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ABSTRACT

In this module the student will combine RL (resistive-inductance) and RC (resistive-capacitive) circuits and learn some of the phenomena which result. The module is divided into four lessons: solving RLC (resistive-inductance-capacitive) circuits, resonant frequency in series circuits, conditions of series resonance, and experiments with series resonance. Each lesson consists of an overview, a list of study resources, lesson narratives, programed instructional materials, and lesson summaries. (Author/BP)

NAVPERS 94558-13a

BASIC ELECTRICITY AND ELECTRONICS

INDIVIDUALIZED LEARNING SYSTEM

MODULE THIRTEEN

SERIES AC RLC CIRCUITS AND RESONANCE

Study Booklet

BUREAU OF NAVAL PERSONNEL

January 1972

OVERVIEW

MODULE THIRTEEN

SERIES AC RLC CIRCUITS AND RESONANCE

In this module you will combine RL and RC circuits and learn some of the phenomena which result.

For you to more easily learn the above, this module has been divided into the following four lessons:

TURN TO THE FOLLOWING PAGE AND BEGIN LESSON I.

NAVPERS 94558-13a

BASIC ELECTRICITY AND ELECTRONICS
INDIVIDUALIZED LEARNING SYSTEM

M O D U L E T H I R T E E N
L E S S O N I

Solving RLC Circuits

Study Booklet

Bureau of Naval Personnel
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OVERVIEW
LESSON 1
Solving RLC Circuits

In this lesson you will study and learn about the following:

- the impedance triangle
- voltage drops
- other circuit quantities
- quality of a coil
- deriving the formula for Q
- the value of Q
- effective or AC resistance
- skin effect
- proximity effect

BEFORE YOU START THIS LESSON, PREVIEW THE LIST OF STUDY RESOURCES ON THE NEXT PAGE.

LIST OF STUDY RESOURCES
LESSON I

<u>Solving RLC Circuits</u>

To learn the material in this lesson, you have the option of choosing, according to your experience and preferences, any or all of the following:

STUDY BOOKLET:

- Lesson Narrative
- Programmed Instruction
- Lesson Summary

ENRICHMENT MATERIAL:

NAVPERS 93400A-1b "Basic Electricity, Alternating Current." <u>Fundamentals of Electronics</u>. Bureau of Naval Personnel. Washington, D. C.: U. S. Government Printing Office, 1965.

AUDIO-VISUAL:

Sound-Slide Presentation - "Solving for Total Impedance and Total Voltage."

YOU MAY NOW STUDY ANY OR ALL OF THE RESOURCES LISTED ABOVE. YOU MAY TAKE THE PROGRESS CHECK AT ANY TIME.

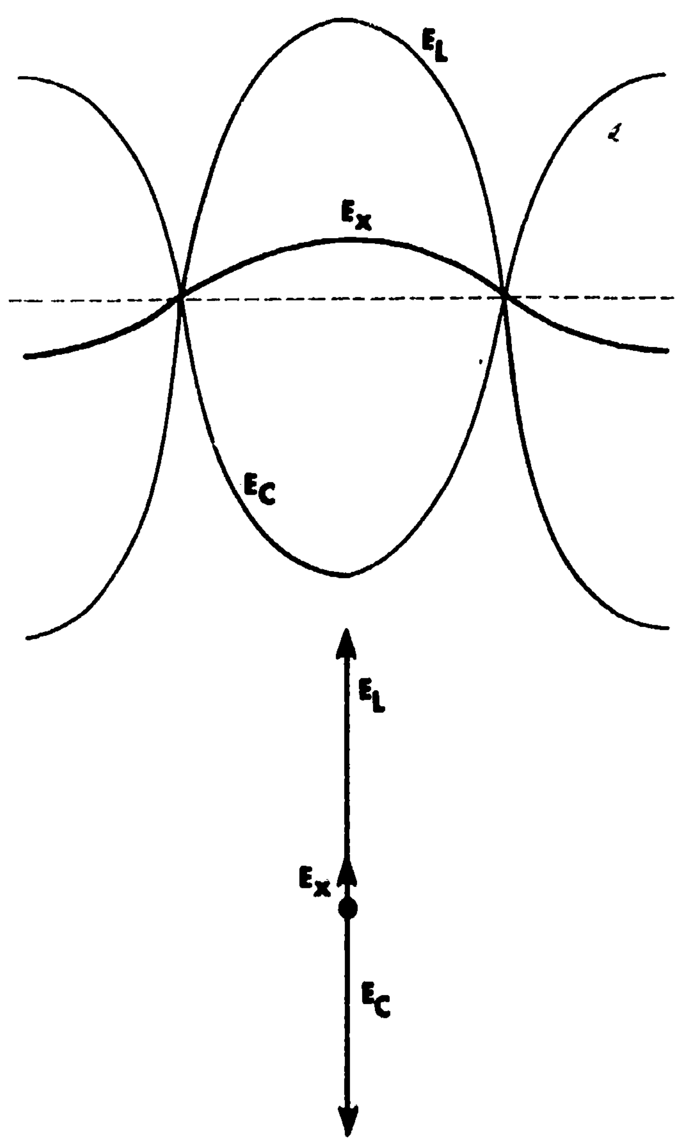
NARRATIVE
LESSON I

Series RLC Circuits

Voltages in Series

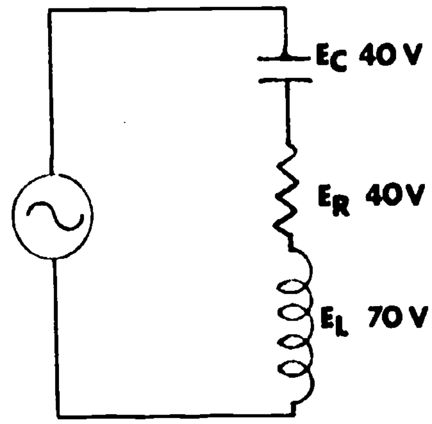
The voltage drops in a series RLC circuit have, of course, phase differences. Because of this, it is necessary to use instantaneous values and graphs or vectors to find the total values in the circuit.

When using vectors, current is used as the reference, for it is the value which has the same phase (is common) in all parts of the circuit. The drop across the resistance is in phase with the current and so is represented with a vector in the standard position. The capacitor voltage drop lags circuit current by 90° and the voltage across the inductor leads current by 90°. Combining either the instantaneous values or the vectors for E_C and E_L reduces the value of the larger one. In other words, you must subtract the smaller value from the larger.



The diagrams on the preceding page show, graphically and by vectors, the addition of 100 volts E_L and 80 volts E_C. They show that the voltages are in direct opposition, so that the resultant voltage (the voltage which affects electron movement) is the difference between E_L and E_C. This circuit, then acts just like a circuit with 20 volts dropped across a single inductor.

Rectangular notation provides a convenient method for finding the total voltage in a series RLC circuit like this one:

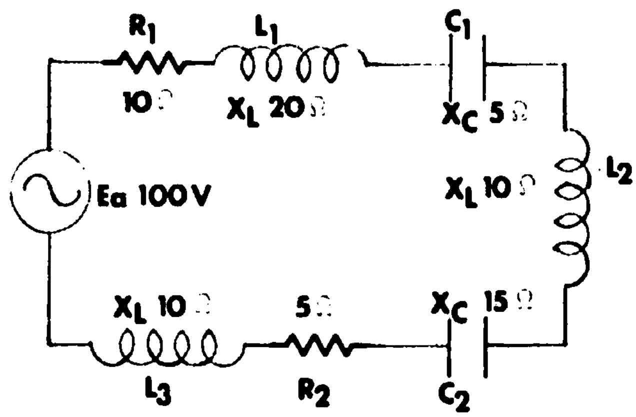


Total voltage is the vector sum of the voltage drops, i. e.,

$$\begin{aligned} E_T &= E_R + jE_L - jE_C \\ &= 40 \text{ v} + j70 \text{ v} - j40 \text{ v} \\ &= 40 \text{ v} + j30 \text{ v} \end{aligned}$$

Impedance

Once again voltage and impedance are directly related, so the impedance can be found in the same way as total voltage. In the circuit diagram on the following page, the values of resistance and reactance are written in rectangular form then added.

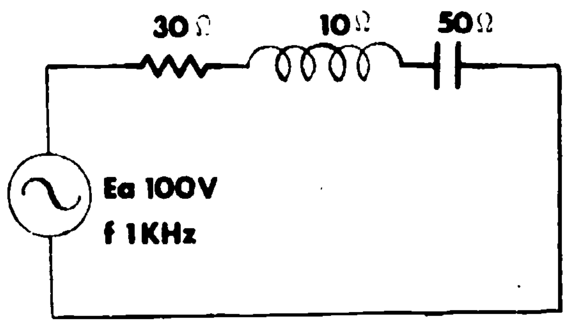


R1	10 + j0
L1	0 + j20
C1	0 - j5
L2	0 + j10
C2	0 - j15
R2	5 + j0
L3	0 + j10
	15 + j20

The circuit impedance is 15 +j20 ohms or, in polar form, $25 \underline{/53.1°}$ ohms. Current is therefore 4 amperes, and the phase angle is -53.1°. So far as the overall effect on the source is concerned, this circuit could be replaced with a 15 ohm resistor in series with an inductor having a reactance of 20 ohms, but the individual components will develop much greater voltage than the source provides to the circuit because of the cancellation between E_L and E_C. For example, E_{L1} is 80 volts, E_{L2} is 40 volts, and E_{L3} is 40 volts for a total of 160 volts dropped across the inductors. This increase in voltage can sometimes be useful, and you will learn more about it later.

Now work the following problems for practice with these concepts.

Solve the RLC circuit shown.



P_t = ______

P_a = ______

$\underline{/\theta}$ = ______

PF = ______

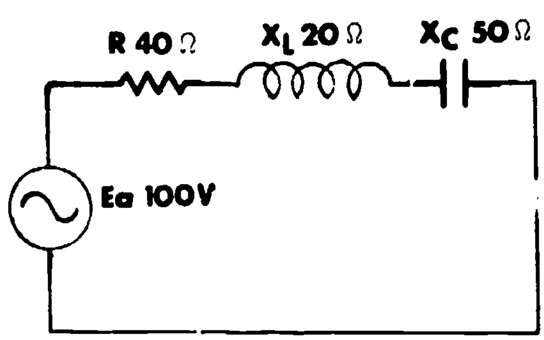
By using the formula $P_t = I^2R$, we find P_t = 120 w.

By the formula $P_a = E \times I$, P_a = 200 va.

You know that $\underline{/\theta}$ = -53.1°, and PF is equal to the COS $\underline{/\theta}$ or 0.6.

Solve this RLC circuit.

Draw the Vector Diagrams



Z_T = ______

I_T = ______

E_R = ______

E_L = ______

E_C = ______

P_t = ______

P_a = ______

PF = ______

$\underline{/\theta}$ = ______

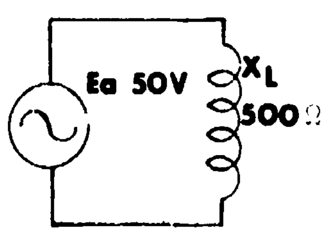
Z_T = 40 Ω - j30 Ω or 50 Ω $\underline{/-36.9°}$; I_T = 2 a; E_R = 80 v; E_L = 40 v; E_C = 100 v; P_t = 160 w; P_a = 200 va; PF = 0.8; $\underline{/\theta}$ = -36.9°

Figure of Merit

The figure of merit (Q) of a reactor is a measure of how close the reactor comes to having all reactance and no resistance. For example, a coil with a high Q is one with very little resistance in its wiring. This means that very little power is dissipated in the coil and nearly all the power delivered to the coil is stored and later returned to the source. Q is defined as the ratio of power stored to power dissipated in the coil $\frac{P_x}{P_t}$. Since the reactive (stored) power is found by I^2X_L and true (dissipated) power by I^2R, this can be written as $Q = \frac{I^2X_L}{I^2R}$. Cancellation of I^2 in the fraction leaves $Q = \frac{X_L}{R}$, a more useful equation for most practical uses.

The figure of merit is seldom used with capacitors, for their construction normally keeps their internal resistance so low that it can be ignored. This is not true of inductors; however, because the wires used to make coils usually have significant amounts of resistance, and the Q of the coil is affected. In most RLC circuits, the coil contains nearly all the resistance of the circuit, so the Q of the coil can be used as the circuit Q.

A coil with a Q of ten or greater is considered a high Q coil. This is because, in most circuits at least, a resistance one-tenth or less of the reactance will have so little effect on the circuit that it can be ignored. Compare these coil values for a pure inductive reactance of 500 ohms with those for a coil containing 500 ohms of inductive reactance in series with 50 ohms of resistance. (NOTE: The dotted line around the inductor and R_{eff} indicate that both are contained in the coil.)

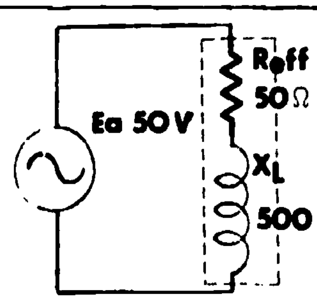


Z = ______

I = ______

$/\theta$ = ______

A



Z = ______

I = ______

$/\theta$ = ______

B

A. Z = 500 Ω

I = 100 ma

$/\theta$ = 90°

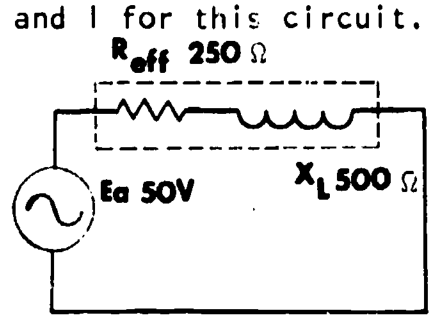
B. Z = 502.3

I = 99.5 ma

$/\theta$ = 84.3°

The differences between these two circuits are smaller than you could detect with normal test equipment and can be considered identical.

Find Q, $\angle\theta$, Z and I for this circuit.



Q = 2; $\angle\theta$ = 63.4°; Z = 559 Ω; I = 89.4 ma

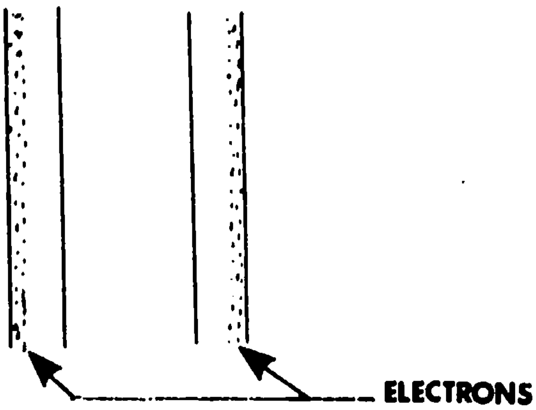
The circuit above illustrates a low Q coil. Compare these values to those for the ideal coil and high Q coil you worked out earlier. As a rule of thumb, if the coil Q is 10 or more (reactance is at least 10 times the resistance), ignore the resistance and treat the coil as a pure inductance. If Q is less than 10, include the resistance in your calculations.

The resistance we have discussed so far has been a constant value unaffected by any condition within a circuit. Unfortunately, this is not exactly the case, and frequency of the applied voltage often affects circuit resistance. When the Q of a circuit is figured, the resistance used must be the actual resistance of the circuit at the applied frequency. This is called the <u>AC resistance</u> or <u>effective resistance</u>. The Q of a circuit is also defined as $\frac{X_L}{R}$ but in this case R includes all the resistance in the circuit, and not just the AC resistance of the coil. Unless otherwise specified, when dealing with series circuits <u>Q</u> means the <u>Q</u> of the circuit rather than the <u>Q</u> of the coil.

AC resistance is the result of two actions in a wire carrying an alternating current. <u>Skin effect</u> results from self induction inside the conductor which causes electrons to crowd away from the center of the wire toward its outer surfaces. This effectively reduces the cross section of the wire, increasing its resistance.

At fairly high frequencies, no electrons travel near the center of the wire, so hollow tubing can be used to save weight and expense in building a circuit without any loss of efficiency.

Proximity effect is the second major factor of AC resistance. The proximity effect occurs when wires carrying AC are placed side by side. The electrons in each wire tend to move away from the adjacent wire like this:



This further reduces the cross-sectional area available for current flow and increases the wire's resistance.

Both skin effect and proximity effect cause resistance to increase with frequency. Since both X_L and R increase as frequency increases, the Q of a circuit will remain nearly constant over a broad frequency range.

AT THIS POINT, YOU MAY TAKE THE PROGRESS CHECK, OR YOU MAY STUDY ANY OF THE OTHER RESOURCES LISTED. IF YOU TAKE THE PROGRESS CHECK AND ANSWER ALL OF THE QUESTIONS CORRECTLY, GO TO THE NEXT LESSON. IF NOT, STUDY ANY METHOD OF INSTRUCTION YOU WISH UNTIL YOU CAN ANSWER ALL THE QUESTIONS CORRECTLY.

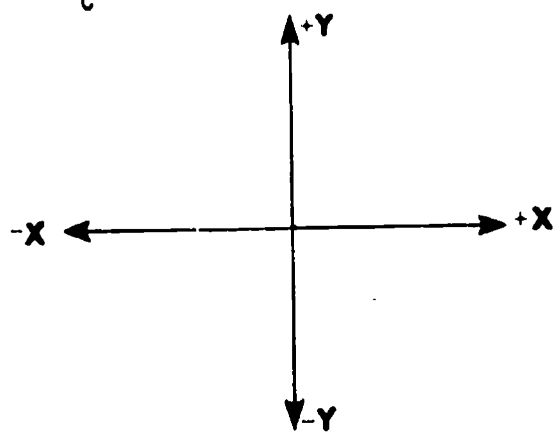
PROGRAMMED INSTRUCTION
LESSON I

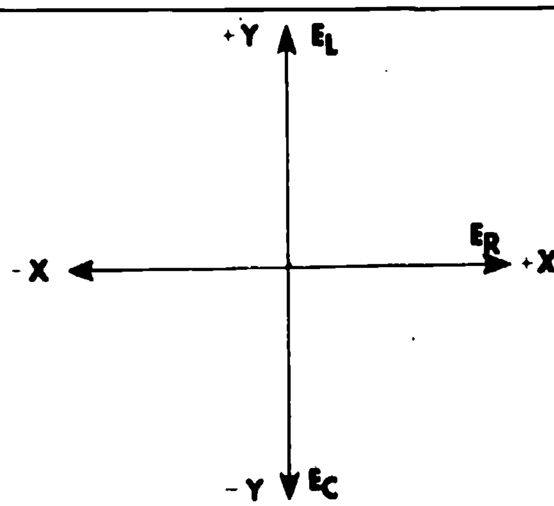
Solving RLC Circuits

TEST FRAMES ARE 11 and 31. AS BEFORE, GO FIRST TO TEST FRAME 11 AND SEE IF YOU CAN ANSWER ALL THE QUESTIONS THERE. FOLLOW THE DIRECTIONS GIVEN AFTER THE TEST FRAME.

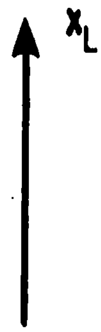
1. Recall that E and I are in phase in a resistive circuit, E leads I in an inductive circuit, and I leads E in a capacitive circuit.

   Plot E_L, E_R, and E_C on the graph shown below.





2. Since the voltage drop across X_L leads the current by 90°, X_L is plotted upward in the <u>+j</u> position on an impedance triangle.

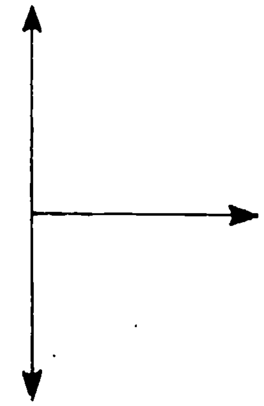


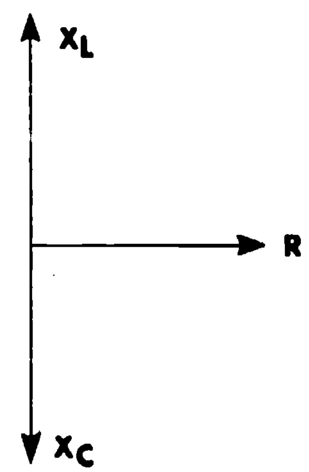
Since the voltage drop across X_C lags the current by 90°, X_C is plotted ____________ or in the ____________ position.

(downward; -j

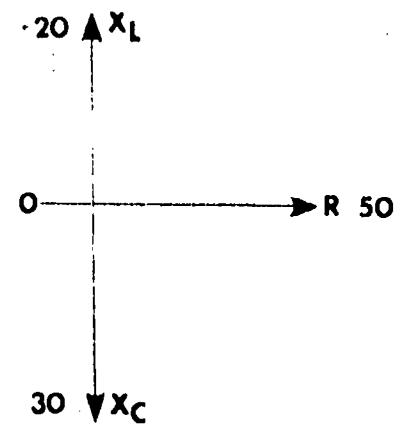
3. Since E and I are in phase through a resistance, R is plotted in the standard vector position.

   On the following graph, indicate where the values of X_L, and X_C, and R are plotted by labeling the respective vectors.

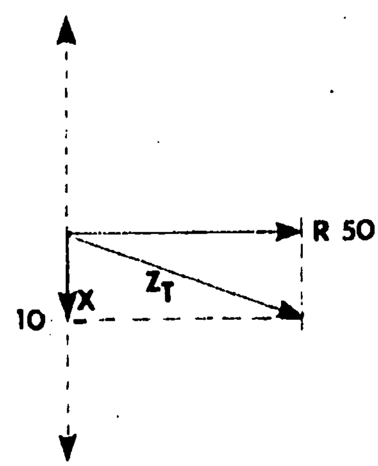




4. Since X_L is plotted opposite X_C, they can be added algebraically.



Draw a vector indicating the resultant value.



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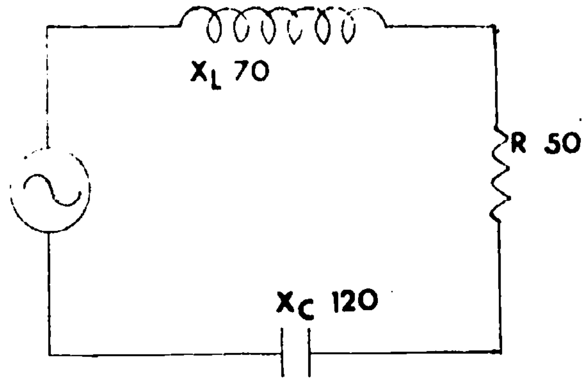
5. Recall from Module Twelve that the j operator is simply another tool for solving AC resistive circuit problems. A +j indicates a 90° counterclockwise rotation or an X_L quantity, a -j indicates a 90° clockwise rotation or an ________ quantity.

(X_C)

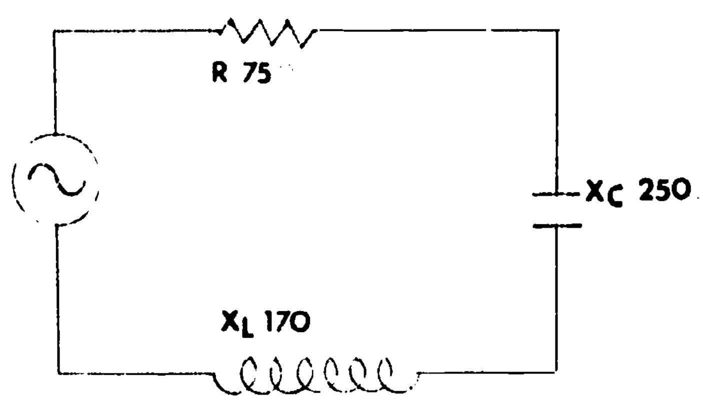
6. To express a series RLC circuit in rectangular notation you must express each component in rectangular notation and algebraically add.



The impedance of this circuit is:

R	50 + j0
X_L	0 + j70
X_C	0 - j120
TOTAL	50 - j50

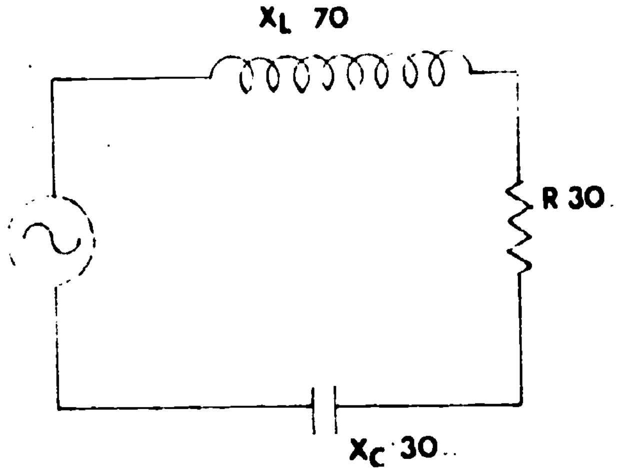
Express the impedance of the following circuit in rectangular notation.



(75 + j0
0 + j170
0 - j250
75 - j80)

7. Algebraic addition enables us to simplify the circuit, leaving either a simple RL or RC circuit to solve.

   Solve the following for all values indicated.



a. $\angle$ = ______

b. Z_T = ______

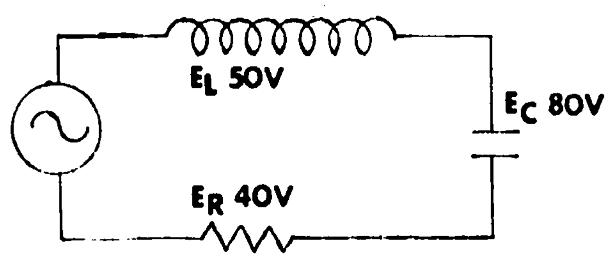
c. sine = ______

d. cosine = ______

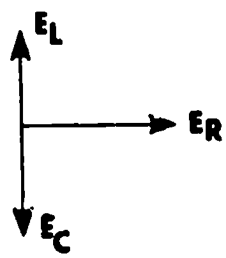
e. TAN = ______

(a. 53.1°; b. 30 + j40 or 50 $\angle$; c. 0.7997; d. 0.6004; e. 1.3319

8. Solving a series RLC circuit for voltage values is accomplished in the same way as solving for impedance.



a. Draw the vector diagram.



b. Express the voltage across each component in rectangular notation.

E_R = 40 v + j0

E_L = 0 + j50 v

E_C = 0 - j80 v

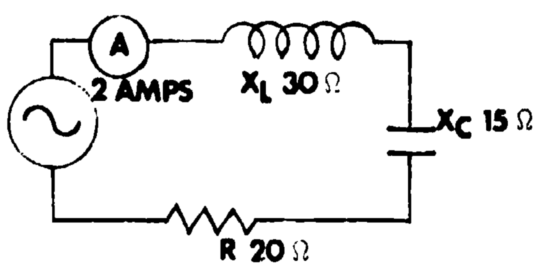
c. Algebraically add 40 v - j30 v

d. Determine the TAN $\underline{/\theta}$ .75

e. Determine $\underline{/\theta}$ -36.9°

f. Determine E_a 50 v

Solve for E_a.



E_a = ________

(50 v $\underline{/36.9°}$)

9. Recall that only resistance dissipates true power.

The formula for true power is:

____a. $P_t = EI$.

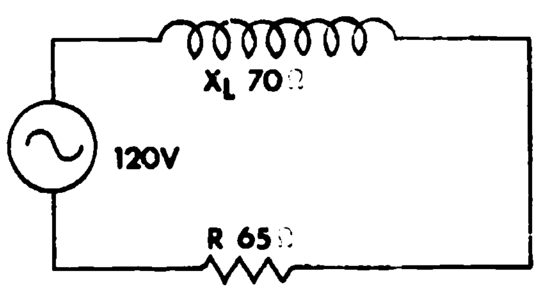
____b. $P_t = IR$.

____c. $P_t = I^2R$.

____d. $P_t = \frac{E}{R}$.

(c)

10. Use of the formula $P = E_a I_T$ results in apparent power. However, recall that in a reactive circuit true power may be determined by the formula $P_t = E_a \times I_T \times \cos \underline{/\theta}$.

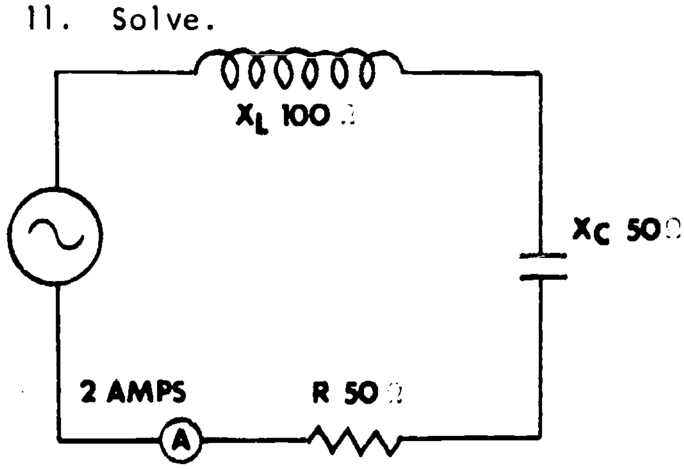


Find P_a ________

Find P_t ________

(150 va; 102 w)

11. Solve.



a. E_a = ________

b. $\underline{/\theta}$ = ________

c. Z_T = ________

d. P_a = ________

e. P_t = ________

(THIS IS A TEST FRAME. COMPARE YOUR ANSWERS WITH THE CORRECT ANSWERS GIVEN AT THE TOP OF THE NEXT PAGE.)

ANSWERS - TEST FRAME 11

a. 141.4 v

b. 45°

c. 70.7 Ω

d. 282.8 va

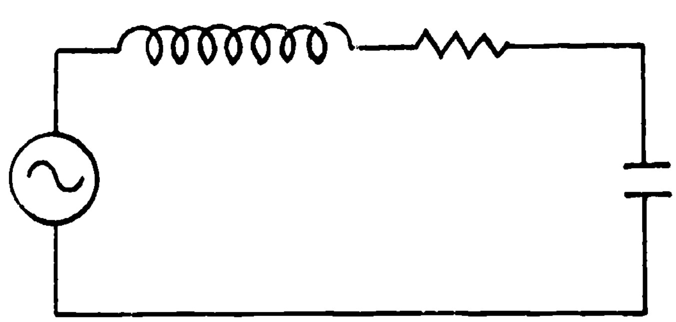
e. 200 w

IF ALL YOUR ANSWERS MATCH THE CORRECT ANSWERS, YOU MAY GO TO TEST FRAME 31. OTHERWISE, GO BACK TO FRAME 1 AND TAKE THE PROGRAMMED SEQUENCE BEFORE TAKING TEST FRAME 11 AGAIN.

12. To this point, we have talked about resistance in a circuit as though it were a fixed value and actually a physical component. Let's take a brief look at what the resistance in a circuit actually is.

Besides the physical resistors, the conductor has resistance and consequently the coil has resistance.

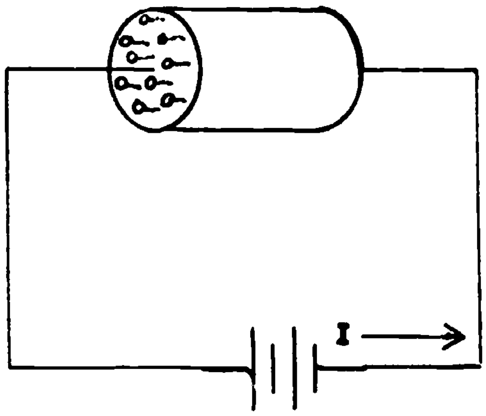
The total resistance in the circuit below is only a physical resistor. ________
true/false

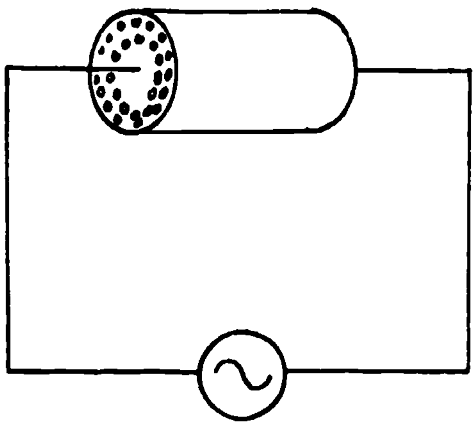


(false)

(We have said in the past that frequency has no effect upon resistance; however, we need to qualify this statement a bit. Frequency does have a slight effect on resistance.)

13. Recall that alternating current causes flux lines to expand and collapse around a conductor. These flux lines induce a voltage into the conductor. The induced voltage opposes the current and tends to decrease it. This induced voltage is greater at the center of the conductor where the flux lines are concentrated. The effect of this is to force the electrons to move towards the skin of the conductor.





By forcing current to flow along the skin of the conductor, the cross-sectional area of the conductor is effectively ________.

(reduced or decreased)

14. When the cross-sectional area of a given conductor decreases, what happens to the resistance of the conductor?

____

(It increases)

15. Increasing frequency increases the skin effect. Thus, the higher the frequency, the ____________ the resistance.

(higher or greater)

16. Because of skin effect, hollow conductors are sometimes used to reduce weight and cost. In a high-frequency circuit a reduction in weight and cost may be obtained by using ________ conductors.

(hollow)

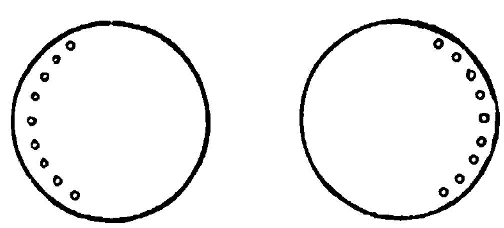
17. There is another factor which causes resistance to increase with frequency. This is called the proximity effect. The proximity effect is also caused by the magnetic field around a conductor. This effect, however, is caused in adjacent conductors.

The proximity effect causes resistance to ______________ when frequency decreases. increase/decrease

---

(decrease)

18. Electrons in conductors placed side by side are forced away from the parts of the conductors nearest each other. Here is an end view of two wires showing electron distribution resulting from both skin effect and proximity effect:



Proximity effect is similar to skin effect in that the effective cross-sectional area of the conductor is __________.

(reduced)

19. Proximity effect causes the resistance of the conductor within a coil to increase as frequency ______________.

(increases)

20. The method used to reduce the problem of proximity effect is to space the turns of the windings farther apart.

This method reduces the amount of voltage induced, thus reducing the increase in ________________.

(resistance)

21. Proximity effect and skin effect cause a conductor to offer more resistance to AC than to DC. Because of this, we call the resistance offered to AC the effective or AC resistance, abbreviated R_{eff} or R_{ac}.

The resistance any inductor offers to AC is somewhat greater than that offered to DC, and it is called the __________ resistance.

(effective or AC)

22. The purpose for discussing the effective resistance of a coil is to discover its effect on the ability of a coil to store energy.

The greater the resistance, the more power ________________.

(lost or dissipated)

23. The ratio of the amount of energy stored in an inductor to the amount of energy lost in the same period of time indicates the quality of a coil. The quality of a coil is also referred to as the figure of merit of a coil and is abbreviated Q.

The Q of a coil may be indicated as: $Q = \frac{P_x}{\_\_}$

(P_t)

24. Using the formula $Q = \frac{P_x}{P_t}$, and substituting the equivalent values for P_x and P_t, results in $Q = \frac{I^2 X_L}{I^2 R_{ac}}$.

(Note that R_{ac} must be used to determine the correct value of true power.)

Deleting the common factors in the numerator and denominator result in the formula for the Q of a coil:

$Q =$ ________

$\frac{X_L}{R_{ac}}$

---

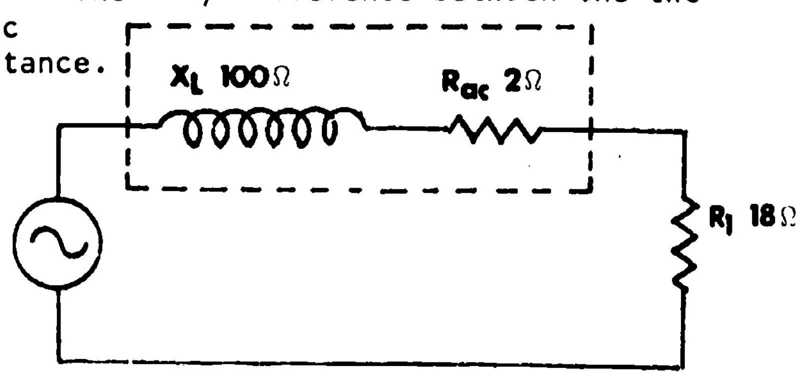
25. Since both X_L and R_{ac} are directly proportional to frequency, how is Q affected by a change in frequency?

____

(It is not appreciably affected.)

---

26. The Q of a series circuit and the Q of a coil are determined in the same way: $Q = \frac{X_L}{R_{ac}}$. The only difference between the two is the value of resistance.



(Note: The dotted line around the inductor and R_{ac} indicate that both are contained in the coil.)

a. Determine the Q of the coil. ________

b. Determine the Q of the circuit. ________

(a. Q of the coil = 50; b. Q of the circuit = 5)

27. Unless otherwise specified, when dealing with series circuits, Q means the Q of the circuit rather than the Q of the coil.

The Q of a series circuit is determined by using the values of the X_L of the coil and ________________.

(total effective circuit resistance)

28. When Q is 10 or greater, the coil is considered to be a high-Q coil. A coil with a Q of less than 10 is a low-Q coil. In solving high-Q circuits, the resistance is normally disregarded and only the reactance is considered.

When solving a circuit with a Q of 2, the resistance is ________________.

(considered or important)

29. You may have recognized that the formulas for the tangent and Q are the same.

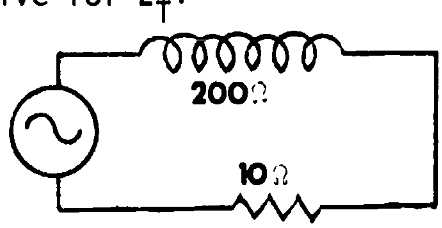
What is the $\angle\theta$ for a circuit with a Q of 10?

(approximately 84.2°)

30. You can see that $\angle\theta$ is very near 90°; therefore, the circuit is almost purely reactive. This is the reason the resistance in a circuit with a Q of 10 or more is disregarded.

(Go to the next frame.)

31. Solve for Z_T:



Z_T = ________________

(THIS IS A TEST FRAME. COMPARE YOUR ANSWER WITH THE CORRECT ANSWER GIVEN AT THE TOP OF THE NEXT PAGE.)

ANSWERS - TEST FRAME 31

200 .. for all practical purposes

IF YOUR ANSWER IS INCORRECT, GO BACK TO FRAME 12 AND TAKE THE PROGRAMMED SEQUENCE.

IF YOUR ANSWER IS CORRECT, YOU MAY TAKE THE PROGRESS CHECK, OR YOU MAY STUDY ANY OF THE OTHER RESOURCES LISTED. IF YOU TAKE THE PROGRESS CHECK AND ANSWER ALL THE QUESTIONS CORRECTLY, GO TO THE NEXT LESSON. IF NOT, STUDY ANY METHOD OF INSTRUCTION YOU WISH UNTIL YOU CAN ANSWER ALL THE QUESTIONS CORRECTLY.

SUMMARY
LESSON 1

Series RLC Circuits

In this lesson you will learn how to solve series RLC (combination resistive-inductive-capacitive) circuits.

Voltage leads current by 90 electrical degrees in a purely inductive circuit, lags current by 90 electrical degrees in a purely capacitive circuit, and is in phase with current in a purely resistive circuit. Since the individual voltages are out of phase, vectors must be used to solve for total voltage.

Impedance must also be solved vectorially because the reactive components (inductors and capacitors) vary in the opposition to circuit current as the applied frequency is changed. Circuit current is used as the reference in all series circuits because it is common to all parts of the circuit. Since E and I are in phase across a resistive component, E_R and I_R are plotted in the standard vector position. Kirchhoff's Voltage Law, "Sum of the voltage drops around a circuit must equal source voltage," still applies, and the instantaneous and vector sums of the voltage drops across the components equal the applied voltage, although the individual voltages across the capacitor and the inductor may be many times the voltage applied.

A coil has a figure of merit called the quality (Q) of the coil. The Q of a coil is X_L divided by the effective resistance of the coil. The effective resistance (R_{eff} or R'_{ac}) varies with frequency as does X_L; therefore, Q does not vary appreciably as frequency changes. The variation of R_{eff} caused by frequency is a result of two factors - the skin effect and the proximity effect.

AT THIS POINT, YOU MAY TAKE THE LESSON PROGRESS CHECK, OR YOU MAY STUDY THE LESSON NARRATIVE OR THE PROGRAMMED INSTRUCTION OR BOTH. IF YOU TAKE THE PROGRESS CHECK AND ANSWER ALL OF THE QUESTIONS CORRECTLY, GO TO THE NEXT LESSON. IF NOT, STUDY ANOTHER METHOD OF INSTRUCTION UNTIL YOU CAN ANSWER ALL THE QUESTIONS CORRECTLY.

BASIC ELECTRICITY AND ELECTRONICS
INDIVIDUALIZED LEARNING SYSTEM

MODULE THIRTEEN

LESSON II

Series AC Circuits at Resonance

Study Booklet

Bureau of Naval Personnel
January 1972

OVERVIEW

Lesson II

Series AC Circuits at Resonance

In this lesson you will study and learn about the following:

- resonant frequency
- circuit analysis at f_o
- voltage gain
- solving for resonant frequency

TURN TO THE FOLLOWING PAGE AND BEGIN LESSON II.

List of Study Resources

Lesson II

Series AC Circuits at Resonance

To learn the material in this lesson, you have the option of choosing, according to your experience and preferences, any or all of the following:

STUDY BOOKLET:

Lesson Narrative

Programmed Instruction

Lesson Summary

ENRICHMENT MATERIAL:

NAVPERS 93400A-1b "Basic Electricity, Alternating Current." Fundamentals of Electronics. Bureau of Naval Personnel. Washington, D. C.: U.S. Government Printing Office, 1965.

Powers of Ten Program.

AUDIO-VISUAL:

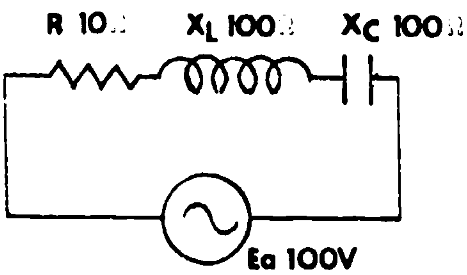
Slide/Sound Presentation - "Factors Affecting Resonant Frequency."

YOU MAY NOW STUDY ANY OR ALL OF THE RESOURCES LISTED ABOVE. YOU MAY TAKE THE PROGRESS CHECK AT ANY TIME.

NARRATIVE
LESSON 11

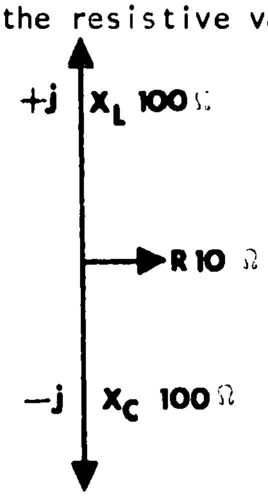
Series AC Circuits at Resonance

Resonant Frequency



Looking at this series RLC circuit, we see that the values of X_L and X_C both equal 100 ohms. Let's draw the impedance vector diagram to compute Z_T, and see what happens.

Current is the common reference and, as always in a series circuit, the resistive values are plotted in the standard position with current.



The X_L vector is rotated 90° counter-clockwise, the X_C vector is rotated 90° clockwise. Since the X_L and X_C vectors are equal and in opposite directions, the voltages across them cancel each other.

$X_L = +\, j100\ \Omega$

$X_C = \underline{-\, j100}\ \Omega$

Reactance $= 0\ \Omega$

For all practical purposes the circuit at this frequency has only one kind of opposition -- resistance.

Resistance is the only thing that limits current, and therefore, the source sees the circuit as being purely resistive. In a purely resistive circuit, all quantities are in phase and the vector representation is R = 10 Ω. The phase angle is 0.

This circuit condition, in which $X_L = X_C$ and $\underline{I}$ is limited only by $\underline{R}$, exists at the <u>resonant frequency</u>. For every value of $\underline{L}$ and $\underline{C}$ in a series circuit, there is a single frequency which causes X_L to equal X_C. That frequency is the resonant frequency of the circuit -- abbreviated f_o.

At f_o, these conditions exist in a series RCL circuit:

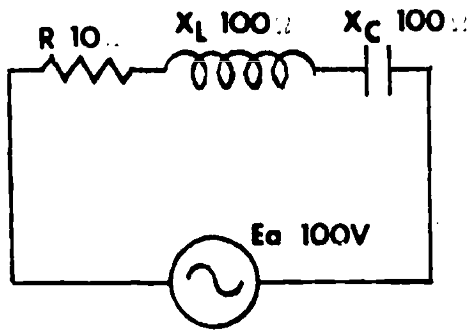
$X_L = X_C$

Z_T is minimum ($\underline{R}$ only)

I_T is maximum (limited only by $\underline{R}$)

Analyzing the Circuit at f_o

Let's look at the circuit again to determine other factors about it. Since at f_o the circuit appears purely resistive, $\angle\theta = 0$, PF is 1, and $Z_T = 10\ \Omega$.



Solve the above circuit.

I_T = ____________

P_t = ____________

P_a = ____________

E_R = ____________

By Ohm's Law, I_T = 10 a; P_t = 1000w; P_a = 1000 va; E_R = 100 v.

Now you may be assuming that since the full applied voltage of 100 volts is dropped across the resistor, there is no voltage drop across the coil or the capacitor, but this is not the case.

Voltage Drops at f_o

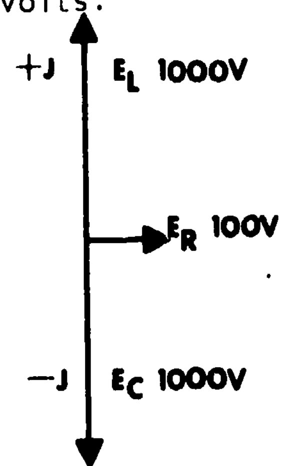
Remember that in AC series circuits which have resistive and reactive components, we must compute the total voltage drops vectorially.

Therefore, with 10 amps current through the coil, $E_L = I \times X_L$

or $E_L = 10 \text{ a} \times 100\ \Omega$

$E_L = 1000 \text{ v}$

We know that X_C is equal to X_L; therefore, E_C also equals 1000 volts.



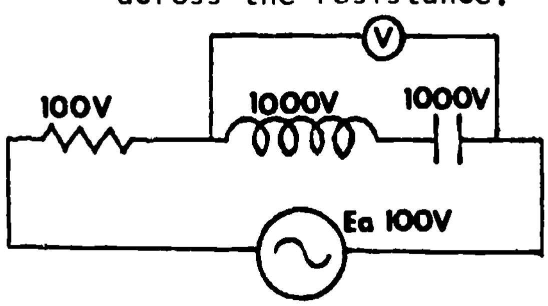
We can plot the voltage vectors this way.

$E_L = + j1000 \text{ v}$

$E_C = \underline{- j1000 \text{ v}}$

Total = 0 v

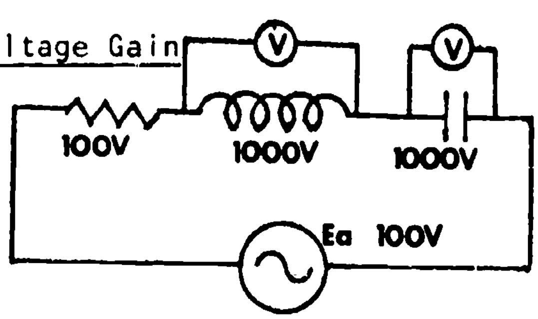
Because these voltages are 180° out of phase, they cancel each other, leaving the full applied voltage of 100 volts dropped across the resistance.



If we connect a voltmeter across both reactive components as shown, the meter indicates 0 volts.

This is true because E_L and E_C are equal and opposite and cancel each other.

<u>Voltage Gain</u>



If, however, we connect meters across the individual components as shown here, the meter reads 1000 volts in each case. This means we can tap off 1000 volts across either the inductor or the capacitor. Notice that the source voltage is only 100 volts, but at f_o the circuit is capable of providing a greater voltage than the amount supplied by the source . .

This increase of voltage is called <u>voltage gain</u>.

Solving for Resonant Frequency

We have said that for every value of L and C, a frequency exists which causes X_L to equal X_C. This is the resonant frequency.

The formula for finding f_o is derived from the formulas for X_C and X_L.

At f_o: $X_L = X_C$

Substituting: $2\pi f_o L = \frac{1}{2\pi f_o C}$

$(2\pi f_o C)\ 2\pi f_o L = 1$

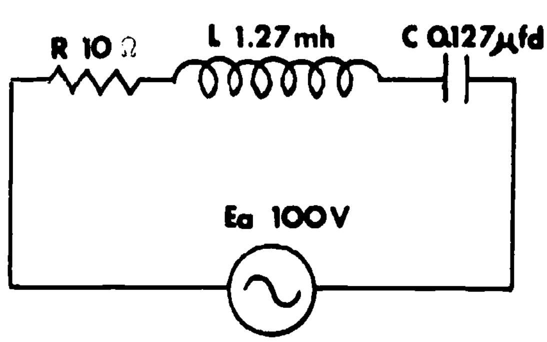
$4\pi^2 f_o^2 LC = 1$

Solving for f_o^2: $f_o^2 = \frac{1}{4\pi^2 LC}$

Taking the square root of both sides: $f_o = \frac{1}{2\pi\sqrt{LC}}$

Simplfying, $f_o = \frac{0.159}{\sqrt{LC}}$

Using this equation, let's find f_o for this circuit.



$$f_o = \frac{0.159}{\sqrt{LC}}$$

$$f_o = \frac{0.159}{\sqrt{(1.27 \times 10^{-3})\ (0.127 \times 10^{-6})}}$$

$$f_o = \frac{0.159}{\sqrt{(1.27 \times 10^{-3})\ (1.27 \times 10^{-7})}}$$

$$f_o = \frac{0.159}{1.27 \times 10^{-5}}$$

$$f_o = 12.5 \times 10^3 \text{ Hz or } 12.5 \text{ KHz}$$

(Note: If you do not understand the procedure for extracting the square root of a number containing a power of ten, refer to the Powers of Ten Program in the reference library.)

Practice:

Solve for f_o when:

1. $L = 100$ mh

   $C = 10\ \mu f$ $f_o =$ ____________

2. $L = 20$ mh

   $C = 50$ pf $f_o =$ ____________

3. $L = 3$ mh

   $C = 120\ \mu f$ $f_o =$ ____________

Check answers, and procedures if necessary, on following pages.

1. Solution:

$$f_o = \frac{0.159}{\overline{LC}}$$

$$f_o = \frac{0.159}{\sqrt{(100 \times 10^{-3})\ (10 \times 10^{-6})}}$$

$$f_o = \frac{0.159}{\sqrt{(100 \times 10^{-3})\ (1 \times 10^{-5})}}$$

$$f_o = \frac{0.159}{\sqrt{100 \times 10^{-8}}}$$

$$f_o = \frac{0.159}{10 \times 10^{-4}}$$

$$f_o = 0.159 \text{ KHz or } 159 \text{ Hz}$$

2. Solution:

$$f_o = \frac{0.159}{\sqrt{(20 \times 10^{-3})\ (50 \times 10^{-12})}}$$

$$f_o = \frac{0.159}{\sqrt{(20 \times 10^{-3})\ (5 \times 10^{-11})}}$$

$$f_o = \frac{0.159}{\sqrt{100 \times 10^{-14}}}$$

$$f_o = \frac{0.159}{10 \times 10^{-7}}$$

$$f_o = 0.159 \text{ MHz or } 159 \text{ KHz}$$

3. Solution: $$f_o = \frac{0.159}{\sqrt{(3 \times 10^{-3})\ (120 \times 10^{-6})}}$$

$$f_o = \frac{0.159}{\sqrt{(3 \times 10^{-3})\ (12 \times 10^{-5})}}$$

$$f_o = \frac{0.159}{\sqrt{36 \times 10^{-8}}}$$

$$f_o = \frac{0.159}{6 \times 10^{-4}}$$

$$f_o = 0.265 \text{ KHz or } 265 \text{ Hz}$$

AT THIS POINT, YOU MAY TAKE THE PROGRESS CHECK, OR YOU MAY STUDY ANY OF THE OTHER RESOURCES LISTED. IF YOU TAKE THE PROGRESS CHECK AND ANSWER ALL OF THE QUESTIONS CORRECTLY, GO TO THE NEXT LESSON. IF NOT, STUDY ANY METHOD OF INSTRUCTION YOU WISH UNTIL YOU CAN ANSWER ALL THE QUESTIONS CORRECTLY.

PROGRAMMED INSTRUCTION
LESSON II

Series AC Circuits at Resonance

THERE ARE NO TEST FRAMES IN THIS PROGRAMMED SEQUENCE.

1. Both X_L and X_C within a series RLC circuit are affected by the frequency of the applied voltage.

(Note: Unless R is specified to be effective or AC resistance, it is considered not to be affected by frequency.)

Changing the frequency of the applied voltage causes:

____a. L and C to change and R to remain the same.

____b. X_L and R to change and C to remain the same.

____c. X_L and X_C to change and R to remain the same.

____d. X_C and R to change and L to remain the same.

- -

(c)

2. If the frequency applied to an RLC series circuit is increased, X_L increases and X_C decreases.

X_L is ____________ proportional, and X_C is ____________ proportional to frequency.

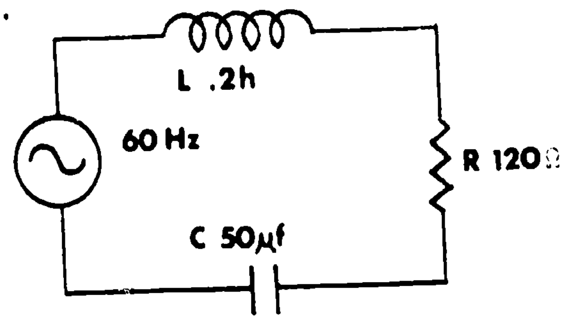
- -

(directly; inversely)

3. As frequency varies, the values of X_L and X_C change.

Solve for X_L and X_C in the circuit below using the applied frequency of 60 Hz in circuit A and 120 Hz in circuit B.

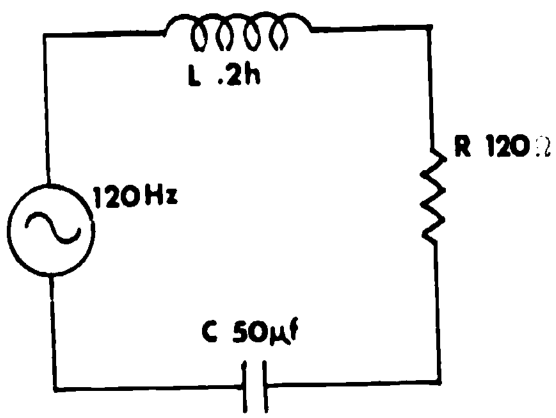
A.



X_L = ____________

X_C = ____________

B.



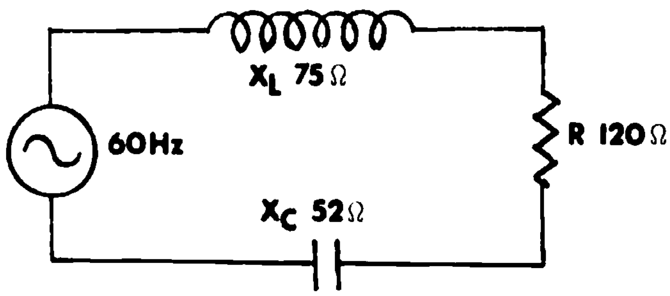
X_L = ____________

X_C = ____________

(A. $X_L = 75\ \Omega$, $X_C = 53\ \Omega$; B. $X_L = 150\ \Omega$, $X_C = 26.5\ \Omega$)

4. Since frequency affects X_L and X_C, it also has an effect on total circuit impedance.

Solve for Z_T and $\underline{/\theta}$ in the circuit below.



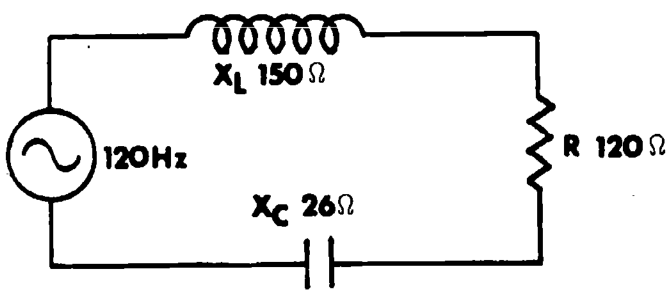
Z_T = ________

$\underline{/\theta}$ = ________

(Z_T = 123 Ω; $\underline{/\theta}$ = 10.8°)

5. A change in the applied frequency results in a corresponding change in Z_T and the circuit phase angle.

Solve the Z_T and $\underline{/\theta}$ in the circuit below.



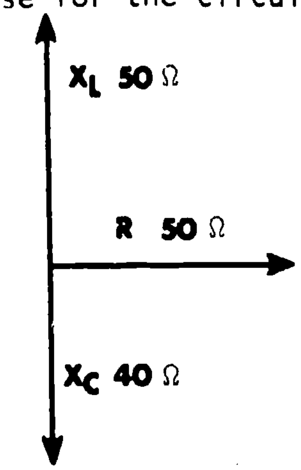
Z_T = ________

$\underline{/\theta}$ = ________

(Z_T = 174 Ω; $\underline{/\theta}$ = 45.9°)

6. When a frequency applied to a series RLC circuit causes X_L to equal X_C, the frequency is known as the <u>resonant frequency</u> for that circuit.

In the circuit represented by the vector diagram below, does frequency increase or decrease for the circuit to become resonant? ________

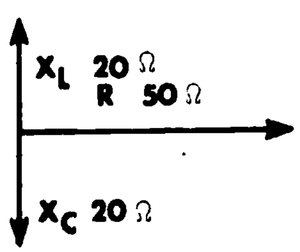


- -

(decrease)

7. Remember, X_L increases as frequency increases; X_C increases as frequency decreases.

The vector diagram below represents a series RLC circuit at resonance. Decreasing frequency has what effect on the circuit?



____a. R↑ X_C↑ X_L↓

____b. R↓ X_C↓ X_L↑

____c. R→ X_C↑ X_L↓

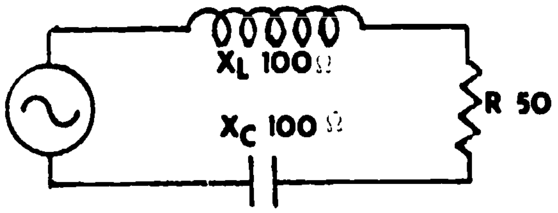
____d. (none of the above)

- -

(c)

8. In a series RLC circuit at resonance, E_L cancels E_C and X_L and X_C no longer affect the total circuit values.

What component represents total impedance of the circuit below when it is at resonance? ________________



- -

(resistor)

9. The resultant of a vector diagram representing an RLC series circuit at resonance is resistance only.

At resonance, the circuit appears purely __________ to the source.

- -

(resistive)

10. The formula used to find the resonant frequency for a given value of L and C is derived from the fact that, at resonance, $X_L = X_C$.

$$X_L = X_C$$

Substituting Formula $2\pi f_o L = \dfrac{1}{2\pi f_o C}$

Cross Multiplying $4\pi^2 f_o^2 LC = 1$

Isolating Frequency $\dfrac{4\pi^2 f_o^2 LC}{4\pi^2 \quad LC} = \dfrac{1}{4\pi^2 LC}$

Resulting in: $f_o^2 = \dfrac{1}{4\pi^2 LC}$

Take Square Root of Both Sides:

$$\sqrt{f_o^2} = \frac{\sqrt{1}}{\sqrt{4\pi^2 LC}}$$

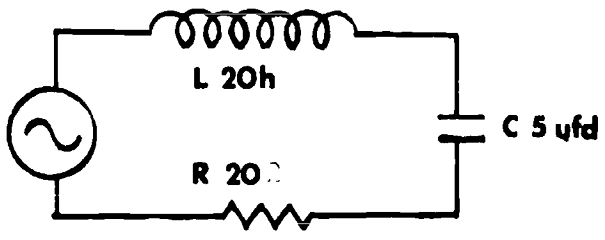
$$f_o = \frac{1}{2\pi\sqrt{LC}}$$

Determine the resonant frequency of a circuit when L = 50 mh and C = 5 ..f. f_o = ______

- -

(318 Hz)

11. Solve for f_o in the circuit shown.



f_o = ______

- -

$$f_o = \frac{0.159}{\sqrt{LC}}$$

$$f_o = \frac{0.159}{\sqrt{20 \times 5 \times 10^{-6}}}$$

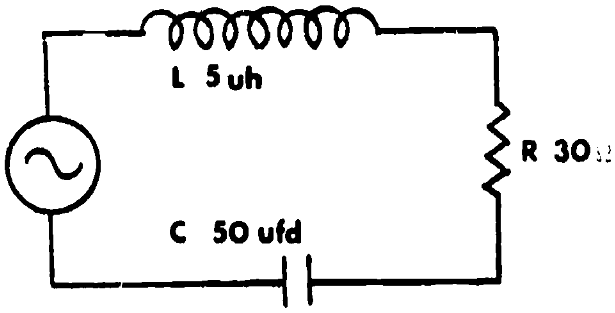
$$f_o = \frac{0.159}{\sqrt{100 \times 10^{-6}}}$$

$$f_o = \frac{0.159}{10 \times 10^{-3}}$$

$f_o = 0.0159 \times 10^3$ or 15.9 Hz

(NOTE: If you do not understand how to extract the square root of numbers including powers of ten, refer to the Powers of Ten Program in the reference library.)

12. Solve for f_o in this circuit.



f_o = ________

(f_o = 10 KHz)

13. At resonance, X_L and X_C are of ________ value.

(equal)

14. Also under these conditions, the circuit impedance is ________.

(minimum)

15. Under resonant conditions, the circuit impedance is equal to circuit ________.

(resistance)

16. Since at resonance Z_T is equal to R, the circuit phase angle at resonance is ________.

(zero)

17. Let's review the characteristics of a series RLC circuit at resonance.

a. The values of X_L and X_C are ____________.

b. Z_T of the circuit is equal to the circuit __________.

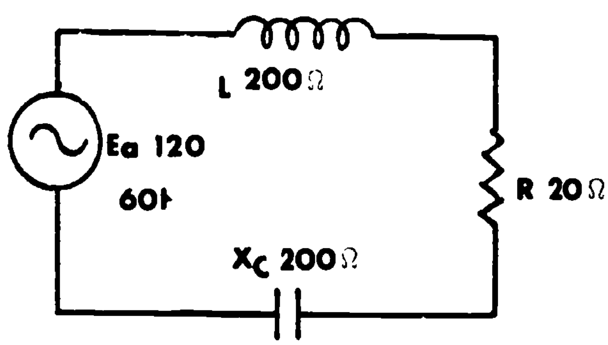
c. Circuit current is ______________.

d. Phase angle between E and I is __________.

- -

(a. equal; b. resistance; c. maximum; d. zero)

18. We know that the circuit shown below is operating at resonance because X_C is equal to ___________.



- -

(X_L)

19. Since the circuit is operating at resonance, Z_T is equal to the _________. By Ohm's Law, current is ___________ amps.

- -

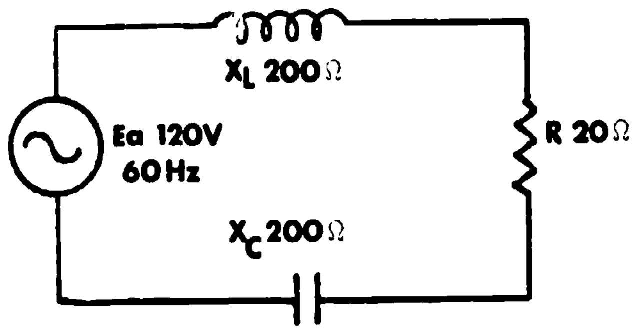
(resistance, 6)

20. Using Ohm's Law, we can solve for the voltage drop across each component.

The voltage drop across the resistor is __________ volts.

The voltage drop across the coil is __________ volts.

The voltage drop across the capacitor is __________ volts.



(120; 1200; 1200)

21. The source voltage is only 120 volts, yet E_L and E_C are 1200 volts each. This voltage gain is characteristic of series RLC circuits at resonance.

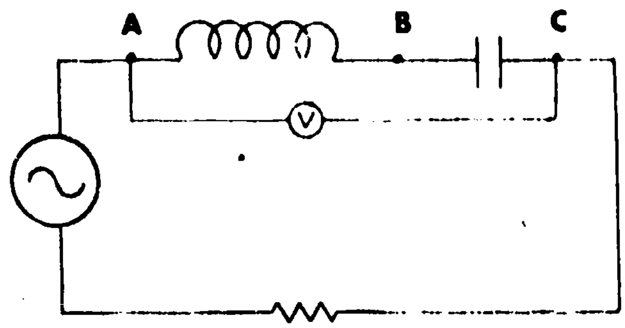
How is it possible to have a voltage drop across a component greater than the applied voltage? ______________________________

____

(The canceling effect of the opposing voltages across the capacitor and the inductor makes it possible.)

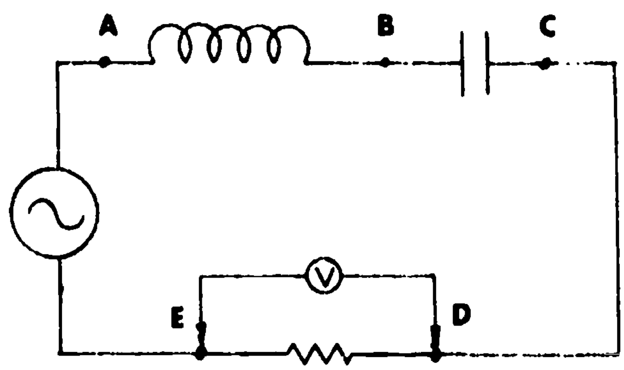
22. If a voltmeter is connected across point A to C in this series RLC circuit operating at resonance, what is the reading?

____



(zero)

23. If the voltmeter is connected from points D to E in the same circuit, the reading is equal to ______________.



(source voltage)

YOU MAY TAKE THE PROGRESS CHECK, OR YOU MAY STUDY ANY OF THE OTHER RESOURCES LISTED. IF YOU TAKE THE PROGRESS CHECK AND ANSWER ALL THE QUESTIONS CORRECTLY, GO TO THE NEXT LESSON. IF NOT, STUDY ANY METHOD OF INSTRUCTION YOU WISH UNTIL YOU CAN ANSWER ALL THE QUESTIONS CORRECTLY.

SUMMARY
LESSON II

Series AC Circuits at Resonance

In this lesson, you will learn that when the value of X_L is equal to the value of X_C, a series RLC circuit is operating at a condition known as resonance. For any combination of values of L and C, there is a frequency which produces a value of X_L that is exactly equal to the value of X_C. The symbol for resonant frequency is f_o.

You will find that the voltage drops across the reactive components at resonance are also exactly equal, and that they are 180° out of phase; thus, they effectively cancel each other, and the entire source voltage is dropped across the circuit resistance.

At resonance, the total impedance of the circuit is equal to the value of R since X_L cancels X_C. Circuit current is maximum at resonance and limited only by the value of R. If the applied frequency is changed from the resonant frequency, Z_T increases and I_T decreases.

The formula for f_o is $f_o = \frac{1}{2\pi\sqrt{LC}}$ $\frac{0.159}{\sqrt{LC}}$. From the formula, you can see that if the value of L or C is changed, the resonant frequency changes. An increase of L or C causes a decrease in f_o.

AT THIS POINT, YOU MAY TAKE THE LESSON PROGRESS CHECK, OR YOU MAY STUDY THE LESSON NARRATIVE OR THE PROGRAMMED INSTRUCTION OR BOTH. IF YOU TAKE THE PROGRESS CHECK AND ANSWER ALL OF THE QUESTIONS CORRECTLY, GO TO THE NEXT LESSON. IT NOT, STUDY ANOTHER METHOD OF INSTRUCTION UNTIL YOU CAN ANSWER ALL THE QUESTIONS CORRECTLY.

NAVPERS 94558-13a

BASIC ELECTRICITY AND ELECTRONICS
INDIVIDUALIZED LEARNING SYSTEM

MODULE THIRTEEN
LESSON III

Resonance in Series AC Circuits

Study Booklet

Bureau of Naval Personnel
January 1972

OVERVIEW

LESSON III

Resonance in Series AC Circuits

In this lesson, you will study and learn about the following:

- current and impedance curves at f_o
- circuit behavior above f_o
- circuit behavior below f_o
- bandwidth
- effects of Q on bandwidth
- practical applications

BEFORE YOU START THIS LESSON, PREVIEW THE LIST OF STUDY RESOURCES ON THE NEXT PAGE.

LIST OF STUDY RESOURCES
LESSON III

Resonance in Series AC Circuits

To learn the material in this lesson, you have the option of choosing, according to your experience and preferences, any or all of the following:

STUDY BOOKLET:

Lesson Narrative

Programmed Instruction

Lesson Summary

ENRICHMENT MATERIAL:

NAVPERS 93400a-1b "Basic Electricity, Alternating Current." Fundamentals of Electronics. Bureau of Naval Personnel. Washington, D.C.: U.S. Government Printing Office, 1965.

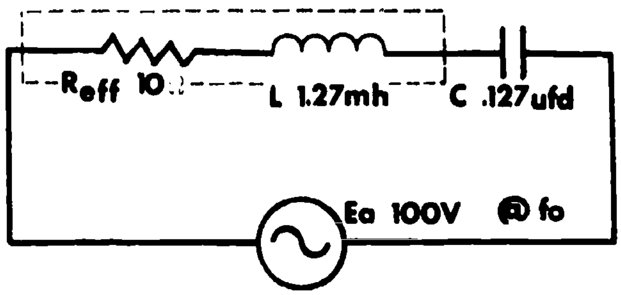
YOU MAY NOW STUDY ANY OR ALL OF THE RESOURCES LISTED ABOVE. YOU MAY TAKE THE PROGRESS CHECK AT ANY TIME.

NARRATIVE
LESSON III

Resonance in Series AC Circuits

In this lesson we will analyze the behavior of a single series RLC circuit as three different frequencies are consecutively applied: resonant frequency, a frequency above f_o, and a frequency below f_o.

This is the circuit we analyzed in the previous lesson.



We know that as this circuit is operating at f_o, these conditions exist:

$$I_T = \text{maximum}$$

$$Z_T = \text{minimum } (\underline{R} \text{ only})$$

$$X_L = X_C$$

$$E_L = E_C$$

By the formula $f_o = \frac{0.159}{\sqrt{LC}}$, we can determine that f_o is 12.5 KHz.

To find the value of X_L, we can use the formula for X_L and use f_o for f:

$$X_L = 2\pi f_o L$$

$$X_L = 98.55 \text{ (approximately 100 ohms)}$$

$$\text{at } f_o,\ X_L = X_C\text{; therefore,}$$

$$X_C = 100 \text{ ohms}$$

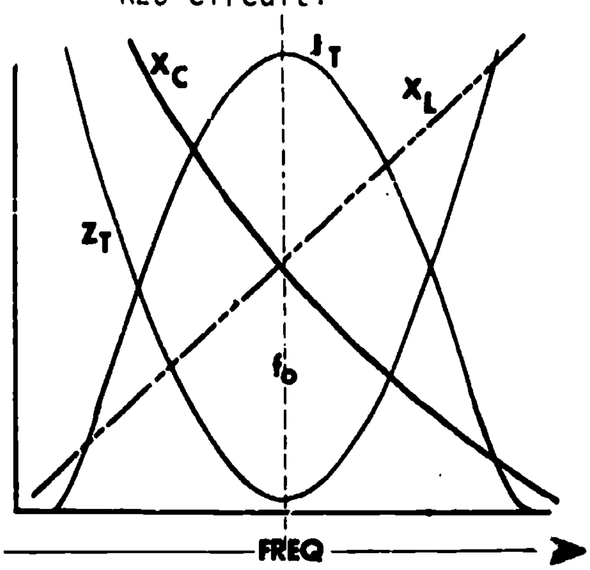
$X_L = + j100$, and $X_C = - j100$; the voltage across them oppose, since they are equal and they cancel each other. This leaves Z_T at a minimum with the resistance of 10 ohms being the only impedance.

Because Z_T is minimum, $\underline{I}$ is maximum at f_o and equal to 10 amps. We know that the source sees a purely resistive circuit. Then $\underline{E}$ and $\underline{I}$ are in phase and $\underline{/\theta}$ is 0. The power factor is 1.

By $P_t = I^2R$, true power equal 1000 w ($10^2 \times 10\ \Omega$). In a purely resistive circuit, true power equals apparent power; therefore, P_a = 1000 va. We can prove this by the formula P = E x I (100 v x 10 a = 1000 va).

Current and Impedance Curves at f_o

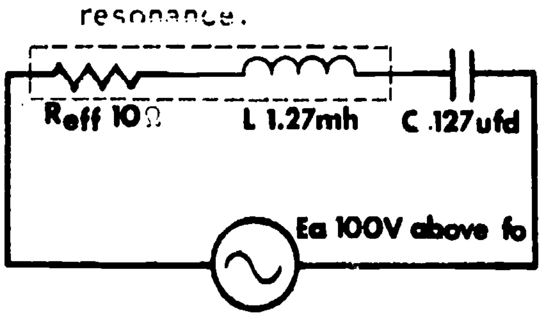
Recall that earlier you saw curves representing the relationship of voltage and frequency for both RL circuits and RC circuits. A similar graph of current shows some relationships in a series RLC circuit.



At the peak of the current curve (maximum current) we have resonant frequency. The lines that are inversely proportional to $\underline{I}$ represent Z_T, so when $\underline{I}$ is maximum, Z_T is at its minimum.

Circuit Behavior Above f_o

Now for a better understanding of what resonance means, we will analyze the same circuit with the applied frequency above resonance.



Assume we have increased the frequency from 12.5 KHz to 13.125 KHz.

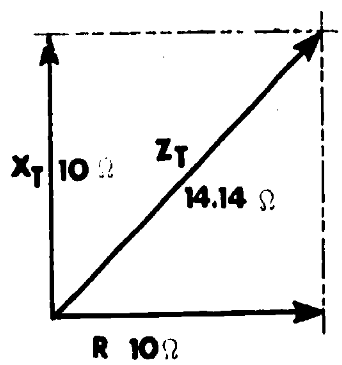
When $\underline{f}$ increases, what happens to:

X_L ______ (Indicate with arrows.)

X_C ______

ANSWERS: $X_L \uparrow$; $X_C \downarrow$

X_L increases to 105 Ω; X_C decreases to 95 Ω. The formulas prove this to you if you are in doubt: $X_L = 2\pi fL$, $X_C = \frac{0.159}{fC}$.



By algebraically adding, we can determine the value and direction of the reactive vector of the impedance vector diagram.

$$X_L = +\,j105\ \Omega$$
$$X_C = \underline{-\,j\ 95\ \Omega}$$
$$\text{reactive vector} = +\,j\ 10\ \Omega$$

Now the source effectively sees a circuit containing only $\underline{R}$ and X_L. Notice that $\underline{R}$ and the effective X_L are equal.

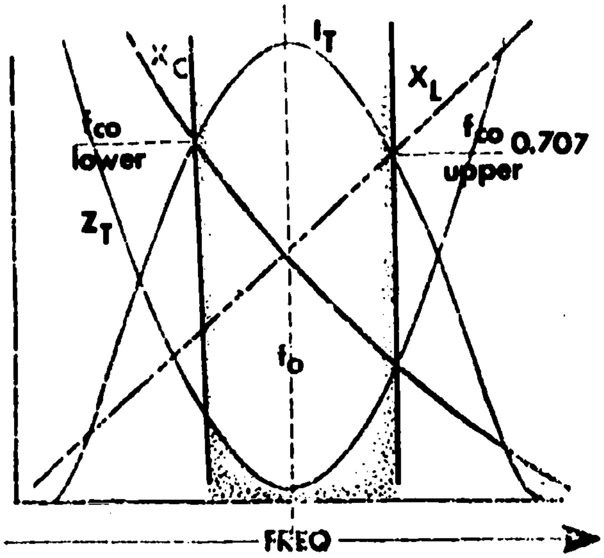
What frequency condition exists when $\underline{R}$ and effective X_L are equal? ____________

What is $\underline{/\theta}$? ______________________

When $R = X_L$, we have the frequency cutoff point (f_{co}). Since the resistance and reactance are equal, $\underline{/\theta}$ is 45°.

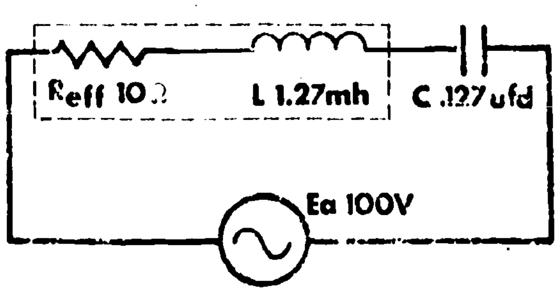
At f_{co}, we are at the half-power point, so P_t is 500 watts, or half of what it was in the purely resistive circuit at f_o.

When frequency reaches a point above resonance where X_L and $\underline{R}$ are equal, we call this the <u>high-frequency cutoff point</u>.



On the current and impedance curve, the high f_{co} point is shown on the high side of f_o. Observe the curve shows that at the upper f_{co} Z_T has increased and I_T has decreased.

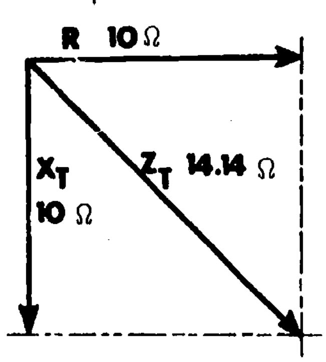
Circuit Behavior Below f_o



Now we will change to a frequency below the resonant frequency. We will use 11.875 KHz as the frequency this time.

We know that when f decreases, X_L decreases and X_C increases. By using the formulas for X_L and X_C, we determine that X_L = 95 Ω and X_C = 105 Ω. Notice these values are just the reverse of the values for X_L and X_C in the circuit operating above f_o.

To find Z_T, we need to use vectors. The reactance vector is determined by algebraically adding.



$X_C = -j105\ \Omega$

$X_L = +j95\ \Omega$; therefore,

$X_T = -j10\ \Omega$

and we draw the vector in the $-j$ position.

Since the resistance and effective reactance are equal, $\angle\theta$ is -45°, and Z_T = 14.14 Ω.

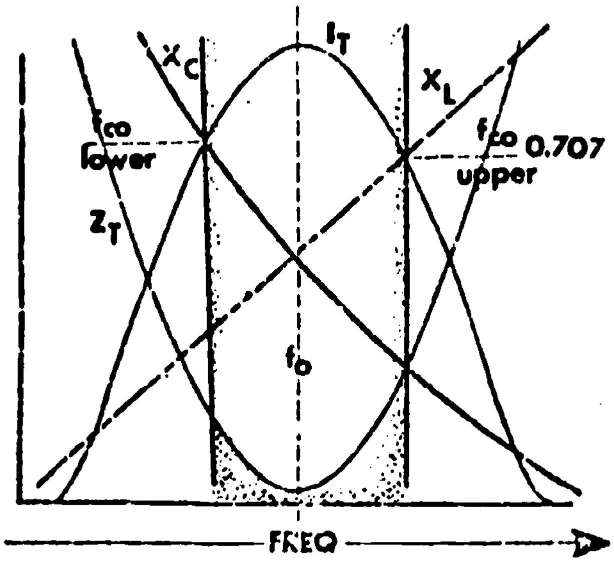
When effective X_C = R, we know that we have reached a f_{co} point -- in this case the lower f_{co} because frequency is below resonant frequency.

We know that at f_{co}:

$\underline{I}$ is 70.7% of maximum and is 7.07 amps.

P_t is half of maximum or 500 watts.

At low frequency cutoff, the source sees an RC circuit.



On the current and impedance curves, the half-power point on the $\underline{I}$ curve for RLC circuits indicates the lower f_{co}.

Here again you see that at low f_{co}, I_T has decreased.

<u>Conclusions</u>

at f_o	upper f_o	lower f_o
Z_T = minimum	Z_T ↑	Z_T ↑
I_T = maximum	I_T ↓	I_T ↓
$\underline{/\theta}$ = 0	$\underline{/\theta}$ = 45°	$\underline{/\theta}$ = -45°

1. Above or below f_o, Z_T will ________ (↑ or ↓).

2. Above or below f_o, I_T will ________ (↑ or ↓).

3. Above f_o, the circuit will appear:

   ____a. purely resistive.

   ____b. RL.

   ____c. RC.

   ____d. RLC.

4. Below f_o, the circuit will appear:

   ____a. purely resistive.

   ____b. RL.

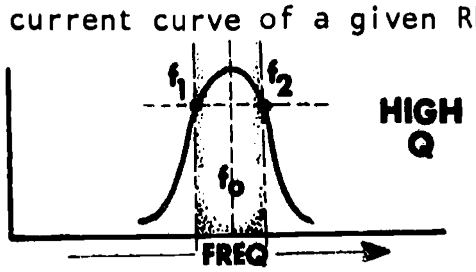
   ____c. RC.

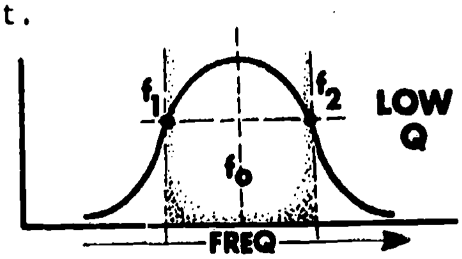
   ____d. RLC.

Answers: 1. ↑ ; 2. ↓ ; 3. b; 4. c

Bandwidth

The curves below indicate the effect of Q on the shape of the current curve of a given RLC circuit.



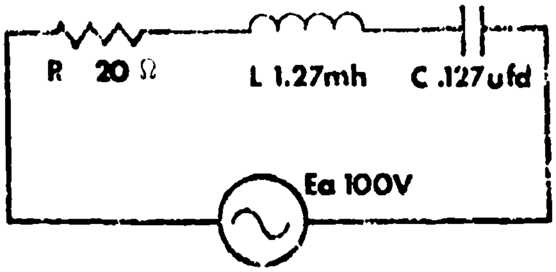


The shaded areas on these curves represent the distance between the upper f_{co} and lower f_{co}. This distance is called the bandwidth of an RLC circuit -- abbreviated BW. Bandwidth is the range of frequencies that a circuit passes with little loss.

Effects of Q on Bandwidth

You know that the Q of a series circuit is determined by the value of X_L and of the effective R. $Q = \frac{X_L}{R_{eff}}$.

If we were to change R in our circuit from 10 Ω to 20 Ω, what would happen to Q? ______________

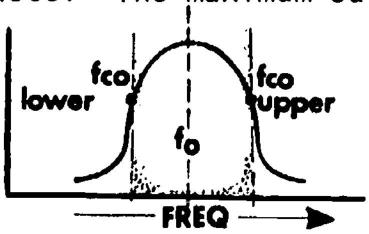


Q would decrease because more resistance had been added and this decreased the amount of power available to be stored in the coil.

Recall that Q does not change much with a change in frequency. If it were possible to change the effective resistance of the circuit, keeping all other values constant, the following would occur.

If the effective resistance increased from 10 ohms to 20 ohms then it would take a higher frequency for X_L to equal the 20 ohms of R. Similarly, we would have to lower frequency more to have X_C equal the 20 ohms of R. The result would be an increased bandwidth, and the resonance curve would look like the one below.

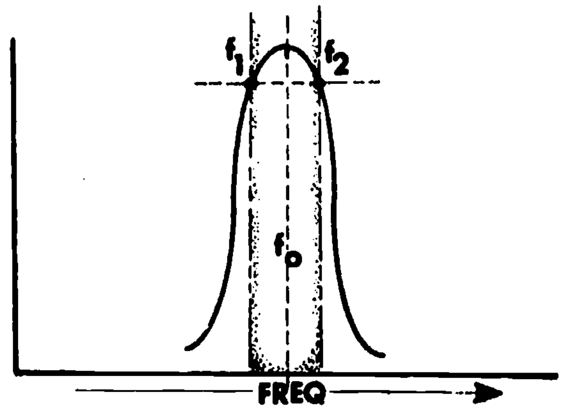
(Note: The maximum current value also decreased.)



The spread between upper f_{co} and lower f_{co} increased, upper f_{co} minus lower f_{co} equals BW; therefore, bandwidth increased.

The circuit conditions at f_{co} are described in Module 12, Lesson VI.

If the effective resistance of our circuit was decreased from 10 ohms to 1 ohm, this would increase Q, and decrease bandwidth. Here I_{max} would increase tenfold.



Therefore, if: Q ↑ then BW ↓

Q ↓ then BW ↑

Q and BW are:

____a. directly proportional.

____b. inversely proportional.

They are inversely proportional.

A second method for computing bandwidth is available if you do not know the upper f_{co} and lower f_{co}.

$$BW = \frac{f_o}{Q}$$

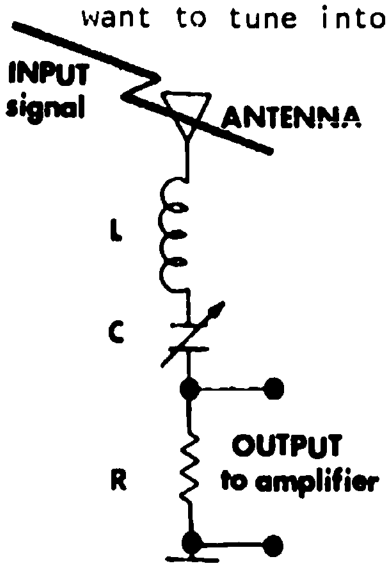
In our circuit, where resonant frequency is 12.5 KHz and Q is 10, use the formula above to solve for bandwidth. BW = ________

Observe that you found BW = 1.25 KHz just as when you subtracted lower f_{co} from upper f_{co}.

A Practical Application

We cannot say that a high Q is necessarily good, or that a low Q is necessarily bad. It depends on the application and on how wide a range of frequencies is desired. If we want a wide range of frequencies, we use a lower Q. Conversely, if we want a narrow bandwidth, we use a higher Q circuit.

For example, let's say you turn on your car radio. You know you want to tune into 1250 KHz on the dial to hear "The Latest News."



The car antenna has a multitude of frequencies to pick up, but you only want 1250 KHz so you can hear "The Latest News," and you want to hear it loud and clear without garble from stations on other frequencies.

You turn the tuning knob to 1250 KHz on the radio dial. This varies the amount of capacitance in the circuit, so that the RLC circuit resonates at 1250 KHz.

Because you do not want to hear the stations at 1350 KHz or 1150 KHz along with "The Latest News," your radio needs a narrow bandwidth which passes only the frequencies you desire.

The tuning circuit of your radio should have:

____a. a high Q.

____b. a low Q.

A high-Q coil passes only a narrow range of frequencies on either side of f_o; therefore, a good radio has a high-Q circuit.

The ability of a circuit to tune a narrow band of frequencies is called selectivity. A circuit with high selectivity has a high Q and narrow bandwidth.

In reality, we seldom see a series circuit used in the radio application just described. At f_o in a series RLC circuit, current is very high; therefore, circuit components have to be large, and expensive to handle the great amount of power. Generally, a parallel circuit is used for such an application and we'll soon be learning about parallel resonant circuits.

Now check yourself on the following questions.

---

1. If Q is decreased, will selectivity increase or decrease?

   ____

2. If Q is decreased, will BW increase or decrease? ________

3. The relationship between Q and Z_T at f_o is:

   ____a. inversely proportional.

   ____b. directly proportional.

4. What is the circuit phase angle at f_o? ____________

5. If you want to design a circuit to pass a frequency range of 950 Hz to 1050 Hz, what must Q be? ______________

Answers:

1. decrease
2. increase
3. a
4. zero
5. Q = 10

In a circuit which has:

upper f_{co} of 1050 Hz

lower f_{co} of $\underline{950 \text{ Hz}}$,

the BW is 100 Hz

If $Q = \frac{f_o}{BW}$, then to find f_o, we simply take half the BW and subtract it from the upper f_{co}:

1050 Hz

$\underline{-\ 50 \text{ Hz } (1/2 \text{ BW})}$

1000 Hz = f_o

Then:

$$Q = \frac{f_o}{BW}$$

$$Q = \frac{1000}{100} = 10$$

AT THIS POINT, YOU MAY TAKE THE PROGRESS CHECK, OR YOU MAY STUDY ANY OF THE OTHER RESOURCES LISTED. IF YOU TAKE THE PROGRESS CHECK AND ANSWER ALL OF THE QUESTIONS CORRECTLY, GO TO THE NEXT LESSON. IF NOT, STUDY ANY METHOD OF INSTRUCTION YOU WISH UNTIL YOU CAN ANSWER ALL THE QUESTIONS CORRECTLY.

PROGRAMMED INSTRUCTION

LESSON III

Resonance in Series AC Circuits

TEST FRAMES ARE 4, 6, 15, 17 AND 26. AS BEFORE, GO FIRST TO TEST FRAME 4 AND SEE IF YOU CAN ANSWER ALL THE QUESTIONS THERE. FOLLOW THE DIRECTIONS GIVEN AFTER THE TEST FRAME.

---

1. Once again, in a series RLC circuit operating at its resonant frequency, X_C and X_L are equal. At resonance, Z_T is:

 ____a. maximum

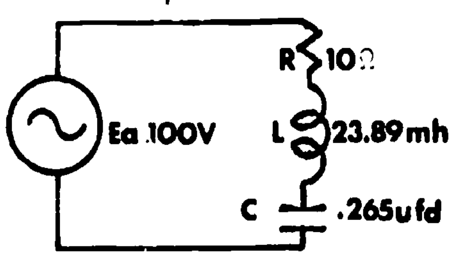
 ____b. minimum

- -

(b)

2. Because the effects of X_L and X_C cancel at resonance, Z is at its minimum value and equal to the value of resistance in the RLC circuit.

 What is I_T (circuit is operating at resonance)?



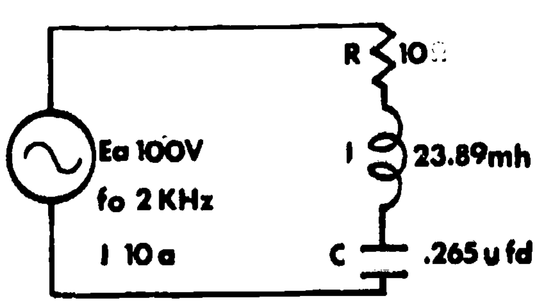
I_T = ____________

- -

(10 amps)

3. In a series RLC circuit at resonance, $X_L = X_C$; and since current is common, this results in $E_C = E_L$. To compute E_C and E_L, it is necessary to find either X_L or X_C, and then multiply this value by I_T.

 Find E_C and E_L.



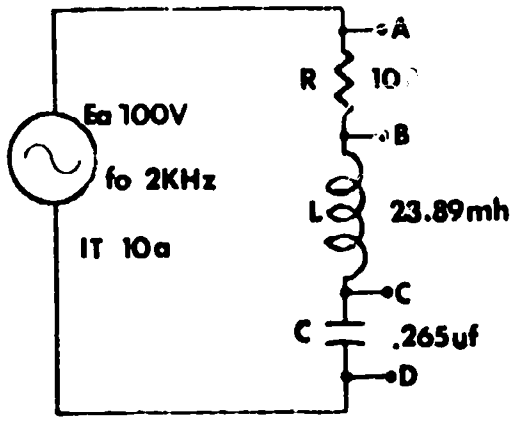
E_C = ____________

E_L = ____________

- -

(3000 volts; 3000 volts)

4. What does a voltmeter read when connected between the following points?



A-B: ______________

B-C: ______________

C-D: ______________

B-D: ______________

A-D: ______________

(THIS IS A TEST FRAME. COMPARE YOUR ANSWERS WITH THE CORRECT ANSWERS GIVEN AT THE TOP OF THE NEXT PAGE.)

ANSWERS - TEST FRAME 4

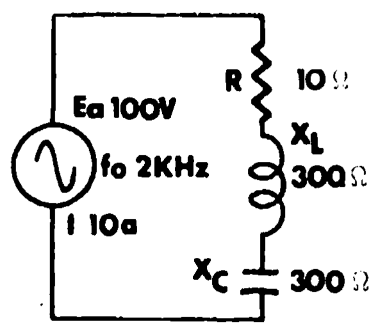
A-B = 100 volts; B-C = 3000 volts; C-D = 3000 volts;

B-D = 0 volts; A-D = 100 volts

IF ALL YOUR ANSWERS MATCH THE CORRECT ANSWERS, YOU MAY GO TO TEST FRAME 6. OTHERWISE, GO BACK TO FRAME 1 AND TAKE THE PROGRAMMED SEQUENCE BEFORE TAKING TEST FRAME 4 AGAIN.

5. Because the reactances effectively cancel at the resonant frequency and the circuit impedance is equal to resistance, the source sees a purely resistive circuit. E_a and I_T are in phase.

Solve for $\underline{/\theta}$ and PF.

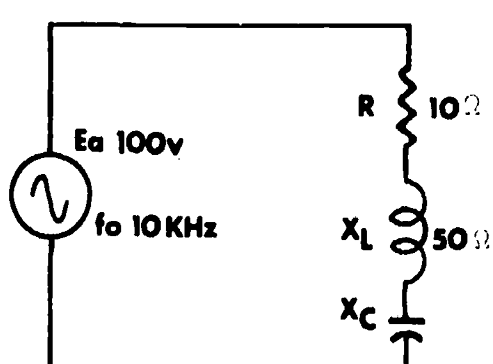


$\underline{/\theta}$ = ______________

PF = ______________

($\underline{/\theta}$ = 0°; PF = 1)

6. Solve for the following values. (Circuit is at resonance.)



a. X_C ______________

b. E_R ______________

c. $\underline{/\theta}$ ______________

d. PF ______________

e. Q ______________

f. Z_T ______________

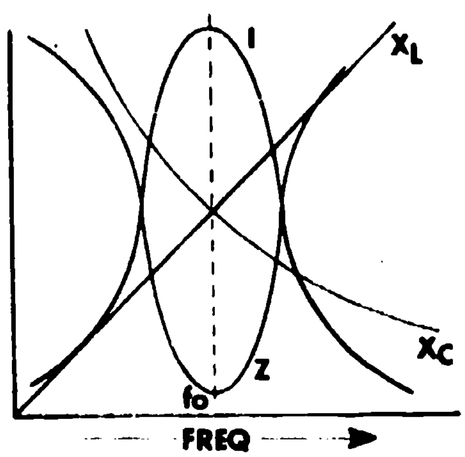
(THIS IS A TEST FRAME. COMPARE YOUR ANSWERS WITH THE CORRECT ANSWERS GIVEN AT THE TOP OF THE NEXT PAGE.)

ANSWERS - TEST FRAME 6

a. 50 Ω; b. 100 v; c. 0°; d. 1; e. 5; f. 10 Ω

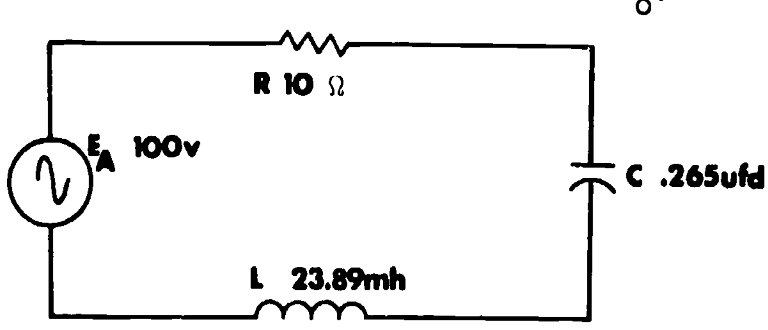
IF ALL YOUR ANSWERS MATCH THE CORRECT ANSWERS, YOU MAY GO TO TEST FRAME 15. OTHERWISE, GO BACK TO FRAME 5 and TAKE THE PROGRAMMED SEQUENCE BEFORE TAKING TEST FRAME 6 AGAIN.

7. The relationship between I and Z can be seen quite well when plotted on a graph.



As you can see from the graph, as frequency increases, X_C starts to decrease and X_L starts to increase. The canceling effect between the two becomes greater and greater until $X_L = X_C$ (at f_o). At this point Z is minimum and I is maximum. If frequency is increased above this point, X_L becomes greater than X_C and E_L is no longer completely cancelled by E_C, so E_R decreases and I decreases.

8. By using the formula to determine F_o, we calculate that F_o for the circuit below is 2 KHz and that at F_o, I = 10a Z = 10Ω.



If frequency is increased by 30 Hz the balance between the reactances is upset. X_L increases to 305 ohms and X_C decreases to 295 ohms.

Compute $\underline{Z}$ and $\underline{I}$ at the increased frequency.

Z = ____________

I = ____________

- -

(14.14 Ω; 7.07 a)

9. At the increased frequency, $\underline{Z}$ is no longer equal to $\underline{R}$, but once again is a combination of resistance and reactance.

   At frequencies above resonance, the circuit appears:

   ____a. resistive.

   ____b. inductive-resistive.

   ____c. capacitive-resistive.

- -

(b) inductive-resistive

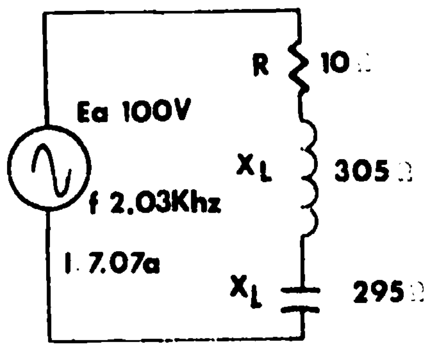
10. At a frequency of 2030 Hz, X_L is 10 ohms greater than X_C, so impedance equals $R + jX_L - jX_C$ or $10 + j10$. Notice that $R = X_L - X_C$. You've seen this condition before. It's known as ____________________.

- -

(f_{co} or cutoff frequency)

11. The circuit is now operating at f_{co} on the high or inductive side of resonance. As the source sees it, the circuit now consists solely of 10 ohms of R and 10 ohms of X_L, even though X_L actually represents total reactance $(X_L - X_C)$. The conditions described in Module Twelve, Lesson VI, for cutoff now exists (series RL circuits).

Solve for the values indicated.



a. $/\theta$ = ________

b. E_L = ________

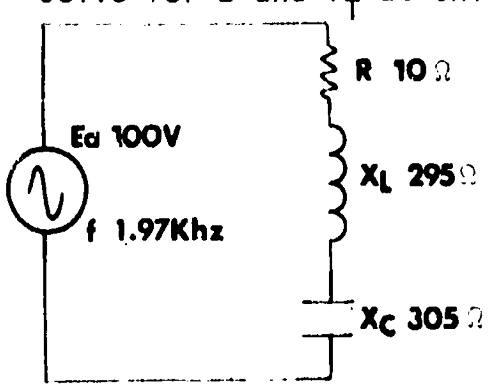
c. E_R = ________

d. P_t = ________

(a. 45°; b. 2156.25v; c. 70.7 volts; d. 500 watts)

12. Frequency is now reduced to 30 Hz below f_o. As this is done, Z decreases to its minimum value (10 ohms, Z=R at f_o), and then increases as frequency goes below f_o to 1.97 KHZ. X_C is now 305 ohms while X_L is 295 ohms. X_C is 10 ohms greater than X_L.

Solve for Z and I_T at this new frequency.



Z = ________

I = ________

(Z = 14.14; I = 7.07 amps)

13. Notice that $\underline{Z}$ and $\underline{I}$ are the same value at 30 Hz below resonance (1.97 KHz) as they did at 30 Hz above f_o. The difference lies in that the circuit appears capacitive and the phase angle is negative.

Which statement is correct?

1. Impedance increases as frequency increases above f_o.
2. Impedance increases as frequency decreases below f_o.

____a. 1

____b. 2

____c. both

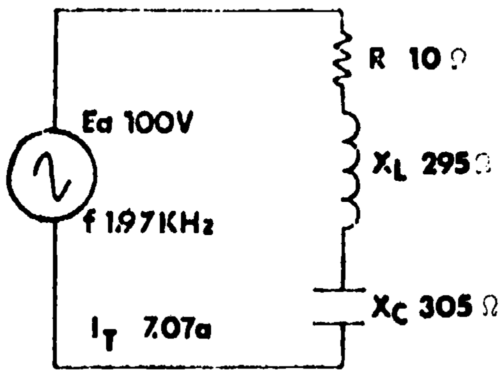
____d. neither

- -

(c) both

14. The circuit now appears resistive-capacitive with 10 ohms resistance and 10 ohms capacitive reactance. Once again resistance equals reactance and the circuit is at cutoff on the low or capacitive side of resonance.

Solve for the following values.



a. $\underline{/\theta}$ = __________

b. E_L = __________

c. E_R = __________

d. P_t = __________

- -

(a. -45°; b. 2085 volts; c. 70.7 volts; d. 500 watts)

15. A series RLC circuit below f_o appears to be:

____a. inductive-resistive.

____b. resistive-capacitive.

____c. purely resistive.

- -

(THIS IS A TEST FRAME. COMPARE YOUR ANSWER WITH THE CORRECT ANSWER GIVEN AT THE TOP OF THE NEXT PAGE.)

ANSWER - TEST FRAME 15

b

IF YOUR ANSWER MATCHES THE CORRECT ANSWER, YOU MAY GO TO TEST FRAME 17. OTHERWISE, GO BACK TO FRAME 7 AND TAKE THE PROGRAMMED SEQUENCE BEFORE TAKING TEST FRAME 15 AGAIN.

16. The extent of the range of frequencies between the upper and lower cutoff frequencies is known as the bandwidth (BW) of the circuit.

Determine the bandwidth of a circuit whose upper f_{co} is 1520 Hz and lower f_{co} is 1480 Hz.

____a. 1520 Hz

____b. 20 Hz

____c. 1480 Hz

____d. 40 Hz

- -

(d) 40 Hz

17. Determine BW if:

upper f_{co} = 13.125 KHz, and

lower f_{co} = 11.875 KHz.

BW = ____________________

- -

(THIS IS A TEST FRAME. COMPARE YOUR ANSWER WITH THE ANSWER GIVEN ON THE TOP OF THE NEXT PAGE.)

ANSWER - TEST FRAME 17

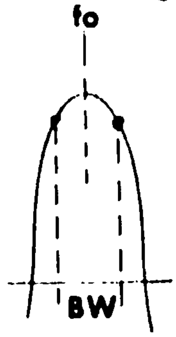
1.250 KHz

IF YOUR ANSWER MATCHES THE CORRECT ANSWER, YOU MAY GO TO TEST FRAME 26. OTHERWISE, GO BACK TO FRAME 16 AND TAKE THE PROGRAMMED SEQUENCE BEFORE TAKING TEST FRAME 17 AGAIN.

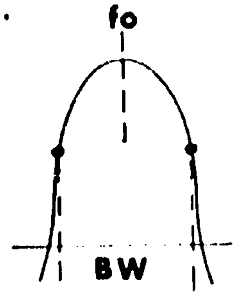
18. Q is a prime factor in determining the bandwidth of a circuit. The higher the Q, the more narrow the bandwidth.

Which drawing depicts a high-Q circuit?

A.



B.

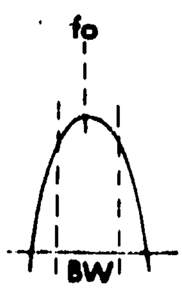


- -

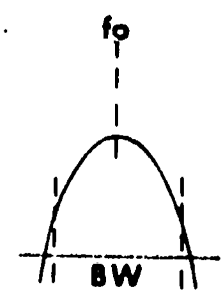
(A)

19. A narrow BW indicates a high-Q or a highly selective circuit. The bandwidth indicates the range of frequencies a circuit will pass with the output taken across the resistor.

A.



B.



Which circuit represented in the above graphs passes the wider range of frequencies? ____ Which graph has the poorer selectivity? ____

- -

(B; B)

20. Bandwidth is inversely proportional to Q. As circuit Q increases, BW ________________.

- -

(decreases)

21. To increase the Q of the circuit, the effective resistance (R_{eff}) must be decreased. As we decrease the R_{eff} value of the circuit, BW ________________.

- -

(decreases)

22. Since decreasing R_{eff} decreases BW and increase circuit Q BW is ____________ proportional to R_{eff} and __________ proportional to Q.

- -

(directly, inversely)

23. To determine the BW of a circuit, you may use the formula

$$BW = \frac{f_o}{Q}.$$

The BW of a circuit with a Q of 100 and an f_o of 12.5 KHz is ____________ Hz.

- -

(125 Hz)

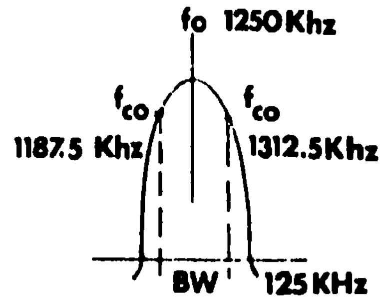
24. A radio circuit with a BW of 125Khz receives a signal from a station 62.5Khz above f_o to 62.5Khz below f_o.

This circuit adjusted to a f_o of 1250 Khz passes a band of frequencies from 1187.5 Khz to __________ Khz.

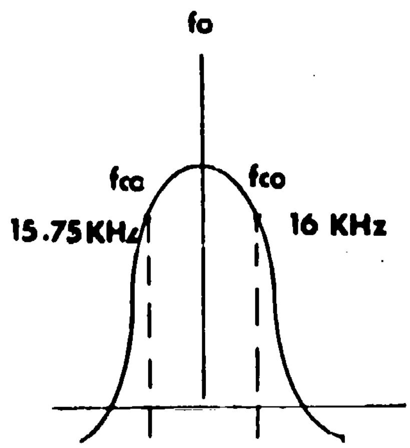
- -

(1312.5)

25. The circuit discussed in the previous frame shown on a resonance curve graph looks like this:



What is the f_o of the circuit represented by the graph below?



f_o = ____________

(15.875KHz)

26. Fill in the following spaces.

1. A series RLC circuit, in which $X_L = X_C$, has total reactance of ________________.

2. If there is zero reactance at resonance, then circuit impedance is equal to the circuit _____________.

3. The equation for finding the resonant frequency is

   $f_o = \frac{1}{2\_\ \overline{\ \ \ \ }}$.

4. In a series RLC circuit at resonance, the phase angle between source voltage and current is _____________.

5. If the Q of a series RLC circuit is increased, the resonance curve is _______________ (steeper/flatter); bandwidth is ______________ (wider/narrower); and selectivity is ____________ (higher/lower).

- -

(THIS IS A TEST FRAME. COMPARE YOUR ANSWERS WITH THE CORRECT ANSWERS GIVEN ON THE TOP OF THE NEXT PAGE.)

ANSWERS - TEST FRAME 26

1. zero; 2. resistance; 3. $\sqrt{LC}$; 4. zero;

5. steeper, narrower, higher

IF ANY OF YOUR ANSWERS IN INCORRECT, GO BACK TO FRAME 18 AND TAKE THE PROGRAMMED SEQUENCE.

IF YOUR ANSWERS ARE CORRECT, YOU MAY TAKE THE PROGRESS CHECK OR YOU MAY STUDY ANY OF THE OTHER RESOURCES LISTED. IF YOU TAKE THE PROGRESS CHECK AND ANSWER ALL THE QUESTIONS CORRECTLY, GO TO THE NEXT LESSON. IF NOT, STUDY ANY METHOD OF INSTRUCTION YOU WISH UNTIL YOU CAN ANSWER ALL THE QUESTIONS CORRECTLY.

SUMMARY
LESSON III

Resonance in Series AC Circuits

You have already studied resonant circuits. You know that resonance occurs when X_L equals X_C in RCL series circuits.

In this lesson, we will take a resonant circuit and vary the frequency above resonance and below resonance to observe the effects of the circuit.

Recall that a circuit at resonance offers minimum impedance to current flow -- total impedance is equal to the value of circuit resistance, so maximum current flows. If frequency is increased above resonance, X_L increases and total impedance increases as it is the vector sum of X_L and R. Circuit current decreases as the frequency increases, and the circuit becomes more inductive.

If we go the other direction from resonance and decrease frequency, the value of X_C increases, again increasing total impedance and decreasing circuit current. The more the frequency is decreased, the more capacitive reactance increases and the more capacitive the circuit appears. The series RCL circuit has a usable output across the resistive component through a central frequency range. The extent of this range is dependent upon the quality (Q) of the circuit. The lower end of this range is called the low-frequency cutoff point and the upper or high end is called the high-frequency cutoff frequency. The range of frequencies between the low f_{co} point and the high f_{co} point is called the circuit bandwidth. Resonant frequency is the center frequency. This range of frequencies that provides a usable output explains why you can hear your favorite radio station around the exact station frequency. When you tune the station in loud and clear, you have adjusted the circuit to resonate at the station frequency. As you adjust above or below this point, the reception gets progressively worse.

The bandwidth of the circuit is dependent upon the circuit Q. Remember, Q is determined by the value of $\frac{X_L}{R_{eff}}$. f_o is determined by $\frac{1}{2\pi\sqrt{LC}}$. Bandwidth (BW) is determined by dividing f_o by the circuit Q, $BW = \frac{f_o}{Q}$.

AT THIS POINT, YOU MAY TAKE THE LESSON PROGRESS CHECK, OR YOU MAY STUDY THE LESSON NARRATIVE OR THE PROGRAMMED INSTRUCTION OR BOTH. IF YOU TAKE THE PROGRESS CHECK AND ANSWER ALL THE QUESTIONS CORRECTLY, GO TO THE NEXT LESSON. IF NOT, STUDY ANOTHER METHOD OF INSTRUCTION UNTIL YOU CAN ANSWER ALL THE QUESTIONS CORRECTLY.

NAVPERS 94558-13a

BASIC ELECTRICITY AND ELECTRONICS
INDIVIDUALIZED LEARNING SYSTEM

MODULE THIRTEEN
LESSON IV

Experiments with Resonance in a Series RCL Circuit

Study Booklet

Bureau of Naval Personnel
January 1972

OVERVIEW

LESSON IV

Experiments With Resonance in a Series RCL Circuit

In this lesson you will perform experiments with a series RCL circuit, using various equipment to see the effects of resonance that you have learned in the previous lessons.

You will use:

- -an audio signal generator.
- -NEAT board 6.
- -a multimeter.

You will:

- -determine f_o.
- -read voltage drops at f_o.
- -see the effects of varying capacitance.

BEFORE YOU START THIS LESSON, PREVIEW THE LIST OF STUDY RESOURCES ON THE NEXT PAGE.

LIST OF STUDY RESOURCES

LESSON IV

Experiments With Resonance in a Series RCL Circuit

Since this lesson consists of experiments, there is only the narrative. There are no other study resources and no progress check.

TURN THE PAGE AND BEGIN THE NARRATIVE.

NARRATIVE

LESSON IV

Experiments With Resonance in a Series RCL Circuit

Now that you understand the theory of resonance in an AC series RCL circuit, you will have the opportunity to see these concepts at work.

For this lesson, go to the materials center and get any of the folowing which you do not have in your carrel.

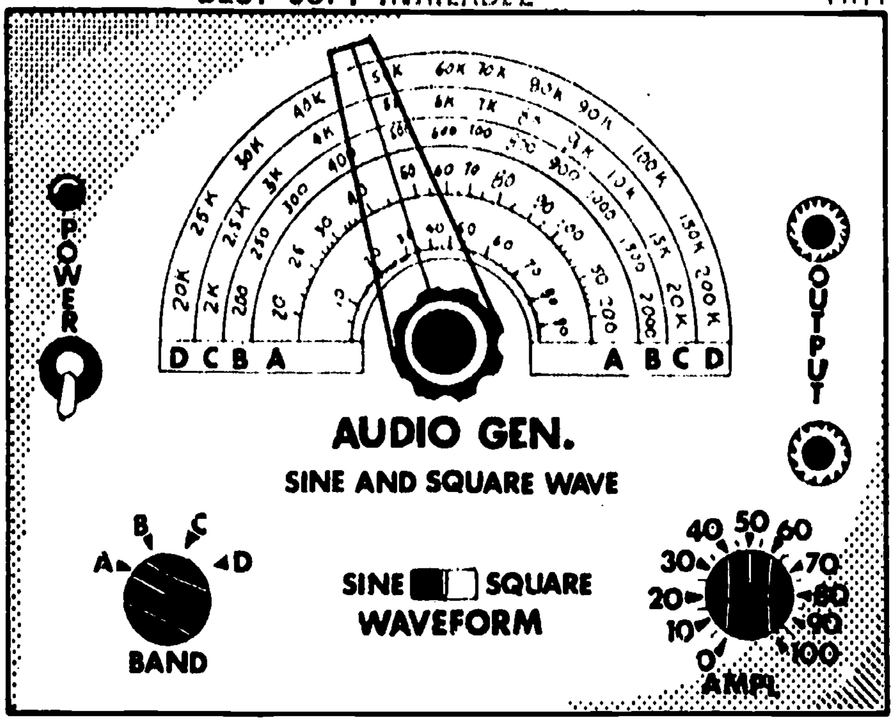
1. an audio signal generator
2. NEAT Board 6 for series resonance
3. a multimeter
4. two sets of test leads

The Audio Signal Generator

The signal generator plugs into your AC wall source and supplies to the NEAT board a controlled signal at any frequency which you determine.

This signal generator can supply a range of frequencies between 20 and 20,000 hertz by generating a sine wave. It is also capable of generating square waves in a frequency range of 60 to 30,000 hertz. The entire frequency spectrum is covered in four ranges for maximum accuracy and readability. For our use in this lesson we will be concerned with only the C scale -- the 2 KHz to 20 KHz range. (Observe the range scale in the center of the generator.)

The output voltage is obtained from the two terminal jacks labeled OUTPUT and located on the right side of the front panel. Study the illustration.



Listed below are the controls of the audio signal generator. Locate these controls on the generator.

1. Power on-off toggle switch (left side of the front panel)

2. Output jacks (right side)

3. Band selector (lower left) to set at A, B, C or D

4. Amplitude Knob (lower right) determines amplitude of output voltage

5. Waveform switch (center bottom) selects sine or square wave

6. Frequency control (center, below range dial) selects output frequency.

One word of caution. <u>DO NOT SHORT OUTPUT TERMINALS</u>. Shorting the output terminals would result in a blown fuse and your signal generator would not operate. (At this point, set the amplitude knob to 0, plug the signal generator into the AC outlet, and turn o the power switch. This is to allow warm-up of the generator while you read the rest of the instructions.)

The NEAT Board

This NEAT board is designed for experimental purposes, and has a series AC circuit built into it. As indicated by the schematic on the front panel, this NEAT board contains a coil and a capacitor. It also has a resistor that can be put in series with the resistive components, depending on the position of S-603.

Notice there are two capacitors, one of a fixed value, and the other a variable capacitor with a knob to vary the amount of

capacitance. The position of S-604 determines which capacitor is included in the circuit at a particular time.

Also observe that S-602 will need to be open if the ammeter is connected in the circuit at the location shown on the schematic. If the ammeter is not in the circuit at that point, S-602 will need to be closed to complete the circuit.

Experiment 1 - Determining f_o

Follow these instructions step by step.

1. Connect one set of test leads to the output jacks of the signal generator. The bottom jack is common (ground).
2. Insert the other end of the leads into terminal points (T_p) for input to the neat board at T_p 601-A and T_p 602-A (common).
3. Turn band selector switch on generator to C position.
4. Turn frequency control fully counterclockwise.
5. Set waveform switch at SINE.
6. Set up the neat board as follows:

   S-601 to ON position (↑)

   S-602 to OFF position (↑) (open)

   S-604 to position A (↑) so that the fixed capacitor is in the circuit

   S-603 to position A (←) so there is no resistor in the circuit
7. Set up your multimeter to read AC voltage on the 50-volt scale.
8. Insert meter leads in multimeter and in neat board in T_p 601-B and T_p 602-B.
9. Now turn the amplitude knob on signal generator until you get a 12-volt reading on the meter. Leave the amplitude knob set in that position.

10. Disconnect the meter plugs from the neat board.

11. Set meter to measure DC on the 100 ma range and insert the meter between T_p 603 (-) and T_p 604 (+). (NOTE: A rectifier in the neat board changes AC to DC, so the ammeter will measure DC; polarity must be observed.)

12. Draw a schematic of the neat board circuit as it is now set up -- including the ammeter.

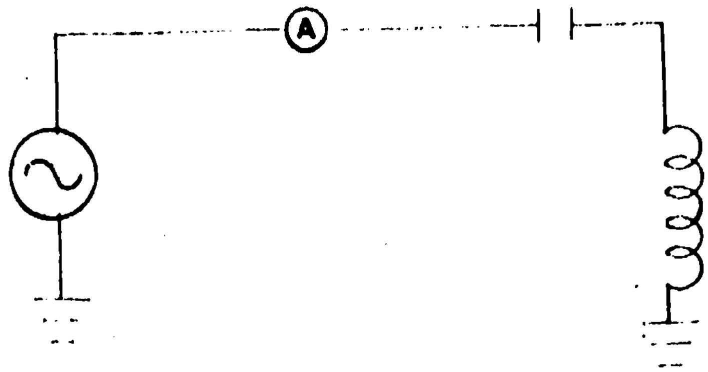
13. Slowly turn the frequency knob clockwise on the signal generator and observe the current reading on the meter.

    Determine from the meter reading at what frequency resonance is reached. Record it. You will be reading f on the C scale.

f_o = ______________

Answers for Experiment 1

(12). Schematic



(13). f_o is approximately 4.3 KHz when I is maximum.

Experiment 2 -- Reading Voltage Drops at f_o

1. Change S-603 to position B. This puts a resistor in series in the circuit.

   Observe the I drops when R is added.

2. Change S-603 back to position A. You have not changed frequency; it is still at f_o.

3. Remove ammeter from circuit. (Remember to observe all safety precautions.)

4. Set S-602 to ON position to close circuit.

5. Set meter to read AC voltage on 250 volt scale.

6. Insert meter probes in T_p 604 - T_p 605 to read the voltage drop across the capacitor. Record.

   E_C = ________

7. Now move probes to T_p 605 - T_p 606. Read the voltage drop across the inductor.

   E_L = ________

   Recall that at f_o, E_C and E_L are approximately equal.

8. Remove voltmeter probes.

Answers to Experiment 2

(6). Approximately 80 v

(7). Approximately 80 v

Experiment 3 -- Varying Capacitance

1. Set up neat board as follows.

   Turn S-602 to OFF (open circuit)

   Turn S-604 to B position.

   Now the variable capacitor is in the circuit, but the fixed capacitor is not.

   Adjust C-601 (variable capacitor knob) to the far left. The capacitor is now at its maximum value.

2. Set up ammeter to read DC current, 100 ma range.

3. Insert meter probes between T_p 603 - T_p 604 (check for proper polarity).

4. Because capacitance has been varied, the circuit will no longer be at f_o. Now slowly turn frequency knob on signal generator back and forth until you find f_o as indicated by the ammeter reading.

5. Now decrease the value of capacitance by turning C-601 to the mid-scale mark (straight up).

   Observe that as capacitance decreases, current:

   ____a. increases.

   ____b. decreases.

   As capacitance decreases X_C:

   ____a. increases.

   ____b. decreases.

   X_C and X_L are no longer equal, so the circuit is not at f_o. X_C is greater than X_L now.

6. Again turn the frequency knob on the signal generator until X_C and X_L are equal.

$$f_o = \underline{\hspace{3cm}}$$

7. Turn amplitude dial to 0.

8. Turn off audio generator power.

9. Disconnect all leads.

10. Return the borrowed equipment to the materials center.

<u>Answers to Experiment 3</u>

(4). f_o = approximately 14 KHz

(5). As <u>C</u> decreases, <u>I</u> decreases.

As <u>C</u> decreases, X_C increases.

(6). f_o = approximately 20 KHz

AT THIS POINT SEE YOUR LEARNING SUPERVISOR FOR FURTHER INSTRUCTIONS.

deg	function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°
0	sin	0.0000	0.0017	0.0035	0.0052	0.0070	0.0087	0.0105	0.0122	0.0140	0.0157
	cos	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999
	tan	0.0000	0.0017	0.0035	0.0052	0.0070	0.0087	0.0105	0.0122	0.0140	0.0157
1	sin	0.0175	0.0192	0.0209	0.0227	0.0244	0.0262	0.0279	0.0297	0.0314	0.0332
	cos	0.9998	0.9998	0.9998	0.9997	0.9997	0.9997	0.9996	0.9996	0.9995	0.9995
	tan	0.0175	0.0192	0.0209	0.0227	0.0244	0.0262	0.0279	0.0297	0.0314	0.0332
2	sin	0.0349	0.0366	0.0384	0.0401	0.0419	0.0436	0.0454	0.0471	0.0488	0.0506
	cos	0.9994	0.9993	0.9993	0.9992	0.9991	0.9990	0.9990	0.9989	0.9988	0.9987
	tan	0.0349	0.0367	0.0384	0.0402	0.0419	0.0437	0.0454	0.0472	0.0489	0.0507
3	sin	0.0523	0.0541	0.0558	0.0576	0.0593	0.0610	0.0628	0.0645	0.0663	0.0680
	cos	0.9986	0.9985	0.9984	0.9983	0.9982	0.9981	0.9980	0.9979	0.9978	0.9977
	tan	0.0524	0.0542	0.0559	0.0577	0.0594	0.0612	0.0629	0.0647	0.0664	0.0682
4	sin	0.0698	0.0715	0.0732	0.0750	0.0767	0.0785	0.0802	0.0819	0.0837	0.0854
	cos	0.9976	0.9974	0.9973	0.9972	0.9971	0.9969	0.9968	0.9966	0.9965	0.9963
	tan	0.0699	0.0717	0.0734	0.0752	0.0769	0.0787	0.0805	0.0822	0.0840	0.0857
5	sin	0.0872	0.0889	0.0906	0.0924	0.0941	0.0958	0.0976	0.0993	0.1011	0.1028
	cos	0.9962	0.9960	0.9959	0.9957	0.9956	0.9954	0.9952	0.9951	0.9949	0.9947
	tan	0.0875	0.0892	0.0910	0.0928	0.0945	0.0963	0.0981	0.0998	0.1016	0.1033
6	sin	0.1045	0.1063	0.1080	0.1097	0.1115	0.1132	0.1149	0.1167	0.1184	0.1201
	cos	0.9945	0.9943	0.9942	0.9940	0.9938	0.9936	0.9934	0.9932	0.9930	0.9928
	tan	0.1051	0.1069	0.1086	0.1104	0.1122	0.1139	0.1157	0.1175	0.1192	0.1210
7	sin	0.1219	0.1236	0.1253	0.1271	0.1288	0.1305	0.1323	0.1340	0.1357	0.1374
	cos	0.9925	0.9923	0.9921	0.9919	0.9917	0.9914	0.9912	0.9910	0.9907	0.9905
	tan	0.1228	0.1246	0.1263	0.1281	0.1299	0.1317	0.1334	0.1352	0.1370	0.1388
8	sin	0.1392	0.1409	0.1426	0.1444	0.1461	0.1478	0.1495	0.1513	0.1530	0.1547
	cos	0.9903	0.9900	0.9898	0.9895	0.9893	0.9890	0.9888	0.9885	0.9882	0.9880
	tan	0.1405	0.1423	0.1441	0.1459	0.1477	0.1495	0.1512	0.1530	0.1548	0.1566
9	sin	0.1564	0.1582	0.1599	0.1616	0.1633	0.1650	0.1668	0.1685	0.1702	0.1719
	cos	0.9877	0.9874	0.9871	0.9869	0.9866	0.9863	0.9860	0.9857	0.9854	0.9851
	tan	0.1584	0.1602	0.1620	0.1638	0.1655	0.1673	0.1691	0.1709	0.1727	0.1745
10	sin	0.1736	0.1754	0.1771	0.1788	0.1805	0.1822	0.1840	0.1857	0.1874	0.1891
	cos	0.9848	0.9845	0.9842	0.9839	0.9836	0.9833	0.9829	0.9826	0.9823	0.9820
	tan	0.1763	0.1781	0.1799	0.1817	0.1835	0.1853	0.1871	0.1890	0.1908	0.1926
11	sin	0.1908	0.1925	0.1942	0.1959	0.1977	0.1994	0.2011	0.2028	0.2045	0.2062
	cos	0.9816	0.9813	0.9810	0.9806	0.9803	0.9799	0.9796	0.9792	0.9789	0.9785
	tan	0.1944	0.1962	0.1980	0.1998	0.2016	0.2035	0.2053	0.2071	0.2089	0.2107
12	sin	0.2079	0.2096	0.2113	0.2130	0.2147	0.2164	0.2181	0.2198	0.2215	0.2232
	cos	0.9781	0.9778	0.9774	0.9770	0.9767	0.9763	0.9759	0.9755	0.9751	0.9748
	tan	0.2126	0.2144	0.2162	0.2180	0.2199	0.2217	0.2235	0.2254	0.2272	0.2290
13	sin	0.2250	0.2267	0.2284	0.2300	0.2318	0.2334	0.2351	0.2368	0.2385	0.2402
	cos	0.9744	0.9740	0.9736	0.9732	0.9728	0.9724	0.9720	0.9715	0.9711	0.9707
	tan	0.2309	0.2327	0.2345	0.2364	0.2382	0.2401	0.2419	0.2438	0.2456	0.2475
deg	function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°

deg	function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°
14	sin	0.2419	0.2436	0.2453	0.2470	0.2487	0.2504	0.2521	0.2538	0.2554	0.2571
	cos	0.9703	0.9699	0.9694	0.9690	0.9686	0.9681	0.9677	0.9673	0.9668	0.9664
	tan	0.2493	0.2512	0.2530	0.2549	0.2568	0.2586	0.2605	0.2623	0.2642	0.2661
15	sin	0.2588	0.2605	0.2622	0.2639	0.2656	0.2672	0.2689	0.2706	0.2723	0.2740
	cos	0.9659	0.9655	0.9650	0.9646	0.9641	0.9636	0.9632	0.9627	0.9622	0.9617
	tan	0.2679	0.2698	0.2717	0.2736	0.2754	0.2773	0.2792	0.2811	0.2830	0.2849
16	sin	0.2756	0.2773	0.2790	0.2807	0.2823	0.2840	0.2857	0.2874	0.2890	0.2907
	cos	0.9613	0.9608	0.9603	0.9598	0.9593	0.9588	0.9583	0.9578	0.9573	0.9568
	tan	0.2867	0.2886	0.2905	0.2924	0.2943	0.2962	0.2981	0.3000	0.3019	0.3038
17	sin	0.2924	0.2940	0.2957	0.2974	0.2990	0.3007	0.3024	0.3040	0.3057	0.3074
	cos	0.9563	0.9558	0.9553	0.9548	0.9542	0.9537	0.9532	0.9527	0.9521	0.9516
	tan	0.3057	0.3076	0.3096	0.3115	0.3134	0.3153	0.3172	0.3191	0.3211	0.3230
18	sin	0.3090	0.3107	0.3123	0.3140	0.3156	0.3173	0.3190	0.3206	0.3223	0.3239
	cos	0.9511	0.9505	0.9500	0.9494	0.9489	0.9483	0.9478	0.9472	0.9466	0.9461
	tan	0.3249	0.3269	0.3288	0.3307	0.3327	0.3346	0.3365	0.3385	0.3404	0.3424
19	sin	0.3256	0.3272	0.3289	0.3305	0.3322	0.3338	0.3355	0.3371	0.3387	0.3404
	cos	0.9455	0.9449	0.9444	0.9438	0.9432	0.9426	0.9421	0.9415	0.9409	0.9403
	tan	0.3443	0.3463	0.3482	0.3502	0.3522	0.3541	0.3561	0.3581	0.3600	0.3620
20	sin	0.3420	0.3437	0.3453	0.3469	0.3486	0.3502	0.3518	0.3535	0.3551	0.3567
	cos	0.9397	0.9391	0.9385	0.9379	0.9373	0.9367	0.9361	0.9354	0.9348	0.9342
	tan	0.3640	0.3659	0.3679	0.3699	0.3719	0.3739	0.3759	0.3779	0.3799	0.3819
21	sin	0.3584	0.3600	0.3616	0.3633	0.3649	0.3665	0.3681	0.3697	0.3714	0.3730
	cos	0.9336	0.9330	0.9323	0.9317	0.9311	0.9304	0.9298	0.9291	0.9285	0.9278
	tan	0.3839	0.3859	0.3879	0.3899	0.3919	0.3939	0.3959	0.3979	0.4000	0.4020
22	sin	0.3746	0.3762	0.3778	0.3795	0.3811	0.3827	0.3843	0.3859	0.3875	0.3891
	cos	0.9272	0.9265	0.9259	0.9252	0.9245	0.9239	0.9232	0.9225	0.9219	0.9212
	tan	0.4040	0.4061	0.4081	0.4101	0.4122	0.4142	0.4163	0.4183	0.4204	0.4224
23	sin	0.3907	0.3923	0.3939	0.3955	0.3971	0.3987	0.4003	0.4019	0.4035	0.4051
	cos	0.9205	0.9198	0.9191	0.9184	0.9178	0.9171	0.9164	0.9157	0.9150	0.9143
	tan	0.4245	0.4265	0.4286	0.4307	0.4327	0.4348	0.4369	0.4390	0.4411	0.4431
24	sin	0.4067	0.4083	0.4099	0.4115	0.4131	0.4147	0.4163	0.4179	0.4195	0.4210
	cos	0.9135	0.9128	0.9121	0.9114	0.9107	0.9100	0.9092	0.9085	0.9078	0.9070
	tan	0.4452	0.4473	0.4494	0.4515	0.4536	0.4557	0.4578	0.4599	0.4621	0.4642
25	sin	0.4226	0.4242	0.4258	0.4274	0.4289	0.4305	0.4321	0.4337	0.4352	0.4368
	cos	0.9063	0.9056	0.9048	0.9041	0.9033	0.9026	0.9018	0.9011	0.9003	0.8996
	tan	0.4663	0.4684	0.4706	0.4727	0.4748	0.4770	0.4791	0.4813	0.4834	0.4856
26	sin	0.4384	0.4399	0.4415	0.4431	0.4446	0.4462	0.4478	0.4493	0.4509	0.4524
	cos	0.8988	0.8980	0.8973	0.8965	0.8957	0.8949	0.8942	0.3934	0.8926	0.8918
	tan	0.4877	0.4899	0.4921	0.4942	0.4964	0.4986	0.5008	0.5029	0.5051	0.5073
27		0.4540	0.4555	0.4571	0.4586	0.4602	0.4617	0.4633	0.4648	0.4664	0.4679
	cos	0.8910	0.8902	0.8894	0.8886	0.8878	0.8870	0.8862	0.8854	0.8846	0.8838
	tan	0.5095	0.5117	0.5139	0.5161	0.5184	0.5206	0.5228	0.5250	0.5272	0.5295
deg	function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°

deg	function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°
	sin	0.4695	0.4710	0.4726	0.4741	0.4756	0.4772	0.4787	0.4802	0.4818	0.4833
28	cos	0.8829	0.8821	0.8813	0.8805	0.8796	0.8788	0.8780	0.8771	0.8763	0.8755
	tan	0.5317	0.5340	0.5362	0.5384	0.5407	0.5430	0.5452	0.5475	0.5498	0.5520
	sin	0.4848	0.4863	0.4879	0.4894	0.4909	0.4924	0.4939	0.4955	0.4970	0.4985
29	cos	0.8746	0.8738	0.8729	0.8721	0.8712	0.8704	0.8695	0.8686	0.8678	0.8669
	tan	0.5543	0.5566	0.5589	0.5612	0.5635	0.5658	0.5681	0.5704	0.5727	0.5750
	sin	0.5000	0.5015	0.5030	0.5045	0.5060	0.5075	0.5090	0.5105	0.5120	0.5135
30	cos	0.8660	0.8652	0.8643	0.8634	0.8625	0.8616	0.8607	0.8599	0.8590	0.8581
	tan	0.5774	0.5797	0.5820	0.5844	0.5867	0.5890	0.5914	0.5938	0.5961	0.5985
	sin	0.5150	0.5165	0.5180	0.5195	0.5210	0.5225	0.5240	0.5255	0.5270	0.5284
31	cos	0.8572	0.8563	0.8554	0.8545	0.8536	0.8526	0.8517	0.8508	0.8499	0.8490
	tan	0.6009	0.6032	0.6056	0.6080	0.6104	0.6128	0.6152	0.6176	0.6200	0.6224
	sin	0.5299	0.5314	0.5329	0.5344	0.5358	0.5373	0.5388	0.5402	0.5417	0.5432
32	cos	0.8480	0.8471	0.8462	0.8453	0.8443	0.8434	0.8425	0.8415	0.8406	0.8396
	tan	0.6249	0.6273	0.6297	0.6322	0.6346	0.6371	0.6395	0.6420	0.6445	0.6469
	sin	0.5446	0.5461	0.5476	0.5490	0.5505	0.5519	0.5534	0.5548	0.5563	0.5577
33	cos	0.8387	0.8377	0.8368	0.8358	0.8348	0.8339	0.8329	0.8320	0.8310	0.8300
	tan	0.6494	0.6519	0.6544	0.6569	0.6594	0.6619	0.6644	0.6669	0.6694	0.6720
	sin	0.5592	0.5606	0.5621	0.5635	0.5650	0.5664	0.5678	0.5693	0.5707	0.5721
34	cos	0.8290	0.8281	0.8271	0.8261	0.8251	0.8241	0.8231	0.8221	0.8211	0.8202
	tan	0.6745	0.6771	0.6796	0.6822	0.6847	0.6873	0.6899	0.6924	0.6950	0.6976
	sin	0.5736	0.5750	0.5764	0.5779	0.5793	0.5807	0.5821	0.5835	0.5850	0.5864
35	cos	0.8192	0.8181	0.8171	0.8161	0.8151	0.8141	0.8131	0.8121	0.8111	0.8100
	tan	0.7002	0.7028	0.7054	0.7080	0.7107	0.7133	0.7159	0.7186	0.7212	0.7239
	sin	0.5878	0.5892	0.5906	0.5920	0.5934	0.5948	0.5962	0.5976	0.5990	0.6004
36	cos	0.8090	0.8080	0.8070	0.8059	0.8049	0.8039	0.8028	0.8018	0.8007	0.7997
	tan	0.7265	0.7292	0.7319	0.7346	0.7373	0.7400	0.7427	0.7454	0.7481	0.7508
	sin	0.6018	0.6032	0.6046	0.6060	0.6074	0.6088	0.6101	0.6115	0.6129	0.6143
37	cos	0.7986	0.7976	0.7965	0.7955	0.7944	0.7934	0.7923	0.7912	0.7902	0.7891
	tan	0.7536	0.7563	0.7590	0.7618	0.7646	0.7673	0.7701	0.7729	0.7757	0.7785
	sin	0.6157	0.6170	0.6184	0.6198	0.6211	0.6225	0.6239	0.6252	0.6266	0.6280
38	cos	0.7880	0.7869	0.7859	0.7848	0.7837	0.7826	0.7815	0.7804	0.7793	0.7782
	tan	0.7813	0.7841	0.7869	0.7898	0.7926	0.7954	0.7983	0.8012	0.8040	0.8069
	sin	0.6293	0.6307	0.6320	0.6334	0.6347	0.6361	0.6374	0.6388	0.6401	0.6414
39	cos	0.7771	0.7760	0.7749	0.7738	0.7727	0.7716	0.7705	0.7694	0.7683	0.7672
	tan	0.8098	0.8127	0.8156	0.8185	0.8214	0.8243	0.8273	0.8302	0.8332	0.8361
	sin	0.6428	0.6441	0.6455	0.6468	0.6481	0.6494	0.6508	0.6521	0.6534	0.6547
40	cos	0.7660	0.7649	0.7638	0.7627	0.7615	0.7604	0.7593	0.7581	0.7570	0.7559
	tan	0.8391	0.8421	0.8451	0.8481	0.8511	0.8541	0.8571	0.8601	0.8632	0.8662
	sin	0.6561	0.6574	0.6587	0.6600	0.6613	0.6626	0.6639	0.6652	0.6665	0.6678
41	cos	0.7547	0.7536	0.7524	0.7513	0.7501	0.7490	0.7478	0.7466	0.7455	0.7443
	tan	0.8693	0.8724	0.8754	0.8785	0.8816	0.8847	0.8878	0.8910	0.8941	0.8972
deg	function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°

deg	function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°
	sin	0.6691	0.6704	0.6717	0.6730	0.6743	0.6756	0.6769	0.6782	0.6794	0.6807
42	cos	0.7431	0.7420	0.7408	0.7396	0.7385	0.7373	0.7361	0.7349	0.7337	0.7325
	tan	0.9004	0.9036	0.9067	0.9099	0.9131	0.9163	0.9195	0.9228	0.9260	0.9293
	sin	0.6820	0.6833	0.6845	0.6858	0.6871	0.6884	0.6896	0.6909	0.6921	0.6934
43	cos	0.7314	0.7302	0.7290	0.7278	0.7266	0.7254	0.7242	0.7230	0.7218	0.7206
	tan	0.9325	0.9358	0.9391	0.9424	0.9457	0.9490	0.9523	0.9556	0.9590	0.9623
	sin	0.6947	0.6959	0.6972	0.6984	0.6997	0.7009	0.7022	0.7034	0.7046	0.7059
44	cos	0.7193	0.7181	0.7169	0.7157	0.7145	0.7133	0.7120	0.7108	0.7096	0.7083
	tan	0.9657	0.9691	0.9725	0.9759	0.9793	0.9827	0.9861	0.9896	0.9930	0.9965
	sin	0.7071	0.7083	0.7096	0.7108	0.7120	0.7133	0.7145	0.7157	0.7169	0.7181
45	cos	0.7071	0.7059	0.7046	0.7034	0.7022	0.7009	0.6997	0.6984	0.6972	0.6959
	tan	1.0000	1.0035	1.0070	1.0105	1.0141	1.0176	1.0212	1.0247	1.0283	1.0319
	sin	0.7193	0.7206	0.7218	0.7230	0.7242	0.7254	0.7266	0.7278	0.7290	0.7302
46	cos	0.6947	0.6934	0.6921	0.6909	0.6896	0.6884	0.6871	0.6858	0.6845	0.6833
	tan	1.0355	1.0392	1.0428	1.0464	1.0501	1.0538	1.0575	1.0612	1.0649	1.0686
	sin	0.7314	0.7325	0.7337	0.7349	0.7361	0.7373	0.7385	0.7396	0.7408	0.7420
47	cos	0.6820	0.6807	0.6794	0.6782	0.6769	0.6756	0.6743	0.6730	0.6717	0.6704
	tan	1.0724	1.0761	1.0799	1.0837	1.0875	1.0913	1.0951	1.0990	1.1028	1.1067
	sin	0.7431	0.7443	0.7455	0.7466	0.7478	0.7490	0.7501	0.7513	0.7524	0.7536
48	cos	0.6691	0.6678	0.6665	0.6652	0.6639	0.6626	0.6613	0.6600	0.6587	0.6574
	tan	1.1106	1.1145	1.1184	1.1224	1.1263	1.1303	1.1343	1.1383	1.1423	1.1463
	sin	0.7547	0.7559	0.7570	0.7581	0.7593	0.7604	0.7615	0.7627	0.7638	0.7649
49	cos	0.6561	0.6547	0.6534	0.6521	0.6508	0.6494	0.6481	0.6468	0.6455	0.6441
	tan	1.1504	1.1544	1.1585	1.1626	1.1667	1.1708	1.1750	1.1792	1.1833	1.1875
	sin	0.7660	0.7672	0.7683	0.7694	0.7705	0.7716	0.7727	0.7738	0.7749	0.7760
50	cos	0.6428	0.6414	0.6401	0.6388	0.6374	0.6361	0.6347	0.6334	0.6320	0.6307
	tan	1.1918	1.1960	1.2002	1.2045	1.2088	1.2131	1.2174	1.2218	1.2261	1.2305
	sin	0.7771	0.7782	0.7793	0.7804	0.7815	0.7826	0.7837	0.7848	0.7859	0.7869
51	cos	0.6293	0.6280	0.6266	0.6252	0.6239	0.6225	0.6211	0.6198	0.6184	0.6170
	tan	1.2349	1.2393	1.2437	1.2482	1.2527	1.2572	1.2617	1.2662	1.2708	1.2753
	sin	0.7880	0.7891	0.7902	0.7912	0.7923	0.7934	0.7944	0.7955	0.7965	0.7976
52	cos	0.6157	0.6143	0.6129	0.6115	0.6101	0.6088	0.6074	0.6060	0.6046	0.6032
	tan	1.2799	1.2846	1.2892	1.2938	1.2985	1.3032	1.3079	1.3127	1.3175	1.3222
	sin	0.7986	0.7997	0.8007	0.8018	0.8028	0.8039	0.8049	0.8059	0.8070	0.8080
53	cos	0.6018	0.6004	0.5990	0.5976	0.5962	0.5948	0.5934	0.5920	0.5906	0.5892
	tan	1.3270	1.3319	1.3367	1.3416	1.3465	1.3514	1.3564	1.3613	1.3663	1.3713
	sin	0.8090	0.8100	0.8111	0.8121	0.8131	0.8141	0.8151	0.8161	0.8171	0.8181
54	cos	0.5878	0.5864	0.5850	0.5835	0.5821	0.5807	0.5793	0.5779	0.5764	0.5750
	tan	1.3764	1.3814	1.3865	1.3916	1.3968	1.4019	1.4071	1.4124	1.4176	1.4229
	sin	0.8192	0.8202	0.8211	0.8221	0.8231	0.8241	0.8251	0.8261	0.8271	0.8281
55	cos	0.5736	0.5721	0.5707	0.5693	0.5678	0.5664	0.5650	0.5635	0.5621	0.5606
	tan	1.4281	1.4335	1.4388	1.4442	1.4496	1.4550	1.4605	1.4659	1.4715	1.4770
deg	function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°

deg	function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°
56	sin	0.8290	0.8300	0.8310	0.8320	0.8329	0.8339	0.8348	0.8358	0.8368	0.8377
	cos	0.5592	0.5577	0.5563	0.5548	0.5534	0.5519	0.5505	0.5490	0.5476	0.5461
	tan	1.4826	1.4882	1.4938	1.4994	1.5051	1.5108	1.5166	1.5224	1.5282	1.5340
57	sin	0.8387	0.8396	0.8406	0.8415	0.8425	0.8434	0.8443	0.8453	0.8462	0.8471
	cos	0.5446	0.5432	0.5417	0.5402	0.5388	0.5373	0.5358	0.5344	0.5329	0.5314
	tan	1.5399	1.5458	1.5517	1.5577	1.5637	1.5697	1.5757	1.5818	1.5880	1.5941
58	sin	0.8430	0.8490	0.8499	0.8508	0.8517	0.8526	0.8536	0.8545	0.8554	0.8563
	cos	0.5299	0.5284	0.5270	0.5255	0.5240	0.5225	0.5210	0.5195	0.5180	0.5165
	tan	1.6003	1.6066	1.6128	1.6191	1.6255	1.6319	1.6383	1.6447	1.6512	1.6577
59	sin	0.8572	0.8581	0.8590	0.8599	0.8607	0.8616	0.8625	0.8634	0.8643	0.8652
	cos	0.5150	0.5135	0.5120	0.5105	0.5090	0.5075	0.5060	0.5045	0.5030	0.5015
	tan	1.6643	1.6709	1.6775	1.6842	1.6909	1.6977	1.7045	1.7113	1.7182	1.7251
60	sin	0.8660	0.8669	0.8678	0.8686	0.8695	0.8704	0.8712	0.8721	0.8729	0.8738
	cos	0.5000	0.4985	0.4970	0.4955	0.4939	0.4924	0.4909	0.4894	0.4879	0.4863
	tan	1.7321	1.7391	1.7461	1.7532	1.7603	1.7675	1.7747	1.7820	1.7893	1.7966
61	sin	0.8746	0.8755	0.8763	0.8771	0.8780	0.8788	0.8796	0.8805	0.8813	0.8821
	cos	0.4848	0.4833	0.4818	0.4802	0.4787	0.4772	0.4756	0.4741	0.4726	0.4710
	tan	1.8040	1.8115	1.8190	1.8265	1.8341	1.8418	1.8495	1.8572	1.8650	1.8728
62	sin	0.8829	0.8838	0.8846	0.8854	0.8862	0.8870	0.8878	0.8886	0.8894	0.8902
	cos	0.4695	0.4679	0.4664	0.4648	0.4633	0.4617	0.4602	0.4586	0.4571	0.4555
	tan	1.8807	1.8887	1.8967	1.9047	1.9128	1.9210	1.9292	1.9375	1.9458	1.9542
63	sin	0.8910	0.8918	0.8926	0.8934	0.8942	0.8949	0.8957	0.8965	0.8973	0.8980
	cos	0.4540	0.4524	0.4509	0.4493	0.4478	0.4462	0.4446	0.4431	0.4415	0.4399
	tan	1.9626	1.9711	1.9797	1.9883	1.9970	2.0057	2.0145	2.0233	2.0323	2.0413
64	sin	0.8988	0.8996	0.9003	0.9011	0.9018	0.9026	0.9033	0.9041	0.9048	0.9056
	cos	0.4384	0.4368	0.4352	0.4337	0.4321	0.4305	0.4289	0.4274	0.4258	0.4242
	tan	2.0503	2.0594	2.0686	2.0778	2.0872	2.0965	2.1060	2.1155	2.1251	2.1348
65	sin	0.9063	0.9070	0.9078	0.9085	0.9092	0.9100	0.9107	0.9114	0.9121	0.9128
	cos	0.4226	0.4210	0.4195	0.4179	0.4163	0.4147	0.4131	0.4115	0.4099	0.4083
	tan	2.1445	2.1543	2.1642	2.1742	2.1842	2.1943	2.2045	2.2148	2.2251	2.2355
66	sin	0.9135	0.9143	0.9150	0.9157	0.9164	0.9171	0.9178	0.9184	0.9191	0.9198
	cos	0.4067	0.4051	0.4035	0.4019	0.4003	0.3987	0.3971	0.3955	0.3939	0.3923
	tan	2.2460	2.2566	2.2673	2.2781	2.2889	2.2998	2.3109	2.3220	2.3332	2.3445
67	sin	0.9205	0.9212	0.9219	0.9225	0.9232	0.9239	0.9245	0.9252	0.9259	0.9265
	cos	0.3907	0.3891	0.3875	0.3859	0.3843	0.3827	0.3811	0.3795	0.3778	0.3762
	tan	2.3559	2.3673	2.3789	2.3906	2.4023	2.4142	2.4262	2.4383	2.4504	2.4627
68	sin	0.9272	0.9278	0.9285	0.9291	0.9298	0.9304	0.9311	0.9317	0.9323	0.9330
	cos	0.3746	0.3730	0.3714	0.3697	0.3681	0.3665	0.3649	0.3633	0.3616	0.3600
	tan	2.4751	2.4876	2.5002	2.5129	2.5257	2.5386	2.5517	2.5649	2.5782	2.5916
69	sin	0.9336	0.9342	0.9348	0.9354	0.9361	0.9367	0.9373	0.9379	0.9385	0.9391
	cos	0.3584	0.3567	0.3551	0.3535	0.3518	0.3502	0.3486	0.3469	0.3453	0.3437
	tan	2.6051	2.6187	2.6325	2.6464	2.6605	2.6740	2.6889	2.7034	2.7179	2.7326
deg	function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°

deg	Function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°
70	sin	0.9397	0.9403	0.9409	0.9415	0.9421	0.9426	0.9432	0.9438	0.9444	0.9449
	cos	0.3420	0.3404	0.3387	0.3371	0.3355	0.3338	0.3322	0.3305	0.3289	0.3272
	tan	2.7475	2.7625	2.7776	2.7929	2.8083	2.8239	2.8397	2.8556	2.8716	2.8878
71	sin	0.9455	0.9461	0.9466	0.9472	0.9478	0.9483	0.9489	0.9494	0.9500	0.9505
	cos	0.3256	0.3239	0.3223	0.3206	0.3190	0.3173	0.3156	0.3140	0.3123	0.3107
	tan	2.9042	2.9208	2.9375	2.9544	2.9714	2.9887	3.0061	3.0237	3.0415	3.0595
72	sin	0.9511	0.9516	0.9521	0.9527	0.9532	0.9537	0.9542	0.9548	0.9553	0.9558
	cos	0.3090	0.3074	0.3057	0.3040	0.3024	0.3007	0.2990	0.2974	0.2957	0.2940
	tan	3.0777	3.0961	3.1146	3.1334	3.1524	3.1716	3.1910	3.2106	3.2305	3.2506
73	sin	0.9563	0.9568	0.9573	0.9578	0.9583	0.9588	0.9593	0.9598	0.9603	0.9608
	cos	0.2924	0.2907	2.2890	0.2874	0.2857	0.2840	0.2823	0.2807	0.2790	0.2773
	tan	3.2709	3.2914	3.3122	3.3332	3.3544	3.3759	3.3977	3.4197	3.4420	3.4646
74	sin	0.9613	0.9617	0.9622	0.9627	0.9632	0.9636	0.9641	0.9646	0.9650	0.9655
	cos	0.2756	0.2740	0.2723	0.2706	0.2689	0.2672	0.2656	0.2639	0.2622	0.2605
	tan	3.4874	3.5105	3.5339	3.5576	3.5816	3.6059	3.6305	3.6554	3.6806	3.7062
75	sin	0.9659	0.9664	0.9668	0.9673	0.9677	0.9681	0.9686	0.9690	0.9694	0.9699
	cos	0.2588	0.2571	0.2554	0.2538	0.2521	0.2504	0.2487	0.2470	0.2453	0.2436
	tan	3.7321	3.7583	3.7848	3.8118	3.8391	3.8667	3.8947	3.9232	3.9520	3.9812
76	sin	0.9703	0.9707	0.9711	0.9715	0.9720	0.9724	0.9728	0.9732	0.9736	0.9740
	cos	0.2419	0.2402	0.2385	0.2368	0.2351	0.2334	0.2317	0.2300	0.2284	0.2267
	tan	4.0108	4.0408	4.0713	4.1022	4.1335	4.1653	4.1976	4.2303	4.2635	4.2972
77	sin	0.9744	0.9748	0.9751	0.9755	0.9759	0.9763	0.9767	0.9770	0.9774	0.9778
	cos	0.2250	0.2232	0.2215	0.2198	0.2181	0.2164	0.2147	0.2130	0.2113	0.2096
	tan	4.3315	4.3662	4.4015	4.4374	4.4737	4.5107	4.5483	4.5864	4.6252	4.6646
78	sin	0.9781	0.9785	0.9789	0.9792	0.9796	0.9799	0.9803	0.9806	0.9810	0.9813
	cos	0.2079	0.2062	0.2045	0.2028	0.2011	0.1994	0.1977	0.1959	0.1942	0.1925
	tan	4.7046	4.7453	4.7867	4.8288	4.8716	4.9152	4.9594	5.0045	5.0504	5.0970
79	sin	0.9816	0.9820	0.9823	0.9826	0.9829	0.9833	0.9836	0.9839	0.9842	0.9845
	cos	0.1908	0.1891	0.1874	0.1857	0.1840	0.1822	0.1805	0.1788	0.1771	0.1754
	tan	5.1446	5.1929	5.2422	5.2924	5.3435	5.3955	5.4486	5.5026	5.5578	5.6140
80	sin	0.9848	0.9851	0.9854	0.9857	0.9860	0.9863	0.9866	0.9869	0.9871	0.9874
	cos	0.1736	0.1719	0.1702	0.1685	0.1668	0.1650	0.1633	0.1616	0.1599	0.1582
	tan	5.6713	5.7297	5.7894	5.8502	5.9124	5.9758	6.0405	6.1066	6.1742	6.2432
81	sin	0.9877	0.9880	0.9882	0.9885	0.9888	0.9890	0.9893	0.9895	0.9898	0.9900
	cos	0.1564	1.1547	0.1530	0.1513	0.1495	0.1478	0.1461	0.1444	0.1426	0.1409
	tan	6.3138	6.3859	6.4596	6.5350	6.6122	6.6912	6.7720	6.8548	6.9395	7.0264
82	sin	0.9903	0.9905	0.9907	0.9910	0.9912	0.9914	0.9917	0.9919	0.9921	0.9923
	cos	0.1392	0.1374	0.1357	0.1340	0.1323	0.1305	0.1288	0.1271	0.1253	0.1236
	tan	7.1154	7.2066	7.3002	7.3962	7.4947	7.5958	7.6996	7.8062	7.9158	8.0285
83	sin	0.9925	0.9928	0.9930	0.9932	0.9934	0.9936	0.9938	0.9940	0.9942	0.9943
	cos	0.1219	0.1201	0.1184	0.1167	0.1149	0.1132	0.1115	0.1097	0.1080	0.1063
	tan	8.1443	8.2636	8.3863	8.5126	8.6427	8.7769	8.9152	9.0579	9.2052	9.3572
deg	Function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°

deg	function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°
84	sin	0.9945	0.9947	0.9949	0.9951	0.9952	0.9954	0.9956	0.9957	0.9959	0.9960
	cos	0.1045	0.1028	0.1011	0.0993	0.0976	0.0958	0.0941	0.0924	0.0906	0.0889
	tan	9.5144	9.6768	9.8448	10.02	10.20	10.39	10.58	10.78	10.99	11.20
85	sin	0.9962	0.9963	0.9965	0.9966	0.9968	0.9969	0.9971	0.9972	0.9973	0.9974
	cos	0.0872	0.0854	0.0837	0.0819	0.0802	0.0785	0.0767	0.0750	0.0732	0.0715
	tan	11.43	11.66	11.91	12.16	12.43	12.71	13.00	13.30	13.62	13.95
86	sin	0.9976	0.9977	0.9978	0.9979	0.9980	0.9981	0.9982	0.9983	0.9984	0.9985
	cos	0.0698	0.0680	0.0663	0.0645	0.0628	0.0610	0.0593	0.0576	0.0558	0.0541
	tan	14.30	14.67	15.06	15.46	15.89	16.35	16.83	17.34	17.89	18.46
87	sin	0.9986	0.9987	0.9988	0.9989	0.9990	0.9990	0.9991	0.9992	0.9993	0.9993
	cos	0.0523	0.0506	0.0488	0.0471	0.0454	0.0436	0.0419	0.0401	0.0384	0.0366
	tan	19.08	19.74	20.45	21.20	22.02	22.90	23.86	24.90	26.03	27.27
88	sin	0.9994	0.9995	0.9995	0.9996	0.9996	0.9997	0.9997	0.9997	0.9998	0.9998
	cos	0.0349	0.0332	0.0314	0.0297	0.0279	0.0262	0.0244	0.0227	0.0209	0.0192
	tan	28.64	30.14	31.82	33.69	35.80	38.19	40.92	44.07	47.74	52.08
89	sin	0.9998	0.9999	0.9999	0.9999	0.9999	1.000	1.000	1.000	1.000	1.000
	cos	0.0175	0.0157	0.0140	0.0122	0.0105	0.0087	0.0070	0.0052	0.0035	0.0017
	tan	57.29	63.66	71.62	81.85	95.49	114.6	143.2	191.0	286.5	573.0
deg	function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°