The establishment and validation of a learning hierarchy for use in the teaching of selected principles and strategies was useful in judging a conditional argument in the familiar content domain. Preservice elementary teachers were instructed in two basic skills: (1) translating a simple argument from verbal to symbolic form, and (2) judging the validity of a simple argument given in symbolic form. Several sequences, with and without a Gagne guided thinking component, were investigated. Subjects attained the first skill readily, while the sequence providing the learning of the second skill prior to the first appeared to be the better one. The guided thinking information appeared to have no significant influence on the attainment of the terminal objective, judging a simple conditional argument presented in verbal form. (Author/SM)
THE DEVELOPMENT AND VALIDATION OF A CURRICULUM HIERARCHY DESIGNED FOR USE IN TEACHING SELECTED PRINCIPLES AND STRATEGIES OF AN ASPECT OF CRITICAL THINKING

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The development of the critical thinking ability of individuals is one of the implicit aims of cognitive activity. Indeed, most educators would agree that critical thinking development is one of the most important outcomes of mathematics education. Under current curricular/instructional circumstances, however, there is evidence (e.g., Ennis and Paulus, 1965; Gardiner, 1966; Paulus, 1967; Roberge, 1970; Roberge and Paulus, 1971; Janson, 1973) that too many elementary and secondary school students and prospective elementary school teachers have significant deficiencies in their critical thinking competencies. While there have been several studies (e.g., Hyram, 1957; Ennis and Paulus, 1965; Ennis, Smith, Wilson, and Finkelstein, 1969; Mason, 1973) that have attempted (with little success) to teach critical thinking, apparently there is no explicit hierarchical learning structure available to the learner which is especially designed for the attainment of critical thinking ability in general. In particular, at present there is no explicit hierarchical learning structure available to assist the learner in the acquisition of the ability to correctly judge the validity of verbal simple deductive arguments, which is one of the important aspects (or behaviors) of critical thinking.

Is it possible to develop and validate a hierarchical learning structure for the attainment of the competency to correctly judge the validity
of verbal simple deductive arguments? In order to answer this question, one approach is to first construct a "theoretically sound" hierarchical learning structure for this competency over a particular content area and then test the structure using precisely stated instructional hypotheses. Accordingly, a major aspect of this study was an attempt to specify, develop, and validate such a hierarchical structure which could be useful to mathematics educators in assisting learners, especially prospective elementary school teachers, in the acquisition of the ability to correctly judge the validity of verbal simple arguments of the conditional type.

In connection with traversing a hierarchical learning structure, there is evidence (Haimer, Lottes, and Klein, 1971, Volume II) to indicate that too often when learners have successfully achieved all the subordinate tasks in a hierarchy they somehow fail to achieve the terminal task. Gagne (1967) has suggested that the reason for this occurrence is the lack of a guided thinking information (GTI) component in the hierarchy prior to performing the terminal task. In this study, the GTI component was a verbal statement identifying for the learner the subordinate tasks (behaviors) he has mastered in the hierarchy. The utility of this notion of a GTI component in the aforementioned hierarchy was also investigated.

The Method

Some Basic Definitions and Notation

For the purposes of this investigation, the term critical thinking was defined as the "correct assessing of statements" (Ennis, 1962, p. 82).

The term simple deductive argument was defined as a chain of reasoning involving statements or propositions, where the first two propositions are
premises (assumed to be true) and the third one is the conclusion drawn from the premises. A simple deductive argument is **conditional** if one of the premises is a conditional statement.

The **verbal form** \((V)\) of a simple deductive argument is one in which each occurrence of an antecedent or consequent component in the argument is presented as a verbal simple sentence in the concrete-familiar content domain. A **symbolic form** \((S)\) of a simple deductive argument is one in which each occurrence of an antecedent or consequent component in the argument is represented by a statement variable.

The following special notation was adopted to facilitate the classification and analysis of the terminal task and the statement of hypotheses given below. Additional information about this notation, and other details of the study, are given in Shipman (1973).

1. \(J:\) A judgment of the validity of a simple deductive argument of the conditional type.
2. \((M,N)\): An objective where \(M\) denotes the **Given** component of the objective and \(N\) denotes the **Required Performance** component of the objective.
3. \(M \rightarrow N\): An instructional sequence whose purpose is the accomplishment of objective \((M,N)\).
4. \(\overline{A} \rightarrow M \rightarrow N\): The achievement of objective \((M,N)\) as a result of interacting with instructional sequence \(M \rightarrow N\) which has been deemed adequate according to some well-formulated criterion; for example, let \(n\) be the number of subjects who fail to reach criterion on a pretest for a given objective, and let \(s\) be the number of subjects reaching criterion after instruction. Then, the instruction is deemed adequate if \(s/n\) is greater than or equal to .80.
 Specification and Development of the Hierarchy

The initial step toward specifying and developing a hierarchical learning structure for the terminal behavior (task), the correct validity judgments of verbal simple conditional arguments, was the use of Gagné's task analysis procedure. (This terminal behavior was denoted as the objective (V,J)). The set of subordinate behaviors (tasks) generated as a result of the task analysis is listed below:

1) the correct translation of verbal simple deductive arguments to symbolic form (denoted as instructional objective (V,S)), and

2) the correct validity judgments of simple deductive arguments given in symbolic form (denoted as instructional objective (S,J)).

In this study, the proposed hierarchical learning structure from these objectives, together with the guided thinking information (GTI), is depicted in Figure 1.

![Figure 1: Proposed Hierarchical Learning Structure](image-url)
The Hypotheses

The general hypotheses probed in this study are stated below:

H1.00: The set of behaviors, \((V,S)\) and \((S,J)\), alone does not constitute a set of sufficient behaviors for the acquisition of the terminal behavior \((V,J)\).

H2.00: The set of behaviors, \((V,S)\) and \((S,J)\), together with guided thinking information, constitutes a sufficient set of subordinate behaviors for the acquisition of the terminal behavior \((V,J)\).

H3.00: The order of acquisition of the behaviors, \((V,S)\) and \((S,J)\), but followed by guided thinking information, is immaterial to the subsequent acquisition of the terminal behavior \((V,J)\).

Based on this set of general hypotheses, a total of 36 consequent hypotheses, each given in both verbal and symbolic forms, was formulated.

Instructional Materials

Carefully controlled and documented\(^1\) instructional episodes for objectives \((V,S)\) and \((S,J)\) were written, involving the contrapositive, converse, and transitivity principles of conditional logic. These three elementary principles are summarized and illustrated in Table 1.

\(^1\)A modified version of the Ruleg system described by Evans, Glaser, and Homme (1962) was used in conjunction with explicit flow diagrams.
Table 1
Three Principles in Conditional Logic

<table>
<thead>
<tr>
<th>Principle</th>
<th>Valid</th>
<th>Symbol form</th>
<th>Sample Verbal form</th>
</tr>
</thead>
<tbody>
<tr>
<td>Contrapositive</td>
<td>Yes</td>
<td>If P, (\neg Q). Not Q. (\therefore), not P.</td>
<td>If the car is black, then it is a police car. The car is not a police car. (\therefore), the car is not black.</td>
</tr>
<tr>
<td>Converse</td>
<td>No</td>
<td>If P, then Q. Q. (\therefore), P.</td>
<td>If the shirt is green, then I will wear it. I will wear the shirt. (\therefore), the shirt is green.</td>
</tr>
<tr>
<td>Transitivity</td>
<td>Yes</td>
<td>If P, then Q. If Q, then R. (\therefore), if P, then R.</td>
<td>If it is Jim, then it is time to go. If it is time to go, then call me. (\therefore), if it is Jim, then call me.</td>
</tr>
</tbody>
</table>

The major component in the instruction on objective \((V,S)\) was the use of simple rules and examples to explicate the concept of translating statements and simple conditional arguments from verbal to symbolic form. On the other hand, the major component in the instruction on objective \((S,J)\) was the use of Venn diagrams to assist the learner in judging the validity of simple conditional arguments given in symbolic form.

In constructing the episodes for these instructional objectives, it was possible to organize sequences of instruction in the following manner for the purpose of setting the framework for validating the hierarchy proposed earlier. From the task analysis of the terminal objective \((V,J)\), the main instructional objectives, \((V,S)\) and \((S,J)\), were arranged in sequential orders \((V,S)\) alone, \((S,J)\) alone, \(((V,S), (S,J))\), and \(((S,J), (V,S))\), respectively. By
adding the guided thinking information (GTI) component to each of the latter two sequential orders, it was possible to formulate the two fundamental instructional sequences, \(((V,S), (S,J), \text{ GTI})\) and \(((S,J), (V,S), \text{ GTI})\), each of which is an ordered sequence. The instructional episodes and sequences were presented on an IBM 1500 Computer-Assisted Instruction (CAI) System.

**Experimental Procedure**

A sample of 115 prospective elementary school teachers (juniors and seniors) in Math Ed 420 course at The Pennsylvania State University were given the \((V,J)\) pretest via CAI terminals. Subjects passing or failing this pretest were randomly assigned by the computer to take either the \((V,S)\) pretest or the \((S,J)\) pretest in order to determine whether they could perform the subordinate tasks without explicit instruction. At this point no further use was made of those subjects who originally passed the \((V,J)\) pretest. They were branched by the computer to the end of the study. Those subjects who originally failed the \((V,J)\) pretest were eligible for one of the aforementioned instructional sequences or treatments.

Non-parametric statistical tests were employed in testing the hypotheses. The Binomial Probability Test was used to test the conditional type directional consequent hypotheses, while the Fisher Exact Probability Test was used to test the directional and non-directional consequent hypotheses involving two independent groups. Both probability tests were applied with a set at .20 since a type 2 error was deemed more costly than a type 1 error. Moreover, no true test of an hypothesis could be made if an adequacy requirement of 80% was not met for each of the main instructional episodes in the
instructional sequence under consideration.

Results

Adequacy of Instructional Episodes

The major instructional objectives involved in this investigation, along with a sample criterion performance item, are indicated in Table 2. In the table, note that Principles 1, 2, and 3 refer to the contrapositive, converse, and transitivity principles, respectively.

Table 2

<table>
<thead>
<tr>
<th>The Major Instructional Objectives Employed in Study</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Objective (V,S)</td>
</tr>
<tr>
<td>A simple deductive argument of the conditional type in verbal form (V) involving principle 1, 2, or 3.</td>
</tr>
</tbody>
</table>

Sample Criterion Performance Item:

Given the verbal argument

If it rains today, then it will snow tomorrow.
It will not snow tomorrow.
It did not rain today.

Which one of the following symbolizes the above argument?

1. If A, then B.
2. If A, then not B.
3. If A, then B.

B. Not A.
Not B.
R.

Not A.
### Table 2 (cont.)

<table>
<thead>
<tr>
<th>B. Objective (S,J)</th>
<th>Required Performance</th>
<th>Criterion</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Given</strong></td>
<td>Select from a set of three alternatives the correct one for the validity judgment (J) of the argument.</td>
<td>2 out of 3 items over principle 1, 2, 3, respectively (no time limit).</td>
</tr>
</tbody>
</table>

**Sample Criterion Performance:**

Suppose you know that

- If P, then Q.
- Not Q.

Then would this be true?

- Not P.

**1.** YES  
2. NO  
3. MAYBE

---

*This statement should be interpreted as 2 for 3 items over prin. 1, 2 for 3 items over prin. 2, and 2 for 3 items over prin. 3.*

**Correct answer.**

Separate instructional episodes were prepared for each objective, and the data relevant to the adequacy of these episodes are displayed in Table 3.
Table 3

<table>
<thead>
<tr>
<th>Objective</th>
<th>Instructional Sequence</th>
<th>m(G&quot;)</th>
<th>m(G')</th>
<th>Adequacy ratio: m(G')/m(G&quot;)</th>
<th>Decision on adequacy of instructional episode</th>
</tr>
</thead>
<tbody>
<tr>
<td>(V,S)</td>
<td>V+S</td>
<td>56</td>
<td>10</td>
<td>.90</td>
<td>Adequate</td>
</tr>
<tr>
<td>(S,J)</td>
<td>S+J</td>
<td>43</td>
<td>42</td>
<td>.50</td>
<td>Not adequate</td>
</tr>
</tbody>
</table>

a Number of students in G", where G" represents students who failed the (V,J) pretest and subsequently encountered the pretest on an instructional objective.

b Number of students in C', where C' represents students in G" who failed the instructional objective pretest.

c Number of students in G' who passed the instructional objective posttest.

d The episode is adequate if adequacy ratio is ≥ .80.

Discussion

Subject to the limitations of generalizability imposed by the characteristics of the basic sample, the specificity of the subject matter, and the inadequacy of the (S,J) instructional episode which probably indicates that the episode was applicable to only a special high ability group of the subjects tested, the following interpretations seem warranted:

1. Neither the achievement of objective (V,S) alone as a result of adequate explicit instruction nor achievement of objective (V,S) without explicit instruction is enough to expect the attainment of objective (V,J). It appears evident that (V,S) to (V,J), without exhibiting some intermediate behavior(s), is not tenable.
2. Achievement to criterion on objective (V,S) without explicit instruction and then achievement to criterion on objective (S,J) with or without the benefit of explicit instruction, does not imply achievement to criterion on the terminal (or transfer) objective (V,J). On the other hand, a post hoc analysis of the data relevant to the {(S,J), (V,S)} instructional sequence suggests that the achievement of objective (S,J) as a result of explicit instruction, followed by the achievement of objective (S,J) as a result of explicit instruction, followed by the achievement of objective (V,S) without explicit instruction, implies the achievement of the terminal objective (V,J). Thus, it appears to be evident that a hierarchical arrangement of objectives from (S,J) to (V,S) to (V,J) is tenable.

3. Mastery of the terminal objective (V,J) does imply mastery of objective (V,S) but does not imply mastery of objective (S,J).

4. Achievement of objective (V,S), followed by the achievement of objective (S,J), both as a result of explicit instruction, followed by guided thinking information (GTI) implies the achievement of the terminal objective (V,J). However, this interpretation must be considered as highly tentative in that the result obtained is based only on a small number of subjects. In light of this, the proportion of subjects achieving the terminal objective (V,J) as a result of the above is significantly greater than the proportion of subjects achieving the terminal objective (V,J) as a result of explicit instruction to criterion on objective (V,S), and being unable to demonstrate mastery of objective (S,J) without explicit instruction.

5. Achievement to criterion on objective (V,S) without explicit instruction and then achievement to criterion on objective (S,J) as a result
of explicit instruction, in any order, followed by guided thinking information
does not imply mastery of the terminal objective (V,J).

Based on the above results, it was not feasible to generalize con-
cerning all the traversing paths in the hierarchical structure proposed
in Figure 1.

Conclusion

In general, the results discussed above were interpreted to indicate
that prospective elementary school teachers, subject to the conditions
of this study, are able to demonstrate mastery of the terminal objective
(V,J) as a result of explicit instruction to criterion on objective (S,J),
followed by achievement to criterion on objective (V,S) without the benefit of
explicit instruction, and without the benefit of guided thinking information
as defined.


