In this module the student will learn the rules that govern the characteristics of parallel circuits; the relationships between voltage, current, resistance and power; and the results of common troubles in parallel circuits. The module is divided into four lessons: rules of voltage and current, rules for resistance and power, variational analysis, and troubleshooting parallel circuits. Each lesson consists of an overview, a list of study resources, lesson narratives, programmed instructional materials, and lesson summaries. (Author/BP)
In this module you will learn the rules that govern the characteristics of parallel circuits; the relationships between voltage, current, resistance, and power; and the results of common troubles in parallel circuits.

For you to more easily learn the above, this module has been divided into the following four lessons:

Lesson I. Rules for Voltage and Current
Lesson II. Rules for Resistance and Power
Lesson III. Variational Analysis
Lesson IV. Troubleshooting Parallel Circuits

TURN TO THE FOLLOWING PAGE AND BEGIN LESSON I.
BASIC ELECTRICITY AND ELECTRONICS
INDIVIDUALIZED LEARNING SYSTEM

MODULE VI
LESSON I

Rules for Voltage and Current

Study Booklet

BUREAU OF NAVAL PERSONNEL
January 1972
OVERVIEW
LESSON 1

Rules for Voltage and Current

In this lesson you will study and learn about the following:

- how Ohm's Law is applied to parallel circuits
- voltage
- current
- practical solutions

Each of the above topics will be discussed in the order listed. As you proceed through this lesson, observe and follow directions carefully.

BEFORE YOU START THIS LESSON, PREVIEW THE LIST OF STUDY RESOURCES ON THE NEXT PAGE.
LIST OF STUDY RESOURCES  
LESSON 1  

Rules for Voltage and Current  

To learn the material in this lesson, you have the option of choosing, according to your experience and preferences, any or all of the following:  

STUDY BOOKLET:  
Lesson Narrative  
Programmed Instruction  
Lesson Summary  

ENRICHMENT MATERIAL:  
NAVPERS 93400A-la "Basic Electricity, Direct Current."  

AUDIO/VISUAL:  
Sound/Slide Presentation "Measurement of I & E in a Parallel Circuit"  

YOU MAY NOW STUDY ANY OR ALL OF THE RESOURCES LISTED ABOVE. YOU MAY TAKE THE PROGRESS CHECK AT ANY TIME.
Rules for Voltage and Current

A Look At Parallel Circuits

In the previous module you learned about Ohm's Law relationships and how they apply to series circuits. You learned certain rules about voltage, current, resistance, and power that are true for series circuits. It is now time to see how Ohm's Law is applied to parallel circuits.

Recall that a parallel circuit is one which has more than one path for current to follow, although it has only one common source. Observing the schematic shown for a parallel circuit, you see one common source and three paths for current flow. Each path or branch supplies a load. Circuit configurations such as this are commonly used to wire homes and buildings.

Suppose that the parallel circuit illustrated above represents the wiring in your kitchen. Then DS1 might be the overhead light, R2 might be the toaster, and R3 the refrigerator. All three loads are fed by the same network of wires, from a common source.

Another wiring example you are probably familiar with is that of strings of Christmas tree lights. These strings may be wired either in series or in parallel as shown:

- In series -- one path for current
- In parallel -- many paths for current

You know that in the series string, if one lamp burns out, it opens the path so that no current can flow, and all of the lights go out. In the parallel string, this is not so. If one bulb in the parallel string burns out, it does not affect the other lights and they continue to glow. To see why this is so, we have to understand the relationships of current, voltage, and resistance in parallel circuits.
Voltage

You recall that in series circuits the same current flows through every component. This is not true of parallel circuits. In a parallel network, voltage is common. That is, across each branch the voltage drop will equal the applied voltage. Because each branch is actually connected across the source, the same difference of potential will exist across each branch.

For example:

![Diagram of parallel circuit]

The applied voltage is 10v; \( E_{R1} \) is 10v; \( E_{R2} \) is 10v; \( E_{R3} \) is 10v; therefore, the voltage drop across \( R4 \) will also be 10v. We can put this in a mathematical expression which reads:

\[
E_{R1} = E_{R2} = E_{R3} = E_{R4} = E_a
\]

The rule for voltage in series circuits (Kirchhoff's Voltage Law) is: "The sum of the voltage drops equals the applied voltage." In series circuits voltage is additive.

In parallel circuits, voltage is not additive. You do not add branch voltages in parallel circuits to solve for \( E_a \). Applied voltage is the same as each branch voltage drop.

In this circuit, what is the value of:

\[
E_{R1} \quad E_{R2} \quad E_{R3} \quad E_a
\]

Because the voltage is common across the elements of a parallel circuit, the voltage drops are all equal to each other, and the applied voltage is equal to any one of the individual voltage drops. In the circuit above, \( E_{R2} \) is 60v; therefore, \( E_{R1}, E_{R3} \), and \( E_a \) are also 60v.

Current

In a series circuit, the same current flows through every component. In a parallel circuit, the current in each branch is determined by the amount of resistance in the branch and the applied voltage.

![Diagram of series circuit]
In the circuit illustrated at the bottom of the previous page, with voltage 50v, total current is 10 amps as it leaves the source and arrives at junction A. Because each of the two branches has the same resistance, the 10 amps divides and 5 amps flows through the R1 branch, and 5 amps flows through the R2 branch. At junction B, the 5 amps from branch 2 joins the 5 amps from branch 1 to add up to a total current of 10 amps returning to the source.

The following two rules apply to current in parallel circuits:

1. Each branch current is determined by the resistance of the branch.
2. Total current is equal to the sum of the branch currents.

Since electrons are neither created nor destroyed as the current divides, the sum of the currents flowing in the branches must be exactly equal to the total current flowing to and from the source.

**Solving for Branch Current**

Ohm's Law is as true for parallel circuits as it is for series circuits. Therefore, by using the Ohm's Law formula \( I = \frac{E}{R} \), you can determine the amount of current through a branch of a parallel network.

\[
\begin{align*}
\text{By Ohm's Law, determine the amount of current through } & R_1 \text{ and } R_2. \\
I_{R1} & = \\
I_{R2} & = \\
\end{align*}
\]

Because voltage is common, if \( E \) is 50v, you know \( E_{R1} \) will be 50v and \( E_{R2} \) will be 50v. Applying Ohm's Law to find \( I \) through \( R_1 \) (or \( I_{R1} \)),

\[
\begin{align*}
I_{R1} & = \frac{E_{R1}}{R_1} \\
I_{R1} & = \frac{50v}{10\Omega} \\
I_{R1} & = 5a \\
\end{align*}
\]

To find \( I_{R2} \):

\[
\begin{align*}
I_{R2} & = \frac{E_{R2}}{R_2} \\
I_{R2} & = \frac{50v}{20\Omega} \\
I_{R2} & = 2.5a \\
\end{align*}
\]
Solving for Total Current

To find total $I$ in a parallel circuit, we add the branch currents. Therefore, we can add $I_{R1}$ and $I_{R2}$ to find total current in the circuit we have been solving.

$$I_T = I_{R1} + I_{R2} + \ldots + I_{Rn} \text{ (any number)}$$

$$I_T = 5a + 2.5a$$

$$I_T = 7.5a$$

The rule for parallel circuits that states the sum of the branch currents equals the total current is an application of what is referred to as Kirchhoff's Current Law; it is actually a statement of the law of conservation of matter, since at branch points, electrons are neither created nor destroyed.

Now solve this circuit.

First you knew that voltage across each branch is equal to the applied voltage of 25v. By using Ohm's Law formula $I = \frac{E}{R}$, you should have determined that:

$$I_{R1} = 5a$$

$$I_{R2} = 2.5a$$

$$I_{R3} = 2.5a$$

Then, because the sum of branch currents equals total current,

$$I_T = 10a.$$
1. Solve for $E_{R3}$ and $E_a$

\[ E_{R3} = \quad E_a = \]

2. Solve for $I_1$, $I_2$, $I_3$ and $I_T$. (Note: $I_1$ can be used just as $I_{R1}$, to indicate $I$ through branch 1.)

\[ I_1 = \quad I_2 = \quad I_3 = \quad I_T = \]

3. Solve for $I_2$

\[ I_2 = \]

Compare your answers with the correct answers on the next page.
Answers to practice problems

1. $E_{R3} = 20v$
   
   $E_a = 20v$

   Voltage is common. The voltage drop across each branch is equal to the applied voltage.

2. $I_1 = 3a$
   
   $I_2 = 6a$

   $I_3 = 2a$

   $I_T = 11a$

   Since you know the values of $E$ and $R$, you can apply Ohm's Law to find branch currents. The sum of the branch currents, then, equals the total current.

3. $I_2 = 4a$

   Since the sum of branch currents equals total current, you can subtract $I_1$ from $I_T$ to solve for $I_2$.

   You could also have solved for voltage across $R1$ ($E = I \times R$). Since $E_{R1}$ is 20v, $E_{R2}$ must also be 20v. Then by Ohm's Law you find:
   
   $I_{R2} = \frac{E_a}{R_2}$

   $I_{R2} = \frac{20v}{5}$

   $I_{R2} = 4$ amps

AT THIS POINT, YOU MAY TAKE THE PROGRESS CHECK, OR YOU MAY STUDY ANY OF THE OTHER RESOURCES LISTED. IF YOU TAKE THE PROGRESS CHECK AND ANSWER ALL OF THE QUESTIONS CORRECTLY, GO TO THE NEXT LESSON. IF NOT, STUDY ANY METHOD OF INSTRUCTION YOU WISH UNTIL YOU CAN ANSWER ALL THE QUESTIONS CORRECTLY.
1. As you recall, a series circuit has only one path for current flow. Because of this, the same current flows through all parts of the circuit. A parallel circuit, on the other hand, has more than one path or branch for current to flow through. As a result, the current divides and a different part of it flows in each of the branches.

   a. A circuit with only one branch is a ________ circuit.
   b. A circuit with two or more paths is a ________ circuit.

(a. series; b. parallel)
2. A parallel circuit has more than one circuit path connected to a common voltage source. A basic parallel circuit is represented in the schematic below, and the two current paths are shown by lines and arrowheads. Some electrons will take Path 1, and others will take Path 2.

![Parallel Circuit Diagram]

Use lines and arrowheads to indicate the two current paths in this circuit:

![Current Path Diagram]

3. Define a parallel circuit. 

---

(This is a test frame. Compare your answer with the correct answer given at the top of the next page.)
A parallel circuit is one which has more than one path for current flow connected to a common source.

IF YOUR ANSWER MATCHES THE CORRECT ANSWER, YOU MAY GO ON TO TEST FRAME 7. OTHERWISE, GO BACK TO FRAME 1 AND TAKE THE PROGRAMMED SEQUENCE BEFORE TAKING TEST FRAME 3 AGAIN.

4. Because there is only one path for current to flow in a series circuit, current is common. Current is not common in a parallel circuit because there is more than one path for current to flow. In a parallel circuit, the applied voltage is common. Look at the schematic below.

As you can see, both ends of each resistor are connected directly to the source, one side to the negative terminal, one to the positive terminal. This means that full source potential is impressed across each of the available current paths.

What value is common in parallel connections?

(The applied voltage across each parallel component.)
5. Source voltage in a parallel circuit is applied across each of the parallel loads. This can be easily seen, for there is almost no resistance between the source and either of the loads; therefore, there is almost no voltage drop between the source and each load.

The equation

\[ E_s = E_1 = E_2 = \ldots = E_n \]

states the above relationship, and indicates that voltage is the constant value in a parallel circuit.

What is the value of source voltage in this circuit? \( E_s = \) ___

(90v - NOTE: At times \( E_S \), \( E_T \), \( E_a \), or \( E_{bb} \) are used to indicate applied voltage.)

6. The resistance contained in the conductors of a circuit is so minute that for all practical purposes, it is neglected. Therefore, we can say that the voltage across each load in a parallel circuit is always ___

a. less than
b. equal to
c. greater than

(b) equal to

7. Solve the circuit for quantities indicated.

(THIS IS A TEST FRAME. COMPARE YOUR ANSWERS WITH THE CORRECT ANSWERS GIVEN AT THE TOP OF THE NEXT PAGE.)
ANSWERS - TEST FRAME 7

a. 60v

b. 60v

IF ALL YOUR ANSWERS MATCH THE CORRECT ANSWERS, YOU MAY GO ON TO TEST FRAME 10. OTHERWISE, GO BACK TO FRAME 4 AND TAKE THE PROGRAMMED SEQUENCE BEFORE TAKING TEST FRAME 7 AGAIN.

8. By now, you should have reached the conclusion that while \( \text{current} \) is common in a series circuit, \( \text{voltage} \) is common in a parallel circuit.

9. If the potential across any path is known or can be computed, source voltage is also known because voltage is the same in each branch of a parallel network.

What is the applied voltage?

\[ E_a = \]

(30v)
10. Solve for $E_a$.

$R_1 \ 50K \Omega$  $R_2 \ 10K \Omega$

$A \ 2 \text{ma}$

$E_a =$
11. Each of the current paths of a parallel circuit is called a branch. The current through each branch is independent of the current in the other branches, and is determined entirely by the source voltage and the resistance in the particular branch.

Ohm's Law will provide the current value once the source voltage and the resistance of the branch are known (I = E/R). Remember that current flow depends upon the voltage applied and the resistance opposing it. In a parallel circuit then, the current in each branch depends on the resistance of that branch.

Find $I_1$ and $I_2$ in the below circuit.

\[ I_1 = 3a; \quad I_2 = 1.5a \]

12. Branch currents are determined by the applied voltage, which is common to all branches of the circuit, and the resistance of the branch.

(resistance)
13. Solve for $I_{R2}$.

\[ I_{R2} = \]
14. Total circuit current ($I_T$) is equal to the sum of the individual branch currents. This is a statement of Kirchhoff's Current Law, expressed mathematically as $I_T = I_1 + I_2 + I_3 + I_4$, etc. (Since electrons are neither created nor destroyed, as the current divides, the sum of the currents in branches must exactly equal the total current.)

What is $I_T$?

![Diagram of circuit with $E_A = 50\text{V}$, $R_1 = 25\Omega$, and $R_2 = 20\Omega$.]

$I_T = \underline{\text{_____}}$

($I_T = 4.5a$; $I_1 = 2a$; $I_2 = 2.5a$)

15. Solve for $I_T$.

![Diagram of circuit with $E_A = 1\text{KV}$, $R_1 = 1\text{K}$, $R_2 = 2\text{K}$, and $R_3 = 2\text{K}$.]

$I_T = \underline{\text{_____}}$

(This is a test frame. Compare your answer with the correct answer given at the top of the next page.)
16. As a review, perhaps a comparison between the two basic types of circuits would be helpful. As you are going through the problems that follow, note the similarities and differences between the two types of circuits.

The two basic types of circuits are _________.
(Either order)
(parallel and series)

17. In a series circuit, current is the common value while the ________ divides among the various load devices.
(voltage)

18. Check the resistor that will have the greatest voltage drop.

(a) R1
(b) R2

(a) R1
19. In a parallel circuit, voltage is the common value while current divides among the branches.

20. Solve for $E_a$.

\[ E_a = \frac{40}{20} = 2 \text{V} \]

21. In a series circuit, $E_a = E_1 + E_2 + E_3$, etc. This is a statement of Kirchhoff’s Voltage law.

22. Solve for $I_T$.

\[ I_T = \frac{100}{25K} = 0.004 \text{A} = 4 \text{mA} \]
23. Solve the circuit for quantities indicated.

\[ \begin{array}{c}
\text{E}_A \\
\text{60V} \\
\text{R}_1 \\
\text{R}_2 \\
\text{R}_3 \\
\text{R}_4 \\
\end{array} \]

\[ \begin{array}{c}
a. l_1 \\
b. l_2 \\
c. l_3 \\
d. l_T \\
\end{array} \]

(a. 3a; b. 6a; c. 2a; d. 11a)

24. One advantage of a parallel circuit over a series circuit becomes readily apparent when the two are compared.

What would happen to lights DS2 through DS5 in each circuit, if DS1 were to burn out (filament opens)?

A. DS1

B. DS2, DS3, DS4, DS5 remain lit

25. Assume all branch resistances to be of equal value.

Solve:

\[ \begin{array}{c}
\text{E}_a \\
100V \\
\text{R}_3 \\
\end{array} \]

\[ \begin{array}{c}
\text{E}_a \\
\text{R}_3 \\
\end{array} \]

(This is a test frame. Compare your answers with the correct answers given at the top of the next page.)
ANSWERS - TEST FRAME 25

\[ E_a = 100v \]
\[ R_3 = 100\Omega \]

If any of your answers are incorrect, go back to frame 16 and take the programmed sequence.

If your answers are correct, you may take the progress check, or you may study any of the other resources listed. If you take the progress check and answer all the questions correctly, go on to the next lesson. If not, study any method of instruction you wish until you can answer all the questions correctly.
Rules for Voltage and Current

You have learned to apply several rules and laws to series circuits, and now it is time to apply similar rules and laws to parallel circuits. A parallel circuit, remember, is one which has more than one current path from a common source. The power outlets in your classroom, barracks, and home are all wired in parallel. Here is a schematic diagram for a parallel circuit with three current paths:

Christmas tree lights are available as either series or parallel strings. In the series string, one open lamp will cause all the lights to go off, but an open lamp in the parallel string will not affect any of the other lamps. This is so because current in a branch of a parallel circuit is independent of the current in any other branch; voltage is the quantity which is common. In the schematic shown above, each component or branch is wired directly to the source, so the voltage applied to each component or branch is the same. Mathematically, the voltages would be written: $E_a = E_1 = E_2 = E_3 = E_4$, etc., where $E_1$ is the voltage applied across the first path for current flow (branch); $E_2$, the voltage across the second branch, etc. Remember that you add voltage drops in a series circuit, but all the voltages in the branches of a parallel circuit have the same value.

In contrast to current in a series circuit, current in a parallel circuit is not a common quantity everywhere in the circuit. Current in each branch depends on the resistance in that branch. This example may help you see this point.

The 50 volts from the source is applied directly to $R_1$ and to $R_2$, thus, according to Ohm's Law, the current through $R_1$ is 5 amps and current through $R_2$ is 2.5 amps. Since both currents are supplied by the battery, current flows to and from the battery is 7.5 amps, which is the sum of the branch currents. Mathematically, the current is: $I_T = I_1 + I_2 + I_3$, etc. You now have two rules for parallel circuits:

1. Voltage is the same across all branches of a parallel circuit ($E_a = E_1 = E_2 = E_3$, etc.)
2. Total current in a parallel circuit is equal to the sum of all branch currents. ($I_T = I_1 + I_2 + I_3$, etc.)

The second rule above is an application of Kirchhoff's Current Law, which is actually a statement of the law of conservation of matter. As an equation, we see $I_T = I_1 + I_2 + I_3 + \ldots + I_n$ where $I_1$ is current in the first branch, $I_2$ is current in the second branch, and $I_n$ is the current through the last branch of whatever number of branches there are. Since electrons are neither created or destroyed as the current divides, the sum of the currents flowing in the branches must be exactly equal to the total current flowing to and from the source.

As a check, solve this problem for.

\[
\begin{array}{c}
\text{RA} \\
20v \\
\end{array}
\begin{array}{c}
\text{R}_1 \\
20\Omega \\
\text{R}_2 \\
5\Omega \\
\text{R}_3 \\
10\Omega \\
\end{array}
\begin{array}{c}
I_1 \\
I_2 \\
I_3 \\
I_T \\
\end{array}
\]

Answers: $I_1 = 1a; I_2 = 4a; I_3 = 2a; I_T = 7a$

AT THIS POINT, YOU MAY TAKE THE LESSON PROGRESS CHECK, OR YOU MAY STUDY THE LESSON NARRATIVE OR THE PROGRAMMED INSTRUCTION OR BOTH. IF YOU TAKE THE PROGRESS CHECK AND ANSWER ALL OF THE QUESTIONS CORRECTLY, GO TO THE NEXT LESSON. IF NOT, STUDY ANOTHER METHOD OF INSTRUCTION UNTIL YOU CAN ANSWER ALL THE QUESTIONS CORRECTLY.
BASIC ELECTRICITY AND ELECTRONICS
INDIVIDUALIZED LEARNING SYSTEM

MODULE SIX
LESSON II

Rules for Resistance and Power

Study Booklet

Bureau of Naval Personnel
January 1972
Rules for Resistance and Power

In this lesson you will study and learn about the following:

- determining branch resistance
- equivalent resistance
- effect of adding resistors
- configurations
- methods of solution for equivalent resistance
- power
- practical application

Each of the above topics will be discussed in the order listed. As you proceed through this lesson, observe and follow directions carefully.

BEFORE YOU START THIS LESSON, PREVIEW THE LIST OF STUDY RESOURCES ON THE NEXT PAGE.
LIST OF STUDY RESOURCES
LESSON II

Rules for Resistance and Power

To learn the material in this lesson, you have the option of choosing, according to your experience and preferences, any or all of the following:

STUDY BOOKLET:
Lesson Narrative
Programmed Instruction
Lesson Summary

ENRICHMENT MATERIAL:
NAVPERS 93400A-la "Basic Electricity, Direct Current."

NAVPERS 93492-7 "Prep Text."

YOU MAY NOW STUDY ANY OR ALL OF THE RESOURCES LISTED ABOVE. YOU MAY TAKE THE PROGRESS CHECK AT ANY TIME.
NARRATIVE
LESSON II

Rules for Resistance and Power

Determining Branch Resistance

We have already discussed methods for solving for voltage and current in parallel circuits. Now we turn our attention to resistance.

When voltage and branch current are known, branch resistance can be determined by applying Ohm's Law. (Just as you did to solve for resistance in series circuits, you use the familiar formula, \( R = \frac{E}{I} \).)

Find the values of:

\[ R_1. \]
\[ R_2. \]

Of course, \( R_1 = \frac{E}{I_{R_1}} \) and \( R_2 = \frac{E}{I_{R_2}} \)

\[ R_1 = \frac{60\text{v}}{3a} \]
\[ R_2 = \frac{60\text{v}}{2a} \]

\[ R_1 = 20\Omega \]
\[ R_2 = 30\Omega \]

Now, using Ohm's Law, we can also determine the total or equivalent resistance of the circuit.

\[ R_T = \frac{E}{I_T} \]

\[ R = \frac{60\text{v}}{5a} \]

\[ R = 12\Omega \]

Observe that although the parallel network had one resistor of 20 \( \Omega \) (\( R_1 \)) and one of 30 \( \Omega \) (\( R_2 \)), total circuit resistance is only 12 \( \Omega \).

A rule to remember here is that total resistance of a parallel network is always less than the smallest resistance of any branch.

Equivalent Resistance or Total Resistance

When we lump the resistances of all branches of a parallel circuit into one imaginary resistor equal to the total resistance of the circuit, we call it the equivalent resistance or total resistance.
The total resistance of resistors in parallel is also referred to as EQUIVALENT RESISTANCE. In many texts the terms total resistance \( R_T \) and equivalent resistance \( R_{eq} \) are used interchangeably.

**Adding Resistors**

Recall that in a series circuit, when we added a resistor, total resistance increased and total current decreased proportionately. Resistances are additive in series circuits.

Discover for yourself what happens when you add a branch resistance to a network.

---

You will need:
- multimeter set as ohmmeter.
- alligator clips for multimeter leads.
- two 10-ohm resistors.

1. Clamp alligator clips of ohmmeter to both leads of one resistor.
2. Record approximate ohmmeter measurement.
3. Clamp the clips of ohmmeter to both leads of both resistors. (This adds a second resistor in parallel.)
4. Record approximate ohmmeter measurement.
5. When you add a branch resistor, what happens to total resistance?

---

Answer - When you added a resistor, the total network resistance decreased. This is true because you added another path for current, which increased the amount of total current with a corresponding decrease in the amount of total resistance.

---

1. If you change this series circuit:
   - from
   - to
   \[ R_T \text{ will } \]
   - increase/decrease
   \[ I_T \text{ will } \]
   - Increase/decrease
2. If you change this parallel circuit:

\[ R_{eq} \quad \text{will} \quad \text{increase/decrease} \]

\[ I_T \quad \text{will} \quad \text{increase/decrease} \]

Answers:
1. \( R_T \) - increase
2. \( R_{eq} \) - decrease
3. \( I_T \) - decrease
4. \( I_T \) - increase

The rule to remember here is that adding a resistor to a parallel network decreases total resistance and provides a new path for current, and total current increases.

A Word About Configurations

From now on some of the schematics we will work with will be unusual configurations. This is being done so you will become familiar with parallel networks as they might appear. You will seldom find parallel networks that look like neat boxes in real situations.

Solving for Equivalent Resistance

To solve parallel circuits for total resistance or \( R_{eq} \), there are several methods we can use. The selection of the method used depends on what information about the circuit is known.

1. Equal Branch Method

To find the \( R_{eq} \) when resistance is equal in each branch of the circuit, divide the ohmic value of one of the resistors by the number of branches.

\[
R_{eq} = \frac{R}{n} \quad \text{(ohmic value of one resistance)}
\]

\[
R_{eq} = \frac{30 \ \Omega}{3} \quad R_{eq} = 10 \ \Omega
\]

The following analysis of the equal branch method may help explain why the method works:
Starting with a 30-volt source and one 30-ohm branch, the total current will be 1 amp.

Adding one branch, total current doubles.

Adding a third branch, total current triples.

So, adding a second equal resistance causes \( R_T \) to be one-half as great, adding a third equal resistance causes \( R_T \) to be one-third as great, etc.

2. **Product Over The Sum Method**

When you have a parallel circuit of two unequal branch resistances, you can use the product over the sum method.

With this method, you divide the product of the branch resistances \( (R_1 \times R_2) \) by the sum of the branch resistances \( (R_1 + R_2) \).

\[
R_{eq} = \frac{R_1 \times R_2}{R_1 + R_2}
\]

For example:

\[
R_{eq} = \frac{20 \Omega \times 30 \Omega}{20 \Omega + 30 \Omega} = \frac{600 \Omega}{50 \Omega} = 12 \Omega
\]

Solve for \( R_{eq} \) in this parallel circuit.

This configuration can be redrawn in box form, thus:

\[
R_{eq} = --
\]
You should have solved for \( R_{eq} \) with the product over the sum method, thus:

\[
R_{eq} = \frac{10\Omega \times 15\Omega}{10\Omega + 15\Omega} = \frac{150\Omega}{25\Omega} = 6\Omega
\]

The next section presents a method for solving for \( R_{eq} \) when you have more than two resistances.

3. Reciprocal Method

Another way to solve for total resistance in parallel circuits when branch resistances are unequal is the reciprocal method. You will probably find this method the most useful in solving for \( R_{eq} \).

We can show that the equivalent resistance of parallel networks is equal to the reciprocal of the sum of the reciprocals, or

\[
R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots + \frac{1}{R_n}}
\]

Notice that the fractions in the denominator are to be added. Since fractions with unlike denominators cannot be added, they must be changed to equivalent fractions with like denominators.

First, find the least common denominator. Then change each of the fractions in the denominator to fractions with this least common denominator. Add these fractions together, and finally invert and simplify as shown in the following example:

Example: To add \( \frac{1}{30} + \frac{1}{40} + \frac{1}{40} \):

\[
\frac{1}{30} = \frac{4}{120} \quad (30 \text{ goes into } 120 \text{ four times, so multiply by 4})
\]

\[
\frac{1}{40} = \frac{3}{120} \quad (40 \text{ goes into } 120 \text{ three times, so multiply by 3})
\]

\[
\frac{1}{40} = \frac{3}{120}
\]

\[
\frac{4}{120} + \frac{3}{120} + \frac{3}{120} = \frac{10}{120}
\]
Reciprocal means opposite, so invert to $\frac{120}{10}$ and divide to get $R_T$ or $R_{eq}$.

$$R_T = \frac{1}{R_1} \frac{1}{R_2} \frac{1}{R_3} = \frac{1}{30} \frac{1}{40} \frac{1}{120} = \frac{1}{120} = \frac{10}{120} = 12 \Omega$$

If you need practice with fractions, study the Mathematics Series Prep Text, Volume 2, Fractions (Part 2).

1. Solve for $R_{eq}$ by using the reciprocal method:

2. Redraw this configuration into box form and solve for $R_{eq}$.

Answers - 1. $4.28 \, k\Omega$ or $4.3 \, k\Omega$
2. $2.7\Omega$

Power in Parallel Circuits

Power computations in a parallel circuit are the same as those used for the series circuit. Since power dissipation in resistors consists of a heat loss, power dissipations are additive regardless of how the resistors are connected in the circuit.

Total true power is equal to the sum of the power dissipated by individual resistors.

$$P_T = P_1 + P_2 + P_3 \cdots P_n \quad (any \ number)$$

In this circuit you can find $P_{R_1}$, $P_{R_2}$, $P_{R_3}$, and solve for $P_T$ even if you do not know total current or total resistance.

You must know the values of branch resistance and branch current for the $V^{2}R$ formula; the values of $E_{B}$ and branch current to use the $P = E \times I$ formula; or the values of $E_{R}$ and branch resistance for $P = \frac{E_{R}^{2}}{R}$.
Recall that voltage is common in parallel circuits; therefore,

\[ P = E \times I \text{ or } P_{R1} = 50 \text{ V} \times 5 \text{ A} = 250 \text{ W} \]
\[ P_{R2} = 50 \text{ V} \times 2 \text{ A} = 100 \text{ W} \]
\[ P_{R3} = 50 \text{ V} \times 1 \text{ A} = 50 \text{ W} \]

\[ P_T = P_1 + P_2 + P_3 = 250 \text{ W} + 100 \text{ W} + 50 \text{ W} = 400 \text{ W} \]

We can prove this correct by applying the power formula and using total current and applied voltage.

\[ P_T = E_a \times I_T \]
\[ P_T = (50 \text{ V}) \times (5 \text{ A} + 2 \text{ A} + 1 \text{ A}) \]
\[ P_T = (50 \text{ V}) \times (8 \text{ A}) \]
\[ P_T = 400 \text{ W} \]

Solving for \( R_T \) and using another power formula, you can also prove this.

\[ P = I^2 R \]
\[ P = 64 \text{ A} \times 6.25 \Omega \]
\[ P = 400 \text{ W} \]

Practice Problems

Solve for the following:

1. [Diagram of a circuit with labels]

   \[ E_{R1} \quad R2 \quad E_a \quad R3 \quad R_T \quad P_T \]

2. [Diagram of a circuit with labels]

   \[ R_{eq} \]
If the resistors in the circuit are all rated at 250w, which resistor or resistors would overheat?

Which resistor will have the greatest voltage drop?

ANSWERS

1. \( E_{R1} = 100v \)  
   \( E_a = 100v \)  
   \( R_T = 16.6 \text{ k}\Omega \)  
   \( R2 = 50 \text{ k}\Omega \)  
   \( R3 = 50 \text{ k}\Omega \)  
   \( P_T = 600 \text{ mw or 0.6w} \)

4. \( E_a = 20v \)  
   \( R4 = 5\Omega \)  
   \( I_{R1} = 2a \)  
   \( I_{R2} = 2a \)  
   \( I_{R3} = 2a \)  
   \( R_T = 2\Omega \)  
   \( P_T = 200w \)
2. \( R_{eq} = 10 \ \Omega \)  
5. \( R_{eq} = 18.6 \ \text{k}\Omega \)

3. \( R_T = 3\Omega \)  
6. \( R_1 \)
   \[ I_T = 4a \]
   \[ I_{R1} = 0.4a \text{ or } 400 \text{ ma} \]
   \[ E_a = 12v \]

7. All three have the same

AT THIS POINT, YOU MAY TAKE THE PROGRESS CHECK, OR YOU MAY STUDY ANY OF THE OTHER RESOURCES LISTED. IF YOU TAKE THE PROGRESS CHECK AND ANSWER ALL OF THE QUESTIONS CORRECTLY, GO TO THE NEXT LESSON. IF NOT, STUDY ANY METHOD OF INSTRUCTION YOU WISH UNTIL YOU CAN ANSWER ALL THE QUESTIONS CORRECTLY.
TEST FRAMES ARE 8, 14, 20, 25, 28, 32 AND 34. AS BEFORE, GO FIRST TO TEST FRAME 8 AND SEE IF YOU CAN ANSWER ALL THE QUESTIONS THERE. FOLLOW THE DIRECTIONS GIVEN AFTER THE TEST FRAME.

1. One simple method of determining total resistance in any circuit is to use Ohm's Law; if source voltage and total circuit current or all branch currents are known, finding total resistance is no real problem: \[ R_T = \frac{E_T}{I_T} \]; however, you must ensure that if you are calculating total resistance you use total values for voltage and current.

   What is \( E_T \)?
   \[ E_T = 150 \text{v} \]

   What is \( I_T \)?
   \[ I_T = 5 \text{a} \]

   What is \( R_T \)?
   \[ R_T = 30 \Omega \]

2. Note that in both circuits, source voltage is the same value, and that in both circuits, branches 1 and 2 have the same resistance. Before solving for the indicated values, check which of the two circuits you believe will have the highest total resistance.

   \[ \text{(Circuit A has the highest total R, in circuit A, } I_T = 5.83 \text{a, } R_T = 17.12 \text{; in circuit B, } I_T = 7.83 \text{a; } R_T = 12.79 \Omega ) \]
3. From the previous frame you can see that the addition of branch 3 to circuit A caused total current to increase and total resistance to decrease. This will occur every time an additional branch is added to the circuit. Total current is equal to the sum of the branch currents. When an additional branch is added, total current increases, and so total resistance must have decreased.

Which of these three circuits has the highest current? __________
Which has the highest resistance? __________

(C, A)

4. If the switch were closed, $I_T$ would __________ by ____ ma.

(increase, 250)
5. By applying Kirchhoff’s Current Law ($I_T = I_1 + I_2 + I_3$) and Ohm’s Law ($R_T = \frac{E_T}{I_T}$) to a parallel circuit, you can see that total current is greater than any of the branch currents; therefore, total resistance must be less than any of the branch resistors.

As the switches are closed in order from left to right, total circuit resistance will:

a. increase.

b. decrease.

c. remain the same.

d. cannot be determined.

(b) decrease

6. What is $R_T$?

a. $435\Omega$

b. $300\Omega$

c. $80\Omega$

d. $30\Omega$

(d) $30\Omega$
7. In this circuit, total resistance will be:

- a. less than R1.
- b. between the values of R1 and R2.
- c. between the values of R2 and R3.
- d. greater than R3.

(a) less than R1

8. If you add a resistor in parallel to a circuit, what will happen to:

- 1. I_T.
- 2. R_T.

- a. remain the same
- b. increase
- c. decrease

(This is a test frame. Compare your answers with the correct answers given at the top of the next page.)
ANSWERS - TEST FRAME 8

1. b - increase
2. c - decrease

IF ALL YOUR ANSWERS MATCH THE CORRECT ANSWERS, YOU MAY GO ON TO TEST FRAME 14. OTHERWISE, GO BACK TO FRAME 1 AND TAKE THE PROGRAMMED SEQUENCE BEFORE TAKING TEST FRAME 8 AGAIN.

9. As you have seen in the previous frames, the total resistance of a circuit is definitely not equal to the sum of the individual values of resistance. Because of this, the term equivalent resistance \((R_e)\) is usually used to refer to the combined resistance of all parts of a parallel circuit. The term total resistance \((R_T)\) indicates the total resistance of any circuit - series, parallel, or series-parallel.

10. By using Ohm's Law and Kirchoff's Current Law, the equivalent resistance of any number of resistors in parallel can be found. As in series circuits, the equivalent of any group of resistors is a single resistor which affects the rest of the circuit in exactly the same way that the group of resistors does. Thus, the equivalent resistance of a parallel circuit is a resistor which draws the same current from the source as does the parallel group.

For example:

Step 1: Find \(I_T\)

\[
I_{R1} = \frac{10 \text{ v}}{20 \Omega} = 0.5 \text{ a}
\]

\[
I_{R2} = \frac{10 \text{ v}}{20 \Omega} = 0.5 \text{ a}
\]

\[
I_T = I_{R1} + I_{R2} \quad \text{(Kirchhoff's Law)}
\]

\[
= 0.5 \text{ a} + 0.5 \text{ a} = 1 \text{ a}
\]
Step 2: Find $R_{eq}$

\[
R_{eq} = \frac{E_s}{I}
\]

\[
= \frac{10 \text{ v}}{1 \text{ a}}
\]

\[
= 10 \Omega
\]

Therefore, the equivalent resistance is 10 ohms; i.e., a single 10-ohm load would affect the source in the same way. We can redraw our original circuit and replace the two 20-ohm resistors with our equivalent (10 $\Omega$) resistor and have an equivalent circuit.

Find the equivalent resistance ($R_{eq}$) of the following parallel resistors and redraw the circuit showing only the source and $R_{eq}$:

\[
R_1 = 15 \Omega
\]

\[
R_2 = 10 \Omega
\]

\[
(6 \text{ ohms})
\]

\[
R_{eq} = \frac{E_s}{I}
\]

\[
= \frac{30 \text{ v}}{I}
\]

\[
= 6 \Omega
\]

11. In some cases you will be given a circuit such as this with no given voltage.

\[
R_1 = 60 \Omega
\]

\[
R_2 = 20 \Omega
\]

\[
R_3 = 10 \Omega
\]

Regardless of the values of the other parallel resistors, you can be sure that the equivalent resistance of this circuit will be less than \(10 \Omega\).
12. The simplest type of parallel circuit to solve is one in which all branches contain equal values of resistance.

With switches 1 and 2 open, circuit resistance is 30 ohms, which is equal to the value of R1. If SW1 is closed, there will be two paths for current, each containing the same opposition. Since \( I = I_1 + I_2 \), total current will double, indicating that total resistance was cut in half. \( R_{eq} \) now equals 15 ohms. If SW2 is closed, \( R_{eq} \) would equal:

a. 5Ω 
b. 7.5Ω 
c. 10Ω 
d. 30Ω

(c) 10 Ω

13. As each succeeding equal branch is added, equivalent resistance decreases a proportional amount. If a second branch is added, \( R_{eq} \) is halved. If a third is added, \( R_{eq} \) will be \( \frac{1}{3} \) of its original value, etc. From this, the formula \( R_{eq} = \frac{R}{n} \) is developed. \( R \) is the resistance of one branch, \( n \) is the number of branches.

Keep in mind that this will only work if all branch resistances are equal.

(5 Ω)

\[
\begin{array}{c}
R_1 \ 60 \Omega \\
R_2 \ 60 \Omega \\
R_3 \ 60 \Omega \\
\hline \\
R_T \ \\
\end{array}
\]

(This is a test frame. Compare your answer with the correct answer given at the top of the next page.)
15. Do not get the idea that all parallel circuits will be drawn in the neat, box-like fashion used up to this point. Actually, there is a wide variety of methods used to represent a circuit schematically. There will occasionally be problems in determining which circuit components are in parallel, and which ones are in series with other components. The simplest way of doing this is to start at the source and trace current through the circuit. At any junction where current can take alternate paths, there are parallel branches.

Trace the current flow in the schematics below. Indicate paths from the source back to the source by drawing in small arrows.

EXAMPLE

A.

B.

C.

D.

(Answers on next page.)
16. At this point you have become familiar with two methods of determining equivalent resistance. The first is **Ohm's Law**, which can be applied if a value for applied voltage is assigned and total current is known; the second, \( R = \frac{V}{I} \), is applicable only if all branches contain equal values of resistance.

Three additional procedures for computing \( R \) will now be presented. These, in addition to those previously learned, will give you several possible ways of solving any parallel resistance problem.
17. A very useful method of solving a parallel network of resistors for \( R_{eq} \) is called the assumed voltage method.

Any value of source voltage can be assigned (if none is given) and individual branch currents calculated (for the assumed voltage).

Adding all the branch currents gives \( I_T \), and Ohm's Law can then be applied to get \( R_T \) (\( R_T = \frac{E}{I_T} \)).

Example:

Any voltage can be assumed, but let's use 75v since the number 75 is divisible by all of the resistor values.

Then, \( I = \frac{E}{R_T} = \frac{75v}{5\Omega} = 15a \)

\[ I_2 = \frac{75v}{25\Omega} = 3a \]

\[ I_3 = \frac{75v}{15\Omega} = 5a \]

\[ I_4 = \frac{75v}{10\Omega} = 7.5a \]

\[ I_T = 30.5a \]

and \( R_T = \frac{E}{I_T} = \frac{75v}{30.5a} \approx 2.5\Omega \) (actually 2.46\( \Omega \))

(\( \approx \) means approximately equal to.)

(NOTE: The only true value in this method is the value of resistance.)

Now, use the assumed voltage method to solve this network for \( R_T \).

18. In the following circuit, no applied voltage is given and the resistance values are not equal.

One possible way of solving this problem is called the reciprocal method. This method can be used with any number of branches and (CONTINUED ON NEXT PAGE)
any values of resistance. The formula is a variation of the assumed voltage method and is derived from a combination of Kirchhoff's Current Law and Ohm's Law.

\[
\frac{E_a}{R_T} = \frac{E_a}{R_1} + \frac{E_a}{R_2} + \frac{E_a}{R_3} \text{ etc.}
\]

dividing both sides of the equations by \(E_a\) yields

\[
\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \text{ etc.}
\]

dividing both sides into 1 yields

\[
R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} \text{ etc.}
\]

Now you can see that all you are really doing is assuming a voltage of \(1v\).

To solve, substitute values in formula:

\[
R_{eq} = \frac{1}{\frac{1}{75} + \frac{1}{50} + \frac{1}{100}}
\]

To combine the fractions, find the lowest common denominator. The most convenient way to do this is to pick the largest denominator and use it as the common denominator. Then determine how many times each of the denominators will go into this number and use these figures as the numerators:

\[
R = \frac{1.3}{100} + \frac{2}{100} + \frac{1.3}{100} = \frac{1}{4.3} = \frac{100}{4.3} = 23.3 \Omega
\]

Instead of dividing the fraction \(\frac{100}{4.3}\) into 1 as the formula implies, simply invert the fraction to \(\frac{100}{4.3}\) and divide 4.3 into 100. Mathematically, both procedures are the same.

\[
\text{Find } R_{eq} \quad \Omega
\]

\[
(16.36 \Omega)
\]
19. Using the reciprocal method, solve for \( R_{eq} \). 

\[ \frac{1}{R_{1}} + \frac{1}{R_{2}} + \frac{1}{R_{3}} = \frac{1}{4.84} \] 

20. Solve for \( R_{eq} \). 

\[ \frac{1}{R_{1}} + \frac{1}{R_{2}} + \frac{1}{R_{3}} + \frac{1}{R_{4}} = \frac{1}{4.84} \] 

(This is a test frame. Compare your answer with the correct answer given at the top of the next page.)
21. A somewhat simplified method of computing equivalent resistance is available, but can only be used with two branches at a time. The method is called product over the sum, represented mathematically as:

\[ R_{eq} = \frac{R_1 \times R_2}{R_1 + R_2} \]

This formula can be applied to a circuit using the steps shown below:

1. Substitute values in formula
   \[ R_{eq} = \frac{30 \times 60}{30 + 60} \]

2. Combine terms
   \[ R_{eq} = \frac{1800}{90} \]

3. Final computation
   \[ R_{eq} = 20 \ \Omega \]

What is \( R_{eq} \). \( \Omega \)

\[ (20 \ \Omega) \]
22. Recall that in a series circuit total resistance could be determined simply by adding all the resistance values in the circuit.

\[ R_T = R_1 + R_2 + R_3 \]

Adding another resistor in series would:

- a. decrease total resistance.
- b. increase total resistance.

(b) increase total resistance

23. Solve for \( R_3 \).

24. In parallel circuits total current is always greater than current in any branch. Consequently, total parallel resistance is always smaller than any branch resistance.

Adding another resistor in parallel would:

- a. increase total resistance.
- b. decrease total resistance.

(b) decrease total resistance
25. Solve for $R_{eq}$. 

\[ R_{2} = \frac{R_{1} \cdot R_{2}}{R_{1} + R_{2}} \]

(This is a test frame. Compare your answer with the correct answer given at the top of the next page.)
26. Power computations in a parallel circuit are the same as those used for series circuits. Since power dissipation in resistors consists of a heat loss, power dissipations are additive regardless of how the resistors are connected in the circuit.

Find $P_T$. $\underline{\text{40w}}$
27. The equations \( P = I^2R, P = \frac{E^2}{R}, \) and \( P = IE \) apply to any resistor or circuit so long as the values used are correct for that resistor or circuit. In other words, if you are looking for the power dissipated by a particular resistor use only the values associated with that resistor.

For example, in this circuit the amount of power dissipated by \( R_1 \) will be 10 watts whether you use \( P_I = EI \), \( P_{R_1} = I^2 R_1 \), or \( P_I = \frac{E^2}{R_1} \) to find the answer, but you will not get the correct answer if you use \( I_T \) or \( I_{R_2} \) or even \( R_2 \) in your calculation. By the same token, total power may be found if \( I_T, R_T \) and \( E \) (any \( E \), since all are the same in parallel) are used.

Find \( P_T \) and \( P_{R_2} \) in the above circuit. \( P_T \), \( P_{R_2} \)

\( P_T = 15 \text{w}, P_{R_2} = 5 \text{w} \) NOTE: Because there will generally be more than one valid method of solving any given problem and you will occasionally be using decimal numbers instead of whole numbers, the answers shown here will not always correspond exactly to the ones you compute.

28. Solve for:
   a. \( P_T \)
   b. \( P_{R_1} \)
   c. \( P_{R_2} \)
   d. \( P_{R_3} \)

(THESE ARE TEST FRAMES. COMPARE YOUR ANSWERS WITH THE CORRECT ANSWERS GIVEN AT THE TOP OF THE NEXT PAGE.)
ANSWERS - TEST FRAME 28

a. 583w
b. 250w
c. 200w
d. 133w

IF ALL YOUR ANSWERS MATCH THE CORRECT ANSWERS, YOU MAY GO ON TO TEST FRAME 32. OTHERWISE, GO BACK TO FRAME 26 AND TAKE THE PROGRAMMED SEQUENCE BEFORE TAKING TEST FRAME 28 AGAIN.

29. If two values from the same part of the circuit are known or can be computed, all of the other values for that part can be found. For example, if $E_1$ and $I_T$ are known, all of the total values can be computed; if $P_{R2}^a$ and $E_{R2}$ are known, all values for $R_2$ can be found.

Solve for:

a. $I_{R4}^b$

b. $P_{R3}$
c. $R_2$
d. $P_{R1}$

(a. 333ma or .33a; b. 112.5w; c. 100Ω; d. 150w)
30. Solve for R2.

\[ \frac{E_A}{50V} \quad R_1 \quad R_2 \quad 40\Omega \quad 2\alpha \]

31. Keep in mind that if you want to find a value at one particular part of a circuit, you must know or be able to compute two other values at that point.

Solve for:

a. \( E_T \)  

b. \( I_T \)  

c. \( R_T \)

32. Solve for:

a. \( P_{R_2} \)  

b. \( P_T \)

(This is a test frame. Compare your answers with the correct answers given at the top of the next page.)
ANSWERS - TEST FRAME 32

a. 10w
b. 60w

IF ALL YOUR ANSWERS MATCH THE CORRECT ANSWERS, YOU MAY GO ON TO TEST FRAME 34. OTHERWISE, GO BACK TO FRAME 29 AND TAKE THE PROGRAMMED SEQUENCE BEFORE TAKING TEST FRAME 32 AGAIN.

33. As you will recall, resistors are rated in watts as well as ohms. The wattage rating of a resistor refers to its _____ heat dissipating capability.

(maximum)

34. Check the resistor or resistors that would overheat.

- a. R1
- b. R2
- c. R3

All resistors are rated at 250w.

(THE NEXT PAGE.)
ANSWER - TEST FRAME 34

a. RI

IF YOUR ANSWER IS INCORRECT, GO BACK TO FRAME 33 AND TAKE THE PROGRAMMED SEQUENCE.

IF YOUR ANSWER IS CORRECT, YOU MAY TAKE THE PROGRESS CHECK, OR YOU MAY STUDY ANY OF THE OTHER RESOURCES LISTED. IF YOU TAKE THE PROGRESS CHECK AND ANSWER ALL THE QUESTIONS CORRECTLY, GO ON TO THE NEXT LESSON. IF NOT, STUDY ANY METHOD OF INSTRUCTION YOU WISH UNTIL YOU CAN ANSWER ALL THE QUESTIONS CORRECTLY.
SUMMARY
LESSON II

Rules for Resistance and Power

In the following parallel circuit, the total resistance can be found by the form of Ohm's Law \( R_T = \frac{E}{I_T} \), just as in a series circuit. When you calculate the total resistance in this circuit, you find that \( R_T \) is only 15 ohms -- less than the value of either \( R_1 \) or \( R_2 \).

\[ \begin{align*}
E_A & = 60V \\
R_1 & = 20\,\Omega \\
R_2 & = 60\,\Omega \\
I_T & = \frac{E_A}{R_T} \\
I_1 & = \frac{E_A}{R_1} \\
I_2 & = \frac{E_A}{R_2}
\end{align*} \]

This rule holds true for any parallel circuit: The total resistance of a parallel circuit is always less than the smallest resistance of any of its branches.

A look at the above circuit will show why this is true. If the circuit contained only \( R_1 \), the total resistance would be 20 ohms and total current 3 amperes. Adding \( R_2 \) in parallel creates a second current path, and total current from the source increases. So as far as the source is concerned, the resistance it sees is reduced, or \( R_T \) has become smaller.

The total resistance of resistors in parallel is also referred to as EQUIVALENT RESISTANCE. In many texts the terms total resistance and equivalent resistances are used interchangeably.

So far, all the schematic diagrams you have seen were drawn in nice, neat straight lines or boxes, but some of the schematics you will be seeing from now on may be drawn in a less orderly manner and not so easily understood. If you find one you do not readily understand, redraw it so that it makes sense to you while keeping all the same connections. A parallel circuit might be drawn like this:

You will obtain even stranger configurations while drawing a diagram from an actual circuit. In actual trouble-shooting techniques you will find, by re-drawing a circuit it will help simplify it for ease in locating the problem.

You have seen how to solve parallel circuits for \( R_T \) by finding total current, then using Ohm's Law to find \( R_T \); now you will see some methods...
which are faster to use. For example, when all the branches of a network are equal in resistance, you can find $R_{eq}$ by dividing the value of one of the resistors by the number of branches.

$$R_{eq} = \frac{10\Omega}{4} = 2.5\Omega$$

A parallel circuit with two unequal branches may be solved by the product over the sum method. The equation for this method is

$$R_{eq} = \frac{R_1 \times R_2}{R_1 + R_2},$$

and all you need to do is insert the values and compute to get the equivalent resistance.

A method which you can use to solve a circuit with any number of branches is the reciprocal of the sum of the reciprocals method, or

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \cdots + \frac{1}{R_n}}.$$

Here is an example of how to use this equation:

$$R_{eq} = \frac{1}{\frac{1}{10\Omega} + \frac{1}{15\Omega} + \frac{1}{30\Omega}}$$

$$= \frac{1}{\frac{3}{30} + \frac{2}{30} + \frac{1}{30}}$$

$$= \frac{1}{\frac{6}{30}}$$

$$= \frac{30}{6}$$

$$= 5\Omega$$

This is a method which you can always depend on to give the equivalent resistance. Be sure to learn it.

Now let's move on to a study of power in parallel circuits. This should be an easy subject, for power in parallel circuits is found in exactly the same way as power in series circuits. Power in the branches can be found by either $P = EI$ or $P = I^2R$ or $P = E^2$, where the branch current, resistance and/or voltage is used in each case. (Remember, $E$ or voltage difference was actually work per unit charge; since $I$ is charge per second, the product of $E$ and $I$ is work per second, or power.) Total power is equal to $EI$; it can also be found by adding the power dissipated in each branch ($P_T = P_1 + P_2 + P_3 + \cdots + P_n$). You may also use either total current or total voltage and equivalent
Six-II

resistance in \( P_T = I_{eq}^2 \times R_{eq} \) or \( P_T = \frac{E_T^2}{R} \). You can prove any of these by making your own parallel circuit and working out the values by all these methods. All of your answers should agree.

AT THIS POINT, YOU MAY TAKE THE LESSON PROGRESS CHECK, OR YOU MAY STUDY THE LESSON NARRATIVE OR THE PROGRAMMED INSTRUCTION OR BOTH. IF YOU TAKE THE PROGRESS CHECK AND ANSWER ALL OF THE QUESTIONS CORRECTLY, GO TO THE NEXT LESSON. IF NOT, STUDY ANOTHER METHOD OF INSTRUCTION UNTIL YOU CAN ANSWER ALL THE QUESTIONS CORRECTLY.
BASIC ELECTRICITY AND ELECTRONICS
INDIVIDUALIZED LEARNING SYSTEM

MODULE SIX
LESSON III

Variational Analysis

Study Booklet

Bureau of Naval Personnel
January 1972
Variational Analysis

In this lesson you will study and learn about the following:

- variable quantities
- changing voltage
- adding or removing a parallel resistor
- changing resistance in an existing branch

BEFORE YOU START THIS LESSON, PREVIEW THE LIST OF STUDY RESOURCES ON THE NEXT PAGE.
LIST OF STUDY RESOURCES
LESSON III

Variational Analysis

To learn the material in this lesson, you have the option of choosing, according to your experience and preferences, any or all of the following:

STUDY BOOKLET:
Lesson Narrative
Programmed Instruction
Lesson Summary

ENRICHMENT MATERIAL:
NAVPERS 93400A-1a "Basic Electricity, Direct Current."

You may now study any or all of the resources listed above. You may take the progress check at any time.
Variational Analysis

In the past three lessons, you have learned the rules that apply to DC parallel circuits. Now we are going to look at the interrelationships between quantities in parallel networks to see what will happen to everything in the circuit if one quantity is changed. To conduct this variational analysis, we will use a simple table as shown.

| \( I_T \) | \( R_T \) | \( P_T \) |
| \( I_{R1} \) | \( I_{R2} \) | \( I_{R3} \) |
| \( E_{R1} \) | \( E_{R2} \) | \( E_{R3} \) |
| \( R_1 \) | \( R_2 \) | \( R_3 \) |

In each empty box, we will indicate with an arrow whether a quantity will increase (+), decrease (−), or remain the same (−).

Variable Quantities

There are three quantities that we can vary by increasing or decreasing:

1. the applied voltage
2. resistance - within an established branch of the network
3. resistance - that calls for adding or deleting a branch of the network

Increasing Voltage

First, we will conduct a variational analysis of this circuit to show what will happen if the applied voltage is increased from 60v to 120v.
In the top square, we indicate the variable, voltage, and that it is being increased (+).

It is immediately obvious that if $E_a$ is increased to 120v, then $E_{R1}$, $E_{R2}$, and $E_{R3}$ will increase to 120v because voltage is common in a parallel circuit. We indicate the voltage drop increase with arrows in the appropriate boxes.

Now, if the applied voltage of this network is increased, then by Ohm's Law we know that branch current will increase in direct proportion to voltage.

Whereas current through each branch was 2 amps, when voltage was doubled, branch current was doubled.

$$I_{R1} = \frac{E_{R1}}{R_1}$$

$$I_{R1} = \frac{120v}{30\Omega} = 4 \text{ amps}$$

Therefore, we enter the appropriate arrows in the boxes to show increased branch current - $I_{R1}$, $I_{R2}$, and $I_{R3}$. 
You know that if branch currents increase, total current also increases. The sum of the branch currents of a parallel network equal total current. We enter the increase arrow accordingly to show $I_T$ has increased from 6 amps to 12 amps.

Since resistance is a physical factor, a change in voltage will not affect the value of the resistance.

Changing the applied voltage does not change the values of $R_1$, $R_2$, or $R_3$, and we fill in the appropriate arrows in the table (→).

As the values of the resistors remain the same, the equivalent resistance of the network remains the same. In the table we indicate that $R_T$ does not change.
What Will Happen to Total Power

When voltage is increased, total power will increase. This is evident when you recall the power formula, $P = E \times I$.

When voltage was increased to 120 volts, total current increased to 12 amps; therefore, total power must also increase.

Now we have completed a variational analysis of the circuit to show symbolically what would happen to every quantity if the applied voltage were increased.

Adding a Resistor

Consider now what will happen to our circuit if we vary resistance by adding another resistance.

If we add another resistor to this circuit, we are adding a fourth branch -- another path for current flow.
When we add $R_4$ to the circuit, we provide another path for current. As current is additive in a parallel circuit, total current will increase.

Recall the rule that branch resistance determines branch current, and you can see that adding $R_4$ will not affect current through the previously established branches. $I_{R_1}$, $I_{R_2}$, and $I_{R_3}$ will remain the same.

The applied voltage is determined by the source; therefore, adding $R_4$ will not change total voltage. Because voltage is common, $V_{R_1}$, $V_{R_2}$, and $V_{R_3}$ will not change.

The only change in voltage will be an additional voltage drop across $R_4$. 

---

<table>
<thead>
<tr>
<th>Effect on Current</th>
<th>Add $R_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_T$</td>
<td>$I_T$</td>
</tr>
<tr>
<td>$R_T$</td>
<td>$R_T$</td>
</tr>
<tr>
<td>$E_T$</td>
<td>$E_T$</td>
</tr>
<tr>
<td>$P_T$</td>
<td>$P_T$</td>
</tr>
<tr>
<td>$I_{R_1}$</td>
<td>$I_{R_1}$</td>
</tr>
<tr>
<td>$I_{R_2}$</td>
<td>$I_{R_2}$</td>
</tr>
<tr>
<td>$I_{R_3}$</td>
<td>$I_{R_3}$</td>
</tr>
<tr>
<td>$E_{R_1}$</td>
<td>$E_{R_1}$</td>
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<tr>
<td>$E_{R_2}$</td>
<td>$E_{R_2}$</td>
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<tr>
<td>$E_{R_3}$</td>
<td>$E_{R_3}$</td>
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<td>$R_1$</td>
<td>$R_1$</td>
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<tr>
<td>$R_2$</td>
<td>$R_2$</td>
</tr>
<tr>
<td>$R_3$</td>
<td>$R_3$</td>
</tr>
</tbody>
</table>
### Narrative

#### What Happens to Resistance

<table>
<thead>
<tr>
<th>Add R4</th>
<th>$I_T$</th>
<th>$R_T$</th>
<th>$E_T$</th>
<th>$P_T$</th>
<th>$I_{R1}$</th>
<th>$I_{R2}$</th>
<th>$I_{R3}$</th>
<th>$E_{R1}$</th>
<th>$E_{R2}$</th>
<th>$E_{R3}$</th>
<th>$R1$</th>
<th>$R2$</th>
<th>$R3$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>↑</td>
<td>↓</td>
<td></td>
<td></td>
<td>↑</td>
<td>↑</td>
<td>↓</td>
<td>↑</td>
<td>↑</td>
<td>↑</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Resistance in branches 1, 2, and 3 will not change because resistance is a physical property.

By the rules for parallel circuits, you know that if you add resistance to the network, the equivalent resistance decreases. $R_T$, then, will show a decrease arrow because we have added a fourth resistor to the network.

#### What Will Happen to Total Power

<table>
<thead>
<tr>
<th>Add R4</th>
<th>$I_T$</th>
<th>$R_T$</th>
<th>$E_T$</th>
<th>$P_T$</th>
<th>$I_{R1}$</th>
<th>$I_{R2}$</th>
<th>$I_{R3}$</th>
<th>$E_{R1}$</th>
<th>$E_{R2}$</th>
<th>$E_{R3}$</th>
<th>$R1$</th>
<th>$R2$</th>
<th>$R3$</th>
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</thead>
<tbody>
<tr>
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<td>↑</td>
<td>↓</td>
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<td>↑</td>
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<td></td>
</tr>
</tbody>
</table>

Total power is directly proportional to total current. When $I_T$ increases, $P_T$ must also increase, so we can complete our second variational analysis table by showing the increase arrow for $P_T$.

### Varying Resistance in an Existing Branch

It is conceivable that you might want to increase or decrease the value of a resistor in an established branch of a parallel network. In such cases, you are not adding or removing a branch in the circuit, but you are changing the value of the resistance within a branch.
Assume that in the following circuit you increase the ohmic value of $R_1$ by replacing the 5-ohm resistor with a 10-ohm resistor.

When you increase resistance within a branch, by Ohm's Law you know that current within the branch will decrease.

Current in Branch 2 will not change.

Total current, then, being the sum of the branch currents, will decrease because $I_{R1}$ decreased.

If $I_T$ decreases, then total power also decreases.

The applied voltage, of course, does not change when resistance changes. Because voltage is common in parallel networks, $E_{R1}$ and $E_{R2}$ will not change either.
Narrative

What Will Happen to Resistance

When voltage is constant, if total current decreases we know by Ohm's Law that $R_T$ must have increased.

We physically increase resistance in branch 1, but branch 2 remains unchanged.

Practice

Recalling the rules for parallel circuits, conduct a variational analysis of this circuit if we replace DS2 with a 150w bulb. (Recall that a 150w bulb has less $R$ than a 20w bulb.)
2. Analyze this circuit, by completing column 1, to show what will happen if \( E_a \) is changed to 50v.

<table>
<thead>
<tr>
<th>( E_a )</th>
<th>1</th>
<th>2</th>
<th>( E_{a_{ADD}} )</th>
<th>( R_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( I_T )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( R_T )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( E_T )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( P_T )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( I_{R1} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( I_{R2} )</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>( I_{R3} )</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>( E_{R1} )</td>
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<tr>
<td>( E_{R2} )</td>
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<tr>
<td>( E_{R3} )</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>( R_1 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( R_2 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( R_3 )</td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

3. Analyze the above circuit, by completing column 2, to show what will happen if we add another 50-ohm resistor in parallel to the network.

Answers:

1. \[ R_{DS2} \]

<table>
<thead>
<tr>
<th>( I_T )</th>
<th>( R_{DS2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_1 )</td>
<td>( R_{DS2} )</td>
</tr>
<tr>
<td>( E_T )</td>
<td>( R_{DS2} )</td>
</tr>
<tr>
<td>( P_T )</td>
<td>( R_{DS2} )</td>
</tr>
<tr>
<td>( R_{DS1} )</td>
<td>( R_{DS2} )</td>
</tr>
<tr>
<td>( R_{DS3} )</td>
<td>( R_{DS2} )</td>
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<tr>
<td>( E_{DS1} )</td>
<td>( R_{DS2} )</td>
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<tr>
<td>( E_{DS2} )</td>
<td>( R_{DS2} )</td>
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<tr>
<td>( E_{DS3} )</td>
<td>( R_{DS2} )</td>
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<tr>
<td>( I_{DS1} )</td>
<td>( R_{DS2} )</td>
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<tr>
<td>( I_{DS2} )</td>
<td>( R_{DS2} )</td>
</tr>
<tr>
<td>( I_{DS3} )</td>
<td>( R_{DS2} )</td>
</tr>
</tbody>
</table>
Opens in Main Lines - Affecting The Whole Network

An open can also occur in the conductor which leads directly to or from the source. How much of the network is affected depends on where the open occurs.

In this example, current does not have a complete path to follow, and it will stop entirely. The entire network is knocked out.

The open in this circuit will allow current flow in branch 1. It will stop current flow to both R2 and R3, thus knocking them out of the network. Total current will be the current through R1. Total current will decrease.

In this schematic the break is in the conductor on the line from the source, but it will affect only current to branch 3. The R1 and R2 branches will function normally.
NOW YOU MAY TAKE THE PROGRESS CHECK, OR YOU MAY STUDY ANY OF THE OTHER RESOURCES LISTED.
1. Before going into variational analysis, you must understand the nature of the values you are dealing with. Recall that voltage is a potential difference that causes current flow. Voltage is said to be an independent variable in that it is controlled from outside the circuit (changing batteries, etc.).

For example, an increase in source voltage would cause an increase in circuit current, but an increase in circuit current would not cause an increase in applied voltage.

A decrease in applied voltage would cause $I_T$ to:

- a. increase.
- b. decrease.
- c. remain the same.

(b) decrease

2. The second circuit value, current, is known as a dependent variable because it is dependent upon applied voltage and circuit resistance.

If applied voltage were doubled and circuit resistance were halved, $I_T$ would:

- a. remain the same.
- b. double.
- c. triple.
- d. quadruple.

(d) quadruple
3. The third circuit value is resistance. As you will recall, resistance is a physical factor. This means that if resistance is to change, something must actually be done to the resistor or circuit (variable resistor, replacement of resistor, etc.).

If circuit current doubles, resistance will:

- a. increase.
- b. decrease.
- c. remain the same.

(c) remain the same

4. Ohm's Law states that current is directly affected by _____ and inversely by resistance. Current is the dependent variable and is affected by both resistance and voltage. On the other hand, current does not determine what applied voltage and circuit resistance will be. They are independent variables.

(voltage)
5. Recall that in a parallel circuit \( R_{\text{eq}} (R_T) \) decreases each time another current path (branch) is added.

Using arrows to show increase (↑), decrease (↓), and remain the same (→), indicate what will happen to each circuit value when SW1 is closed.

a. \( E_a \) →
b. \( R_T \) ↑
c. \( I_T \) ↑
d. \( R_1 \) →
e. \( R_2 \) →
f. \( R_3 \) →
g. \( I_{R1} \) →
h. \( I_{R2} \) →
i. \( I_{R3} \) ↓

---

(a. \( E_a \) →
b. \( R_T \) ↑
c. \( I_T \) ↑
d. \( R_1 \) →
e. \( R_2 \) →
f. \( R_3 \) →
g. \( I_{R1} \) →
h. \( I_{R2} \) →
i. \( I_{R3} \) ↓)
6. Any variation within an existing branch will cause a corresponding change in $R_{eq}$ and $I_T$. For example, if branch 1 resistance were increased, this would cause an increase in $R_T$ and a decrease in $I_T$.

Using arrows, indicate the changes which would occur if the value of $R_3$ were decreased.

- $I_T$ (up)
- $R_T$ (up)
- $I_{R_1}$ (down)
- $I_{R_2}$ (down)
- $I_{R_3}$ (up)
- $I_{R_4}$ (down)
7. A change in applied voltage will cause a direct change in all values of current, voltage, and power within the circuit.

Using arrows, indicate the changes which would take place if source voltage were doubled.

a. $I_T$ 

b. $R_{eq}$ 

c. $R_1$ 

d. $R_2$ 

e. $R_3$ 

f. $I_{R1}$ 

g. $I_{R2}$ 

h. $I_{R3}$ 

---

(a. $I_T$ ↑

b. $R_{eq}$ ↑

c. $R_1$ ↑

d. $R_2$ ↑

e. $R_3$ ↑

f. $I_{R1}$ ↑

g. $I_{R2}$ ↑

h. $I_{R3}$ ↑)
8. Place arrows to show increase (+), decrease (-), or same (-) in each box of the variational analysis table in accordance with these questions.

<table>
<thead>
<tr>
<th>Ea</th>
<th>Add R4</th>
<th>R1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_T$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E_T$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_T$</td>
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<td>$P_T$</td>
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<tr>
<td>$I_{R1}$</td>
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<td>$I_{R2}$</td>
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<tr>
<td>$I_{R3}$</td>
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<tr>
<td>$R1$</td>
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<td>$R2$</td>
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<tr>
<td>$R3$</td>
<td></td>
<td></td>
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<tr>
<td>$E_{R1}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E_{R2}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E_{R3}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. What will happen to the other quantities if total $E$ is increased to 100v?

b. What will happen to the other quantities if a 50-ohm resistor is added in parallel?

c. What will happen to the other quantities if $R1$ is replaced with a 20-ohm resistor?
ANSWERS - TEST FRAME 8

<table>
<thead>
<tr>
<th></th>
<th>a.</th>
<th>b.</th>
<th>c.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_T$</td>
<td>$\uparrow$</td>
<td>$\uparrow$</td>
<td>$\uparrow$</td>
</tr>
<tr>
<td>$E_T$</td>
<td>$\uparrow$</td>
<td>$\rightarrow$</td>
<td>$\rightarrow$</td>
</tr>
<tr>
<td>$R_T$</td>
<td>$\rightarrow$</td>
<td>$\uparrow$</td>
<td>$\uparrow$</td>
</tr>
<tr>
<td>$P_T$</td>
<td>$\uparrow$</td>
<td>$\uparrow$</td>
<td>$\uparrow$</td>
</tr>
<tr>
<td>$I_{R1}$</td>
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<td>$\rightarrow$</td>
<td>$\rightarrow$</td>
</tr>
<tr>
<td>$I_{R2}$</td>
<td>$\uparrow$</td>
<td>$\rightarrow$</td>
<td>$\rightarrow$</td>
</tr>
<tr>
<td>$I_{R3}$</td>
<td>$\uparrow$</td>
<td>$\rightarrow$</td>
<td>$\rightarrow$</td>
</tr>
<tr>
<td>$R1$</td>
<td>$\rightarrow$</td>
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<tr>
<td>$R2$</td>
<td>$\rightarrow$</td>
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<tr>
<td>$R3$</td>
<td>$\rightarrow$</td>
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<tr>
<td>$E_{R1}$</td>
<td>$\uparrow$</td>
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<tr>
<td>$E_{R2}$</td>
<td>$\uparrow$</td>
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<td>$\rightarrow$</td>
</tr>
<tr>
<td>$E_{R3}$</td>
<td>$\uparrow$</td>
<td>$\rightarrow$</td>
<td>$\rightarrow$</td>
</tr>
</tbody>
</table>

If all your answers match the correct answers, you may go on to Test Frame 12. Otherwise, go back to Frame 1 and take the programmed sequence before taking Test Frame 8 again.
9. Parallel circuits have one distinct advantage over series circuits in that an open in one branch will not affect the operation of the other branches and a variation of resistance (within reasonable limits) will not affect the other branches.

If DS2 burns out, DS1 and DS3 will:

- a. become dimmer.
- b. become brighter.
- c. not change.

(C) not change

10. Check the arrow that indicates what will happen to the ammeter reading if R1 opens.

- a. 
- b. 
- c. 

(a)
11. Should an open occur within a branch of a parallel circuit, only that branch would be affected; should the open isolate more than one branch, only the branches beyond the open will be affected. All others will function normally.

Indicate what would happen to each light bulb if SW3 were closed.

1. DS1
2. DS2
3. DS3
4. DS4

a. brighten  b. dim  c. go out  d. remain the same

(1. d; 2. d; 3. a; 4. a)

12. Match the lettered choice in column B to the meter symbols in column A to indicate what will happen to each meter in this circuit if an open occurs at the X.

A
1. M1
2. M2
3. M3
4. M4

B
a. +
b. +
c. +
d. goes to zero

(THIS IS A TEST FRAME. COMPARE YOUR ANSWERS WITH THE CORRECT ANSWERS GIVEN AT THE TOP OF THE NEXT PAGE.)
ANSWERS - TEST FRAME 12

1. c
2. a
3. d
4. d

IF ALL YOUR ANSWERS MATCH THE CORRECT ANSWERS, YOU MAY GO ON TO TEST FRAME 20. OTHERWISE, GO BACK TO FRAME 9 AND TAKE THE PROGRAMMED SEQUENCE BEFORE TAKING TEST FRAME 12 AGAIN.

13. Although an open in a branch will decrease total current, it will not stop all circuit action. To completely de-energize a circuit, the open must occur at an electrically common point (point through which all circuit current must pass).

If SW1 were opened, what would happen to DS1, DS2, and DS3.

14. State how many amps the ammeter will read in this circuit.

_____ amps.
15. An open in a parallel circuit will not necessarily have the same effect as it would in a series circuit. To completely stop all circuit action in parallel, the open must occur at an electrically common point, whereas in a _______ circuit, an open anywhere would de-energize the entire circuit.

(series)

16. A short circuit is an accidental low-resistance path for current flow. Regardless of the type of circuit, a short will have the same effect, an increase in total current.

What is I_T?

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>83 ma</td>
</tr>
<tr>
<td>b</td>
<td>100 ma</td>
</tr>
<tr>
<td>c</td>
<td>300 ma</td>
</tr>
<tr>
<td>d</td>
<td>2500 ma</td>
</tr>
</tbody>
</table>

(d) 2500 ma

17. As you will recall, R_T in a parallel circuit is always _______ than the least branch resistance. If a direct short is placed across a parallel circuit at any point, the total resistance is effectively reduced to 0. In the case of a partial short, R_T would be something less than normal.

(smaller, or less)

18. Solve for R_T.

R_T _______
19. If a circuit is shorted, it will pass an abnormally large current. Due to the extremely small resistance of the short (usually considered 0), all of the current will flow through the short, and none will flow through the load.

Indicate how each value will be affected if SW₂ is closed.

20. Match the letter choice in column B to the meter symbols in column A to indicate what will happen to each meter in this circuit if the short occurs as indicated.

*(THIS IS A TEST FRAME. COMPARE YOUR ANSWERS WITH THE CORRECT ANSWERS GIVEN AT THE TOP OF THE NEXT PAGE.)*
ANSWERS - TEST FRAME 20

1. a

2. c

If all your answers match the correct answers, you may go on to test frame 25. Otherwise, go back to frame 13 and take the programmed sequence before taking test frame 20 again.

21. Recall that a fuse is placed in a circuit to protect against excessive current. To do this, the fuse must be placed in series with the source.

At which point could the fuse be placed?

- a. A
- b. B
- c. C
- d. D
- e. any of the above

(e) any of the above
22. In a series circuit, the fuse can be placed at any point and still be in series with the source. To protect a parallel circuit, you must ensure that the fuse is placed at an electrically common point so that all the circuit current passes through it.

Where could the fuse be placed to protect the entire circuit?

- a. A
- b. B
- c. C
- d. D
- e. E

(a. A; e. E)

23. Recall that fuses are rated in amps; if the rated value is exceeded, the filament burns out, and the circuit is de-energized.

The fuse is rated at 15 amps. Will this circuit remain operative?

- a. yes
- b. no

(b) no

24. One of the most common reasons for a fuse to blow in a parallel circuit is excessive current caused by adding too many load devices.

Adding additional loads in parallel causes \( I \) to \( \text{increase/decrease} \).

(Increase)
25. Check the probable explanations of why the fuse would blow when SW3 is closed.

- a. The fuse must be defective.
- b. Current will exceed the fuse rating.
- c. There are too many loads.
- d. SW3 must be defective.
ANSWERS - TEST FRAME 25

b. Current will exceed the fuse rating.
c. There are too many loads.

---

IF ANY OF YOUR ANSWERS ARE INCORRECT, GO BACK TO FRAME 21 AND TAKE THE PROGRAMMED SEQUENCE.

IF YOUR ANSWERS ARE CORRECT, YOU MAY TAKE THE PROGRESS CHECK, OR YOU MAY STUDY ANY OF THE OTHER RESOURCES LISTED. IF YOU TAKE THE PROGRESS CHECK AND ANSWER ALL THE QUESTIONS CORRECTLY, GO ON TO THE NEXT LESSON. IF NOT, STUDY ANY METHOD OF INSTRUCTION YOU WISH UNTIL YOU CAN ANSWER ALL THE QUESTIONS CORRECTLY.
Variational Analysis

A quick way to improve your understanding of how circuit quantities interact is to use variational analysis. In variational analysis, one value in a circuit is caused to change and the effect of this change on all other circuit quantities is examined. A table is made up listing all measurable values in a circuit, and arrows are used to show what changes take place. An arrow pointing upward indicates increase (+); an arrow downward, decrease (−); and a horizontal arrow, no change (0). Variational analysis usually starts with an assumed change in either voltage or resistance, the quantities we can physically change in a circuit.

Here is an example of changing the applied voltage in a circuit from 60v to 120v.

First, show in the top square the value which has changed and whether it increases or decreases; then mark in arrows which show how the remaining values are affected. You need not work out values unless you are uncertain of the answer; you should be able to fill in the blanks from your knowledge of Ohm's Law and circuit rules.
Another change could be the addition of another branch to the circuit; here is an analysis of such a change:

<table>
<thead>
<tr>
<th></th>
<th>Add R4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_T$</td>
<td>↑</td>
</tr>
<tr>
<td>$R_T$</td>
<td>↑</td>
</tr>
<tr>
<td>$P_T$</td>
<td>↑</td>
</tr>
<tr>
<td>$I_{R1}$</td>
<td>→</td>
</tr>
<tr>
<td>$I_{R2}$</td>
<td>→</td>
</tr>
<tr>
<td>$I_{R3}$</td>
<td>→</td>
</tr>
<tr>
<td>$E_{R1}$</td>
<td>→</td>
</tr>
<tr>
<td>$E_{R2}$</td>
<td>→</td>
</tr>
<tr>
<td>$E_{R3}$</td>
<td>→</td>
</tr>
<tr>
<td>$R_1$</td>
<td>→</td>
</tr>
<tr>
<td>$R_2$</td>
<td>→</td>
</tr>
<tr>
<td>$R_3$</td>
<td>→</td>
</tr>
</tbody>
</table>

A third change we could make is to vary one of the resistance values in an existing circuit. Suppose that the resistance of $R_3$ in the circuit below is changed from 60 ohms to 200 ohms. For this example, you fill in the boxes.
AT THIS POINT YOU MAY TAKE THE LESSON PROGRESS CHECK, OR YOU MAY STUDY THE LESSON NARRATIVE OR THE PROGRAMMED INSTRUCTION OR BOTH. IF YOU TAKE THE PROGRESS CHECK AND ANSWER ALL OF THE QUESTIONS CORRECTLY, GO TO THE NEXT LESSON. IF NOT, SELECT ANOTHER METHOD OF INSTRUCTION UNTIL YOU CAN ANSWER ALL THE QUESTIONS CORRECTLY.
BASIC ELECTRICITY AND ELECTRONICS
INDIVIDUALIZED LEARNING SYSTEM

MODULE SIX
LESSON IV

Troubleshooting Parallel Circuits

Study Booklet

Bureau of Naval Personnel
January 1972
Troubleshooting Parallel Circuits

In this lesson, you will study and learn about the following:

- shorts
- opens
- practical experiments in troubleshooting

Each of the above topics will be discussed in the order listed. As you proceed through this lesson, observe and follow directions carefully.

BEFORE YOU START THIS LESSON, PREVIEW THE LIST OF STUDY RESOURCES ON THE NEXT PAGE.
LIST OF STUDY RESOURCES

LESSON IV

Troubleshooting Parallel Circuits

To learn the material in this lesson, you have the option of choosing, according to your experience and preferences, any or all of the following:

STUDY BOOKLET:
Lesson Narrative
Lesson Summary

ENRICHMENT MATERIAL:
NAVPERS 93400a-la "Basic Electricity, Direct Current."

YOU MAY NOW STUDY ANY OR ALL OF THE RESOURCES LISTED ABOVE. YOU MAY TAKE THE PROGRESS CHECK AT ANY TIME.
Troubleshooting Parallel Circuits

Do all of the following experiments as you go through this narrative.

Experiment #1 - Voltage

1. Using Practice Board 0-1 and one dry cell, construct the following circuit:

   ![Circuit Diagram]

2. Set up multimeter as voltmeter.
3. Close switch.
4. Measure voltage across the terminals of the source. $E = \underline{\phantom{1234}}$
5. Measure voltage across the branch with DS1, between T2 and T7. $E_a = \underline{\phantom{1234}}$
6. Measure voltage across the branch with DS2, between T3 and T6. $E_a = \underline{\phantom{1234}}$

You have observed that the voltage drop across each parallel branch is, for all practical purposes, equal to the applied voltage. In a parallel circuit, voltage is common to all branches.

Experiment #2 - Current

1. Using Practice Board 0-1 set up as for experiment 1, connect multimeter as ammeter in series with DS1.
2. Close switch.
3. Record current in branch 1.
4. Open switch.
5. Connect ammeter in series with DS2.
7. Record current in branch 2.

8. Open switch.

9. Compute total current by adding branch currents.
   \[ I_T = \]

10. Now prove your answer in No. 9 is true by connecting ammeter between \( T_1 \) and \( T_2 \) to read total current.


12. Record ammeter reading.
   \[ I_T = \]

13. Open switch.

This experiment proves visually to you that the sum of the branch currents equals total current. You can also see this by doing another experiment.

**Experiment #3**

1. With Practice Board 0-1 set as in experiment #2, connect ammeter to measure total current.

2. Close switch.

3. Unscrew bulb in branch 2. Does bulb in branch 1 go out? Does bulb in branch 1 get dimmer or brighter?

You have seen that an open branch in a parallel circuit does not affect another branch. Current through each branch is determined by the branch resistance and the applied voltage.

4. Now, tighten bulb in branch 2. Read total current with both bulbs drawing current.
   \[ I_T = \]

5. Loosen bulb in branch 1 and read total current.
   \[ I_T = \]

Total current has decreased, because one branch current has been subtracted from it.

6. Tighten bulb 1, loosen bulb 2 and read total current.
   \[ I_T = \]

7. Open switch.

Experiments have shown you that in parallel circuits:

1. Voltage is common.
2. Branch current is determined by branch resistance and applied voltage.
3. Total current is equal to the sum of the individual branch currents.

Troubleshooting

You recall from our study of series circuits that there are two problems that can cause a malfunction in circuits. These are short circuits and open circuits.

Shorts

When a short occurs in a parallel circuit, it places a direct short across the power lines -- the wire leading from the source and the wire leading back to the source.

The exception to this is when part of a component is shorted. Then we have a partial short.

Experiment #4

Let's create a direct short!

1. Using Practice Board 0-1 and one dry cell, construct the following circuit:

   ![Circuit Diagram]

2. Close switch.
3. Cause a short across the lamp in branch 1 by placing a wire between terminals T2 and T7.
4. What happens to the light in branch 1? To the light in branch 2?
5. What path must current be taking?
6. Remove the wire causing the short.
7. Open switch.

You observed that when a short was caused across branch 1, all circuit current followed the path through the short. We say that current follows the path of least resistance.

When there is no circuit resistance, current will be very high, because nothing is limiting the amount of current flow (except internal source resistance and the resistance of the wire).
Notice there is a short across R2 in this schematic. Current will take the path of least resistance -- all circuit current will go through the short. Then, because current will increase tremendously, it will exceed the fuse rating. The fuse will blow, and current will stop.

If there were no fuse, the wires would become overheated and catch fire, thus a short could cause an open.

If the conductor opened at point X, then R1 and R3 could resume normal operation, with branch resistance controlling branch current.

If, however, the wire overheated and opened at point Z, all current in the network would stop. This is a main-line open which we will discuss shortly.

Branch Opens

Branch one in the following parallel network has a burned out resistor. This causes one branch of the network to be open. This open does not affect branch 2 or 3. It will, however, cause total current to decrease to 4a, because there is no current through R1.

Total resistance was 10Ω, but with R1 out of the picture, total resistance increases to 15Ω.

If an open occurred as shown in the schematic above:

I_T would \((\text{increase/decrease})\).

R_T would \((\text{increase/decrease})\).

P_T would \((\text{increase/decrease})\).

I_T would decrease; R_T would increase; P_T would decrease.
Recalling all you have learned about shorts and opens in parallel networks, practice these problems, then check your answers against the correct answers. Be sure to study the schematics carefully.

1. Which branch or branches will be knocked out of the circuit by the problem observable in the schematic?

(a) [Diagram]

(b) [Diagram]

2. Compare these quantities to the normal circuit condition. Will they increase, decrease, or stay the same?

\[ I_T \]

\[ R_T \]

\[ E_A \]

Check answers on next page.
Narrative

ANSWERS:

1. (a) R2
   (b) R1, R2 and R3

2. No doubt you caught the open (R1) in the circuit and if you're really observant, you noticed that the third leg of the network does not have a resistor in it. Therefore, it is a direct short. All current will flow through the short, bypassing the other two branches. Therefore:

   \[ I_T \text{ increase} \]
   \[ R_T \text{ decrease} \]
   \[ E_a \text{ same} \]

   When current exceeds fuse rating -- fuse will blow and then everything will stop.

Experiment #5

Locating Shorts With a Meter

1. Construct a parallel circuit on Practice Board 0-1 according to the schematic.
2. Leave switch open.
3. Connect a wire across DS2 between T3 and T6.

This creates a short in branch 2 of the network. Assuming we do not know where the short is, let's hunt for it. We will do this by systematically checking each branch of the circuit.

4. Take a visual check looking for signs of overheating, such as charred parts or smoke damage. If the visual check does not locate the problem, then set up the ohmmeter.
5. Because a parallel circuit has more than one path for current flow, it is difficult to locate a short unless
we check each branch systematically. Operating a circuit with a short in it may cause considerable damage to good components, so most troubleshooting should be done with an ohmmeter. Let's begin by making sure the circuit has no voltage across the branches, then connecting the ohmmeter across the loads. For this circuit, clip the test leads to points T2 and T7.

What is the ohmmeter reading? \( 0 \Omega \)

6. Place probes across T2 and T7. You measure 0 but you know that there should be some resistance in the bulb.
7. Disconnect the wire from DS1 to T2.
8. Measure across T2 and T7 again. You still get a 0 reading indicating the circuit still is shorted; therefore, the trouble does not occur in this branch.
9. Reconnect DS1, disconnect DS2 and the wire shorting it.
10. Measure across DS1. The meter reads some resistance, indicating that the short has been isolated. You have located the short as being in branch 2.
11. Now the problem can be corrected and the circuit restored to normal.

AT THIS POINT, YOU MAY TAKE THE PROGRESS CHECK, OR YOU MAY STUDY ANY OF THE OTHER RESOURCES LISTED. IF YOU TAKE THE PROGRESS CHECK AND ANSWER ALL OF THE QUESTIONS CORRECTLY, YOU HAVE MASTERED THE MATERIAL AND ARE READY TO TAKE THE MODULE TEST. SEE YOUR CLASSROOM SUPERVISOR.

IF YOU DECIDE NOT TO TAKE THE PROGRESS CHECK AT THIS TIME, OR IF YOU MISSED ONE OR MORE QUESTIONS, STUDY ANY METHOD OF INSTRUCTION YOU WISH UNTIL YOU HAVE ANSWERED ALL THE PROGRESS CHECK QUESTIONS CORRECTLY. THEN SEE YOUR CLASSROOM SUPERVISOR AND ASK TO TAKE THE MODULE TEST.
Troubleshooting Parallel Circuits

Short circuits and open circuits are the sources of trouble in parallel circuits just as they were in series circuits. A direct short in a parallel circuit will bypass the current around all the branches, for current always follows the path of least resistance. The direct short will cause very high current flow which will cause the internal resistance of the source to drop all of the source voltage. If the circuit is not protected by fuses, the source or wiring will soon be damaged.

Short circuits in parallel circuits are usually located with an ohmmeter, if fire and smoke do not show the source of the trouble first. The procedure used is to connect the ohmmeter across the de-energized circuit, then disconnect the branches, one at a time, until the short (0) reading disappears. The defect in that branch can then be corrected and the circuit restored to operation.

Open circuits may affect all or any part of a parallel circuit; that is, one branch may open or the entire circuit may be opened. In the diagram below, the open in branch 1 affects only three factors in the circuit, $I_1$, $I_2$, and $R_3$. If the open occurs at the point marked $X$, however, $I_T$ drops to 0 and the entire circuit is dead.

Open circuits may be found using an ohmmeter and disconnecting branches again -- this time you must watch for a branch which does not cause a change in the resistance reading.

The narrative for this lesson contains a variety of experiments which you should perform if you have not previously worked with circuits.

AT THIS POINT, YOU MAY TAKE THE LESSON PROGRESS CHECK, OR YOU MAY STUDY THE LESSON NARRATIVE. IF YOU TAKE THE PROGRESS CHECK AND ANSWER ALL OF THE QUESTIONS CORRECTLY, GO TO THE NEXT LESSON. IF NOT, STUDY ANOTHER METHOD OF INSTRUCTION UNTIL YOU CAN ANSWER ALL THE QUESTIONS CORRECTLY.