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ABSTRACT

The module covers series circuits which contain both resistive and reactive components and methods of solving these circuits for current, voltage, impedance, and phase angle. The module is divided into six lessons: voltage and impedance in AC (alternating current) series circuits, vector computations, rectangular and polar notation, variational analysis of series RL (resistive-inductive) circuits, frequency discrimination in RL circuits, and series RC (resistive capacitive) circuits. Each lesson consists of an overview, a list of study resources, lesson narratives, programed instructional materials, and lesson summaries. (Author/BP)

NAVPERS 94558-12a

BASIC ELECTRICITY AND ELECTRONICS

INDIVIDUALIZED LEARNING SYSTEM

MODULE TWELVE

SERIES AC RESISTIVE-REACTIVE CIRCUITS

Study Booklet

BUREAU OF NAVAL PERSONNEL

January 1972

O V E R V I E W.
MODULE TWELVE
SERIES AC RESISTIVE-REACTIVE CIRCUITS

In this module you will learn about series circuits which contain both resistive and reactive components. You will discover methods of solving these circuits for current, voltage, impedance and phase angle. You will also learn some of the uses of this type circuit.

For you to more easily learn the above, this module has been divided into the following six lessons.

TURN TO THE FOLLOWING PAGE AND BEGIN LESSON I.

NAVPERS 94558-12a

BASIC ELECTRICITY AND ELECTRONICS

INDIVIDUALIZED LEARNING SYSTEM

MODULE TWELVE

LESSON I

Voltage and Impedance in AC Series Circuits

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OVERVIEW

LESSON I

Voltage and Impedance in AC Series Circuits

In this lesson you will study and learn about the following:

-voltage drops in AC Series Circuits

-Ohm's Law for AC Series Circuits

BEFORE YOU START THIS LESSON, PREVIEW THE LIST OF STUDY RESOURCES ON THE NEXT PAGE.

LIST OF STUDY RESOURCES
LESSON I

Impedance

To learn the material in this lesson, you have the option of choosing, according to your experience and preferences, any or all of the following:

STUDY BOOKLET:

Lesson Narrative
Programmed Instruction
Lesson Summary

ENRICHMENT MATERIAL:

NAVPERS 93400A-1b "Basic Electricity, Alternating Current." Fundamentals of Electronics. Bureau of Naval Personnel. Washington, D.C.: U.S. Government Printing Office, 1965.

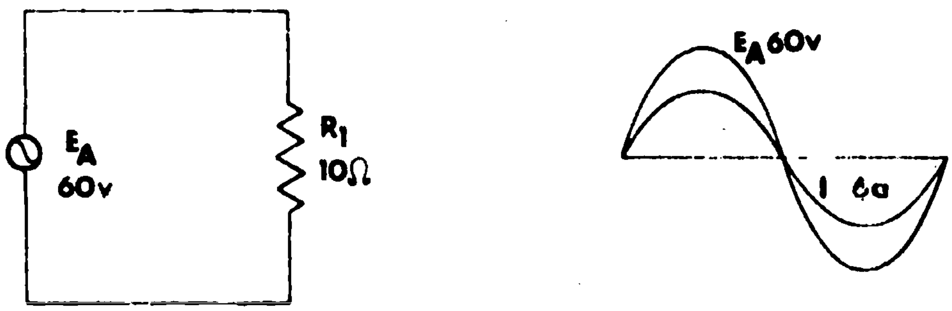
YOU MAY NOW STUDY ANY OR ALL OF THE RESOURCES LISTED ABOVE. YOU MAY TAKE THE PROGRESS CHECK AT ANY TIME.

NARRATIVE
LESSON I

Voltage and Impedance in AC Series Circuits

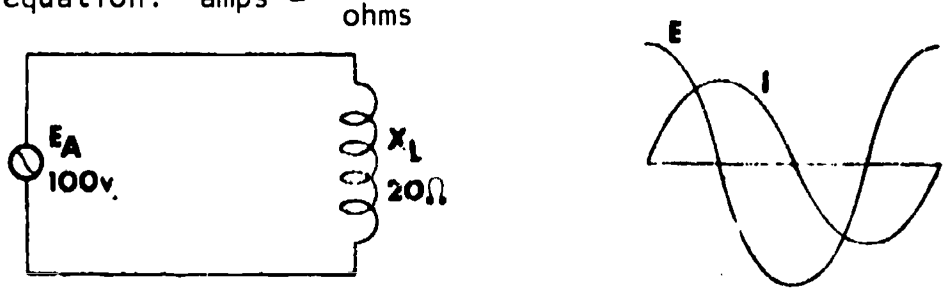
You have learned how R, L, and C individually affect AC current flow, phase angle, and power in AC circuits. You will now discover how the various combinations of R, L, and C affect series circuits.

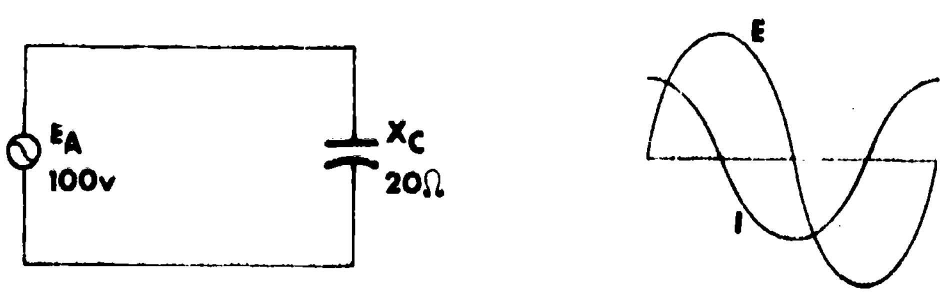
All electrical circuits contain some amounts of resistance (R), inductance (L), and capacitance (C). Inductance and capacitance cause inductive and capacitive reactance (X_L, X_C) which, like resistance, opposes the flow of AC current. Each of these oppositions will develop a voltage drop. The combination of these voltage drops will, according to Kirchhoff's Voltage Law, equal the applied voltage. If two of the factors (L, C, or R) are negligible the circuit can be considered pure or ideal. For example, a purely resistive circuit may be represented as follows:



Computations in this type of circuit are not difficult. Ohm's Law can be applied to find I, the power formula I^2R or I x E can be used to find P_T, and as you can see from the illustration, applied voltage and circuit current are in phase and the circuit phase angle ($\angle\theta$) is 0°.

If the circuit is purely reactive (capacitive or inductive), circuit values can still be found in the same manner. Remember, anything measured in ohms (R, X_C or X_L) can be used in the Ohm's Law equation: $\text{amps} = \frac{\text{volts}}{\text{ohms}}$

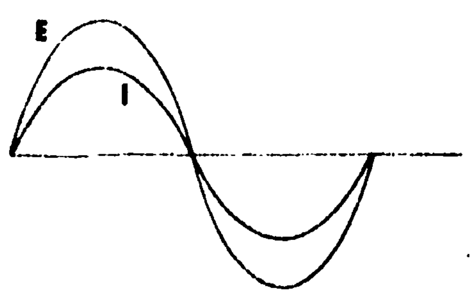




There are, of course, some significant differences between these reactive circuits and the resistive circuit previously explained. The most notable difference between resistive and reactive circuits is the circuit phase angle between applied voltage (E) and circuit current (I). In the inductive circuit, as you can see from the illustration, voltage leads current by 90° (ELI). In the capacitive circuit the reverse is true; current leads voltage by 90° (ICE).

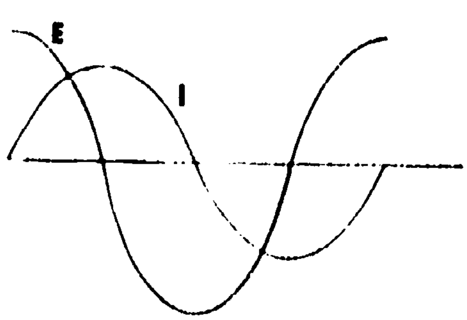
Recall that vectors can be used to represent the magnitude and phase of voltage and current. For example, these are vector diagrams for the three circuits you've just seen:

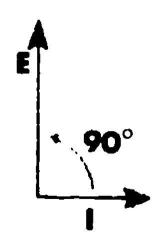
Purely Resistive Circuit:



Notice that voltage and current are both plotted on the same line because they are in phase.

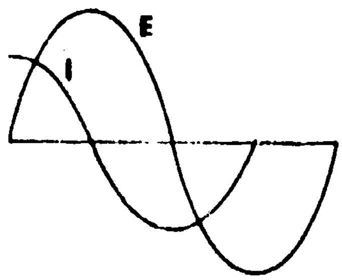
Purely Inductive Circuit:

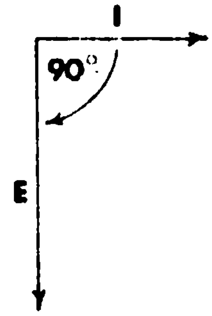




Current is plotted on the reference line and voltage is plotted 90° ahead, showing that E leads I by 90°.

Purely Capacitive Circuit:

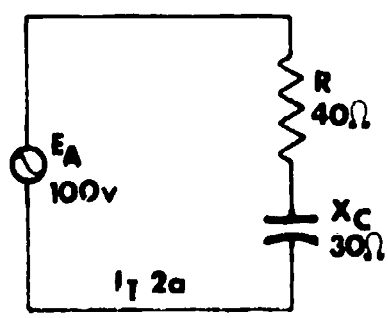


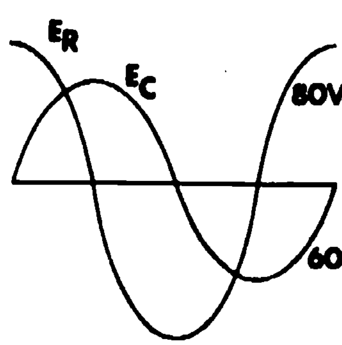


Current is once again plotted on the reference line, but voltage is now shown 90° behind indicating that I leads E by 90°.

In each of the preceding examples, current is used as the reference in the vector diagrams. Since there is only one path for current in a series circuit the current in every component is in phase with the current in every other component. This makes current a common reference for all other values in a series circuit.

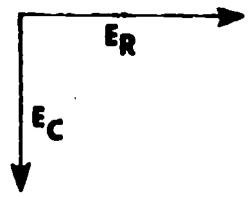
In a resistive-reactive AC circuit the voltage drops which occur across the resistance and the reactance are not in phase, that is, they do not occur to the same extent at the same instant of time. Why? You can readily see this if you will recall that the voltage across either a capacitor or an inductor is out of phase with the current through them. Then, if the current through the resistor and the reactive component is the same current and therefore in phase, then the voltage drops across the two components must be out of phase. For example, in the circuit shown, the voltage across the resistor reaches its maximum value 90° before the voltage across the capacitor reaches its maximum value.





Since E_C lags circuit current by 90° and E_R is in phase with current, E_C will lag E_R by 90°.

The vector diagram which represents this relationship is drawn like this:



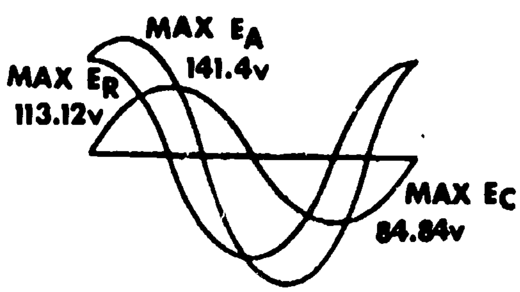
E_R is plotted on the reference line because it is in phase with circuit current. E_C is plotted 90° behind E_R because it lags current by 90°.

By applying Ohm's Law to this circuit we can determine the values of E_C and E_R.

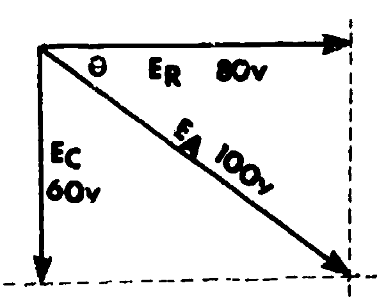
$$E_C = I \times X_C \qquad\qquad E_R = I \times R$$

$$2 \times 30 = 60\ v \qquad\qquad 2 \times 40 = 80\ v$$

If you are at all observant you have noticed that the sum of the two voltage drops apparently does not equal the applied voltage. This is true if you consider the effective values of E_C and E_R (60 v + 80 v); however, the instantaneous values of E_C and E_R can be added directly and they will equal the applied voltage at a given instant of time. This is shown in the following illustration.

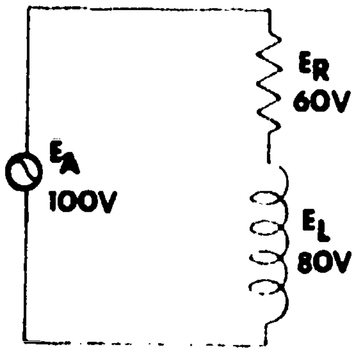


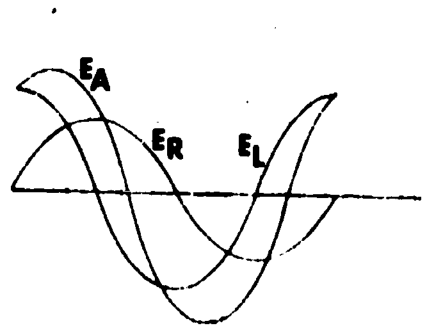
The vector diagram representing this circuit is:

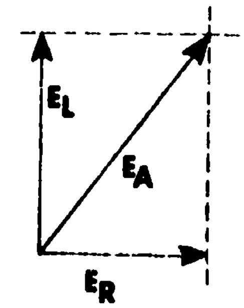


Once again notice the positions of E_R and E_C. The angle labeled $\underline{/\theta}$ (angle theta) represents the circuit phase angle which is the number of degrees by which the applied voltage leads or lags (in this case lags) the circuit current.

If the circuit were resistive-inductive (RL) instead of resistive-capacitive (RC) the 90° phase shift between the individual voltage drops would still exist but would be a leading instead of a lagging angle because voltage leads current in a inductive circuit.



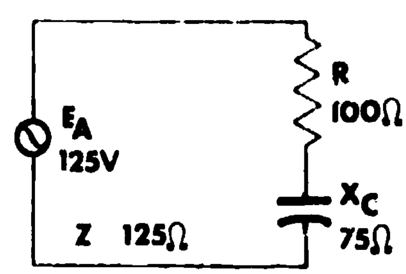




Impedance

In a resistive-reactive AC circuit the total opposition can no longer be considered strictly as resistance or as reactance but must be thought of as a combination of X_L, X_C and R. For this reason a new term is introduced, impedance. Impedance is defined as the total opposition a circuit offers to alternating current. The letter designation for impedance is Z. Z represents an opposition to current and, just like X_C, X_L and R, impedance is measured in ohms.

Solve:



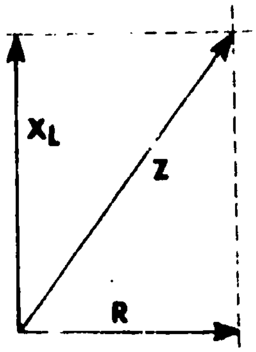
I_T =

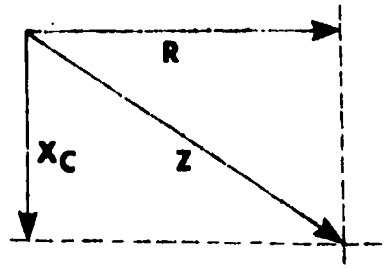
E_C =

E_R =

Answer: I_T = 1a; E_C = 75 v; E_R = 100 v

As you can see from the circuit above, impedance is not equal to the direct sum of the oppositions within the circuit, first as the applied voltage is not equal to the direct sum of the voltage drops. Since the voltage drop across each component in a series circuit is in direct proportion to the imepdance of that component, the individual values of opposition (X_C, X_L, and R) may be vectored to obtain total impedance just as the voltage drops are vectored to obtain total voltage. The vector diagrams representing the impedance of an RL and an RC circuit are drawn as show at the top of the next page.





Note that X_L and X_C are plotted in the same relationship to R as E_L and E_C are to E_R. The mathematical processes you will learn in following lessons for combining out of phase values will apply equally well to solving for E_a or for Z.

AT THIS POINT, YOU MAY TAKE THE PROGRESS CHECK, OR YOU MAY STUDY ANY OF THE OTHER RESOURCES LISTED. IF YOU TAKE THE PROGRESS CHECK AND ANSWER ALL OF THE QUESTIONS CORRECTLY, GO TO THE NEXT LESSON. IF NOT, STUDY ANY METHOD OF INSTRUCTION YOU WISH UNTIL YOU CAN ANSWER ALL THE QUESTIONS CORRECTLY.

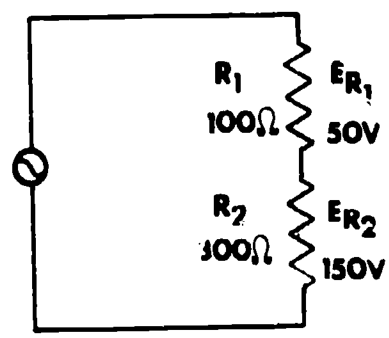
PROGRAMMED INSTRUCTION
LESSON I

Voltage and Impedance in AC Series Circuits

THIS PROGRAMMED SEQUENCE HAS NO TEST FRAMES.

1. All circuits contain some amount of resistance, inductance and capacitance. If two of these factors are negligible the circuit may be considered pure and is solved by applying procedures you've already learned.

Solve:



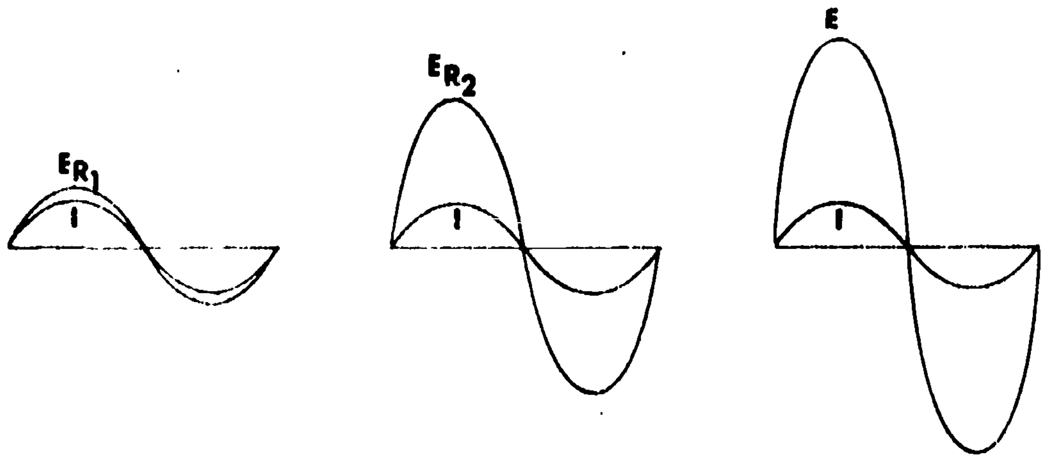
E_a = ________

I = ________

R_T = ________

(200 v; 0.5 a; 400 Ω)

2. In a purely resistive circuit, such as the one in frame 1, the applied voltage is in phase with the circuit current which is in phase with the individual voltage drops E_{R1} and E_{R2}.



In a resistive circuit the applied voltage is in phase with the circuit current. This implies a circuit phase angle of ____________ degrees.

---

(0)

---

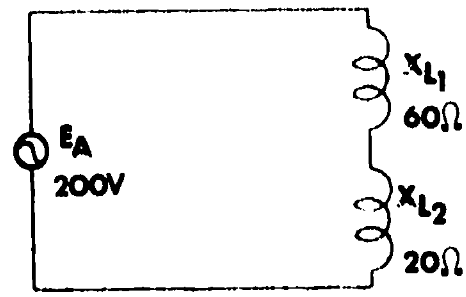
3. Computations in a ideal resistive circuit are quite simple. Ohm's Law is applied to find current. R_T is equal to the ______ of the resistances and according to Kirchhoff's Voltage Law E_a is equal to the __________ of the voltage drops.

(sum; sum)

---

4. If the circuit is purely reactive (capacitive or inductive) circuit values can be found in the same manner as in a purely resistive circuit, using Ohm's Law and Kirchhoff's Voltage Law.

Solve.

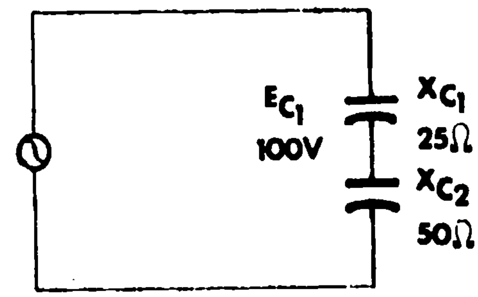


X_{LT} = ______

I = ______

E_{L1} = ______

E_{L2} = ______



X_{CT} = ______

I = ______

E_{C2} = ______

E_a = ______

- -

(80 Ω; 2.5 a; 150 v; 50 v; 75 Ω; 4 a; 200 v; 300 v)

5. Although most circuit computations are carried out the same way in both purely reactive and purely resistive circuits there are some significant circuit differences, the primary one being the difference in phase between applied voltage and circuit current.

The difference in phase between voltage and current in a purely reactive circuit is:

_____ a. 0°
_____ b. 45°
_____ c. 90°
_____ d. 180°

- -

(c) 90°

6. Vectors are often used to represent the magnitude and phase relationship of voltage and current in a circuit. When dealing with relative quantities, one must be chosen as a reference to which all other quantities may be compared.

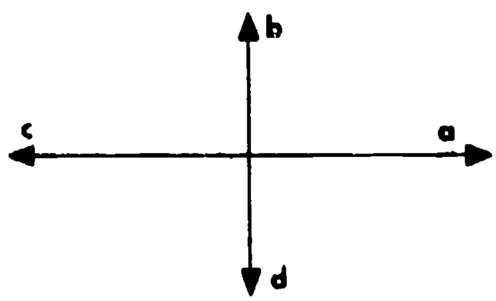
   The ideal reference is a quantity which is common to all parts of a circuit. For a series circuit the reference value is:

   ____ a. voltage
   ____ b. current
   ____ c. resistance
   ____ d. reactance

(b) current

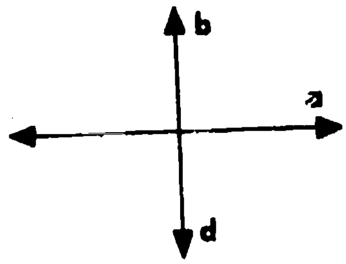
---

7. In which position of the vector diagram is the reference value (current) placed.



(a) [If you were unable to answer this question review the lesson on vectors in Module Nine.]

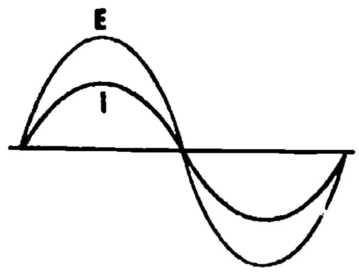
8. On which axis would a value be placed to indicate it was 90° ahead of (leading) the reference? __________
90° behind (lagging)? __________



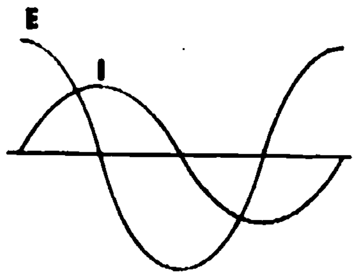
(b; d) [If you were unable to answer this question, review the lesson on vectors in Module Nine.]

9. Below are the voltage and current sine waves for three circuits. Beside each, draw the vector representation using current as the reference.

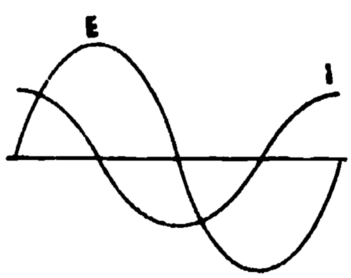
a.



b.

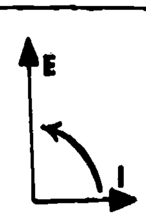


c.

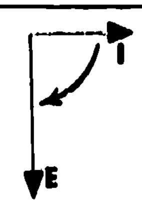


(a.

b.



c.



)

10. When vectors are used to describe a series circuit, __________ is used as the reference because it is the ________ circuit value.

- -

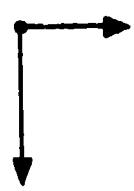
(current; common)

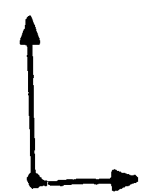
11. When resistance and reactance are placed in series, with an AC voltage applied, the voltage drops which occur across the individual components are 90° out of phase.

Which vector diagram(s) could possibly be used to represent the phase difference between resistive and reactive voltage drops.

___ a. 

___ c. 

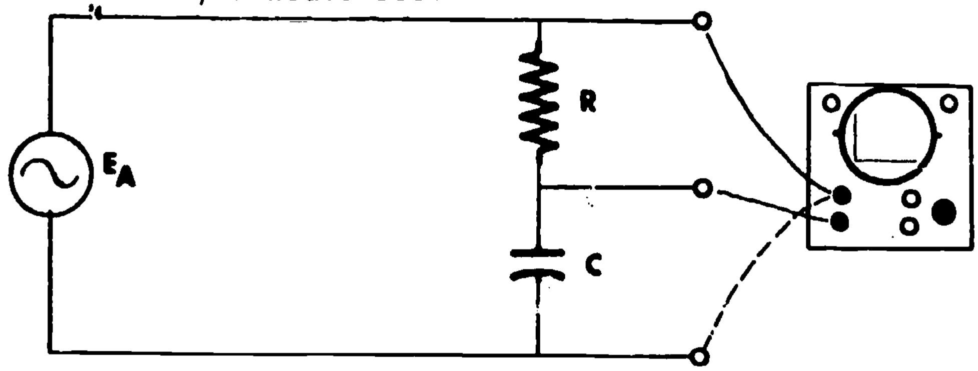
___ b. 

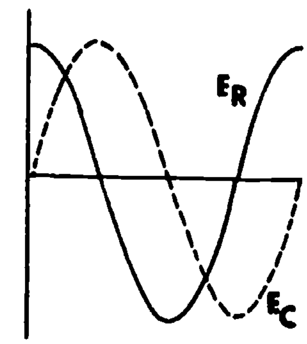
___ d. 

- -

(b; d)

12. If you were to view the voltage drops across a capacitor and a resistor at the same time, with an oscilloscope, this is what you would see.

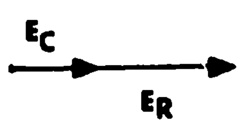




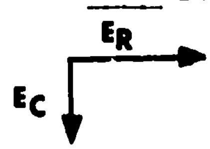
Notice that E_R is at its maximum value while E_C is at its minimum. 90° later E_C reaches its maximum value and E_R has decreased to its minimum value.

Which of these vector diagrams correctly represents the phase relationship between E_C and E_R?

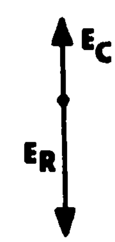
____ a.



____ b.



____ c.



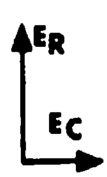
- -

(b)

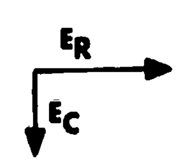
13. As you can see, E_R reaches its maximum value 90° before E_C. The proper way of expressing this for a series circuit is E_C lags E_R by 90°. Current is the reference point and E_R is in phase with current.

Both of these diagrams show a 90° phase shift with E_R ahead. Which one is correct if current is the reference?

____ a.



____ b.



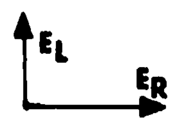
- -

(b)

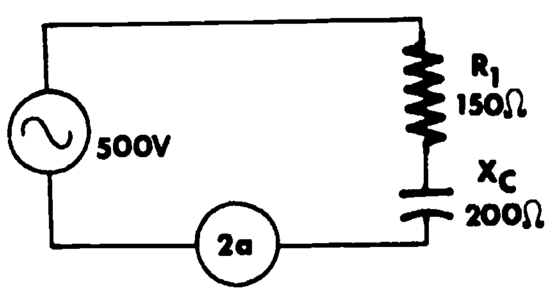
14. A series circuit containing a coil and a resistor also has a 90° phase difference between the voltage drops. The voltage drop (E_L) across the inductor leads the voltage drop across the resistor.

Draw the vector diagram representative of a resistive-inductive circuit with current as the reference:

- -



15. Solve for:



E_R = ________

E_C = ________

- -

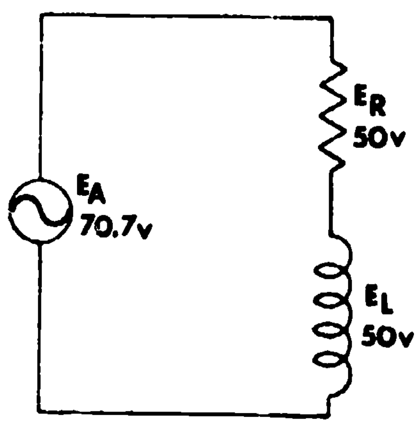
(300 v; 400 v)

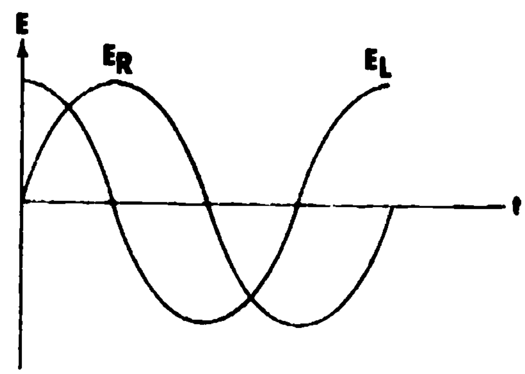
16. Unless you are completely unobservant you noticed that the sum of E_R and E_C (300 v + 400 v) is greater than the applied voltage (500 v). This apparent contradiction of Kirchhoff's Voltage Law can be explained by the 90° phase difference between E_R and E_C. By observation you can see that the combination of out of phase voltages such as E_C and E_R will be ____________ (less/greater) than either individual value and ____________ (less/greater) than the direct sum of the two.

(greater; less)

---

17. The values given on the schematic do not reach maximum at the same time. In other words, E_L will reach its maximum 90° before E_R; when E_R is maximum, E_L will have decreased to 0 v.



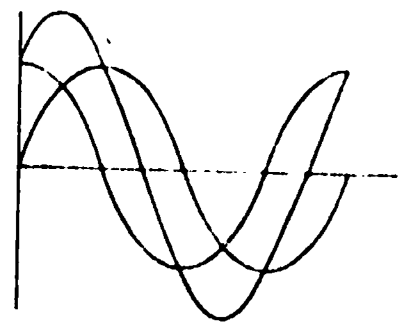


What will E_R be when E_L has increased to maximum ____________.

(0 v)

18. E_R and E_L, when properly combined (not directly added), will equal source voltage. The total effective voltage across R and L taken together will be greater than the effective voltage across either R or L, but it will be less than the direct sum of the two effective voltages.

Label E_L, E_R, E_a



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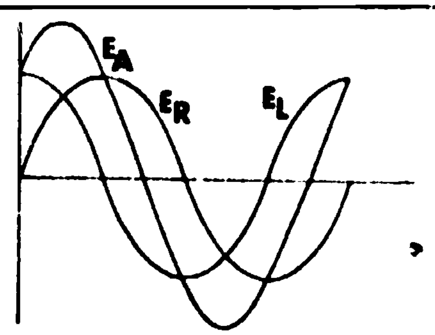
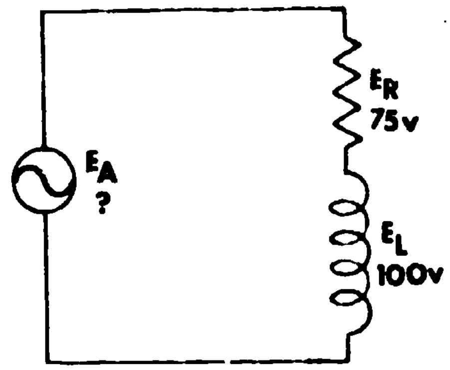


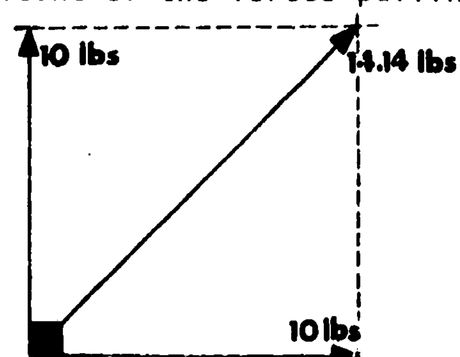
19. Kirchhoff's Voltage Law is true for all circuits <u>if properly applied</u>. for example an RL circuit:



From previous frames you should recognize that E_a is not 175 volts. E_a must be greater than the largest voltage drop but less than the direct sum of the two. At any instant of time, however, the instantaneous value of E_R added to the instantaneous value of E_L will equal E_a. The values normally given are effective values, not instantaneous. Some system for combining these out of phase effective values is required unless we want to change them to instantaneous values for direct addition.

(GO TO NEXT FRAME.)

20. The combination of out of phase voltage drops is usually accomplished by vector addition, this process is comparable to finding the resultant of two forces pulling on an object at an angle of 90°

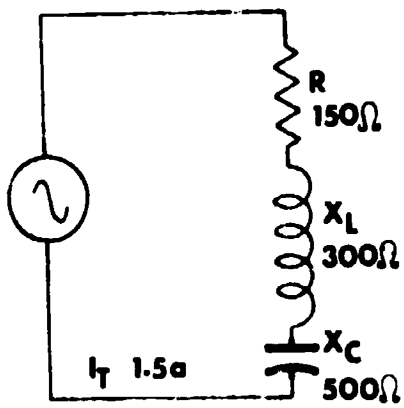


As you can see, the resultant force acting on the object will not be in the direction either force is pulling, but will be somewhere between the two. The strength of the resultant force will be greater than either of the pulling forces but __________ than the direct sum of the two.

(less)

21. The voltage drops which occur in any circuit are caused by the oppositions to current within that circuit. Each of these oppositions (X_C, X_L, R) are measured in ohms and each can be used in the Ohm's Law equation amp = $\frac{\text{volts}}{\text{ohms}}$.

Solve:



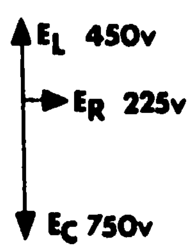
E_R = ________

E_L = ________

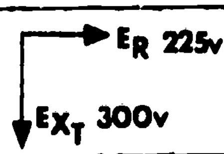
E_C = ________

(225 v; 450 v; 750 v)

22. Each voltage drop is out of phase with the other two as shown by this vector diagram.



Since this is a series circuit, current is the reference and E_R is plotted in the reference position. E_L and E_C are then plotted leading and lagging E_R. As you can see E_L and E_C are 180° out of phase (directly opposing). They tend to cancel one another, eliminating the effects of one of the reactive components. Draw the voltage vector diagram for the above sample which results from the canceling reactive voltage drops.



23. The phase differences between resistive and reactive voltage drops in an AC circuit are caused by the different types of opposition offered by the reactances (X_L and X_C) and the resistance (R).

Each type of opposition is different because it opposes current flow for different reasons and is affected by different factors.

Match the determining factors to the value:

____ 1. X_C	a.	frequency
____ 2. X_L	b.	capacitance
____ 3. R	c.	inductance
	d.	resistance

(1. a and b; 2. a and c; 3. d)

24. The total opposition in a resistive-reactive circuit can be considered neither pure resistance nor pure reactance, but must be thought of as a combination of the two.

Which factors will affect total opposition in an AC circuit:

____ a. total current
____ b. frequency
____ c. applied voltage
____ d. capacitance
____ e. resistance
____ f. inductance

(b. frequency; d. capacitance; e. resistance; f. inductance)

25. Total opposition in an AC circuit is referred to as impedance
Impedance represents a proper combination of:

_______________ and _______________

(resistance; reactance)

26. Total opposition in an AC circuit is called:

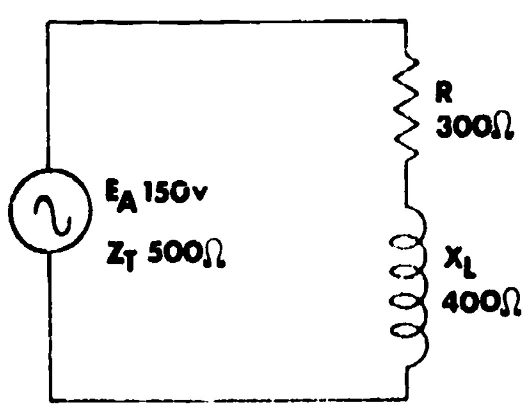
____ a. resistance.
____ b. capacitance.
____ c. inductance.
____ d. impedance.

(d) impedance

27. Impedance is measured in ohms and is designated by the letter Z.

Ohm's Law can be written as $I = \frac{E}{Z}$

What is I_T? _______________



(300 ma)

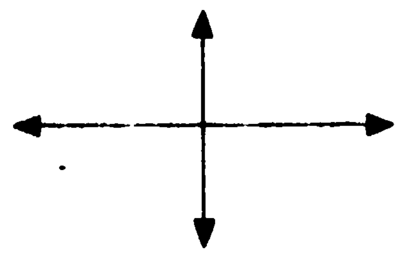
28. Thus, the total opposition in any AC circuit can be referred to as impedance (Z). If the circuit is purely inductive, capacitive or resistive, Z is equal to X_L, X_C, or R respectively. Should the circuit be resistive and reactive, the impedance is a special combination of R and X. Regardless of the type of circuit you are dealing with, ____________ is the correct term to use when referring to total opposition to alternating current.

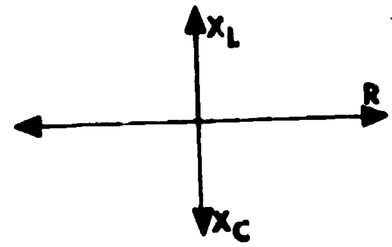
(impedance)

29. Since the voltage drop across each component is directly proportional to the opposition of that component, the individual oppositions may be vectored to obtain total impedance.

The impedance vector diagram for a series circuit is identical to the voltage vector diagram.

Indicate the placement of X_L, R, and X_C on the vector diagram.





IF ANY OF YOUR ANSWERS IS INCORRECT, GO BACK AND TAKE THE PROGRAMMED SEQUENCE AGAIN.

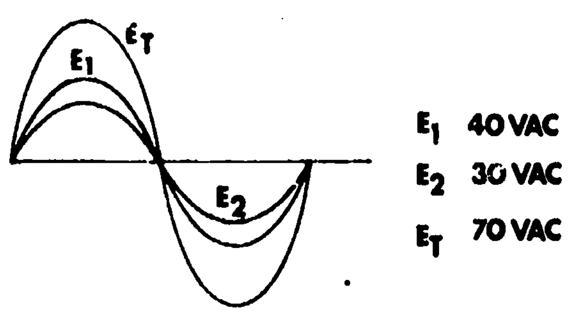
IF YOUR ANSWERS ARE CORRECT, YOU MAY TAKE THE PROGRESS CHECK, OR YOU MAY STUDY ANY OF THE OTHER RESOURCES LISTED. IF YOU TAKE THE PROGRESS CHECK AND ANSWER ALL THE QUESTIONS CORRECTLY, GO TO THE NEXT LESSON. IF NOT, STUDY ANY METHOD OF INSTRUCTION YOU WISH UNTIL YOU CAN ANSWER ALL THE QUESTIONS CORRECTLY.

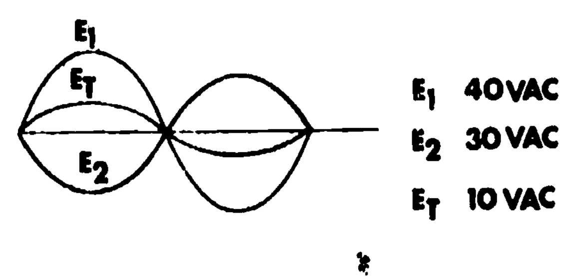
SUMMARY
LESSON 1

Voltage and Impedance in AC Series Circuits

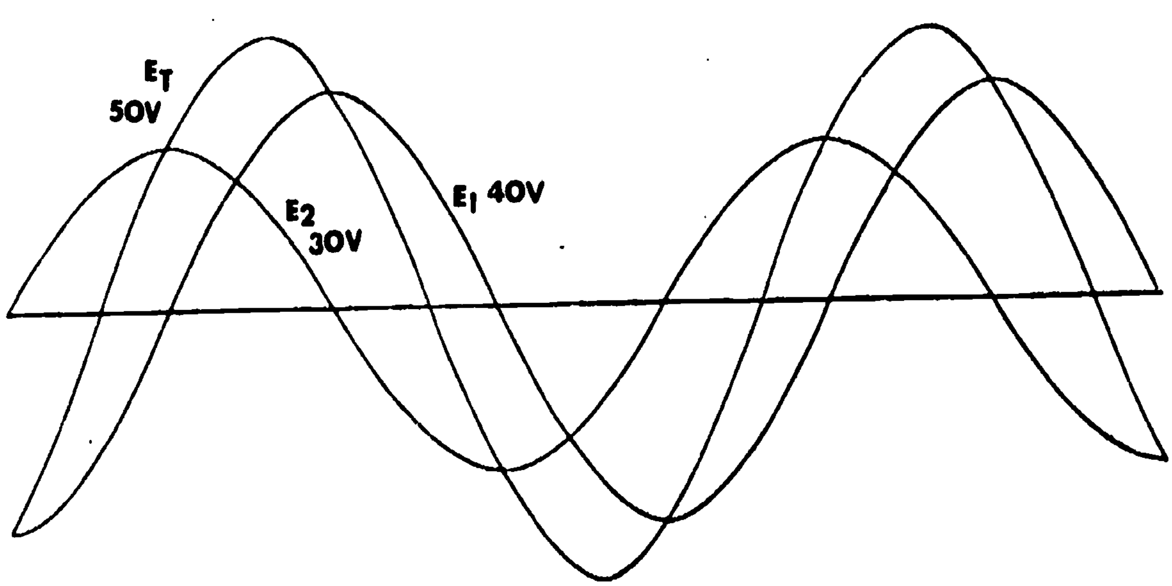
In Module Two, you learned that cells can be connected in series so that their voltages either aid or oppose each other. The total voltage from these series-connected cells is equal to the difference in voltages when the cells oppose and equal to the sum of the voltages when they aid.

AC voltages in series act in the same way, but the instantaneous values must be added together algebraically to find the total voltage. The output of two alternators running at exactly the same frequency and connected in series will depend on the phase relationship of the voltages. If the alternators are operating in phase, their combined output is equal to the sum of their voltages; 180° out of phase, the total output is the difference in their voltages. Waveform A below shows two voltages in phase and the total or resultant voltage. Waveforms B shows two voltages 180° out of phase.

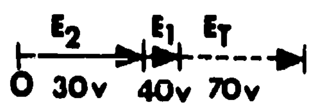




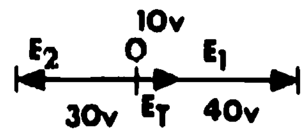
The resultant voltage shown above can be found by arithmetic addition, or subtraction, but voltages with any other phase relationship are more complicated to work with. Here two voltages 90° out of phase are added together using instantaneous values.



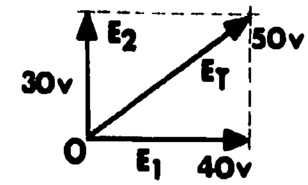
Finding the total voltage this way is slow, difficult, and usually inaccurate. A far simpler way to determine the resultant is to use vectors to represent the voltages. Using E_1 as the reference vector in each of the three cases at the bottom the last page yields these results:



IN PHASE



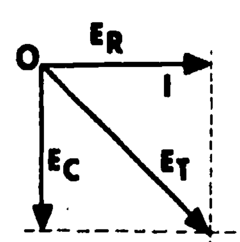
180° OUT OF PHASE



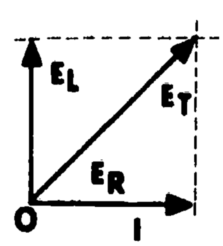
90° OUT OF PHASE

In a series AC circuit, the voltage drops across the loads must also be added vectorially. The voltages across two resistors in series are in phase, the voltages across two inductors are in phase and the voltages across two capacitors are in phase. Because of the phase shift between current and voltage in an inductor or capacitor, the voltages across a resistor and a capacitor (or inductor) connected in series will be out of phase.

Current is the same throughout a series circuit, so the current through any component is in phase with the current through any other components in series. For this reason, current is always used as the reference in the vector analysis of a series AC circuit. A vector diagram of the voltage drops across a capacitor and resistor in series looks like this:



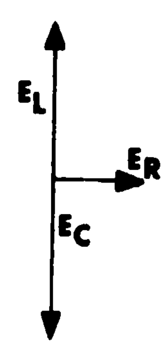
Remember ELI the ICE man, current through the capacitor leads the voltage drop across the capacitor and the voltage drop across a resistor is always in phase with the current through it. The vector diagram for voltages in an AC circuit containing both inductance and resistance would be:

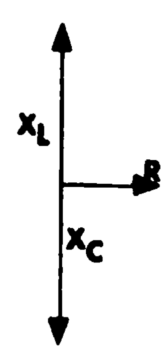


The angle between the voltage and current in a circuit is called the phase angle of the circuit and is represented by the Greek letter θ. Recall that the phase angle in a purely inductive circuit is a positive 90°, for a purely capacitive circuit, it is negative 90° and for a circuit containing only resistive, it is 0°. In the succeeding lessons, you will learn more about vectors in AC circuits, and some better methods for finding the resultant of two vectors at right angles.

As you saw in the special statements $I = \frac{E}{X_L}$ and $I = \frac{E}{X_C}$, Ohm's Law applies to AC circuits as well as to DC circuits. In series circuits that combine resistance and reactance, total current will be determined by the total opposition to current flow, and this opposition is called impedance. The capital letter Z is the symbol for impedance, so Ohm's Law for AC circuits is written $I_T = \frac{E_a}{Z}$. Impedance includes the cases of resistance alone, capacitance alone, inductance alone, or any combination of resistance, inductance, and capacitance. Transposing Ohm's Law, you will find that $E_a = IZ$, and the voltage across a pure inductance or capacitance can be found from $E_L = IX_L$ or $E_C = IX_C$.

You have learned that voltages in AC series circuits are represented by vectors that show voltage phase relationships with respect to current, and that adding out-of-phase voltages arithmetically gives false answers. From this it follows that adding resistance and reactance arithmetically will provide a false impedance value. Since voltage and impedance are proportional, however, it is possible to vector the impedance in a series circuit much as the voltages are vectored:





The resultant of the first vector group is total voltage, that of the second group is total opposition or impedance.

AT THIS POINT, YOU MAY TAKE THE LESSON PROGRESS CHECK, OR YOU MAY STUDY THE LESSON NARRATIVE OR THE PROGRAMMED INSTRUCTION OR BOTH. IF YOU TAKE THE PROGRESS CHECK AND ANSWER ALL OF THE QUESTIONS CORRECTLY, GO TO THE NEXT LESSON. IF NOT, SELECT ANOTHER METHOD OF INSTRUCTION UNTIL YOU CAN ANSWER ALL THE QUESTIONS CORRECTLY.

NAVPERS 94558-12a

BASIC ELECTRICITY AND ELECTRONICS
INDIVIDUALIZED LEARNING SYSTEM

M O D U L E   T W E L V E
L E S S O N   I I

Vector Computations

Study Booklet

Bureau of Naval Personnel
January 1972

OVERVIEW
LESSON II

Vector Computations

In this lesson, you will study and learn about the following:

- -when vectors can be added
- -Pythagorean theorem
- -relationships in a triangle
- -practical application

BEFORE YOU START THIS LESSON, PREVIEW THE LIST OF STUDY RESOURCES ON THE NEXT PAGE.

LIST OF STUDY RESOURCES
LESSON II

Vector Computations

To learn the material in this lesson, you have the option of choosing, according to your experience and preferences, any or all of the following:

STUDY BOOKLET:

Lesson Narrative
Programmed Instruction

Lesson Summary

ENRICHMENT MATERIAL:

NAVPERS 93400A-1b "Basic Electricity, Alternating Current." Fundamentals of Electronics. Bureau of Naval Personnel. Washington, D.C.: U.S. Government Printing Office, 1965.

NAVPERS 93400A-8 "Tables and Master Index." Fundamentals of Electronics. Bureau of Naval Personnel. Washington, D.C.: U.S. Government Printing Office, 1965.

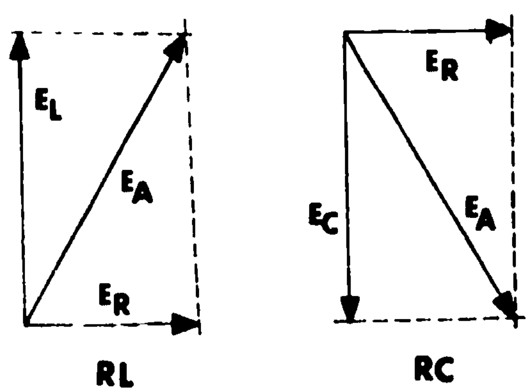
YOU MAY NOW STUDY ANY OR ALL OF THE RESOURCES LISTED ABOVE. YOU MAY TAKE THE PROGRESS CHECK AT ANY TIME.

NARRATIVE
LESSON II

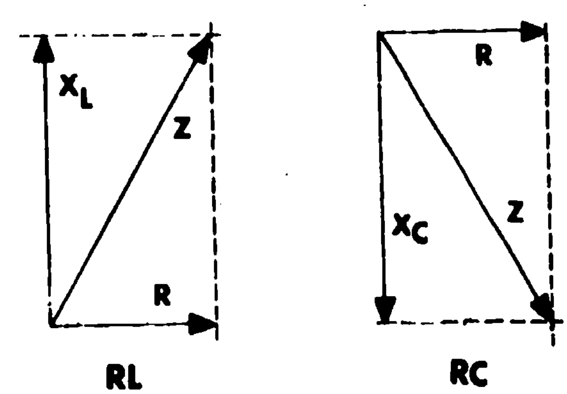
Vector Computations

Most practical circuits will contain all three of the basic electrical properties -- resistance, inductance, and capacitance. Because the voltages developed by these quantities are not in phase with each other, new methods of computing circuit values must be learned.

You've seen how vector diagrams can be used to represent voltages (E_L, E_C, E_R) and oppositions (X_L, X_C, R) in series AC circuits.



VOLTAGE VECTOR DIAGRAMS

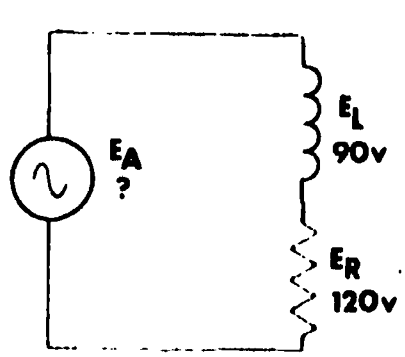


IMPEDANCE VECTOR DIAGRAMS

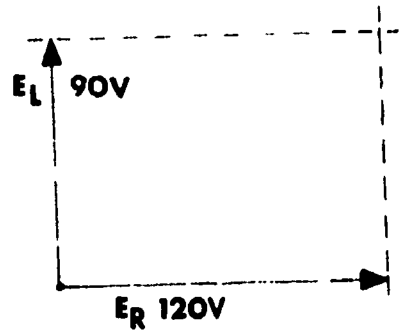
You will now learn the mathematical techniques that are used to combine out of phase values by vectorial addition.

Graphical Analysis

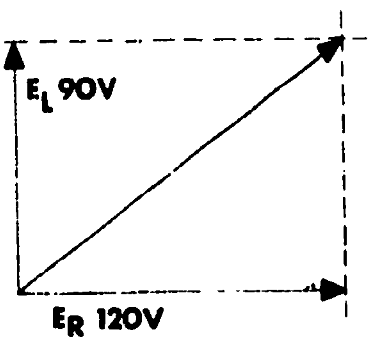
Graphical analysis involves the use of graph paper or drawing to scale.



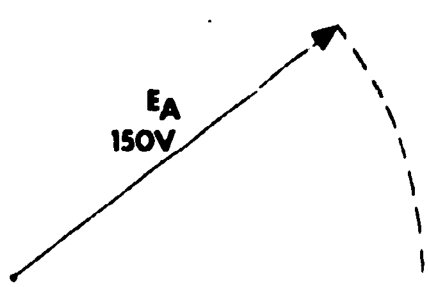
1. Draw the vectors representing each value to scale and in the proper position (as described in Lesson I).



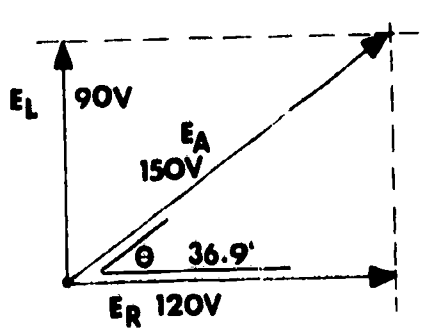
2. Construct a parallelogram from the existing sides. (Draw connecting lines equal in length and parallel to the existing sides.



3. Draw the resultant vector from the lower left corner (apex or zero point) to the upper right-hand corner.



4. The length of the resultant vector represents the combined value of the sides which is the value of E_a. Measure the resultant and compare it to the scale.

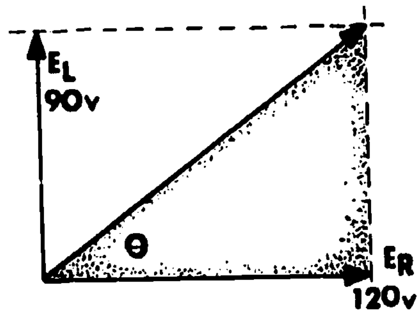


5. The angle between the base line and the resultant represents the circuit phase angle, $\underline{/\theta}$. Measure the angle. (The process is the same if resistance and reactance were to be combined to find impedance.)

While this is a long, slow, and usually inaccurate process, it is a good idea to make a rough sketch of the vectors for all circuits you solve.

Pythagorean Theorem

By looking at the parallelogram which is constructed from the original vector diagram, you can see that two right triangles are formed by the sides and the resultant.



The triangle we are concerned with is the one which contains the circuit phase angle, $\angle\theta$ (shaded area). The base line which represents the resistive value (R or E_R) forms one side of this triangle; the side opposite $\angle\theta$ represents the reactive value (X_L or E_L). Since the figure is a parallelogram, the side opposite $\angle\theta$ is identical to the vector representing the resistive value. The resultant vector which is the unknown value forms the hypotenuse. The problem now resolves into finding the length of one side of a right triangle when the length of the other two sides are known. One way of doing this is by applying the Pythagorean Theorem, which states that the length of the longest side is equal to the square root of the sum of the squares of the other two sides.

For our purposes, this can be written as: $E_a = \sqrt{E_R^2 + E_L^2}$ or, if impedance is desired: $Z = \sqrt{R^2 + X_L^2}$.

For the example shown:

$$E_R = 120 \text{ v} = 12 \times 10^1$$

$$E_L = 90 \text{ v} = 9 \times 10^1$$

$$E_a = \sqrt{(12 \times 10^1)^2 + (9 \times 10^1)^2}$$

$$E_a = \sqrt{144 \times 10^2 + 81 \times 10^2}$$

$$E_a = \sqrt{225 \times 10^2}$$

$$E_a = 15 \times 10^1 = 150 \text{ v}$$

The only drawback to the Pythagorean Theorem is that the value of $\underline{/\theta}$ cannot be determined and as you will soon see this value can be quite important.

Trigonometry

In order to determine the exact value of $\underline{/\theta}$ and the resultant vector (E_a or Z), it is necessary that you become familiar with certain mathematical relationships which exist between the sides and angles of a right triangle. The mathematical expressions known as trigonometric functions will be given in terms of the voltage and impedance triangle.

1. The sine of the circuit phase angle $\underline{/\theta}$ expresses the relationship between the reactive value and the resultant (E_a or Z).

$$\text{sine (SIN)} = \frac{E_L}{E_a} \text{ or } \frac{X_L}{Z}$$

2. The cosine of $\underline{/\theta}$ expresses the relationship between the resistive value and the resultant.

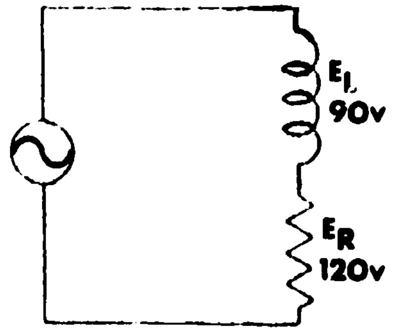
$$\text{cosine (COS)} = \frac{E_R}{E_a} \text{ or } \frac{R}{Z}$$

3. The tangent of $\underline{/\theta}$ expresses the relationship between the reactive value and the resistive value.

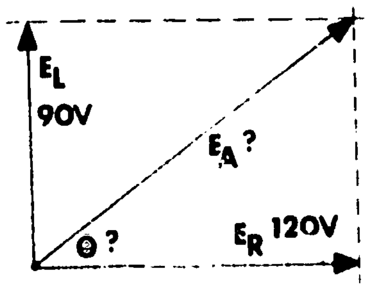
$$\text{tangent (TAN)} = \frac{E_L}{E_R} \text{ or } \frac{X_L}{R}$$

By using these equations it is possible to completely solve either the voltage or impedance triangle for $\underline{/\theta}$ and any unknown side when two values are known.

Example:



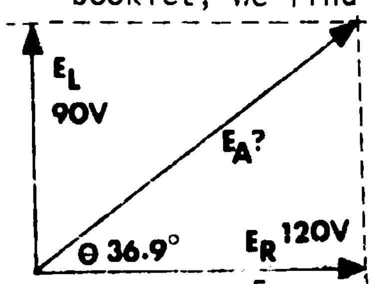
To help you keep track of all values, draw a sketch of the vector diagram.



For this example, the only equation which has both the known values is the equation for the tangent.

$$\text{TAN} = \frac{E_L}{E_R} = \frac{90}{120} = .75$$

By referring to the trig tables located in the back of the study booklet, we find that $\underline{/\theta}$ is 36.9°.



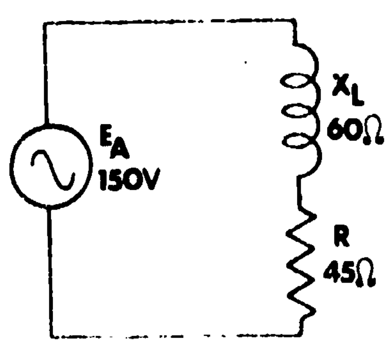
Located directly above the TAN in the trig tables are the sine and cosine for each angle. To complete the problem, either the sine or cosine may be used.

$$\text{SIN} = \frac{E_L}{E_a} : E_a = \frac{E_L}{\text{SIN}} : \frac{90}{.6004} = 150$$

or

$$\text{COS} = \frac{E_R}{E_a} : E_a = \frac{E_R}{\text{COS}} : \frac{120}{.7997} = 150$$

Solve:



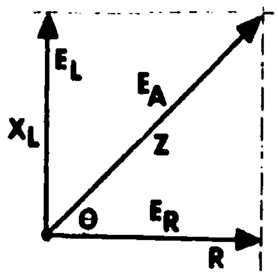
$\underline{/\theta}$ = ______

Z = ______

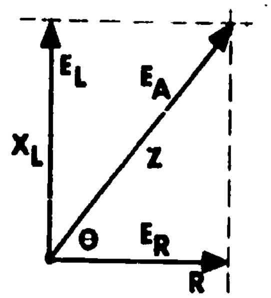
I = ______

$\underline{/\theta}$ = 53.1°; Z = 75 Ω; I = 2 a

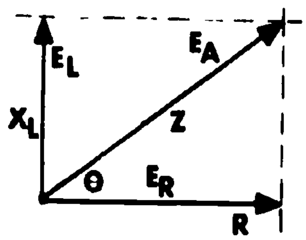
Regardless of the method used to solve the voltage or impedance triangle, there are four rules that can be applied to check the answers:



1. If the resistive value and the reactive value are equal, $\angle\theta$ must equal 45°



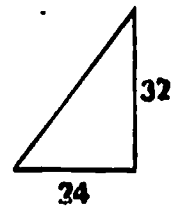
2. If the reactive value is greater than the resistive value, $\angle\theta$ will be greater than 45°.



3. If the resistive value is greater than the reactive value, $\angle\theta$ will be less than 45°.

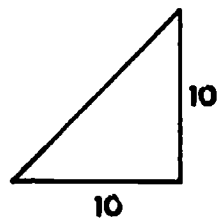
4. In all cases, the length of the resultant will be greater than the length of the sides but less than the direct sum of the two.

Practice.

1. 

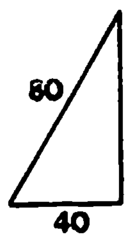
Find hyp = ______

$\angle\theta$ = ______

2. 

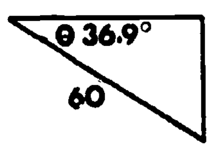
Find hyp = ______

$\angle\theta$ = ______

3. 

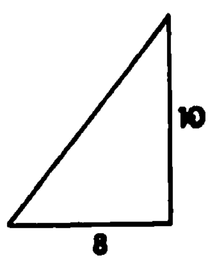
Find opp ≈ ______

$\angle\theta$ = ______

4. 

Find adj = ______

opp = ______

5. 

Find hyp ≈ ______

$\angle\theta$ = ______

Answers:

1. hyp = 40

   $\underline{/\theta}$ = 53.1°

2. hyp = 14.14

   $\underline{/\theta}$ = 45°

3. opp ≈ 69.28

   $\underline{/\theta}$ = 60°

4. adj = 48

   opp = 36

5. hyp ≈ 12.8

   $\underline{/\theta}$ = 51.4°

AT THIS POINT, YOU MAY TAKE THE PROGRESS CHECK, OR YOU MAY STUDY ANY OF THE OTHER RESOURCES LISTED. IF YOU TAKE THE PROGRESS CHECK AND ANSWER ALL OF THE QUESTIONS CORRECTLY, GO TO THE NEXT LESSON. IF NOT, STUDY ANY METHOD OF INSTRUCTION YOU WISH UNTIL YOU CAN ANSWER ALL THE QUESTIONS CORRECTLY.

PROGRAMMED INSTRUCTION
LESSON II

Vector Computations

THERE ARE NO TEST FRAMES IN THIS PROGRAMMED SEQUENCE.

1. Individual voltage drops in resistive-reactive circuits cannot be added directly to find total voltage (E_a) because of the ________ degree ________ ____________ between the values.

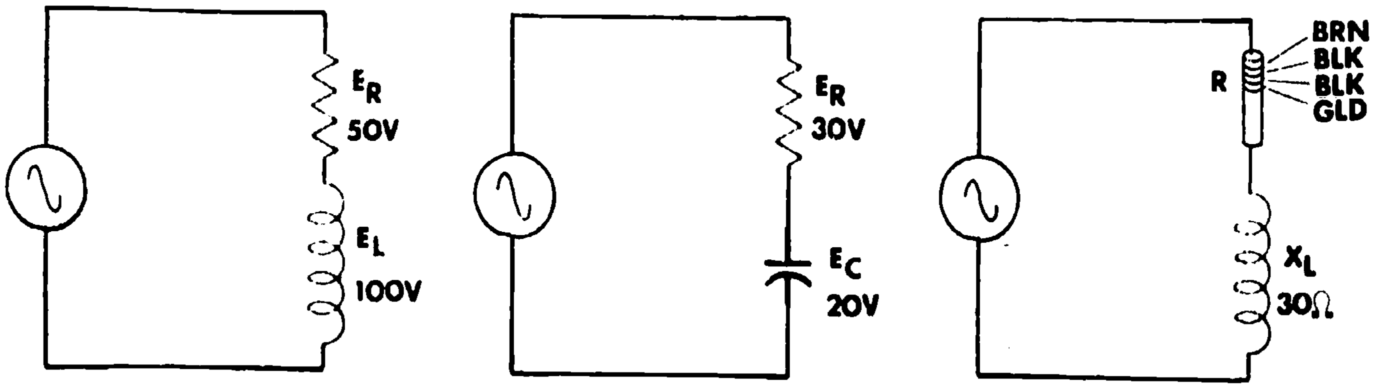
(90; phase difference)

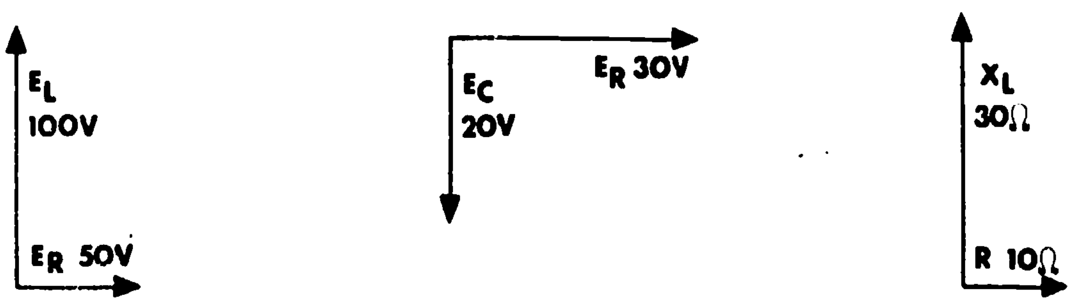
2. Since the voltage drop across each component in a series circuit is directly proportional to the opposition of that component, total impedance of a resistive-reactive circuit is found by ________________ addition.

(vectorial)

3. Vectors may be used to show the phase relationship between out-of-phase values.

Draw and label the vector diagrams for the circuits shown below.

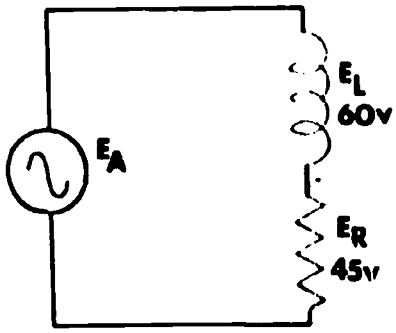


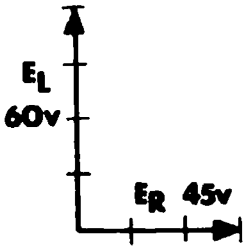


4. Since direct addition is not possible between out-of-phase values, vectorial addition must be used to combine these values. The problem resolves itself to constructing the voltage or impedance vector diagram and solving for a resultant vector which will represent the value of ____________ or ____________.

(E_a ; Z -- [either order])

5. One method of adding vectors is by graphic analysis. To do this the vectors must be drawn to scale. For example:

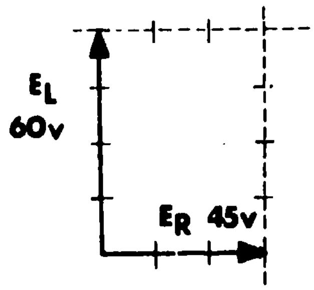




What is the value of each graduation mark? ____________

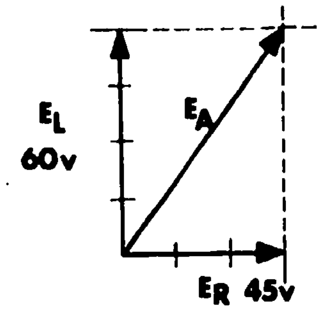
(15 v. The choice of scale is up to each individual so long as the same scale is used for the entire diagram.)

6. The next step is to construct a parallelogram from the original vector diagram. (A parallelogram is a four-sided figure in which the opposite sides are equal in length and parallel.)



(GO TO NEXT FRAME)

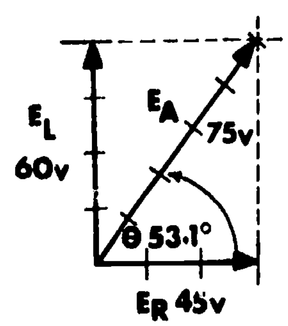
7. Once the parallelogram is constructed, draw the resultant vector from the lower left-hand corner (apex or zero point) to the upper right-hand corner. The length of this line when compared to the original scale gives the value of E_a.



What is the value of E_a? ____________

(75 v)

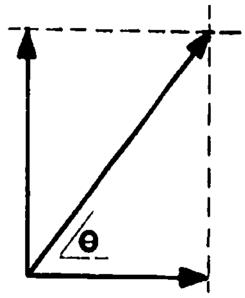
8. The angle between the base line (reference) and the resultant vector represents the circuit phase angle, $\underline{/\theta}$.



In this example, the applied voltage ____________ the circuit current by ____________.
leads/lags

(leads; 53.1°) NOTE: Graphic analysis can only be used when accuracy is not important and the necessary equipment is available.

9. When the parallelogram is constructed and the resultant vector drawn, two triangles are formed. The one we are concerned with contains $\underline{/\theta}$.



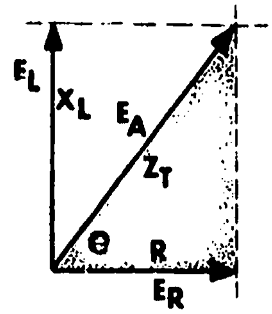
a. The base line of this triangle represents the value of __________ or __________.

b. The vertical side being equal in length to the side opposite it represents __________ or __________.

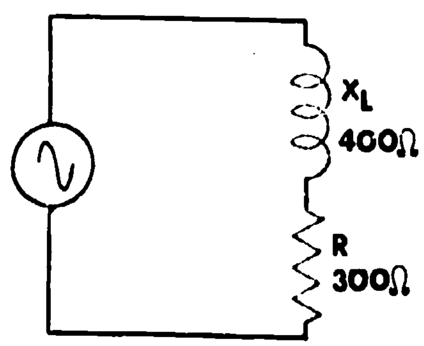
c. The resultant vector represents the value of __________ or __________.

(a. E_R, R; b. E_L, X_L; c. E_a, Z)

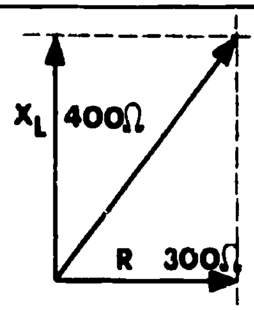
10. Mathematically, the problem can be resolved to solving for angles and sides within a right triangle.



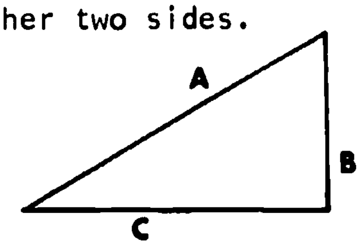
The information usually given will provide the values for the base and vertical sides of the triangle. The desired information will be the length of the resultant and the angle between it and the base line or angle theta.



Draw the vector diagram, construct the parallelogram, and label all known values. (It is not necessary to draw to exact scale.)



11. There are definite mathematical relationships between the sides of a right triangle. One of these relationships is known as the Pythagorean Theorem, which states that the length of the longest side is equal to the square root of the sum of the squares of the other two sides.

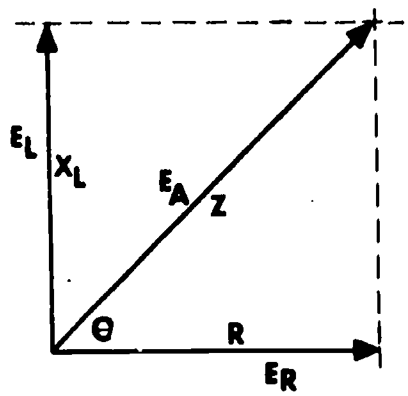


The length of side _____ is equal to the square root of the sum of the squares of sides _____ and _____.

- -

(a; b: c)

12. For our purposes, the Pythagorean Theorem can be stated in terms of the electrical values plotted on the vector diagrams.



The resultant vector is the longest side of the triangle and represents the value of E_a, so the applied voltage is equal to the square root of the sum of the squares of the other two sides which represent E_L and E_R. Written in the form of an equation:

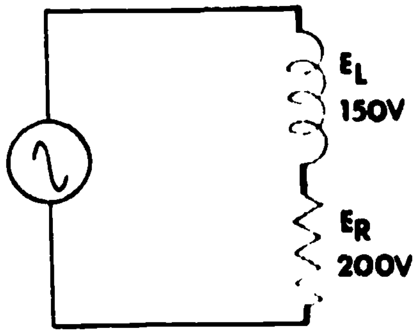
$$E_a = \sqrt{E_L^2 + E_R^2}$$

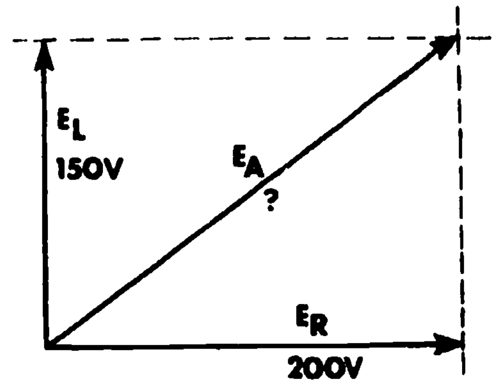
Write the equation which would be used to find impedance: _____

- -

($Z = \sqrt{X_L^2 + R^2}$)

13. This example illustrates the solution of a voltage vector diagram using the Pythagorean Theorem.





$$E_a = \sqrt{E_L^2 + E_R^2}$$

$$E_a = \sqrt{(15 \times 10^1)^2 + (20 \times 10^1)^2}$$

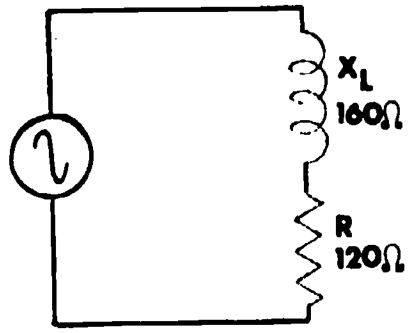
$$E_a = \sqrt{225 \times 10^2 + 400 \times 10^2}$$

$$E_a = \sqrt{625 \times 10^2}$$

$$E_a = 25 \times 10^1$$

$$E_a = 250$$

Solve for Z.

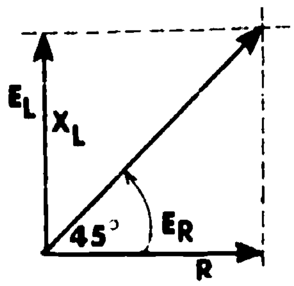


- -

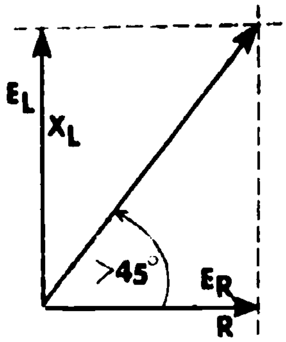
(200 Ω)

14. The Pythagorean Theorem can be quite useful in solving electrical circuits. It does, however, have one important drawback; it does not tell us the number of degrees in $\angle\theta$. This value is quite important in AC circuit computations.

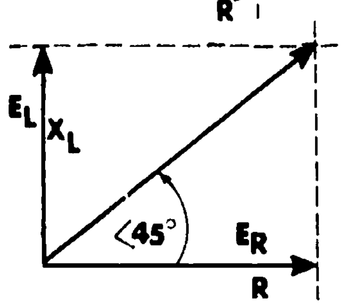
The length of the sides of the triangle determines the size of $\angle\theta$.



If the reactive and resistive values are equal, $\angle\theta$ will be 45° or just half-way between 0° and 90°.



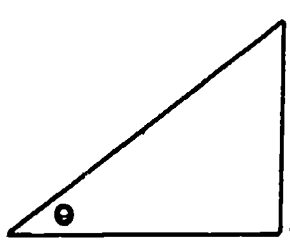
If the reactive value is greater than the resistive value, $\angle\theta$ will be greater than 45°.



If the reactive value is less than the resistive value, $\angle\theta$ will be less than 45°.

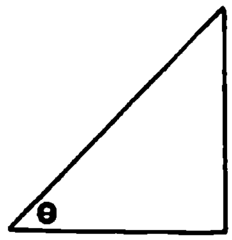
In the following examples, determine if $\angle\theta$ is greater than, equal to, or less than 45°.

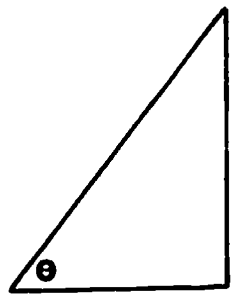
a.

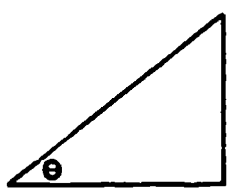


____

14. cont.

b.  ____________

c.  ____________

d.  ____________

- -

(a. less; b. equal; c. greater; d. less)

15. As the adjacent side of a right triangle increases, /θ ____________.
increases/decreases

As the opposite side of a right triangle increases, /θ ____________.
increases/decreases

- -

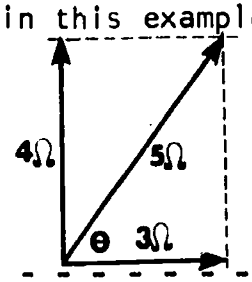
(decreases; increases)

16. The mathematics of solving a triangle is known as trigonometry. To understand trigonometry, you must become familiar with three mathematical expressions relating the three sides and $\underline{/\theta}$ of a right triangle. For convenience, the sides of the triangle will be given in terms of the electrical value they represent.

The sine of the $\underline{/\theta}$ expresses the relationship between the reactive value (E_L or X_L) and the resultant vector (E_a or Z). The sine is determined by dividing the reactive value by the value of the resultant vector.

$$SIN\ \underline{/\theta} = \frac{E_L}{E_a} \text{ or } \frac{X_L}{Z}$$

What is the sine of $\underline{/\theta}$ in this example?



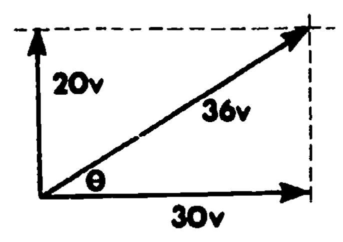
SIN = ______

(0.8000)

17. The cosine of the $\underline{/\theta}$ expresses the relationship between the resistive value and the resultant. The cosine is determined by dividing the resistive value by the value of the resultant.

$$COS\ \underline{/\theta} = \frac{E_R}{E_a} \text{ or } \frac{R}{Z}$$

What is the cosine of $\underline{/\theta}$ in this illustration?



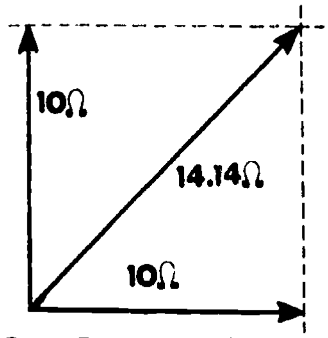
COS = ______

(.8333)

18. The reactive value divided by the resistive value is equal to a number called the tangent of $\underline{/\theta}$.

$$\tan\underline{/\theta} = \frac{E_L}{E_R} \text{ or } \frac{X_L}{R}$$

What is the tangent of $\underline{/\theta}$ in the illustration?



TAN = ______

(1.0000) [Note: When computing trig values, carry the numbers to four decimal places to ensure reasonable accuracy.]

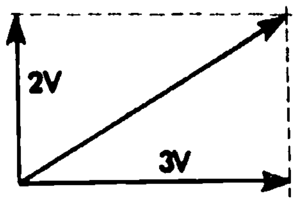
19. Match the functions in column B to the trigonometric equations in column A.

A.	B.
____ 1. $\frac{E_L}{E_a}$	a. SIN $\underline{/\theta}$
____ 2. $\frac{R}{Z}$	b. TAN $\underline{/\theta}$
____ 3. $\frac{E_L}{E_R}$	c. COS $\underline{/\theta}$

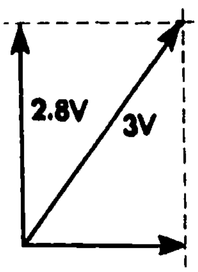
(1. a; 2. c; 3. b)

20. By using one or a combination of the trigonometric functions, it is possible to completely solve any right triangle if any two sides or one side and /θ are known.

Which trigonometric functions would you use to solve for /θ in the following examples?

a. 

______

b. 

______

(a. TAN: b. SIN)

21. Now that you know what the trigonometric functions are and how to solve for their numerical value, here is what you do with them. In the back of this booklet is a set of trigonometric tables. The column on the left is divided into degrees increasing in value toward the bottom of the page.

TABLE OF TRIGONOMETRIC FUNCTIONS

deg	Function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°
0	sin	0.0000	0.0017	0.0035	0.0052	0.0070	0.0087	0.0105	0.0122	0.0140	0.0157
	cos	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999
	tan	0.0000	0.0017	0.0035	0.0052	0.0070	0.0087	0.0105	0.0122	0.0140	0.0157
1	sin	0.0175	0.0192	0.0209	0.0227	0.0244	0.0262	0.0279	0.0297	0.0314	0.0332
	cos	0.9998	0.9998	0.9998	0.9997	0.9997	0.9997	0.9996	0.9996	0.9995	0.9995
	tan	0.0175	0.0192	0.0209	0.0227	0.0244	0.0262	0.0279	0.0297	0.0314	0.0332
2	sin	0.0349	0.0366	0.0384	0.0401	0.0419	0.0436	0.0454	0.0471	0.0488	0.0506
	cos	0.9994	0.9993	0.9993	0.9992	0.9991	0.9990	0.9990	0.9989	0.9988	0.9987
	tan	0.0349	0.0367	0.0384	0.0402	0.0419	0.0437	0.0454	0.0472	0.0489	0.0507

Across the top, the table is divided into tenths of a degree (0.0° - 0.9°). To the right of each whole degree and underneath each tenth of a degree is a group of three numbers; these numbers represent the sine, cosine and tangent of that angle.

If the tangent of an unknown angle is known to be 0.0297 what is the angle?

(1.7°)

22. Using the trigonometric table, find the angle for each of the following values.

a. COS = 0.9304 $\angle\theta$ = ______
b. SIN = 0.7455 $\angle\theta$ = ______
c. TAN = 0.6494 $\angle\theta$ = ______
d. COS = 0.9690 $\angle\theta$ = ______
e. TAN = 5.4486 $\angle\theta$ = ______

(a. 21.5°; b. 48.2°; c. 33.0°; d. 14.3°; e. 79.6°)

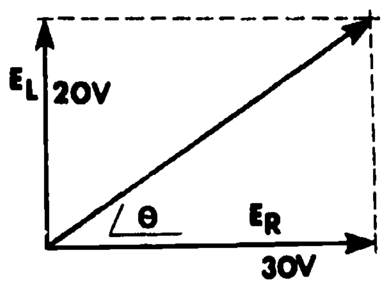
23. By reversing the process the numerical value of the functions can be found if the angle is known.

Find the functions indicated.

a. SIN $\underline{/73.1^\circ}$ ________
b. COS $\underline{/45^\circ}$ ________
c. TAN $\underline{/36.9^\circ}$ ________
d. SIN $\underline{/23.5^\circ}$ ________
e. TAN $\underline{/42.2^\circ}$ ________

(a. 0.9568; b. 0.7071; c. 0.7508; d. 0.3987; e. 0.9067)

24. By manipulating the three trigonometric equations, it is possible to find any value on the triangle if any other two values are known. The values we will usually be looking for are $\underline{/\theta}$ and the value of the resultant.



The reactive and resistive values are known. Which trigonometric function would be used to find $\underline{/\theta}$? ________

What is the value of the function? ________

What angle does this number represent? ________

(tangent = $\frac{opp}{adj}$; 0.6666; $\underline{/\theta}$ = 33.7°)

25. $\underline{/\theta}$ was found to be 33.7°. By looking at the two numbers directly above the tangent, we find that the cosine of 33.7° is 0.8320 and the sine is 0.5548. Either of these can be used to find the value of E_a.

$$SIN = \frac{E_L}{E_a} \quad \text{or} \quad E_a = \frac{E_L}{SIN} \quad E_a = \frac{20}{0.5548} = 36.05$$

$$COS = \frac{E_R}{E_a} \quad \text{or} \quad E_a = \frac{E_R}{COS} \quad E_a = \frac{30}{0.8320} = 36.05$$

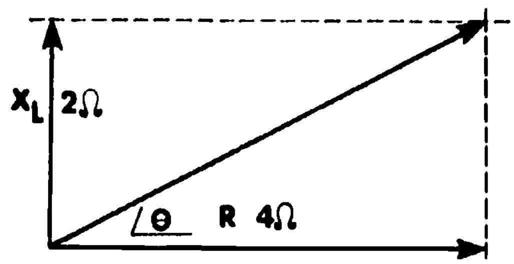
What is $\underline{/\theta}$ and E_a?

$\underline{/\theta}$ = ______

E_a = ______

(33.7°; 36.05v)

26. Solve.



$\underline{/\theta}$ = ________

Z = ________

($\underline{/\theta}$ = 26.6°; Z = 4.46)

YOU MAY NOW TAKE THE PROGRESS CHECK OR YOU MAY STUDY ANY OF THE OTHER RESOURCES LISTED. IF YOU TAKE THE PROGRESS CHECK AND ANSWER ALL THE QUESTIONS CORRECTLY, GO TO THE NEXT LESSON. IF NOT, STUDY ANY METHOD OF INSTRUCTION YOU WISH UNTIL YOU CAN ANSWER ALL THE QUESTIONS CORRECTLY.

SUMMARY
LESSON 11

Vector Computations

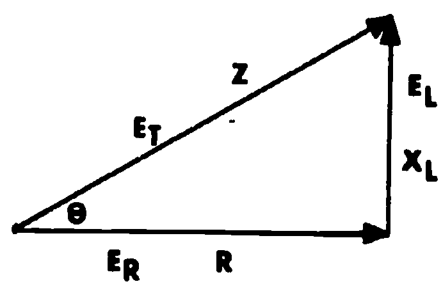
You have seen how vectors can be used to represent voltage and impedance in series AC circuits, and it is time now to learn more about how to manipulate the vectors. One way, of course, is to make a scale drawing of the vectors and measure the needed values. While this is a long, slow, and usually inaccurate process, it is a good idea to make a rough sketch of the vectors for all circuits you solve. It helps you keep the process straight in your mind. Another method is to use the Pythagorean theorem ($X^2 + R^2 = Z^2$), but this gives only the magnitude of the vector and tells nothing about its direction.

Trigonometric functions can easily provide both the length and direction of any needed vectors when two quantities are known. The trigonometric functions of the phase angle θ are:

tangent (TAN) $\underline{/\theta} = \frac{E_L}{E_R} = \frac{X_L}{R}$

sine (SIN) $\underline{/\theta} = \frac{E_L}{E_T} = \frac{X_L}{Z}$

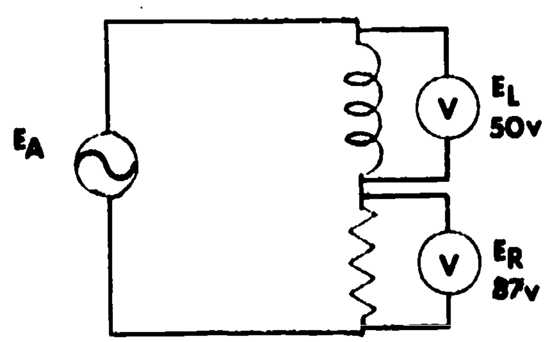
cosine (COS) $\underline{/\theta} = \frac{E_R}{E_T} = \frac{R}{Z}$



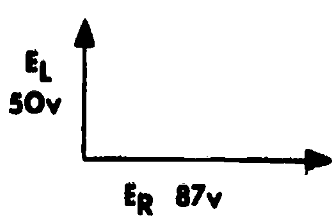
A table of the values of these functions is located at the back of this module.

An example of the use of trigonometric functions is:

Find E_a:



First, sketch the known vectors.



Second, use the TAN function to determine $\underline{/\theta}$.

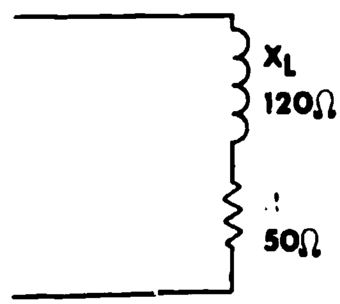
$$\text{TAN } \underline{/\theta} = \frac{E_L}{E_R} = \frac{50}{87} = .57$$

From the table $\underline{/\theta} = 30°$

Transpose the second equation: $E_T = \frac{E_L}{\text{SIN } \underline{/\theta}}$

$$= \frac{50}{\text{SIN } 30°} = \frac{50}{0.5} = 100 \text{ v}$$

Here is a solution for impedance:



$$\text{TAN } \underline{/\theta} = \frac{X_L}{R} = \frac{120}{50} = 2.4$$

From the table $\underline{/\theta} = 67.3°$

$$Z = \frac{X_L}{\text{SIN } \underline{/\theta}} = \frac{120}{.9225} = 130\ \Omega$$

Similar methods can be used to solve for one vector when one vector and the resultant are known, or when the resultant and the phase angle are known.

Some rules to help you solve AC series circuit problems: remember first to sketch the vectors; then

1. if the reactance (reactive voltage) is greater than the resistance (resistive voltage) the phase angle will be greater than 45°.

2. if the reactance and resistance are equal, $\underline{/\theta}$ will be 45°.

3. if the resistance is larger than the reactance, $\underline{/\theta}$ will be less than 45°.

Another quick check to avoid some gross mistake is that the resultant will be larger than either vector, but less than the sum of the vectors.

The following lessons will go more deeply into solving these problems and will give you more practice with them. If you want to try some problems now, there are several practice examples at the end of the narrative for this lesson.

AT THIS POINT, YOU MAY TAKE THE LESSON PROGRESS CHECK, OR YOU MAY STUDY THE LESSON NARRATIVE OR THE PROGRAMMED INSTRUCTION OR BOTH. IF YOU TAKE THE PROGRESS CHECK AND ANSWER ALL OF THE QUESTIONS CORRECTLY, GO TO THE NEXT LESSON. IF NOT, STUDY ANOTHER METHOD OF INSTRUCTION UNTIL YOU CAN ANSWER ALL THE QUESTIONS CORRECTLY.

NAVPERS 94558-12a

BASIC ELECTRICITY AND ELECTRONICS
INDIVIDUALIZED LEARNING SYSTEM

MODULE TWELVE
LESSON III

Rectangular and Polar Notation

Study Booklet

Bureau of Naval Personnel
January 1972

OVERVIEW

LESSON III

Rectangular and Polar Notation

In this lesson, you will study and learn about the following:

- rectangular notation
- the j operator
- positive and negative angles
- polar notation
- converting notation
- mathematical computations

BEFORE YOU START THIS LESSON, PREVIEW THE LIST OF STUDY RESOURCES ON THE NEXT PAGE.

LIST OF STUDY RESOURCES

LESSON III

Rectangular and Polar Notation

To learn the material in this lesson, you have the option of choosing, according to your experience and preferences, any or all of the following:

STUDY BOOKLET:

- Lesson Narrative
- Programmed Instruction
- Lesson Summary

ENRICHMENT MATERIAL:

- NAVPERS 93400A-1b "Basic Electricity, Alternating Current." Fundamentals of Electronics. Bureau of Naval Personnel. Washington, D.C.: U.S. Government Printing Office, 1965.
- NAVPERS 93400A-8 "Tables and Master Index." Fundamentals of Electronics. Bureau of Naval Personnel, Washington, D.C.: U.S. Government Printing Office, 1965.

YOU MAY NOW STUDY ANY OR ALL OF THE RESOURCES LISTED ABOVE. YOU MAY TAKE THE PROGRESS CHECK AT ANY TIME.

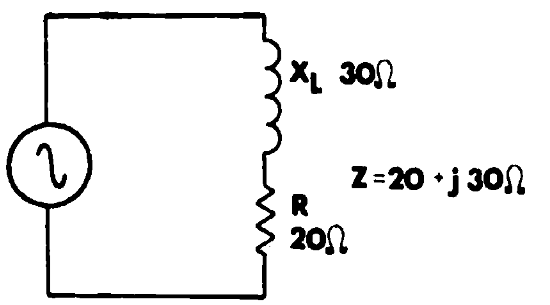
NARRATIVE
LESSON III

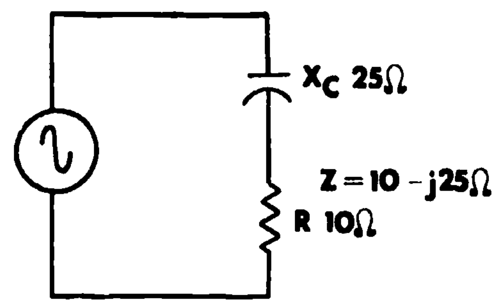
Rectangular and Polar Notation

Rectangular and polar notation are simpler means of using trig to solve AC circuits.

Looking first at rectangular notation, let's see how it makes our job easier. Rectangular notation states the ohmic value of the circuit components in two parts, the resistive value always being stated first and the reactance value second.

The impedance of the following circuits are written in rectangular notation as shown.

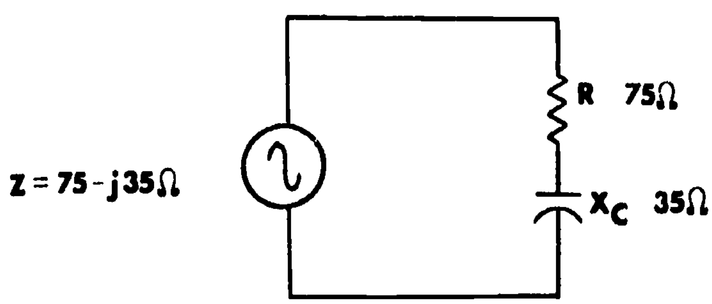




Notice all inductive values are indicated as +j while capacitive values are indicated as ______________.

(-j)

With rectangular notation indicating the following values, draw a circuit corresponding to the values indicated.



With circuit values given in rectangular notation, a complicated circuit can be easily reduced through addition.

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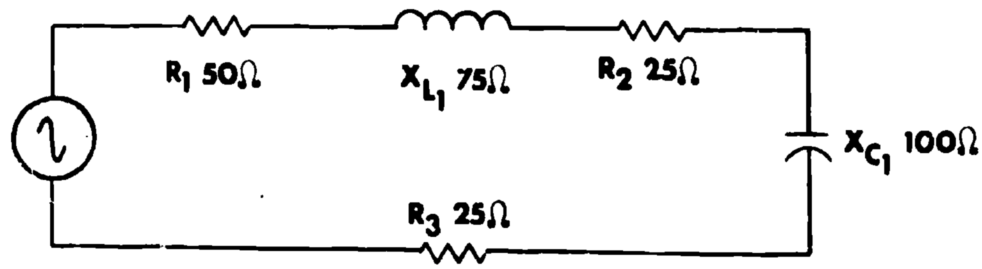
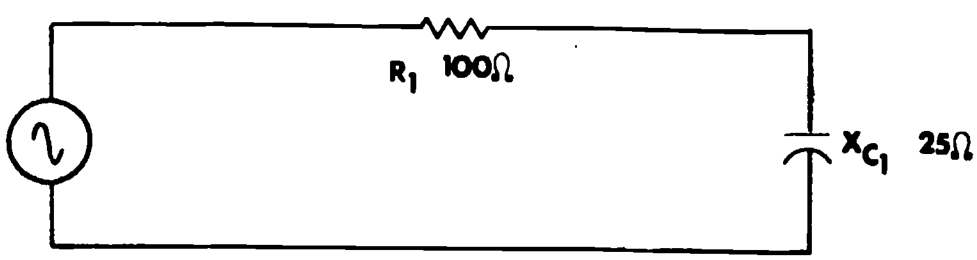


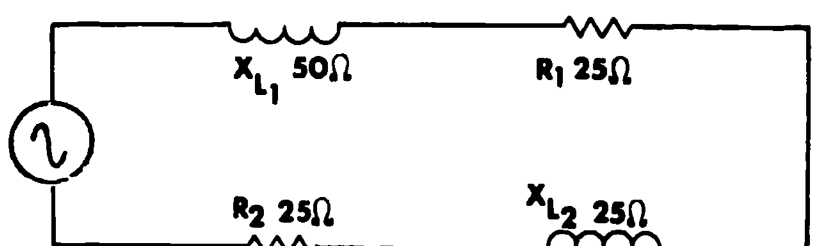
$50 + j75 + 25 - j100 + 25 = Z_T$. Add all resistances together and all reactances (j-values) together, and the total value is:

$$Z_T = 100 - j25$$

The +j values will cancel the -j values; thus we have an equivalent circuit containing 100 ohms of resistance and 25 ohms of capacitive reactance.



Looking at a circuit that contains inductive and resistive opposition

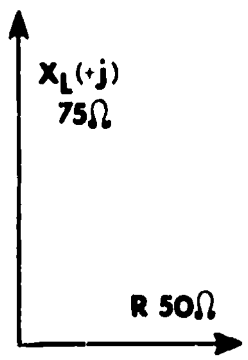


in series, when written in rectangular notation the circuit would be written as __________.

$25 + j50 + 25 + j25$

Adding these values in rectangular notation, we would end up with $50 + j75$. Since the 50 denotes resistance, it would be plotted horizontally on a vector. ⟶ The +j value, the inductive reactance, is plotted vertically upward.

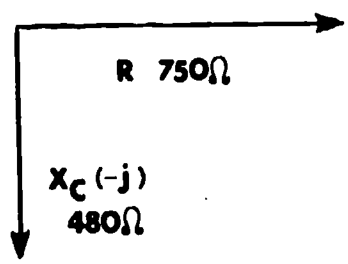
50 + j75Ω



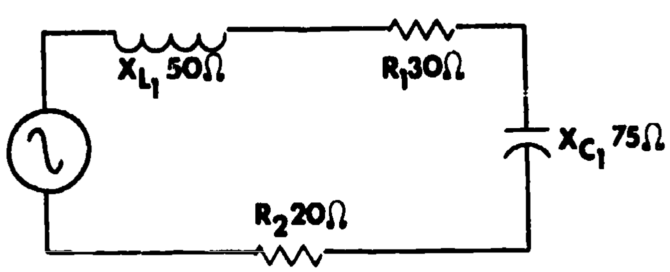
Draw a vector diagram representing the following number in rectangular notation:

750 - j480Ω

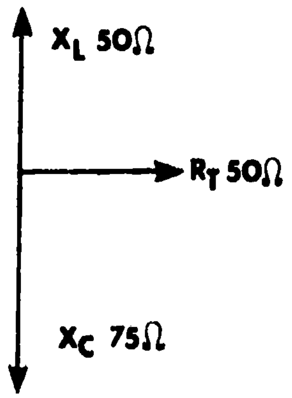
The resistive value, 750, is plotted horizontally, while the -j480 identifies the circuit component as capacitive reactance, and is plotted downward.



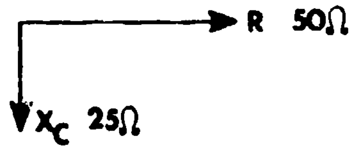
In a series circuit containing X_L and X_C the larger value will dominate and the resulting circuit will appear inductive or capacitive.



Represented in rectangular notation, this circuit would be:



After performing the indicated functions, you should end up with 50 - j25. The -j implies a capacitive circuit, and the vector diagram is:



When the circuit has been reduced by rectangular notation, it can then be converted into a vector resultant and easily solved with the usual trig functions.

Rectangular notation is highly useful if the individual component values are known. However, if only the resultant value is known, with its angle, theta, we have what is called polar notation.

Rectangular notation gives the values of the resistive and reactive components of a vector, while polar notation gives only the value of the resultant and its phase angle. In this configuration, vectors can be multiplied or divided with ease.

If a circuit has a total impedance of 65 ohms and an angle of 35.5°, we would put it in polar notation, 65 $\Omega\underline{/35.5^\circ}$. If this circuit has a source voltage of 100 volts and we are solving for total current, the problem could be laid out like this:

$$\frac{E_a}{Z_T} = \frac{100\ \underline{/0^\circ}}{65\ \underline{/35.5^\circ}} \qquad I = \frac{100}{65\underline{/35.5^\circ}} = 1.53\ a\ \underline{/-35.5^\circ}$$

To bring the $\underline{/35.5^\circ}$ above the line in division we change the sign to $\underline{/-35.5^\circ}$. In multiplying or dividing vector quantities, the angles are treated like powers of ten, that is, the angles are added algebraically for multiplication, and the angle in the denominator is subtracted algebraically from the angle in the numerator for division.

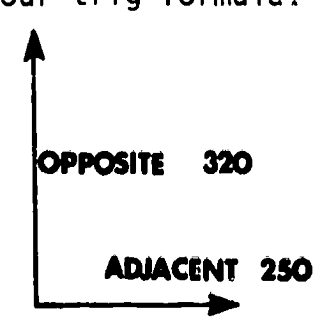
We have solved not only for the total current in the circuit (1.53 a), but have also found that the circuit current is lagging the source voltage by 35.5°.

In converting from rectangular notation, 250 + j320, to polar notation, 405 $\underline{/52.1^\circ}$, we simply perform the function indicated by our trig formula.

$$\text{SIN} = \frac{\text{opposite}}{\text{hypotenuse}} \qquad S = \frac{o}{h} \qquad S = \frac{X_L}{Z}$$

$$\text{COSINE} = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad C = \frac{a}{h} \qquad C = \frac{R}{Z}$$

$$\text{TANGENT} = \frac{\text{opposite}}{\text{adjacent}} \qquad T = \frac{o}{a} \qquad T = \frac{X_L}{R}$$



As the formula dictates for finding the tangent, we divide the reactive side by the resistive side, the values listed in rectangular notation. 320 divided by 250 gives us a tangent of 1.28. Looking at our trig table we find we have an angle of 52.1°. We also find we have a SIN function of 0.7891 and a COS function of 0.6143. Here again the formula will indicate what to do, find the resultant by dividing the reactive side by the SIN value.

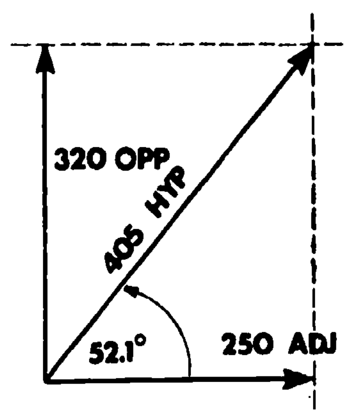
$$\frac{320}{.7891} = Z$$

In performing the indicated functions we derive the resultant value.

SIN function first ---- $\frac{320.000}{0.7891} = 405$

COS function next ---- $\frac{250.000}{0.6143} = 405$

We now have a triangle containing the following values, resistive, reactive, and resultant, as well as the angle.



Taking the value of the resultant and its respective angle of displacement, we can put the problem in polar notation, $405 \underline{/52.1^\circ}$. In this form, you can see that you already have the value of the resultant and the angle.

Following directions, work the following problem.

370 + j440

Draw the angle indicated.

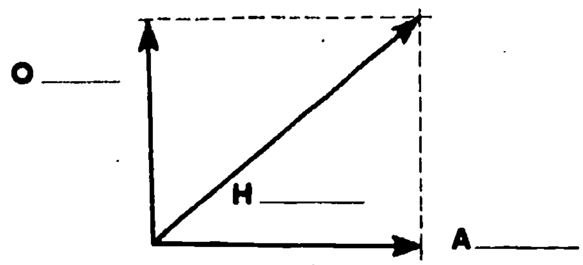
List the trig functions.

Perform the math indicated by the formula.

List the following values. TAN = ______ SIN = ______ COS = ______

Using the SIN function, the resultant is __________.
Using the COS function, the resultant is __________.

Fill in the values on the resulting triangle.

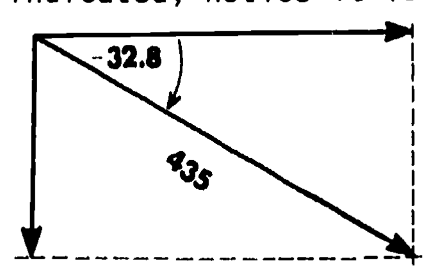


Write the values in polar notation.

$575\angle 49.9^\circ$

Remember, in polar notation we are simply given the resultant value and its respective angle. And again, we can depend on the formula to show us what steps to take to solve for our unknown values of the resistive and reactive sides.

Taking the following problem in polar form, $435\angle -32.8^\circ$, first let's draw out the triangle indicated; notice it is a negative angle.



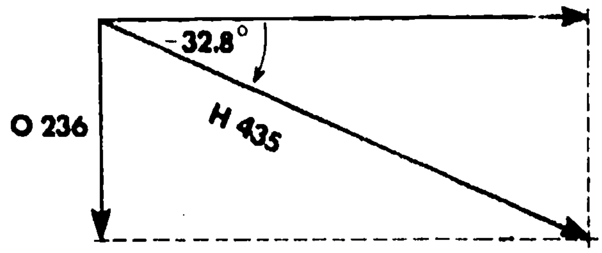
Now let's go to our trig table and find the SIN and COS function for the angle of -32.8°. SIN = 0.5417; COS = 0.8406.

Our formula tells us we can do some simple multiplication and solve our problem.

SIN x resultant = reactive value

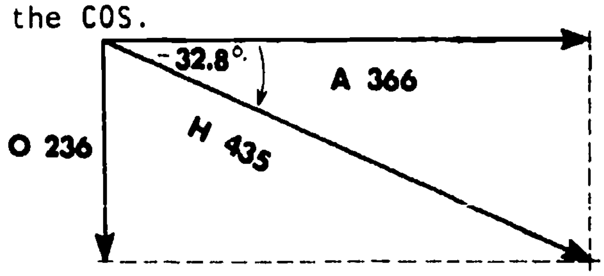
COS x resultant = resistive value.

First, work the SIN.



$$\begin{array}{r} .5417 \\ \underline{\times 435} \\ 236 \end{array}$$

Next, work the COS.



$$\begin{array}{r} .8406 \\ \underline{\times 435} \\ 366 \end{array}$$

Now this problem can be written in rectangular notation as 366 - j236 or in polar form as 435 /-32.8°.

Using what has been covered, convert 750 /42.6° to rectangular form. Follow directions and solve for the values indicated.

Draw a vector representing the values given.

List the trig functions.

On the trig sheet, find angle indicated and list SIN = ______

COS = ______

Solve for the resistive side. ________

Solve for the reactive side. __________

Draw a vector representing the values of the resistive and reactive sides.

Indicate angle theta and the resultant.

Show vector triangle in rectangular form.

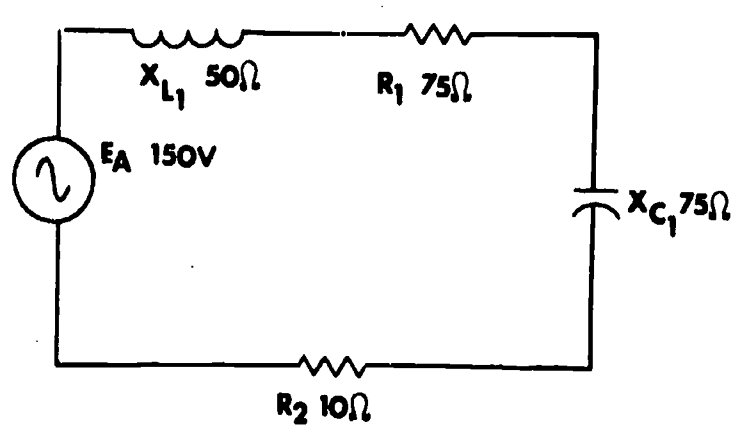
552 + j508

Solve.

Z_T = ______

I_T = ______

$\underline{/}$ = ______



Rectangular notation =

Vector representation =

Polar notation = __________ $\underline{/-16.4°}$

Triangle representation =

Solve for I_T in the preceding problem using polar notation.

$$I_T = \frac{150}{88\ \underline{/-16.4°}} \qquad I_T = \underline{1.7A}\ \underline{/16.4°}$$

In this circuit we found the current ______________ the source voltage.
leading/lagging

- -

(leading)

Since we have a combination of resistive and reactive components in a circuit, we must know how much true power is dissipated and how much reactive power is returned to the circuit. A ratio of true power to apparent power is the circuit's <u>power factor</u> (PF).

This power factor can be determined by the formula $PF = \frac{P_t}{P_a}$, which is also equal to the cosine of the circuit phase angle. True power may be computed using the formula $P_t = E \times I \times COS\ \underline{/\theta}$. Reactive power is $P_x = E_a \times I_T \times SIN\ \underline{/\theta}$. Apparent power is simply $E_a \times I_T$.

AT THIS POINT, YOU MAY TAKE THE PROGRESS CHECK, OR YOU MAY STUDY ANY OF THE OTHER RESOURCES LISTED. IF YOU TAKE THE PROGRESS CHECK AND ANSWER ALL OF THE QUESTIONS CORRECTLY, GO TO THE NEXT LESSON. IF NOT, STUDY ANY METHOD OF INSTRUCTION YOU WISH UNTIL YOU CAN ANSWER ALL THE QUESTIONS CORRECTLY.

PROGRAMMED INSTRUCTION
LESSON III

Rectangular and Polar Notation

THERE ARE NO TEST FRAMES IN THIS PROGRAMMED SEQUENCE.

1. Rectangular notation (30 + J20) states the ohmic value of the circuit components in two parts. The resistive value always being stated first and the reactance value second.

   In rectangular notation the __________ value is given first.

(resistive)

2. The term J (J operator) is used to express the reactive component. All inductive values are indicated as +J while capacitive values are indicated as -J.

   The reactive component is indicated by the prefix j and a __________ or __________ sign.

(+; -; either order)

3. A +j indicates the vector must rotate counterclockwise and a -j indicates the vector must rotate clockwise.

   The sign of j operator denotes the __________ the vector is rotated.

(direction)

4. A +j indicates counterclockwise rotation while a __________ indicates clockwise rotation.

(-j)

5. Capacitive reactance is indicated as a -j for its vector rotates clockwise.

Inductive reactance (in ohms) is indicated by a +j and is plotted as ____________ rotation.

(counterclockwise)

6. The reactive component (capacitive or inductive) is always the value attached to the j operator and indicated with a + or - sign.

A -j indicates a ____________ reactance and is plotted clockwise.

(capacitive)

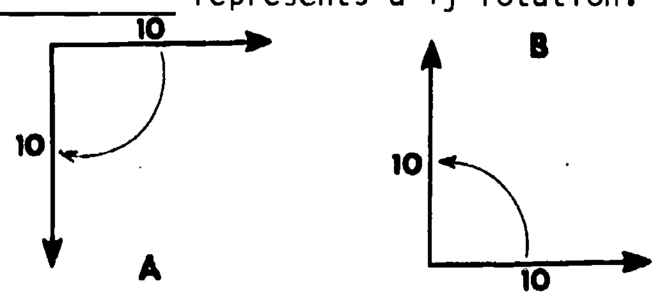
7. Actually rectangular notation is another method of giving us the two sides of a triangle.

The reactive side of a triangle is represented by a + _____ or - _____

(j; j)

8. Rectangular notation is drawing a vector triangle with numbers and letters.

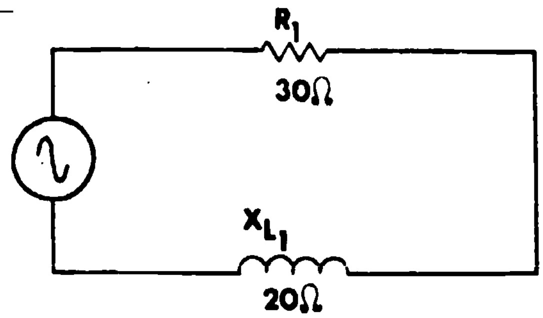
Drawing ____________ represents a +j rotation.



(B)

9. To eliminate drawing a circuit or a right triangle we simply state the values of the circuit given in rectangular notation.

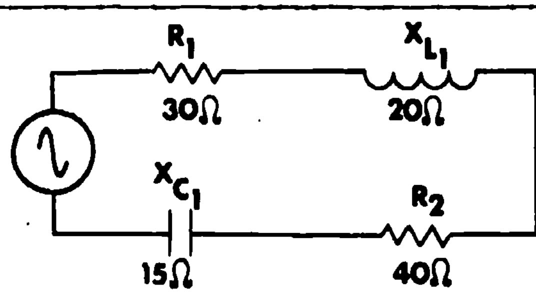
   Indicate the below circuit using rectangular notation.



(30 + j20)

10. By looking at a statement in rectangular notation you know immediately what values the respective vectors must represent.

    Draw a circuit indicated by the values in rectangular notation.
    30 + j20 + 40 - j15



11. Trig functions and algebraic addition are still required when using rectangular notation.

In using rectangular notation with trigonometry, what would be the resultant from the reactive values of + j40 and -j20?

____

(+j20)

12. Normal addition will not work on the reactive components in rectangular notation for their signs are not always the same.

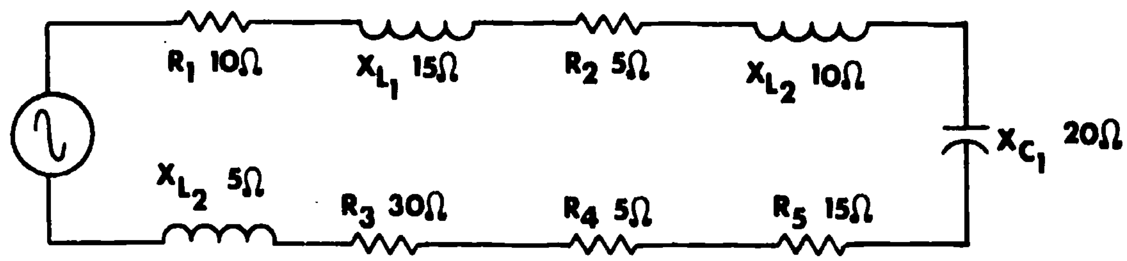
Triangles are easily added in rectangular notation. Normal addition is applied to the resistive values and the algebraic addition is applied to the __________ values.

(reactive)

13. Adding values in rectangular notation will reduce a large series circuit to something much simpler.

Using rectangular notation, add up the values of the circuit below.

____

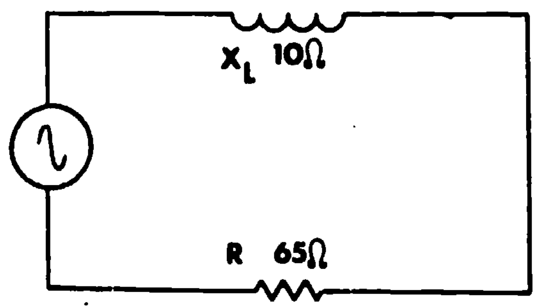


(65 + j10)

14. Values in rectangular notation represent an actual circuit containing resistance and reactance.

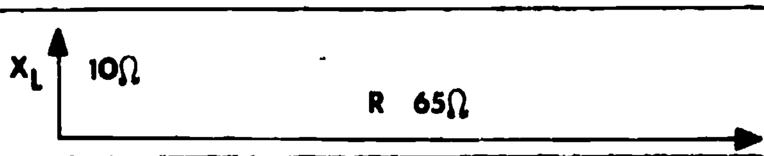
Draw a circuit indicated by the resultant values obtained from frame #13.

- -



15. Draw a vector diagram representing the circuit in frame #14.

- -



16. Rectangular notation in giving resistive and reactive values is a numerical vector.

In rectangular notation, values of the ________ and __________ sides are indicated as is the direction of _________ of the vector.

- -

(opposite; adjacent; rotation)

17. Vectors may be added only in rectangular notation.

Complete the following problems.

a.		b.		c.	
	33 + j70		70 - j50		20 - j30
	45 - j35		40 + j80		15 - j10
	15 - j20		60 + j10		10 - j10
	93 _ ___		___ _ 40		__ _ ___

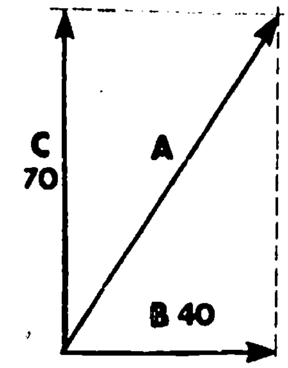
- -

(+j15; 170 + j; 45 - j50)

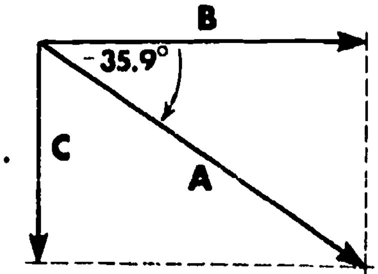
18. Polar notation of a vector is a vector identified by the value of the resultant and its angle theta.

Which of the below drawings could be written directly in polar notation?

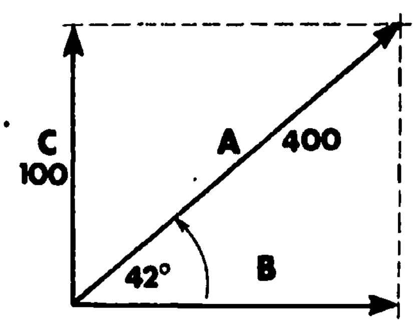
____ a.



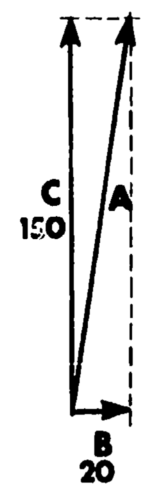
____ b.



____ c.



____ d.



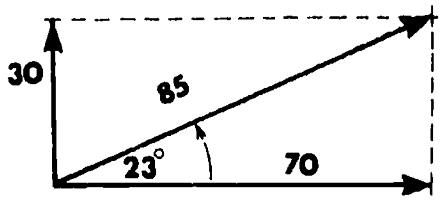
(c) 400 /42°

---

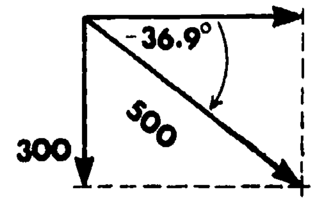
19. In multiplying vectors they must be in polar form.

Write the following resultant vector in their polar form.

a.



b.



______ ______

(a. $85 \underline{/23^\circ}$; b. $500 \underline{/-36.9^\circ}$)

20. To multiply a vector in polar form regular multiplication is first performed on the values preceding the angle.

Perform the math indicated on the preceding problem.

$$\begin{array}{ll} 85 & \underline{/23^\circ} \\ \underline{500} & \underline{/-36.9^\circ} \end{array}$$

$$(42500 \; \frac{/23^\circ}{/-36.9^\circ}$$

21. With the angles now we simply perform algebraic addition which, since their signs are different, means we really subtract.

Algebraically add -36.9° and 23° ______

(-13.9°)

22. The answer is of a negative value because the largest angle was in a negative direction. (42500 /-13.9°)

Of the following angles what would the resultants be positive or negative?

a. /-45°
/ 35°
____

b. /20°
/40°
____

c. /-40°
/-28°
____

- -

(a. /-10°; b. /60°; c. /-68°)

23. In polar form then two distinct operations are performed, multiplication and algebraic addition.

Compute the following polar notations.

a. 7 /30° x 5 /15° = ________ /45°

b. 15 /10 x 2 /-15 = 30 /____

- -

(a. 35 /45°; b. 30 /-5)

24. All vectors must be in polar form for multiplication.

Complete the following:

a. 10 /40° x 5 /15° = ____ /55°

b. 15 /-20° x 2 /50° = 30 /____

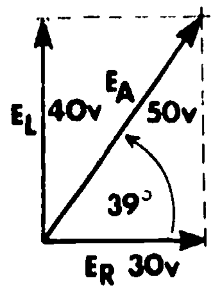
c. 20 /-30° x 3 /-20° = ________

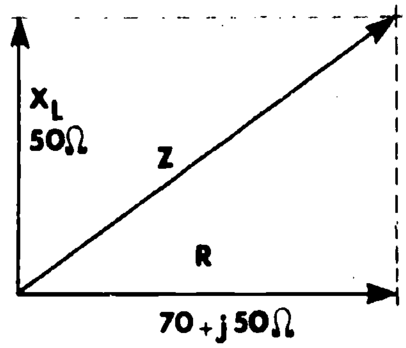
- -

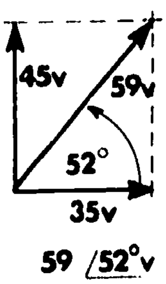
(a. 50; b. /30° ; c. 60 /-50°)

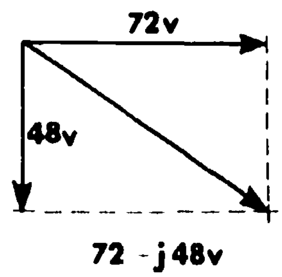
25. As with multiplication, division also can be performed only when the vector is in polar form.

Which of the following vectors are divisible in their present states?

____ a. 

____ b. 

____ c. 

____ d. 

- -

(a; c)

26. In division of polar forms the one confusing part is in the division of the angles, when the lower angle is brought above the line its signs changed and it's then added to the upper angle.

$$\frac{14\,\underline{/30}}{2\,\underline{/15}} = \frac{14}{2} = 7, \frac{\underline{/30}}{\underline{/15}}$$

The 15 in coming above the line becomes a -15 which is then "added" to a +30 so we end up with $7\underline{/15^\circ}$.

Divide the following angles.

a. $\dfrac{\underline{/10}}{\underline{/-5}}$ = ______ b. $\dfrac{\underline{/-15}}{\underline{/\ 5}}$ = ______ c. $\dfrac{\underline{/10}}{\underline{/10}}$ = ______

(a. $\underline{/15}$; b. $\underline{/-20}$; c. $\underline{/0^\circ}$)

27. Division of vectors is performed only in polar notation.

Complete the following problems.

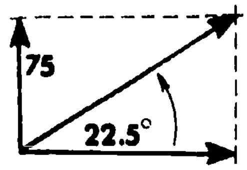
a. $\dfrac{15\,\underline{/30^\circ}}{5\,\underline{/20^\circ}}$ = ____ $\underline{/10^\circ}$ b. $\dfrac{100\,\underline{/-45^\circ}}{20\,\underline{/\ 45^\circ}}$ = 5 $\underline{/-\quad}$

c. $\dfrac{30\,\underline{/30^\circ}}{15\,\underline{/-15^\circ}}$ = ____ $\underline{/\quad}$ d. $\dfrac{25\,\underline{/-30^\circ}}{5\,\underline{/-45^\circ}}$ = ____ $\underline{/\quad}$

(a. 3; b. 90°; c. 2 $\underline{/45^\circ}$; d. 5 $\underline{/15^\circ}$)

28. When given a problem in polar notation the sine or cosine of the indicated angle theta can be used to find the resistive and reactive values. The SIN and COS values are listed in the trig table in the back of this booklet.

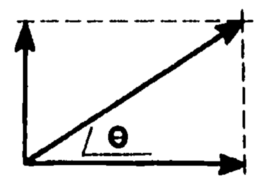
List the SIN and COS of the following angles.

a. 

SIN = ____
COS = ____

b. 320 $\underline{/-68.2°}$
SIN = ____
COS = ____

c. 100 $\underline{/90°}$
SIN = ____
COS = ____

d. 

SIN = ____
COS = ____

$\underline{/\theta}$ = 29°

hyp= 48

- -

(a. SIN 0.3827, COS 0.9239; b. SIN 0.9285, COS 0.3714;
c. SIN 1.000, COS 0000; d. SIN 0.4848, COS .8746)

29. Polar notation represents the value of the resultant and its angle theta.

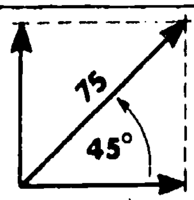
Draw the vectors representing the notations given.

a. 75 $\underline{/45°}$ b. 250 $\underline{/-32.5°}$

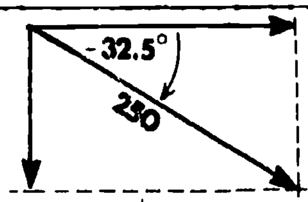
c. 150 $\underline{/65°}$ d. 48 + j55

- -

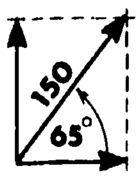
a.



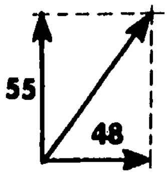
b.



c.



d.



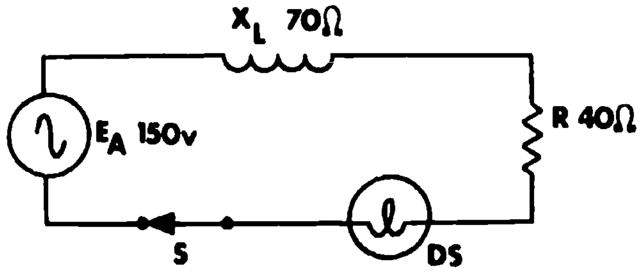
30. In converting from rectangular to polar form the following relations are used.

____ a. $X_L = Z \times \text{SIN}\ \theta$

____ b. $R = Z \times \text{COS}\ \theta$

____ c. $\text{TAN}\ \theta = \frac{X_L}{R}$

Which formula would be used in the first step to find the polar coordinates for the below circuit?



40 + j70Ω

(c)

31. By dividing X_L by R $\frac{(70)}{(40)}$, we determine the tangent function for this right triangle.

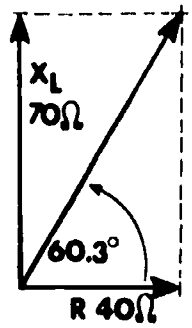
From the trig table, what is the angle? ____________

(60.3°)

32. Drawing a vector triangle to represent the above circuit values, we have all the important information listed except the SIN and COS of 60.3°.

List the SIN = ________

COS = ________



(SIN = 0.8686; COS = 0.4955)

33. The next step is to find the total impedance **Z** of the circuit.

Which of the following formulas could be used in solving for **Z**?

____ a. $X_L = Z \times \text{SIN } \underline{/\theta}$

____ b. $\text{TAN } \underline{/\theta} = \frac{X_L}{R}$

____ c. $R = Z \times \text{COS } \underline{/\theta}$

(a. $X_L = Z \times \text{SIN } \underline{/\theta}$; and/or c. $R = Z \times \text{COS } \underline{/\theta}$)

34. If formula a is used, we simply divide X_L (70 Ω) by the SIN $\underline{/\theta}$ to find **Z**.

Complete the following division: $0.8686 \overline{)70}$

(80.5)

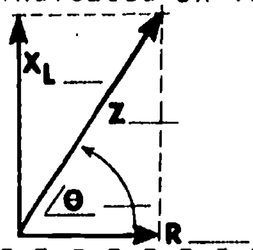
35. Had you chosen to use formula c, R divided by the COS yields the same value for $\underline{Z}$.

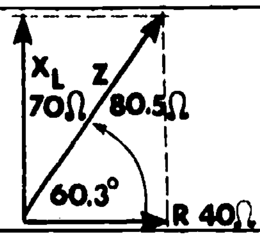
Complete the following division: $0.4955\,\overline{)40}$

(80.5Ω)

36. You now know X_L, R, Z, and the $\underline{/\theta}$, not to mention the SIN, COS and TAN of $\underline{/\theta}$.

Place correct values as indicated on vector diagram.





37. The last step in conversion from rectangular form to polar form is to show the circuit impedance in its polar form.

Which of the following correctly shows the impedance in polar form?

____ a. 40 - j70Ω ____ b. 60 $\underline{/-80}$Ω

____ c. 40 + j70Ω ____ d. 80.5 $\underline{/60.3°}$Ω

(d) 80.5 $\underline{/60.3°}$ Ω

38. In rectangular form only the resistive and reactive values are shown. In polar form only the resultant $\underline{Z}$ and the angle theta are shown.

Which of the forms are correctly shown?

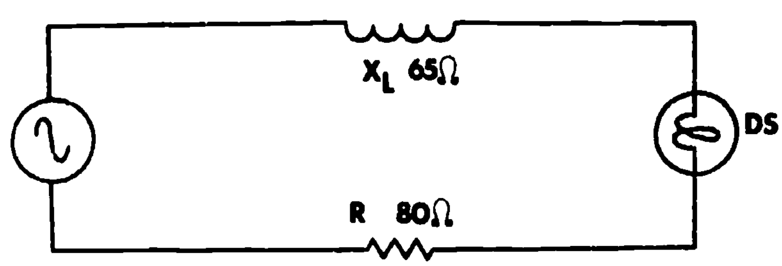
____ a. $X_L + jR$ ____ b. $X_L \underline{/\theta}$ ____ c. $R + jX_L$

____ d. $Z \underline{/\theta}$ ____ e. $\underline{/R - jX_L}$ ____ f. $R - jX_C$

(c. $R + jX_L$; d. $Z \underline{/\theta}$; f. $R - jX_C$)

---

39. The following summarizes the formulas required and the proper steps to follow to convert the following problem from rectangular notation to polar notation.



$\text{TAN } \underline{/\theta} = \frac{X_L}{R}$

$X_L = Z \text{ SIN } \underline{/\theta}$

$R = Z \text{ COS } \underline{/\theta}$

a. Solve for the circuit tangent = ____
b. Solve for the circuit angle = ____
c. Solve for SIN = ____
d. Solve for COS = ____
e. Solve for $\underline{Z}$ using either formula 1 or 2. Z $\underline{Z}$ = ____
f. Show impedance in rectangular form = ____
g. Show impedance in polar form = ____

(a. 0.8125; b. 39.1°; c. 0.6307; d. 0.7760; e. 103; f. 80 + j65; g. 103 $\underline{/39.1°}$)

40. In converting a problem from polar to rectangular form, we use the same three basic formulas as before.

List three formulas pertaining to Z, X_L, and R. SIN, COS, and TAN.

a. ______
b. ______
c. ______

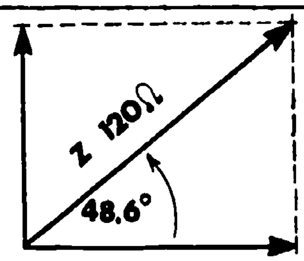
- -

(a. $\frac{X_L}{Z_T}$ = SIN $\underline{/\theta}$; b. $\frac{R}{Z_T}$ = COS $\underline{/\theta}$; c. $\frac{X_L}{R}$ = TAN $\underline{/\theta}$)

41. The problem is in polar form giving the value of $\underline{Z}$ and the phase angle. 120 $\underline{/48.6°}$.

From the above information sketch a vector triangle showing the vector length and angle.

- -



42. We have the value of $\underline{Z}$ and its angle theta. To convert to rectangular notation, we have to solve for X_L and R.

From Frame #40, select the two or more formulas you will need to solve for X_L and R.

a. ______
b. ______

- -

(a. $\frac{X_L}{Z_T} = \text{SIN } \underline{/\theta}$; b. $\frac{R}{Z_T} = \text{COS } \underline{/\theta}$)

43. It is necessary to know the SIN and COS of 48.6° for conversion.

In the trig table locate and list:

a. SIN = ______
b. COS = ______

- -

(a. 0.7501; b. 0.6613)

44. We have the SIN - COS, angle and the value of $\underline{Z}$. We still must solve for X_L and R.

Using the formula $X_L = Z \text{ SIN } \underline{/\theta}$, perform the indicated math and determine X_L ______. When SIN $\underline{/\theta}$ is 0.7506 and Z is 120.

- -

($X_L = 90\ \Omega$)

45. You now have found the +j value of the circuit. To complete the conversion, you still must solve for ______.

- -

(R)

46. Using the formula for R, you can complete the transition from polar to rectangular form.

Using the correct formula, solve for R ____________

(R = Z COS $\underline{/\theta}$ = 79.4)

47. We started with a problem in polar form and by using simple math and a trig table, we have successfully converted it into its rectangular form.

Write the just completed problem in its polar form ________
rectangular form ________

(120 $\underline{/48.6}$; 79.4 + j90)

48. We continually listed the SIN and COS of the angle during conversion, the COS has another important job. The COS $\underline{/\theta}$ equals circuits power factor (PF).

What is the power factor of a circuit having an angle of 45°?

PF = ____________

(0.707)

49. Power factor is a ratio of power used to power supplied; measure of the efficiency of the circuit.

What was the power factor of the circuit in Frame #39? ________

(0.776)

50. If the apparent power supplied by the source and the amount of power dissipated by the load resistance is known, power factor can be computed using this formula: $\frac{P_t}{P_a} = PF$.

What is the PF of a circuit with 600 va supplied and 300 watts actually dissipated? ______________

(0.5)

IF YOUR ANSWERS ARE CORRECT, YOU MAY TAKE THE PROGRESS CHECK, OR YOU MAY STUDY ANY OF THE OTHER RESOURCES LISTED. IF YOU TAKE THE PROGRESS CHECK AND ANSWER ALL THE QUESTIONS CORRECTLY, GO ON TO THE NEXT LESSON. IF NOT, STUDY ANY METHOD OF INSTRUCTION YOU WISH UNTIL YOU CAN ANSWER ALL THE QUESTIONS CORRECTLY.

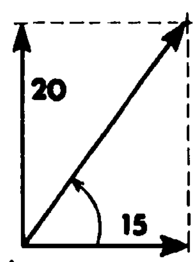
SUMMARY
LESSON III

Rectangular and Polar Notation

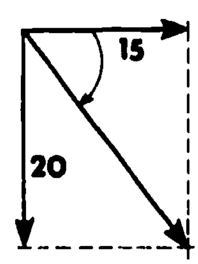
A resultant of two vectors at right angles can be described in two important ways - Rectangular Notation and Polar Notation. Each of these systems has some very important advantages, so we will study both of them.

Rectangular notation is based on the graphs you probably studied in high school and grade school. Any point can be described as "so many units left or right of zero and so many units up or down." The directions are called negative when they are left or down from zero and positive when they are right or up from zero. This is the system we have been using for our vectors with E_R (R) as positive horizontal, E_L (X_L) as positive vertical, and E_C (X_C) as negative vertical. We will not normally use the negative horizontal part.

Rectangular notation is a simple system for describing the location of a point such as the end of a vector. The horizontal position is written as a plain number and the vertical position is indicated by a number preceded by the letter j. For example, 15 + j20 describes this vector:

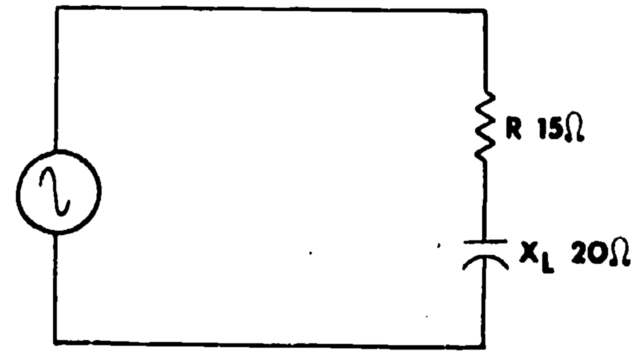


And 15 - j20 means this one:

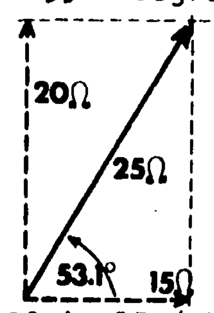


We explain this system by defining the j operator as a function which causes a vector to rotate 90°.

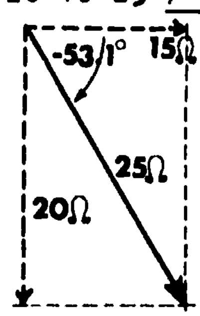
In electrical units, a series circuit containing 100 ohms of resistance and 75 ohms of inductive reactance can now be described as having an impedance of 100 + j75 ohms. Capacitive reactance (or capacitive voltage drop) is shown by the negative j, as in 20 - j15, which describes the circuit at the top of the next page.



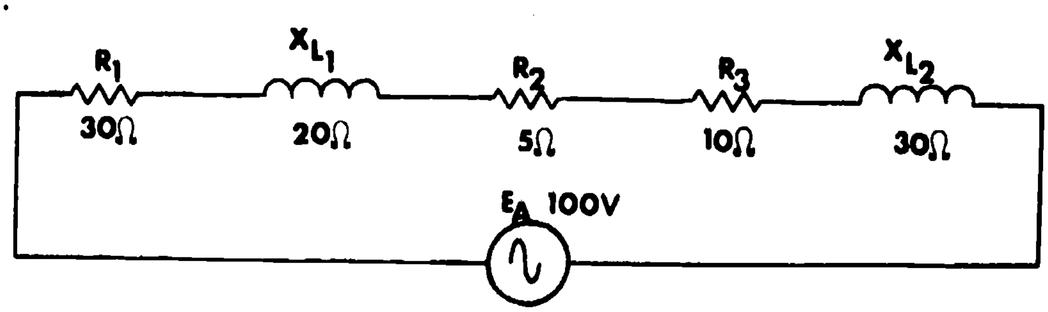
Polar notation uses the phase angle, θ, and the length of the resultant to describe the location of a point. For example, the vector 15 + j20 used in the last paragraph is 25 $\underline{/53.1°}$ (read "25 at an angle of 53.1 degrees" or just "25 at 53.1 degrees") in polar notation.



For another example, 15 - j20 is 25 $\underline{/-53.1°}$ in polar notation.



The greatest advantage of rectangular notation is that any values stated in this form can be added together easily. The series circuit below can easily be solved for total impedance using the rectangular form.

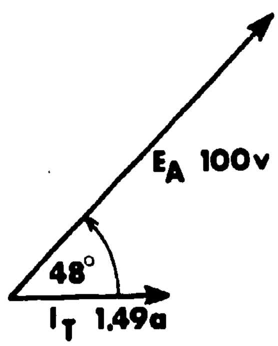


$$Z = 30 + j20 + 5 + 10 + j30 = 45 + j50 \text{ ohms}$$

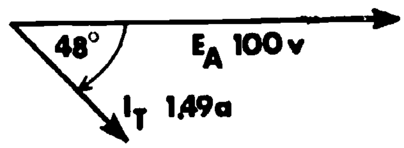
(All the numbers without j's are added together then all the numbers with j's are added algebraically.)

Vectors in polar form are easily multiplied or divided. In the example above, total current can be found by dividing the applied voltage by the impedance ($I_T = \frac{E_a}{Z}$), but this comes out $I_T = \frac{100}{45 + j50}$, a division which is very difficult to perform. If the impedance is converted to polar notation (TAN $\underline{/\theta} = \frac{X_L}{R} = \frac{50}{45} = 1.111$; $\underline{/\theta} = 48°$; $Z = \frac{X_L}{\text{SIN } \underline{/\theta}} = \frac{50}{.743} = 67.3\ \Omega\ \underline{/48°}$), the division can be done easily.

$I_T = \frac{100}{67.3\ \underline{/48°}} = 1.49\ \underline{/-48°}$ a. For division (or multiplication) the numbers are divided normally, and the angles are treated exactly like powers of ten; that is, change the sign of any angle in the divisor and add algebraically. In this example, the current is 1.49 amperes and it lags the voltage by 48 degrees. Since current is the reference for series circuits, we usually change the reference to show the voltage vector leading current by 48°.



instead of



Polar and rectangular notation and the ability to convert between them easily make AC problems very little harder to solve than DC problems.

Converting from polar form to rectangular form is very simple. Recall that COS $\underline{/\theta} = \frac{E_R}{E_T}$ and SIN $\underline{/\theta} = \frac{E_L}{E_T}$. Multiplying through by E_T we get $E_R = E_T$ COS $\underline{/\theta}$ and $E_L = E_T$ SIN $\underline{/\theta}$, so the rectangular form can be written E_T COS $\underline{/\theta}$ + jE_T SIN $\underline{/\theta}$. To convert 71 v $\underline{/45°}$ to rectangular form, first find COS $\underline{/\theta}$ (0.707) and SIN $\underline{/\theta}$ (0.707) in the tables. Then E_T COS $\underline{/\theta}$ = 50 volts and E_T SIN $\underline{/\theta}$ = 50 volts and the rectangular form is 50 + j50 volts. Another example:

Convert 35 $\underline{/-33.7°}$ to rectangular form.

1. COS $\underline{/\theta}$ = 0.8320
2. SIN $\underline{/\theta}$ = -0.5548 (the sine of a negative angle will be negative)
3. 35 v COS $\underline{/\theta}$ = 29 v
4. 35 v SIN $\underline{/\theta}$ = -j19 v
5. 29 - j19 v

Remember to use the rectangular form of vectors when you want to add or subtract and the polar from when you want to divide or multiply.

Recall that, the resistance in a circuit dissipates all the power (true power) and the voltage and current which seem to show power dissipated in reactance (reactive power) really show an exchange of energy between the reactance and the source. The vector sum of the reactive power and the true power is called apparent power, and it can be found by multiplying the applied voltage by the total current ($P_a = E_a \times I_T$).

The power factor of an AC circuit is the ratio of true power to apparent power ($PF = \frac{P_t}{P_a}$). This number represents the portion of the total available power which is actually dissipated in the circuit. Another formula for true power derived from $PF = \frac{P_t}{P_a}$ is $PF = \cos \angle\theta$, a much easier way to determine power factor when impedance or total voltage is expressed in polar form.

There are practice problems available from your instructor if you would like to try solving some circuits using these methods.

AT THIS POINT, YOU MAY TAKE THE LESSON PROGRESS CHECK, OR YOU MAY STUDY THE LESSON NARRATIVE OR THE PROGRAMMED INSTRUCTION OR BOTH. IF YOU TAKE THE PROGRESS CHECK AND ANSWER ALL OF THE QUESTIONS CORRECTLY, GO TO THE NEXT LESSON. IF NOT, STUDY ANOTHER METHOD OF INSTRUCTION UNTIL YOU CAN ANSWER ALL THE QUESTIONS CORRECTLY.

NAVPERS 94558-12a

BASIC ELECTRICITY AND ELECTRONICS

INDIVIDUALIZED LEARNING SYSTEM

MODULE TWELVE

LESSON IV

Variational Analysis of Series RL Circuit

Study Booklet

Bureau of Naval Personnel

January 1972

OVERVIEW

LESSON IV

Variational Analysis of Series RL Circuit

In this lesson you will study and learn about the following:

- -result of changing frequency
- -result of changing resistance
- -result of changing applied voltage
- -result of changing inductance

BEFORE YOU START THIS LESSON, PREVIEW THE LIST OF STUDY RESOURCES ON THE NEXT PAGE.

LIST OF STUDY RESOURCES

LESSON IV

Variational Analysis of Series RL Circuit

To learn the material in this lesson, you have the option of choosing, according to your experience and preferences, any or all of the following:

STUDY BOOKLET:

Lesson Narrative

Programmed Instruction

Lesson Summary

ENRICHMENT MATERIAL:

NAVPERS 93400A-1b "Basic Electricity, Alternating Current." Fundamentals of Electronics. Bureau of Naval Personnel. Washington, D.C.: U.S. Government Printing Office, 1965.

YOU MAY NOW STUDY ANY OR ALL OF THE RESOURCES LISTED ABOVE. YOU MAY TAKE THE PROGRESS CHECK AT ANY TIME.

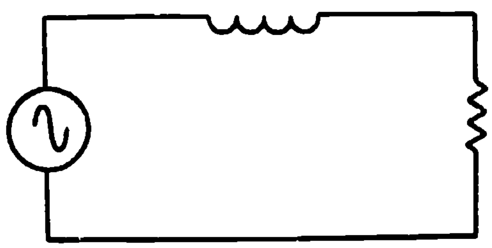
NARRATIVE
LESSON IV

Variational Analysis of Series RL Circuit

Equally as important as vector representation is a thorough understanding of what happens to other quantities in a series AC RL circuit if one quantity is changed.

There are four quantities that, when changed, may cause other RL circuit quantities to increase or decrease. These four variables are: (1) frequency, (2) resistance, (3) applied voltage, and (4) inductance. Let's analyze what happens if any one of these variables is increased or decreased.

Changing Frequency



Assume that f is increased in this circuit, and analyze what would happen to each of the quantities listed in the variational analysis table.

A good starting point is to ask what quantity changes first.

To help you recall the quantity relationships, here are the formulas that are pertinent to this circuit.

$$X_L = 2\pi fL$$

$$I_T = \frac{E_a}{Z_T}$$

$$P_t = I_T{}^2R = I_T \times E_a \times \text{COS } \underline{/\theta}$$

$$P_x = I_T \times E_a \times \text{SIN } \underline{/\theta} = I^2X_L$$

$$P_a = E \times I$$

$$Z_T = R + jX_L$$

f	↑
Z_T	
X_L	
I_T	
E_R	
E_L	
P_T	
P_X	
P_A	
$\underline{/\theta}$	
PF	
R	
L	

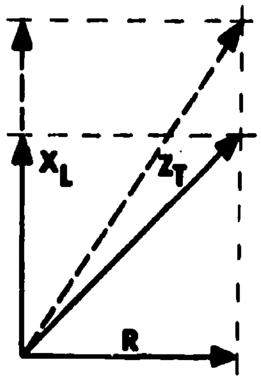
What Happens to X_L

If f increases, the first quantity affected is X_L. The formula for X_L verifies that X_L is directly proportional to f; so if f increases, X_L increases. This is indicated in the table beside X_L with an arrow showing increases (↑).

f	↑
Z_T	
X_L	↑

What Happens to Z_T

If X_L increases, what happens to total impedance? The best way to see this is to sketch the impedance triangle for an RL circuit. Always draw the triangle; it is very helpful in seeing the relationships. Extend the X_L vector to indicate an increase, and draw the new hypotenuse. Observe that when X_L increases, Z_T increases. This is indicated by placing an arrow in the box (↑).



f	↑
Z_T	↑
X_L	↑

What Happens to I_T

By Ohm's Law, as total opposition increases, total current decreases. Therefore, the arrow beside I_T indicates a decrease (↓).

f	↑
Z_T	↑
X_L	↑
I_T	↓
E_R	

What Happens to P_t and P_a

After determining what happens to I_T, it is a simple matter to skip ahead to true power and apparent power and mark these items. Because the formula for true power is $P_t = I_T^2 R$, if $\underline{I}$ decreases, true power decreases.

Similarly, by the formula $P_a = E_a \times I_T$, if current decreases, apparent power decreases.

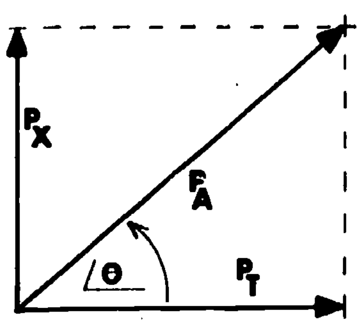
A rule to remember in analyzing a series RL circuit is that whatever I does -- increase or decrease -- P_a and P_x also does.

$P_a = I \times E$

$P_x = I \times E \times \text{SIN } \underline{/\theta}$

$P_t = I \times E \times \text{COS } \underline{/\theta}$

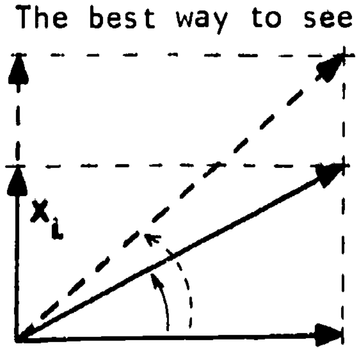
or I^2R



f	↑
Z_T	↑
X_L	↑
I_T	↓
E_R	
E_L	
P_T	↓
P_X	↓
P_A	↓
$\underline{/\theta}$	
PF	
R	
L	

What Happens to $\underline{/\theta}$ and PF

The best way to see what happens to $\underline{/\theta}$ is to look at the impedance triangle. Observe that as X_L increases, $\underline{/\theta}$ increases.



Whatever $\underline{/\theta}$ does, power factor does the opposite; therefore, PF decreases. This is quite understandable when you consider the formula $PF = \frac{P_t}{P_a}$.

If X_L increases, the ratio of reactance to resistance increases. Since a purely reactive circuit has a PF of 0, the more reactive a circuit is, the smaller the PF. If reactance increases, PF decreases.

f	↑
Z_T	↑
X_L	↑
I_T	↓
E_R	
E_L	
P_T	↓
P_X	↓
P_A	↓
$\angle\theta$	↑
PF	↓
R	
L	

<u>What Happens to Voltage Drops</u>

E_R and E_L are the two circuit quantities most likely to cause confusion. However, they are really no problems if you remember the two rules for voltage drops in a series circuit:

(1) The greatest voltage drop occurs across the greatest opposition.

(2) The vector sum of the voltage drops is equal to the applied voltage.

If I_T decreases, E_R decreases ($E_R = I \times R$). If E_R decreases and E_a remains constant, then E_L increases, since the vector sum of the voltage drops equals E_a.

What Happens to R and L

f	↑
Z_T	↑
X_L	↑
I_T	↓
E_R	↓
E_L	↑
P_T	↓
P_X	↓
P_A	↓
$\underline{/\theta}$	↑
PF	↓
R	→
L	→

Because resistance and inductance are physical properties, they can only be changed physically. Increasing f affects neither R nor L.

Decreasing R

To analyze what happens if resistance varies, decrease the amount of resistance in the same AC RL series circuit.

Keep these formulas in mind:

$$X_L = 2\pi f L$$

$$I_T = \frac{E_a}{Z_T}$$

$$P_t = I_T^2 R = I_T \times E_a \times \mathrm{COS}\ \underline{/\theta}$$

$$P_x = I_T \times E_a \times \mathrm{SIN}\ \underline{/\theta} = I^2 X_L$$

$$P_a = E \times I$$

$$Z_T = R + jX_L$$

R	↓
Z_T	↓
X_L	→
I_T	↑
E_R	↓
E_L	↑
P_T	*
P_X	↑
P_A	↑
$\underline{/\theta}$	↑
PF	↓
R	↓
L	→

*In a series RC circuit, varying R causes the true power to change in a complex way, so you are not expected to solve this special case.

If resistance is decreased:

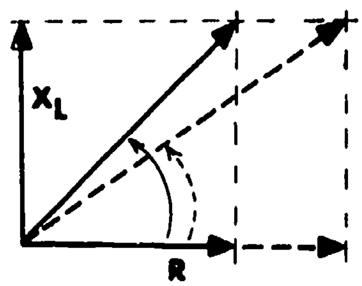
Z_T decreases.

X_L is not affected.

I_T increases (Ohm's Law).

P_t, P_x and P_a do what I_T does.

$\underline{/\theta}$ increases.



To see this clearly, draw the impedance triangle.

Decrease the R vector to show the resistance decrease. Note that $\underline{/\theta}$ increases as R decreases.

PF decreases.

L remains the same.

E_L increases (because I_T increases).

E_R decreases, because the vector sum of the voltage drops must equal E_a.

Conduct a variational analysis to show what happens if

(1) E_a' is increased.

(2) $\underline{L}$ is decreased.

	1. $E_A \uparrow$		2. $L \downarrow$
Z_T			
X_L			
I_T			
E_R			
E_L			
P_T			
P_X			
P_A			
$\angle\theta$			
PF			
R			
L			

Recall the pertinent formulas, and in each case draw either the impedance or voltage triangle.

Check answers on next page.

Answers to Variational Analysis Table:

	1. $E_A\uparrow$	2. $L\downarrow$
Z_T	$\rightarrow$	$\downarrow$
X_L	$\rightarrow$	$\downarrow$
I_T	$\uparrow$	$\uparrow$
E_R	$\uparrow$	$\uparrow$
E_L	$\uparrow$	$\downarrow$
P_T	$\uparrow$	$\uparrow$
P_X	$\uparrow$	$\uparrow$
P_A	$\uparrow$	$\uparrow$
$\angle\theta$	$\rightarrow$	$\downarrow$
PF	$\rightarrow$	$\uparrow$
R	$\rightarrow$	$\rightarrow$
L	$\rightarrow$	$\downarrow$

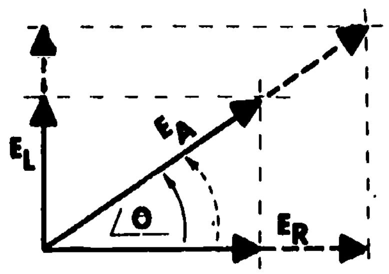
Increasing E_a

Remember that we have been stressing that if E_a is increased, X_L, Z_T, R, and L all remain the same.

Since E_a is increased and Z_T is not changed, I_T increases according to Ohm's Law.

If I_T increases, then both E_L and E_R increases. P_t, P_x and P_a increase as does I_T.

Draw the voltage triangle to see what happens to /θ. Notice θ does not change when you increase E_a because E_R and E_L increase proportionally.



Since /θ doesn't change, PF remains the same.
Being physical properties, R and L do not change.

Decreasing L

When L is decreased;

X_L decreases.

Z_T decreases.

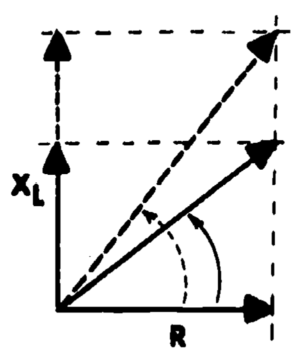
Therefore, I_T increases.

E_R increases because I_T increases.

E_L decreases.

P_t, P_x and P_a increases as I.

/θ decreases as shown in the triangle.



PF increases since /θ decreases, and R remains the same.

Conduct a variational analysis of an AC RL series circuit by placing arrows in the table to show what happens to circuit quantities.

IF:		f ↓	R ↑	E_A ↑	L ↑
THEN:	Z_T				
	X_L				
	I_T				
	E_R				
	E_L				
	P_T				
	P_X				
	P_A				
	$\angle\theta$				
	PF				
	R				
	L				

Check answers on next page.

Answers to Variational Analysis Table:

	f ↓	R ↑	E_A ↑	L ↑
Z_T	↓	↑	→	↑
X_L	↓	→	→	↑
I_T	↑	↓	↑	↓
E_R	↑	↑	↑	↓
E_L	↓	↓	↑	↑
P_T	↑		↑	↓
P_X	↑	↓	↑	↓
P_A	↑	↓	↑	↓
$\angle\theta$	↓	↓	→	↑
PF	↑	↑	→	↓
R	→	↑	→	→
L	→	→	→	↑

AT THIS POINT, YOU MAY TAKE THE PROGRESS CHECK, OR YOU MAY STUDY ANY OF THE OTHER RESOURCES LISTED. IF YOU TAKE THE PROGRESS CHECK AND ANSWER ALL OF THE QUESTIONS CORRECTLY, GO TO THE NEXT LESSON. IF NOT, STUDY ANY METHOD OF INSTRUCTION YOU WISH UNTIL YOU CAN ANSWER ALL THE QUESTIONS CORRECTLY.

PROGRAMMED INSTRUCTION
LESSON IV

Variational Analysis of Series RL Circuit

To fully understand series RL circuits, a good understanding of the relationship between resistance, inductance, frequency, current, and power is necessary.

We have worked the different equations, diagrams, and rules that pertain to an RL circuit. The characteristics of the different circuit components react differently to changes in frequency. Voltage changes also cause changes to some circuit conditions.

Vectors, right triangles, and formulas have all been used by you and are vital for solving reactive circuits. Another method to present circuit condition changes, when one of the fundamental factors is varied, is variational analysis. It clearly shows each component's dependence on the other components and circuit factors.

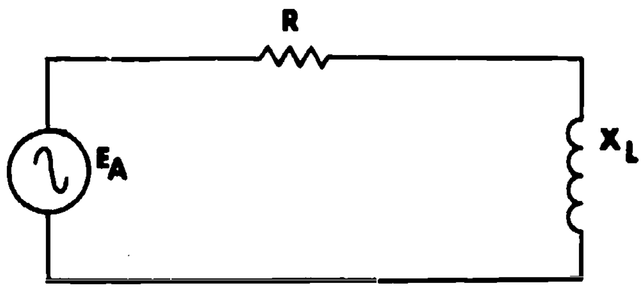
The fundamental factors or variables are:

1. frequency.
2. voltage (source).
3. resistance.
4. inductance.

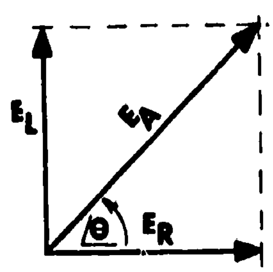
Remember: When working on a complex circuit have the equations and vectors in front of you; don't try to remember all the components' actions.

TEST FRAMES ARE 4 AND 18. AS BEFORE, GO FIRST TO TEST FRAME 4 AND SEE IF YOU CAN ANSWER ALL THE QUESTIONS THERE. FOLLOW THE DIRECTIONS GIVEN AFTER THE TEST FRAME.

1. Using the basic AC RL circuit with two components below,

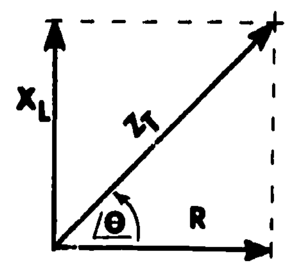


draw a voltage vector triangle and label the quantities (E_a, E_R and E_L). Indicate angle theta.



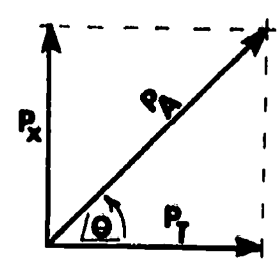
2. Using the same circuit, draw an impedance vector diagram. Indicate X_L, R, Z_T, and the angle theta.

- -



3. Again, using the same circuit, draw a power vector diagram. Indicate P_t, P_x, P_a and the angle theta.

- -



4. Let's not stop with vectors; let's list all the major equations we can use to simplify solving a series AC RL circuit.

Complete the following equations using the dependent variables indicated in parentheses.

X_L = ______________ (fL)

Z_T = ______________ (R, jX_L) = ______________ (R, X_L)

COS $\underline{/\theta}$ = ______________ (R, Z_T)

E_a = ______________ (I, Z_T) = ______________ (E_R, E_L)

E_R = ______________ (I, R)

E_L = ______________ (I, X_L)

P_t = ______________ (E, COS $\underline{/\theta}$, I = ______________ (I, R)

P_a = ______________ (E_a, I) = ______________ (Z_T, I)

P_x = ______________ (E_a, I_T, SIN $\underline{/\theta}$)

PF = ______________ (P_t, P_a) = ______________ (COS $\underline{/\theta}$)

- -

(THIS IS A TEST FRAME. COMPARE YOUR ANSWERS WITH THE CORRECT ANSWERS GIVEN AT THE TOP OF THE NEXT PAGE.)

ANSWERS - TEST FRAME 4

$X_L = 2\pi fL$

$Z_T = R + jX_L = \sqrt{(R)^2 + (X_L)^2}$

$\cos \underline{/\theta} = \frac{R}{Z_T}$

$E_a = I \times Z_T = \sqrt{(E_R)^2 + (E_L)^2}$

$E_R = IR$

$E_L = I\ X_L$

$P_t = E_a \times I_T \times \cos \underline{/\theta} = I_T{}^2R$

$P_a = E_a \times I = I^2Z_T$

$P_x = I_T \times E_a \times \sin \underline{/\theta}$ or I^2X_L

$PF = \frac{P_t}{P_a} = \cos \underline{/\theta}$

You may use this answer page for reference in studying the rest of this lesson.

IF YOUR ANSWERS ARE NOT CORRECT, REVIEW LESSONS I TO IV OF THIS MODULE. IF THEY ARE CORRECT, PROCEED TO FRAME 18.

5. Using vectors will increase the ease and accuracy of solving for variables.

   What happens to X_L if frequency is increased?

   Formula: $X_L = 2\pi fL$

   X_L ________

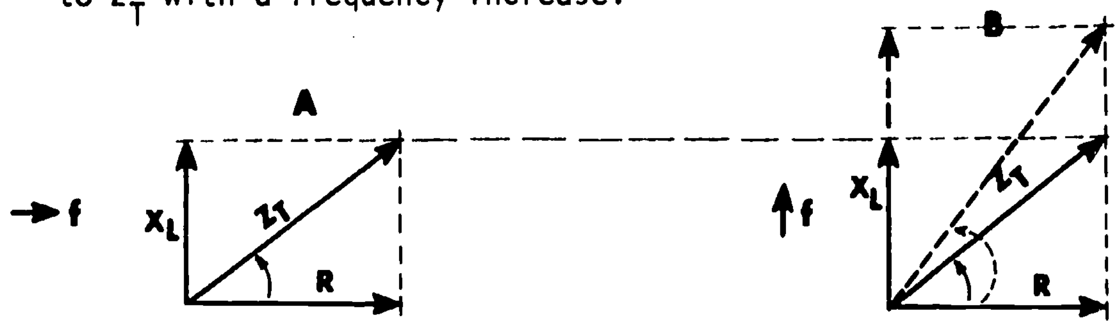
(increases)

6. In an RL circuit, what happens to <u>R</u> with a frequency increase?

______________ __

(remains the same)

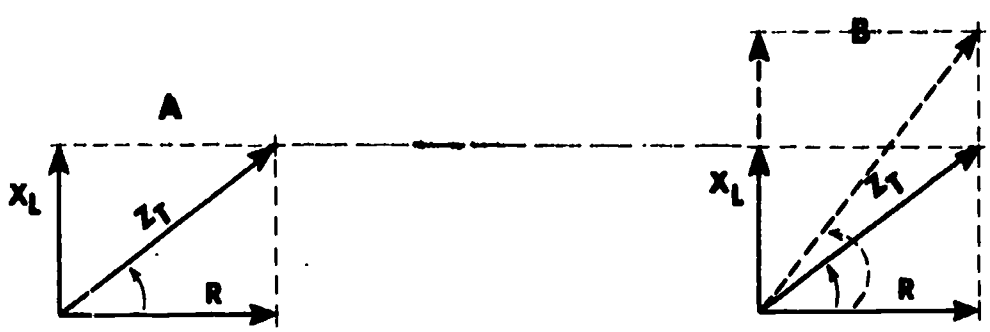
7. Now we can draw an impedance vector and determine what happens to Z_T with a frequency increase.



Comparing drawing B to drawing A, you see that X_L has increased and Z_T has ____________.

(increased)

8. Using vector diagram A again as the reference, you can see that the phase angle has changed with a frequency increase.

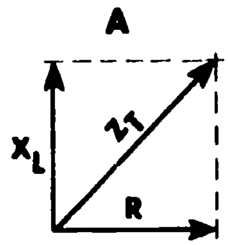


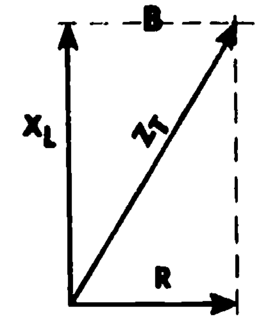
With an increase in frequency, X_L increases, Z_T increases, and $\underline{/\theta}$ ______________.

(increases)

9. Since current thr.gh the circuit depends on E_a and Z_T, and Z_T depends on X_L and frequency, changing frequency has far reaching effects.

In reference to vector diagram A, the current in the circuit in vector diagram B has ______________.

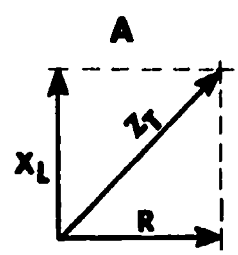


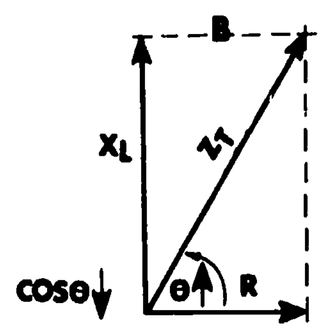


(decreased; $I_T = \frac{E_a}{Z}$, $Z_T \uparrow$, $E_a \rightarrow$, $I_T \downarrow$)

---

10. Since P_t and P_a depend on I_T, a frequency change has other effects.





f↑, $X_L\uparrow$, $Z_T\uparrow$, $\underline{/\theta}\uparrow$, and $I_T\downarrow$, so true power ($E_a \times I_T \times \cos \underline{/\theta}$) through the circuit ______________.

(decreases; $E_a \times I_T \times \cos \underline{/\theta} = P_t$, $E_a\rightarrow$, $I_T\downarrow$, $\cos \underline{/\theta}\downarrow = P_t\downarrow$)

11. True power decreases with a frequency increase, so it stands to reason that P_a and P_x also are affected.

P_a and P_x __________, since P_a = __________ and P_x = __________.

(decrease) ($E_a \times I_T = P_a$) ($E_a \times I_T \times \text{SIN } \underline{/\theta}$)

---

12. With a frequency increase, you have seen that $X_L\uparrow$, $Z_T\uparrow$,

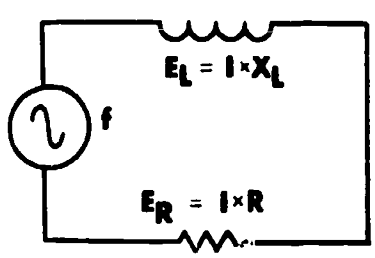
$I_T\downarrow$, $\underline{/\theta}\uparrow$, COS $\downarrow$, $P_t\downarrow$, $P_a\downarrow$, $P_x\uparrow$ and of course $R\rightarrow$.

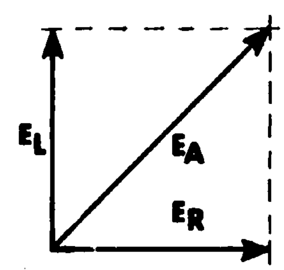
Why is R not affected by the frequency change? __________

(R is a physical factor)

---

13. Now, look at E_a, E_R, and E_L with a frequency change.



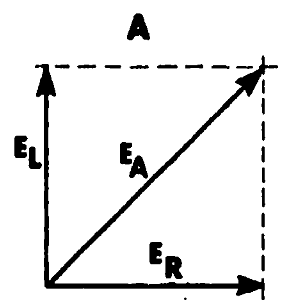


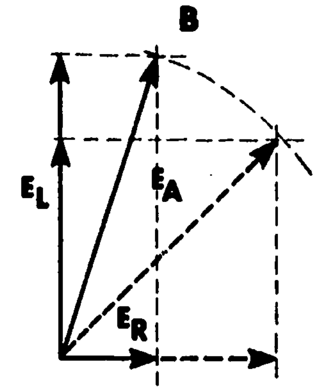
First using the formulas for E_R, E_L, and E_a, and then voltage vector diagrams, determine what happens to the three factors.

Increasing frequency causes I_T to __________.
I_T __________ causes E_R to decrease.
E_L __________ because E_R __________ and E_a __________ because it was not physically changed.

(decrease; decreasing; increases; decreases; remains the same)

14. Look at the vector diagrams below to see the effect of an increase in frequency upon the voltage in a series RL circuit.





Vector diagram B in reference to A shows that E_a ________ even through E_L increases and E_R decreases.

(remains the same)

15. Increasing frequency has increased Z_T and E_a has remained constant.

Recalling what you have learned thus far, anytime opposition is raised and source voltage held constant, then I_T must ________.

(decrease)

16. Using a basic series AC RL circuit, let's draw a table listing all circuit components and factors; indicate with arrows what happens throughout the circuit when <u>frequency</u> is increased.

f	R	L	X_L	Z_T	I_T	E_A	E_L	E_R	$\angle\theta$	P_X	P_T	P_A	PF	$\cos\theta$
↑														

(Use vectors, formulas, or whatever means you best understand to determine what happens. Start with the basic component, i.e., <u>R</u>. You know <u>R</u> is a physical property and not affected by frequency, so it remains unchanged →.)

- -

---

f	R	L	X_L	Z_T	I_T	E_A	E_L	E_R	$\angle\theta$	P_X	P_T	P_A	PF	$\cos\theta$
↑	→	→	↑	↑	↓	→	↑	↓	↑	↓	↓	↓	↓	↓

---

17. Again using a basic RL circuit, determine what happens to the various factors when <u>L</u> is <u>decreased</u> ↓.

L	R	f	X_L	Z_T	I_T	E_A	E_L	E_R	$\angle\theta$	P_X	P_T	P_A	PF	$\cos\theta$
↓														

As before, use all means available to solve and understand why these effects take place.

- -

L	R	f	X_L	Z_T	I_T	E_A	E_L	E_R	$\angle\theta$	P_X	P_T	P_A	PF	$\cos\theta$
↓	→	→	↓	↓	↑	→	↓	↑	↓	↑	↑	↑	↑	↑

---

18. As a review, let's conduct a variational analysis of a series AC RL circuit.

Place arrows in table to correctly indicate changes.

	Z_T	X_L	I_T	E_L	E_A	E_R	$\angle\theta$	$\cos\theta$	P_X	P_T	P_A	PF	L
f ↓													
E_A ↑													
L ↓													
f ↑													
R ↑													
L ↑													

(THIS IS A TEST FRAME. COMPARE YOUR ANSWERS WITH THE CORRECT ANSWERS GIVEN AT THE TOP OF THE NEXT PAGE.)

ANSWERS - TEST FRAME 18

	Z_T	X_L	I_T	E_L	E_A	E_R	$\angle\theta$	cos θ	P_X	P_T	P_A	PF	L
f ↓	↓	↓	↑	↓	→	↑	↓	↑	↑	↑	↑	↑	→
E_A ↑	→	→	↑	↑	↑	↑	→	→	↑	↑	↑	→	→
L ↓	↓	↓	↑	↓	→	↑	↓	↑	↑	↑	↑	↑	↓
f ↑	↑	↑	↓	↑	→	↓	↑	↓	↓	↓	↓	↓	→
R ↑	↑	→	↓	↓	→	↑	↓	↑	↓	[redacted]	↓	↑	→
L ↑	↑	↑	↓	↑	→	↓	↑	↓	↓	↓	↓	↓	↑

- -

IF ANY OF YOUR ANSWERS IS <u>INCORRECT</u>, GO BACK TO FRAME 5 AND TAKE THE PROGRAMMED SEQUENCE.

IF YOUR ANSWERS ARE CORRECT, YOU MAY TAKE THE PROGRESS CHECK, OR YOU MAY STUDY ANY OF THE OTHER RESOURCES LISTED. IF YOU TAKE THE PROGRESS CHECK AND ANSWER ALL THE QUESTIONS CORRECTLY, GO TO THE NEXT LESSON. IF NOT, STUDY ANY METHOD OF INSTRUCTION YOU WISH UNTIL YOU CAN ANSWER ALL THE QUESTIONS CORRECTLY.

SUMMARY
LESSON IV

Variational Analysis of Series RL Circuit

There are four circuit values that, when changed, may cause other circuit quantities to vary. These variables are: frequency, resistance, applied voltage, and inductance.

A convenient starting point in our study is to determine what circuit factors are affected directly.

X_L is directly related to frequency, so if frequency is increased X_L increases proportionally.

$$X_L = 2\pi f L$$

↑ ↑

Inductive reactance is a part of the total opposition to current flow; therefore, if X_L increases, the total opposition, Z, must increase.

$$Z = R + jX_L$$

↑ ↑

If impedance increases, circuit current decreases.

$$I = \frac{E}{Z}$$

↓ ↑

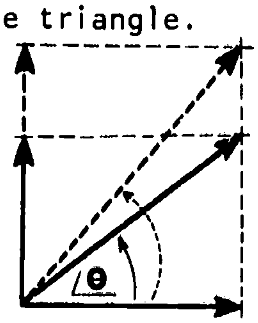
After determining what happens to I_T, it is a simple matter to determine what happens to P_t, P_x and P_a.

$$\underset{\downarrow}{P_t} = \underset{\downarrow}{I_T}^2 R = \underset{\downarrow}{I_T} \times E_a \times \text{COS } \underline{/\theta}$$

$$\underset{\downarrow}{P_x} = \underset{\downarrow}{I_T} \times E_a \times \text{SIN } \underline{/\theta} = I^2 X_L$$

$$\underset{\downarrow}{P_a} = \underset{\downarrow}{I_T} \times E_a$$

The best way to see what happens to $\underline{/\theta}$ and PF is to consider the impedance triangle.



As the reactive side of the triangle increases, the circuit phase angle increases. As $\underline{/\theta}$ increases, COS θ decreases. Since PF is equal to COS $\underline{/\theta}$, PF decreases.

E_R and E_L are the values most likely to cause confusion. However, there should be no real problem if you remember the rules concerning voltage drops in series circuits.

1. The greatest voltage drop occurs across the greatest opposition.
2. The vector sum of the voltage drops equals the applied voltage ($E_a = E_R + jE_L$).

If I_T decreases and R does not change, E_R decreases. Then, since E_a does not change, E_L increases.

Looking at these changes in block form, you can get a clearer picture of what happens to an RL circuit when frequency is increased.

f	↑
X_L	↑
R	→
Z	↑
I_T	↓
P_A	↓
P_X	↓
P_T	↓
$\angle\theta$	↑
PF	↓
E_R	↓
E_L	↑
L	→

The next variable to be considered is resistance.

If resistance is increased the total circuit opposition, increases.

$$Z = R + jX_L$$

↑ ↑

When impedance increases, circuit current decreases.

$$\underset{\downarrow}{I} = \frac{E}{\underset{\uparrow}{Z}}$$

If circuit current decreases, P_t, P_x and P_a also decrease.

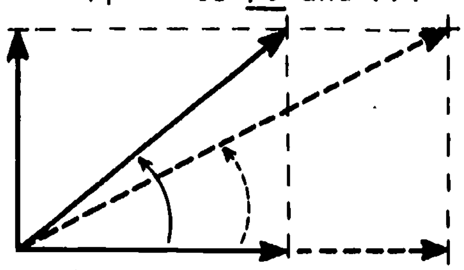
$$\underset{\downarrow}{P_t} = \underset{\downarrow}{I_T}^2 R = \underset{\downarrow}{I_T} \times E_a \times \cos \underline{/\theta}$$

(Note in first formula that the decrease is the square of the current.)

$$\underset{\downarrow}{P_x} = \underset{\downarrow}{I_T} \times E_a \times \sin \underline{/\theta} = I^2 X_L$$

$$\underset{\downarrow}{P_a} = \underset{\downarrow}{I} \times E$$

Once again the impedance triangle may be used to visualize what happens to $\underline{/\theta}$ and PF.



As you can see, the circuit becomes more resistive and $\underline{/\theta}$ decreases. Cosine $\underline{/\theta}$ and PF increases.

Since X_L doesn't change and I_T decreases, E_L decreases. If E_L decreases, then E_R increases.

$$\underset{\rightarrow}{E_a} = \underset{\uparrow}{E_R} + j\,\underset{\downarrow}{E_L}$$

In block form, this is what happens to a series RL circuit when resistance is increased:

R	↑
X_L	→
Z_T	↑
I_T	↓
P_T	*
P_X	↓
P_A	↓
$\underline{/\theta}$	↓
PF	↑
E_R	↑
E_L	↓
L	→

* Changing resistance in a series circuit affects power in a complex way, so you will not be required to answer this question.

Variations in inductance affect circuit conditions in the same manner as a comparable change in frequency because inductive reactance is directly proportional to both.

Changes in circuit values caused by variations in E_a are perhaps the easiest to predict. Any change in E_a causes a comparable change in all voltage drops, circuit current, and power. PF does not change because the circuit phase angle does not change.

Here in block form is a list of the changes in circuit conditions for variations in L and E_a.

	E_A ↑	L ↓
Z	→	↓
X_L	→	↓
I_T	↑	↑
P_T	↑	↑
P_X	↑	↑
P_A	↑	↑
$\angle\theta$	→	↓
E_R	↑	↑
E_L	↑	↓
PF	→	↑
L	→	↓
R	→	→

AT THIS POINT, YOU MAY TAKE THE LESSON PROGRESS CHECK, OR YOU MAY STUDY THE LESSON NARRATIVE OR THE PROGRAMMED INSTRUCTION OR BOTH. IF YOU TAKE THE PROGRESS CHECK AND ANSWER ALL OF THE QUESTIONS CORRECTLY, GO TO THE NEXT LESSON. IF NOT, STUDY ANOTHER METHOD OF INSTRUCTION UNTIL YOU CAN ANSWER ALL THE QUESTIONS CORRECTLY.

NAVPERS 94558-12a

BASIC ELECTRICITY AND ELECTRONICS
INDIVIDUALIZED LEARNING SYSTEM

MODULE TWELVE
LESSON V

Frequency Discrimination in RL Circuits

Study Booklet

Bureau of Naval Personnel

January 1972

OVERVIEW
LESSON V

Frequency Discrimination in RL Circuits

In this lesson you will study and learn about the following:

- -effects of frequency on RL circuits
- -frequency cutoff point
- -determining f_{co}
- -filters

BEFORE YOU START THIS LESSON, PREVIEW THE LIST OF STUDY RESOURCES ON THE NEXT PAGE.

LIST OF STUDY RESOURCES

LESSON V

Frequency Discrimination in RL Circuits

To learn the material in this lesson, you have the option of choosing according to your experience and preferences, any or all of the following:

STUDY BOOKLET:

- Lesson Narrative
- Programmed Instruction
- Lesson Summary

ENRICHMENT MATERIAL:

NAVPERS 93400A-1b "Basic Electricity, Alternating Current." Fundamentals of Electronics. Bureau of Naval Personnel. Washington, D.C.: U.S. Government Printing Office, 1965.

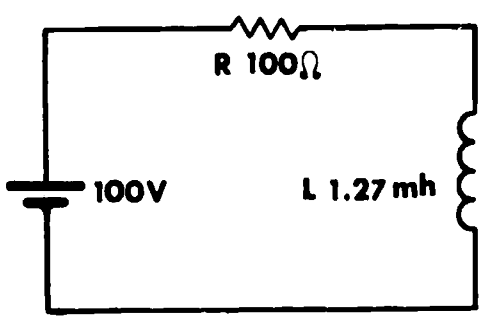
YOU MAY NOW STUDY ANY OR ALL OF THE RESOURCES LISTED ABOVE. YOU MAY TAKE THE PROGRESS CHECK AT ANY TIME.

NARRATIVE
LESSON V

Frequency Discrimination in RL Circuits

At this point, consider what happens in a series RL circuit when the frequency is varied.

Series DC RL Circuit



First, consider a circuit with a DC source after five time constants.

How much X_L do we have in the above circuit? ____________

What is total impedance? ________

This is a DC circuit so the applied frequency is zero. The formula for X_L is $X_L = 2\pi fL$; therefore, if there is no frequency, there can be no X_L. X_L is zero. Recall that X_L is the opposition an inductor offers to AC.

Since the circuit contains no X_L, the only opposition in the circuit is 100 Ω of resistance. Z_T is 100 Ω.

For the circuit above, determine the values for:

I. ____________

E_R. ____________

E_L. ____________

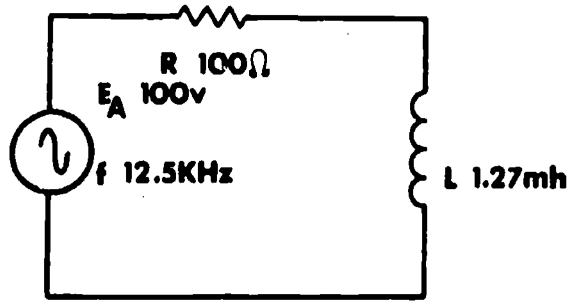
P_T. ____________

By Ohm's Law, if E_a is 100 v and Z_T is 100 Ω, then I is 1 amp. Since X_L is 0, there will be no voltage drop across the coil and the full 100 v is dropped across R.

Power equals I^2R ($1^2 \times 100$ Ω) or 100 watts.

A Circuit With AC Applied

Now, if you replace the battery in this RL circuit with an AC source, you increase the frequency from 0 to some value. Let's see how this affects circuit quantities.



X_L: As frequency increases, X_L increases from 0 to some value which can be computed by the formula:

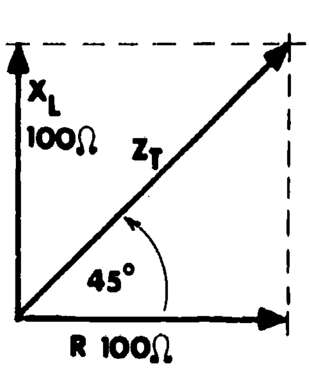
$$X_L = 2\pi f L$$

$$X_L = (6.28)\ (12.5 \times 10^3)\ (1.27 \times 10^{-3})$$

$$X_L \simeq 100\ \Omega$$

Notice that in this circuit and at this particular frequency, X_L and R are equal: $X_L = 100\Omega$ $R = 100\Omega$

Z_T: To find total impedance in the circuit, we construct the LR impedance triangle.



By dividing either the SIN θ or COS θ (which are equal in triangles of equal sides) into either R or X_L ($Z = \frac{X_L}{\text{SIN } \theta}$ or $Z = \frac{R}{\text{COS } \theta}$), we arrive at the value of the hypotenuse, Z_T.

$$Z_T = \frac{100\Omega}{0.7071}$$

$$Z_T = 141.4\Omega$$

$\underline{\theta}$: Angle theta of an RL circuit with <u>equal</u> resistance and inductive reactance is always 45°.

$\underline{I_T}$: By Ohm's Law, if Z_T is 141.4 Ω then;

$$I_T = \frac{100\ v}{141.4\ \Omega} = 0.707 \text{ amps}$$

$$E_R = 100\ \Omega \times 0.707 \text{ amps} = 70.7\ v$$

$$\theta = 45°$$

$$PF = \cos\theta \text{ or } 0.707$$

$$P_a = E \times I \text{ or } 70.7\ va$$

$$P_x = E \times I \times \sin\underline{/\theta} \text{ or } 70.7 \text{ vars}$$

$$P_t = I^2R \text{ or } 50\ w$$

or

$$P_t = P_a \cos\underline{/\theta} \text{ or } 50\ w$$

<u>Comparing Effects of Frequency</u>

Compare the values of quantities in an RL circuit with 0 frequency (DC) and with some frequency applied (AC).

NO FREQUENCY	FREQUENCY APPLIED	CHANGE
$f = 0$	$f = 12.5$ KHz	↑
$X_L = 0$	$X_L = 100\ \Omega$	↑
$Z_T = 100\ \Omega$	$Z_T = 141.4\ \Omega$	↑
$I_T = 1a$	$I_T = 0.707\ a$	↓
$E_R = 100\ v$	$E_R = 70.7\ v$	↓
$E_L = 0$	$E_L = 70.7\ v$	↑
$P = 100\ w$	$P_t = 50\ w$	↓

Frequency Cutoff Point

Notice particularly that the applied frequency has reached a point where E_R and E_L are equal, and X_L and R are equal. When the circuit reaches this condition, it is at the frequency cutoff (f_{co}) point.

Another term often used to designate f_{co} is the half-power point. You can see in the comparison above that P_t in the circuit with DC applied is 100 w, and in the circuit at f_{co}, P_t is exactly one-half of that value, or 50 w.

For every given value of resistance and inductance in a circuit, there is a frequency cutoff point.

When f_{co} is reached in a series RL circuit, these five conditions exist:

$$X_L = R$$

$$E_L = E_R \text{ and } 70.7\% \text{ of } E_a$$

$$I_T = 70.7\% \text{ of its maximum value}$$

$$\underline{/\theta} = 45°$$

$$P_t = 50\% \text{ of its maximum value}$$

Determining f_{co}

You can determine the cutoff frequency of a given RL circuit by substituting the value of R for X_L in the formula: $X_L = 2\pi fL$.

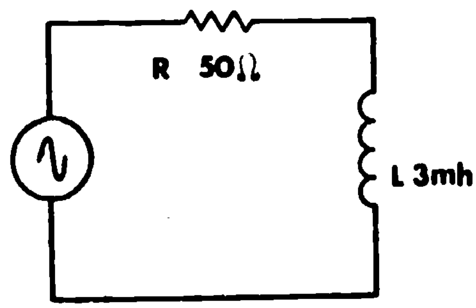
You know that $X_L = R$ @ f_{co}

therefore, $R = 2\pi f_{co}L$

Now you can transpose this formula to isolate f_{co}.

$$f_{co} = \frac{R}{2\pi L}$$

Determine the frequency of the half-power point of this circuit.



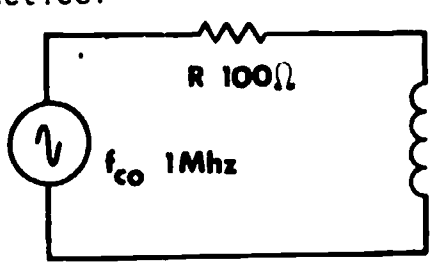
The half-power point, or f_{co}, of this circuit is found by:

$$f_{co} = \frac{R}{2\pi L}$$

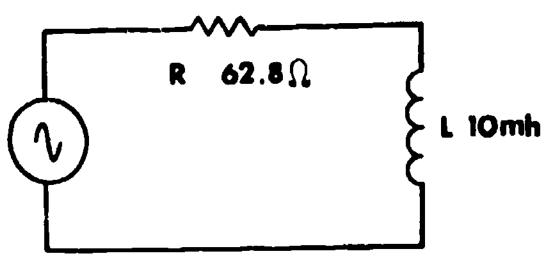
$$f_{co} = \frac{50\ \Omega}{(6.28)\ (3 \times 10^{-3})}$$

$$f_{co} = 2.6\ \text{KHz}$$

Practice:

1. 

Find X_L at f_{co}. ______

2. 

Find f_{co}. ______

3. Which of the below conditions exists at f_{co} in a series RL circuit?

   ____ a. E_R is 50% of E_a.

   ____ b. I_T is 50% of its maximum value.

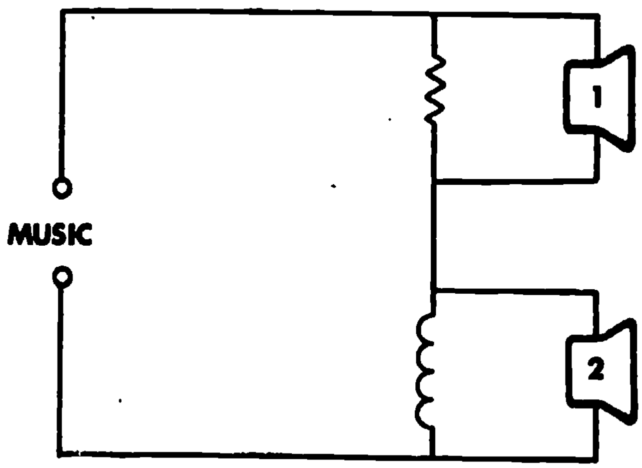
   ____ c. P_t is 70.7% of its maximum value.

   ____ d. P_t is 50% of its maximum value.

Answers: 1. $X_L = 100\Omega$; 2. $f_{co} = 1$ KHz; 3. d

Filters

Cutoff frequency is important in many kinds of equipment. You are probably familiar with hi-fi speaker systems that use speakers called woofers and tweeters. Woofers reproduce the low-frequency sounds and tweeters supply the high-frequency sounds. Filter circuits are used to separate the high from the low tones.

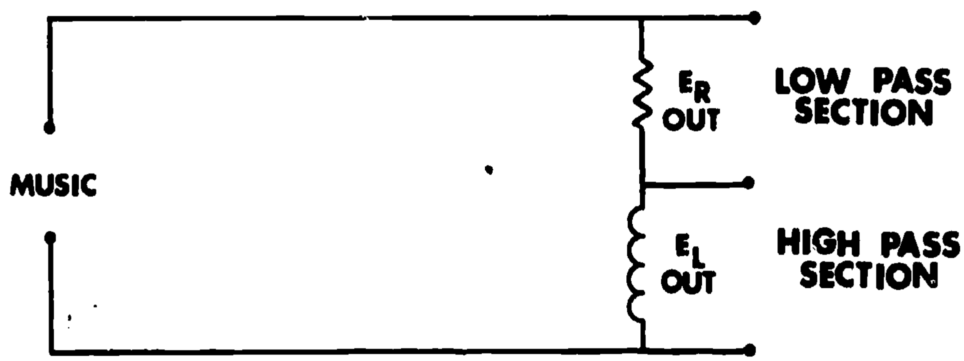


A series RL circuit performs this filtering job easily. Wiring the woofer (speaker 1) across the resistor and the tweeter (speaker 2) to the coil separates the high- and low-frequency signals fed to the speakers.

At frequencies above f_{co}, E_L is greater than E_R and the volume from the tweeter is greater than that from the woofer. At frequencies near f_{co}, the output from the speakers is about equal, and at lower frequencies, the woofer supplies the most sound. In this way, the series RL circuit is used to discriminate between high and low frequencies.

Types of Filters

- If the frequencies below f_{co} are the desi·ed frequencies, then the output is taken across the resistance.



This is called a low-pass filter. It mostly passes the low frequencies and discriminates against the high frequencies. This circuit configuration is also called a high-frequency discriminator, or a high-frequency attenuator (meaning the high frequencies do not develop enough voltage across the resistor for a useable output.)

On the other hand, when the frequencies above f_{co} are the desired frequencies, the output is taken across the coil. This is a high-pass filter. It passes the majority of the high frequencies and discriminates against the low frequencies. For this reason, it is called a low-frequency discriminator, or a low-frequency attenuator.

Your hi-fi has a network that separates the high frequencies from the low frequencies and supplies the signals to speakers designed to work well in the different ranges.

---

1. In a series RL circuit, if the output is taken across the coil it is called: (check all correct answers)

   ____ a. a low-pass filter
   ____ b. high-frequency attenuator
   ____ c. a high-pass filter
   ____ d. low-frequency discriminator

2. In a series RL circuit, if you want a low-frequency attenuator, which component do you take the output from? ____________

3. A series RL circuit used as a low-pass filter has an output from the:

_____ a. coil
_____ b. resistor
_____ c. capacitor

Answers: 1. c and d; 2. coil; 3. b

AT THIS POINT, YOU MAY TAKE THE PROGRESS CHECK, OR YOU MAY STUDY ANY OF THE OTHER RESOURCES LISTED. IF YOU TAKE THE PROGRESS CHECK AND ANSWER ALL OF THE QUESTIONS CORRECTLY, GO TO THE NEXT LESSON. IF NOT, STUDY ANY METHOD OF INSTRUCTION YOU WISH UNTIL YOU CAN ANSWER ALL THE QUESTIONS CORRECTLY.

PROGRAMMED INSTRUCTION
LESSON V

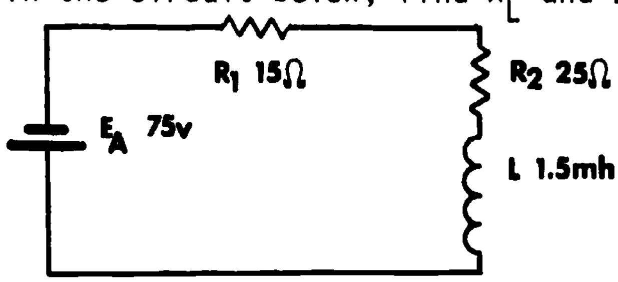
Frequency Discrimination in RL Circuits

TEST FRAMES ARE 7, 14, 19 AND 23. AS BEFORE, GO FIRST TO TEST FRAME 7 AND SEE IF YOU CAN ANSWER ALL THE QUESTIONS THERE. FOLLOW THE DIRECTIONS GIVEN AFTER THE TEST FRAME.

---

1. Inductive reactance is a direct function of frequency ($X_L = 2\pi fL$). In other words, there has to be a constantly changing current before a coil offers opposition to current other than the resistance of the wire from which it was constructed.

   In the circuit below, find X_L and Z_T.



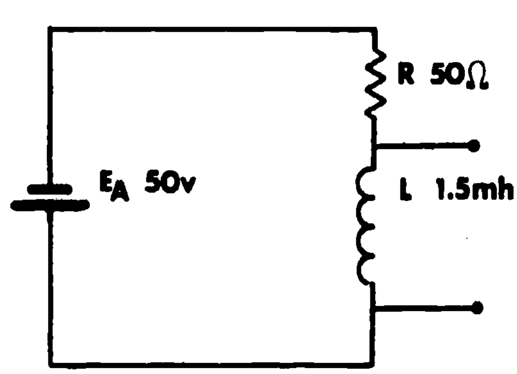
X_L = ________

Z_T = ________

($X_L = 0$; $Z_T = 40\ \Omega$)

---

2. Still dealing with a DC circuit, if there is no X_L, what is the voltage drop across the coil? ________ (The circuit has been turned on for 5 minutes.)

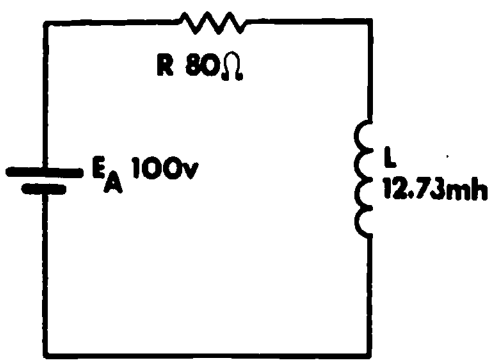


---

(0)

3. If the frequency of the applied voltage is 0 (DC), the coil offers no opposition to current. So for all practical purposes the circuit can be considered purely resistive.

Solve for the indicated values.



a. X_L = ______
b. Z_T = ______
c. I_T = ______
d. E_R = ______
e. E_L = ______
f. $\underline{/\theta}$ = ______
g. P_t = ______

(a. 0; b. 80Ω; c. 1.25 a; d. 100 v; e. 0; f. 0°; g. 125 w)
(Keep these values in mind. They will be used for comparisons later in the lesson.)

4. If the DC source is replaced by an AC source and frequency increases from 0 to some value, X_L increases to some measurable value.

What happens to E_L as frequency increases?

____ a. increases
____ b. decreases
____ c. remains the same

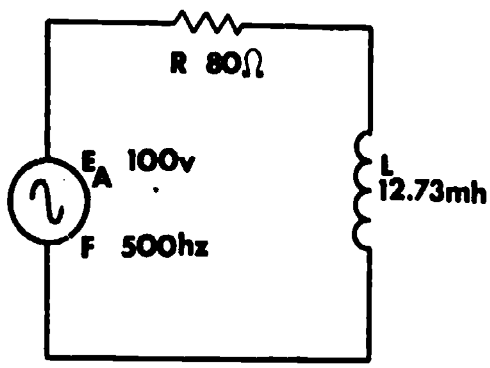
(a) increases

5. Since $E_a = E_R + jE_L$, what happens to E_R as frequency increases?

____ a. increases
____ b. decreases
____ c. remains the same

(b) decreases

6. Using the circuit in frame 3, but with an applied voltage at a frequency of 500 Hz, solve for the same values and compare the results with those of frame 5 to see how circuit values react to changes in frequency.



a. X_L = ____________

b. Z_T = ____________

c. I_T = ____________

d. E_R = ____________

e. E_L = ____________

f. $\underline{/\theta}$ = ____________

g. P_t = ____________

- -

(a. 40Ω; b. 89.3Ω; c. 1.12a; d. 89.6; e. 44.8v; f. 26.6°; g. 100.3w)

7. Using arrows, indicate increase ↑, decrease ↓, or remain the same → for the following values of a series RL circuit when the frequency of the applied voltage is increased.

a. X_L ____________

b. Z_T ____________

c. I_T ____________

d. E_R ____________

e. E_L ____________

f. $\underline{/\theta}$ ____________

g. P_t ____________

- -

(THIS IS A TEST FRAME. COMPARE YOUR ANSWERS WITH THE CORRECT ANSWERS GIVEN AT THE TOP OF THE NEXT PAGE.)

ANSWERS - TEST FRAME 7

a. ↑

b. ↑

c. ↓

d. ↓

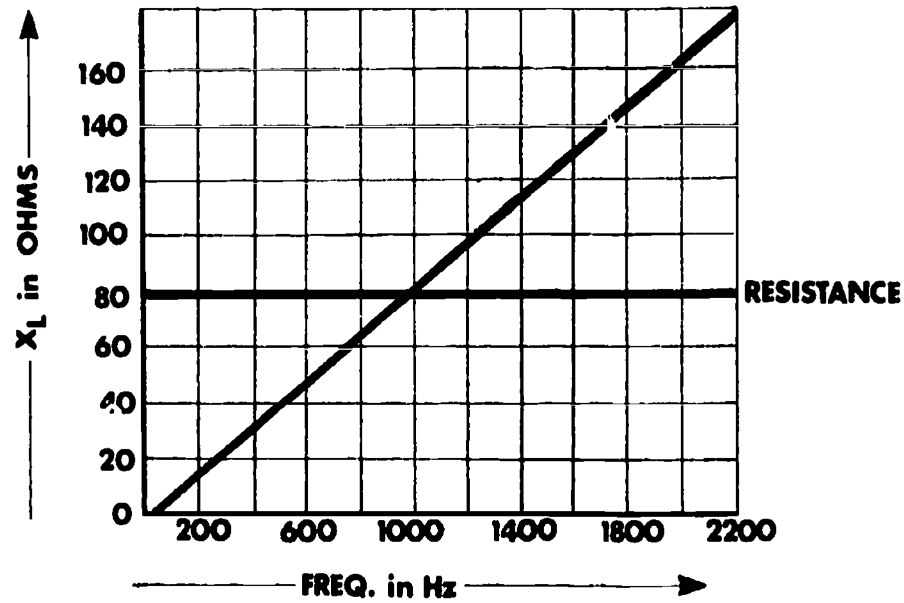
e. ↑

f. ↑

g. ↓

IF ALL YOUR ANSWERS MATCH THE CORRECT ANSWERS, YOU MAY GO TO TEST FRAME 14. OTHERWISE, GO BACK TO FRAME 1 AND TAKE THE PROGRAMMED SEQUENCE BEFORE TAKING TEST FRAME 7 AGAIN.

8. As frequency continues to increase, a point is reached in any RL circuit where $X_L = R$. This can be represented graphically like this:

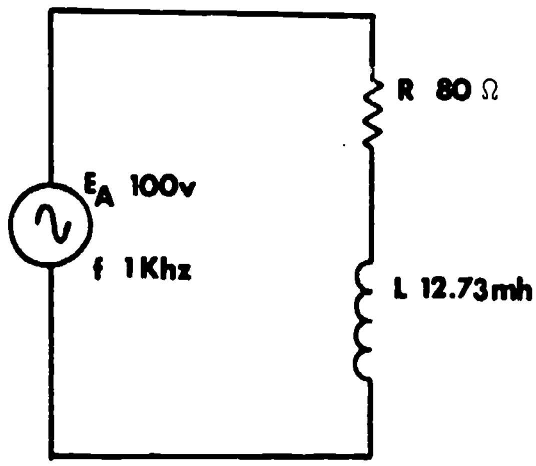


For the circuit represented by this graph, at what frequency does $X_L = R$? ________________

What is the ohmic value of X_L? __________

(1 KHz; 80 Ω)

9. Solve:



a. X_L = ______

b. Z_T = ______

c. I_T = ______

d. E_L = ______

e. E_R = ______

f. $\underline{/\theta}$ = ______

g. P_t = ______

h. P_x = ______

(a. 80Ω; b. 113Ω; c. 884ma; d. 70.7v; e. 70.7v; f. 45°; g. 62.5w; h. 62.5 vars)

10. The point at which $X_L = R$ is called the <u>cutoff frequency</u>, f_{co}. Since $X_L = R$ at f_{co}, what is the relationship between E_L and E_R?

______

($E_L = E_R$)

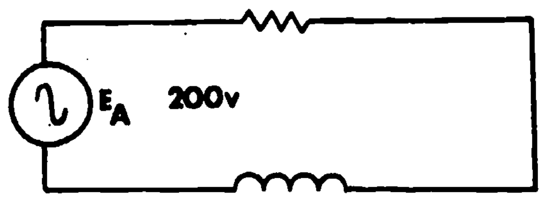
11. $E_L = E_R$ and $X_L = R$ at f_{co}. This makes the circuit phase angle

______.

(45°)

12. The circuit phase angle at f_{co} is 45°. Recall that to compute E_L and E_R if E_a and $\underline{/\theta}$ are known, you multiply E_a by the sine and cosine of $\underline{/\theta}$. $E_L = \overline{SIN}\ \underline{/\theta} \times E_a$, $E_R = COS\ \underline{/\theta} \times E_a$. At 45° both the sine and cosine of $\underline{/\theta} = \overline{0.707}$.

Solve for E_L and E_R when circuit is at cutoff.



E_L = ________

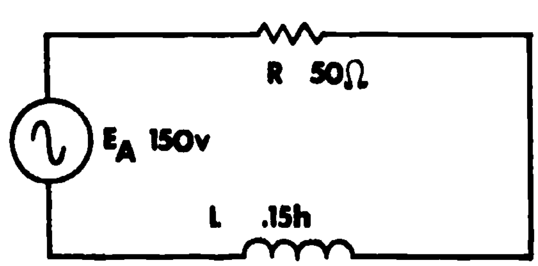
E_R = ________

(141.4 v)

---

13. There is one more term used to describe f_{co}, the half-power point. If you compare P_t, which you computed in frame 3 (125 watts with a DC source), to the P_t computed in frame 9 (62.5 watts at 1 KHz which is the cutoff frequency for that circuit), you will see how this term came about.

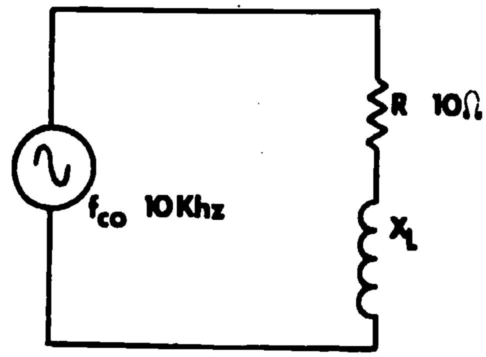
What is P_t at f_{co}?



P_t at f_{co} = ______

(225 w; one-half of the DC power dissipation, 450 w.)

14. Solve (Circuit is operating at f_{co}).



a. X_L = ______

b. $\underline{/\theta}$ = ______

(THIS IS A TEST FRAME. COMPARE YOUR ANSWERS WITH THE CORRECT ANSWERS GIVEN AT THE TOP OF THE NEXT PAGE.)

ANSWERS - TEST FRAME 14

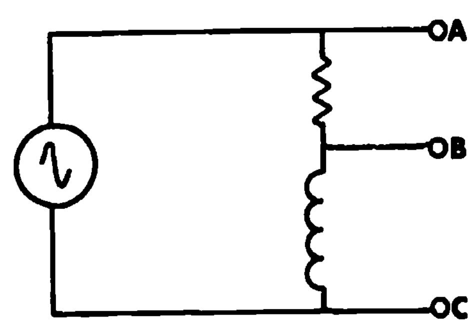
a. 10 Ω

b. 45°

IF ALL YOUR ANSWERS MATCH THE CORRECT ANSWERS, YOU MAY GO TO TEST FRAME 19. OTHERWISE, GO BACK TO FRAME 8 AND TAKE THE PROGRAMMED SEQUENCE BEFORE TAKING TEST FRAME 14 AGAIN.

15. To understand the term cutoff, you may think of the RL circuit as a variable voltage divider, with changes in frequency causing the output voltages taken across the resistor and coil to vary.

At f_{co}, E_L and E_R are equal. At frequencies above cutoff, the greatest output is between points:



____ a. A and B

____ b. B and C

(b) B and C

16. At frequencies above cutoff, E_L is ________ (greater/less) than E_R.

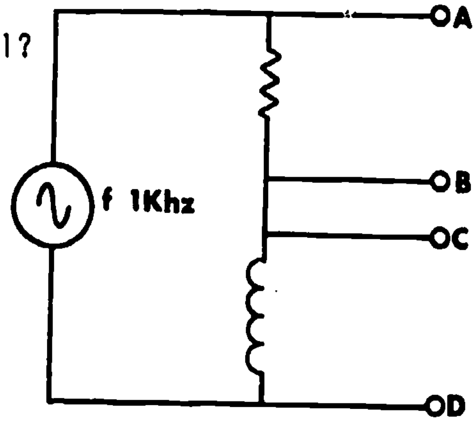
At frequencies below cutoff, E_R is ________ (greater/less) than E_L.

(greater; greater)

17. Cutoff frequency is a point below which the output across the coil is not considered useful and above which the output across the resistor is not considered useful.

Which output is useful?

$f_{co} = 1.5$ KHz



____ a. A to B

____ b. B to C

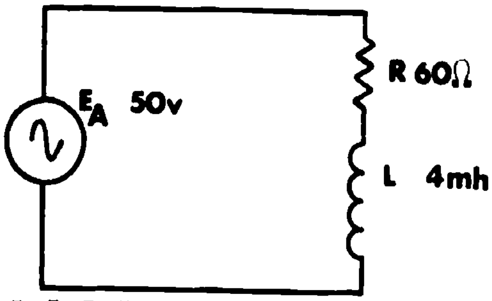
____ c. C to D

(a) A to B

18. A formula for finding f_{co} is derived from the fact that $R = X_L$ at f_{co}. By substituting R for X_L in the formula $X_L = 2\pi fL$, then solving for frequency, we arrive at an equation for finding the frequency at which R is equal to X_L.

$$f_{co} = \frac{R}{2\pi L}$$

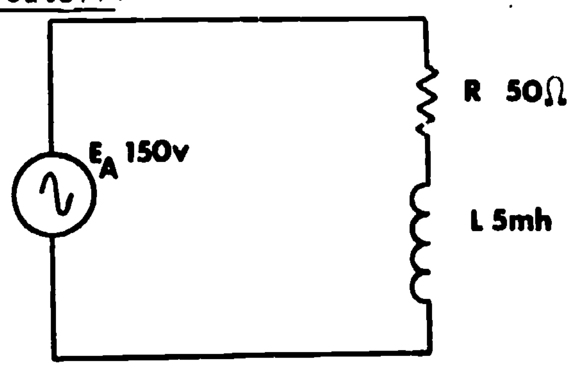
What is f_{co} for this circuit?



f_{co} = ________

(2.38 KHz)

19. Solve for f_{co}, E_L, E_R and P_t at f_{co}. <u>Remember the conditions which exist at cutoff</u>:



a. f_{co} = _____

b. E_L = _____

c. E_R = _____

d. P_t = _____

(THIS IS A TEST FRAME. COMPARE YOUR ANSWERS WITH THE CORRECT ANSWERS GIVEN AT THE TOP OF THE NEXT PAGE.)

ANSWERS - TEST FRAME 19

a. 1.59 KHz

b. 106 v

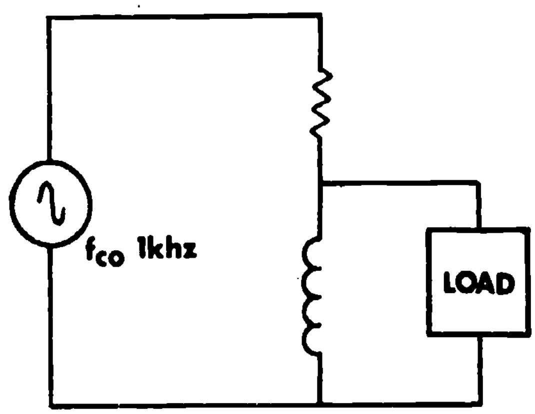
c. 106 v

d. 225 w

IF ALL YOUR ANSWERS MATCH THE CORRECT ANSWERS, YOU MAY GO TO TEST FRAME 23. OTHERWISE, GO BACK TO FRAME 15 AND TAKE THE PROGRAMMED SEQUENCE BEFORE TAKING TEST FRAME 19 AGAIN.

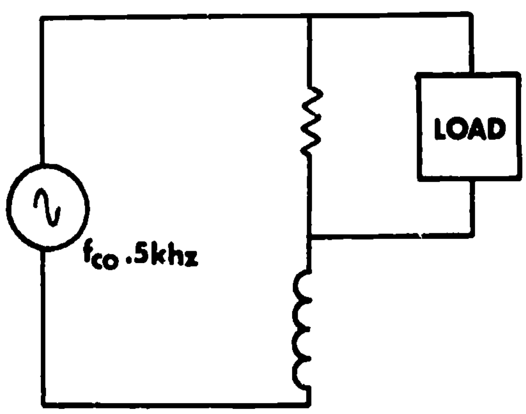
20. Because a resistive-reactive circuit reacts differently to different frequencies, it is capable of discriminating against certain frequencies while passing others.

For example:



The cutoff frequency for this circuit is 1 KHz. With the output taken across the coil, the load functions properly only when the frequency is above 1 KHz.

This circuit is called a high-pass filter or a low-frequency discriminator.

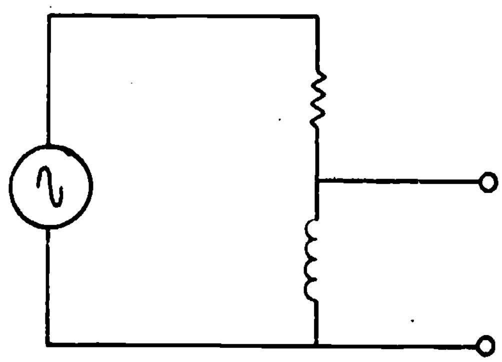


With the output taken across the resistor, the load functions properly at frequencies __________ 500 Hz?
above/below

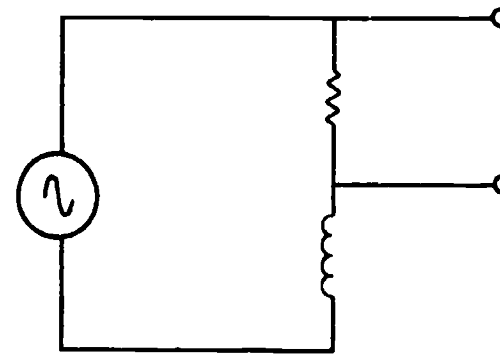
- -

(below)

21. The action described in frame 19 is known as filter action. The positioning of the output leads determines whether the filter circuit passes high frequencies or low frequencies.



High-Pass Filter
(low-frequency discrimination)



Low-Pass Filter
(high-frequency discrimination)

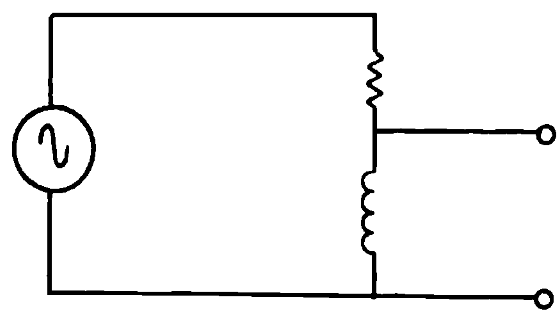
The term pass, when used in connection with a filter circuit, does not mean that circuit current stops above f_{co} in the case of a low-pass filter or below f_{co} in the case of a high-pass filter. It merely means that the output voltage is not high enough to be useful.

The terms discriminate and attenuate, when used in reference to a filter circuit, both mean that those particular frequencies are greatly reduced. These two words are used interchangeably.

If a filter attenuates high frequencies, the ________ frequency output will be useable. high/low

(low)

22. Draw an RL series circuit as a low frequency attenuator.



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23. When the output is taken across the coil, a series RL circuit is called a: (check all correct answers)

____ a. low pass filter

____ b. high-frequency attenuator

____ c. high pass filter

____ d. low-frequency discriminator

(THIS IS A TEST FRAME. COMPARE YOUR ANSWERS WITH THE CORRECT ANSWERS GIVEN AT THE TOP OF THE NEXT PAGE.)

ANSWERS - TEST FRAME 23

c. high pass filter

d. low-frequency discriminator

IF ANY OF YOUR ANSWERS IS INCORRECT, GO BACK TO FRAME 20 AND TAKE THE PROGRAMMED SEQUENCE.

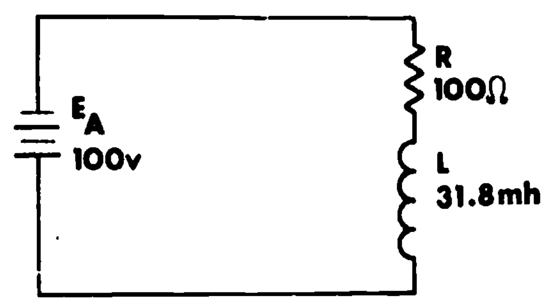
IF YOUR ANSWERS ARE CORRECT, YOU MAY TAKE THE PROGRESS CHECK, OR YOU MAY STUDY ANY OF THE OTHER RESOURCES LISTED. IF YOU TAKE THE PROGRESS CHECK AND ANSWER ALL THE QUESTIONS CORRECTLY, GO TO THE NEXT LESSON. IF NOT, STUDY ANY METHOD OF INSTRUCTION YOU WISH UNTIL YOU CAN ANSWER ALL THE QUESTIONS CORRECTLY.

SUMMARY
LESSON V

Frequency Discrimination in RL Circuits

Because of the type of opposition offered to alternating current by a coil, a circuit containing inductance reacts to a change in frequency. Thus, a properly designed RL circuit may be utilized to differentiate between frequencies. In this way, the RL circuits may be used to eliminate undesired frequencies while developing a useable output for the desired frequencies.

To understand this operation it is first necessary to consider the reaction of a coil to changes in frequency. As you know by now, inductive reactance is directly related to the frequency of the applied voltage. First consider a circuit with a DC potential. (Assume the circuit has been energized for at least five time constants.)



With frequency equal to 0, X_L is equal to 0. The impedance of the circuit is equal to 100 Ω, the value of circuit resistance. Other circuit values are:

$$I_T = 1 \text{ amp}$$

$$P_t = 100 \text{ watts}$$

$$P_a = 100 \text{ va}$$

$$P_x = 0$$

$$E_L = 0$$

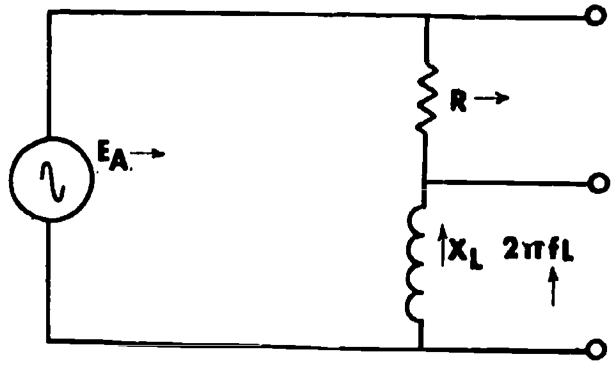
$$E_R = 100 \text{ volts}$$

$$\underline{/\theta} = 0°$$

$$PF = 1$$

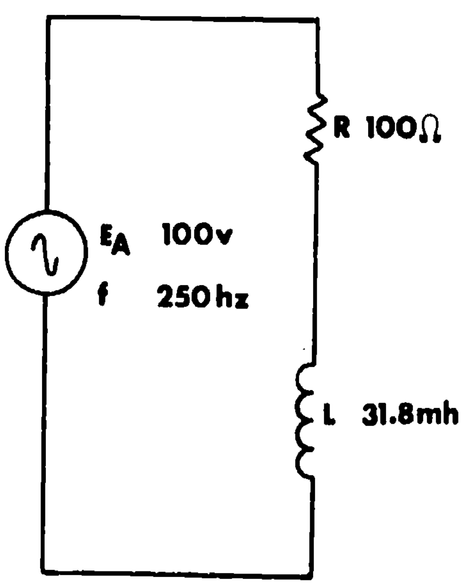
Before circuit values are computed at an increased frequency, consider wh is going to happen to the outputs taken across the coil

(E_L) and across the resistor (E_R).



As frequency increases, the opposition offered by the coil increases; consequently there is a value of voltage developed across the coil. As E_L increases E_R must decrease because $\rightarrow E_a = \downarrow E_R + jE_L \uparrow$.

As you learned in the previous lesson, a change in frequency affects many circuit values, not just X_L. Consider the original circuit with the frequency increased from 0 to 250 Hz.



$X_L = 50\ \Omega$

$Z = 112\ \Omega$

$I_T = 895$ ma

$P_a = 89.5$ va

$P_t = 80$ w

$P_x = 40.2$ vars

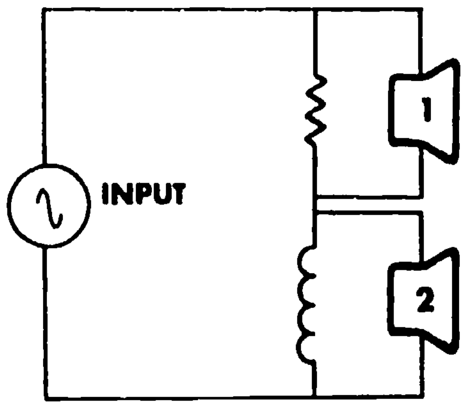
$E_L = 44.6$ v

$E_R = 89.5$ v

$\underline{/\theta} = 26.6°$

PF = 0.89

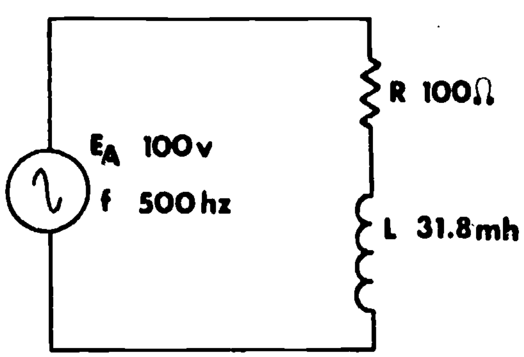
The changes you are primarily interested in at this time are E_L and E_R. It was mentioned earlier that an RL series circuit can differentiate between frequencies. As frequency varies, the outputs taken across the resistor and the coil also vary. For example:



As the frequency of the input increases, the voltage across the coil increases while the voltage across the resistor decreases. If the input frequency decreases, the reverse occurs.

Assuming speakers are connected across both the resistor and the coil, the volume of each varies as the voltage across each component varies.

As the frequency increases, a point is reached where $X_L = R$. This frequency is termed cutoff frequency, designated f_{co}. The cutoff frequency for a given RL circuit may be determined by using the formula: $f_{co} = \frac{R}{2\pi L}$.



At 500 Hz, this circuit is at f_{co}. Compare the circuit values which now exist to those which existed with DC and 250 Hz AC applied.

$R = 100\ \Omega$

$X_L = 100\ \Omega$

$Z = 141.4\ \Omega$

$I_T = 707$ ma

$P_a = 70.7$ va

$P_x = 50$ vars

$P_t = 50$ w

$E_L = 70.7$ v

$E_R = 70.7$ v

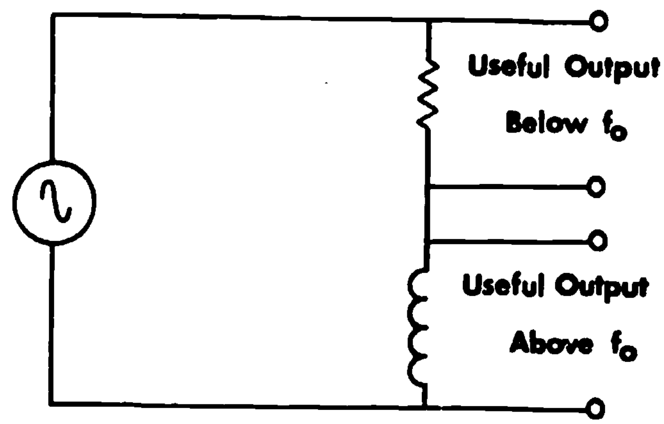
$\underline{/\theta} = 45°$

PF = 0.707

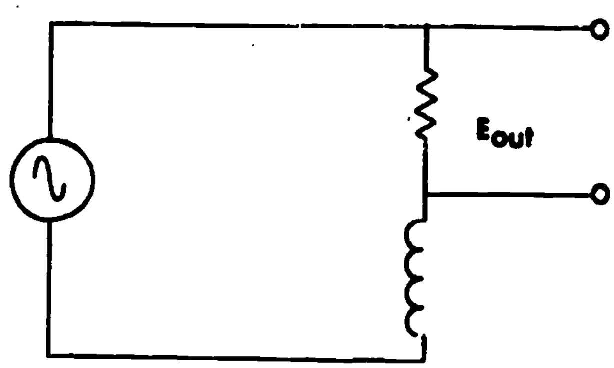
Notice that at this frequency (f_{co}), $E_L = E_R$ and both are equal to 70.7% of E_a.

By comparing P_t at f_{co} to P_t with DC applied, you see that the power dissipated at f_{co} is exactly one-half of that dissipated with DC applied. For this reason, f_{co} is also referred to as the half-power point.

Do not let the term cutoff confuse you; it does not mean that all circuit action stops at this frequency. f_{co} is merely a term assigned to the frequency below which the output across the coil is not considered useful and above which the output across the resistor is not considered useful.

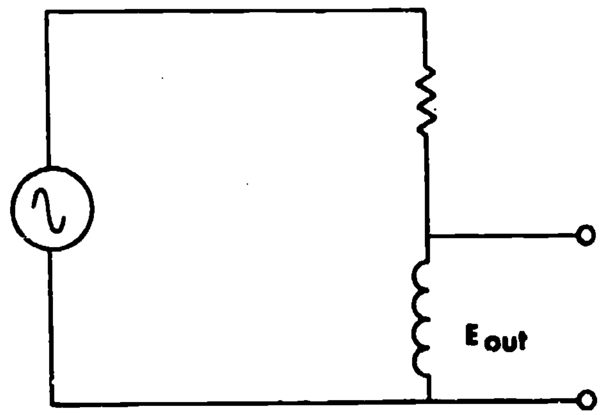


The ability of a resistive-reactive circuit to differentiate between frequencies is known as filter action. Circuits which make use of this ability are called filter circuits. If the output is taken across the resistor as shown, a useful output would be developed at frequencies below the cutoff frequency of the circuit. This circuit is called a low-pass filter or a high-frequency discriminator.



LOW-PASS FILTER
High-Frequency Discriminator

If the output is taken across the coil, the circuit is called a high-pass filter or low-frequency discriminator because a useful output is developed at frequencies above the f_{co} of the circuit.



HIGH-PASS FILTER
Low-Frequency Discriminator

AT THIS POINT, YOU MAY TAKE THE LESSON PROGRESS CHECK, OR YOU MAY STUDY THE LESSON NARRATIVE OR THE PROGRAMMED INSTRUCTION OR BOTH. IF YOU TAKE THE PROGRESS CHECK AND ANSWER ALL OF THE QUESTIONS CORRECTLY, GO TO THE NEXT LESSON. IF NOT, STUDY ANOTHER METHOD OF INSTRUCTION UNTIL YOU CAN ANSWER ALL THE QUESTIONS CORRECTLY.

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BASIC ELECTRICITY AND ELECTRONICS
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L E S S O N   V I

Series RC Circuits

Study Booklet

Bureau of Naval Personnel
January 1972

OVERVIEW
LESSON VI

Series RC Circuits

In this lesson you will study and learn about the following:

- -vector diagrams for series RC circuits
- -variational analysis of series RC circuits
- -frequency discrimination in RC circuits
- -RC filter circuits

BEFORE YOU START THIS LESSON, PREVIEW THE LIST OF STUDY RESOURCES ON THE NEXT PAGE.

LIST OF STUDY RESOURCES
LESSON VI

Series RC Circuits

To learn the material in this lesson, you have the option of choosing, according to your experience and preferences, any or all of the following:

STUDY BOOKLET:

- Lesson Narrative
- Programmed Instruction
- Lesson Summary

ENRICHMENT MATERIAL:

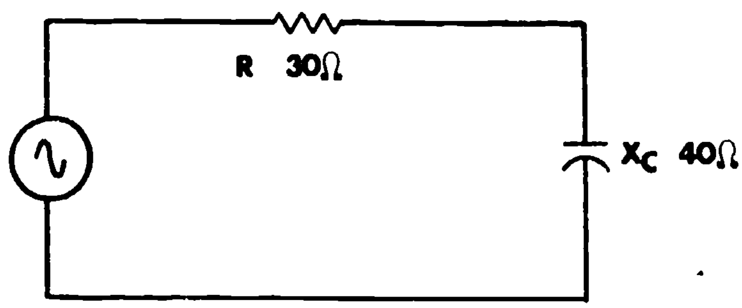
NAVPERS 93400A-1b "Basic Electricity, Alternating Current." Fundamentals of Electronics. Bureau of Naval Personnel. Washington, D.C.: U.S. Government Printing Office, 1965.

YOU MAY NOW STUDY ANY OR ALL OF THE RESOURCES LISTED ABOVE. YOU MAY TAKE THE PROGRESS CHECK AT ANY TIME.

NARRATIVE
LESSON VI

Series RC Circuits

We have indicated that we must use vector addition to solve circuits which have out-of-phase quantities caused by resistance and reactance. We have already studied circuits which have both resistive and inductive components. Now, we turn our attention to series AC circuits with both resistance and capacitance.

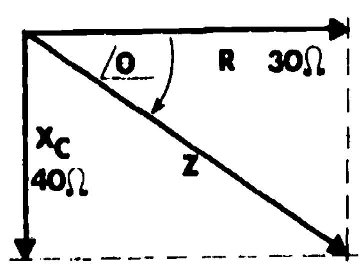


Plotting RC Circuits

The only difference between the vectorial representation of a series RL circuit and a series RC circuit is in the direction of the angle. Angle theta in an RC circuit is always negative.

As in any series circuit, I is common and is plotted on the adjacent side with resistive quantities.

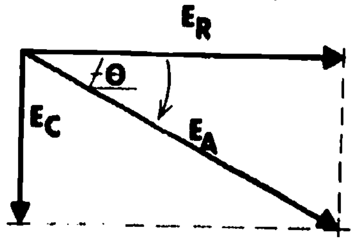
The phase relationship between the E_C and I in the circuit is 90° with I leading voltage (ICEman); therefore, we plot X_C in a 90° clockwise (-j) position.



The hypotenuse indicates Z_T.

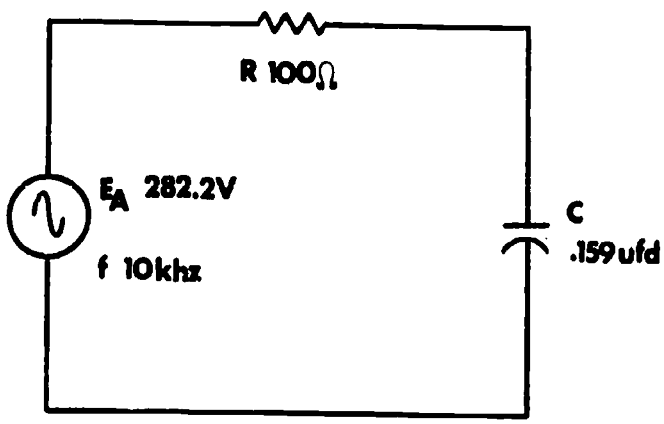
This triangle can be expressed in rectangular notation as 30Ω - j40Ω. X_C is always a -j quantity, and R is always plotted on the adjacent side and listed first in rectangular notation.

Obviously, this is a 3-4-5 triangle, so the hypotenuse (Z_T) is 50 ohms and angle theta is -53.1°. Notice that /θ is now a negative angle. This is the only difference between an RC impedance, power or voltage triangle or an LC triangle.



The voltage triangle is shown at the left.

Solve for the following.



X_C = ______

Z_T = ______

TAN θ = ______

SIN θ = ______

COS θ = ______

I_T = ______

E_R = ______

E_C = ______

P_t = ______

P_a = ______

P_x = ______

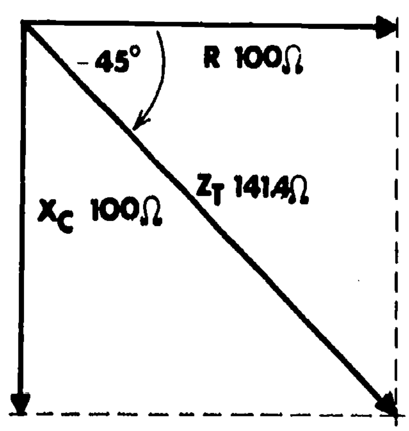
= ______

PF = ______

Recall $X_C = \frac{.159}{fC}$

Round answers off to nearest whole number. Draw Vector Diagram.

Since X_C rounded off is 100 ohms, the triangle has equal sides and is -45°.



Z_T = 141.4 ohms

TAN $\underline{/\theta}$ = 1 (this is true for $\underline{/}$'s with equal sides)

SIN $\underline{/\theta}$ = .707

COS $\underline{/\theta}$ = .707

$$I_T = \frac{E_a}{Z_T} = \frac{282.8v}{141.4} = 2a$$

$E_C = 200v$

$E_R = 200v$

$P_t = I^2R = 4 \times 100\Omega = 400w$ or $P_t = E.I.COS \underline{/\theta} = 400w$

$P_a = E \cdot I = 282.8v \times 2a = 565.6$ va

$P_x = I^2X_C = 4$ a $\times 100\ \Omega = 400$ vars

$\underline{/\theta} = -45°$

PF = .7 (COS θ)

What would $\underline{/\theta}$ be if X_C in the above circuit were 50Ω?
What would the power factor be? $\underline{/\theta}$ = ______; PF = ______

$\underline{/\theta} = -26.6°$

PF = 0.89

Variational Analysis of Series RC Circuits

You can analyze quantities in an RC series AC circuit just as you did for RL circuits. Remember to draw the impedance, power or voltage triangle, and remember that $\underline{/\theta}$ is negative.

With this formula as a reminder, indicate on the table what will happen in an RC circuit if f decreases.

$$X_C = \frac{.159}{fC}$$

Draw Triangle.

f	↓
Z_T	
X_C	
I_T	
E_R	
E_C	
P_T	
P_X	
P_A	
$\angle\theta$	
PF	
R	
C	

If frequency is decreased:

X_C will increase ↑

Z_T will increase ↑

I_T will decrease ↓

E_R will decrease ↓

E_C will increase ↑

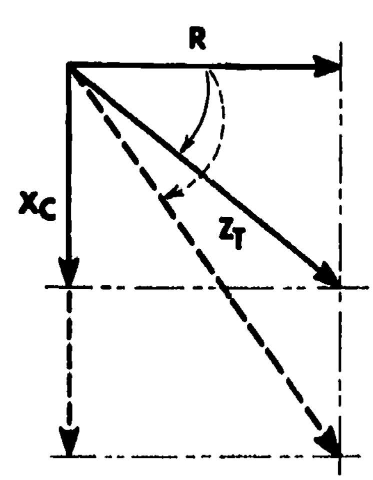
P_t, P_x and P_a will decrease ↓

$\angle\theta$ will increase ↑

PF will decrease ↓

R will remain the same →

C will remain the same →



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Complete this table by inserting arrows to indicate the changes that will take place in a series RC circuit.

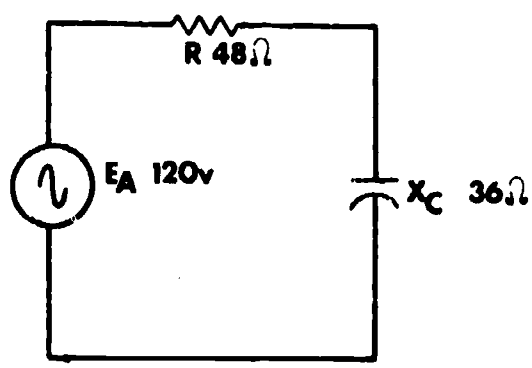
	E_A↑	R↑	C↓	f↑
Z_T				
X_C				
I_T				
E_R				
E_C				
P_T		*		
P_X				
P_A				
$\angle\theta$				
PF				
R				
C				

*In a series RC circuit, varying R causes the true power to change in a complex way, so you are not expected to solve this special case.

	$E_A\uparrow$	$R\uparrow$	$C\downarrow$	$f\uparrow$
Z_T	$\rightarrow$	$\uparrow$	$\uparrow$	$\downarrow$
X_C	$\rightarrow$	$\rightarrow$	$\uparrow$	$\downarrow$
I_T	$\uparrow$	$\downarrow$	$\downarrow$	$\uparrow$
E_R	$\uparrow$	$\uparrow$	$\downarrow$	$\uparrow$
E_C	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$
P_T	$\uparrow$	*	$\downarrow$	$\uparrow$
P_x	$\uparrow$	$\downarrow$	$\downarrow$	$\uparrow$
P_A	$\uparrow$	$\downarrow$	$\downarrow$	$\uparrow$
$/\theta$	$\rightarrow$	$\downarrow$	$\uparrow$	$\downarrow$
PF	$\rightarrow$	$\uparrow$	$\downarrow$	$\uparrow$
R	$\rightarrow$	$\uparrow$	$\rightarrow$	$\rightarrow$
C	$\rightarrow$	$\rightarrow$	$\downarrow$	$\rightarrow$

Try these problems.

1.



Find:

Z_T = ______

I_T = ______

E_R = ______

E_C = ______

PF = ______

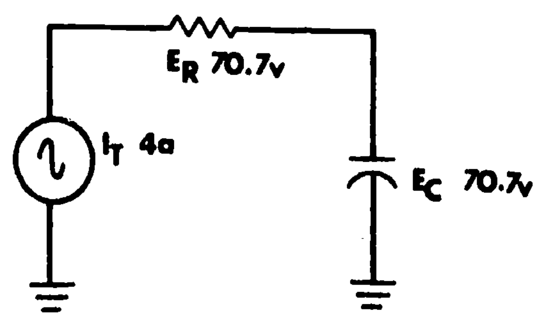
P_t = ______

P_x = ______

P_a = ______

$\angle\theta$ = ______

2.



Find:

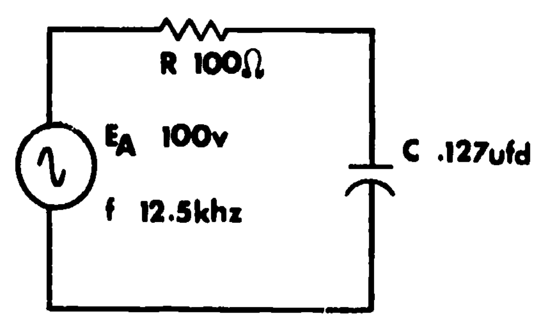
Z_T = ______

I_T = ______

E_a = ______

$\angle\theta$ = ______

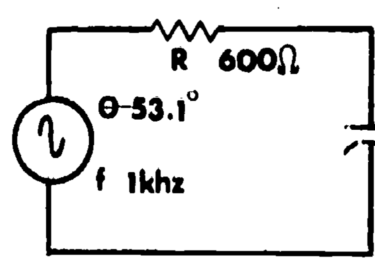
3.



Find:

Z_T = ______

4.



Find:

X_C = ______

Z_T = ______

1. $Z_T = 60\Omega$

   $I_T = 2a$

   $E_R = 96v$

   $E_C = 72v$

   $PF = 0.80$

   $P_t = 192w$

   $P_x = 144$ vars

   $P_a = 240$ va

   $\underline{/\theta} = -36.9°$

2. $Z_T = 25\Omega$

   $I_T = 4a$

   $E_a = 100v$

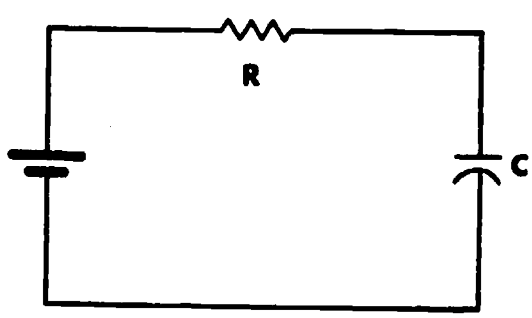
   $= -45°$

3. $Z_T = 141.4\Omega$

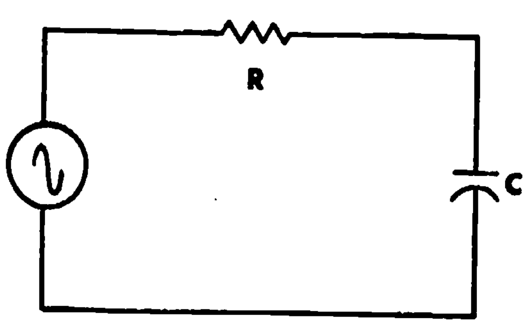
4. $X_C = 800\Omega$

   $Z_T = 1,000\Omega$

Frequency Discrimination in RC Circuits



Analyze what happens to quantities in this circuit with DC applied. After five time constants, there is no current flow and full applied voltage appears across the capacitor.



If you replace the battery in this RC circuit with an AC source, the capacitor reacts to changing AC voltage and there is some value of X_C.

As the frequency of the applied voltage is increased, X_C decreases. The formula for X_C shows that X_C is inversely proportional to frequency.

$$X_C = \frac{1}{2\pi fC}$$

You can see that X_C is high at low frequencies, and is low at high frequencies. In an RL circuit, X_L is high at high frequencies and low at low frequencies.

Current in a circuit is determined by opposition, as the table at the top of the next page shows.

	Low Frequency	High Frequency
RL Circuit	X_L - minimum	maximum
	I - maximum	minimum
RC Circuit	X_C - maximum	minimum
	I - minimum	maximum

Increasing f

As the applied frequency increases in a series RC circuit:

X_C decreases.

Z_T decreases.

I_T increases.

E_R increases.

E_C decreases ($E_a = E_R - jE_C$).

Frequency Cutoff

There is a frequency for which the value of X_C equals the value of R. This is called the frequency cutoff point (f_{co}), or the half-power point for the series RC circuit.

The five conditions which exist in series RC circuits at f_{co} are:

$X_C = R$

$E_C = E_R$ and is equal to 70.7% of E_a

I_T = 70.7% of its maximum value

$\angle\theta = -45°$

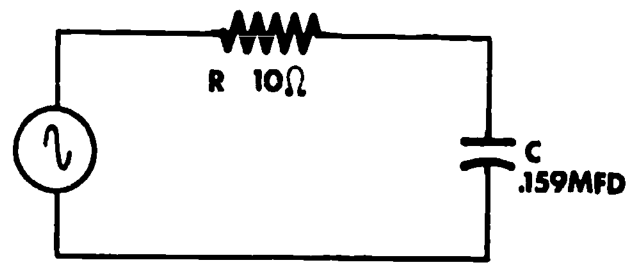
P_t = 50% of its maximum DC value.

$P_t = P_x$

Determining f_{co} for RC Circuits

Since X_C is equal to R at f_{co}, the formula $X_C = \frac{.159}{fC}$ can be modified to $R = \frac{.159}{f_{co}C}$. Transposing this formula to find f_{co} for an RC circuit yields: $f_{co} = \frac{.159}{RC}$

Find f_{co} for this circuit.



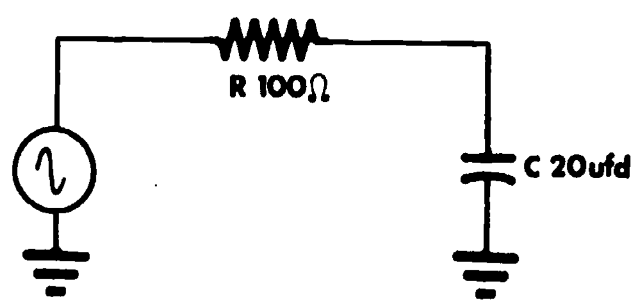
f_{co} = ______

By the formula $f_{co} = \frac{.159}{(.159 \times 10^{-6})\ (10)}$

f_{co} = 100 KHz

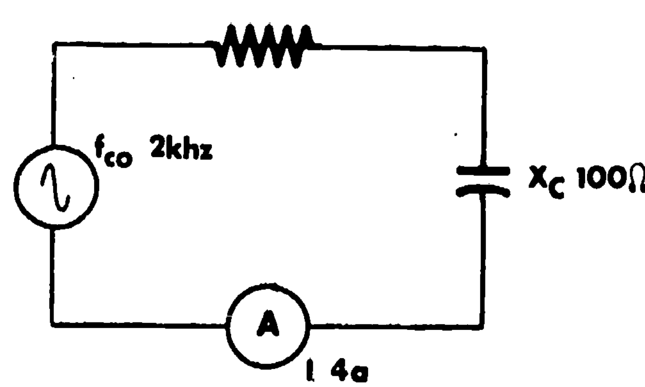
Practice.

1.



f_{co} = ______

2.



P_t = ______

---

1. $f_{co} = 79.5$ Hz

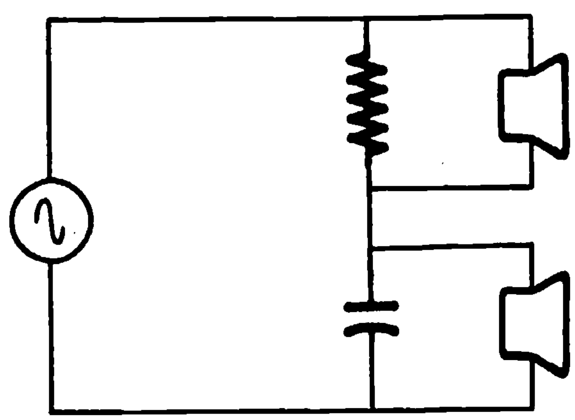
2. $P_t = 1{,}600$ w

RC Circuit Filters

Just as series RL circuits are used as filters, so are series RC circuits.

As frequency increases, X_C and E_C decrease. When the output is taken across the capacitor, the circuit discriminates against high frequencies; therefore, it is a low-pass filter, or high-frequency attenuator.

As frequency increases and E_C decreases, E_R increases. Therefore, when the output is taken across the resistor, the circuit discriminates against low frequencies, and it is a high-pass filter.

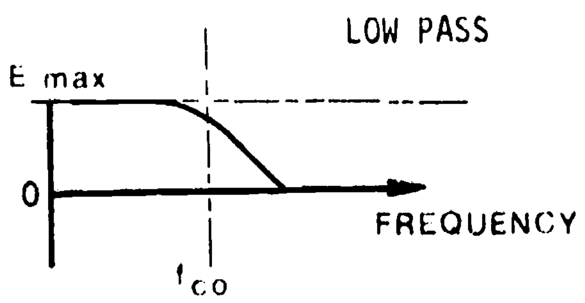


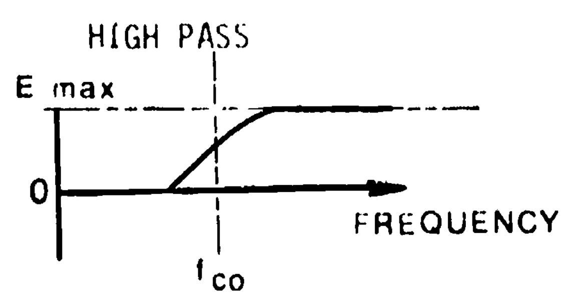
High-pass filter
Low-frequency discriminator, or
Low-frequency attenuator

Low-pass filter
High-frequency discrimination, or
High-frequency attenuator

Voltage Graphs

A graph of the variation of output voltages from low-pass and high-pass filters as frequency is changed looks like these curves:





At the cutoff frequency, output voltage is 70.7 percent of the applied voltage, power is half of maximum, and circuit reactance equals circuit resistance.

AT THIS POINT, YOU MAY TAKE THE PROGRESS CHECK, OR YOU MAY STUDY ANY OF THE OTHER RESOURCES LISTED. IF YOU TAKE THE PROGRESS CHECK AND ANSWER ALL OF THE QUESTIONS CORRECTLY, YOU MAY TAKE THE MODULE TEST. IF NOT, STUDY ANY METHOD OF INSTRUCTION YOU WISH UNTIL YOU CAN ANSWER ALL THE PROGRESS CHECK QUESTIONS CORRECTLY, BEFORE YOU TAKE THE MODULE TEST.

PROGRAMMED INSTRUCTION
LESSON VI

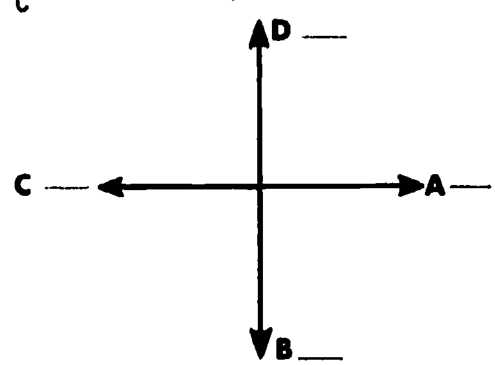
Series RC Circuits

TEST FRAMES ARE 3, 7, 11, 16 AND 24. AS BEFORE, GO FIRST TO TEST FRAME 3 AND SEE IF YOU CAN ANSWER ALL THE QUESTIONS THERE. FOLLOW THE DIRECTIONS GIVEN AFTER THE TEST FRAME.

---

1. When dealing with a series RC circuit, current is still used as the reference and is plotted in the standard vector position. The phase relationship between capacitor voltage (E_C) and circuit current is 90°.

   Indicate where E_C should be polotted.



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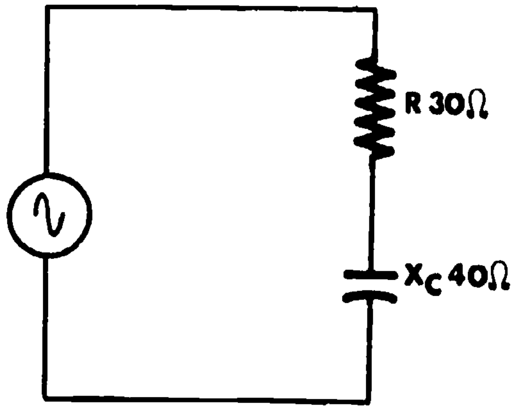
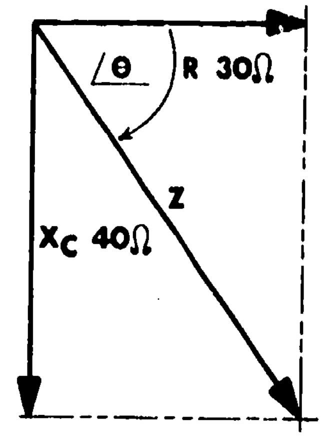
(B)

2. Once the values are plotted on the vector diagram, the impedance, power or voltage triangle can be constructed.





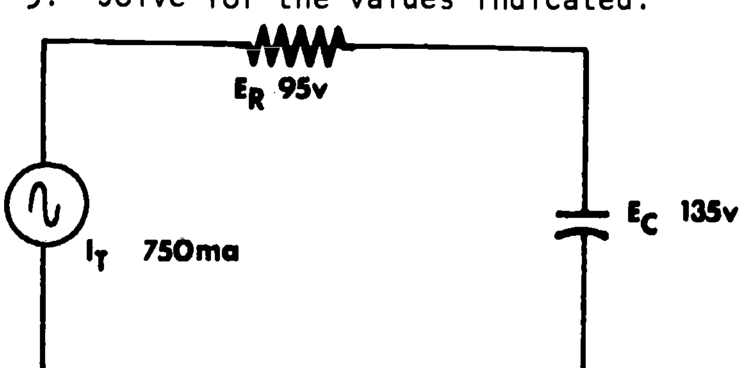
Impedance can be computed in the same manner as in an RL circuit, the only difference being that /θ is negative indicating a lagging phase angle.

What is the impedance of the circuit above? Give your answer in both rectangular and polar notation.

Z = ______________ Z = ______________

(30Ω - j40Ω; 50Ω /-53.1°)

3. Solve for the values indicated.



a. E_A = ______

b. Z_T = ______

c. /θ = ______

(THIS IS A TEST FRAME. COMPARE YOUR ANSWERS WITH THE CORRECT ANSWERS GIVEN AT THE TOP OF THE NEXT PAGE.)

ANSWERS - TEST FRAME 3

a. 165 v

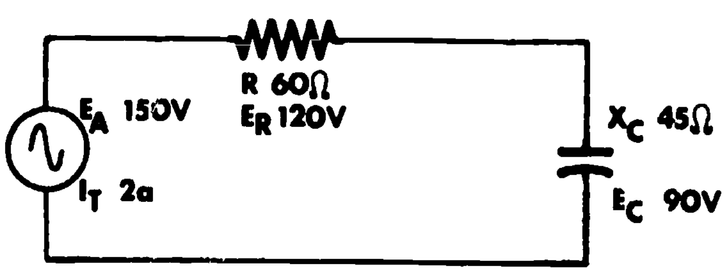
b. 220 Ω

c. -54.8°

IF ALL YOUR ANSWERS MATCH THE CORRECT ANSWERS, YOU MAY GO ON TO TEST FRAME 7. OTHERWISE, GO BACK TO FRAME 1 AND TAKE THE PROGRAMMED SEQUENCE BEFORE TAKING TEST FRAME 3 AGAIN.

4. Recall that power is not dissipated by a reactive component. In an RC circuit some power is dissipated by the resistor (P_t), and some is stored by the capacitor (P_x) and returned to the circuit. The vector sum of these equal P_a, the total available power.

Solve for the values indicated.



a. P_t = ______

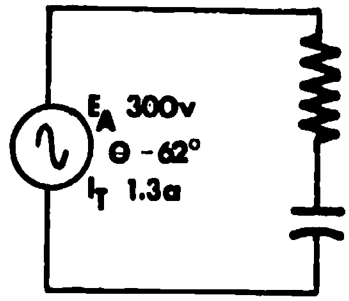
b. P_x = ______

c. P_a = ______

(a. 240w; b. 180 vars; c. 300 va)

5. The power factor is still equal to $\frac{P_t}{P_a}$ or, perhaps more conveniently, PF equals cosine $\underline{/\theta}$.

Solve for PF

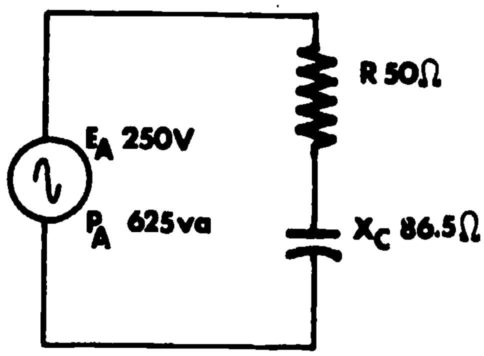


PF = ______

(0.4695)

6. Remember, if two values are known, whether they be total values or values for individual components, the circuit can usually be solved.

Solve for the values indicated.



a. I_T = ______

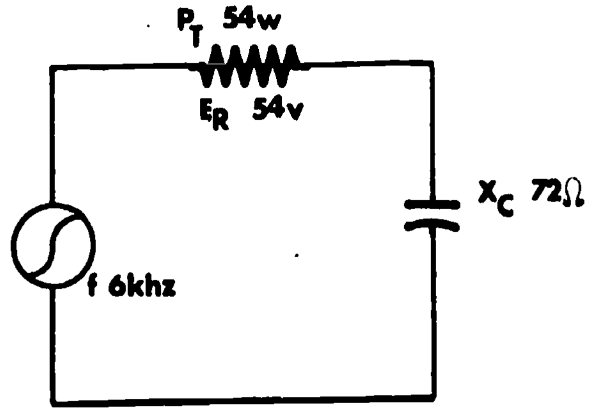
b. Z = ______

c. E_R = ______

d. E_C = ______

(a. 2.5a; b. 100Ω; c. 125v; d. 216v)

7. Solve for the values indicated.



a. Z_T = ______

b. I_T = ______

c. P_a = ______

d. $\underline{/\theta}$ = ______

e. PF = ______

f. E_C = ______

g. R = ______

h. E_a = ______

(THIS IS A TEST FRAME. COMPARE YOUR ANSWERS WITH THE CORRECT ANSWERS GIVEN AT THE TOP OF THE NEXT PAGE.)

ANSWERS - TEST FRAME 7

a. 90 Ω

b. 1 a

c. 90 va

d. -53.1°

e. 0.60

f. 72 v

g. 54 Ω

h. 90 v

IF ALL YOUR ANSWERS MATCH THE CORRECT ANSWERS, YOU MAY GO TO TEST FRAME 11. OTHERWISE, GO BACK TO FRAME 4 AND TAKE THE PROGRAMMED SEQUENCE BEFORE TAKING TEST FRAME 7 AGAIN.

---

8. Variational analysis of a series RC circuit is conducted in the same manner as in a series RL circuit. Remember, however, that a change in frequency has a different effect on X_C than it does on X_L. The reason is that X_L is <u>directly</u> proportional to frequency while X_C is <u>inversely</u> proportional to frequency.

Indicate with arrows what happens to the following circuit values if frequency is increased in a series RC circuit.

X_C ______

Z_T ______

I_T ______

($X_C\downarrow$; $Z_T\downarrow$; $I_T\uparrow$)

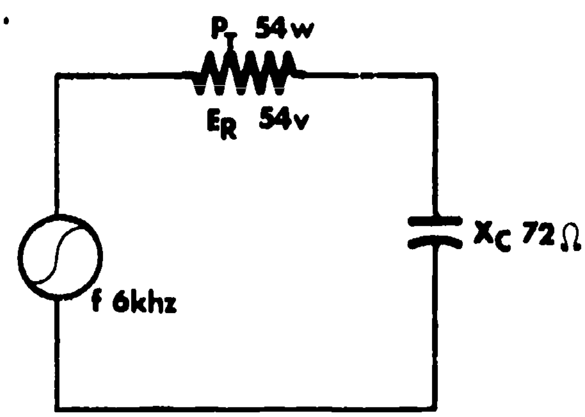
9. As frequency increases, X_C decreases. Since resistance is not affected by frequency and $Z = R - jX_C$, the circuit appears more resistive with an increase in the applied frequency.

As frequency increases /θ ______________.

(decreases ↓)

---

10. Study this circuit.



If frequency is increased, true power:

____ a. increases.
____ b. decreases.
____ c. does not change.

(a) increases

11. Using the table below, indicate what happens to each of the circuit values if frequency is decreased. Keep in mind the factors which are strictly physical and the relationship between frequency and X_C.

f	↓
Z_T	
X_C	
I_T	
E_R	
E_C	
P_T	
P_X	
P_A	
$\angle\theta$	
PF	
R	
C	

(THIS IS A TEST FRAME. COMPARE YOUR ANSWERS WITH THE CORRECT ANSWERS GIVEN AT THE TOP OF THE NEXT PAGE.)

ANSWERS - TEST FRAME 11

f	↓
Z_T	↑
X_C	↑
I_T	↓
E_R	↓
E_C	↑
P_T	↓
P_X	↓
P_A	↓
$\angle\theta$	↑
PF	↓
R	→
C	→

IF ALL YOUR ANSWERS MATCH THE CORRECT ANSWERS, YOU MAY GO TO TEST FRAME 16. OTHERWISE, GO BACK TO FRAME 8 AND TAKE THE PROGRAMMED SEQUENCE BEFORE TAKING TEST FRAME 11 AGAIN.

12. A variation in resistance has the same effect in a series RC circuit as it does in a series RL circuit.

If resistance is increased, what happens to these circuit values?

a. X_C ____________

b. Z_T ____________

c. I_T ____________

d. $\underline{/\theta}$ ____________

- -

(a. →; b. ↑; c. ↓; d. ↓)

13. An increase in applied voltage causes an increase in all values of power P_a, P_x, P_t), voltage (E_a, E_R, E_C), and current within the circuit. PF does not change with variations in applied voltage.

Indicate what happens to the following circuit values if the applied voltage is decreased.

a. X_C ____________

b. Z ____________

c. I_T ____________

d. $\underline{/\theta}$ ____________

e. P_t ____________

f. P_x ____________

- -

(a. →; b. →; c. ↓; d. →; e. ↓; f. ↓)

14. Since X_C is inversely proportional to <u>C</u> as it is to <u>f</u>, any variation in capacitance has the same effect upon circuit conditions in a series RC circuit as a corresponding change in frequency.

An increase in <u>C</u> causes:

a. X_C ______

b. Z_T ______

c. I_T ______

d. P_t ______

e. P_a ______

f. $\underline{/\theta}$ ______

- -

(a. ↓; b. ↓; c. ↑; d. ↑; e. ↑; f. ↓)

15. If at this point you are still having trouble with variational analysis, perhaps a review of procedure will help. Let's increase frequency in a series RC circuit, and by applying the proper equation to each value, determine what happens to each of the values in the circuit.

IF: $\boxed{f\uparrow}$

THEN: / BECAUSE:

Z_T	$\downarrow$	$Z_T = R - jX_C\downarrow$
X_C	$\downarrow$	$X_C = 1/2\pi f\uparrow C\rightarrow$
I_T	$\uparrow$	$I_T = \rightarrow E_A / Z_T\downarrow$
E_R	$\uparrow$	$E_R = \uparrow I_T \cdot R\rightarrow$
E_C	$\downarrow$	$\rightarrow E_A = \uparrow E_R - jE_C\downarrow$
P_T	$\uparrow$	$P_T = \uparrow I_T \cdot E_R\uparrow$
P_X	$\uparrow$	$P_X = \uparrow (I_T)^2 \cdot X_C\downarrow$
P_A	$\uparrow$	$P_A = \uparrow I_T \cdot E_A\rightarrow$
$\angle\theta$	$\downarrow$	$\tan\theta = \downarrow X_C / R\rightarrow$ **
PF	$\uparrow$	$PF = \cos\theta = \rightarrow R / Z\downarrow$ ***
R	$\rightarrow$	R = *
C	$\rightarrow$	C = *
E_A	$\rightarrow$	E_A = *

*Physical property which is not affected by frequency.

**The tangent function is used to determine $\angle\theta$.

***PF is equal to the COS θ. As the $\angle\theta$ approaches 90°, the COS approaches 0.

This procedure, with slight modification, can be applied to any problem.

16. Complete this table to show what happens in a series RC circuit when the indicated changes are made.

	$E_A \uparrow$	$R \downarrow$	$C \uparrow$	$f \downarrow$
Z_T				
X_C				
I_T				
E_R				
E_C				
P_T				
P_X				
P_A				
$/\theta$				
PF				
R				
C				

(THIS IS A TEST FRAME. COMPARE YOUR ANSWERS WITH THE CORRECT ANSWERS GIVEN AT THE TOP OF THE NEXT PAGE.)

ANSWERS - TEST FRAME 16

	E_A↑	R↓	C↑	f↓
Z_T	→	↓	↓	↑
X_C	→	→	↓	↑
I_T	↑	↑	↑	↓
E_R	↑	↓	↑	↓
E_C	↑	↑	↓	↑
P_T	↑		↑	↓
P_X	↑	↑	↑	↓
P_A	↑	↑	↑	↓
$\angle\theta$	→	↑	↓	↑
PF	→	↓	↑	↓
R	→	↓	→	→
C	→	→	↑	→

IF ALL YOUR ANSWERS MATCH THE CORRECT ANSWERS, YOU MAY GO TO TEST FRAME 24. OTHERWISE, GO BACK TO FRAME 12 AND TAKE THE PROGRAMMED SEQUENCE BEFORE TAKING TEST FRAME 16 AGAIN.

17. A series RC circuit exhibits frequency discrimination similar in many respects to that encountered in series RL circuits. The terms cutoff frequency and half-power point have the same meaning.

At f_{co} in a series RC circuit:

a. X_C = ______

b. E_C = ______

c. $\underline{/\theta}$ = ______

d. P_t = ______ of maximum DC value

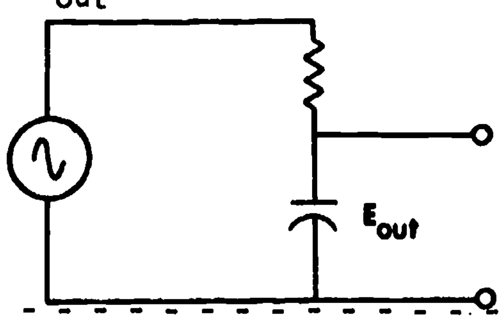
e. E_R = ______ of E_a

f. E_C = ______ of E_a

(a. R; b. E_R; c. 45°; d. 50%; e. 70.7%; f. 70.7%)

18. Series resistive-reactive circuits can be used as variable voltage dividers. The voltage developed across a reactive component depends on the amount of opposition offered by that component, which in turn depends on the frequency of the applied voltage.

E_{out} would be greatest at:



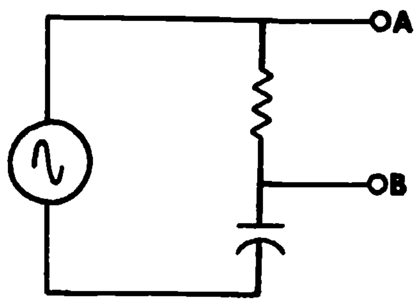
___a. frequencies above f_{co}.

___b. frequencies below f_{co}.

(b)

19. As frequency decreases, X_C increases because of the inverse relationship between the two. Since circuit current decreases, the voltage drop across the resistor decreases as frequency decreases. As E_R decreases, E_C must increase because $E_a = E_R - jE_C$.

If frequency is increased in the circuit below, the voltage measured between points A and B:



___a. increases.

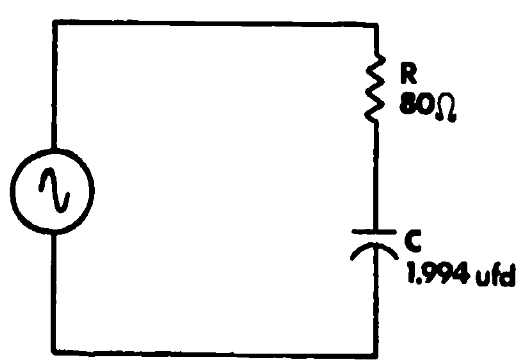
___b. decreases.

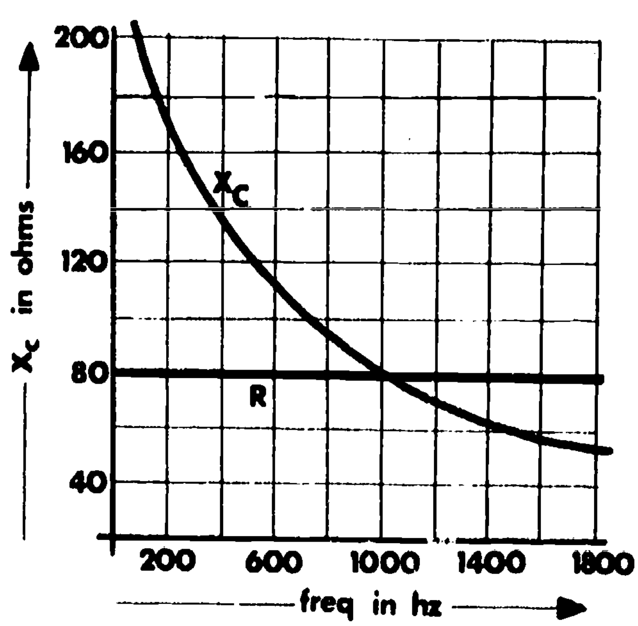
___c. remains the same.

(a)

20. In series RC circuits, just as in series RL circuits, a point is reached where reactance equals resistance ($X_C = R$). That, of course, is f_{co} for this circuit.

What is f_{co} for this circuit?





(1 KHz)

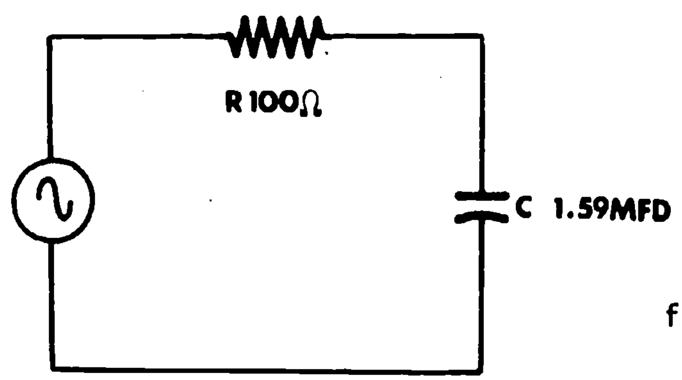
21. Because $R = X_C$ at f_{co}, a formula can be derived to find this frequency. By subsituting R for X_C in the capacitive reactance equation, we have:

$$R = \frac{1}{2\pi f_{co} C}$$

Then by transposing the formula to find f_{co}, we have:

$$f_{co} = \frac{1}{2\pi RC} \text{ or } f_{co} = \frac{0.159}{RC}$$

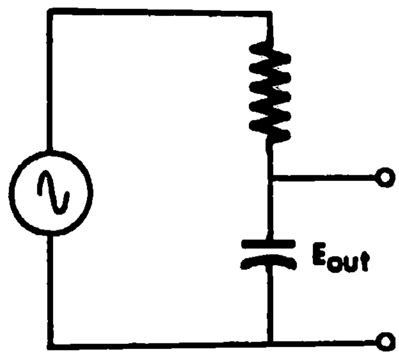
Solve for f_{co}.



f_{co} = ______

(1KHz)

22. A capacitor develops the greatest voltage at low frequencies, so if a series RC circuit is used to pass low frequencies to some type of load, the output must be taken across the capacitor.



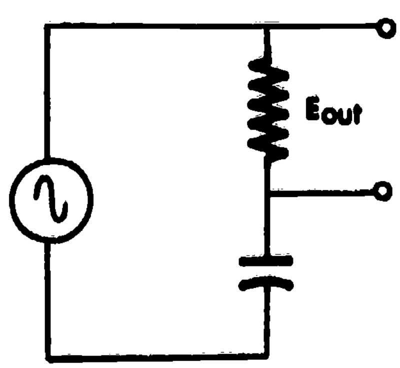
Low-pass filter (high-frequency discriminator)

This circuit would develop a useful output at frequencies ________ f_{co}.
above/below

(below)

23. If it is desired to use a series RC circuit as a high-pass filter (low-frequency discriminator), the output would be taken across the resistor. Due to the decrease in X_C as frequency increases, the greater voltage is developed across the resistor at frequencies above cutoff.

This circuit will produce a useable output voltage at frequencies ________ f_{co}.
above/below

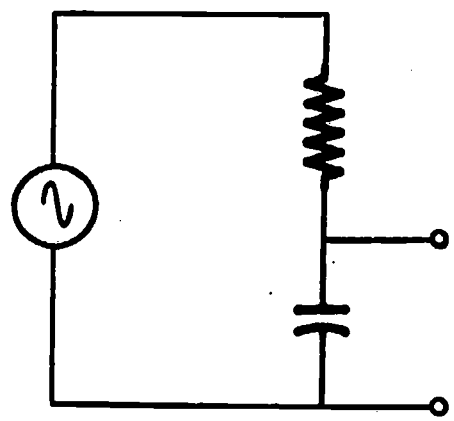


(above)

24. Draw a series RC circuit and indicate the necessary connections for use as a high-frequency discriminator.

(THIS IS A TEST FRAME. COMPARE YOUR ANSWERS WITH THE CORRECT ANSWERS GIVEN AT THE TOP OF THE NEXT PAGE.)

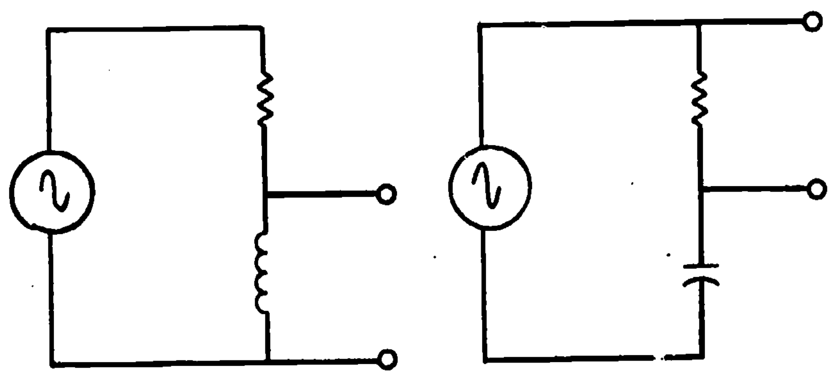
ANSWERS - TEST FRAME 24.



IF YOUR ANSWER MATCHES THE CORRECT ANSWER, YOU MAY GO TO FRAME 25. OTHERWISE, GO BACK TO FRAME NUMBER 17 AND TAKE THE PROGRAMMED SEQUENCE BEFORE TAKING TEST FRAME 24 AGAIN.

25. Notice the difference between series RL and RC filter circuits. In a series RL circuit, the greater voltage is developed across the reactive device (coil) at frequencies above f_{co}, while in a series RC circuit, due to the inverse relationship between frequency and X_C, the greater voltage is developed across the capacitor at frequencies below f_{co}.

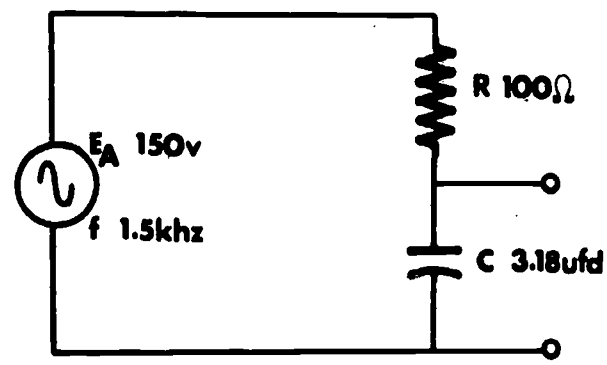
Draw a series RL and a series RC circuit, and indicate the output connections necessary to utilize them as high-pass filters.



26. To determine if the circuit will produce a useable output at a particular frequency, first determine if you are dealing with a high- or low-pass filter. Second, compute f_{co}. Third, compare the desired operating frequency with f_{co}. Fourth, determine whether the load will receive more voltage above or below that particular frequency.

What type of filter is shown in the schematic below? ________

Is there a useable output at the applied frequency? ________



- -

(low-pass; no)

YOU MAY NOW TAKE THE PROGRESS CHECK, OR YOU MAY STUDY ANY OF THE OTHER RESOURCES LISTED. IF YOU TAKE THE PROGRESS CHECK AND ANSWER ALL OF THE QUESTIONS CORRECTLY, YOU HAVE MASTERED THE MATERIAL AND ARE READY TO TAKE THE MODULE TEST. SEE YOUR LEARNING SUPERVISOR.

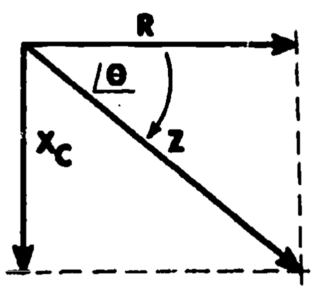
IF YOU DECIDE NOT TO TAKE THE PROGRESS CHECK AT THIS TIME, OR IF YOU MISSED ONE OR MORE QUESTIONS, STUDY ANY METHOD OF INSTRUCTION YOU WISH UNTIL YOU HAVE ANSWERED ALL THE PROGRESS CHECK QUESTIONS CORRECTLY. THEN SEE YOUR LEARNING SUPERVISOR AND ASK TO TAKE THE MODULE TEST.

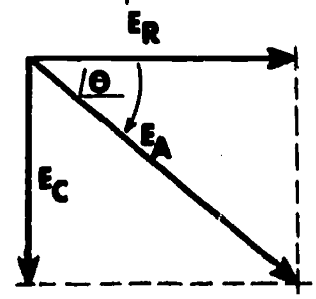
SUMMARY
LESSON VI

Series RC Circuits

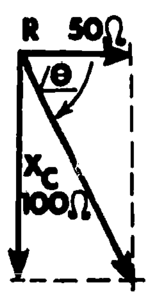
As you learned in the lessons on RL circuits; vectors must be used to solve resistive-inductive circuits. This is also true for resistive-capacitive circuits.

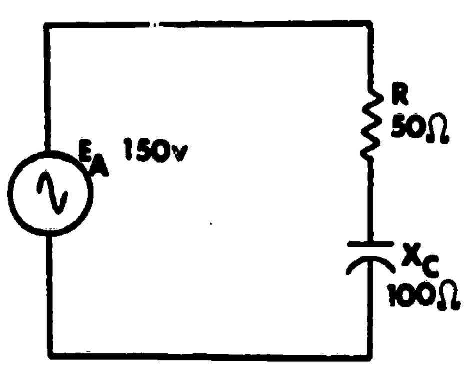
The only difference between solving RC circuits and RL circuits is the direction of the resultant vector of impedance and voltage.





As you see, the vectors representing X_C and E_C are rotated clockwise and plotted 90° behind the reference line. The reason for this is I leads E by 90° in a capacitive circuit and in a series circuit current is the reference value. In a resistive-capacitive circuit is a negative angle. Other than the negative phase angle, an RC circuit is solved in the same manner as an RL circuit.





$$\text{TAN } \angle\theta = \frac{\text{opp}}{\text{adj}} = \frac{100}{50} = 2$$

$$= -63.4°$$

$$\text{hyp} = \frac{\text{opp}}{\text{SIN } \angle\theta} = \frac{100}{.8942} = 113$$

$$Z = 113\Omega$$

$$I = \frac{E}{Z} = \frac{150}{113} = 1.33\text{a}$$

$$P_a = I \cdot E_a = 1.33 \times 150 = 199 \text{ va}$$

$$P_t = I \cdot E \cdot \text{COS } \angle\theta = I^2R$$

$$P_x = I \cdot E \cdot \text{SIN } \angle\theta = I^2X_C$$

$$P_t = 1.33 \times 150 \times .4478 = 89\text{w}$$

$$\text{PF} = \text{COS } \angle\theta = .4478$$

Variational analysis of series RC circuits is conducted in the same way as for series RL circuits. Do not expect all RL circuit values to react in the same way as the corresponding value in the RC circuit, however. Remember, X_C is inversely proportional to frequency while X_L is directly proportional to frequency.

Any variation in frequency causes X_C to change in the reverse direction. For example, if frequency increases, X_C decreases $(\downarrow X_C = \frac{1}{2\pi f \uparrow C})$. Impedance is equal to the vectorial sum of resistance and capacitive reactance $(Z = R - jX_C)$. So any change in X_C causes a corresponding change in Z_T $(\underset{\downarrow}{Z} = R - j\underset{\downarrow}{X_C})$. After change to impedance and capacitive reactance has been determined, all other circuit values change as they do in an RL circuit.

f	↓
X_C	↑
R	→
Z	↑
I	↓
P_A	↓
P_X	↓
P_T	↓
E_A	→
E_C	↑
$\angle\theta$	↑
PF	↓
C	→

A change in capacitance causes circuit conditions to react in the same way as a comparable change in frequency.

C	▲
X_C	▼
R	→
Z	▼
I	▲
P_A	▲
P_x	▲
P_T	▲
E_R	▲
E_C	▼
θ	▼
PF	▲

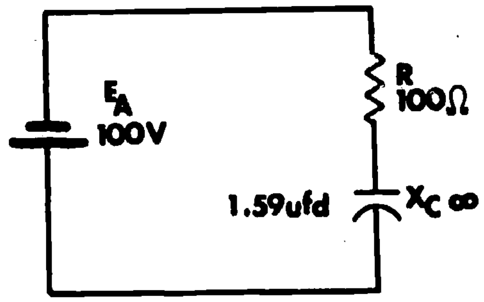
Again the variations in the voltage drops may be the most confusing. Just remember to determine which opposition is unchanged (with X_C or R), then determine the direction of the change in circuit current. From this information, the change in the voltage drop across the unchanged opposition can be determined; then the voltage drop across the remaining opposition changes in the opposite direction ($E_a = E_R - jE_C$).

The other two variable factors (E_a and R) have exactly the same effect on RC circuits as on RL circuits.

	E_A↑	R↓
Z	→	↓
I	↑	↑
P_T	↑	*
P_x	↑	↑
P_A	↑	↑
E_R	↑	↓
E_C	↑	↑
$\angle\theta$	→	↑
PF	→	↓

RC circuits may also be used as filter circuits.

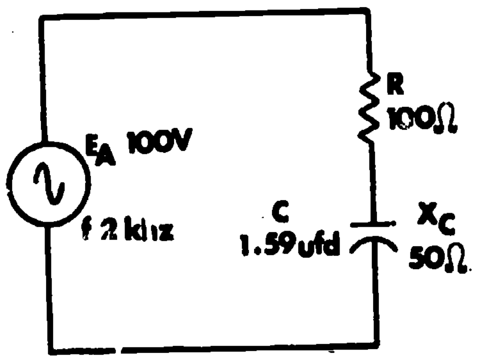
*Changing resistance in a series circuit affects power in a complex way, so you will not be required to answer this question.



With a DC potential applied, there is no current flow, and the entire source voltage appears across the capacitor. E_R is 0.

If the DC source is replaced by an AC source and frequency is increased, X_C decreases to some measurable value and there is some current flow.

At 2000 Hz these circuit values exist:



$X_C = 50\Omega$

$Z = 113\Omega$

$I_T = 885$ ma

$E_R = 88.5$ v

$E_C = 44.2$ v

$P_a = 88.5$ va

$P_x = 39.16$ vars

$P_t = 79$ w

$\underline{/\theta} = -26.6°$

PF = .8942

As frequency decreases, a point is reached where $X_C = R$.
As in RL circuits, this condition is called <u>frequency</u> <u>cutoff</u> (f_{co}).
At f_{co} in a series RC circuit:

$X_C = R$

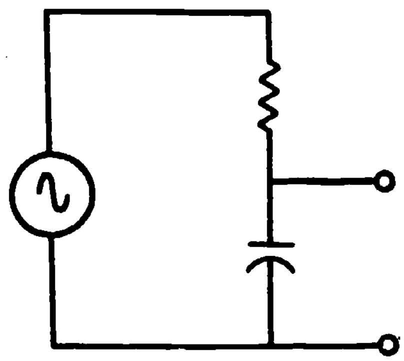
$E_C = E_R = 70.7\%$ of E_a

P_t = 1/2 maximum DC value (half-power point)

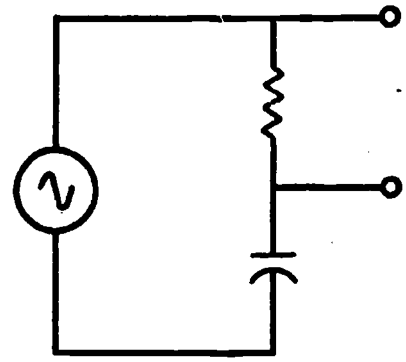
$\underline{/\theta} = -45°$

The formula for determining cutoff frequency is $f_{co} = \frac{1}{2\pi RC}$

The output connections for RC filters are:



Low-pass filter
(high-frequency discriminator)



high-pass filter
(low-frequency discriminator)

AT THIS POINT, YOU MAY TAKE THE LESSON PROGRESS CHECK, OR YOU MAY STUDY THE LESSON NARRATIVE OR THE PROGRAMMED INSTRUCTION OR BOTH. IF YOU TAKE THE PROGRESS CHECK AND ANSWER ALL OF THE QUESTIONS CORRECTLY, YOU MAY TAKE THE MODULE TEST. IF NOT, STUDY ANOTHER METHOD OF INSTRUCTION UNTIL YOU CAN ANSWER ALL PROGRESS TEST QUESTIONS CORRECTLY. THEN YOU ARE READY FOR THE MODULE TEST.

deg	function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°
0	sin	0.0000	0.0017	0.0035	0.0052	0.0070	0.0087	0.0105	0.0122	0.0140	0.0157
	cos	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999
	tan	0.0000	0.0017	0.0035	0.0052	0.0070	0.0087	0.0105	0.0122	0.0140	0.0157
1	sin	0.0175	0.0192	0.0209	0.0227	0.0244	0.0262	0.0279	0.0297	0.0314	0.0332
	cos	0.9998	0.9998	0.9998	0.9997	0.9997	0.9997	0.9996	0.9996	0.9995	0.9995
	tan	0.0175	0.0192	0.0209	0.0227	0.0244	0.0262	0.0279	0.0297	0.0314	0.0332
2	sin	0.0349	0.0366	0.0384	0.0401	0.0419	0.0436	0.0454	0.0471	0.0488	0.0506
	cos	0.9994	0.9993	0.9993	0.9992	0.9991	0.9990	0.9990	0.9989	0.9988	0.9987
	tan	0.0349	0.0367	0.0384	0.0402	0.0419	0.0437	0.0454	0.0472	0.0489	0.0507
3	sin	0.0523	0.0541	0.0558	0.0576	0.0593	0.0610	0.0628	0.0645	0.0663	0.0680
	cos	0.9986	0.9985	0.9984	0.9983	0.9982	0.9981	0.9980	0.9979	0.9978	0.9977
	tan	0.0524	0.0542	0.0559	0.0577	0.0594	0.0612	0.0629	0.0647	0.0664	0.0682
4	sin	0.0698	0.0715	0.0732	0.0750	0.0767	0.0785	0.0802	0.0819	0.0837	0.0854
	cos	0.9976	0.9974	0.9973	0.9972	0.9971	0.9969	0.9968	0.9966	0.9965	0.9963
	tan	0.0699	0.0717	0.0734	0.0752	0.0769	0.0787	0.0805	0.0822	0.0840	0.0857
5	sin	0.0872	0.0889	0.0906	0.0924	0.0941	0.0958	0.0976	0.0993	0.1011	0.1028
	cos	0.9962	0.9960	0.9959	0.9957	0.9956	0.9954	0.9952	0.9951	0.9949	0.9947
	tan	0.0875	0.0892	0.0910	0.0928	0.0945	0.0963	0.0981	0.0998	0.1016	0.1033
6	sin	0.1045	0.1063	0.1080	0.1097	0.1115	0.1132	0.1149	0.1167	0.1184	0.1201
	cos	0.9945	0.9943	0.9942	0.9940	0.9938	0.9936	0.9934	0.9932	0.9930	0.9928
	tan	0.1051	0.1069	0.1086	0.1104	0.1122	0.1139	0.1157	0.1175	0.1192	0.1210
7	sin	0.1219	0.1236	0.1253	0.1271	0.1288	0.1305	0.1323	0.1340	0.1357	0.1374
	cos	0.9925	0.9923	0.9921	0.9919	0.9917	0.9914	0.9912	0.9910	0.9907	0.9905
	tan	0.1228	0.1246	0.1263	0.1281	0.1299	0.1317	0.1334	0.1352	0.1370	0.1388
8	sin	0.1392	0.1409	0.1426	0.1444	0.1461	0.1478	0.1495	0.1513	0.1530	0.1547
	cos	0.9903	0.9900	0.9898	0.9895	0.9893	0.9890	0.9888	0.9885	0.9882	0.9880
	tan	0.1405	0.1423	0.1441	0.1459	0.1477	0.1495	0.1512	0.1530	0.1548	0.1566
9	sin	0.1564	0.1582	0.1599	0.1616	0.1633	0.1650	0.1668	0.1685	0.1702	0.1719
	cos	0.9877	0.9874	0.9871	0.9869	0.9866	0.9863	0.9860	0.9857	0.9854	0.9851
	tan	0.1584	0.1602	0.1620	0.1638	0.1655	0.1673	0.1691	0.1709	0.1727	0.1745
10	sin	0.1736	0.1754	0.1771	0.1788	0.1805	0.1822	0.1840	0.1857	0.1874	0.1891
	cos	0.9848	0.9845	0.9842	0.9839	0.9836	0.9833	0.9829	0.9826	0.9823	0.9820
	tan	0.1763	0.1781	0.1799	0.1817	0.1835	0.1853	0.1871	0.1890	0.1908	0.1926
11	sin	0.1908	0.1925	0.1942	0.1959	0.1977	0.1994	0.2011	0.2028	0.2045	0.2062
	cos	0.9816	0.9813	0.9810	0.9806	0.9803	0.9799	0.9796	0.9792	0.9789	0.9785
	tan	0.1944	0.1962	0.1980	0.1998	0.2016	0.2035	0.2053	0.2071	0.2089	0.2107
12	sin	0.2079	0.2096	0.2113	0.2130	0.2147	0.2164	0.2181	0.2198	0.2215	0.2232
	cos	0.9781	0.9778	0.9774	0.9770	0.9767	0.9763	0.9759	0.9755	0.9751	0.9748
	tan	0.2126	0.2144	0.2162	0.2180	0.2199	0.2217	0.2235	0.2254	0.2272	0.2290
13	sin	0.2250	0.2267	0.2284	0.2300	0.2318	0.2334	0.2351	0.2368	0.2385	0.2402
	cos	0.9744	0.9740	0.9736	0.9732	0.9728	0.9724	0.9720	0.9715	0.9711	0.9707
	tan	0.2309	0.2327	0.2345	0.2364	0.2382	0.2401	0.2419	0.2438	0.2456	0.2475
deg	function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°

deg	function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°
	sin	0.2419	0.2436	0.2453	0.2470	0.2487	0.2504	0.2521	0.2538	0.2554	0.2571
14	cos	0.9703	0.9699	0.9694	0.9690	0.9686	0.9681	0.9677	0.9673	0.9668	0.9664
	tan	0.2493	0.2512	0.2530	0.2549	0.2568	0.2586	0.2605	0.2623	0.2642	0.2661
	sin	0.2588	0.2605	0.2622	0.2639	0.2656	0.2672	0.2689	0.2706	0.2723	0.2740
15	cos	0.9659	0.9655	0.9650	0.9646	0.9641	0.9636	0.9632	0.9627	0.9622	0.9617
	tan	0.2679	0.2698	0.2717	0.2736	0.2754	0.2773	0.2792	0.2811	0.2830	0.2849
	sin	0.2756	0.2773	0.2790	0.2807	0.2823	0.2840	0.2857	0.2874	0.2890	0.2907
16	cos	0.9613	0.9608	0.9603	0.9598	0.9593	0.9588	0.9583	0.9578	0.9573	0.9568
	tan	0.2867	0.2886	0.2905	0.2924	0.2943	0.2962	0.2981	0.3000	0.3019	0.3038
	sin	0.2924	0.2940	0.2957	0.2974	0.2990	0.3007	0.3024	0.3040	0.3057	0.3074
17	cos	0.9563	0.9558	0.9553	0.9548	0.9542	0.9537	0.9532	0.9527	0.9521	0.9516
	tan	0.3057	0.3076	0.3096	0.3115	0.3134	0.3153	0.3172	0.3191	0.3211	0.3230
	sin	0.3090	0.3107	0.3123	0.3140	0.3156	0.3173	0.3190	0.3206	0.3223	0.3239
18	cos	0.9511	0.9505	0.9500	0.9494	0.9489	0.9483	0.9478	0.9472	0.9466	0.9461
	tan	0.3249	0.3269	0.3288	0.3307	0.3327	0.3346	0.3365	0.3385	0.3404	0.3424
	sin	0.3256	0.3272	0.3289	0.3305	0.3322	0.3338	0.3355	0.3371	0.3387	0.3404
19	cos	0.9455	0.9449	0.9444	0.9438	0.9432	0.9426	0.9421	0.9415	0.9409	0.9403
	tan	0.3443	0.3463	0.3482	0.3502	0.3522	0.3541	0.3561	0.3581	0.3600	0.3620
	sin	0.3420	0.3437	0.3453	0.3469	0.3486	0.3502	0.3518	0.3535	0.3551	0.3567
20	cos	0.9397	0.9391	0.9385	0.9379	0.9373	0.9367	0.9361	0.9354	0.9348	0.9342
	tan	0.3640	0.3659	0.3679	0.3699	0.3719	0.3739	0.3759	0.3779	0.3799	0.3819
	sin	0.3584	0.3600	0.3616	0.3633	0.3649	0.3665	0.3681	0.3697	0.3714	0.3730
21	cos	0.9336	0.9330	0.9323	0.9317	0.9311	0.9304	0.9298	0.9291	0.9285	0.9278
	tan	0.3839	0.3859	0.3879	0.3899	0.3919	0.3939	0.3959	0.3979	0.4000	0.4020
	sin	0.3746	0.3762	0.3778	0.3795	0.3811	0.3827	0.3843	0.3859	0.3875	0.3891
22	cos	0.9272	0.9265	0.9259	0.9252	0.9245	0.9239	0.9232	0.9225	0.9219	0.9212
	tan	0.4040	0.4061	0.4081	0.4101	0.4122	0.4142	0.4163	0.4183	0.4204	0.4224
	sin	0.3907	0.3923	0.3939	0.3955	0.3971	0.3987	0.4003	0.4019	0.4035	0.4051
23	cos	0.9205	0.9198	0.9191	0.9184	0.9178	0.9171	0.9164	0.9157	0.9150	0.9143
	tan	0.4245	0.4265	0.4286	0.4307	0.4327	0.4348	0.4369	0.4390	0.4411	0.4431
	sin	0.4067	0.4083	0.4099	0.4115	0.4131	0.4147	0.4163	0.4179	0.4195	0.4210
24	cos	0.9135	0.9128	0.9121	0.9114	0.9107	0.9100	0.9092	0.9085	0.9078	0.9070
	tan	0.4452	0.4473	0.4494	0.4515	0.4536	0.4557	0.4578	0.4599	0.4621	0.4642
	sin	0.4226	0.4242	0.4258	0.4274	0.4289	0.4305	0.4321	0.4337	0.4352	0.4368
25	cos	0.9063	0.9056	0.9048	0.9041	0.9033	0.9026	0.9018	0.9011	0.9003	0.8996
	tan	0.4663	0.4684	0.4706	0.4727	0.4748	0.4770	0.4791	0.4813	0.4834	0.4856
	sin	0.4384	0.4399	0.4415	0.4431	0.4446	0.4462	0.4478	0.4493	0.4509	0.4524
26	cos	0.8988	0.8980	0.8973	0.8965	0.8957	0.8949	0.8942	0.8934	0.8926	0.8918
	tan	0.4877	0.4899	0.4921	0.4942	0.4964	0.4986	0.5008	0.5029	0.5051	0.5073
	sin	0.4540	0.4555	0.4571	0.4586	0.4602	0.4617	0.4633	0.4648	0.4664	0.4679
27	cos	0.8910	0.8902	0.8894	0.8886	0.8878	0.8870	0.8862	0.8854	0.8846	0.8838
	tan	0.5095	0.5117	0.5139	0.5161	0.5184	0.5206	0.5228	0.5250	0.5272	0.5295
deg	function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°

deg	function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°
	sin	0.4695	0.4710	0.4726	0.4741	0.4756	0.4772	0.4787	0.4802	0.4818	0.4833
28	cos	0.8829	0.8821	0.8813	0.8805	0.8796	0.8788	0.8780	0.8771	0.8763	0.8755
	tan	0.5317	0.5340	0.5362	0.5384	0.5407	0.5430	0.5452	0.5475	0.5498	0.5520
	sin	0.4848	0.4863	0.4879	0.4894	0.4909	0.4924	0.4939	0.4955	0.4970	0.4985
29	cos	0.8746	0.8738	0.8729	0.8721	0.8712	0.8704	0.8695	0.8686	0.8678	0.8669
	tan	0.5543	0.5566	0.5589	0.5612	0.5635	0.5658	0.5681	0.5704	0.5727	0.5750
	sin	0.5000	0.5015	0.5030	0.5045	0.5060	0.5075	0.5090	0.5105	0.5120	0.5135
30	cos	0.8660	0.8652	0.8643	0.8634	0.8625	0.8616	0.8607	0.8599	0.8590	0.8581
	tan	0.5774	0.5797	0.5820	0.5844	0.5867	0.5890	0.5914	0.5938	0.5961	0.5985
	sin	0.5150	0.5165	0.5180	0.5195	0.5210	0.5225	0.5240	0.5255	0.5270	0.5284
31	cos	0.8572	0.8563	0.8554	0.8545	0.8536	0.8526	0.8517	0.8508	0.8499	0.8490
	tan	0.6009	0.6032	0.6056	0.6080	0.6104	0.6128	0.6152	0.6176	0.6200	0.6224
	sin	0.5299	0.5314	0.5329	0.5344	0.5358	0.5373	0.5388	0.5402	0.5417	0.5432
32	cos	0.8480	0.8471	0.8462	0.8453	0.8443	0.8434	0.8425	0.8415	0.8406	0.8396
	tan	0.6249	0.6273	0.6297	0.6322	0.6346	0.6371	0.6395	0.6420	0.6445	0.6469
	sin	0.5446	0.5461	0.5476	0.5490	0.5505	0.5519	0.5534	0.5548	0.5563	0.5577
33	cos	0.8387	0.8377	0.8368	0.8358	0.8348	0.8339	0.8329	0.8320	0.8310	0.8300
	tan	0.6494	0.6519	0.6544	0.6569	0.6594	0.6619	0.6644	0.6669	0.6694	0.6720
	sin	0.5592	0.5606	0.5621	0.5635	0.5650	0.5664	0.5678	0.5693	0.5707	0.5721
34	cos	0.8290	0.8281	0.8271	0.8261	0.8251	0.8241	0.8231	0.8221	0.8211	0.8202
	tan	0.6745	0.6771	0.6796	0.6822	0.6847	0.6873	0.6899	0.6924	0.6950	0.6976
	sin	0.5736	0.5750	0.5764	0.5779	0.5793	0.5807	0.5821	0.5835	0.5850	0.5864
35	cos	0.8192	0.8181	0.8171	0.8161	0.8151	0.8141	0.8131	0.8121	0.8111	0.8100
	tan	0.7002	0.7028	0.7054	0.7080	0.7107	0.7133	0.7159	0.7186	0.7212	0.7239
	sin	0.5878	0.5892	0.5906	0.5920	0.5934	0.5948	0.5962	0.5976	0.5990	0.6004
36	cos	0.8090	0.8080	0.8070	0.8059	0.8049	0.8039	0.8028	0.8018	0.8007	0.7997
	tan	0.7265	0.7292	0.7319	0.7346	0.7373	0.7400	0.7427	0.7454	0.7481	0.7508
	sin	0.6018	0.6032	0.6046	0.6060	0.6074	0.6088	0.6101	0.6115	0.6129	0.6143
37	cos	0.7986	0.7976	0.7965	0.7955	0.7944	0.7934	0.7923	0.7912	0.7902	0.7891
	tan	0.7536	0.7563	0.7590	0.7618	0.7646	0.7673	0.7701	0.7729	0.7757	0.7785
	sin	0.6157	0.6170	0.6184	0.6198	0.6211	0.6225	0.6239	0.6252	0.6266	0.6280
38	cos	0.7880	0.7869	0.7859	0.7848	0.7837	0.7826	0.7815	0.7804	0.7793	0.7782
	tan	0.7813	0.7841	0.7869	0.7898	0.7926	0.7954	0.7983	0.8012	0.8040	0.8069
	sin	0.6293	0.6307	0.6320	0.6334	0.6347	0.6361	0.6374	0.6388	0.6401	0.6414
39	cos	0.7771	0.7760	0.7749	0.7738	0.7727	0.7716	0.7705	0.7694	0.7683	0.7672
	tan	0.8098	0.8127	0.8156	0.8185	0.8214	0.8243	0.8273	0.8302	0.8332	0.8361
	sin	0.6428	0.6441	0.6455	0.6468	0.6481	0.6494	0.6508	0.6521	0.6534	0.6547
40	cos	0.7660	0.7649	0.7638	0.7627	0.7615	0.7604	0.7593	0.7581	0.7570	0.7559
	tan	0.8391	0.8421	0.8451	0.8481	0.8511	0.8541	0.8571	0.8601	0.8632	0.8662
	sin	0.6561	0.6574	0.6587	0.6600	0.6613	0.6626	0.6639	0.6652	0.6665	0.6678
41	cos	0.7547	0.7536	0.7524	0.7513	0.7501	0.7490	0.7478	0.7466	0.7455	0.7443
	tan	0.8693	0.8724	0.8754	0.8785	0.8816	0.8847	0.8878	0.8910	0.8941	0.8972
deg	function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°

deg	function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°
	sin	0.6691	0.6704	0.6717	0.6730	0.6743	0.6756	0.6769	0.6782	0.6794	0.6807
42	cos	0.7431	0.7420	0.7408	0.7396	0.7385	0.7373	0.7361	0.7349	0.7337	0.7325
	tan	0.9004	0.9036	0.9067	0.9099	0.9131	0.9163	0.9195	0.9228	0.9260	0.9293
	sin	0.6820	0.6833	0.6845	0.6858	0.6871	0.6884	0.6896	0.6909	0.6921	0.6934
43	cos	0.7314	0.7302	0.7290	0.7278	0.7266	0.7254	0.7242	0.7230	0.7218	0.7206
	tan	0.9325	0.9358	0.9391	0.9424	0.9457	0.9490	0.9523	0.9556	0.9590	0.9623
	sin	0.6947	0.6959	0.6972	0.6984	0.6997	0.7009	0.7022	0.7034	0.7046	0.7059
44	cos	0.7193	0.7181	0.7169	0.7157	0.7145	0.7133	0.7120	0.7108	0.7096	0.7083
	tan	0.9657	0.9691	0.9725	0.9759	0.9793	0.9827	0.9861	0.9896	0.9930	0.9965
	sin	0.7071	0.7083	0.7096	0.7108	0.7120	0.7133	0.7145	0.7157	0.7169	0.7181
45	cos	0.7071	0.7059	0.7046	0.7034	0.7022	0.7009	0.6997	0.6984	0.6972	0.6959
	tan	1.0000	1.0035	1.0070	1.0105	1.0141	1.0176	1.0212	1.0247	1.0283	1.0319
	sin	0.7193	0.7206	0.7218	0.7230	0.7242	0.7254	0.7266	0.7278	0.7290	0.7302
46	cos	0.6947	0.6934	0.6921	0.6909	0.6896	0.6884	0.6871	0.6858	0.6845	0.6833
	tan	1.0355	1.0392	1.0428	1.0464	1.0501	1.0538	1.0575	1.0612	1.0649	1.0686
	sin	0.7314	0.7325	0.7337	0.7349	0.7361	0.7373	0.7385	0.7396	0.7408	0.7420
47	cos	0.6820	0.6807	0.6794	0.6782	0.6769	0.6756	0.6743	0.6730	0.6717	0.6704
	tan	1.0724	1.0761	1.0799	1.0837	1.0875	1.0913	1.0951	1.0990	1.1028	1.1067
	sin	0.7431	0.7443	0.7455	0.7466	0.7478	0.7490	0.7501	0.7513	0.7524	0.7536
48	cos	0.6691	0.6678	0.6665	0.6652	0.6639	0.6626	0.6613	0.6600	0.6587	0.6574
	tan	1.1106	1.1145	1.1184	1.1224	1.1263	1.1303	1.1343	1.1383	1.1423	1.1463
	sin	0.7547	0.7559	0.7570	0.7581	0.7593	0.7604	0.7615	0.7627	0.7638	0.7649
49	cos	0.6561	0.6547	0.6534	0.6521	0.6508	0.6494	0.6481	0.6468	0.6455	0.6441
	tan	1.1504	1.1544	1.1585	1.1626	1.1667	1.1708	1.1750	1.1792	1.1833	1.1875
	sin	0.7660	0.7672	0.7683	0.7694	0.7705	0.7716	0.7727	0.7738	0.7749	0.7760
50	cos	0.6428	0.6414	0.6401	0.6388	0.6374	0.6361	0.6347	0.6334	0.6320	0.6307
	tan	1.1918	1.1960	1.2002	1.2045	1.2088	1.2131	1.2174	1.2218	1.2261	1.2305
	sin	0.7771	0.7782	0.7793	0.7804	0.7815	0.7826	0.7837	0.7848	0.7859	0.7869
51	cos	0.6293	0.6280	0.6266	0.6252	0.6239	0.6225	0.6211	0.6198	0.6184	0.6170
	tan	1.2349	1.2393	1.2437	1.2482	1.2527	1.2572	1.2617	1.2662	1.2708	1.2753
	sin	0.7880	0.7891	0.7902	0.7912	0.7923	0.7934	0.7944	0.7955	0.7965	0.7976
52	cos	0.6157	0.6143	0.6129	0.6115	0.6101	0.6088	0.6074	0.6060	0.6046	0.6032
	tan	1.2799	1.2846	1.2892	1.2938	1.2985	1.3032	1.3079	1.3127	1.3175	1.3222
	sin	0.7986	0.7997	0.8007	0.8018	0.8028	0.8039	0.8049	0.8059	0.8070	0.8080
53	cos	0.6018	0.6004	0.5990	0.5976	0.5962	0.5948	0.5934	0.5920	0.5906	0.5892
	tan	1.3270	1.3319	1.3367	1.3416	1.3465	1.3514	1.3564	1.3613	1.3663	1.3713
	sin	0.8090	0.8100	0.8111	0.8121	0.8131	0.8141	0.8151	0.8161	0.8171	0.8181
54	cos	0.5878	0.5864	0.5850	0.5835	0.5821	0.5807	0.5793	0.5779	0.5764	0.5750
	tan	1.3764	1.3814	1.3865	1.3916	1.3968	1.4019	1.4071	1.4124	1.4176	1.4229
	sin	0.8192	0.8202	0.8211	0.8221	0.8231	0.8241	0.8251	0.8261	0.8271	0.8281
55	cos	0.5736	0.5721	0.5707	0.5693	0.5678	0.5664	0.5650	0.5635	0.5621	0.5606
	tan	1.4281	1.4335	1.4388	1.4442	1.4496	1.4550	1.4605	1.4659	1.4715	1.4770
deg	function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°

deg	function	0.0°	0.1°	0.2°	0 3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°
56	sin	0.8290	0.8300	0.8310	0.8320	0.8329	0.8339	0.8348	0.8358	0.8368	0.8377
	cos	0.5592	0.5577	0.5563	0.5548	0.5534	0.5519	0.5505	0.5490	0.5476	0.5461
	tan	1.4826	1.4862	1.4938	1.4994	1.5051	1.5108	1.5166	1.5224	1.5282	1.5340
57	sin	0.8387	0.8396	0.8406	0.8415	0.8425	0.8434	0.8443	0.8453	0.8462	0.8471
	cos	0.5446	0.5432	0.5417	0.5402	0.5388	0.5373	0.5358	0.5344	0.5329	0.5314
	tan	1.5399	1.5458	1.5517	1.5577	1.5637	1.5697	1.5757	1.5818	1.5880	1.5941
58	sin	0.8480	0.8490	0.8499	0.8508	0.8517	0.8526	0.8536	0.8545	0.8554	0.8563
	cos	0.5299	0.5284	0.5270	0 5255	0.5240	0.5225	0.5210	0.5195	0.5180	0.5165
	tan	1.6003	1.6066	1.6128	1.6191	1.6255	1.6319	1.6383	1.6447	1.6512	1.6577
59	sin	0.8572	0.8581	0.8590	0.8599	0.8607	0.8616	0.8625	0.8634	0.8643	0.8652
	cos	0.5150	0.5135	0.5120	0.5105	0.5090	0.5075	0.5060	0.5045	0.5030	0.5015
	tan	1.6643	1.6709	1.6775	1.6842	1.6909	1.6977	1.7045	1.7113	1.7182	1.7251
60	sin	0.8660	0.8669	0.8678	0.8686	0.8695	0.8704	0.8712	0.8721	0.8729	0.8738
	cos	0.5000	0.4985	0.4970	0.4955	0.4939	0.4924	0.4909	0.4894	0.4879	0.4863
	tan	1.7321	1.7391	1.7461	1.7532	1.7603	1.7675	1.7747	1.7820	1.7893	1.7966
61	sin	0.8746	0.8755	0.8763	0.8771	0.8780	0.8788	0.8796	0.8805	0.8813	0.8821
	cos	0.4848	0.4833	0.4818	0.4802	0.4787	0.4772	0.4756	0.4741	0.4726	0.4710
	tan	1.8040	1.8115	1.8190	1.8265	1.8341	1.8418	1.8495	1.8572	1.8650	1.8728
62	sin	0.8829	0.8838	0.8846	0.8854	0.8862	0.8870	0.8878	0.8886	0.8894	0.8902
	cos	0.4695	0.4679	0.4664	0.4648	0.4633	0.4617	0.4602	0.4586	0.4571	0.4555
	tan	1.8807	1.8887	1.8967	1.9047	1.9128	1.9210	1.9292	1.9375	1.9458	1.9542
63	sin	0.8910	0.8918	0.8926	0.8934	0.8942	0.8949	0.8957	0.8965	0.8973	0.8980
	cos	0.4540	0.4524	0.4509	0.4493	0.4478	0.4462	0.4446	0.4431	0.4415	0.4399
	tan	1.9626	1.9711	1.9797	1.9883	1.9970	2.0057	2.0145	2.0233	2.0323	2.0413
64	sin	0.8988	0.8996	0.9003	0.9011	0.9018	0.9026	0.9033	0.9041	0.9048	0.9056
	cos	0.4384	0.4368	0.4352	0.4337	0.4321	0.4305	0.4289	0.4274	0.4258	0.4242
	tan	2.0503	2.0594	2.0686	2.0778	2.0872	2.0965	2.1060	2.1155	2.1251	2.1348
65	sin	0.9063	0.9070	0.9078	0.9085	0.9092	0.9100	0.9107	0.9114	0.9121	0.9128
	cos	0.4226	0.4210	0.4195	0.4179	0.4163	0.4147	0.4131	0.4115	0.4099	0.4083
	tan	2.1445	2.1543	2.1642	2.1742	2.1842	2.1943	2.2045	2.2148	2.2251	2.2355
66	sin	0.9135	0.9143	0.9150	0.9157	0.9164	0.9171	0.9178	0.9184	0.9191	0.9198
	cos	0.4067	0.4051	0.4035	0.4019	0.4003	0.3987	0.3971	0.3955	0.3939	0.3923
	tan	2.2460	2.2566	2.2673	2.2781	2.2889	2.2998	2.3109	2.3220	2.3332	2.3445
67	sin	0.9205	0.9212	0.9219	0.9225	0.9232	0.9239	0.9245	0.9252	0.9259	0.9265
	cos	0.3907	0.3891	0.3875	0.3859	0.3843	0.3827	0.3811	0.3795	0.3778	0.3762
	tan	2.3559	2.3673	2.3789	2.3906	2.4023	2.4142	2.4262	2.4383	2.4504	2.4627
68	sin	0.9272	0.9278	0.9285	0.9291	0.9298	0.9304	0.9311	0.9317	0.9323	0.9330
	cos	0.3746	0.3730	0.3714	0.3697	0.3681	0.3665	0.3649	0.3633	0.3616	0.3600
	tan	2.4751	2.4876	2.5002	2.5129	2.5257	2.5386	2.5517	2.5649	2.5782	2.5916
69	sin	0.9336	0.9342	0.9348	0.9354	0.9361	0.9367	0.9373	0.9379	0.9385	0.9391
	cos	0.3584	0.3567	0.3551	0.3535	0.3518	0.3502	0.3486	0.3469	0.3453	0.3437
	tan	2.6051	2.6187	2.6325	2.6464	2.6605	2.6746	2.6889	2.7034	2.7179	2.7326
deg	function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°

deg	function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°
	sin	0.9397	0.9403	0.9409	0.9415	0.9421	0.9426	0.9432	0.9438	0.9444	0.9449
70	cos	0.3420	0.3404	0.3387	0.3371	0.3355	0.3338	0.3322	0.3305	0.3289	0.3272
	tan	2.7475	2.7625	2.7776	2.7929	2.8083	2.8239	2.8397	2.8556	2.8716	2.8878
	sin	0.9455	0.9461	0.9466	0.9472	0.9478	0.9483	0.9489	0.9494	0.9500	0.9505
71	cos	0.3256	0.3239	0.3223	0.3206	0.3190	0.3173	0.3156	0.3140	0.3123	0.3107
	tan	2.9042	2.9208	2.9375	2.9544	2.9714	2.9887	3.0061	3.0237	3.0415	3.0595
	sin	0.9511	0.9516	0.9521	0.9527	0.9532	0.9537	0.9542	0.9548	0.9553	0.9558
72	cos	0.3090	0.3074	0.3057	0.3040	0.3024	0.3007	0.2990	0.2974	0.2957	0.2940
	tan	3.0777	3.0961	3.1146	3.1334	3.1524	3.1716	3.1910	3.2106	3.2305	3.2506
	sin	0.9563	0.9568	0.9573	0.9578	0.9583	0.9588	0.9593	0.9598	0.9603	0.9608
73	cos	0.2924	0.2907	2.2890	0.2874	0.2857	0.2840	0.2823	0.2807	0.2790	0.2773
	tan	3.2709	3.2914	3.3122	3.3332	3.3544	3.3759	3.3977	3.4197	3.4420	3.4646
	sin	0.9613	0.9617	0.9622	0.9627	0.9632	0.9636	0.9641	0.9646	0.9650	0.9655
74	cos	0.2756	0.2740	0.2723	0.2706	0.2689	0.2672	0.2656	0.2639	0.2622	0.2605
	tan	3.4874	3.5105	3.5339	3.5576	3.5816	3.6059	3.6305	3.6554	3.6806	3.7062
	sin	0.9659	0.9664	0.9668	0.9673	0.9677	0.9681	0.9686	0.9690	0.9694	0.9699
75	cos	0.2588	0.2571	0.2554	0.2538	0.2521	0.2504	0.2487	0.2470	0.2453	0.2436
	tan	3.7321	3.7583	3.7848	3.8118	3.8391	3.8667	3.8947	3.9232	3.9520	3.9812
	sin	0.9703	0.9707	0.9711	0.9715	0.9720	0.9724	0.9728	0.9732	0.9736	0.9740
76	cos	0.2419	0.2402	0.2385	0.2368	0.2351	0.2334	0.2317	0.2300	0.2284	0.2267
	tan	4.0108	4.0408	4.0713	4.1022	4.1335	4.1653	4.1976	4.2303	4.2635	4.2972
	sin	0.9744	0.9748	0.9751	0.9755	0.9759	0.9763	0.9767	0.9770	0.9774	0.9778
77	cos	0.2250	0.2232	0.2215	0.2198	0.2181	0.2164	0.2147	0.2130	0.2113	0.2096
	tan	4.3315	4.3662	4.4015	4.4374	4.4737	4.5107	4.5483	4.5864	4.6252	4.6646
	sin	0.9781	0.9785	0.9789	0.9792	0.9796	0.9799	0.9803	0.9806	0.9810	0.9813
78	cos	0.2079	0.2062	0.2045	0.2028	0.2011	0.1994	0.1977	0.1959	0.1942	0.1925
	tan	4.7046	4.7453	4.7867	4.8288	4.8716	4.9152	4.9594	5.0045	5.0504	5.0970
	sin	0.9816	0.9820	0.9823	0.9826	0.9829	0.9833	0.9836	0.9839	0.9842	0.9845
79	cos	0.1908	0.1891	0.1874	0.1857	0.1840	0.1822	0.1805	0.1788	0.1771	0.1754
	tan	5.1446	5.1929	5.2422	5.2924	5.3435	5.3955	5.4486	5.5026	5.5578	5.6140
	sin	0.9848	0.9851	0.9854	0.9857	0.9860	0.9863	0.9866	0.9869	0.9871	0.9874
80	cos	0.1736	0.1719	0.1702	0.1685	0.1668	0.1650	0.1633	0.1616	0.1599	0.1582
	tan	5.6713	5.7297	5.7894	5.8502	5.9124	5.9758	6.0405	6.1066	6.1742	6.2432
	sin	0.9877	0.9880	0.9882	0.9885	0.9888	0.9890	0.9893	0.9895	0.9898	0.9900
81	cos	0.1564	1:1547	0.1530	0.1513	0.1495	0.1478	0.1461	0.1444	0.1426	0.1409
	tan	6.3138	6.3859	6.4596	6.5350	6.6122	6.6912	6.7720	6.8548	6.9395	7.0264
	sin	0.9903	0.9905	0.9907	0.9910	0.9912	0.9914	0.9917	0.9919	0.9921	0.9923
82	cos	0.1392	0.1374	0.1357	0.1340	0.1323	0.1305	0.1288	0.1271	0.1253	0.1236
	tan	7.1154	7.2066	7.3002	7.3962	7.4947	7.5958	7.6996	7.8062	7.9158	8.0285
	sin	0.9925	0.9928	0.9930	0.9932	0.9934	0.9936	0.9938	0.9940	0.9942	0.9943
83	cos	0.1217	0.1201	0.1184	0.1167	0.1149	0.1132	0.1115	0.1097	0.1080	0.1063
	tan	8.1443	8.2636	8.3863	8.5126	8.6427	8.7769	8.9152	9.0579	9.2052	9.3572
deg	function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°

deg	function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°
84	sin	0.9945	0.9947	0.9949	0.9951	0.9952	0.9954	0.9956	0.9957	0.9959	0.9960
	cos	0.1045	0.1028	0.1011	0.0993	0.0976	0.0958	0.0941	0.0924	0.0906	0.0889
	tan	9.5144	9.6768	9.8448	10.02	10.20	10.39	10.58	10.78	10.99	11.20
85	sin	0.9962	0.9963	0.9965	0.9966	0.9968	0.9969	0.9971	0.9972	0.9973	0.9974
	cos	0.0872	0.0854	0.0837	0.0819	0.0802	0.0785	0.0767	0.0750	0.0732	0.0715
	tan	11.43	11.66	11.91	12.16	12.43	12.71	13.00	13.30	13.62	13.95
86	sin	0.9976	0.9977	0.9978	0.9979	0.9980	0.9981	0.9982	0.9983	0.9984	0.9985
	cos	0.0698	0.0680	0.0663	0.0645	0.0628	0.0610	0.0593	0.0576	0.0558	0.0541
	tan	14.30	14.67	15.06	15.46	15.89	16.35	16.83	17.34	17.89	18.46
87	sin	0.9986	0.9987	0.9988	0.9989	0.9990	0.9990	0.9991	0.9992	0.9993	0.9993
	cos	0.0523	0.0506	0.0488	0.0471	0.0454	0.0436	0.0419	0.0401	0.0384	0.0366
	tan	19.08	19.74	20.45	21.20	22.02	22.90	23.86	24.90	26.03	27.27
88	sin	0.9994	0.9995	0.9995	0.9996	0.9996	0.9997	0.9997	0.9997	0.9998	0.9998
	cos	0.0349	0.0332	0.0314	0.0297	0.0279	0.0262	0.0244	0.0227	0.0209	0.0192
	tan	28.64	30.14	31.82	33.69	35.80	38.19	40.92	44.07	47.74	52.08
89	sin	0.9998	0.9999	0.9999	0.9999	0.9999	1.000	1.000	1.000	1.000	1.000
	cos	0.0175	0.0157	0.0140	0.0122	0.0105	0.0087	0.0070	0.0052	0.0035	0.0017
	tan	57.29	63.66	71.62	81.85	95.49	114.6	143.2	191.0	286.5	573.0
deg	function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°