}4 \\ \underline{-2\ 8} \end{array}$$

and use these non-standard numerals for carrying out the work.

Note that, if one uses (say) MAB blocks, both standard and non-standard numerals correspond to physical realities:

${}^{5}\cancel{6}\,{}^{1}4$ correspond to 

whereas the standard numeral

64 corresponds to 

and the standard numerals are precisely those where we have used the minimum number of pieces of wood.

3. Comment. Students should learn to relate each operation to various concrete situations or actions. This correspondence is one-many or even many-many.

<u>Each different "meaning" will suggest its own algorithm.</u>

Thus, the <u>subtractive</u> meaning of division,

$8 \div 2$ — "I have \$8 in the bank; I take out \$2 each week. For how many weeks can I do this?"

clearly suggests some sort of <u>subtractive</u> algorithm.

$$\begin{array}{rl}
2\overline{)\,8} & \\
\underline{-2} & 1 \text{ week} \\
6 & \\
\underline{-2} & 1 \text{ more week} \\
4 & \\
\underline{-2} & 1 \text{ more week} \\
2 & \\
\underline{-2} & \underline{1} \text{ more week} \\
0 & 4 = \text{total number of weeks}
\end{array}$$

Again, the "comparison" meaning of subtraction suggests the "cashier's algorithm":

$18 - 13$ — "Mary has 18 marbles; Kathy has 13. Who has more? How <u>many</u> more?"

suggesting

$$\begin{aligned}
13 + 1 &= 14 \\
+ 2 &= 15 \\
+ 3 &= 16 \\
+ 4 &= 17 \\
+ 5 &= 18
\end{aligned}$$

(Cf. "\$2.63 plus 2¢ is \$2.65, plus 10¢ is \$2.75, plus 25¢ is \$3.00, plus \$2.00 is \$5.00")

B. 1. Measurement and the Metric System.

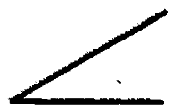
Linear measure, addition and subtraction of lengths, perimeter, area, surface area, volume, weight, density, capacity, time, coinage, different units of measure, metric units, comparison between units, practical arithmetic, practical geometry, error, precision, accuracy.

2. Comment. Obviously it's very important that children know the <u>kind</u> of thing they're measuring, so they realize that square centimeters will never "fill up" a volume, etc.

This is especially true in the case of angles. Many children want to know "how far" an angle goes, which, in their sense, is unanswerable.

3. Comment. This problem is often easily handled if treated in context: how many cartons will fill a warehouse? how much paint will cover a wall? etc. Angles are easy if we are dealing with actual rotations, either of a wheel, or of Papert's turtles, etc.

4. Comment: Speaking of angles, we might contrast the static angle



which is a union of rays with a common end-point, vs. a dynamic angle, which measures (say) the rotation of a wheel. It is important for children (and teachers) to work first with the dynamic angle, which is very sensible if taken in context. Once you understand dynamic angles, then static angles are easy.

But if you start with static angles, you will continually have trouble with angles of zero degrees, or 180 degrees, or angles more than 360^{o}, or with negative angles.

This is probably a case of Polya's Inventor's Paradox.

C. 1. Mathematical Sentences and Systems.

Integers, rationals, real numbers, exponents, roots and radicals, operations, postulates, clock arithmetic, polynomials, matrices, open sentences, linear equations and inequalities, systems of equations, polynomial equations.

2. Comment. This involves an important dilemma. Do we do it intuitively first (which I prefer), or do we begin with the final elegant abstract result? The intuitive treatment, if it is honest, must be somewhat messy, because we do not at the outset see the problem with maximum abstract clarity. (I'll return to this point below, in the case of functions.) For example, we can look at equations such as

$$x + 3 = 2$$

$$x + 3 = 3$$

$$x + 3 = x + 6$$

$$x^2 = 2$$

$$x^2 = -4$$

and ask whether they have solutions, and from such considerations (or others) we can search for "new kinds of numbers."

Alternatively, we can begin with precise delineations of the various systems, which would never be a starting point in an honest gradual development.

D. 1. Geometry.

Shapes (planar and solid), form and patterns, congruences, paralle_s, similarity, transformations (isometries, symmetry, enlargements, distortion), tessellations, topology, regions, polyhedra, deltahedra.

2. Comment. I would not use traditional Euclidean synthetic geometry as the basis for school geometry in grades 2 through 9. Traditional "10th grade" geometry uses formal logical proofs, therefore (necessarily) complex verbal statements, and therfore precise verbal defintions.

This mode of operation is very foreign to children. Their language is vague and impressionistic, but their perceptions are nonetheless sharp. Children are oriented toward doing, as in the famous child's definition of a hole as "A hole is to dig."

Now, fortunately, there are geometries that are oriented toward doing. Simple parts of analytic geometry are, since they involve acts like counting in order to plot points. Marion Walter's "milk carton cutting" is action-oriented, and excellent for visualization (and other mathematical aspects). Geoboards are excellent. Edith Biggs has much good action-oriented material. Formulas for area and volume are good for students in a cumulative curriculum who have come to feel at home with variables, functions, etc. Even vector geometry can be used to good advantage. So, of course, can Papert's "turtle geometry," except that you probably need to have a turtle, and that means you need to have a computer!

E. 1. Relations and Graphs.

Pictographs, bar graphs, circle graphs, binary relations, networks, graphs of relations, functions, formulas, sequences, coordinates,

coordinate graphs, interpretation of graphs, extrapolation, interpolation.

2. Remark. The concept of function is a good place to describe my final dilemma: do we develop the ideas gradually, in a (rough) mimicking of a (simplified) "historical" development, or do we begin with the best 1974 abstraction? The abstraction version is easier to read. It begins with something clear and definite, such as: "A function $\mathcal{F}$ is a set of ordered pairs of numbers such that, if $(a,b) \in \mathcal{F}$, and if $(a,c) \in \mathcal{F}$, then $b = c$."

But beginning this way distorts completely the nature of math, which is essentially an unending struggle against problems that are not well understood.

We reveal the true nature of math much better if we use a more gradual, intuitive, "successive approximations" approach, e.g., with:

> "y is a function of x if the value of y is known when the value of x is given."

But what do we mean by "known"? If $y(n)$ is the nth prime, is $y(10^{23})$ "known"? Is $J_0(0.17893)$ "known"?

Note the conflict in usage between scientists and mathematicians: The scientist will often say: "in an open system, the pressure is not a function of the temperature," meaning that, if the system is open to atmospheric pressure, its pressure won't change when the temperature changes. (Cf. also economists, etc.) The pressure is "known".

From all this messiness, we gradually refine the current 1974 abstraction.

As usual, I urge us to settle the dilemma by compromise. Maybe the book will give the modern abstraction first, but then it will say: "This idea did not spring full-grown from the head of some mathematician. It has developed gradually, from the following history: . . ."

IX. Summary.

This paper is only a set of footnotes to the Hoffer - Nelson paper. I recommend their plan very highly, and do not wish to suggest that we change it.

WORKING GROUP RECOMMENDATIONS

CONTENT

The group discussed the word content from two points of view: first as the mathematical topics which should be covered in the resource materials and secondly as the composition and structure of the books. The question was raised whether the mathematical topics to be covered should be those in Dr. Davis' position paper or the ten topics listed in the project description or some other list. The group then set about to resolve this question and make the following recommendations concerning the content of the resources.

Recommendations:

1. It was emphasized that these should be teacher-oriented resources so the books should cover all the topics of a middle school program and yet the size of each book must be a consideration. In some cases topics can be developed for the teacher in less space than topics which are covered for students. The student pages should serve as examples of the main messages to the teacher. There should be a balance between the materials for teachers and for students. It was recommended to place an emphasis on the teacher materials and include a sufficient amount of sample student pages.

2. The group supported the topical list of books presented in the project description and suggested to weave throughout the books the topics which were presented in Dr. Davis' position paper. It was also suggested that there could be some practical value in producing an overview book which

might contain four main sections dealing with the following categories of topics which are listed in the project description: (a) Number, including Number Sense and Arithmetic Skills, Mathematical Sentences and Systems, and Number Patterns and Theory; (b) Geometry, including Measurement and the Metric System, Geometry, and Ratio, Proportion and Scaling; (c) Statistics, including Relations and Graphs, Statistics and Information Organization, and Probability and Expectation; and (d) Mathematics in Science and Society. In establishing a working priority, the group recommended that the resource: Number Sense and Arithmetic Skills be developed early as well as a book which attacks the problem of the "junior high slump", which was alluded to in Dr. Davis' paper. Some group members felt that the resource: Measurement and the Metric System should be developed early because of its timeliness.

3. It was emphatically recommended that the project should not repeat the mistakes of prior curriculum projects. That the project should:

   a. Avoid flashy irrelevancies such as a superficial use of sets and flow charts or so-called laboratory activities such as making Christmas tree ornaments without discussing geometric concepts.

   b. Relate a topic to other parts of the curriculum. For example, if the distributive law is introduced in arithmetic, it should be pointed out to teachers how the law relates to topics in algebra or is used elsewhere in arithmetic.

   c. Stress priorities. The resources should make clear to the teachers what is fundamental and why it is important at the middle school level. The project should decide on a core of important concepts and do these well.

4. It was suggested that teachers generally do not look in depth at the mathematics they attempt to teach. This is especially true of arithmetic. The resources could provide teachers with a penetrating analysis of questions and comments which arise in the learning and teaching of arithmetic. There is mathematics to be learned here as well as an awareness of potential learning difficulties.

5. The project should approach arithmetic as applied mathematics. A page could begin with an interesting problem for teachers and which might be applicable to students. This could be followed by a class, small group, or individual student activity involving, for example, carpentry or cooking, which may in turn lead to more mathematical questions.

6. With regard to the alleged junior high slump, it was recommended that the resources should encourage teachers to guide students into explorations of their own which might decrease teacher supervision and demands. It was suggested that too many adult demands on junior high age children may well cause or contribute to the slump. Such explorations may be patterned after computer related activities where young students devote vast amounts of energy and time in writing and running computer programs on topics of their choice with neglible interference from teachers.

7. The group felt strongly that information about computers and calculators should be in the resources but there was no clear decision whether or not the project should develop a separate book in these areas. The group did recommend that ideas about computers and calculators should be interspersed throughout all of the resources in much the same way that applications and problem-solving should be woven throughout. Some group members felt that a separate book on computers should be developed in addition to the inclusion of computer ideas in the other topical resources.

8. It was recommended that the project attempt to organize the resources so that teachers will be able to up-date the materials by incorporating new things which appear commercially or in journals. It was suggested that an outgrowth of the project could be a clearinghouse of organized resources and possibly information and services to teachers nationwide.

9. It was recommended to build inservice potential into the resources in such a way to release the creative power within teachers. For example, there could be a sequence which begins with tear-off sheets for students and proceeds to creative activities with increasing teacher involvement and leading to open-ended or comprehensive problem-solving situations. There

was a warning not to mislead teachers with too polished worksheets which work too, well so that they do not foresee the difficulty in producing similar things on their own.

MODELS, PROBLEMS, AND APPLICATIONS OF MATHEMATICS*

By MAYNARD THOMPSON
Department of Mathematics
Indiana University
Bloomington, Indiana

The curriculum projects of the late fifties and sixties were still young when some mathematicians and mathematics educators proposed that the mathematics community reconsider the role of applications in mathematics education ([E], [P]**). The proposals were, however, for a type of application and a point of view that were rather different from the traditional use of mathematics as a tool in engineering and the physical sciences. Briefly, there is some elaboration in Section 1, the concern was with the discussion of honest or genuine or real problems arising outside of mathematical models. The report of the Snowmass Conference held in June 1973 concluded that although considerable attention had been directed to these issues, little actual change had taken place in the classroom ([S], Chapter V).

Section 1 of this paper is a selective review of the major features of the proposed uses of applications in mathematics instruction. Section 2 is a discussion of the curricular implications of these suggestions and contains some examples. The relation between the recommendations and certain current projects is considered in Section 3.

1. The Process of Applied Mathematics.

The traditional vehicle to present applications of mathematics is the

*Pre-Conference Paper prepared for a Conference on Topical Resource Books for Mathematics Teachers, Eugene, Oregon, June 9-12, 1974.

**Letters in brackets refer to the references.

"word" problem of the mathematics textbook. The simplest such problems require the use of elementary arithmetic, algebra, geometry, and probability and statistics to aid in the tasks which must be performed by citizens in the seventies (and beyond). A few examples: computing the price per unit in the grocery store, finding yearly depreciation when completing an income tax return, determining the material necessary to build a fence, and evaluating costs of loans and rates of return at savings institutions. There is no serious dispute on the worth of such problems. They are and will continue to be an important part of the mathematics curriculum.

Another type of word problem, roughly the next level of sophistication above the simple ones just mentioned, is one involving a situation in everyday life or another academic area in which a translation from everyday language to mathematical language is necessary. Usually the translation from the terms used in stating the problem to the mathematical symbols is completely routine, and no more or no less information is provided than is necessary to solve the problem. Frequently the situation makes little or no sense is a real problem and any relation to a question which might actually arise is kept hidden from the student. Textbooks abound with constant current rivers, constantly accelerating balls, production schedules given by polynomial functions, precisely known demands, etc. Many of these situations could lead to honest and quite interesting problems, but when presented without any comment on the often gross idealizations, they have little value as examples of applied mathematics. Unfortunately they masquerade in many classrooms as examples of how mathematics is actually used. Today's students are too knowledgeable to be misled by such claims.

The sort of activity that is truly representative of the use of mathematics in contributing to our understanding of a situation rarely occurs in textbooks, even very recent ones, and only occurs in the latest generation of project-produced materials. This activity is basically the construction and study of mathematical models. Indeed "applied mathematics is the art of model building." Although the words are those of Arthur Engel ([E], p 258), the sentiment has been expressed at least since the thirties. The multistage model building process has been described in many ways (see for example [K] p 55-56 or [T], Part II) and will be considered here only in broadest terms. Typically one

begins with a situation and a concern. The concern may not be a specific problem, in fact, in most cases, it will not be. For example, the situation might be that of owning an automobile and the concern that of determining the best time to trade it. The formulation of a specific problem is an important, frequently difficult, and sometimes highly creative aspect of applied mathematics. In the example above, one has to assign a meaning to the term "best", and the meaning will clearly depend on the individual. For some, dependability may be the foremost consideration, for others, economy, and for others, style or safety features. In other situations the problem may be the very general one of understanding the reason that certain data occur. One can view elementary population genetics in this way. Incidentally, elementary Mandelian genetics provides a very attractive model building opportunity. The situation is an inherently interesting one, it can be presented with minimal biological content, and it is open to a variety of simulation activities. There is a very nice program on PLATO for computer simulation.

There is a significant shift of emphasis here from the "given a problem--solve it" point of view to that of "given a situation--study it." It is another example of the emphasis on understanding and deemphasis of the answer as the primary objective that is a common ingredient of modern mathematics programs. The goals of teaching for understanding rather than technical skill --for example, improved retention and the ability to apply mathematical knowledge in new situations--can also result from careful study of applications. One of the objectives of all mathematics instruction is to develop in the student the ability to understand and work with abstractions. However, more is wanted than the ability, say, to recognize that a certain operation is commutative. There is, to be sure, some talent involved in checking whether an axiom is satisfied. However, we ought to be concerned with deeper insight, with the ability to examine a situation where the mathematical content is not at all clear and to recognize that the concept of commutativity helps in understanding what goes on. The desire of students to know what a certain mathematical idea means, how a technique works, and why it is useful, requires careful cultivation. It is all too easy to omit the time-consuming discussion that provides the proper context for thinking about a situation. The urge to "get to the point" quickly must be restrained.

In summary, the phrase "the process of applied mathematics" as used here refers to much more than the simple word problems or the translation problems of typical textbooks. It is a more general activity than solving mathematical problems. It includes the identification and formulation of precise problems in situations which may be only partially understood, the solution of these problems (involving computations where appropriate), comparison of results with observations or experiments, and finally the drawing of conclusions. It may be necessary to cycle through the process a number of times before the conclusions are satisfactory.

2. Implications for Instruction.

The modeling activities described in Section 1 suggest some changes in how instructional time is used and the mathematics that is taught.

First, at least a portion of the time devoted to mathematics will be spent in a rather different way than has been traditional. Unstructured situations arising outside mathematics will be discussed and there will be a certain amount of time that appears marginally productive from a mathematical point of view. Such activity is almost always a part of any use of mathematics outside the classroom, and consequently provides valuable experience. Much of the lack of recognition of the utility of mathematics by the average citizen may originate in the lack of any classroom experiences that remotely approximate the way mathematics is used outside academe.

Second, some mathematical ideas will arise that are not a part of the traditional curriculum. The most important of these are the concepts of randomness, probability and statistics. Even very unsophisticated data collection and organization activities can profit from a knowledge of elementary statistics, and many real problems from everyday life have a stochastic character that is completely lost if one is restricted to thinking in deterministic terms. An example cited by Henry Pollak is useful here. Consider the question, "what is the best way to get to the airport?" The immediate task is to decide on a meaning for "best". In some instances it might mean the minimum distance route, in others the route of minimum time or expected time. After some personal investigation Pollak concludes that frequently best means the route of minimum variance in time, not necessarily

minimum expected time. Thus two of the three criteria mentioned are stochastic ones.

This example illustrates another characteristic of many problems that arise in real situations, namely, they are questions of decision. Their study involves notions of optimization and the analysis of complex systems that are entirely outside the traditional school mathematics. Since most decision making is a decision making under uncertainty, it is crucial that a probabilistic (and/or statistical) mode of thinking be developed. Monte Carlo simulation of systems which cannot be analyzed at the level of the student provide a realistic way of obtaining information on a situation that is otherwise intractable. For example, it is common to have trinkets included in boxes of breakfast cereal. Suppose there are six different trinkets, how many boxes of cereal should you expect to purchase before you have all six trinkets? Here is a very down-to-earth problem--certainly one which has occurred to every parent--and whose solution is beyond the usual school mathematics. However, the process can be simulated either with a spinner or a table of random numbers. The results of a modest number of simulations give a means of estimating a quantity whose determination is otherwise impossible. The results of a computer simulation of opening boxes until all six trinkets have been obtained 200 times are presented on the next page.

number of boxes	frequency	histogram
6	3	xxx
7	9	xxxxxxxxx
8	10	xxxxxxxxxx
9	18	xxxxxxxxxxxxxxxxxx
10	11	xxxxxxxxxxx
11	17	xxxxxxxxxxxxxxxxx
12	18	xxxxxxxxxxxxxxxxxx
13	19	xxxxxxxxxxxxxxxxxxx
14	11	xxxxxxxxxxx
15	15	xxxxxxxxxxxxxxx
16	12	xxxxxxxxxxxx
17	13	xxxxxxxxxxxxx
18	6	xxxxxx
19	5	xxxxx
20	4	xxxx
21	5	xxxxx
22	4	xxxx
23	3	xxx
24	3	xxx
25	4	xxxx
26	1	x
27	2	xx
28	2	xx
29	1	x
30	0	
31	0	
32	1	x
33	1	x
34	0	
35	0	
36	2	xx

Other types of content that often arise in modeling are combinatorics and graphs. Many decision problems involve enumeration of possibilities, at least at the level of special cases, and for this a little knowledge of combinatorics is very helpful. Counting problems also arise in computing probabilities. A graph is a mathematical structure of great generality whose utility as a modeling device is becoming increasingly apparent. Graphs (or networks) have been standard tools of mathematical programmers since W.W. II, and their use in the social sciences (as sociograms) is almost as old. They also provide a convenient mathematical structure to use in studying everyday problems. An example that I have used several times is that of scheduling a tournament. How should a tournament be arranged? This is an example of the

sort of unstructured and somewhat vague situation that can lead to a number of interesting questions. What is the goal? Is it fairness, efficiency, or compatibility with a school schedule? One precise question is how many periods are necessary to schedule a round robin tournament with a given number, say n, of teams? A more complicated question is how can a schedule be determined? A combinatorial argument shows that the answer to the first question is $\frac{n(n-1)}{2[n/2]}$, where [X] denotes the greatest integer in X . In working with students one would normally encounter the problem for a specific value of n and no effort would be directed to obtaining the general result. The development of an algorithm for actually assigning terms to time slots is an interesting and worthwhile problem. Algorithms exist based on modular arithmetic, basket weave techniques for selecting subsets of entries in a matrix, and connecting vertices in a regular n-gon.

This brings up an important point. Frequently the utility of mathematical techniques applied to real problems hinges on the existence of algorithms. The simplex method of linear programming is an outstanding example of an algorithm which, because of the ease and efficiency of computer implementation, led to the widespread use of a particular mathematical model. Another example of a problem in which an algorithm is essential for the model to be of any value is that of determining the least cost path in a capacitated network (e.g. [TMMW],). The latter is an example of a problem for which the existence of a solution is obvious (there are only finitely many paths) but the utility of the model depends on the existence of an efficient method of determining the minimum cost path. The difference in the effort needed to solve this problem by efficient and inefficient means is several orders of magnitude.

It seems clear that calculators and computers are certain to have a significant influence on mathematics instruction in the seventies. The computational capability provided by even small hand held calculators substantially enlarges the class of situations appropriate for study. Students can process real rather than fabricated data, and large scale computations become realistic. Nonnumerical computing, e.g. sorting and logical operations, also provides access to new applications. Flow charting which is almost indispensible in computing, will become a standard part of

the curriculum.

The consideration of large scale problems will further stimulate the development of group or team approaches to applied problems. A team effort is a common way of approaching a problem outside the classroom and experiences along these lines are a worthwhile part of mathematics education.

There are a number of other areas which will receive increased emphasis: the science/art of making good approximations and estimations, methods of sampling and data collection, the basic mathematics necessary for solving everyday problems [B], and the development of strategies for problem solving, to name a few. However, all proposed modifications must contribute to the fundamental aim of mathematics education--to change the student, to influence his view of the world around him and how he reacts to it. To this end we should study important problems and we should make the effort to convince the student that they are important.

3. Current Efforts.

There has been significant progress in providing at least sample materials involving applications for students and teachers. Although the following list is by no means exhaustive, it does indicate the scope of past and present efforts.

THE MAN-MADE WORLD [TMMW] was created by the Engineering Concepts Curriculum Project and contains many examples of engineering and operations research oriented applied mathematics. The emphasis is on using mathematical and scientific techniques to understand the problems of a technically oriented society.

The Huntington Two Computer Project has written student-teacher-resource materials and simulation programs for problems in the social, life, and physical sciences. Many of these modules can be used as introductions to the area or as examples of computer simulation.

The ASA-NCTM STATISTICS BY EXAMPLE [SE], and the accompanying volume STATISTICS: A GUIDE TO THE UNKNOWN [SGU] provide background articles and student material for teaching statistics. The MAA and NCTM have undertaken a similar project for a sourcebook in applied mathematics for use by secondary teachers.

The USMES project has prepared a number of interdisciplinary units which are based on comprehensive studies of real problems of practical significance. Sample titles of existing units give a flavor of the type of situation being considered: Traffic Flow, Pedestrian Crossings, Lunch Lines, Play Area Design and Use, Consumer Research-Product Testing, Designing for Human Proportions, and Weather Prediction.

There are also SMSG materials, especially Studies in Mathematics, Vol. XX, Mathematical Uses and Models in Our Everyday World by Max Bell, Supplementary and Enrichment Series SP-26, The Mathematical Theory of the Struggle for Life, and SP-23, Radioactive Decay.

It is clear even from this brief list, that there are materials available. However, it is also clear, as noted in the Project Description, that they are not widely used. Efforts to provide teachers with information and resources which will facilitate and encourage use of existing and proposed materials should prove profitable.

REFERENCES

[B] Bell, Max. A Short and Tentative List of What is "Really" Wanted as a Minimum Residue for Everyman from the School Mathematics Experience, 1972.

[E] Engel, Arthur. "The Relevance of Modern Fields of Applied Mathematics for Mathematics Education," Educational Studies in Mathematics 2 (1969), 257-269.

[P] Pollak, H. O. "How Can We Teach Applications of Mathematics," Educational Studies in Mathematics 2 (1969), 393-404.

[S] Report of the Conference on the K-12 Mathematics Curriculum, Snowmass, Colorado, June 21-24, 1973.

[SE] Statistics by Example. Part 1, Exploring Data; Part 2, Weighing Chances; Part 3, Detecting Patterns; Part 4, Finding Models, ed. by Frederick Mosteller et al.,Reading, Mass.: Addison-Wesley Publishing Co., 1973.

[T] Thompson, Maynard. "The Role of Applications in Mathematics Education," Colloquium for Prospective Supervisors of Mathematics, Teachers College, Columbia University, 1970.

[TMMW] The Man-Made World, prepared by the Engineering Concepts Curriculum Project, New York: McGraw-Hill, 1971.

[USMES] The USMES Guide, Education Development Center, Newton, Mass., 1974.

WORKING GROUP RECOMMENDATIONS

APPLICATIONS

The working group noted that the term "application" is ambiguous in the mind of the teacher. For example, the term may be interpreted as the reproduction of an old process in a new setting such as applied to a different set of data. The meaning adopted by the working group, however, was the use of mathematics in "real-world" situations, issues and problems. The group asserted that superimposing the structure of mathematics on the world of students is a motivational technique and that this should be based on criteria which relates to the life-space of the learners including their personal, peer group and family interests. The degree to which an application is personally and/or socially significant is a crucial variable in deciding its suitability for inclusion into a school setting. As it was discussed in Dr. Thompson's position paper, the group fully agreed that applications should involve an extensive use of problem-solving processes and the use of model-building as a way of specifying and representing a problem.

Recommendations:

1. Involving teachers in applications and comprehensive problem-solving is difficult but once they have adopted an attitude and operating policy for comprehensive problem solving this opens the way to applications of various types and levels. Another problem is that mathematics teachers tend to avoid interdisciplinary or multidisciplinary activities either because of an overemphasis on skill building or a lack of knowledge in other areas or because of a fear of conflicting with another teacher's

domain(eg., should a discussion of batting averages be the domain of a physical education teacher?) It was recommended that the project present, in the resources, materials which encourage teachers to develop, within themselves, a new sensibility toward applications. They need to believe that applications are useful to motivate and synthesize specific concepts. They need to believe that they have the knowledge and techniques to apply the mechanics of comprehensive problem-solving situations with their students.

2. The group's deliberations focused on the role of skill-building. A point was made that skill-building requires separate demands quite apart from applications, that students will not be able to cope with applications if the applications are too involved and if the students lack sufficient skills to solve the inherent problems. The question was raised whether mathematics should be emphasized in school curricula or should mathematics be an outgrowth of a problem situation. The group seemed to agree that "schools teach skills which students never use and expect students to use skills they were never taught." It was recommended that the project develop materials which tie skill-building with applications at various levels.

3. In order to determi. appropriate topics and activities for applications, the group recommended a two-pronged approach. Students should play an integral part in the formulation of the problem - applications should be about things which interest the students and they should be involved in decision-making processes. Also it was recommended that, whether students fully accept it or not, initially, there are things which they will need in order to function effectively in society about which they should have some knowledge. Students should also understand the role that mathematics may play in the formulation and solution of societal problems.

4. The group recommended that structure should be encouraged for applications in school curricula. Teachers could analyze their current program and de ide upon a necessary or consensus sequence of learning in mathematics and agreed upon domains of concern in mathematics. They could also appraise the general arenas of student activity from which applications

may be drawn. Then a matrix may be devised which pairs a topic from the course of study (for example, percents) with an area of student interest (for example, sports) and establishes possible activities for applications in the entry of the matrix which relates these. The project could include a sample of such a matrix in the resource materials.

5. It was recommended that the resources contain numerous examples of applications and problem-solving situations. Descriptions of activities should be provided for teachers to help them develop the mechanics of using applications with their students. As examples, there could be suggested questions for teachers to pose to their students to think about over the weekend: there could be questions which involve a 10-minute discussion in class; there could be questions which involve an entire class period, as well as 2-3 days activitiesand eventually long-range activities which may take weeks or a full term to complete. In this way, teachers could start small and develop their teaching techniques for problem-solving and applications.

6. It was recommended that the examples of applications which are provided in the resources should not be sterile or contrived such as income tax exercises or questions about the amount of concrete needed for a walkway. An appropriate activity at least includes: student appeal, concept development, laboratory activity, problem-solving and skill development. The project staff should establish criteria questions which may be applied in deciding upon the inclusion or exclusion of an application. Some examples are:

   a. What mathematical topics can be connected to it?
   b. What levels of understanding are involved or necessary?
   c. What resources are needed (materials, maps, etc.)?
   d. Does it utilize student-generated data?, readily available data?, published data?
   e. Will students perceive it as problematic l?
   f. To what degree is it related to the life-space of the adolescent?
   g. To what degree does it have appeal and interest to teachers and students?

h. Does it relate to socio-emotional concerns of early adolescents?

i. Does it require acceptance of new patterns of classroom behavior?

j. To what extent is it appropriate to "concrete operational" thinking stage, to "formal operational" thinking stage, or to the transition from the former to the latter?

k. Is it an "open" investigation?

l. Does it lend itself to "chunking", that is, to the unifying of ideas?

7. It was recommended that the project staff should be familiar with various studies and curriculum projects. In some of these there are excellent examples of student-generated data which could provide prototypes for similar activities involving mathematics. Examples may be found in the American Political Behavior Project, in the Biological Science Curriculum Study, in the National Social and Emotional Concerns of Early Adolescents, in guides from Unified Science and Mathematics in Elementary Schools, and in the Man-Made World Project. Students may be interested in music, sports, personal and physical measurements, identification of people, suicides, etc. These provide potential topics for student-generated data.

8. It was recommended that the resources contain references for teachers of other sources from which they could select information for and examples of applications in which they could involve their students. These references could include mention of inexpensive sources such as world almanacs and publications from local, state and federal governmental agencies.

DEVELOPING TEACHER COMPETENCIES
FOR EVALUATING MATHEMATICAL LEARNING

By LARRY L. HATFIELD
Visiting Associate Professor
Department of Mathematical Education
Teachers College
Columbia University

PRERAMBLE*

The preparation of this "position paper" was a challenging but exciting task, reflecting my feelings about the topical resource books project. The ideas to be found herein certainly need to be discussed and amplified far beyond what I have managed to accomplish. Hopefully, our working group(s) will attempt this during the Advisory Conference. Many leads to reasonable and useful references have been cited; I hope the reader will be able to find and review (especially) the starred references prior to our Eugene meeting.

SOME PREMISES AND ASSUMPTIONS

At the outset I wish to voice certain initial points of view which I have employed in constructing the ideas which follow. My aim here is to try to clarify and focus some of the fundamental realities, purposes and perspectives that I see in the production of the evaluation component of these resouce books.

1. The meaningful and useful measurement of children's mathematical knowledges, attitudes and developmental readiness can be impossibly complex. A great variety of intervening factors can essentially invalidate any results obtained

*"Misspelling" intended by author.

during an assessment activity. Yet educators do attempt to find our more about a child's knowledges, thinking processes, or feelings. Of course, our approaches to evaluation must necessarily be pragmatic: ideas, materials and methods that can be immediately and directly applied to a wide variety of practicing teachers. At the same time we must exercise caution to avoid misleading ourselves and our future users with an over-simplified, naturalistic view of the evaluation task.

2. Evaluation, to most teachers, seems to be synonymous with testing aimed at grading and classification, which in turn is embodied almost without exception in teacher-made, post-unit or post-course (summative), paper-and-pencil tasks of recognition or memory (recall), comprehension and (sometimes) application. A less-recognized but widely practiced approach to evaluation is the administration of commercially available "standardized" tests whose results are rarely but often erroneously used by teachers. We must develop for teachers a considerably broadened viewpoint of evaluation activities in these resource books. The purposes, techniques and timing of evaluation as well as the analysis, interpretation and application of evaluation results must be clarified and operationalized. To the extent feasible the approaches to evaluation must be offered in teacher-ready ideas and materials.

3. The fundamental notion of evaluation adopted for this paper is the characterization presented by Hartung (1961). Taking as undefined the terms behavior and situation, he defined three key terms:

i) experience is behavior in specified situations;

ii) objectives are desired experiences;

iii) evaluation is the process of finding the extent to which actual experiences conform to objectives.

He asserts that "...the purpose of education is to change students from a given state of experience to a desired state by means of a variety of appropriate learning experiences, some of which may be used as a basis for evaluation of achievement....Evaluation is the means we use to discover where we stand on the path between present experience and the objective" (Hartung, 1961, p. 23).

4. The primary concern of evaluation is its use in improving the student's learning and the teaching. Evaluation is to be viewed as a system of quality

control whereby teachers and students may frequently and dependably determine whether the teaching-learning process is effective or not, and if necessary, what changes should be made to improve its effectiveness. This view implicitly recognizes that education can produce significant changes in learners and explicitly sees the need for evidence that certain changes are indeed taking place in learners as well as the need to determine the extent of change in individual students.

Bloom, Hastings and Madaus (1971) make a distinction between the teaching-learning process and the evaluation process. Summative evaluation is viewed as apart from the instructional process with a clear intent to make a judgment after instruction or learning has taken place. Formative evaluation is an integral part of instruction, using a variety of means to gather samples of behavior in order to provide feedback information for facilitating learning and instruction. Thus, formative evaluation should facilitate the pacing of learning, aid in pinpointing gaps in the student's knowledge or difficulties in the learning due to the student's background, provide reinforcement to and build the confidence of students, set the stage for a complete diagnosis of the student's difficulties and their probable causes, and provide feedback to the teacher to be used as a basis for modifying the instruction.

5. Wilson (1971)*, Weaver (1970), Epstein (1968), and Avital and Shettleworth (1968)* have described levels of cognitive behavior in mathematics. Used in a double classification scheme with different types of mathematical content, one has the basis for a two-way stratification of outcomes of mathematics instruction. Such schemes have been used to formulate the evaluation activities of several large scale studies of mathematical achievement (see Husén, 1967; Romberg and Wilson, 1968; Foreman and Mehrens, 1971). The availability and application of such a scheme as Wilson's (1971) can help to guide teachers in writing or choosing test items to reflect the balanced treatment of all cognitive levels. Such a guide should probably also be used as a kind of template to fit over the goals, objectives, and methods of instruction to assure that a balanced emphasis occurs here, too.

---

*References so noted are recommended for study prior to the Advisory Conference.

LISTENING TO "NOISE"

Noise is all around us, battering our senses and numbing our brains. Think of "noise" broadly enough to include not only the auditory din of a busy metropolitan thoroughfare but the visual gruel served to us through most of the commercial television programming. Environmentalists discuss noise pollution and its effects. But there are subtleties that make elusive the classification of such environmental stimuli. Noise is like weeds: dandelions are considered gourmet delicacies or nutritional sources by the Euell Gibbons but are attacked as pests by the Scotts, the Greenfields, and the Orthos. What is excluded or controlled in one context can be sought after in another.

All measurement is accompanied by "noise", the factors which appear as interruptive, irrelevant sources of interference or distortion. Scientists have learned to filter or dampen out the noise accompanying their experiments. At the same time, important information revealing sources of error and interesting new variables have been found in what had been considered to be noise. Radio astronomers have discovered distant stellar systems in the "noise" of their observations. Einstein first tested his great theory of relativity through experiments that sought to measure important distortions in light waves coming from a distant star. Science, as a search for patterns, consistencies and predictions, has often considered "experimental noise" as an important source of information.

Classroom measurement of mathematical learning in the form of objective paper-and-pencil tasks dramatically dampens-out most of the "noise" that exists in the teaching-learning context. It is exactly this noise that we need to seriously consider in producing a more sensitive and effective learning environment. As I see it, the major task of the evaluation aspect of the resource books will be to sensitize middle school teachers to the existence of important (heretofore) "noise" in their evaluation approach and to assist them in using the new information spawned by attending to, sometimes measuring, and reflectively analyzing this "noise". In addition, attention should be given in these resource books to helping teachers use more completely the ("noise" dampened) information now generated in their objective tests.

Types and sources of "noise"

I attempt here to freely identify some typically overlooked but potentially significant items of information for the evaluation of learning and teaching. (Perhaps some attention can be given in our working group to sharpening these foci, considering others, culling those which cannot be operatior .ized, etc.) Ignored or dampened-out "noise" in the traditional evaluation context, these items represent sources of distortion, delimitation or error in measuring student mathematical knowledges and feelings.

1. Cognitive development.

Almost certainly some poor performances are directly due to the child's inability to deeply learn (operationalize) particular mathematical knowledges because of the student's state of cognitive development. Piaget and a large cast of his disciples have convincingly argued for the importance of considering the cognitive development of children in relation to learning. This point of view would integrally connect competence (as opposed to performance) in mathematics with the presence of prerequisite cognitive structures. Experience results in learning only to the extent that the elements of the experience are assimilable to the existing cognitive structure. For example, this theory identifies the emergence of a proportionality schema as the stage of formal operations is obtained (us .ily about the ages of middle school children) (see Lovell, 1971). It is entirely possible that the serious difficulties that many children encounter in work with fractions, rational numbers, ratio, proportion, probability, and geometric similarity may be due in part to their stage of readiness to assimilate the instruction being given.

Piaget's theory of cognitive development has perhaps shown how terribly complex the growth of mathematical ideas can be, but it has not demonstrated how it can be made easier. Beilin (1971) has noted that the direct translation of the clinical method, which is useful for researchers as a technique in the discovery of the constructive processes, to a technology of educational instruction is at this point unwarranted. Education is "becoming increasingly organized in terms of mass technology, with more children per teacher, more materials and more instrumentation per class. The time spent by a teacher with an individual child is constantly diminishing. Individualized instruction is increasingly a myth, and instrumented individualization is also a myth, as

has been pointed out by Piagetians themselves. The Piagetian method, on the other hand, places its primary emphasis on one-to-one teacher-student interaction with individualized teacher response. This is economically and tactically impossible in today's increasingly crowded schools. What is needed, on the other hand, is vigorous pursuit of how 'constructive' educational approaches can be realized with groups of children, which would require instructional strategies different from the one-on-one 'clinical method'" (Beilin, p. 115).

Some writers and curricula have sought to incorporate Piaget's problem situations and methods directly into teaching situations (see Copeland, 1974; Nuffield Project Teacher's Guides, 1967; Dienes, 1963; Adler, 1966). Sinclair (1971)* observes that "a child's reactions to a few Piagetian tasks will enable a well-trained psychologist to give a fair description of that child's intellectual level; but teaching the solutions of these same Piagetian tasks to a group of children does not mean that the children will thereby attain the general intellectual level of the child who can solve the tasks independently" (pp. 1-2).

What, then, can <u>we</u> reasonably consider from developmental psychology for use by teachers? First, an attempt might be made to describe in these resource books the major features of Piaget's theory (see Sinclair, 1971)*. Surely this presentation must only highlight this detailed theory and particularly emphasize the concrete and formal operations stages. Secondly, an effort might be made to discuss the content knowledges of the key mathematical strands (e.g. number and operation, measurement, geometry, functions and relations, ratio and proportion, probability and statistics) in terms of the necessary underlying cognitive structures. Third, describe diagnostic activities (group as well as individual) for observing whether a child has acquired notions judged to be basic to understanding the new content (see Riesman, 1972; Steffe, 1971)*. I recognize that, done carefully and completely, this development will require considerable writing effort. I hope we would view the importance of considering selected developmental phenomena to be sufficient to warrant the expenditure of effort.

2. Learning versus testing.

Achievement testing usually addresses itself to finding out what students know and can do. However, consider the incongruence that may occur between the situations of learning and testing, particularly when instruction seeks

to employ guided discovery, laboratory or manipulative activities, or real-world challenges. To be sure, most tests seem to require that the student transfer learnings obtained in these more action-oriented group settings to independent paper-and-pencil tasks. Yet, care and consideration must be used for developing situations of evaluation that are reasonably similar in design and conduct to the concrete, active, constructive approach of the instruction. This becomes especially important when processes of problem identification and definition as well as strategies (heuristics) for approaching problems are part of the goals of instruction. Generally, only inadequate efforts have been given toward devising testing situations which focus on higher cognitive processes such as analysis, synthesis, creativity, and problem solving. An interesting example can be found with the Unified Science and Mathematics for Elementary Schools (USMES) project evaluation, wherein group challenges with various materials are used as a direct parallel to the type of context used in the instruction. Other excellent examples can be found in the test items offered by Wilson (1971)* and Avital and Shettleworth (1968)*. It appears that important emphases on manipulatives, on applied challenges, and on comprehensive problem solving strategies are intended in these resource books. Therefore, suggestions for examining student performances and difficulties in a fashion that reflects these emphases must be offered.

3. Wastebasket "noise".

Swart (1974) has echoed the point of view that what children are evaluated on is what they learn in mathematics. And, furthermore, he contends that the vast majority of classroom instruments (tests and daily work) involve almost entirely computation exercises. These instruments very likely mask or distort what the child really "knows" about these exercises. Probing beyond the written solution will often reveal a child has either no concrete, meaningful representation for the exercise (e.g. 2x3 as 2 sets of 3 blocks) or an incorrect representation (e.g. 2x3 as a set of 2 blocks and a set of 3 blocks).

Examining the strategies and meanings employed by students in obtaining correct as well as wrong answers can be most enlightening. Lankford (1974)* describes the computational strategies of seventh grade pupils from 6 varied school systems. He interviewed and recorded 176 subjects by presenting a set of computational exercises and encouraging them to describe aloud their

thinking as they computed. His report illustrates what a teacher may learn about a pupil's thinking through such oral interviews:

> Through individual oral interviews with several pupils in a class a teacher will soon become aware, maybe to his surprise, of wide variations in computational strategies employed by his pupils. He can then plan his teaching either to encourage and reinforce those variations that are acceptable or to try to achieve more uniformity in computational practice for his pupils by getting them to adopt and use a single strategy approved by him. (Lankford, 1974, p. 32.)

Ashlock (1972)* has emphasized the identification of error patterns in the written computation work of students. Citing a study by Roberts (1968), he establishes the value of looking for erroneous algorithmic techniques which students employ with some consistency. He encourages teachers to consider possible reasons for the adoption of defective algorithms and to provide remedial instruction which may recognize some of the possible sources of error. Providing for mastery of basic facts, emphasizing meaning in the development of an algorithm, allowing flexibility by encouraging alternative algorithms, and encouraging skill in estimation (to be able to check if an answer is reasonable) are presented as important factors in teaching computational procedures. Numerous specific activities are described to deal with the remediation task implied by each illustrated error pattern. Ashlock's ideas will surely be helpful in formulating similar emphases in the resource books. The identification and analysis of student errors represents a major, generalizable perspective for diagnostically-based instruction; major attention should be given to applying these ideas to the learning of other items of knowledge besides computation (see Riesman, 1972)*.

4. Sighs, frowns, tears, and fingernails.

Probably the deepest conviction that I hold about mathematics learning is that it ought to be challenging, exciting, fun, and worthwhile (relevant?), not only for the teacher but more importantly for the child. I am confident most teachers would be able to agree with this point of view. And I believe that the character of learning must especially be this way for middle school-aged children. The emergence of adolescence can be an especially difficult emotional time for many girls and boys. I have heard many discouraging

statements from middle school and junior high school teachers about how children lose the fresh, uninhibited joy for learning so characteristic of younger children. The factors contributing to this phenomenon, be it real, would certainly be many and complex.

The measurement of affective constructs, such as attitudes, interests, anxieties, self-concepts, values, and appreciations, is more difficult by at least another order of magnitude than cognitive outcomes. Corcoran and Gibb (1961)* present an excellent discussion of methods used in appraising attitudes in the learning of mathematics. While they voice caution in the interpretation of attitude data due to serious, unanswered issues of validity and reliability, they do encourage the varied accumulation of attitude information over time by teachers. Aiken (1972)* has carefully reviewed the research on attitudes toward mathematics, himself conducting several studies in this area.

I have developed this past year (through my own research on problem solving processes) an even greater skepticism about the self-reporting scales that seem to be propagating in the measurement of attitudes. Basically, I am wondering why these scale authors are not seeking corroborating information to help support the student's own description of his feelings. Certainly other overt performances can also be used to help make inferences about how a student feels about mathematics, the instruction, and himself as a learner of mathematics.

Establishing guidelines and concrete strategies and materials for assessing affective constructs should certainly be considered for these resource books. Yet I believe that we must emphasize the development of good feelings in students and perhaps worry less about being able to measure our results. I have an idea that the teacher (if he is honest with himself) will recognize behavior indicative of positive attitudes and self-concepts (or their absence) more dependably than the existing instrumentation will reliably provide. But what of this "development of good feelings"? I view this as the overall goal of the resource books. And as Neale (1969) has correctly pointed out, not only do attitudes affect achievement, but achievement (success) also affects attitudes.

5. "Who? Me?"

The teacher, as central figure in the teaching-learning scene, cannot avoid becoming a part of the "noise" in evaluation. Though not intending to do so, the teacher can easily generate distortion and error in the classroom measurement of a student's knowledge or feelings. For example, he may be

i) using tests that do not reflect the objectives of learning,

ii) writing poor (confusing, incorrect, too difficult) items,

iii) building student anxiety to the point of a debilitating effect during the test (massing evaluation, one-trial opportunity, test length, emphasizing grading and peer competition, caustic personality),

iv) incompletely, incorrectly, ineffectively teaching the content and processes required for performing on the tests,

v) ignoring valuable group and individual information by failing to analyze in detail the performances (item by student) obtained on the test.

These resource books should probably discuss the technical and mechanical aspects of test construction, administration, and analysis. Hopefully, without becoming pedantic, useful ideas and suggestions can be written to include the importance of measuring different cognitive levels, the variety of test-formats for measuring achievement, the dos-and-don'ts of good test item writing, the importance of measuring the typically overlooked outcomes (retention, transfer, attitudes, learning how to read and study mathematics, creativity and productive thinking) and the student-by-item analysis aimed at assessing the test itself (Johnson and Rising, 1972*; Merwin, 1961; Wilson, 1971*).

6. A "noisy" potpourri.

Many other factors related to the child, life in classrooms, the teacher, the school, the community, and, indeed, our society might be viewed as measurement "noise". Among these one might consider:

i) the "cognitive style" of the learner and the teacher (e.g. reflectiveness/impulsiveness or analytical; see Cathcart and Liedtke, 1969);

ii) biophysical basis for learning (e.g. short- vs. long-term

memory mechanisms; see Davis, 1970);

iii) the achievement motive of the student;

iv) teacher expectation and perception of the child;

v) attitudes of the teacher toward mathematics, teaching, children;

vi) teacher philosophy or belief-system about mathematical education;

vii) understanding of the teacher about mathematics, the teaching-learning process, children's development and cognition.

"PACKAGING" THE COMPLEX

My purpose in writing this paper has been to stimulate some initial thoughts toward developing the evaluation component of the proposed resource books. It should be clear by now that this paper has not formulated the detailed plan or contents of the evaluation activity of these resource books; that must be done during and after the Advisory Conference.

However, some previously described points are re-emphasized here to summarize my position.

1. What is done by the teacher in evaluation will reflect that teacher's viewpoint and understanding of the broad goals of mathematical education, the teaching-learning process, and the purposes and strategies of evaluation. The other working groups will be dealing with the difficult challenges of packaging content and pedagogy directed to influencing these teacher viewpoints and understanding. To the extent that evaluation activities mirror the rest of the classroom learning-teaching paradigm, then our material on evaluation must reflect the materials produced in the other working groups. In particular the emphases on applications, "new" content, interdisciplinary and real world problem solving, laboratory approaches to learning, and the affective outcomes of life in mathematics classrooms must be recognized.

2. Having teachers focus on testing as more than a basis for grading may be a trivial, but nonetheless, profound result of our resource books. Once a teacher "opens up" to other purposes, then one can emphasize the questions

of cognitive development, formative evaluation, diagnostic testing, error analysis, feelings and interests, and testing-as-learning/learning-as-testing.

3. Unlike the self-contained classrooms of primary school teachers, departmentalized middle school teachers may typically not "get close" to their students. We must encourage teachers to (somehow) arrange to use individual interview techniques of diagnosing student content strategies, understandings, disabilities, and feelings.

4. The use of formative evaluation techniques should lead to testing as a step in the teaching-learning process. Perhaps teachers will learn to use objectives-based, criterion referenced tests for some (appropriate) outcomes. A model of repeated testing opportunities may be important in any "mastery" learning scheme. The no-risk, low-threat nature of offering alternate versions of a test through a repeated testing approach can permit the focus to be on identifying those items of knowledge on which additional study may be required.

SUGGESTIONS FOR FURTHER READING

Adler, Irving. "Mental growth and the art of teaching," The Arithmetic Teacher, November 1966, pp. 576-584.

Aiken, Lewis R., Jr. "Research on attitudes towards mathematics," The Arithmetic Teacher, March 1972, pp. 229-234.

Ashlock, Robert B. Error Patterns in Computation. Columbus, Ohio: Charles E. Merrill Publishing Co., 1972.

Avital, Shmeul M. and Sara J. Shettleworth. Objectives for Mathematics Learning: Some Ideas for the Teacher. Toronto: The Ontario Institute for Studies in Education, 1968.

Beilin, Harry. "The Training and Acquisition of Logical Operations," Piagetian Cognitive-Development Research in Mathematical Education (Rosskopf, Steffe and Taback, editors). Washington, D.C.: NCTM, 1971, pp. 81-124.

Bloom, Benjamin S., J. Thomas Hastings and George F. Madaus. Handbook on Formative and Summative Evaluation of Student Learning. McGraw-Hill Book Company, 1971.

Carry, L. Ray. "A critical assessment of published tests for elementary school mathematics," The Arithmetic Teacher, January 1974, pp. 14-18.

Cathcart, George W. and Werner Liedtke. "Reflectiveness/impulsiveness and mathematics achievement," The Arithmetic Teacher, November 1969, pp. 563-567.

Copeland, Richard J. How Children Learn Mathematics--Teaching Implications of Piaget's Research (Second Edition). New York: The Macmillan Company, 1974.

Corcoran, Mary and E. Glenadine Gibb. "Appraising Attitudes in the Learning of Mathematics," Evaluation in Mathematics (Twenty-sixth Yearbook). Washington, D.C.: NCTM, 1961, pp. 105-122.

Davis, Robert B. "Research in Mathematics Education," an unpublished paper delivered to the Special Interest Group for Research in Mathematics Education, 1970 Annual Meeting of the American Educational Research Association.

Dienes, Z. P. An Experimental Study of Mathematics Learning. London: Hutchinson Educational, Ltd., 1963.

______. Building Up Mathematics (Revised Edition). London: Hutchinson Educational, Ltd., 1964.

Epstein, Marion. "Testing in Mathematics: Why? What? How?" The Arithmetic Teacher, April 1968, pp. 311-319.

Foreman, Dale I. and William A. Mehrens. "National assessment in mathematics," The Mathematics Teacher, March 1971, pp. 193-199.

Gray, Roland F. "An approach to evaluating arithmetic understandings," The Arithmetic Teacher, March 1966, pp. 187-191.

Hartung, Maurice L. "Basic Principles of Evaluation," Evaluation in Mathematics (Twenty-sixth Yearbook). Washington, D.C.: NCTM, 1961, pp. 21-42.

Husén, Torsten, ed. International Study of Achievement in Mathematics (Vol I, II). New York: John Wiley & Sons, 1967.

Johnson, Donovan A. and Gerald R. Rising. Guidelines for Teaching Mathematics (Second Edition). Belmont, California: Wadsworth Publishing Company, 1972.

Lankford, Francis G., Jr. "What can a teacher learn about a pupil's thinking through oral interviews," The Arithmetic Teacher, January 1974, pp. 26-32.

Lovell, Kenneth. "Proportionality and Probability," Piagetian Cognitive-Development Research and Mathematical Education (Rosskopf, Steffe, and Taback, editors). Washington, D.C.: NCTM, 1971, pp. 136-148.

Mathews, Geoffrey. "The Nuffield Mathematics Teaching Project," The Arithmetic Teacher, February 1968, pp. 101-102.

Merwin, Jack C. "Constructing Achievement Tests and Interpreting Scores," Evaluation in Mathematics (Twenty-sixth Yearbook). Washington, D.C.: NCTM, 1961, pp. 43-70.

Neale, Daniel C. "The role of attitudes in learning mathematics," The Arithmetic Teacher, December 1969, pp. 631-640.

Nuffield Mathematics Teaching Project - Teacher's Guides. New York: John Wiley & Sons, 1967f.

Reisman, Fredericka K. A Guide to the Diagnostic Teaching of Arithmetic. Columbus, Ohio: Charles E. Merrill Publishing Company, 1972.

Roberts, Gerhard H. "The failure strategies of third grade arithmetic pupils," The Arithmetic Teacher, May 1968, pp. 442-446.

Romberg, Thomas A. and James W. Wilson. "The development of mathematics achievement tests for the National Longitudinal Study of Mathematical Abilities," The Mathematics Teacher, May 1968, pp. 489-495.

Sinclair, Hermine. "Piaget's Theory of Development: The Main Stages," Piagetian Cognitive-Development Research and Mathematical Education (Rosskopf, Steffe, and Taback, editors). Washington, D.C.: NCTM, 1971, pp. 136-148.

Steffe, Leslie P. "Thinking about measurement," The Arithmetic Teacher, May 1971, pp. 332-338.

Swart, William L. "Evaluation of mathematics instruction in the elementary classroom," The Arithmetic Teacher, January 1974, pp. 7-11.

Tyler, Ralph W., Robert M. Gagné and Michael Scriven. Perspectives of Curriculum Evaluation (#1, AERA Monograph Series on Curriculum Evaluation). Chicago: Rand McNally & Company, 1967.

Weaver, J. Fred. "Evaluation and the classroom teacher," Mathematics Education (Sixty-ninth Yearbook, Part I, National Society for the Study of Education). Chicago: The University of Chicago Press, 1970, pp. 335-366.

Wilkinson, Jack D. "Teacher-directed evaluation of mathematics laboratories," The Arithmetic Teacher, January 1974, pp. 19-24.

Wilson, James W. "Evaluation of Learning in Secondary School Mathematics," Handbook on Formative and Summative Evaluation of Student Learning (Bloom, Hastings, & Madaus, editors). New York: McGraw-Hill Book Company, 1971.

WORKING GROUP RECOMMENDATIONS

DIAGNOSIS AND EVALUATION

The working group readily accepted the positions which were expressed in Dr. Hatfield's paper and set about to provide the project staff with specific recommendations. It was pointed out early in the discussion that the project has a two-pronged potential: first, to effect, or at least to encourage, a radical transformation of schools; and secondly, to take a more pragmatic view of organizing materials to stimulate teachers' thinking about evaluation. It was recognized that the former is a massive task but that the project may in some way contribute to encouraging a new way of thinking among teachers which would promote among their students insightful problem-solving and reflective thinking.

The main thread throughout the working group's deliberations was to find ways to encourage teachers to more closely look at and understand their students with regard to student attitude as well as to concept formulation and application. The group was well aware that not all teachers will want to evaluate students, other than for grades by means of paper and pencil tests. However, it was strongly held that examples of a variety of evaluation techniques should be described in the resource books from which teachers may select.

Recommendations:

1. An overview of the philosophy, purpose, and use of evaluation techniques as a part of the learning process should appear in the resources. This discussion should include:

a. The role of testing, the technical and mechanical aspects of test construction, administration, and analysis; and

b. suggestions which focus on the importance of measuring different cognitive levels, retention, ability to read and study mathematics, transfer,attitudes, creativity and productive thinking.

As a result of this discussion, and in subsequent references throughout the resource books, the teacher should be encouraged to view evaluation not as a method of obtaining course grades but as a process which encompasses the idea of cognitive development, formative and summative evaluation, diagnostic testing (clinical as well as paper and pencil), error analysis, feelings and interests, and testing-as-learning/learn as tested.

Although evaluation should play an important role in these resource books, it must be emphasized that evaluation should support, but not dominate, the instructional process. Moreover, since a thorough treatment of this topic may appear to be imposing as a single unit, perhaps a broad overview of evaluation should appear as an appendix section to the first book with references to this section appearing throughout all of the books. (see Comments - Part A for specific suggestions).

2. To encourage teachers to employ varied and effective techniques of diagnosing and evaluating several examples, capsule descriptions and case studies should be included in the resources. Exemplars of a variety of types of tests and testing situations should be an integral part of the resource books as well as concrete methods for interpreting the results of the tests. If evaluation is to become an important part of the instructional program, references to, hints about, and examples of good evaluation techniques and instruments should appear regularly (and often) throughout all of the resource books. (see Comments - Part B for specific suggestions). Moreover, when nonstandard testing situations are suggested, recommendations for classroom management should be included.

3. Evaluation should include the affective as well as the cognitive domain. (see Comments - Part C for specific suggestions).

Comments:

A. What is the purpose of evaluation? (Why evaluate?)

   1. To determine a "grade" for the student.
   2. To determine "placement" of the student in the mathematics curriculum.
   3. To find out what the student has or has not learned.
   4. To find out if the teacher has done a good job of teaching.
   5. To improve student learning and teacher teaching.
   6. To find out if the evaluation techniques are effective or defective, so as to improve them.

B. What are some evaluation techniques? (When and where should they be used?)

   1. Give the students problems to work, and then walk around to help students and find out their difficulties. Routinely sit down, watch and listen to students work out problems.
   2. Administer a paper-pencil test.
   3. Provide oral exercises or contests for the entire class.
   4. Give problem situations that produce sequences and patterns where computation skills (or whatever needs to be evaluated) can be noticed and checked. (see <u>Finite Differences</u>, Creative Pub. Co.)
   5. Administ standardized tests and utilize an item analysis.
   6. Give an activity or game that will test certain concepts or skills. Observe students as they do the activity or game.
   7. Have a group of students working at the chalkboard show their work to the teacher and other students.
   8. Find ways to get students to display their work by use of overhead transparencies, bulletin boards, classroom displays, slides, pictures, etc.
   9. Let students help other students, so they are evaluating themselves. The first time some people really learn mathematics is when they have to teach it to others. Let students explain things to their partner or groups or the entire class.
   10. Have students write comments and observations about how they worked

their mathematics problems, describe patterns they see, perhaps formulate conjectures or more problems.

11. Interview students to see what processes they are employing. (see Lankford: The Arithmetic Teacher, January 1974.) (Note: The particular mathematics topics to be studied may suggest which evaluation techniques would be most effective and useful)

C. What are some affective evaluation techniques?

1. Observe class attendance. Not the students' behavior and working habits in tasks given in the classroom.
2. Walk around the classroom and listen to comments.
3. Lead a class discussion where children can talk about their feelings and behavior. Encourage the students to ask for as well as lead such discussions.
4. Generate written and verbal comments from students. Have the student build a long-term documentation of what he does by writing a book or diary or project reports with his or her thoughts. A diary can give students the chance to write and express their feelings privately where only the teacher reads the diary.
5. Talk to students in a small group or individually where a teacher can show empathy and compassion for the children.
6. Have students fill out a questionnaire on their attitudes and feelings about mathematics. Ask students to identify mathematical topics they understand and enjoy.
7. Have students write comments and observations on their daily assignments and quizzes as they react to them. For example, when taking a paper-pencil test, the students may circle words, phrases and problems they don't understand, even though they may try the problem.
8. Note the students' willingness to cooperate and behave acceptably with the class and teacher.
9. Look for students exhibiting a sense of humor and a willingness to talk about their activities outside of the school.
10. Note the students' ambition and initiative to try mathematical problems and to get involved in mathematics.

11. Note the correlation between student friendliness to the teacher outside of the classroom and teacher friendliness to students outside the classroom.

D. How can certain learning disabilities be recognized?

1. Illustrate Piaget's theories in the form of examples with pictures; give specific mathematical examples which illustrate how students at different"stages" react to a given situation. These examples would cover how to teach certain mathematical topics in light of, for example, Piaget's learning theories and how to test children to see if they understand Piagetian mathematical concepts. Other learning theorists' ideas should also be incorporated.
2. Have students discuss what they are doing when they work a problem.
3. Look for erroneous algorithmic techniques in the students' working of problems. What are some erroneous algorithmic techniques that students use? (see Ashlock: Error Patterns in Computation.)

DIDACTICS

By WILLIAM M. FITZGERALD
Department of Mathematics
Michigan State University
East Lansing, Michigan

Didactics: The science or art of instruction or education.
(Funk and Wagnalls, Standard College Dictionary, 1968)

Our first task should be to narrow the focus of our consideration. The topic as defined above is a bit more broad than is fruitful.

When we consider the eventual goal of the project we try to envision some "Teacher Resource Books" which will in some way help a junior high school mathematics teacher become a "better" or "more effective" teacher in the classroom. Implicit is the assumption that the past two decades of curriculum activity have produced an enormous amount of stuff which is available to a teacher for his potential self-improvement. The central task is to devise a medium to convey to the teacher that information which will be useful to him. So, what information should be conveyed?

Should we describe to teachers what is good and effective teaching of junior high school mathematics?

I think we should and the best description of what I regard as good instruction is contained in the report on IntermediateMathematics Methodology from O.I.S.E.[1] in which we find:

"The focus of instruction is the student. With this in mind, the objective of instruction should be to impart knowledge and:
1) to develop, by making use of the inherent curiosity of the student,

an interest in inquiry and an ability to investigate;
2) to develop whatever creative talents the student may have;
3) to develop in the student a sense of self-confidence in himself, and in his power to deal analytically with situations;
4) to develop an ability to communicate with others."

The report continues and describes ways to conduct mathematics classes and assess the effects of good teaching. Accompanying this paper are excerpts from that report selected by Oscar Schaaf.

Are there research results available which would be helpful to teachers in junior high school classrooms?

Many people have thought long and hard about how to measure various aspects of the dynamics of classrooms. Yamamoto[2] characterized these attempts as 1) cognitive systems, 2) affective systems and 3) multi-dimensional systems. Probably a teacher who understands something about these measurement systems will be able to be more effective in the classroom than one who doesn't. For example, a teacher who knows what an $^{i}/_{d}$ ratio and an $^{I}/_{D}$ ratio is in Flanders' Verbal Interaction Analysis System would probably have a better appreciation for the dynamics of a classroom.[3]

But, as we approach the task of trying to help a teacher be more effective, the results of the research to date are not very helpful. The present state of affairs is summarized well in the last chapter of Yamamoto.

Can we help teachers develop congruence between their teaching practices and the psychological and philosophical assumptions with which they concur?

How a teacher behaves in a classroom is the function of many things which include his own value and belief systems. It seems that a teacher who is aware of his own belief system and his own value system will be able to move toward a teaching pattern which is more nearly congruent with those systems.

Certainly no single learning theory is going to be adequate to provide guidance in the immense variety of complexities which inhabit a typical

classroom. However, a catalog of the various ways one can view man, his basic nature, and how one learns could be helpful for a teacher to arrive at the congruence discussed above. One such catalog is shown in the chart (see page 68) which comes from Bigge[4].

One particular desirable aspect of the congruence between a teacher's behavior and his belief systems is the relation between his evaluation system and his value system. Much criticism of common teaching and testing practices cite the excessive influence of early S-R psychologists. Brownell, long ago, described instructional weaknesses which exist in many classrooms yet today.[5] The weaknesses were listed as follows:

1) Our attention as teachers is directed away from the processes by which children learn, while we are overconcerned about the product of learning.

2) Our pace of instruction is too rapid, while we fail to give learners the aid they need to forestall or surmount difficulty.

3) We provide the wrong kinds of practice to promote sound learning.

4) Our evaluation of error and treatment of error are superficial.

Systematic attempts to help teachers provide and evaluate higher level cognitive instruction are of relatively recent origin. The taxonomy of the cognitive domain by Bloom[6] has helped focus thinking in all areas of school subjects, and work such as that of Avital and Shettleworth[7] can be helpful specifically to mathematics teachers attempting to raise the level of their intellectual interaction with their students.

Still more recently developed is the taxonomy of the affective domain. The implications for teachers of the increased attention to affective aspects of classroom teaching are far from clear at this point in time but the work of Reisman[8] is a good beginning in that direction. She discusses the levels of human needs as described by Maslow, Krathwohl and Carl Rogers and concludes:

> "Since attitudes and emotions seem to be directly involved in learning mathematics, a diagnostic strategy of teaching mathematics must include skills in investigating a student's affective domain." (p.111)

Table 1.1
Representative Theories of Learning and Their Implications for Education

	theory of learning I	psychological system or outlook II	conception of man's basic moral and actional nature III	basis for transfer of learning IV	emphasis in teaching V	key persons VI	contemporary exponents VII
Mental discipline theories of mind substance family	1. Theistic mental discipline	Faculty psychology	*Bad active* mind substance continues active until curbed	Exercised faculties, automatic transfer	Exercise of faculties—the "muscles" of the mind	St. Augustine John Calvin J. Edwards	Many Hebraic-Christian fundamentalists
	2. Humanistic mental discipline	Classicism	*Neutral-active* mind substance to be developed through exercise	Cultivated mind or intellect	Training of intrinsic mental power	Plato Aristotle	M. J. Adler Robert M. Hutchins
	3. Natural unfoldment	Romantic naturalism	*Good-active* natural personality to unfold	Recapitulation of racial history, no transfer needed	Negative or permissive education	J. J. Rousseau F. Froebel Progressivists	P. Goodman J. Holt A. H. Maslow
	4. Apperception or Herbartianism	Structuralism	*Neutral passive* mind composed of active mental states or ideas	Growing apperceptive mass	Addition of new mental states or ideas to a store of old ones in subconscious mind	J. F. Herbart E. B. Titchener	Many teachers and administrators
S-R (stimulus-response) conditioning theories of behavioristic family	5. S-R bond	Connectionism	*Neutral-passive* or *reactive* organism with many potential S-R connections	Identical elements	Promotion of acquisition of desired S-R connections	E. L. Thorndike	A. I. Gates J. M. Stephens
	6. Conditioning (with no reinforcement)	Behaviorism	*Neutral-passive* or *reactive* organism with innate reflexive drives and emotions	Conditioned responses or reflexes	Promotion of adhesion of desired responses to appropriate stimuli	J. B. Watson	E. R. Guthrie
	7. Conditioning through reinforcement	Reinforcement	*Neutral-passive* organism with innate reflexes and needs with their drive stimuli	Reinforced or conditioned responses	Successive, systematic changes in organisms' environment to increase the probability of desired responses	C. L. Hull	B. F. Skinner K. W. Spence
Cognitive theories of Gestalt-field family	8. Insight	Gestalt psychology	*Neutral-active* being whose activity follows psychological laws of organization	Transposition of insights	Promotion of insightful learning	M. Wertheimer K. Koffka	W. Köhler
	9. Goal-insight	Configurationalism	*Neutral-interactive,* purposive individual in sequential relationships with environment	Tested insights	Aid students in developing high-quality insights	B. H. Bode R. H. Wheeler	E. E. Bayles
	10. Cognitive-field	Field psychology or positive relativism	*Neutral interactive,* purposive person in simultaneous mutual interaction with psychological environment including other persons	Continuity of life spaces, experience, or insights	Help students restructure their life spaces—gain new insights into their contemporaneous situations	Kurt Lewin E. C. Tolman Gordon W. Allport Adelbert Ames, Jr. J. S. Bruner Hadley Cantril	R. G. Barker M. L. Bigge A. W. Combs H. F. Wright

It is sobering to consider the distance between a typical middle school mathematics classroom as it exists today and those conditions which would have to exist to fulfill the implications of Carl Rogers' works which would

> "permit the student at any level to be in real contact with the reluctant problems of his existance so that he perceives those problems and issues which he wishes to resolve." (p.118)

As if the task of a teacher were not sufficiently difficult when all of the cognitive and affective factors need to be considered, there is also a wealth of information about the tools of teaching which help to enrich a teachers repertoire. The effective teacher is familiar with the specialized media for teaching mathematics and also with the more generalized media described in a reference such as Wittich and Schuller.[9]

In the last analysis the key to the existence of good, effective teachers in the classroom will lie in the desire on the part of the teacher to be effective and in some systematic procedures for helping teachers to assess and improve themselves. A good beginning in this direction appeared in two issues of Learning in April and May, 1974. In those articles, Curwin and Fuhrmann discussed how a teacher might assess his own value system and teaching skills.[10]

There may be other aspects of didactics which deserve consideration by the project and if so, they will probably emerge during the conference. In the mean time the bibliography will serve as an initial suggested reading list for the project staff.

REFERENCES

[1] Intermediate Mathematics Methodology, Report of the Intermediate Mathematics Methodology Committee, Ontario Institute for Studies in Education, Curriculum Series/4, May, 1968.

[2] Yamamoto, Kaoru, Teaching, Houghton Mifflin, 1969.

[3] Flanders, Ned, Analyzing Teaching Behavior, Addison Wesley, 1970.

[4] Bigge, Morris L., Learning Theories for Teachers, Harper and Row Pub., 1971.

[5] Brownell, William A., "The Progressive Nature of Learning in Mathematics," Teaching Mathematics: Psychological Foundations, Crosswhite, Higgins, Osborne, and Shumway, Jones Pub. Co., 1973.

[6] Bloom, Benjamin, "Taxonomy of Educational Objectives: Cognitive Domain," Teaching Mathematics: Psychological Foundations, Crosswhite, Higgins, Osborne, and Shumway, Jones Pub. Co., 1973.

[7] Avital, Shmuel and Shettleworth, Objectives for Mathematics Learning - Some Ideas for the Teacher, The Ontario Institute for Studies in Education, Bulletin No. 3.

[8] Reisman, Fredricka K., A Guide to the Diagnostic Teaching of Arithmetic, Charles E. Merrill Pub. Co., 1972.

[9] Wittich, Walter and Schuller, Charles, Instructional Technology, Its Nature and Use, Harper and Row Pub., 1973.

[10] Curwin, Richard and Fuhrmann, Barbara, "Is Your Teaching What You Think It Is?" Learning, April and May, 1974.

WORKING GROUP RECOMMENDATIONS

DIDACTICS

In reacting to Dr. Fitzgerald's position paper the group noted that teachers generally do not perceive of psychology as a tool in instruction. Teachers are often textbook bound - placing an overemphasis on drill while lacking an attitude toward problem-solving. Teachers must be viewed as managers of the instructional process - managers of time, space, materials, environment, equipment and people. Each teacher has a theory of instruction which is unique, and they should be provided with tools to enable them to make decisions about the learning situations they provide for students which are compatible with their theory of instruction.

The purpose of the didactics component of the resources should be to provide teachers with a store of procedures to use in planning for instruction. These procedures should be justifiable in the sense that they have a theoretical and/or empirical base. They should be communicated by example in order to increase the probability of teachers using them. The following recommendations contain likely candidates for these procedures. They are not ordered by importance nor is the list complete.

Recommendations:

1. The resources should encourage teachers to use a variety of embodiments for each concept. Some should be in a figural mode, others in a verbal mode and still others in a symbolic mode. The resources should include specific examples of learning theories and how children learn mathematics.

2. The resources should show teachers how they could distribute (as opposed to massing) practice over a period of time. Most drill and practice can be accomplished through a variety of pleasurable activities and investigations. Problem-solving can be interwoven with drill, and the resources should contain examples of this.

3. Teachers should introduce most concepts at a concrete level. The resources should show how this can be done for a variety of mathematical concepts and emphasize the mental processes which are required for transition to formal levels.

4. Teachers who are aware of, for example, Flanders' Verbal Interaction Analysis System or transactional analysis may have a better appreciation of the dynamics of a classroom. The resources should make teachers aware of the various interactions within their classrooms.

5. Different techniques for managing students in a classroom should be stressed. This could involve using mathematics laboratory approaches, small group or team work and soliciting feedback from students. The resources should encourage teachers to be group facilitators.

6. It was recommended that the resources emphasize concepts which teachers will realize are important. Irrelevant dimensions of a concept may be seen as such, if the teachers are provided with a rich enough set of positive attributes. Teachers should provide children with both positive and negative exemplars of concepts.

7. The resources should encourage teachers to use positive reinforcement in a selective but essentially intermittent fashion for each learner.

8. The resources should provide examples of how teachers could use perceptual models for concepts which are consistent with the concepts.

9. Given more than one procedure or algorithm as the goal of an instruction

sequence, teachers should be made aware of interfering factors of learning one for the other. The resources should plant questions, such as: "Have you thought about ...?" These would help teachers become aware of the learning difficulties which are experienced by their students.

10. Teachers should be encouraged to provide their students with the opportunity to assume responsibility for making and testing their own mathematical conjectures.

11. The resources should encourage teachers to provide their students with the opportunity to set goals for their own learning. Also, students should know what they are expected to learn and be provided with some alternative ways for obtaining these goals.

12. Teachers should be encouraged to adjust instruction according to what the child knows. But a lack of readiness is not a valid reason for "protecting" children from mathematical experiences. For example, if students are having trouble with some aspect of computation, they should not always be expected to demonstrate complete mastery before being exposed to other interesting mathematical ideas.

TEACHER UTILIZATION

By ALBERT P. SHULTE
Oakland Schools
Pontiac, Michigan

PART I

In any curriculum endeavor, teachers play the key role. No matter what the quality of text materials, no matter what supplementary resources are provided, no matter what manipulative or audio-visual devices are available, no matter what special facilities are present in the school, what the teacher is and does determines the success or failure of the program. Of course, a skilled teacher, as any skilled practitioner of his art, can function more effectively given better materials and resources. However, a master teacher can overcome deficiencies in the text, in the school environment, and to some extent, in the backgrounds of the learners themselves.

Recognizing the importance of the teacher, it is imperative that this project build as much as possible on the expertise and the strengths of the teachers who will be involved. They should be involved in all stages of the project, not just handed materials and told to go teach them. They must feel themselves active participants in constructing, testing out, and revising the topical resource books.

In addition, the teachers must be specially trained to carry out any teaching strategies which would not be in their normal bag of tricks. They must be provided with the opportunity to work with the activities in the resource books in much the same way that their students will, with the major

difference the faster rate at which teachers can absorb the material.

It is my understanding that teachers will be doing a great deal of the work of putting together the resource books. This means that special care must be taken to get the maximum out of the participating teachers in the minimum time. This may require the setting of fairly stringent deadlines - developing quality materials tends to take much longer than anticipated, and a tight deadline schedule keeps the materials flowing.

In this project, there are six major ways in which I see teachers functioning effectively, in addition to their role of trying out the resource books with children in actual classroom situations. It is to these uses of teachers that the rest of the paper will be directed.

1. <u>The teacher is a source of ideas about appropriate topics and approaches.</u>

Not all teachers will be effective in the actual process of writing - putting words down on paper. However, most, if not all, of the teachers involved will be rich sources of ideas for content which should be included, for teaching strategies which are effective for presenting particular concepts, for developing games and activities which will promote learning.

Those teachers that are particularly fertile sources of ideas should have some time where they can interact with each other and with the persons who are writing the resource books. They can serve as a creative influence, by brainstorming or providing ideas, which others can refine and set down on paper.

2. <u>The teachers, with appropriate experience, can be highly effective writers.</u>

In the Oakland County Mathematics Project, as we proceeded to fill staff positions, we sought people with particular interest and experience with non-college aspiring students and with creative ideas. We did not look primarily for past evidence of writing skill, although several staff members

had some previous writing experience. The assumption was that given the ideas and the interest in the students, writing skill could be developed. This turned out to be a valid assumption - the staff developed into accomplished writers in a relatively short time.

However, this brings up a word of caution. The project writers were employed full-time to write, to develop associated audio-visual and manipulative materials, and to conduct in-service sessions. They did not also have the responsibility to teach classes. It is unrealistic, in my judgement, to expect teachers to teach part of the day and prepare materials during another part. When teachers are teaching classes, they tend to be, first and foremost, teachers and tend to spend most of their non-teaching professional time thinking about the classes they are teaching, rather than turning their minds off to the classroom and on to the writing task.

Then, how can teachers be used effectively in the writing task? In oneof several ways. An obvious solution is for the teachers to write during the summer, developing materials that will be tried out and modified as a result of classroom use during the school year.

Another solution might be to rotate teachers from the classroom on a regular basis. Teachers could be released from classroom teaching for periods of three or four weeks, during which time they have the responsibility for writing particular sections of the resource books. Possibly one or two teachers could be employed full-time as rotating teachers, to go in and take the writer's classes to provide them the necessary release time. Or, possibly some teachers could be paired in a one-month-on, one-month-off fashion, so that both are teaching and both are writing.

The periodic rotation of the teacher from classroom activities to writing and back again has some obvious advantages. It keeps the writer in close contact with children, so that the materials developed are more realistic. It gives him the opportunity to try out his ideas in a classroom known to him, which means that the classroom tone is already set,

and the halo effect may be lessened. It also breaks up the writing task, so that it does not become as tedious.

3. The teacher should serve as a reactor to drafts of all materials produced.

Drafts of all sample units should be circulated to teachers and writers. The teachers will be able to react to the materials from the point of view of the children they are teaching.

Again, to insure the rapid flow of reactions and suggestions back to the writers, fairly stringent deadlines should be set. Provision should be made for teachers and writers to meet together to discuss the materials after the reaction time. Many reactions can be presented more effectively and forcefully in verbal interaction than through the written word.

4. The teacher is a source of feedback as to the effectiveness of the resource books.

There should be regular opportunities for teachers to provide feedback on the materials. Our Oakland County experience was that this is best accomplished by brief meetings, possibly as a regular part of in-service sessions. It is useful to group teachers who have been teaching the same material together, and have them interact directly with the author of that material.

In addition, teachers should be encouraged to write comments in a copy of the resource book, and return these notes to the authors for use in revision.

5. The teacher can serve as a suggester of alternate teaching strategies or supplementary topics.

This can occur at any stage, but will probably happen most frequently at two times. (1) When the teacher reads a draft of a topic, he may think of other effective ways to approach the same topic. (2) When

teaching the materials, he may be forced to develop alternate strategies, finding that those suggested in the teaching materials don't do the job with his class.

In any case, teachers should be encouraged to share their approaches to project topics on a regular basis.

6. The teacher may provide special expertise in particular areas.

Every effort should be made to learn these particular strengths and build on them. This might imply designating teachers with a particular flair for geometry or measurement (for example) to write in those areas. Some teachers may be particularly strong in developing games or audio-visual materials; others may be particularly effective in teaching in a laboratory fashion.

Since the teacher is the key to the project, each of the teachers must be made to feel it is their project. They should feel free to make suggestions, to try innovative approaches, to develop materials beyond those suggested to them, and to disagree with the mathematicians and mathematics educators who will be advising them. A project that is developed for teachers and in large part by teachers has great potential for improving the instruction of mathematics at the middle school level.

PART II

How to Provide for Optimal Use of the Resource Books by Teachers

In order to ensure that teachers will make effective use of the resource books, it is not enough simply to prepare the materials, try them out in classrooms with especially interested, highly motivated and able teachers, and to certify them as classroom-proven. Most teachers, to be able to use the resource books in anything like the manner for which they are designed,

will have to undergo a major change in their point of view. They will have to move from being curriculum followers (where the curriculum consists of covering as many pages as possible in the current text) to being curriculum planners. Teachers will have to make decisions about the particular topics to be emphasized in their class that year, the level of achievement to be expected in those topics, and the type of classroom organization and topic sequence that will best accomplish these aims.

This will be a big move for most teachers. It cannot take place simply by osmosis, by putting the materials in their hands and telling them how nice it would be to teach from a set of resources instead of a textbook. They must have planned in-service experiences in such areas as: selecting appropriate educational objectives (in the broad sense as well as the behavioral sense); planning approaches to particular topics (including laying out the learning sequence, selecting the teaching strategies, selecting or preparing materials for related activities, and evaluating the progress made); diagnosing student difficulties and prescribing particular remedial activities; allowing different students to use algorithms, activities, techniques.

One of the common findings of most curriculum projects is that teachers should experience the activities that they will be using with the children. Of course, the time frame can be much reduced, and there should be follow-up discussions of possible problems, of alternative strategies, and of reasons for using these particular approaches. However, the involvement of the teacher in these activities is a key to his acceptance of their suitability for the students he teaches.

School districts who undertake to use these resource materials should commit themselves to regular meetings of the mathematics teachers, for the purpose of mutually planning the curriculum, both at a particular grade level and over a span of grades. The teachers in that district should reach agreement as to the basic sequence, and as to what topics could be set aside for enrichment (and at what level), so that the teachers do not simply teach their "hobby" areas, and so that students do not arrive at a particular

grade level having seen all the "goodies" that the new teacher has to offer. These regular meetings should also be used for discussion of the program and the way in which it is working, for adding local techniques of materials found to be particularly effective, and for modifications in the program as the need for change becomes apparent.

A technique that seems particularly well adapted for local districts is to provide teachers a week or two of basic orientation to a new program in the late summer, before they are to begin teaching the new materials. This will provide an overview of the whole program, allow the teachers to become accustomed to the resources and their organization, allow them to do some of the basic planning, and allow them to pursue in depth the first few topics for the year. Throughout the year, in-service sessions should be held a few weeks before the materials are to be used in class - as close to the actual time of use as possible.

For wide dissemination of the materials, possibly the best technique would be to train key teachers in local districts in a comprehensive summer program and send them back to the local districts to conduct the in-service sessions in those districts. This would require commitment from those districts in providing materials, secretarial help, release time, and the opportunity for the key teachers to visit classes to see how the resource booklets are being used.

Publicizing the resource books developed by this project through articles in periodicals is a necessary thing. However, it is by no means sufficient. Until the typical middle school teacher becomes a person who actively plans his own curriculum and selects his objectives, materials, and teaching strategies, it will not suffice to say through a national publication "Here is a new set of exciting materials! Come and try them!"

Characteristics of Resource Books

Insofar as possible, the style of the resource materials, when addressing the teacher, should be conversational. The materials must show how the

particular activities fit into the overall picture, the purpose of each activity or teaching strategy should be set forth, and possible pitfalls should be pointed out. The mathematics should be correct but not pedantic.

Topics should be presented in as great detail as possible. It is not sufficient to outline things in a sketchy fashion.

Teachers are sensitive to format. The materials must be easy to read, appealing to the eye, and it must be possible to find things rapidly. The most popular single format with teachers is to present student pages in reduced form, with the teacher notes or information "wrapped around" the margin of the student page. In such a format, it is also helpful to provide answers on the reduced student page.

Providing the materials in loose-leaf form in ring binders, with some blank pages at the end of each topic, will allow the teachers to add their own comments, and make useful additions to the materials. It will also allow them to reorganize the sequence of topics to fit the needs of their particular class or their teaching style.

Teachers should be encouraged to share their own successful techniques, additions to or adaptations of the materials with other teachers in their districts and with the project staff. Bulletins could be released from the project periodically, featuring these innovations, giving the teachers who had developed the ideas appropriate credit.

The resource books should indicate the preferred class organization for particular topics or activities (e.g., groups of 4, groups of 2, whole-class instruction). When students are working in groups, the roles of each group member should be specified as much as possible. Teachers will need guidance in organizing their students for different tasks. Classroom management is one of the big problems in any program which moves away from "all eyes on the teacher and chalkboard up front."

Probably the most important characteristic of the materials in the

resource books is that they deal with topics perceived by the middle school teachers as important. There will be little problem in getting teachers to use materials related to the teaching of fractions and decimals, since these areas are seen as part of the backbone of mathematical work at the middle school level. The materials on ratio, measurement, geometry, probability and statistics will require more selling, or the topics treated should be handled in an especially appealing manner, since teachers perceive these as less in the mathematical mainstream.

Since middle school teachers focus primarily on teaching of computational skills, particular attention should be given in in-service sessions to the need for developing problem-solving skills. Most teachers will give lip service to the need for students to solve problems, but when the chips are down, they will spend a much larger percentage of their teaching time on computational skill. If problem-solving is to be a major thrust of the materials, this must be "sold to the teachers from the beginning. Also, teachers must be led to see that greater emphasis on problem-solving does not mean less performance in computation. Of course, one can argue that there is no point in being able to compute if one cannot apply this skill to solving problems, but many, if not most, middle school teachers think of computational skill as an end in itself.

The applications to other subject areas (science, social studies, or what have you) must also contain sufficient detail that the teacher will feel comfortable handling the related discussions. Many teachers hesitate to use applications because they are afraid of the questions interested students may raise that they cannot answer. This also highlights the need for a good bibliography in each of the areas of application.

If possible the applications should be real and realistic, not too contrived or "hokey". This is somewhat a limiting factor, since real applications tend to have all sorts of non-mathematical complications.

WORKING GROUP RECOMMENDATIONS

TEACHER UTILIZATION

The title: Teacher Utilization was interpreted by the working group in the following two ways: as teachers would be used on the staff in developing the resources, and how the materials could be put to use by inservice teachers. These interpretations were treated separately in Dr. Shulte's position paper which was positively received by the members of the working group. It was evident that the group had more interest in the second interpretation and this is exhibited in their recommendations to follow. The part of Dr. Shulte's paper which dealt with the first interpretation has been very helpful to the project director in planning for and identifying staff to work on the project.

It was noted by one group member early in the deliberations that the main purpose of the project is to "open up" and make more flexible teaching techniques. This comment received general agreement by the group and seemed to pervade the entire discussion.

Recommendations:

1. It was recommended that the student materials in the resources be directed for use with grades five through eight. Secondary school teachers could possibly use these materials in general mathematics at all levels.

2. Humanize the materials. Make the activities relevant for children - the human body is especially relevant to children of this age. (a good source

is You and Me by Richard L. Kimball). The materials could be written to encourage teachers to look more closely at their environment and identify things with which children can become involved. Various questions and arithmetical activities can arise from a field trip or a discussion of local businesses or athletic events. Encourage teachers to listen to the children for other sources of questions and mathematical activities. Suggest that students write their own stories which may dramatize a particular arithmetic problem in order to more personally involve the children in the mathematics.

3. It was recommended that the resources be written so that they can easily fit into an existing program. This may be done by identifying the topics which are involved in the student pages. It was suggested that a separate pamphlet be written which correlates the resources with major textbook series. This may aid those teachers who are "textbook prone" to open up and try new approaches.

4. A variety of approaches should be included. There may be sample lessons with specific directions which teachers could follow. One could start with standard large group instruction and lead to various models of instruction. A simulated activity could be described with a suggested teacher's dialogue provided. Teacher pages with attention-getting (Aha!) questions or ice-breakers could be presented together with suggested techniques for follow-up so that a traditional teacher would become involved in and experience more teaching options. It was suggested that most teachers have difficulty in getting started on something new and then later recognizing potential extensions. This should be considered in developing the resources.

5. The materials should encourage teachers to use alternatives to paper and pencil activities. There should be suggestions for scrounging hints - for sources of free and inexpensive materials and suggestions for how teachers and their students could make and use manipulatives.

6. It was recommended that the project not attempt to define nebulous "demonstrated competencies" but rather include the kinds of activities which

incorporate skill building and emphasize children's involvement in the activities. It was suggested that for ease in teacher use a reference system should be used which identifies the activities as, for example, diagnostic, concept forming, drill and practice, and evaluation.

7. For teacher convenience and to encourage more extensive use it was recommended that certain activities be repeated in several resources as they may pertain to the topic of the resource.

8. It was very strongly recommended that the resources encourage teachers to become curriculum planners - at least with reference to their own classes.

The working group discussed other important considerations such as the use of mini-calculators, the current over-emphasis on basic skills and various strategies for disseminating and implementing the resources. These topics are also reported on by other working groups.

WORKING GROUP RECOMMENDATIONS

PROJECT DEVELOPMENT AND STAFFING POLICY

The group discussed the organization of the project and the use of teachers as writers. Experiences from three previous curriculum projects were represented in the deliberations. The following suggestions were made:

1. Writers working in isolation tend to be far less effective than having people work together on a task. Experience has shown that teams of two or three writers seem to be most effective.
2. It is good to have a writer-coordinator on the project, someone who knows the overall picture for the project and who coordinates the different teams.
3. The entire staff should meet as the need demands in order to discuss questions about the project.
4. The writers should be out in the schools using the materials with children or observing the manner in which other teachers use the materials.
5. It is wise to have people other than the original writers do revisions and editorial tasks related to the materials.
6. Assign writers special subject matter responsibilities so they can make suggestions for the inclusion of ideas to other teams.
7. It is essential to have an adequate staff do the job intended in the project. Plan to have at least one full-time artist on the project.
8. Teachers will develop as writers. They will learn from their experiences and improve their methods of writing and their use of laboratory techniques, puzzles and the like.
9. The staff should know the bounds of their working conditions. There should be guidelines and rules.

10. Maintain stringent deadlines and clamp down on them. Avoid Hemingway's desease - revise, revise, revise.

WORKING GROUP RECOMMENDATIONS

FORMAT AND STYLE

The group discussed the design, appearance and format of the resource materials. Comparisons were made between existing materials and the way teachers respond to different types of commercial materials. The group agreed that the project staff should decide ultimately on the format and style that the resources take. The following suggestions were made:

1. The resources should be attractive. They should include several diagrams, easily read narratives and enjoyable examples and activities.
2. The resources should be easy to use by teachers and to adapt to classroom activities.
3. It should be easy for teachers to personalize the resources - to add new materials as they are encountered and continually up-date the materials. For this purpose it would be best to leave the resources unbound.
4. The staff should consider several packaging possibilities such as loose-leaf notebook form or indexed file boxes.
5. A package could consist of a collection of (i) card files, perhaps color coded according to topic, which are designed for teacher use and so indicated; (ii) a loose-leaf collection of student materials which could be keyed to overview books for teachers; and (iii) bound overview books which might be similar to the Time-Life series book Mathematics.
6. There could be four overview books which deal with learning theories, use of manipulatives, diagnosis and evaluation, and student self-concepts with respect to the different mathematical topics. The overview books could deal with the topics: Number, Geometry, Probability and Statistics, and

Applications.

7. Other media should be considered as possible additions to or inclusions in the resource packages, such as slides, cassettes or video tapes. A lively cassette package with investigations for students might be well received.
8. Size and shape of the resources must be carefully considered.
9. The indexing system should not be too complicated and it is probably wise to avoid an excessive use of color for coding purposes.
10. Avoid the use of educational jargon and deep philosophical considerations. The teachers should not be intimidated by the resources.

WORKING GROUP RECOMMENDATIONS

PROBLEM-SOLVING

The group discussed the role of problem-solving in mathematics courses generally and the way the project should approach problem-solving. More questions were raised than were answered in an attempt to give focus and definition to the topic. Professor Wicklegren's book How to Solve Problems was referred to often during the deliberations as the group searched for some structure for problem-solving activities at the middle school level in mathematics. The following suggestions were made:

1. The project should encourage teachers to involve themselves more in problem-solving and to develop reflective thinking on the part of their students. Problem-solving should be one of the important goals of education.
2. The group suggested that the CUPM recommendations for teacher training may have done a great deal of harm due to the overemphasis on mathematical content while avoiding the methods and processes of problem-solving. Teachers generally do not have an awareness or positive attitude toward open investigations, and this seems to be encouraged by the actions of university personnel toward teachers. It may be that the materials developed by the project could induce changes in pre-service programs, both in the mathematics and methods courses which are taken by teachers.
3. The way people are tested tends to discourage problem-solving. This is especially true of standardized tests in mathematics. The project should make a study of this as well as look for ways to test problem-solving abilities.
4. Another inhibitor of problem-solving activities is that teachers and students

tend to be answer oriented. It is likely that there is security and satisfaction in "getting the answer." The project staff should consider psychological aspects of problem-solving. Problem-solving activities incorporated in the resources might be written to encourage confidence in an ability to tackle problems on the part of teachers as well as students.

5. Most of the available curriculum materials and textbooks are not oriented toward problem-solving. It will probably be necessary for the staff to do less synthesizing in this area and do more creative writing. Look for ways to establish proper environment for students to begin and become involved in problem-solving activities which are close, initially, to traditional settings and topics.
6. Problem-solving should be closely interconnected with content ideas and possibly interrelating mathematics and other subjects.
7. There is a need to crystallize some methods for problem-solving. The project should attempt to establish some structure for problem-solving working in the framework of what schools are expected to do and ways to do it. A small set of rules would be best for such a structure to increase the chances that people will be able to remember and apply them.
8. People need lots of experience in working a variety of problems in order to develop the psychological and intellectual muscle to solve problems. These should be provided by the project.
9. It is not necessary to work on problems in isolation. The resource materials should encourage various formats for problem-solving activities, such as small group interaction, students working with their friends or obtaining information and ideas from others.
10. To plan for the organization of the problem-solving component in the resource materials the staff could use the following outline:
    a. Search through the literature and organize the available information on problem-solving.
    b. Invite consultants to help in developing a structure for problem-solving.
    c. Identify some good problems and try them out with children. Do a lot of "kid-watching" to see what happens.
    d. Establish proper settings for problem-solving activities.
    e. Do a sales job which proves that problem-solving is a legitimate

objective in education.

f. Ignite the idea of problem-solving within teachers and provide them with good materials to use with their students.

WORKING GROUP RECOMMENDATIONS

CALCULATORS

The group discussed a variety of calculating devices which could be used in schools and the manner in which they could be used by students. It was pointed out that the project is timely because of the accessibility of inexpensive hand calculators and the increasing momentum in the country toward the metric system. With the metric system of measurement there will likely be an increased emphasis on decimals which are handled so readily with electronic calculators. The group made the following suggestions:

1. The project should consider the inclusion into the resources of a variety of calculators - even some of the classical ones. Some of these illustrate mathematical concepts more easily than electronic calculators. As examples, an abacus may be used for place-value, a mechanical device with gears may be used to show borrowing or carrying, a slide-rule may be used for approximation. Other calculators which should be considered are cash registers, Napier's rods, Papy's minicomputer, Whitney's computer, nomographs, tables and the human hands, to mention a few. There are several others which are available commercially. Hints should be given to teachers to demonstrate how they can make some of these devices inexpensively.

2. Teachers would probably welcome activity pages which involve students in some creative uses of calculators.

3. Electronic calculators will be used increasingly in the schools, but it is not at all clear what affect these will have on arithmetic programs.

The project should look for ways which influence meaningful use of these instruments in the schools, such as:

a. Students could use calculators to check the results of their hand computations.

b. Teachers could use calculators to reinforce certain arithmetical ideas.

c. Students could use calculators to decide on which operation to use. An example was given where a group of four students work together on a problem and a calculator is passed from one to the other - each student working part of the problem and deciding which button to press.

d. Calculators could be used to derive formulas or algorithms.

e. Calculators could be used to illustrate how the distributive law, for example, is used to devise easy ways to compute.

f. Calculators could be used to set the stage for limits with iterative processes.

g. Calculators could be used on real world problems which may involve prohibitive hand calculations.

WORKING GROUP RECOMMENDATIONS

COMPUTERS

The group began by discussing some problems and practices of computer uses in the schools and quickly incorporated the role of electronic calculators into the deliberations. It was pointed out by those closest to recent technological developments that hand calculators will soon be available which are equivalent to the large scale computers of just a few years ago. Also, inexpensive hand calculators are already available, and it is likely that these will effect an easy transition to a more widespread use of computers with larger capabilities into school curricula. The following suggestions were made:

1. The increased availability of computers and calculators will likely have a dramatic effect on the mathematics content which will be taught in schools. The project should bring this message to teachers and encourage a re-appraisal of the topics presently taught and the processes involved.
2. With an increasing use of the metric system decimals will likely take on more importance and fractions less importance in school curricula. The resources should emphasize the uses of calculators with whole numbers and decimal numbers and the way fractions may be viewed within these contexts.
3. There could be an increased emphasis placed on problem-solving in the schools as the demand for computational skills diminishes. The project should encourage this and provide teachers with materials which use calculators and computers in problem-solving activities.
4. There seems to be a resistance on the part of teachers, generally, to use computers in their classes. This even applies to those teachers who have computer literacy. This may be caused by lack of adequate facilities as

well as management problems and lack of time for planning. The project could provide teachers with examples and sufficient student materials which encourage them to more actively include these tools into their curriculum. An example was described of a mathematics course involving computers in which the teacher assigned problems to small groups of students. The students were to plan an algorithm which would solve the problem. They showed their algorithm to the teacher and when the teacher felt that the algorithm was developed sufficiently, the students were allowed to run their program. Examples such as these could be described in the resources.

5. The resources should contain problem situations for students for which there is no right answer and for which iterative processes might be used to arrive at an acceptable approximation. Such examples illustrate a need for the computational potential of calculators and computers which extend human abilities. For so many problems in the real world there is no "exact answer." The resources should remind and make teachers aware of this.
6. Calculators should be used at the lowest levels in the schools and continued throughout. The project should survey what has been done with young children as well as obtain information from on-going projects. For example, the project should try to obtain information from the on-going project at Indiana University which uses hand calculators in problem-solving. John Kelley at the University of California, Berkeley is doing some work with calculators as well as Tom Romberg and Fred Weaver at the University of Wisconsin and Larry Hatfield and Les Steffe at the University of Georgia.
7. Computer simulations are excellent opportunities for students to personalize their learning experiences in school. Some students who fit the pattern of the "junior high slump," which was described in Dr. Davis' paper will reverse themselves with activities on computers. The resources could alert teachers to this possibility of more effectively venting student energies.
8. There are "computer bums" in schools which teachers could effectively utilize in their classes. These are students with vast interests in computers and who have developed rather sophisticated knowledge - sometimes to the exclusion of doing other school work. These students could form a "computing squad" at their school which help other students and teachers with computing. The resources could describe such possibilities and encourage them.

L I S T O F P A R T I C I P A N T S

Karen Billings, Roosevelt Junior High School,
School District 4J, Eugene, Oregon

Lud Braun, Director, Huntington II Project,
State University of New York, Stony Brr New York

Janet Brougher, Project Staff, Mathematics Resource Project,
University of Oregon, Eugene, Oregon

Robert Davis, Director, Curriculum Laboratory,
University of Illinois, Urbana, Illinois

Benjamin (Rudy) Dudley, Junior High Mathematics Supervisor,
School District of Philadelphia, Philadelphia, Pennsylvania

William Fitzgerald, Department of Mathematics,
Michigan State University, East Lansing, Michigan
(University of Oregon, 1973-1974)

Jane Donnelly Gawronski, Department of Education,
San Diego City Schools, San Diego, California

Jay Greenwood, Mathematics Specialist,
Multnomah County Intermediate Education District, Portland, Oregon

John Haas, Director, Center for Education in the Social Sciences,
University of Colorado, Boulder, Colorado

Alice Hart, Department of Mathematics Education,
University of Illinois at Chicago Circle, Chicago, Illinois

Larry Hatfield, Mathematics Education Department,
University of Georgia, Athens, Georgia
(Teachers College Columbia University, 1973-1974)

Jill Hermanson, Project Staff, Mathematics Resource Project,
University of Oregon, Eugene, Oregon

Alan Hoffer, Project Director, Mathematics Resource Project,
University of Oregon, Eugene, Oregon

Shirley Ann Hoffer, Coordinator, Mathematics Resource Center,
University of Oregon, Eugene, Oregon

Judy Johnson, Mathematics Laboratory Specialist,
Eugene School District 4J, Eugene, Oregon

Wes Johnson, Mathematics Director, Basic Skills Department,
Seattle Public Schools, Seattle, Washington

Eugene Maier, Director, Oregon Mathematics Education Council,
University of Oregon, Eugene, Oregon

Scott McFadden, Cal Young Junior High School,
Eugene School District 4J, Eugene, Oregon

Sue McGraw, LaGrande High School,
LaGrande School District, LaGrande, Oregon

Mary Montgomery. Research and Development Center for Cognitive Learning,
University of Wisconsin, Madison, Wisconsin

Gary Musser, Department of Mathematics,
Oregon State University, Corvallis, Oregon

Leonard (Ted) Nelson, Project Assistant Director, Mathematics Resource Project,
University of Oregon, Eugene, Oregon

Alan Osborne, College of Education,
Ohio State University, Columbus, Ohio

Oscar Schaaf, Mathematics Supervisor, Eugene School District 4J and
College of Education, University of Oregon, Eugene, Oregon

Fred Schippert, Supervisor, Junior High School Mathematics,
Detroit Public Schools, Detroit, Michigan

Albert Shulte, Director, Oakland County Mathematics Project,
Oakland Schools, Pontiac, Michigan

Ross Taylor, Department of Secondary Mathematics,
Minneapolis Public Schools, Minneapolis, Minnesota

Heyward Thomas, Instructional Services Center,
Atlanta Public Schools, Atlanta, Georgia

Maynard Thompson Department of Mathematics,
Indiana University, Bloomington, Indiana

Pat Tuel, Pelton Junior High School,
San Francisco Unified School District, San Francisco, California

Irvin Vance, Outreach Director, Project ONE,
Education Development Center, Newton, Massachusetts

Wayne Wickelgren, Department of Psychology,
University of Oregon, Eugene, Oregon

Willie Williams, Department of Mathematics Education,
Florida International University, Miami, Florida

Jim Young, Willamette High School, Bethel School District 52 and Project Staff,
Mathematics Resource Project, University of Oregon, Eugene, Oregon

I KNOW THAT YOU BELIEVE YOU UNDERSTAND
WHAT YOU THINK I SAID, BUT
I AM NOT SURE THAT YOU REALIZE THAT
WHAT YOU HEARD IS NOT WHAT I MEANT.