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ABSTRACT

A series of 10 teacher-prepared Learning Activity Packages (LAPs) in advanced algebra and trigonometry, these units cover absolute value, inequalities, exponents, radicals, and complex numbers; functions; higher degree equations and the derivative; the trigonometric functions; graphs and applications of the trigonometric functions; sequences and series; permutations, combinations, and probability; descriptive statistics; special theorems and functions; and matrices and vectors. The units each contain a rationale for the material being covered; lists of behavioral objectives; a list of reading assignments, problem sets, tape recordings, and filmstrips that accompany the unit; a student self-evaluation problem set; suggestions for advanced study; and references. (DT)

LEARNING ACTIVITY PACKAGE

Absolute Value, Inequalities, Exponents, Radicals, and Complex Numbers

Ninety Six High School · Ninety Six, S.C.

ALGEBRA 124

REVIEWED BY

LAP NUMBER 476

WRITTEN BY Bill Holland

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RATIONALE

Much of the world of mathematics is a world of numbers, and in order to work with numbers effectively, we must know the rules that govern their use. You already have had a good deal of experience with the basic rules of algebra; in fact, you are probably so familiar with them that you apply them mechanically without thinking about them. For this reason, we are going to use the rules of algebra in an unfamiliar setting. We are going to use these rules to solve equations involving absolute values and also inequalities. Then we are going to undergo a thorough review of the Laws of Exponents and radicals. Finally we are going to study a set of numbers that allow us to determine the solution set of the relatively simple equation $x^2 + 1 = 0$: the set of Complex Numbers.

SECTION I

BEHAVIORAL OBJECTIVES:

By the completion of the prescribed course of study, you will be able to:

1. Answer questions and/or solve problems relating to the definition of the absolute value of a real number and the following properties of the absolute value Given: a,b, are real numbers:

   a) $|-a| = |a|$
   b) $|a| > 0, a \neq 0$
   c) $|0| = 0$
   d) $|ab| = |a||b|$
   e) $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$
   f) $|a^2| = a^2$

2. Determine the solution set of a given inequality when the inequality is:

   a) a linear inequality
   b) a fractional inequality
   c) a second or third degree inequality
   d) an inequality involving absolute values

RESOURCES I

I. Readings:

1. White: # 1 ____; # 2 p. 107
2. Rees: # 1 p. 14; # 2 pps. 75-76, 80-87, 214-219
3. Vance: # 1 pps. 76-77; # 2 pps. 184-188
4. Fisher: # 1 pps. 26-28; # 2 pps. 22-25, 29-31
5. Dolciani: # 1p. 50; # 2 pps. 50-55, 57-59

II. Problems:

1. White: # 1___; # 2 pps. 108-109 exs. A(14-18), [illegible]
2. Rees: # 1 ___; # 2 p. 77 exs. 1-20 (odd numbers), pps. [illegible] exs. 1-20 (odd numbers), p. 219 exs. 1-20 (odd numbers)
3. Vance: # 1 pps. 77-78 exs. 6-7; # 2 p. 78 exs. 12,[illegible], p. 18[illegible] exs. [illegible]
4. Fisher: # 1 p. 29 exs 1-4; # 2 pps. 25-26 ex. 2, pps. [illegible] exs. [illegible]
5. Dolciani: # 1 ___; # 2 p. 55 Exs. [illegible], p. [illegible] exs. [illegible]

Self Evaluation I

1. Answer the following true or false. Given: $-a$, b are real numbers:

   a) $-a < 0$

   b) $|a + b| = |a| + |b|$

   c) $|a^3| = a^3$

   d) $|a| = -a$ if $a < 0$

2. Solve the following for all values of x:

   a) $|6x - 3| = 6$

   b) $|4 - 6x| = -4$

   c) $|5 - 3x| = 0$

   d) $|x - 1| = 1 - x$

3. Determine the solution sets for x in the following:

   a) $7x - 3 \geq 2$

   b) $2 - 6x \leq 8$

   c) $|3x - 6| \geq 3$

   d) $x^2 + 6x \geq -5$

   e) $\frac{x + 3}{x - 4} \leq 0$

   f) $x^3 + 2x^2 + x \leq 0$

If you have mastered the Behavioral Objectives, take your Progress Test.

SECTION II

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BEHAVIORAL OBJECTIVES:

By the completion of the prescribed course of study, you will be able to:

3. Demonstrate your understanding of the following laws of positive integral exponents by solving problems and/or simplifying * expressions relating to them. Given- a,b are real numbers and m and n are positive integers:

   a) $a = a^1$

   b) $a^m a^n = a^{m+n}$

   c) $(ab)^n = a^n b^n$.

   d) $\frac{a^m}{a^n} = a^{m-n}$, $m > n$

   e) $\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$, $m < n$

   f) $a^0 = 1$, $a \neq 0$

   g) $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$, $b \neq 0$

   h) $(a^m)^n = a^{mn}$

4. Simplify * any expression containing positive and negative integral exponents and write the expression without negative exponents using the laws in Objective 1 and the law $a^{-t} = \frac{1}{a^t}$

5. Demonstrate your understanding of the symbol $\sqrt[k]{a}$ by being able to determine the principal k th. root of a.

6. Simplify an expression written with a fractional exponent by using the law $a^{\frac{j}{k}} = \sqrt[k]{a^j} = \left(\sqrt[k]{a}\right)^j$

7. Write any expression written with a fractional exponent in radical form and vice-versa.

8. Demonstrate that you understand the laws of integral exponents also hold for fractional exponents by simplifying * expressions that have fractional exponents using the laws of exponents from Objective 3.

9. Use the law $\sqrt[k]{ab} = \sqrt[k]{a}\ \sqrt[k]{b}$ to:

   a) remove rational factors from the radicand

   b) Simplify * expressions involving products of radicals of the same order

   c) insert rational factors into the radicand

10. Use the law $\sqrt[k]{\frac{a}{b}} = \frac{\sqrt[k]{a}}{\sqrt[k]{b}}$ to:

    a) Simplify * expressions involving quotients of radicals of the same order

    b) rationalize a monomial denominator of a quotient

11. Use the law $\sqrt[k]{\sqrt[j]{a}} = \sqrt[kj]{a}$ to change the order of a given radical.

12. Determine the sum or difference of a given set of radicals.

13. Rationalize the denominator of an expression when the denominator is a binomial.

* Note: To <u>simplify</u> an exponential expression or a radical is to write the expression so that:

a) each exponent is written as a positive exponent

b) no real number is written in exponential notation

c) each exponent of the radicand is a natural number less than the index

d) there is no radical in the denominator

e) there are no unnecessary parenthesis

I Readings:

1. White: # 3, 4 pps. 3-4; # 5 ___ ; # 6 - # 13 pps. 6-8

2. Rees: # 3 pps. 98-100; # 4 pps. 101-105; # 5 - # 8 pps. 106-109; # 9 pps. 110-111; # 10 pps. 111-112; # 11 p. 112; # 12 p. 114; # 13 p. 115

3. Vance: # 3, 4 p. 59; # 5 - # 8 pps. 60-61; # 9 - # 10 pps. 62-63, 65-66, # 11___ ; # 12 p. 64; # 13 p. 66

4. Fisher: # 3, 4 pps. 32-35; # 5 ___ ; # 11 ___; # 12 ___; # 6 - # 10, # 13 pps. 36-39

II. Problems:

1. White: # 3, 4 p. 5 exs. 1-26 (even numbers) ; # 5 ___; # 6 - # 13 p. 9 exs. A(1-20)

2. Rees: # 3 pps. 100-101 exs. 1-44 (even numbers) ; # 4 pps. 105-106 exs. 1-44 (even numbers); # 5 - # 8 exs. 1-34 (even numbers); # 9 - # 11 pps. 112-114 exs. 1-56 (even numbers); # 12 - # 13 pps. 115-116 exs. 1-40 (even numbers)

3. Vance: # 3, 4 p. 60 exs. 1-12; # 5 - # 8 pps. 61-62 exs. 1-28; # 9 - # 10 exs. 1-30 (even numbers), p. 66 exs. 1-24; # 11 ___; # 12 pps. 64-65 exs. 1-20 ; # 13 p. 67 exs. 25-41

4. Fisher: # 3, 4 pps. 35-36 exs. 1-4; # 5 ___; # 11___; # 12___; # 6 - # 10, # 13 pps. 39-40 exs. 1, 3-6

SELF EVALUATION II

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1. Simplify the following expressions:

   a) $\dfrac{3x^{-2}y}{27xy^{-5}}$ c) $(5x^2y^{-3}z)^2(\frac{1}{5}x^{-4}yz^{-2})^{-3}$

   b) $(3^{\frac{1}{2}}a^5b^{-\frac{7}{2}}c^{-\frac{3}{2}})^2$ d) $\dfrac{(3x^2y^{-6}z^3)^3}{(27x^2y\,z^{-3})^3}$

2. Write the following without using exponents or radicals:

   a) $(0.81)^{\frac{1}{2}}$ c) $(0.001)^{\frac{4}{3}}$ e) $\sqrt[3]{27x^6y^{12}z^3}$

   b) $(-32)^{-\frac{3}{5}}$ d) $\sqrt[3]{(64)^5}$ f) $\sqrt{\dfrac{9x^6y^2z^{10}}{25x^8y^{12}z^{-4}}}$

3. Simplify the following expressing your answer with rational exponents:

   a) $\sqrt{xy^2}\cdot\sqrt{x^2y}\cdot\sqrt{x^3y^4}$ b) $\dfrac{\sqrt{a^3b^{-2}c}}{\sqrt{a^{-2}b^{-2}c^3}}$

4. Simplify the following:

   a) $(3^{\frac{1}{3}}x^{\frac{2}{3}}y^{\frac{3}{4}}z^{-\frac{1}{2}})^{\frac{3}{4}}(3^{\frac{4}{5}}x^{\frac{1}{2}}y^{-\frac{2}{3}}z^{\frac{5}{4}})^{-\frac{2}{3}}$

   b) $\dfrac{16^{\frac{3}{4}}a^{\frac{5}{7}}b^{-\frac{5}{4}}}{32^{\frac{2}{5}}a^{-\frac{7}{2}}b^{-\frac{2}{5}}}$ c) $\left(\dfrac{a^{3x-5}}{a^x}\right)^{\frac{3x}{2}}$

5. a) Simplify the following by removing rational factors from the radicand where possible

   1) $\sqrt[3]{243}$ 3) $\sqrt{98x^7yz^4}$

   2) $\sqrt[5]{64x^6}$ 4) $\sqrt[3]{24x^5y^8z^{12}}$

   b) In each of the following insert the rational factor into the radicand

   1) $\sqrt{2x\,6x^3}$ 2) $3ab\sqrt[4]{2a^3b^2}$

6. Express each of the following as a single radical and simplify:

   a) $\sqrt[3]{\sqrt{64x^{12}}}$ b) $\sqrt[4]{\sqrt[3]{16x^{24}y^6}}$

SELF EVALUATION II (cont.)

7. Reduce the order of the following and simplify:

   a) $\sqrt[8]{81\ x^{10}y^{14}z^{20}}$ b) $\sqrt[6]{64\ x^{8}y^{12}z^{15}}$

8. Make all combinations that are possible in the following by symplifying:

   a) $\sqrt{48} + \sqrt{27} - \sqrt{75}$ c) $\sqrt[3]{16} + \sqrt[3]{54} - \sqrt[3]{128}$

   b) $\sqrt{\frac{9}{x}} + \sqrt{\frac{54}{x^3}}$ d) $3t^2\ \sqrt{16m^5n^8t^7} + 2m\ \sqrt{36m^9n^{16}t^{11}}$

9. Rationalize the denominators in the following

   a) $\sqrt{\frac{3x^5}{2y^7}}$ d) $\frac{\sqrt{5x}}{\sqrt{5} - \sqrt{2}}$

   b) $\sqrt[3]{\frac{5}{7x^2}}$ e) $\frac{5x}{\sqrt{3x} + \sqrt{5y}}$

   c) $\frac{6}{\sqrt{x} - \sqrt{y}}$ f) $\frac{3\sqrt{x} + 5\sqrt{y}}{\sqrt{x} + \sqrt{y}}$

If you have mastered the Behavioral Objectives, take your Progress Test.

ADVANCED STUDY I

1. The integers $a = 0$ and $b = 0$ satisfy the equation $ab = a + b$. Are there any others?

2. Given x and y are positive numbers. Show that $\sqrt{x + y} < \sqrt{x} + \sqrt{y}$.

3. Prove: If $\sqrt{(x + c)^2 + y^2} + \sqrt{(x - c)^2 + y^2} = 2a$ and $a^2 = b^2 + c^2$, then $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

4. Is 6 the only value of $(x^2 + 6x + 9)^{\frac{1}{2}} - (x^2 - 6x + 9)^{\frac{1}{2}}$? Justify your answer.

5. Rationalize the denominator in the fraction $\frac{1}{5 - \sqrt[3]{5}}$.

SECTION III

BEHAVIORAL OBJECTIVES:

By the completion of the prescribed course of study, you will be able to:

14. a) Define the set of complex numbers.

    b) Determine a relation between the set of complex numbers and the set of real numbers based on the definition given in part (a)

    c) Answer questions and/or solve problems relating to the definition given in part (a)

15. Simplify positive integral powers of the imaginary unit i.

16. Express any complex number written in the form a + bi as an ordered pair and vice-versa:

17. Solve problems relating to the definition of equality of two complex numbers.

18. Solve problems relating to the definition of the conjugate of a complex number.

19. Given an expression involving square roots of real numbers, write the expression in simplified form so that the radicand is a positive integer.

20. Given a pair of complex numbers:

    a) compute their sum algebraically

    b) determine the additive inverse of either

    c) compute their difference algebraically

    d) compute their product algebraically

    e) determine the multiplicative inverse of either

    f) determine their quotient algebraically

21. a) Graph any complex number

    b) Give a geometric interpertation of the sum or difference of two complex numbers:

22. Solve problems relating to the definition of the modulus of a complex number.

RESOURCES III

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I. Readings:

1. Vannatta: # 14 p. 3, # 15 p. 23; # 16 ___; # 17 ___; #18 ___; # 19 ___, # 20 pps. 23-24; # 21 - # 22; pps. 25-28

2. Rees: # 14 p. 315, # 15 ___; # 16 pps. 317-318; # 17 p. 316; # 18 p. 318; # 19 ___; # 20 pps. 316-317; # 21 ___; # 22___

3. Vance: # 14 p. 163; # 15___; # 16 p. 163; # 17 ___; # 18 p. 164; # 19 ___; # 20 pps. 163-164; # 21 p. 165; # 22 p. 166

4. Fisher: # 14 pps. 180-181; # 15 ___; # 16 ___; # 17 p. 181; # 18 pps. 183-185; # 19___; # 20 pps. 181-182, p. 185; #21-#22 pps. 186-187

5. Dolciani # 14 p. 256; # 15 ___; # 16 p. 256; # 17 p. 256; # 18 p. 261; # 19 pps. 264-265; # 20 pps. 256-257, pps. 259-262; # 21 ___; # 22 p. 258

6. White: # 14 pps. 76-77; # 15 pps. 79-80; # 16 ___; # 17___; # 18 p. 80; # 19 p. 77; # 20 pps. 77-78, pps. 80-81; # 21 pps. 83-84, pps. 85-87; # 22___

II. Problems:

1. Vannatta: # 14 p. 24 exs. 1-19, # 15___; # 16 ___; # 17___; # 18___; # 19 ___; # 20 pps. 24-25 exs. 20-21; # 21 p. 28 exs. 1,4,5; # 22 p. 28 exs. 2,3 (modulus only)

2. Rees: # 14___; # 15___; # 16 ___; # 17 p. 319 exs. 1-8; # 18 p. 320 exs. 49-52; # 19 ___; # 20 p. 319 exs. 9-48 (odd numbers); # 21 ___; # 22 ___

3. Vance: # 14___; # 15 p. 164 exs. 4-5; # 16___; # 17 ___; # 18 ___; # 19 ___; # 20 p. 164 exs. 1-4; # 21 pps. 165-166 exs. 19-20, # 22 exs. 1-6

4. Fisher: # 14___, # 15___; # 16___; # 17___; # 18___ pps. 185-186 exs. 1-2, 4; # 19 ___; # 20 p. 183 exs. 1,2,4, p. 186 exs. 3, 3-10; # 21 . # 22 p. 189 exs. 1,3

5. Dolciani: # 14___; # 15___; # 16 ___; # 17___, # 18 ___; # 19 p. 263 exs. 13-24; # 20 p. 258 exs. 1-6, 21-24, p. 263 exs. 1-30 (even numbers); # 21 ___; # 22 p. 258 ex. 27

6. White # 14___; # 15___; # 16___; # 17___; # 18___; # 19 - # 20 pps. 78-79 exs. A(1-5), B(1-10), pps. 81-82 exs. A (1-15), B (1-10); # 21 pps. 84-85 exs. 1-14, p. 87 exs. 1-4, p. 87 exs. 1-4; # 22

SELF EVALUATION III

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1. State which of the following are complex numbers:

   a) 5
   b) $-6\frac{1}{3}$
   c) $(-4,-2)$
   d) $\sqrt{-13}$
   e) $7i$
   f) $3 + \sqrt{17}\, i$

2. What is the identify element for multiplication in the set of complex numbers?

3. Simplify the following:

   a) i^{19}
   b) i^{36}
   c) i^{4n+3} (n a positive integer)

4. Express the following complex numbers as ordered pairs:

   a) 3
   b) $\sqrt{-69}$
   c) $5-3i$
   d) $2i-6$
   e) $-6i$

5. Find replacements of the variables so that the following equations are true:

   a) $3 + xi = 2y - 6i$
   b) $7x + 5yi = 14i - 15$
   c) $(a,9) = (11,5b)$
   d) $(3 + x, -4) = (5 + 2x, 8y)$

6. Prove the following If u and v are two complex numbers, then $\overline{u + v} = \bar{u} + \bar{v}$

7. Simplify the following:

   a) $\sqrt{-108} + \sqrt{-192} - \sqrt{-75}$
   b) $\sqrt{-50} - \sqrt{-32} + \sqrt{-8}$

8. Combine the following as indicated:

   a) $[(3 + 2i)(6 - 4i)] + [(2 - 6i) - (4i + 3)]$
   b) $[(3 - \sqrt{-4}) + (\sqrt{-169} + 6i)][(2 + 5i) + (5 - 4i)]$
   c) $[(6 + 3i^3) - (5i^4 - 3i)] + [(2i) + (3i)]$
   d) $[(3,-2)(5, 4)] + [(-2,-1) - (4,-3)]$

9. For the following complex numbers
   a) graph them on a complex plane
   b) graph their conjugate
   c) determine their modulus

SELF EVALUATION III (cont.)

1) $3 + 5i$

2) $3 - \sqrt{-25}$

3) $(8,-6)$

4) $(0,7)$

5) $\sqrt{-36}$

If you have mastered the Behavioral Objectives, take your LAP Test.

ADVANCED STUDY II

1. We say that the set of real numbers is ordered since for any two real numbers a and b we can determine whether $a < b$, $a > b$, or $a = b$. Is the set of complex numbers ordered? Justify your answer. If your answer was no, try to determine a way to set up an order relationship for the set of complex numbers.

2. Suppose z_1 and z_2 are two complex numbers such that $3z_1 + 5z_2 = 3 - 6i$ and $5z_1 - 3z_2 = 3i - 5$. Determine a value for z_1 and z_2.

3. Let z_1 and z_2 be two complex numbers that are not real. If $z_1 z_2$ and $\frac{z_1}{z_2}$ are real numbers, what can you say about z_1 and z_2? Justify your answer.

REFERENCES

I. Textbooks:

1. Vannatta, Carnahan, Fawcett: Advanced High School Mathematics, Expanded Edition (Charles E. Merrill Publishing Co., 1965)

2. White: Advanced Algebra (Allyn and Bacon, Inc., 1952).

3. Rees, Sparks: Algebra and Trigonometry, 2nd Edition, (McGraw-Hill, Inc., [illegible]).

4. Vance: Modern Algebra and Trigonometry, 2nd Edition, (Addison-Wesley Publishing Co., Inc., 1968).

5. Fisher, Ziebur: Integrated Algebra and Trigonometry, 2nd Edition, (Prentice-Hall, Inc., 1967).

6. Dolciani, Beckenback, Donnelly Jurgensen, Wooton: Modern Introductory Analysis, (Houghton Mifflin Co., 1970).

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FUNCTIONS

F(X)=X

Ninety Six High School

Ninety Six, S.C.

ALGEBRA 124

REVIEWED BY

CAT NUMBER 117

WRITTEN BY William Holland

RATIONALE

Functions are one of the most important mathematical tools we have. They are used in Physics, Chemistry, and Biology. They are used in Statistics, all types of engineering, and in computer programming. But they are not restricted to use in the sciences. Such diverse fields as Economics and Music make extensive use of functions.

In this LAP you will analyze several different functions, study the relations between them, and some of their applied uses.

SECTION 1: General Functions

BEHAVIORAL OBJECTIVES:

By the completion of the prescribed course of study, you will be able to:

1. a. Define a function.

   b. Determine whether a given relation is a function.

2. Determine the domain of any given function.

3. Compute f(a) for any given function (f), where a is any variable.

4. Sketch a graph of any given function.

5. Find the zeroes of any given function.

6. a. State and prove the distance formula.

   b. Apply the distance formula to find the distance between any two given points.

7. a. Define a direct variation relationship.

   b. Solve problems relating to direct variation.

8. Define a linear function.

9. Determine the slope of any line given the equation of the line or two points on the line.

10. Determine the x and y intercepts of the graph of any linear function.

11. Determine the equation of a line given two points on the line or a point on the line and the slope of the line.

12. a. Define an inverse variation relationship.

    b. Solve problems relating to inverse variation.

I. **Readings:**

1. Vannatta: # 1 - # 3 pps. 36-38; # 4 pps. 38-39; # 5 p. 54; # 6 pps. 39-41; # 7 _____; # 8 p. 33; # 9 pps. 42-44 # 10 pps. 45-46; # 11 pps. 46-50, # 12 _____

2. Rees: # 1 - # 3 pps. 117-123; # 4 pps. 126-128; # 5 p. 128; # 6 p. 300; # 7 pps. 291-293; # 8 pps. 128-130; # 9 _____; # 10 p. 129; # 11_____; # 12 pps. 291-293

3. Vance: # 1 - # 3 pps. 93-97; # 4 pps. 100-103; # 5 p. 101; # 6 pps. 85-86; # 7 p. 198; # 8 p. 169; # 9 _____; # 10 _____; # 11_____; # 12 pps. 198-199

4. Fisher: # 1 - # 3 pps. 43-50; # 4 pps. 58-61; # 5 p. 69; # 6 pps. 51-56; # 7 pps. 62-64; # 8 - # 11 pps. 65-69; # 12 pps. 70-72

5. Dolciani: # 1- # 3 pps. 215-217, 222; # 4 pps. 222-223; # 5 _____; # 6 pps. 168-169; # 7 _____; # 8 p. 219; # 9 - # 11 pps. 203-206; # 12 _____

II. Problems:

1. Vannatta: # 1 - # 3 p. 38 exs. 1-10; # 4 p. 39 exs. 1-6; # 5 p. 54 exs. 1-12; # 6 p. 41 exs. 1-10; # 7 _____ ; # 8 _____ ; # 9 pps. 44-45 exs. 1-10; # 10 p. 46 exs. 1-10; # 11 pps. 50-51 exs. 1-20; # 12_____

2. Rees: # 1 - # 3 pps. 123-124 exs. 1-36; # 4 - # 5 p. 130 exs. 5-12; # 6_____; # 7 p. 296 exs. 13,15, 17-20; # 8 _____; # 9 _____; # 10 p. 130 exs. 21-24; # 11 _____; # 12 pps. 296-297 exs. 14, 21-24

3. Vance: # 1 - # 3 pps. 97-98 exs. 1-13; # 4 p. 103 exs. 1,2,7,8; # 5_____; # 6 p. 86 exs. 1-12; # 7 pps. 199-200 exs. 5,7,8,9,13,14,16; # 8_____; # 9_____; # 10_____; # 11_____; # 12 pps. 199-200 exs. 1,2,3,10,17

4. Fisher: # 1 - # 3 pps. 47-48 exs. 1-5, pps. 50-51 exs. 1-4; # 4 p. 61 exs. 2,3; # 5 _____; # 6 p. 57 exs. 1,3-5; # 7 p. 64 exs. 1-3, 5-9; # 8 - # 11 exs. 1-3, 6; # 12 p. 73 exs. 1-3, 8-10

5. Dolciani: # 1 - # 3 pps. 217-218 exs. 1-16; pps. 224-225 exs. 1-25, 35-38; # 4 exs. 39-44; # 5_____; # 6 pps. 169-170 exs. 1-14, 19-23; # 7_____; # 8_____; # 9 - # 11 pps. 207-208 exs. 11-18, 25-27; # 12 _____

III. Audio:

1. Wollensak Teaching Tape C-3852: Graphing Linear Functions
2. Wollensak Teaching Tape C-3854: The Slope of a Line
3. Wollensak Teaching Tape C-3855: Slope Intercept Form

IV. Visual:

Filmstrip 1142: Direct Variation

SELF-EVALUATION I

1. a. Define a function?

   b. Determine whether the following relations are functions:

      (1) $\{(3,1), (4,2), (4,3), (5,4)\}$

      (2) $f(x) = \dfrac{3x - 9}{x - 5}$

      (3) $y \geq x + 5$

      (4) $f(x) = [\,x\,]$

2. Find the domain of the following functions.

   a. $f(x) = x$

   (b) $y = \dfrac{3x - 2}{x + 5}$

   (c) $f(x) = |x|$

   (d) $y = \dfrac{7x - 9}{x^2 - 81}$

3. a. (1) Given $f(x) = |x|$ Compute $f(0)$, $f(-1)$, and $f(-9)$

      (2) Given $f(x) = [\,x\,]$ Compute $f(1)$, $f(3\frac{1}{2})$, and $f(-5\frac{1}{3})$

      (3) Given $f(x) = \dfrac{7x - 9}{x + 3}$ Compute $f(-3)$, $f(0)$, and $f(5)$

   b. Given $f(x) = [\,x\,]$, which of the following relations hold for f?

      (1) $f(x^2) = (f(x))^2$ (2) $f(x + y) = f(x) + f(y)$

      (3) $f(4x) = 4f(x)$

4. Sketch a graph of the following functions.

   (a) $f(x) = [\,x\,]$
   (b) $f(x) = |x|$
   (c) $f(x) = 3x - 2$
   (d) $f(x) = -2x + 5$

5. Find the zeroes of the functions given in problem 4.

6. a. State and prove the distance formula.

   b. Find the distance between the following pairs of points.

      1) (0,3) and (-1,-2)

      2) (-2,-5) and (4,2)

   c. Determine whether the points A(3,-2), B(-4,5) and C(4,-3) are collinear.

7. a. What does it mean to say y is directly proportioned to x.

   b. The point (0,-3) belongs to the graph of the equation $y = f(x)$ and y is directly proportional to x. Find the formula for $f(x)$.

   c. Given y is directly proportional to x and $y = f(x)$. Does $f(a+b) = f(a) + f(b)$ for any two numbers a and b. Justify your answer.

8. Define a linear function?

9. a. Find the slope of the line given by the equation:

      1) $f(x) = -\frac{1}{2}x - 6$

      2) $y = 11x + 17$

   b. Find the slope of the line passing through the given points.

      1) (-1,0) and (3,2)

      2) (4,3) and (1,-1)

10. Find the x and y intercepts of the graphs of the following functions:

    a. $f(x) = -3x - 2$

    b. $f(x) = -x + 7$

11. a. Find the equation of the line passing through the points (3,1) and (-2,0).

    b. Find the equation of the line passing through the points (-1,4) with slope -3.

12. a. What does it mean to say x is inversely proportional to y?

    b. If y is inversely proportional to x and the graph of the equation $y = f(x)$ contains the point [illegible], what is the formula for $f(x)$?

c. If y is inversely proportional to x and $y=f(x)$ does $f(ab) = a \cdot f(b)$ where a and b are any non-zero numbers? Justify your answer.

IF YOU HAVE MASTERED THE BEHAVIORAL OBJECTIVES IN THIS SECTION, TAKE THE PROGRESS TEST.

SECTION II: Exponential and Logarithmic Functions

BEST COPY AVAILABLE

BEHAVIORAL OBJECTIVES:

By the completion of the prescribed course of study, you will be able to:

13. Define an exponential function and solve problems relating to this definition.
14. Sketch a graph for an indicated domain of any given exponential function.
15. Define a logarithmic function and solve problems relating to this definition.
16. a) State and/or prove the following properties of logarithms:

    1. If M and N are positive numbers and b is any base, then

       $\text{Log}_b MN = \text{Log}_b M + \text{Log}_b N$

    2. If N is a positive number, p is any real number, and b is any base, then

       $\log_b N^p = p\log_b N$

    3. If M and N are positive numbers and b is any base, then

       $\text{Log}_b \frac{M}{N} = \text{Log}_b M - \text{Log}_b N$

    4. If a and b are two bases, and N is any positive number, then

       $\text{Log}_a N = \frac{\text{Log}_b N}{\text{Log}_b a}$

    b) Solve problems and equations relating to the properties in part (a).

17. a) Determine the characteristic and mantissa of a common logarithm.

    b) Use tables to determine:

    1. The value of log N when N is a three digit number.
    2. N to three digits when log N is given

18. Use the method of linear interpolation to:

    a) Determine log N when N is a four digit number.
    b) Determine N when log N is given, but is not listed in the table of logarithms.

19. Sketch a graph for an indicated domain of any given logarithmic function.
20. Solve problems involving numerical computations using logarithms.
21. Solve any given exponential or logarithmic equation.

RESOURCES II

I. Readings:

1. Rees: # 13 - # 14 pps. 374-376; # 15 - # 16 pps. 354-357, 373-374; # 17 - # 18 pps. 363-367, 368-369; # 19 pps. 375-376; # 20 pps. 370-372; # 21 pps. 377-378

2. Vance: # 13 - # 14 pps. 305-306, # 15 - # 16 pps. 313-316, 325-326; # 17 - # 18 pps. 316-319; # 19 pps. 313-314; # 20 pps. 320-324; # 21 pps. 326-327

3. Fisher: # 13 - # 14 pps. 77-80; # 15 pps. 81-83; # 16 pps. 84-86; # 17 pps. 91-93; # 18 pps. 94-97; # 19 pps. 87-90; # 20 pps. 98-100; # 21 pps. 101-103

4. Dolciani: # 13 - # 14 pps. 327-332k 338-341; # 15 pps. 353-355; #16 pps. 360-362; # 17 - # 18 pps. 356-359; # 19_____; # 20 p. 359; # 21 _____

II. Problems:

1. Rees: # 13 - # 14 p. 377 exs. 25-28; # 15 - # 16 pps. 357-359 exs. 1-68 (every third exercise), p. 377 exs. 7-16; # 17 - # 18 pps. 367-368 exs. 1-67 (every third exercise), p. 369 exs. 1-48 (every second exercise) ; # 19 p. 377 exs. 17-24; # 20 pps. 372-373 exs. 1-48 (every second exercise); # 21 p. 379 exs. 9-24

2. Vance: # 13 - # 14 p. 306 exs. 1-6; # 15 - # 16 p. 316 exs. 1-21, p. 327 exs. 1-10; # 17 - # 18 p. 318 exs. 1-24, pps. 319-320 exs. 1-20; # 19 p. 316 exs. 22-25; # 20 pps. 321-322 exs. 1-12, pps. 324-325 exs. 1-4, 8; # 21 pps. 327-328 exs. 16-23, 26-29

3. Fisher: # 13 - # 14 p. 80 exs. 1-4; # 15 p. 83 exs. 1-3, 5, 9, 10; # 16 pps. 86-87 exs. 1-6; # 17 pps. 93-94 exs. 1-5, 8; # 18 p. 97 exs. 1-4; # 19 pps. 90-91 exs. 1,2,8,9; # 20 p. 100 exs. 1-9; # 21 pps. 103-104 exs. 1-4

4. Dolciani: # 13 - # 14 p. 332 exs. 1-8, pps. 341-342 exs. 1-3, 6; # 15 p. 355 exs. 1-24; # 16 p. 362 exs. 1-8; # 17- # 18 p. 359 exs. 1-12; # 19_____; # 20 pps. 359-360 exs. 13-24; # 21 p. 360 exs. 33-40

1. a. Define an exponential function.

   b. Find the bases of the exponential functions whose graphs contain the following points:

      1) $(2,8)$

      2) $(-2,\frac{1}{2})$

   c. Why is the base (b) of the exponential functions restricted to positive numbers?

2. Sketch the graphs of the following functions.

   a. $y = 2^x$

   b. $y = 5^x$

3. a. Define a logarithmic function?

   b. Write the following equations in logarithmic form:

      1. $9^2 = 81$

      2. $a^3 = B$

   c. Solve the following equations:

      1. $\log_3 x = 5$

      2. $\log_{16} 4 = x$

      3. $\log_x a = 0$

      4. $2^{\log_2 x} = 5$

4. a. Prove the following:

   If M and N are positive numbers and b is any base, then

   $\log_b M \cdot N = \log_b M + \log_b N$

   b. Given that $\log_b 3 = .7$ $\log_b 10 = 1.35$ and $\log_b 7 = .95$,

   find the number $\log_b \left(\frac{70}{3}\right)^{\frac{11}{4}}$

5. Sketch the graphs of the following functions:

   a. $y = \log_2 x$

   b. $y = \log_{10} 3x$

6. a. Approximate the following logarithms:

      1. log 3.634

      2. log 7.675

   b. Find N if:

      1. log N = 2.1652

      2. log N = 1.3511

   c. Find the number x

      1. $10^x = 7.22$

      2. $10^x = .3976$

7. a. Using logarithms solve the following problems.

      1. $\frac{632 \times 1.35}{4,611 \times .005}$

      2. $(2\frac{1}{3})^{\frac{1}{5}}$

   b. Determine which number is larger, $32^{\frac{5}{4}}$ or $28^{\frac{4}{3}}$

   c. Solve for x

      1. $5^x = 17$

      2. $x^{-3} = 4$

      3. $3 \cdot 4^x = 5 \cdot 6^{2x}$

      4. $\log(x + 1) - \log(x - 1) = 1$

IF YOU HAVE MASTERED THE BEHAVIORAL OBJECTIVES, TAKE THE LAP TEST.

REFERENCES

I. Resources:

1. Vanatta, Carnahan, Fawcett: Advanced High School Mathematics, Expanded edition (Chalres E. Merrill Publishing Company, 1965)

2. Rees and Sparks: Algebra and Trigonometry, 2nd edition (McGraw-Hill Book Company, 1965)

3. Fisher and Zicbur: Integrated Algebra and Trigonometry, 2nd edition (Prentice-Hall, Inc., 1967)

4. Vance: Modern Algebra and Trigonometry, 2dn Edition (Additon-Wesley Publishing Co., Inc., 1968)

5. Dolciani, Beckenbach, Donnelly, Jurgensen, Wooton: Modern Introductory Analysis (Houghton Mifflin Co., 1970)

L

A

PACKAGE

HIGHER DEGREE EQUATIONS AND THE DERIVATIVE

Ninety Six High School · Ninety Six, S.C.

Algebra 124

REVIEWED BY

48

Bill Holland

RATIONALE

In previous math courses you have learned how to determine the roots of linear equations, quadratic equations and selected higher degree equations. In this LAP you will study methods that will enable you to determine whether a given equation has any real roots. If an equation has real roots and they are rational, you will learn a method to determine their value and if they are irrational, you will learn to approximate their value.

Also, in this LAP you will be introduced to the underlying foundations of the calculus - the limit and the derivative.

Behavioral Objectives

By the completion of the prescribed course of study, you will be able to:

1. Define a polynomial of degree n and solve problems relating to this definition.
2. Derive the quadratic formula and determine the roots of a second degree equation by factoring the equation or by using the quadratic formula.
3. Use the discriminant to determine the nature of the roots of a second degree equation.
4. Use the method of synthetic division to divide a first degree polynomial into any higher degree polynomial.
5. State and prove the Remainder Theorem and solve problems relating to this theorem.
6. State and prove the Factor Theorem and its converse and solve problems relating to these two theorems.
7. State the Fundamental Theorem of Algebra and solve problems relating to this theorem and its corollary.
8. State the Rational Root Theorem and solve problems relating to this theorem.
9. Demonstrate your understanding of the statement of Descartes Rule of Signs by being able to determine the possible numbers of positive and negative roots of a given polynomial equation.
10. Isolate the real roots of any given polynomial equation between two consecutive integers.
11. Determine integral upper and lower limits (bounds) for the real roots of any given polynomial equation.
12. Solve problems involving imaginary roots of polynomial equations.

RESOURCES I

BEST COPY AVAILABLE

I. Readings:

1. Vannatta: Advanced High School Mathematics - #1 ____; #2 p. 54; #3 ____; #4 pp. 54-56; #5, #6 pp. 56-59; #7 pp. 59-60; #8 pp. 60-62; #9 pp. 62-63; #10, #11 pp. 64-67; #12 pp. 67-78.

2. Rees: Algebra and Trigonometry - #1 p. 11; #2 pp. 191-193; #3 pp. 206-207; #4 pp. 334-336; #5 pp. 332-333; #6 pp. 333-334; #7 p. 340; #8 pp. 345-348; #9 ____; #10 p. 343; #11 pp. 341-343; #12 ____.

3. Fisher: Integrated Algebra and Trigonometry - #1 p. 198; #2, #3 pp. 206-210; #4 pp. 200-201; #5, #6 pp. 217-218; #7 pp. 220-221; #8 pp. 227-230; #9 ____; #10 pp. 231-232; #11 ____; #12 pp. 223-226.

4. Vance: Modern Algebra and Trigonometry - #1 p. 246; #2 pp. 49-50, pp. 179-183; #3 p. 192; #4 pp. 45-46; #5, #6 pp. 246-248; #7 pp. 250-251; #8 pp. 253-256; #9 ____; #10 ____; #11 p. 249; #12 pp. 260-261.

5. Dolciani: Modern Introductory Analysis - #1 pp. 230-231; #2 pp. 267-270; #3 p. 276; #4 p. 236; #5 p. 235; #6 pp. 238-240; #7 pp. 269-270; #8 pp. 241-243; #9 pp. 245-246; #10 pp. 289-291; #11 pp. 244-245; #12 pp. 275-276.

II. PROBLEMS:

1. Vannatta: Advanced High School Mathematics - #1 ____; #2 p. 54 ex. 4-12; #3 ____; #4 p. 56 ex. 1-8; #5 p. 59 ex. 1-10; #6 p. 59 ex. 11-20; #7 ____; #8 p. 62 ex. 1-10; #9 p. 63 ex. 1-10; #10, #11 p. 67 ex. 1-12; #12 pp. 68-69 ex. 1-10.

2. Rees: Algebra and Trigonometry - #1 ____ #2 pp. 198-199 ex. 1-12, 25-48 (every third exercise); #3 ex. 1-20 (even numbered exercises); #4 p. 337-338 ex. 25-40; #5, #6 pp. 336-337 ex. 1-24; #7 ____; #8 pp. 348-349 ex. 1-32 (even numbered exercises); #9 ____; #10, #11 p. 344 ex. 21-32; #12 ____.

3. Fisher: Integrated Algebra and Trigonometry - #1 p. 201 ex. 1-4; #2, #3 pp. 210-211 ex. 1, 3, 4, 5, 11; #4 p. 202 ex. 8; #5, #6 p. 219 ex. 1-3, 6-8; #7 pp. 22-223 ex. 1-2, 6,10; #8 p. 230 ex. 1-3; #9 ____; #10 p. 233 ex. 1-2; #11 ____; #12 p. 226-227 ex. 1-3,5,6.

4. Vance: Modern Algebra and Trigonometry - #1 ____; #2 p. 50 ex. 3, 11, 12, 23, 24, 26, 27, 29, 30, 37, 38, 40-44, pp. 183-184 ex. 1-4, 7-9, 13, 19, 21; #3 ____; #4 pp. 46-47 ex. 1-12(even numbered exercises); #5, #6 ex. 1-14; #7 p. 251 ex. 1-5; #8 p. 256 ex. 1-14 (even numbered exercises); #9 ____; #10 ____; #11 p. 251 ex. 1-15; #12 p. 262 ex. 17-26.

5. Dolciani: Modern Introductory Analysis - #1 p. 232 ex. 1-6; #2 ex. 1-5, 6-8; #3 ____; #4 p. 237 ex. 9-14; #5 p. 237 ex. 23; #6 p. 240 ex. 1-8, 13-14; #7 ____; #8 pp. 243-244 ex. 1-10; #9, #11 p. 246 ex. 1-12; #10 pp. 291-292 ex. 1-8; #12 pp. 276-277 ex. 1-8.

1. a. Define a polynomial of degree n.

   b. For each of the following polynomials state the degree of the polynomial and tell the constant term.

      1. $9x^5 - 7x^3 + 3x - 5 = 0$

      2. $7 - 9x + 16x^3 = 0$

      3. $8t^4 - 7t^2 - 5t = 0$

2. Determine the roots of the following second degree equation.

   a. $x - 7 + \frac{12}{x} = 0$

   b. $-3x^2 - 5x - 1 = 0$

   c. $\frac{3}{x - 1} + 7x = -2$

3. What is the nature of the roots in the following equations.

   a. $x^2 - x - 2 = 0$

   b. $\frac{x - 2}{x - 1} + 5x - 7 = 0$

4. Simplify the following using synthetic division:

   $(4x^5 + 1) \div (x + 3)$

5. Use the Remainder Theorem to determine P(r) for the given values of P(x) and r.

   a) $P(x) = x^4 - 3x^2 + 6x - 8 \quad r = 2$

   b) $P(x) = x^{17} + 1 \quad r = 1$

6. a. State and prove the Factor Theorem.

   b. Use the Factor Theorem to show that $x - a$ is a factor of $x^{17} - a^{17}$.

7. Form the equation which has $(x - 3)$, $(x - 4)$, and $(x + 1)$ as linear factors.

8. Find the rational roots of the following equations:

   a. $x^3 + 5x^2 + x - 7 = 0$

   b. $12x^3 + 16x^2 - 7x - 6 = 0$

SELF-EVALUATION I (cont')

9. Determine the maximum and minimum number of positive and negative roots of the following equations.

   a) $5x^3 - 7x^2 + 6x - 9 = 0$

   b) $10x^5 - x^3 - x^2 - 5x + 1 = 0$

10. Isolate the real roots of the following equation between two consecutive integers.

    a) $x^3 - 3x^2 - x + 2 = 0$

    b) $x^4 - 2x^3 - 10x^2 - 8x + 4 = 0$

11. For the equations listed in exercise 10, determine the least upper limit and the greatest lower limit of the real roots.

12. a. Must the equation $ax^3 + bx^2 + cx + d = 0$ have any real roots if a, b, c, and d are all real numbers? Justify your answer.

    b. Form the cubic equation two of whose roots are 3 and $1 + i$

If you have mastered all the Behavioral Objectives, take your Progress Test.

ADVANCED STUDY I

1. For the following equations, determine all values of k for which the solutions are real numbers.

   a. $2x^2 + 2kx + 10 = 0$

   b. $3kx^2 - 6x + 9k = 0$

2. Suppose $P(x) = ax^2 + bx + c$ where $a < 0$ and b and c are real numbers. What conditions are necessary for both roots to be positive? Both roots to be negative? Both roots to be of opposite signs.

3. Find a quadratic polynomial whose zeroes are the cubes of the zeroes of $x^2 + 7x - 9$.

4. The square of twice a certain number is larger than the sum of the number and 1. Which numbers possess this property?

5. Solve the equation $4^{3x} - 2^{3x + 1} + 1 = 0$.

SECTION II

Behavioral Objectives

By the completion of the prescribed course of study, you will be able to:

13. Determine the slope of a line tangent to a curve at a point on the curve.
14. Determine the equation of a line tangent to a curve at a point on the line.
15. Determine the derivative of a given polynomial.
16. Find the extreme points of a given polynomial and determine whether they are maxima or minima.
17. Sketch a graph of any given equation for an indicated domain.
18. Approximate a real root of a polynomial equation to a specified degree of accuracy.
19. Determine the given number of roots of any given real numbers.

RESOURCES II

I. READINGS:

1. Vannatta: Advanced High School Mathematics - #13 pp. 69-72; #14 pp. 72-73; #15 ____; #16 pp. 73-76; #17 pp. 77-78; #18 pp. 79-80; #19 pp. 85-86.
2. Rees: Algebra and Trigonometry - #17 pp. 338-339.
3. Fisher: Integrated Algebra and Trigonometry - #17 pp. 209-210.
4. Vance: Modern Algebra and Trigonometry - #17 pp. 251-252.
5. White: Advanced Algebra - #13, #14 pp. 268-272; #15 pp. 272-274; #16 pp. 275-280, 283-287; #17 ____; #18 pp. 294-295; #19 ____.

6. Dolciani: Modern Introductory Analysis - #13, #14 pp. 293-295, 297-298; #15 pp. 300-302; #16, #17 pp. 304-307; #18 ____; #19 ____.

II. PROBLEMS:

1. Vannatta: Advanced High School Mathematics - #13 p. 72 ex. 1-10; #14 p. 73 ex. 1-8; #15 ____; #16 pp. 76-77 ex. 1-6; #17 p. 79 ex. 1-10; #18 p. 80 ex. 1-8; #19 p. 86 ex. 1-4

2. Rees: Algebra and Trigonometry - #17 p. 344 ex. 1-12 (even numbered exercises).

3. Fisher: Integrated Algebra and Trigonometry - #17 p.210 ex. 9.

4. Vance: Modern Algebra and Trigonometry - #17 p. 253 ex. 1-15 (odd numbered exercises).

5. White: Advanced Algebra - #13, #14 p. 272 ex. 1-6; #15 ____; #16 p. 280 ex. 1-3, p 287 ex. 1-6; #17 ____; #18 p. 295 ex. 1-3; #19 ____.

6. Dolciani: Modern Introductory Analysis - #13, #14 p. 296 ex. 1-12, 15-16, p. 299 ex. 1-8 (part b only), 9-12; #15 pp. 302-303 ex. 1-16; #16, #17 p. 307 ex. 1-12 (parts a, b, and d only); #18 ____; #19 ____.

1. Determine the slope of a line tangent to the graph of the curve at a point with given abscissa:

   a. $y = x^2 - 9x + 3$ abscissa = 2

   b. $y = 2x^3 - 7x$ abscissa = -1

2. Determine the equation of the line tangent to the graph of the equation $y = x^3 - 7x + 5$ at the point (1,-1).

3. a. Determine the derivatives of the following polynomials:

      1. $y = 7x - 11$

      2. $y = 3x^2 - 10$

   b. Determine the derivatives of the polynomials you found in part a.

4. Determine the coordinates of the maxima and minima of the equation $y = x^3 - 3x - 1$.

5. Sketch a graph of the equation $y = x^3 - 6x - 4$ for the domain $-3 \leq x \leq 3$.

6. Find the value of a root of the equation $y = x^3 + 4x - 1$ correct to one decimal place.

7. Find the three cube roots of 2 graphically.

IF YOU HAVE MASTERED ALL THE BEHAVIORAL OBJECTIVES, TAKE THE LAP TEST.

1. In a 220-volt circuit having a resistance of 20 ohms, the power W in watts when a current I is flowing is given by $W = 220I - 20I^2$. Determine the maximum power that can be delivered by this circuit.

2. What is the minimum velocity of a particle whose velocity with respect to time is given by the equation $y = t(t - 3)$?

3. Determine the derivatives of the following equations with respect to x.

   a) $y = \sqrt[3]{x^3 + 5x^2 - 7x + 2}$

   b) $y = (x^3 - 9x^2 + 5x)(3x^4 - 7x^2 - 8x + 6)$

   c) $y = \frac{2x^2 - 8}{7x - 5}$

4. Evaluate each of the following integrals.

   a) $\int \frac{1}{2} d.$

   b) $\int (7x - 2)dx$

   c) $\int 2x(3 - 4x^2)dx$

   d) $\int \sqrt{x^3}\, dx$

   e) $\int \frac{x - 1}{x^5}\, dx$

   f) $\int \frac{1}{\sqrt{7x - 9}}\, dx$

5. The slope of a tangent line to a curve at the point (-1,3) is $2x^2 - 5x$. Determine the equation of this curve. Also, write the equations of the tangent and normal line to this curve at the point with abscissa 4.

RESOURCES

I. Textbooks:

Vannatta, Carnahan, Fawcett: Advanced High School Mathematics, Expanded Edition (Charles E. Merrill Publishing Co., 1965).

Rees and Sparks: Algebra and Trigonometry (McGraw-Hill, Inc., 1969).

Fisher and Ziebur: Integrated Algebra and Trigonometry, 2nd Edition (Prentice-Hall, Inc., 1967).

Vance: Modern Algebra and Trigonometry (Addison-Wesley Publishing Company, Inc., 1968).

White: Advanced Algebra (Allyn and Bacon, Inc., 1962).

Dolciani, Beckenbach, Donnelly, Jurgensen, Wooton: Modern Introductory Analysis (Houghton Mifflin Company, 1970).

BEST COPY AVAILABLE

LEARNING

ACTIVITY

PACKAGE

THE TRIGONOMETRIC FUNCTIONS

Ninety Six High School - Ninety Six, S.C. -

Algebra 124

REVIEWED BY

LAP NUMBER 49

WRITTEN BY Bill Holland

51173 3½

RATIONALE

Trigonometry is perhaps the most easily applied branch of mathematics studied on the secondary level. In short order, you will be able to easily solve problems that without trigonometry would be extremely challenging or impossible; problems dealing with subjects ranging from civil engineering to ballistics, from biology to automotive engineering.

In this LAP you will be introduced to the trigonometric functions and will learn to perform basic trigonometric analysis.

SECTION I

Behavioral Objectives

By the completion of the prescribed course of study, you will be able to:

1. Define the following functions* and solve problems relating to these definitions:

   a) sine
   b) cosine
   c) tangent
   d) cotangent
   e) secant
   f) cosecant

2. Write the value of any function of the following angles without using tables: 0^0, 30^0, 45^0, 60^0, 90^0.
3. Use tables to determine the value of a function of a given acute angle.
4. State and prove any cofunction relation and solve problems relating to these relations.
5. State and prove any reciprocal relation and solve problems relating to these relations.
6. State and prove any quotient (ratio) relation and solve problems relating to these relations.
7. State and prove any Pythagorean relation and solve problems relating to these relations.

* Unless otherwise stated, the word function in this LAP refers only to trigonometric functions.

I. READINGS:

1. Vannatta: Advanced High School Mathematics - #1 pp. 106-109, 111-112; #2 pp. 109-111; #3 pp. 113-114; #4 p. 115; #5 p. 116; #6, #7 pp. 117-119.

2. Hooper: A Modern Course in Trigonometry - #1 pp. 6-8, 14-15, 16-17, 18-19, 20-21, 24; #2 pp. 25-26; #3 pp. 9-10, 12-13, 15, 19; #4 - #7 pp. 21-22, 102-105.

3. White: Advanced Algebra - #1 pp. 18-21, 26-29; #2 p. 30; #3 ____; #4 p. 27; #5 - #7 pp. 36-39.

4. Rees: Algebra and Trigonometry - #1 pp. 141-142, 147-154; #2 pp. 155-158; #3 pp. 173-175; #4 ____; #5 - #7 pp. 160-162.

II. PROBLEMS:

1. Vannatta: Advanced High School Mathematics - #1 p. 109 ex. 1-6, p. 112 ex. 3-8; #2 p. 111 ex. 1-8; #3 pp. 114-115 ex. 1 - 18; #4 pp. 115-116 ex. 1-8; #5 p. 116 ex. 1-13; #6 - #7 p. 119 ex. 1-12.

2. Hooper: A Modern Course in Trigonometry - #1 p. 24 ex.1-12; #2 pp. 27-28 ex. 1-33 (odd numbered exercises); #3 p. 11 ex. 1-2, p. 14 ex. 1-10, p. 16 ex. 1-3 p. 17 ex. 2-6, p. 20 ex. 2-5; #4-7 p. 23 ex. 12-45 (even numbered exercises), p. 106 ex. 1-4.

3. White: Advanced Algebra - #4 - #7 p. 42 ex. 1-10.

4. Rees: Algebra and Trigonometry - #1 pp. 154 ex. 11, 13, 21, 25, 29, 32-33, 40; #2 p. 159 ex. 26-27, 29-32; #3 pp. 175-176 ex. 1-8, 13-24, 29-36; #4 ____; #5 - #7 ____.

III. AUDIO:

1. Wollensak Teaching Tape C-3711: The Sine Function

2. Wollensak Teaching Tape C-3712: The Cosine Function

3. Wollensak Teaching Tape C-3713: The Tangent Function

IV. VISUAL:

Filmstrip 514: Introductory Trigonometry

SELF-EVALUATION I

1. a) Define the following functions:

   1. tangent

   2. secant

   3. sine

   b) 1. Given that (3,4) is a point on the terminal side of angle α, find the value of the six functions of α.

   2. If $\sin \theta = \frac{2}{3}$, then what are the values of the other functions of θ?

2. Find the following without using tables:

   a. $\sin 30^0$

   b. $\tan 90^0$

   c. $\dfrac{\sec 0^0 + \cos 90^0}{\tan 30^0 \cdot \csc 45^0}$

   d. $\dfrac{\tan 60^0 + \cot 30^0}{1 - \sin 60^0 \cdot \sin 30^0}$

3. a. Evaluate the following using tables:

   1. $\sin 40^0 30' =$

   2. $\tan 53^0 13' =$

   3. $\cos 73^0 49' =$

   b. Find the following angles from the given numerical value.

   1. $\cot x = 1.4019$

   2. $\cos x = .8066$

   3. $\sin x = .7465$

SELF-EVALUATION I (Cont')

4. a. If $\sin 39^0 = .6293$, then $\cos 51^0 =$

   b. If $\tan 21^0 = .3839$, then $\cot 69^0 =$

5. a. If $\sin A = \frac{\sqrt{3}}{2}$, then $\csc A =$

   b. If $\tan B = \frac{1}{2}$, then $\cot B =$

6. a. Prove $\tan \theta = \frac{\sin \theta}{\cos \theta}$

   b. If $\cos A = .2$ and $\sin A = .5$, then $\cot A =$

7. a. Prove $\sin^2 \theta + \cos^2 \theta = 1$

   b. If $\cos A = \frac{2}{3}$, then $\sin A =$

   c. If $\csc T = 1$, then $\cot T =$

If you have mastered all the Behavioral Objectives, take the Progress Test.

ADVANCED STUDY I

1. Evaluate the following using logarithms:

   a) $\sqrt{\sin 33^{\circ}20' \tan 57^{\circ}40'}$

   b) $(\cos 39^{\circ}10')^{\frac{4}{3}} \cdot (\sin 77^{\circ}40')^{\frac{5}{8}}$

   c) $\left(\dfrac{\tan 37^{\circ}40' \cot 49^{\circ}30'}{\sin 23^{\circ}50' \cos 88^{\circ}10'}\right)^{\frac{1}{4}}$

   d) $(\sec 27^{\circ}40')^{8} \cdot (\csc 78^{\circ}50')^{9}$

SECTION II

Behavioral Objectives

By the completion of the prescribed course of study, you will be able to:

8. Define a radian and solve problems involving radians and degrees.
9. Determine the value of a positive or negative angle of any size.
10. a. Determine the values of the functions of the following quadrantal angles without using tables: 0^0, 90^0, 180^0, 270^0, 360^0.
    b. Prove and/or apply statements relating to functions of multiples of the quadrantal angles given above.
11. State and prove formulas relating to the sum or difference of two angles and solve problems relating to these formulas: e. g. $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$.
12. State and prove formulas relating to double angles and half angles and solve problems relating to these formulas: e.g. $\sin 2\theta = 2\sin\theta\cos\theta$, $\tan\frac{x}{2} = \frac{1 - \cos x}{\sin x}$.
13. State and prove formulas that transform a sum or difference of functions into a product of functions and solve problems relating to these formulas: e.g. $\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$.

I. READINGS:

1. Vannatta: Advanced High School Mathematics - #8 pps. 123-124; #9 pp. 126-130; #10 p. 131; #11 pp. 133-135; #12 pp. 136-138; #13 p. 139.

2. Hooper: A Modern Course in Trigonometry - #8 pp. 70-73; #9 pp. 36-49; #10 pp. 53-55; #11 pp. 183-186; #12 pp. 187-190; #13 pp. 192-195.

3. White: Advanced Algebra - #8 pp. 22-23; #9, #10 pp. 30-34; #11 pp. 63-69; #12 pp. 70-73; #13 ____.

4. Rees: Algebra and Trigonometry - #8 pp. 142-144; #9 pp. 168-172; #10 pp. 157-158; #11, #12 pp. 300-313; #13 ____.

II. PROBLEMS:

1. Vannatta: Advanced High School Mathematics - #8 pp. 124-125 ex. 1-6; #9 p. 130 ex. 1-12, 23-31; #10 pp. 131-132 ex. 1-5; #11 pp. 135-136 ex. 1-8; #12 p. 138 ex. 1-9; #13 pp. 139-140 ex. 1-10.

2. Hooper: A Modern Course in Trigonometry - #8 pp. 73-74 ex. 1-39; #9 p. 44 ex. 1-5, pp. 45-46 ex. 1-2, p. 48 ex. 1-24, p. 50 ex. 1-22, p. 74 ex. 56-70; #10 p. 55 ex. 1-10; #11 p. 74 ex. 40-55, pp. 184-185 ex. 1-10, 15-16, 26-29, pp. 186-187 ex. 1-10, 15-16; #12 p. 188 ex. 1, 5-9, p. 191 ex. 1-4, 6-7, 9, 11, 13; #13 p. 193 ex. 1-16, pp. 195-196 ex. 1-16, 23-26.

3. White: Advanced Algebra - #8 p. 25 ex. 1-27; #9, #10 pp. 35-36 ex. A (1,2,4), B (1-3); #11 p. 35 ex. 3 pp. 69-70 ex. 1-15; #12 p. 73 ex. 1-14; #13 ____.

4. Rees: Algebra and Trigonometry - #8 pp. 144-145 ex. 1-28; #9 p. 172 ex. 1-32; #10 pp. 158-159 ex. 1-28; #11, #12 pp. 303-304 ex. 1-8, 13-16, p. 307 ex. 1-8, 13-16, pp. 311-312 ex. 1-8, 13-16; #13 ____.

SELF-EVALUATION II

BEST COPY AVAILABLE

1. a. Define a radian.

   b. Change the following degrees to radians.

      1. 61^0 2. 153^0 3. 333^0

   c. Change the following radians to degrees.

      1. $\frac{\pi}{9}$ 2. $\frac{7\pi}{10}$ 3. 1.9

2. a. Prove: If $A > 0$, then $\cos(-A) = \cos A$.

   b. Find the value of the following:

      1. $\sin 334^0$
      2. $\tan 179^0$
      3. $\csc(-327^0)$
      4. $\sec(-100^0)$
      5. $\cot 385^0$
      6. $\cos(-450^0)$

3. a. Write the value of the following without using tables:

      1. $\sin 0 \cdot \cos 270^0 - \sec 180^0 \cdot \tan 0^0$
      2. $\cot 90^0 \cdot \sin 360^0 + \csc 270^0 \cdot \cos 0^0$

   b. Prove: $\sin(k \cdot 90^0) = 0$ if k is an even integer.

   c. For what values of k does $\cos(k \cdot 90^0) = 0$?

4. a. Prove: $\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$

   b. If $\sin x = \frac{2}{3}$ and $\cos y = \frac{4}{5}$, then $\sin(x + y) =$

   c. Find $\cos \frac{5\pi}{12}$ without using the tables.

   d. Evaluate tan ([illegible] + [illegible]) [illegible]

5. a. Prove $\tan 2x = \dfrac{2 \tan x}{1 - \tan x}$.

   b. Find $\tan 22^\circ 30'$ without using tables.

   c. If x is acute and $\cos x = \dfrac{2}{3}$, find the value of $\cos 2x$. What quadrant does the terminal side of angle 2x lie in?

6.a) Prove $\sin x + \sin y = 2 \sin \dfrac{x + y}{2} \cdot \dfrac{x - y}{2}$.

  b) Express $\cos 21^\circ - \cos 15^\circ$ as a product of functions.

  c) Express $-2 \sin 6x \sin 3x$ as a sum of functions.

7. Mark the following true or false.

_______ a) $\tan (x + y) = \tan x + \tan y$

_______ b) $2 \cos \frac{x}{2} = \cos x$

_______ c) $\tan (-x) = -\tan x$

_______ d) $\sin 2x - 2 \sin x$

_______ e) $2 \tan \frac{x}{2} = \tan x$

_______ f) $2 \left(\frac{\sin x}{2}\right) = \sin x$

_______ g) $\sin x + \sin y = \sin (x + y)$

_______ h) $\cos x = \sin (90^\circ - x)$

_______ i) $\tan 2x = 2 \tan x$

_______ j) $\cos (-x) = -\cos x$

IF YOU HAVE MASTERED THE BEHAVIORAL OBJECTIVES, TAKE THE PROGRESS TEST.

1. Describe the variation of the functions sin x, cos x, tan x, cot x, sec x, and csc x as x varies in the following manner:

   a) $0^0 \leq x \leq 90^0$

   b) $90^0 < x \leq 180^0$

   c) $180^0 < x \leq 270^0$

   d) $270^0 < x \leq 360^0$

2. Determine all values of x such that $0^0 \leq x \leq 360^0$ and for which the following relations hold:

   a) $\tan (x + y) - \tan x + \tan y$

   b) $\cos (x + y) = \cos x + \cos y$

   c) $\sin (x + y) = \sin x + \sin y$

3. Work the following problems:

   a) A plane is 2000 ft above the sea when it is 5 miles from the shore. Then it climbs steadily at an angle of 15^0 with the horizontal, flying in a straight line toward the shore. What height above sea level, to the nearest foot, will its altimeter record as it passes over the coast?

   b) To determine the width of a river, a spot directly opposite a tree on the farther bank is chosen on a straight stretch of the river. An observer then walks 50 yards along the bank and finds that the angle between the bank and the direction of the tree is 32^0. To the nearest foot, how wide is the river at the point where the tree stands?

   c) If a man 5 feet 8 inches tall casts a shadow 20 feet long, what is the angle of elevation of the sun?

SECTION III

Behavioral Objectives

By the completion of the prescribed course of study, you will be able to:

14. Verify identities and/or prove statements relating to the identities given in objectives 4 - 7 and the formulas given in objectives 11 - 13.
(a)
e.g. Develop a formula for sin 3A in terms of sin A and cos A.

(b) $$\frac{2 \cot x}{1 + \cot^2 x} = \sin 2x$$

RESOURCES III

I. READINGS:

1. Vannatta: Advanced High School Mathematics - #14 pp. 119-121, pp. 140-141.

2. Hooper: A Modern Course in Trigonometry - #14 pp. 107-108.

3. White: Advanced Algebra - #14 pp. 40-41, pp. 73-74.

4. Rees: Algebra and Trigonometry - #14 p. 162, pp. 163-165, p. 313.

5. Vance: Modern Algebra and Trigonometry - #14 pp. 124-126.

II. PROBLEMS:

1. Vannatta: Advanced High School Mathematics - #14 pp. 122-123 ex. 1-30, p. 136 ex. 9-12, p. 138 ex. 10-15, p. 140 ex. 11-15, p. 141 ex. 1-14.

2. Hooper: A Modern Course in Trigonometry - #14 pp. 108-109 ex. 1-28, p. 185 ex. 17-25, p. 187 ex. 11-16, pp. 188-189 ex. 10, 22-34, pp. 191-192 ex. 5, 8, 10, 12, 14-20, pp. 195-196 ex. 17-22, 31-32.

3. White: Advanced Algebra - #14 p. 42 ex. 11-30, p. 75 ex. 1-13.

4. Rees: Algebra and Trigonometry - #14 pp. 162-163 ex. 1-24, pp. 165-166 ex. 1-28.

5. Vance: Modern Algebra and Trigonometry - #14 pp. 126-127 ex. 7-50 (odd numbered exercises), pp. 133-134 ex. 18-26, 28-32, pp. 140-141 ex. 12-30, p. 143 ex. 17-24.

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SELF-EVALUATION III

1. Derive a formula for csc $(x + y)$ in terms of csc x, csc y, sec x, and sec y.

2. Show that $\sin(45^{0} + x) - \sin(45^{0} - x) = \sqrt{2} \sin x$.

3. Verify the following identities:

   a) $\tan x = \dfrac{1 - \cos 2x}{\sin 2x}$

   b) $\dfrac{1 + \tan^2 x}{\tan^2 x} = \csc^2 x$

   c) $\cot^2 A = \dfrac{\cos A}{\sec A - \cos A}$

   d) $\dfrac{\sin 3B - \sin B}{\cos^2 B - \sin^2 B} = 2 \sin B$

   e) $\dfrac{\sin x}{1 + \cos x} + \dfrac{\cos x}{\sin x} = \sec x$

IF YOU HAVE MASTERED ALL THE BEHAVIORAL OBJECTIVES IN THE LAP, TAKE THE LAP TEST.

ADVANCED STUDY III

1. Derive a formula for sin 5A in terms of A.

2. Verify any of the following identities:

   a) $\dfrac{\cot^3 x}{1 + \cot^2 x} + \dfrac{\tan^3 x}{1 + \tan^2 x} = \dfrac{\cos^4 x + \sin^4 x}{\sin x \cos x}$

   b) $\sin^3 x - \cos^3 x = \sin x\,(1 + \sin x \cos x) - \cos x\,(1 + \sin x \cos x)$

   c) $(\sin A + \cos A)^2 + (\sin B + \cos B)^2 = 2(1 + \sin A \cos A + \sin B \cos B)$

   d) $\dfrac{\sin x + \sin 3x + \sin 5x}{\cos x + \cos 3x + \cos 5x} = \tan 3x$

REFERENCES

I. Textbooks

1. Vannatta, Carnahan, Fawcett, Advanced High School Mathematics, expanded edition (Charles E. Merrill Publishing Co., 1965).

2. Hooper and Griswold, A Modern Course in Trigonometry, (Henry Holt, and Company, Inc., 1959).

3. White, Advanced Algebra (Allyn and Bacon, Inc., 1961).

4. Rees and Sparks, Algebra and Trigonometry, 2nd edition (McGraw-Hill, Inc., 1959).

5. Vance: Modern Algebra and Trigonometry, 2nd edition (Addison-Wesley Publishing Co., 1968).

II. Audio

1. Wollensak Teaching Tape C-3711: The Sine Function

2. Wollensak Teaching Tape C-3712: The Cosine Function

3. Wollensak Teaching Tape C-3713: Tangent Function

III. Visual

Filmstrip: Introductory Trigonometry (Colonial Films)

LEARNING ACTIVITY PACKAGE

GRAPHS AND APPLICATIONS OF THE TRIGONOMETRIC FUNCTIONS

Ninety Six High School - Ninety Six, S.C.

Trigonometry

REVIEWED BY ____________

LAP NUMBER 50

WRITTEN BY Bill Holland

51173 4

RATIONALE

When you first studied functions, you learned to do several things with them. You learned to determine their domain and range, you learned to compute $f(a)$ for any given function f, and any real variable a in the domain of f, and you learned how to sketch a graph of any given function.

In the previous LAP you were introduced to the trigonometric functions. You learned their definitions and how to find the value of any given trigonometric function evaluated at a given angle. Also, you were taught how to change trigonometric equations using proven trigonometric identities.

In this LAP you will learn how to sketch a graph of any given trigonometric function. Also, you will learn how the trigonometric functions apply to solve problems of everyday life.

Behavioral Objectives

By the completion of the prescribed course of study, you will be able to:

1. Determine the amplitude and fundamental period of any given trigonometric function.

2. Sketch a graph of the following trigonometric functions for any indicated domain:

   a) y = Asinkx
   b) y = Acoskx
   c) y = Atankx
   d) y = Acotkx
   e) y = Aseckx
   f) y = Acsckx

3. Sketch a graph of any given compound function by the addition of ordinates or the product of ordinates method.

RESOURCES

I. Readings.

1. Vanatta - Advanced High School Mathematics: #1 pps. 172-174; #2 pps. 168-171, 174-176; #3 pps. 177-179.

2. Griswold - A Modern Course in Trigonometry: #1 p. 85; #2 pps. 80-92; #3 p. 93.

3. Vance - Modern Algebra and Trigonometry: #1,2 pps. 330-331, 332-333; #3 p. 333.

4. Rees - Algebra and Trigonometry: #1 pps. 177-180; #2 pps. 180-184; #3 pps. 185-188.

5. Fisher - Integrated Algebra and Trigonometry: #1,2 pps. 143-147; #3 ___.

II. Problems.

1. Vanatta - Advanced High School Mathematics: #1 p. 174 ex. 1-12; #2 pps. 171-172 ex. 1-7, p. 176 ex. 1-9; #3 p. 179 ex. 1-8.

2. Griswold - A Modern Course in Trigonometry: #1 ___; #2 pps. 90-91 ex. 1-3; #3 p. 93 ex. 1-8.

3. Vance - Modern Algebra and Trigonometry: #1,2 p. 331 ex. 1-8; #3 p. 334 ex. all odd problems.

4. Rees - Algebra and Trigonometry: #1 ___; #2 p. 184 ex. 1-16; #3 pps. 188-189 ex. 1-12, 29-36.

5. Fisher - Integrated Algebra and Trigonometry: #1,2 p. 147 ex. 1-3.

SELF-EVALUATION I **BEST COPY AVAILABLE**

1. **Determine the amplitude and fundamental period of the following trigonometric functions whenever possible.**

a) $y = -7 \sin \frac{x}{4}$

b) $y = \frac{3}{2} \csc \frac{x}{2}$

c) $y = -\tan \frac{x}{2\pi}$

d) $y = \pi \cos \frac{3\pi x}{5}$

e) $y = \sqrt{5} \cot \frac{6x}{5}$

f) $y = \sec 7\pi x$

2. Sketch a graph of the following trigonometric functions for the indicated domain:

a) $y = 3\sin x$ from $-\pi$ to π

b) $y = -2\tan x$ from π to 2π

c) $y = \frac{1}{2}\cos 3x$ from $-\frac{\pi}{2}$ to $\frac{3\pi}{2}$

d) $y = -\sec x$ from 0 to 2π

e) $y = 2\cot(x + 2\pi)$ from $\frac{\pi}{2}$ to $\frac{5\pi}{2}$

f) $y = \csc(-x)$ from $-\frac{\pi}{4}$ to $\frac{5\pi}{4}$

3. Sketch a graph of the following compound functions:

a) $y = x - \cos x$

b) $y = 2(2x + \sin x)$

c) $y = \sin x + \sin 2x$

d) $y = 3\sin 2x - 4\cos 2x$

e) $y = 5x - 3\sin 2x$

ADVANCED STUDY

1. Sketch a graph of the following pairs of functions. Each pair of functions should be sketched on the same graph with different color pens.

   a) cosx, sinx from -2π to 2π

   b) tanx, cotx from -2π to 2π

   c) secx, cscx from -2π to 2π

   What comparisons can you deduce about each pair of functions? Write the cosine function in such a way that it will be equal to the sine function.

2. Sketch a graph of the following functions:

   a) $y = 2x^2 - 4\cos 2x$ from -2π to 2π

   b) $y = \sin^2 x + 2\cos 2x$ from -2π to 2π

   c) $y = 2\sin 2\pi x + \cos^2 2x$ from -2π to 2π

   d) $y = \csc x + \sec x$ from -2π to 2π

   d) $y = x \cdot \sin^2 2x + 2x\cos^2 2x$ from -2π to 2π

SECTION 2

Behavioral Objectives

By the completion of the prescribed course of study, you will be able to:

1. a. Define the composite function of two functions.

   b. Work problems relating to composite functions.

2. a. Define the inverse of a function.

   b. Work problems relating to the inverse of a function.

3. a. Define the following functions:

      1. inverse of sinx
      2. inverse of cosx
      3. inverse of tanx
      4. inverse of cotx
      5. inverse of secx
      6. inverse of cscx

   b. Work problems relating to the above definitions.

4. Determine principal and general values of the functions arcsinx, arccosx, and arctanx.

5. Sketch a graph of the relations arcsinx, arccosx, arctanx, arccotx, arcsecx, arccscx for any indicated domain.

6. Determine the solution set of any given linear trigonometric equation.

7. Determine the solution set of any given quadratic trigonometric equation.

8. Determine the solution set of a trigonometric equation which is neither linear nor quadratic.

RESOURCES 2

I. Readings:

1. Vannatta - Advanced High School Mathematics: #1 ___; #2 p. 100; #3 pps. 180-182; #4 pps. 183-184; #5 pps. 185-186; #6 p. 187; #7 pps. 188-189; #8 pps. 189-192.

2. Griswold - A Modern Course in Trigonometry: #1 ___; #2,3 pps. 94-96; #4 ___; #5, pps. 96-97; #6,7,8 pps. 109-112.

3. Vance - Modern Algebra and Trigonometry: #1 pps. 99-100; #2 pps. 265-267; #3,4,5 pps. 270-275; #6 ___; #7 ___; #8 ___.

4. Rees - Algebra and Trigonometry: #1 ___; #2 pps. 132-134; #3,4,5 pps. 445-449; #6,7 pps. 439-441; #8 pps. 441-443.

5. Fisher - Integrated Algebra and Trigonometry: #1 ___; #2 pps. 342-345; #3,4,5 pps. 347-355; #6,7,8 pps. 356-358.

II. Problems:

1. Vanatta - Advanced High School Mathematics: #1 ___; #2 ___; #3 pps. 182-183 ex. 1-26; #4 pps. 184-185 ex. 1-20; #5 pps. 186-187 ex. 1-4; #6 p. 188 ex. 1-10; #7 p. 189 ex. 1-12; #8 p. 192 ex. 1-19.

2. Griswold - A Modern Course in Trigonometry: #1 ___; #2,3 p. 96 ex. 1-20; #4 ___; #5 p. 97 ex. 1-8; #6,7,8 pps. 113-114 all even numberd exercises.

3. Vance - Modern Algebra and Trigonometry: #1 p. 100 ex. 1-12; #2 p. 268 ex. 1-12; #3,4,5 pps. 273-274 ex. 1-24, pps. 275-276 ex. 1-18, 36-45; #6 ___; #7 ___; #8 ___.

4. Rees - Algebra and Trigonometry: #1 ___; #2 pps. 139-140 ex. 9-20; #3,4,5 p. 450 ex. 1-24; #6,7 p. 441 ex. 1-24; #8 p. 444 ex. 1-30.

5. Fisher - Integrated Algebra and Trigonometry: #1 ___; #2 p. 346 ex. 1-3, 6; #3,4,5 p. 350 ex. 1-5, p. 355 ex. 1-4; #6,7,8 p. 358 ex. 1-5.

SELF-EVALUATION 2

BEST COPY AVAILABLE

1. a. Define the composite function of two functions.

   b. Find gof for the following combinations:

      1) $f(x) = 3x - 9$ $\quad g(x) = x^2$

      2) $f(x) = \frac{4}{x}$ $\quad g(x) = x^2 - 16$

      3) $f(x) = x^2$ $\quad g(x) = \sqrt{x}$

      4) $f(x) = x^2 - 9$ $\quad g(x) = x + 16$

2. a. Define the inverse of a function.

   b. Determine f^{-1} for the following functions and tell the domain and range of f^{-1}.

      1) $f(x) = x - 4$

      2) $f(x) = x^2 - 2$

      3) $f(x) = \frac{2x}{x - 6}$

3. a. Define the following functions:

      1) inverse of sinx

      2) arctanx

      3) $\sec^{-1}x$

   b. Find the values of the following:

      1) $x = \arccos 0 \quad 0^\circ \leq x < 360^\circ$

      2) $x = \arctan 3 \quad 0^\circ \leq x < 360^\circ$

      3) $x = \text{arcsec}\, 4 \quad 0^\circ \leq x < 360^\circ$

      4) $x = \arcsin(-.5) \quad 0^\circ \leq x < 360^\circ$

      5) $\tan(\arctan\frac{3}{4}) = x$

      6) $\cos(\text{arccot}\frac{2}{3}) = x$

      7) $\sec(\arccos\frac{4}{5}) = x$

      8) $\tan(\arcsin\frac{2}{3}) + \cot(\arccos\frac{3}{4}) = x$

4. a. Give principal values in the following exercises:

1. $\text{Arcsin}\ \frac{\sqrt{2}}{2}$ 3. $\text{Arccos}\ (\frac{\sqrt{3}}{-2})$

2. $\text{Arctan}\frac{5}{6}$ 4. Arcsin(.8829)

b. Give general values in the following exercises.

1) $\arctan\frac{1}{2}$ 3) $\arcsin(-\frac{3}{4})$

2) $\text{arcsec}\ \frac{\sqrt{3}}{2}$ 4) $\text{arccsc}(-\frac{2}{3})$

5. Sketch a graph of the following relations.

1. arcsinx $-180^\circ \leq x \leq 180^\circ$

2. arctanx $-360^\circ \leq x \leq 180^\circ$

3. arcsecx $-90^\circ \leq x \leq 90^\circ$

6. Solve the following equations for x in degrees.

a. $5\sin x + 1 = 0$

b. $\sqrt{3}\tan x + 3 = 0$

c. $5\cos x - \sqrt{5} = 0$

d. $\sin 3x - .5 = 0$

7. Solve the following equations for x in degrees.

a) $2\cos^2x - 5\cos x + 2 = 0$

b) $\sqrt{3}\tan^2x + 2\tan x - \sqrt{3} = 0$

c) $3\sin^2x - 7\sin x = 3$

d) $\cos x\cot x - \cot x = 0$

8. Solve the following equations for principal values of x.

a) $5\sin^2x + 2\csc x + 4 = 0$

b) $\cot x + 1 = \sin x$

c) $\cos x - 1 + \tan x = 0$

d) $6\sin x - 8\cos x = 5$

1. Sketch a graph of the following relations on the same set of coordinate axes within the given values of x using a different colored pen for each one.

a) arcsin2x   d) arccot2x

b) arccos2x   e) arcsec2x $-360^\circ \leq x \leq 360^\circ$

c) arctan2x   f) arccsc2x

2. Determine the inverse of the following functions and also determine a suitable domain and range so that the inverse will be a function. Then graph the inverse function.

a) $f(x) = |x|$

b) $f(x) = \sqrt[3]{x - 3}$

c) $f(x) = \dfrac{\sqrt{x^2 - 4}}{\sqrt{x^3 - 3}}$

d) does $f(x) = [x]$ have an inverse function? Justify your answer.

3. How many times will a line parallel to the x or to the y axis intersect a function if the function has an inverse? Explain.

4. If a function is increasing (i.e. if $x_1 < x_2$, then $f(x_1) < f(x_2)$) or if a function is decreasing (i.e. if $x_1 < x_2$, then $f(x_1) > f(x_2)$) the can you say its inverse is decreasing or is increasing? Justify your answer.

SECTION

Behavioral Objectives

By the completion of the prescribed course of study, you will be able to:

1. Solve word problems that require construction of right triangles and use of trigonometric functions.

2. a. State and prove the Law of Sines. $\left(\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}\right)$
   b. Work problems relating to the Law of Sines.

3. a. State and prove the Law of Cosines ($c^2 = a^2 + b^2 - 2ab\cos C$)
   b. Work problems relating to the Law of Cosines.

4. a. State and prove a formula that will determine the area of a triangle given two sides and the included angle.
   b. Find the area of a triangle given two sides and the included angle.

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RESOURCES

I. Readings.

1. Vanatta - Advanced High School Mathematics: #1 pp. 144-147; #2 pp. 150-154; #3 pp. 154-156; #4 pp. 157-158.

2. Griswold - A Modern Course In Trigonometry: #1 ___; #2 pp. 142-151; #3 pp. 152-158; #4 pp. 158-161.

3. Vance - Modern Algebra and Trigonometry: #1,2 pp. 364-368; #3 pp. 372-376; #4 ___.

4. Rees - Algebra and Trigonometry: #1 pp. 383-389; #2 pp. 393-395, 397-399; #3 pp. 399-400; #4 pp. 395-396.

5. Fisher - Integrated Algebra and Trigonometry: #1 ___; #2 pp. 166-169; #3 pp. 170-172; #4 pp. 173-176.

II. Problems.

1. Vanatta - Advanced High School Mathematics: #1 pp. 147-149 ex. 1-21; #2 p. 154 ex. 1-8; #3 p. 157 ex. 1-10; #4 p. 158 ex. 1-8.

2. Griswold - A Modern Course In Trigonometry: #1 ___; #2 p. 145 ex. 1-8, pp. 146-146 ex. 1-10, p. 150 ex. 1-10, p. 152 ex. 11-17; #3 pp. 154-155 ex. 1-10, p. 157 ex. 1-10; p. 161 ex. 6-10.
3. Vance - Modern Algebra and Trigonometry: #1,2 pp. 368-369 ex. 1-17, p. 376 ex. 1-3, 5; #3 pp. 377-378 ex. 6-9, 14-19; #4 ___.

4. Rees - Algebra and Trigonometry: #1 pp. 385-386 ex. 1-20, pp. 389-391 ex. 1-2, 5-6, 10-17, 20, 22; #2 pp. 396-397 ex. 1-22, pp. 400-401 ex. 1-12; #3 p. 401 ex. 13-20; #4 p. 401 ex. 21-28.

5. Fisher - Integrated Algebra and Trigonometry: #1 ___; #2 pp. 169-170 ex. 2-3, 5-8; #3 pp. 172-173 ex. 1-3, 5-6, 8, 10; #4 p. 176 ex. 1, 3, 7.

I. a. From a firetower 99.8 feet above the level of the ground, the angle of depression of a tree is $15^{\circ}30'$. How far is the tree from a point directly under the point of observation?

b. Two poles are on horizontal ground and a person is standing between them. He is 104 feet from one pole and finds the angle of elevation to its top to be $15^{\circ}13'$. He is 55 feet from the other pole and finds the angle of elevation to its top to be $29^{\circ}50'$. Which pole is taller and by how much?

c. A 5.3 inch chord subtends a central angle of $11^{\circ}50'$ in a circle. What is the diameter of the circle?

d. To determine the height of a tree two points A and B were located on level ground in line with the tree and the angles of elevations were measured at each point. The angle at A was $55^{\circ}10'$ and the angle at B was $105^{\circ}40'$. The distance from A to B was 320 feet. How tall is the tree?

II. a. State and prove the Law of Sines.

b. In triangle ABC, $A = 77^{\circ}24'$, $a = 344$ feet, and $c = 406$ feet. Find B, C, b.

c. An apartment building stands on the side of a ravine that has a uniformly sloped side. At a time when the sun has an angle of elevation of $55^{\circ}12'$ the shadow of the building extends down the side of the ravine. If the side of the ravine has an angle of $9^{\circ}10'$, find the length of the shadow.

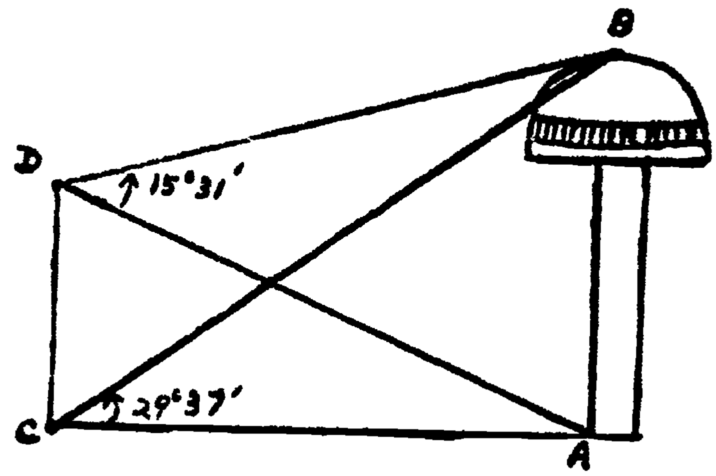
III. a. State and prove the Law of Cosines.

b. The sides of a parallelogram are 13 in. and 55 in. Find the length of each diagonal if the smaller angle is 32°.

IV. a. State and prove an area formula for triangles.

b. In triangle ABC, $a = 38.84$ cm, $c - 27.2$ cm, and $B = 62^{\circ}32'$. Find the area of triangle ABC.

1. In order to find the height of watertower AB, the angle of elevation to the top B is measured by means of a transit from point C, whose distance from the watertower is not known. Then the transit is turned through a horizontal angle of 90° and point D is located. At D the angle of elevation of the top of the watertower is again measured. Find the height of the watertower if $\angle ACB = 29°37'$ and $\angle ADB = 15°31'$, and CD = 200.0 feet.



2. Find a derivation of Hero's formula for the area of a triangle

($A = \sqrt{s(s - a)(s - b)(s - c)}$), study it, and then derive this formula for your teacher.

3. Derive the following:

   a. Law of Cosecants

   b. Law of Secants

   c. Law of Tangents

   d. Law of Cotangents

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1. Vanatta, Carnahan, Fawcett - Advanced High School Mathematics, Expanded Edition, (Charles E. Merrill Publishing Co., 1965)

2. Griswold, Hooper - A Modern Course in Trigonometry, (Henry Holt and Company, 1959)

3. Vance - Modern Algebra and Trigonometry, 2nd edition, (Addison-Wesley Publishing Company Inc., 1968)

4. Rees, Sparks - Algebra and Trigonometry, 2nd edition, (McGraw-Hill, Inc., 1969)

5. Fisher, Ziebur - Integrated Algebra and Trigonometry, 2nd edition, (Prentice-Hall, Inc., 1967)

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LEARNING

ACTIVITY

PACKAGE

SEQUENCES AND SERIES

Ninety Six High School - Ninety Six, S.C.

Algebra 124

REVIEWED BY

LAP NUMBER 51

WRITTEN BY Bill Holland

RATIONALE

Ancient Greek philosophers argued that in a race involving a hare and a tortoise the hare could not catch the tortoise if the tortoise were allowed a head start. Their reasoning was as follows: suppose the hare is ten times as fast as the tortoise and also suppose that the tortoise is allowed to start one foot head of the hare. Then when the hare travels the one foot that was between him and the hare, the tortoise has traveled one-tenth of a foot; when the hare travels the one-tenth of a foot that was between himself and the tortoise, the tortoise has traveled one-hundredth of a foot. Each time the hare travels the distance that was between himself and the tortoise, the tortoise has traveled one-tenth of that distance. Hence, the hare will never catch the tortoise.

This argument puzzled philosophers for ages. They knew that the hare would catch the tortoise, but they could not see any flaw in the above agrument.

In this LAP we will learn what sequences and series are. We will learn how to find the sum of selected finite series. Finally we will study a topic that will enable us to show directly that the hare does in fact catch the tortoise--convergent infinite series.

SECTION I

Behavioral Objectives

By the completion of the prescribed course of study, you will be able to:

1. Determine for any given arithmetic sequence
   a. the first term of the sequence
   b. the common difference of the terms of the sequence
   c. any term in the sequence which is not given
2. Determine the means of any arithmetic sequence and/or work problems relating to the means of an arithmetic sequence.
3. Derive a formula that will enable you to determine the sum of the terms in an arithmetic series and work problems relating to this formula.
4. Determine for any given geometric sequence:
   a. the first term of the sequence
   b. the common ratio of the terms of the sequence
   c. any term in the sequence which is not given
5. Determine the means of any geometric sequence and/or work problems relating to the means of a geometric sequence.
6. Derive a formula that will enable you to determine the sum of the terms in a geometric series and work problems relating to this formula.
7. Determine the amount earned when a principal is invested at a certain interest rate r and is compounded k times per year.

Resources

I. READINGS:

1. Vanatta - #1 pp. 209-211; #2 p. 211; #3 p. 212; #4 pp. 213-214; #5 pp. 214-215; #6 p. 215; #7 pp. 218-219.

2. Rees - #1 pp. 409-411; #2 pp. 422-423; #3 pp. 411-414; #4 pp. 415-416; #5 pp. 423-424; #6 pp. 416-419; #7 ____.

3. Vance - #1 pp. 173-175; #2 ____; #3 p. 175; #4 pp. 307-308; #5 ____; #6 p. 308; #7 pp. 322-324.

4. Fisher - #1 pp 324-325; #2 p. 332; #3 pp. 325-327; #4 pp. 328-329; #5 pp. 333-334; #6 pp. 329-331; #7 ____.

5. Dolciani - #1 pp. 75-80; #2 pp. 80-81; #3 p. 81; #4 pp. 83-84; #5 pp. 84-85; #6 pp. 86-87; #7 ____.

II. PROBLEMS:

1. Vanatta - #1 p. 210 ex. 1-15 (odd numbered problems), p. 211 ex. 1-10; #2 p. 212 ex. 1-7; #3 p. 213 ex. 1-5; #4 p. 213 ex. 1-5, p. 214 ex. 1-5; #5 p. 215 ex. 1-5; #6 p. 216 ex. 1-5; #7 p. 219 ex. 1-3.

2. Rees - #1 pp. 414-415 ex. 1-10, 13, 15; #2 p. 424 ex. 1-4; #3 pp. 414-415 ex. 11-12, 14, 16-28; #4 p. 421 ex. 1-10, 13, 15; #5 p. 424 ex. 5-8; #6 pp. 421-422 ex. 11-12, 14, 16-24, 29-36; #7 ____.

3. Vance - #1, #3 p. 176 ex. 1-16; #2 p. 176 ex. 17-20; #4, #6 p. 309 ex. 1-16, 21-27; #5 ex. 16-18; #7 p. 324 ex. 1-3.

4. Fisher - #1 p. 337 ex. 1; #2 p. 335 ex. 1-2; #3 pp. 327-328 ex. 2-5, 9, 11; #4, #6 p. 331 ex. 1-5; #5 p. 335 ex. 3, 4; #7 ____.

5. Dolciani - #1 p. 82 ex. 1, 4-16, 19-20; #2 p. 82 ex. 2; #3 p. 82 ex. 3, 17-18, 21-22, 24-25; #4 p. 86 ex. 1, 3, 5, 7, 13, 19-21, 24; #5 p. 86 ex. 2, 4; #6 pp. 86-87 ex. 6, 8-12, 14, 16, 22-23; #7 ____.

SELF-EVALUATION I

1. Determine the 20th term in the sequence -5, 3, 11, ...

2. Insert five arithmetic means between -2 and 4.

3. Find the sum of 31 terms of the series -2 + 1 + 4 + ...

4. Determine the 9th term of the sequence -2, 4, -8, ...

5. Insert two geometric means between 8 and 64.

6. Find the sum of 11 terms of the series $\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \ldots$

7. Find the compound amount of $1500 invested for 2 years at 5%, interest being compounded quarterly.

ADVANCED STUDY

1. a. Extend the harmonic progression $1 + \frac{5}{6} + \frac{5}{7} +$ --- to four more terms.

   b. Insert two harmonic means between ½ and $\frac{1}{3}$.

   c. Show that x, y, z is a geometric progression if y - x, 2y, y - z is a harmonic progression

2. Find the compound amount at the end of fourteen years on an original principal of $1,000 compounded continuously.

3. Does the series 1 - 1 + 1 - 1 + 1 ... have a sum. If so, what is it? Justify your answer.

4. A series of squares is drawn by connecting the midpoints of the sides of a four-inch square, then the midpoints of the sides of the second square, and so on. Find the approximate sum of the areas of the square.

SECTION II

Behavioral Objective

By the completion of the prescribed course of study, you will be able to:

8. Derive a formula that will enable you to determine the sum of an infinite geometric series and work problems relating to this formula.

9. Determine whether any given geometric series is convergent or divergent.

10. Demonstrate your understanding of the comparison test for convergence of series by using it to determine whether a given series is convergent or divergent.

11. Demonstrate your understanding of the ratio test for convergence of series by using it to determine whether a given series is convergent or divergent.

RESOURCES

I. Readings:

1. Vanatta - #8 pp. 216-217; #9 pp. 219-221; #10 pp. 221-224; #11 p. 225.
2. Rees - #8 pp. 419-421; #9 ____; #10 ____; #11 ____.
3. Vance - #8 pp. 310-312; #9 ____; #10 ____; #11 ____.
4. Fisher - #8 pp. 334-335; #9 ____; #10 ____; #11 ____.
5. Dolciani - #, #9 pp. 101-103; #10 ____; #11 ____.

II. Problems:

1. Vannatta - #8 p. 218 ex. 1-8; #9 p. 221 ex. 1-5; #10 pp. 224-225 ex. 1-8; #11 p. 226 ex. 1-6.
2. Rees - #8 p. 421 ex. 21-24; #9 ____; #10 ____; #11 ____.
3. Vance - #8 pp. 312-313 ex. 1-13, 17-18; #9 ____; #10 ____; #11 ____.
4. Fisher - #8 p. 335 ex. 6, 8; #9 ____; #10 ____; #11 ____.
5. Dolciani - #8, #9 pp. 104-106 ex. 5-22, 29-30, 37, 39-43; #10 ____; #11 ____.

SELF-EVALUATION II

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1. a. Determine the sum of the following series:

$$\frac{1}{3} + \frac{2}{9} + \frac{6}{27} + \cdots$$

   b. A ball is dropped from a height of one foot. It then hits the floor and rebounds to one-half its original height, hits the floor again and then rebounds to one-half the height it had bounced the first time, etc. Neglecting external forces, how far will the ball travel while it bouncing?

2. Are the following series convergent or divergent? Justify your answer:

   a. $\frac{1}{2} + \frac{3}{8} + \frac{6}{24} + \cdots$

   b. $\frac{1}{9} + \frac{3}{18} + \frac{9}{36} + \cdots$

   c. $\frac{1}{3} + \frac{4}{9} + \frac{16}{27} + \cdots$

3. Use the comparison test to determine whether the following series are convergent or divergent.

   a. $1 + \frac{1}{2^{1/2}} + \frac{1}{3^{1/2}} + \cdots$

   b. $\frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \cdots$

   c. $\frac{1}{2} + \frac{1}{2 \cdot 3} + \frac{1}{2 \cdot 3 \cdot 3} + \cdots$

   d. $\frac{1}{3} + \frac{1}{3 \cdot 2} + \frac{1}{3 \cdot 3} + \cdots$

4. Use the ratio test to determine whether each of the following series are convergent or divergent.

   a) $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$

   b) $\frac{3}{2 \cdot 3} + \frac{3}{2 \cdot 3 \cdot 5} + \frac{3}{2 \cdot 3 \cdot 5 \cdot 7} + \cdots$

   c) $\frac{2}{2^2} + \frac{3}{2^3} + \frac{4}{2^4} + \cdots$

   d) $\frac{1}{5} + \frac{3}{5^3} + \frac{5}{5^5} + \cdots$

ADVANCED STUDY

1. If $a_1, a_2, a_3, \ldots$ is an infinite sequence such that the series $a_1 + a_2 + a_3 + \ldots$ converges, does the series $|a_1| + |a_2| + |a_3| + \ldots$ also converge. Justify your answer.

2. If b_1, b_2, b_3 is an infinite sequence such that the series $|b_1| + |b_2| + |b_3| + \ldots$ converges, does the series $b_1 + b_2 + b_3 + \ldots$ also change? Justify your answer.

3. If $c_1, c_2, c_3, \ldots$ is an infinite sequence such that the series $c_1 + c_2 + c_3 + \ldots$ converges, does the series $(-c_1) + (-c_2) + (-c_3) + \ldots$ also converge? Justify your answer.

4. We have studied two tests that will enable us to determine whether an infinite series converges. You are to state and prove a theorem that will give another test to determine whether an infinite series converges, and then give an example of this test.

SECTION III

Behavioral Objectives

By the completion of the prescribed course of study, you will be able to:

12. Use the binomial formula to write the expansion of any given binomial and/or determine any given term of a binomial in the form $(x + y)^n$.
13. Solve problems relating to factorial notation.
14. Demonstrate your understanding of the principle of mathematical induction by proving statements using mathematical induction.
15. Demonstrate your understanding of Pascal's triangle by being able to write it for any given value of n and/or using it to write an expansion of any given binomial.

RESOURCES

I. READINGS:

1. Vanatta - #12 pp. 226-228, 229; #13 p. 228; #14 pp. 230-234; #15 p. 235.
2. Rees - #12, #13 pp. 431-433, 435; #14 pp. 426-429; #15 ____.
3. Vance - #12 pp. 287-289; #13 p. 232; #14 pp. 295-297; #15 ____.
4. Fisher - #12 pp. 285-288; #13 pp. 275-276; #14 pp. 320-323; #15 ____.
5. Dolciani - #12 ____; #13 p. 89; #14 pp. 69-73; #15 p. 90.

II. PROBLEMS:

1. Vanatta - #12 p. 228 ex. 1-9, p. 229 ex. 1-6; #13 pp. 228-229, ex. 1-22; #14 p. 233 ex. 1-8, p. 235 ex. 1-2; #15 p. 235 ex. 1-4.
2. Rees - #12, #13 pp. 435-436 ex. 1-28; #14 pp. 429-430 ex. 1-24; #15 ____.
3. Vance #12 p. 290 ex. 1-24; #13 ____; #14 pp. 297-298 ex. 1-15, 18-20; #15 ____.
4. Fisher - #12 p. 289 ex. 1-3, 6-7; #13 pp. 276-277 ex. 1-2; #14 pp. 323-324 ex. 1,5,8,9; #15 ____.
5. Dolciani - #12 ____; #13 pp. 92-93 ex. 1-4, 9-16, 21-26; #14 pp. 73-74 ex. 13-24; #15 ____.

SELF-EVALUATION III

1. a. Use the binomial formula to write the expansion of $(2x - 3y)^7$.

   b. List the fifth term of $(x - 4y)^6$.

2. a. Give a numerical value for each of the following:

   1) $\frac{2!}{3!}$ 2) $(4!)$ 3) $\frac{16!}{15!}$

   b. Simplify each of the following where a and b are positive integers, $a > b$, and $a > 1$.

   1) $\frac{a!}{(a - 1)!}$ (2) $\frac{(a - b)!}{(a - b + 1)!}$ (3) $\frac{(a - 2)!(a + 1)!}{(a - 1)!\, a!}$

3. Prove: $1 + 3 + 5 + \ldots + (2n - 1) = n^2$

4. Use Pascal's triangle to write an expansion of $(x - 3)^7$.

1. If $a_1, a_2, a_3, ---, a_n$ is a sequence, determine what is meant by $\sum_{i=1}^{n} a_i$. Then use mathematical induction to prove the following statements.

   a) If $a_1, a_2, a_3, ---, a_n$ and $b_1, b_2, b_3, ---, b_n$ are two sequences then $\sum_{i=1}^{n} (a_i+b_i) = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i$

   b) If $a_1, a_2, a_3, ---, a_n$ is a sequence and k is any fixed number, then $\sum_{i=1}^{n} ka_i = k \sum_{i=1}^{n} a_i$

2. Use mathematical induction to prove that

   a) $n^2 - 3n + 4$ is an even number

   b) $2n^3 - 3n^2 + n$ is divisible by 6

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1. Vanatta, Carnahan, Fawcett - Advanced High School Mathematics, Expanded Edition (Charles E. Merrill Publishing Co., 1965).
2. Rees, Sparks - Algebra and Trigonometry, 2nd Edition (McGraw-Hill, Inc., 1969).
3. Vance - Modern Algebra and Trigonometry, 2nd Edition (Addison-Wesley Publishing Co., Inc., 1968).
4. Fisher, Ziebur - Integrated Algebra and Trigonometry, 2nd Edition (Prentice-Hall, Inc., 1967).
5. Dolciani, Beckenbach, Donnelly, Jurgensen, Wooton - Modern Introductory Analysis (Houghton Mifflin Co., 1970).

LEARNING

ACTIVITY

PACKAGE

PERMUTATIONS, COMBINATIONS, AND PROBABILITY

Ninety Six High School
Ninety Six, S.C.

Algebra 124

REVIEWED BY

LAP NUMBER 52

WRITTEN BY Bill Holland

RATIONALE

In the theory of probability, in statistics, industry, and in the sciences, it is frequently necessary to calculate the number of ways that the elements in a set can be arranged or to determine the number of ways the elements of a set can be combined into subsets. For example, a telephone company must provide each subscriber with a unique number, and a state government has a similiar problem in assigning license numbers for vehicles. We shall be concerned with problems of this nature in this LAP and proceed from them to some of the very useful concepts of the theory of probability.

BEHAVIORAL OBJECTIVES

By the completion of the prescribed course of study, you will be able to:

1. Solve problems relating to the Fundamental Principle of Choice.
2. Determine the number of permutations of n elements of a set taken r at a time where $r \leq n$.
3. Determine the number of distinct permutations of n elements of a set taken r at a time if two or more of these elements are alike and $r \leq n$.
4. Determine the number of combinations of n elements of a set taken r at a time where $r \leq n$.
5. Solve word problems involving simple event probability in which the occurrence of any event is equally likely.
6. Solve word problems involving the probability of mutually exclusive events.
7. Solve word problems involving the probability of independent or dependent events.
8. Use the Binomial Theorem to determine the probability of events or to prepare a binomial probability table.

RESOURCES

I. Readings:

1. Vannatta: Advanced High School Mathematics - #1 pp. 238-239; #2 pp. 240-241; #3 pp. 242-243; #4 pp. 243-245; #5 pp. 246-248; #6, #7 pp. 248-251; #8 pp. 253-257.

2. Rees: Algebra and Trigonometry - #1 pp. 455-456; #2, #3 pp. 457-459; #4 pp. 461-462; #5 pp. 464-465; #6 pp. 469-470; #7 pp. 470-472; #8 ___.

3. Vance: Modern Algebra and Trigonometry - #1 pp. 278-279; #2, #3 pp. 280-282; #4 pp. 284-285; #5-#8 ____.

4. Fisher: Integrated Algebra and Trigonometry - #1 pp. 273-276; #2, #3 pp. 277-281; #4 pp. 282-284; #5 pp. 294-297; #6, #7 pp. 298-302; #8 pp. 304-308.

5. Dolciani: Modern Introductory Analysis - #1 ___; #2, #3 pp. 610-612; #4 pp. 613-615; #5 pp. 599-601; #6 pp. 602-605; #7 pp. 607-608; #8 pp. 621-623.

II. Problems:

1. Vannatta: Advanced High School Mathematics - #1 p. 239 ex. 1-5; #2 pp. 241-242 ex. 1-20; #3 p. 243 ex. 1-5; #4 pp. 245-246 ex. 1-20; #5 p. 248 ex. 1-5; #6, #7 pp. 251-253 ex. 1-12; #8 pp. 257-258 ex. 1-10.

2. Rees: Algebra and Trigonometry - #1 ___; #2, #3 pp. 459-460 ex. 1-28 (even numbered exercises); #4 pp. 462-463 ex. 1-28 (odd numbered exercises); #5 ex. 1-20; #6, #7 pp. 473-475 ex. 1-32 (even numbered exercises); #8 ___.

3. Vance: Modern Algebra and Trigonometry - #1 pp. 279-280 ex. 1-12; #2, #3 p. 283 ex. 1-20; #4 pp. 285-286 ex. 1-19; #5-#8 ___.

4. Fisher: Integrated Algebra and Trigonometry - #1 pp. 276-277 ex. 1-10; #2, #3 pp. 281-282 ex. 1, 3-10; #4 pp. 284-285 ex. 1-12; #5 pp. 297-298 ex. 1-14; #6, #7 pp. 303-304 ex. 1-12, 14; #8 pp. 308-309 ex. 1-10.

5. Dolciani: Modern Introductory Analysis - #1 ___; #2, #3 pp. 612-613 ex. 1-16; #4 pp. 616-617 ex. 1-18; #5 pp. 601-602 ex. 1-12; #6 pp. 605-606 ex. 1-16; #7 pp. 608-609 ex. 1-12; #8 pp. 623-624 ex. 1-8.

SELF-EVALUATION

1. a. If the first digit cannot be equal to zero, how many five digit numbers can be formed?

   b. A penny, nickel, dime, and quarter are flipped simultaneously. How many different ways can the coins land?

2. If a coach has fifteen football players, how many different lineups can he make (one lineup is different from another if one player is at a different position in one than he is in another)?

3. Show that $r! \cdot P(n,n-r) = P(n,n)$.

4. How many distinct permutations can be made from the letters of the word COMBINATION?

5. If a convex polygon has 10 vertices, how many diagonals can be drawn?

6. How many different committees of 6 Americans, 5 Chinese, and 7 Negroes can be selected from a group of 17 Americans, 10 Chinese, and 12 Negroes?

7. How many football games are played in the Big Eight if each team plays all the other teams once?

8. If 11 coins are tossed at the same time, what is the probability that 5 of them will come up tails?

9. In a drawer a man has 7 blue socks and 9 green socks. What is the probability he will get a pair that matches if he selects 3 socks from the drawer at random?

10. It has been determine experimentally that the success of an event is .75. What is the probability of 3 successful events in 4 trials?

ADVANCED STUDY

1. A diagonal of a polygon is a line that joins 2 non-adjacent vertices. How many diagonals does an n-sided polygon have?

2. a. Prove the relation $C(n,r+1) = \frac{n-r}{r+1} C(n,r) \quad 0 \le r < n$.

   b. Prove $C(n,r) + C(n,r-1) = C(n+1,r)$.

3. A baby has 11 letter blocks that consist of four S's, four I's, two P's, and one M. The baby places the blocks all in a row and all right side up. What is the probability he will spell the word MISSISSIPPI. If he selects 3 blocks and places them right side up in a row, what is the probability that he will spell the word IMP?

4. Three boxes each contain 5 white, 3 red, and 2 blue poker chips. One chip is selected at random from the first box and placed in the second. Two chips are then selected at random from the second box and placed in the third box. Finally 3 chips are selected at random from the third box. What is the probability that all three chips will be of a different color?

5. If you glance at your watch, what is the probability the second hand will be exactly at the 30 second mark? What is the probability it will be between the 29 second and 31 second mark?

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1. Vannatta, Carnahan, Fawcett - Advanced High School Mathematics, expanded edition (Charles E. Merrill Publishing Co., 1965).
2. Rees, Sparks - Algebra and Trigonometry, 2nd edition (McGraw-Hill Book Co., 1969).
3. Vance - Modern Algebra and Trigonometry, 2nd edition (Addison-Wesley Publishing Co., Inc., 1968).
4. Fisher, Ziebur - Integrated Algebra and Trigonometry, 2nd edition (Prentice-Hall, Inc., 1967).
5. Dolciani, Beckenbach, Donnelly, Jurgensen, Wooton - Modern Introductory Analysis (Houghton Mifflin Co., 1970).

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LEARNING

ACTIVITY

PACKAGE

DESCRIPTIVE STATISTICS

Ninety Six High School - Ninety Six, S.C.-

Algebra 124

REVIEWED BY

LAP NUMBER 53

WRITTEN BY Bill Holland

51173 3½

RATIONALE

It has often been said that one cannot be an intelligent member of society today without some understanding of statistics. In the ever-increasing complexity of modern society, we have a penchant for taking data, organizing it, and drawing whatever conclusions we may from it. When a student takes the Scholastic Aptitude Test his performance is ranked by the use of statistics. Statistics can tell us what the probability of our living another ten years is. Pollsters use statistics to tell us what we do and do not like. Statistics determines the success or failure of any television program. Some presidents have even watched polls based on random sampling to help them decide on a popular course of action to follow.

In this LAP we will be concerned with the tools of statistics. We will then use these to see how Mr. Gallup can predict that 55% of the people will vote for a certain candidate.

BEHAVIORAL OBJECTIVES

By the completion of the prescribed course of study, you will be able to:

1. Compute the arithmetic mean, mode, and median of any given set of data as indicated.
2. Compute the geometric, harmonic, or quadratic mean of any given set of data as indicated.
3. Compute the mean deviation and/or the semi-interquartile range for a given set of data.
4. Compute the standard deviation for a given set of data.
5. Construct a frequency distribution for a given set of data and from this frequency distribution,
   a. Sketch a histogram and/or frequency polygon.
   b. Compute the arithmetic mean, median, and standard deviation.
6. Demonstrate your understanding of normal distribution by sketching a graph of a normal curve using formulas or binomial expansion coefficients and/or solve problems relating to normal distribution.
7. Demonstrate your ability to interpret normally distributed data using standard deviations by being able to determine the percent of the data that falls in a certain range or the probability that a datum will fall within a certain range.
8. Compute for any random sample of data the standard error of the mean and establish a level of confidence about the sample mean.

RESOURCES

I. Readings:

1. Vannatta: Advanced High School Mathematics - #1 pp. 260-264; #2 pp. 264-265; #3 pp. 266-267; #4 pp. 267-278; #5 pp. 269-276; #6 pp. 276-280; #7 pp. 280-282; #8 pp. 283-285.

2. White: Advanced Algebra - #1 pp. 302-305; #2 ___; #3 ___; #4 pp. 313-314; #5 pp. 305-312; #6, #7 pp. 315-319; #8 pp. 319-322.

II. Problems:

1. Vannatta: Advanced High School Mathematics - #1 p. 262 ex. 1-5, p. 263 ex. 1-5, p. 264 ex. 1-3; #2 p. 265 ex. 1-5; #3 p. 267 ex. 1-3; #4 p. 268 ex. 1-3; #5 p. 271 ex. 1-2, p. 272 ex. 1-4, p. 276 ex. 1; #6 p. 278 ex. 1-2, p. 280 ex. 1-2; #7 pp. 282-283 ex. 1-4; #8 pp. 285-286 ex. 1-4.

2. White: Advanced Algebra - #1, #5 pp. 308-309 ex. 1-5, pp. 312-313 ex. 1-3; #2 ___; #3 ___; #4 - #8 pp. 322-323 ex. 1-5.

SELF-EVALUATION

1. The grades scored on a test by a twelfth grade class are as follows: 85, 89, 93, 89, 95, 74, 79, 93, 89, 100, 81, 94, 76, 89, 93, 79, 81, 87.

   a) Compute the mean grade.

   b) Compute the median grade.

   c) Determine the modal grade.

2. Compute the quadratic mean of 7.5, 8.9, 4.5, 3.7, 8.3, 5.4, 6.2, and 7.1.

3. Compute the mean deviation and semi-interquartile range of the following numbers: 32, 88, 67, 72, 85, 56, 93, 81, 48, 57, 63, 79, 89, 39.

4. Compute the standard deviation of the numbers in Ex. 3. Which measure of variability is greater?

5. Make a frequency distribution of the following weights in grams of selected materials: 3.2, 5.7, 4.3, 6.8, 2.1, 2.7, 3.5, 3.9, 2.6, 4.7, 4.1, 6.3, 5.9, 2.4, 4.9, 6.5, 4.2, 3.1, 2.9, 4.3, and 6.7. From this frequency distribution,

   a) Construct a frequency polygon.

b) Compute the mean, media, and standard deviation.

6. For the function $y = ke^{-hx^2}$, let $h = \frac{3}{2}$, $k = 11$, and construct the graph.

7. The mean of a set of normally distributed numbers is 82 and the standard deviation is 6.

   a) What percent of the numbers fall in the range from 73 to 91?

   b) What percent fall in the range from 80 to 84?

   c) What is the probability that a number selected at random from the data will be [illegible] than [illegible]?

   d) What is the probability that a number selected at random from the data will be less than 64?

8. A random sample of [illegible] students from [illegible] High School shows a mean heght of [illegible] inches and a standard deviation of 1.6 inches.

   a) Find the standard error of the mean.

   b) What is the range about the mean of the sample that will give a 90% level of confidence that the true mean will fall within it?

ADVANCED STUDY

1. Determine the scores made on the Scholastic Aptitude Test by the graduating class of 1973 at Ninety Six High School. Then

   a. Compute the mean score

   b. Compute the median score.

   c. Compute the modal score.

   d. Which one of the averages seems to give a better representation of the data? Why?

   e. Make a frequency distribution for this data.

   f. From this frequence distribution, construct a frequency polygon.

   g. Are the scores normally distributed? If they are not, give possible reasons why not.

   h. Compute the standard deviation for the scores.

   i. What is the probability that a score picked at a random will be less than 820?

   j. Find a range about the mean which will include 90% of the scores.

2. Determine the heights of 50 randomly selected individuals at Ninety Six High School.

   a. Compute the mean height.

   b. Compute the standard deviation.

   c. Compute the standard error of the mean.

   d. What is a range of heights about the sample mean of the data that will give a .95 probability that the true mean will fall within it?

   e. Select 20 more individuals at random and determine whether their heights fall within the range.

   f. Based on part (e) what conclusions can you draw about the sample mean of your data?

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1. Vannatta, Carnahan, Fawcett: Advanced High School Mathematics, Expanded Edition (Charles E. Merrill Publishing Co., Inc., 1965).
2. White: Advanced Algebra (Allyn and Bacon, Inc., 1962.)

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LEARNING

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SPECIAL THEOREMS AND FUNCTIONS

Ninety Six High School - Ninety Six, S.C. -

ALGEBRA 124

REVIEWED BY

LAP NUMBER 54

WRITTEN BY Bill Holland

51173 2½

RATIONALE

In a previous LAP you learned that the set of real numbers is a proper subset of the set of complex numbers and that any complex number can be expressed in the form a + bi where a is the real part and bi is the imaginary part. This form is referred to as the rectangular form of a complex number and is sometimes expressed as (a, b). There are many applications of complex numbers that are associated with the amplitude of the complex number. One section of this LAP will be devoted to developing a form to express complex numbers using trigonometric functions.

In a LAP on trigonometric functions, you learned how to determine the function of an angle, but you did not learn how all these values were arrived. You will study series in this LAP that will enable you to compute any of the six functions to a desired degree of accuracy.

SECTION I

BEHAVIORAL OBJECTIVES

By the completion of the prescribed course of study, you will be able to:

1. Take a given complex number and:

   a) plot it on a rectangular coordinate system
   b) compute its modulus and/or draw a line on a coordinate system to represent its modulus
   c) determine its amplitude correct to the nearest 10 minutes by use of trigonometric tables

2. Express a complex number given in rectangular form in trigonometric form and vice-versa.

3. Compute the product and quotient of any two complex numbers expressed in trigonometric form.

4. State and prove DeMoivre's Theorem and apply the statement of this theorem to determine the value of a complex number raised to the p^{th} power.

5. Apply the statement of DeMoivre's Theorem to determine the p^{th} root of any given complex number.

SECTION I

RESOURCES

I. Readings:

1. Vanatta: #1 pp. 25-28, pp. 183-184; #2 pp. 289-291; #3 pp. 291-293; #4 pp. 294-295; #5 pp. 295-297.

2. Rees: #1-#2 pp. 320-321; #3 pp. 321-322; #4 p. 324; #5 pp. 324-326.

3. White: #1-#2 pp. 83-84, pp. 88-90; #3 pp.91-92; #4-#5 pp. 93-96.

4. Vance: #1-#3 pp. 166, 353-358; #4-#5 pp. 361-363.

5. Fisher: #1-#3 pp. 186-189; #4-#5 pp. 190-194.

6. Dolciani: #1-#3 pp. 258, 493-496; #4-#5 pp. 498-501.

II. Problems:

1. Vannatta: #1 p. 38 ex. 1-2, p. 184 ex. 3, 7, 8; #2 p. 291 ex. 1-14; #3 p. 293 ex. 1-11; #4 p. 295 ex. 1-8; #5 p. 297 ex. 1-8.

2. Rees: #1-#3 pp. 323-324 ex. 1-48 (every third exercise); #4-#5 pp. 330-331 ex. 1-8, 17-36 (odd numbered exercises).

3. White: #1-#2 p. 84 ex. 1, 3, 5, 7, pp. 90-91 ex. 1-20 (even numbered exercises); #3 pp. 92-93 ex. 1-8; #4-#5 p. 96 ex. 1-18 (even).

4. Vance: #1-#3 p. 359 ex. 17-21, 23; #4-#5 pp. 363-364 ex. 1-19 (odd numbered exercises).

5. Fisher: #1-#3 pp. 189-190 ex. 2, 8-9; #4-#5 p. 194 ex. 1, 3.

6. Dolciani: #1-#3 p. 497 ex. 1-24 (even numbered exercises); #4-#5 p. 502 ex. 1-6, 11-18.

SELF-EVALUATION I

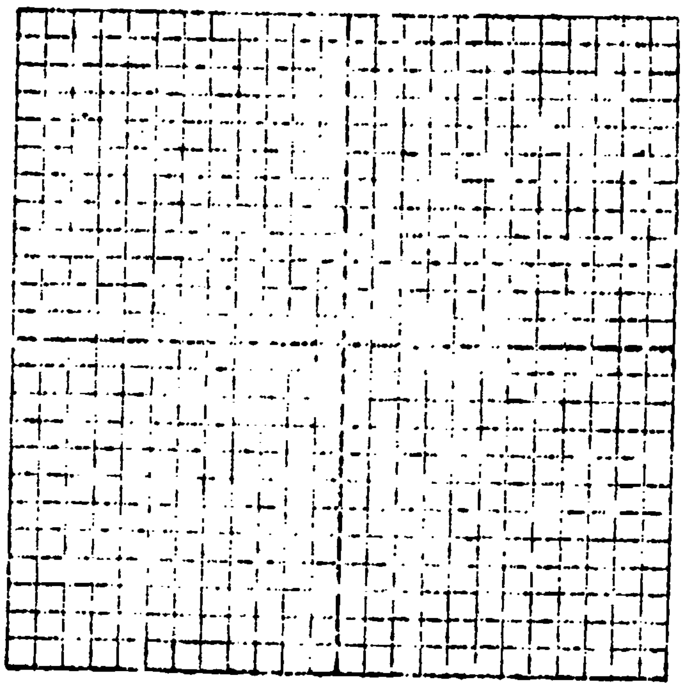
1. For the given complex numbers:

   a) plot them on the graph

   b) compute their modulus and draw a line on the graph to represent their modulus

   c) determine their amplitude.

   (1) $3 - 4i$ (2) $6i - 5$ (3) -5 (4) $8i$



2. Express the complex numbers given in problem 1 in trigonometric form.

3. Express the following complex numbers in rectangular form:

   a) $3(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6})$ (b) $2(\cos 0 + i \sin 0)$

   c) $3\sqrt{3}\,(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4})$

4. Simplify the following and express your answer in the form a + bi.

(a) $2\sqrt{3}\,(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}) + \sqrt{27}\,(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12})$

(b) $\dfrac{6\sqrt{2}\,(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}) \cdot 5\,(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4})}{7(\cos\pi + i\sin\pi) \cdot 9\sqrt{2}\,(\cos\frac{5\pi}{8} + i\sin\frac{5\pi}{8})}$

5. State and prove DeMoivre's Theorem.

6. Simplify the following and express your answer in rectangular form:

(a) $(4 + 5i)^4$ (b) $\left[7\,(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4})\right]^6$

7. Determine one root in each of the following:

(a) $(1 - i)^{\frac{1}{3}}$ (b) $(-6)^{\frac{1}{4}}$ (c) $\left[3(\cos\pi + i\sin\pi)\right]^{\frac{1}{2}}$

IF YOU HAVE MASTERED ALL THE OBJECTIVES, TAKE YOUR PROGRESS TEST.

ADVANCED STUDY I

1. a) Prove: The reciprocal of $r(\cos\theta + i\sin\theta)$ is $\frac{1}{r}(\cos\theta - i\sin\theta)$

   b) State the conditions under which the conjugate and reciprocal of a complex number are equal.

2. Suppose Z is a complex number such that $Z^6 = 1$. If $R = Z^5 + Z^4 + Z^3 + Z^2 + Z + 1$, show that $RZ = R$. What can you conclude about R?

3. Let U and V denote the points representing $u = r(\cos\theta + i\sin\theta)$ and $v = s(\cos x + i\sin x)$ where θ and x are acute angles. Let O denote the origin, A the point (1,0) and P the point that represents the product uv. Show that triangle OVP is similar to triangle OAV.

4. Apply the binomial theorem and DeMoivre's Theorem to $(\cos\theta + i\sin\theta)^3$ to prove that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$ and $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$.

5. Let x_1 and x_2 be real numbers. Prove:

   (a) $e^{ix_1} \cdot e^{ix_2} = e^{i(x_1 + x_2)}$ (b) $\frac{e^{ix_1}}{e^{ix_2}} = e^{i(x_1 - x_2)}$

SECTION II

Behavioral Objectives

By the completion of the prescribed course of study, you will be able to:

6. Evaluate $\lim_{\alpha \to 0}$ for any quotient involving α, $\sin k\alpha$, $\cos k\alpha$, and $\tan k\alpha$.
7. Determine a value for $\sin x$ and $\cos x$ to a specified number of decimal places by using the trigonometric series.
8. Use the exponential series to compute e^x correct to a specified number of decimal places.
9. Derive a formula to determine the amount of money A you have if you invest a principal P for k number of years when the interest rate is r and the interest is compounded continuously and where rk = 1, 2, 3, 4, 5 and solve problems relating to this formula.
10. Derive Euler's Formulas and solve problems relating to these formulas.

Resources

I. Reading:

1. Vannatta: #6 pp. 297-299; #7 pp. 299-301; #8 pp. 301-302; #9 pp. 303-304; #10 pp. 304-305.

II. Problems:

1. Vannatta: #6 p. 299 ex. 1-14; #7 p. 301 ex. 1-6; #8 p. 302 ex. 1-2; #9 p. 304 ex. 1-3; #10 p. 306 ex. 1-11.

SELF-EVALUATION II

1. Evaluate $\lim_{\alpha \to 0} \frac{\tan 2\alpha}{\alpha}$.

2. Compute the following correct to three decimal places using the trigonometric series:

   (a) $\sin \frac{\pi}{8}$ (b) $\cos \frac{5\pi}{12}$

3. Compute e^3 correct to three decimal places.

4. If $600 is invested at 8% compounded continuously for 12½ years, what amount of many do you have at the end of this time?

5. Derive the formula $\cos \alpha = \frac{e^{i\alpha} + e^{-i\alpha}}{2}$.

6. a) Use Euler's Formulas to show that $\cos 2x = 2\cos^2 x - 1$.

   b) Express the following in exponential form:

   $3\left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right)$

   c) $\log_e(-2) = ?$

IF YOU HAVE MASTERED THE OBJECTIVES, TAKE YOUR LAP TEST.

ADVANCE STUDY II

1. a) Compute $\sec \frac{\pi}{12}$ correct to four decimal places.

   b) Compute $\csc \frac{\pi}{8}$ correct to four decimal places.

   c) Compute e^4 correct to three decimal places.

   d) Compute $\tan \frac{5\pi}{6}$ correct to four decimal places.

   e) Compute $\cot \frac{7\pi}{8}$ correct to four decimal places.

2. Evaluate the following:

   a) $\lim\limits_{\alpha \to 0} \dfrac{\sin 2\alpha}{2\alpha}$

   b) $\lim\limits_{\alpha \to 0} \dfrac{\sin^2 \alpha + 2\alpha \cos^2 \alpha}{\alpha}$

   c) $\lim\limits_{\alpha \to 0} \dfrac{1}{\alpha \sqrt{\cot^2 \alpha + 1}}$

   d) $\lim\limits_{\alpha \to 0} \dfrac{\tan^2 \alpha}{\alpha^2}$

   e) $\lim\limits_{\alpha \to 0} \dfrac{\tan 2\alpha + \dfrac{1 - \cos \alpha}{\sin \alpha}}{\alpha}$

   f) $\lim\limits_{\alpha \to 0} \dfrac{1}{\alpha \sqrt{\csc^2 \alpha - 1}}$

3. a) Prove: $\tan \alpha = \dfrac{e^{i\alpha} - e^{-i\alpha}}{ie^{i\alpha} + ie^{-i\alpha}}$

b) Use Euler's Formulas to prove $\sin(x + y) = \sin x \cos y + \cos x \sin y$.

c) Use Euler's Formulas to prove $\sin x + \sin y = 2 \sin \frac{x + y}{2} \cos \frac{x - y}{2}$.

d) Use Euler's Formulas to prove $\cos (x - y) = \cos x \cos y + \sin x \sin y$.

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2. Rees, Sparks: Algebra and Trigonometry, 2nd Edition (McGraw-Hill, Inc., 1969).

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LEARNING

ACTIVITY

PACKAGE

MATRICES AND VECTORS

Ninety Six High School - Ninety Six, S.C.

Algebra 12

REVIEWED BY

LAP NUMBER 55

WRITTEN BY Bill Holland

11772

RATIONALE

You have solved systems of linear equations using various methods in your previous algebra courses. In this LAP, we will represent systems of linear equations as rectangular arrays of numbers. We shall study some properties of these rectangular arrays and will use these properties to solve our systems.

Also, we will study one row matrices called vectors. We will study some of their properties and see how vectors are applied to solve problems in physics.

SECTION I

Behavioral Objectives

By the completion of the prescribed course of study, you will be able to:

1. Write the dimensions of a given matrix, write the position of a given element of a matrix, and write the transpose of a given matrix.
2. Solve problems relating to addition of matrices of the appropriate order.
3. Solve problems relating to the product of a scalar and a matrix and/or of matrices of the same order.
4. Determine the additive and multiplicative inverses of a matrix whenever they exist and solve problems relating to these inverses.
5. Solve systems of linear equations using matrices with row transformations.

RESOURCES I

I. Readings:

1. Vannatta: #1 pp. 388-390; #2 pl 390; #3 pp. 391-392; #4 pp. 393-396; #5 pp. 397-400.

2. Rees: #1 p. 281; #2 pp. 281-282; #3 pp. 282-283; #4 pp. 284-287; #5 ____.

3. Vance - #1 pp. 214, 217; #2 pp. 214-215; #3 pp. 215, 217-219; #4 pp. 215, 222-224; #5 ____.

4. Berman: #1 pp. 543-544; #2 pp. 545-546; #3 pp. 546, 548-551; #4 pp. 554-556, 558-559; #5 ____.

5. Sharron: #1-#2 pp. 633-637, 655; #3 pp. 639-641, 642-644, 646-648; #4 pp. 654-656; #5 ____.

6. Fisher: #1-#4 ____; #5 pp. 246-250.

II. Problems:

1. Vannatta: #1-#2 pp. 390-391 ex. 1-8; #3 pp. 392-393 ex. 1-8; #4 pp. 396-397 ex. 1-10; #5 p. 400 ex. 1-6.

2. Rees: #1 p. 287 ex. 1-4; #2 pp. 287-288 ex. 5-8; #3 p. 288 ex. 13-20; #4 p. 289 ex. 21-32; #5 ____.

3. Vance: #1 p. 216 ex. 1-4; #2-#3 pp. 216-217 ex. 5-30, pp. 219-221 ex. 1-14, 18-29; #4 pp. 224-226 ex. 1-9, 11-13, 15-18; #5 ____.

4. Berman: #1 p. 544 ex. 1-22, p. 545 ex. 1-8; #2-#3 pp. 546-548 ex. 1-30, pp. 551-552 ex. 1-14; #4 pp. 556-557 ex. 1-19, p. 559 ex. 1-10; #5 ____.

5. Sharron: #1-#2 p. 638 ex. 1-27; #3 p. 641 ex. 1-24, p. 645 ex. 1-23, p. 648 ex. 1-28; #4 pp. 656-657 ex. 1-23; #5 ____.

6. Fisher: #1-#4 ____; #5 p. 250 ex. 1-5.

SELF-EVALUATION I

1. a) If you multiply a 3 x 5 matrix times a 5 x 2 matrix, what are the dimensions of the resultant matrix?

   b) The element a_{53} is an element of some m x n matrix where $m > 5$ and $n > 3$. What is the position of this element in this matrix?

   c) If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $A^t = ?$

   d) If $A = \begin{bmatrix} -3 & 5 & -2 \\ 6 & -4 & 3 \\ 7 & 6 & -4 \end{bmatrix}$, show that $(-A)^t = -(A^t)$.

2. Determine a single matrix equal to each of the following:

   a) $\begin{bmatrix} -2 & 4 & -7 \\ 3 & -5 & 6 \end{bmatrix} - \begin{bmatrix} -4 & -8 & 3 \\ 3 & 6 & 7 \\ 8 & -5 & 1 \end{bmatrix}$

   b) $3\begin{bmatrix} 5 & -4 \\ -2 & 6 \\ 3 & 7 \end{bmatrix} + 5 \begin{bmatrix} a & -4 \\ -3 & -9 \\ c & -b \end{bmatrix}$

3. If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, and $B = \begin{bmatrix} e & f \\ g & h \end{bmatrix}$, does $A + B = B + A$? Justify your answer.

4. a) If $A = \begin{bmatrix} -3 & 7 \\ 6 & -9 \end{bmatrix}$, then $(A^t)^2 = ?$

   b) If $A = \begin{bmatrix} 6 & -3 & 5 \\ 2 & 7 & 9 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 1 \\ 6 & 0 \\ -9 & 3 \end{bmatrix}$, then $AB = ?$

   c) If $A = \begin{bmatrix} 1 & 3 \\ -1 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 0 \\ 5 & -1 \end{bmatrix}$, does $AB = BA$? Justify your answer.

5. If X is a 2 x 3 matrix, solve for X in the following:

$$X + \begin{bmatrix} -5 & 6 & -3 \\ 2 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -3 & 4 \end{bmatrix}$$

6. Determine the inverse of the following matrices whenever possible. If it doesn't exist, tell why.

a) $\begin{bmatrix} 3 & 6 \\ -2 & 7 \end{bmatrix}$

b) $\begin{bmatrix} 2 & 5 \\ 4 & 10 \end{bmatrix}$

c) $\begin{bmatrix} 3 & -7 & 5 \\ 6 & 4 & 0 \\ -9 & 2 & 1 \end{bmatrix}$

d) $\begin{bmatrix} 3 & -5 & 7 \\ 4 & 6 & -1 \end{bmatrix}$

7. If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $ad - bc \neq 0$, show $(A^t)^{-1} = (A^{-1})^t$.

8. If X is a 2 x 1 matrix, solve the following equation:

$$\begin{bmatrix} 3 & 0 \\ 7 & -1 \end{bmatrix} X = \begin{bmatrix} 5 \\ -2 \end{bmatrix}$$

9. A man had nickels, dimes, quarters, and half dollars worth \$5.10. The nickels and dimes were worth \$1.10 and the quarters and half dollars were worth \$4.00. He had total of 32 coins. Write equations for the above informations (you should have four equations), and find the common solution of these equations using matrices. Solve the matrices using row transformations.

IF YOU HAVE MASTERED THE BEHAVIORAL OBJECTIVES, TAKE YOUR PROGRESS TEST.

ADVANCED STUDY I

1. An abelian group consists of the following:

   a) A set G

   b) An operation * which associates with each pair of elements x, y in G an element $x * y$ in G such that

      1) $x * y = y * x$

      2) $x * (y * z) = (x * y) * z$

      3) there is an element e in G such that $e * x = x * e = x$ for every x in G.

      4) to each element x in G there corresponds an element x^{-1} in G such that $x * x^{-1} = x^{-1} * x = 3$

   Prove that the set of 2 x 2 matrices forms an abelian group if the operation * is addition.

   Prove that this set does not form a group if the operation * is multiplication.

2. Suppose A is a 2 x 1 matrix and B is a 1 x 2 matrix. Prove that $C = AB$ is not invertible.

3. Let A be an n x n matrix. Prove that if A is invertible and $AB = 0$ for some n x n matrix B, then $B = 0$.

4. A man has nickels, dimes, quarters, half dollars, dollars, and five dollar bills worth $24.65. Twice the value of his dollars decreased by 5 times the value of his quarters is $1.75. The value of his dollars and five dollars is $19.00. Twice the value of his half dollars plus three times the value of his nickels decreased by six times the value of his dimes is $1.20. He has a total of 36 pieces of money. Write equations for the above information, express the equations in matrix form and solve the matrix using row transformations.

SECTION II

Behavioral Objectives

By the completion of the prescribed course of study, you will be able to:

6. Expand the determinant of any given square matrix up to order four and/or solve problems relating to determinants.

7. Solve problems and/or answer questions relating to the following properties of determinants:

   a) If the rows of a given determinant appear as columns of a second, then the two determinants are equal.
   b) If two rows or two columns of a determinant are identical, the determinant is equal to zero.
   c) If the elements of any row or column of a determinant are all zero, the value of the determinant is zero.
   d) If each element of a row (or column) is multiplied by the cofactor of the corresponding element of another row (or column) and the products are added, the sum is zero.
   e) If each element of a row or a column of a determinant is multiplied by the same constant k, the determinant is multiplied by k.
   f) If k times each element of any row or column of the determinant D is added to the corresponding element of another row or column of D, the determinant obtained is equal to D.
   g) If two adjacent rows or columns of a determinant are interchanged, the value of the determinant thus obtained is the negative of the value of the original determinant.

8. Determine the simultaneous solution of a system of linear equations by use of Cramer's rule.

RESOURCES II

I. Reading:

1. Rees: #6 pp. 263-267; #7 pp. 268-273; #8 pp. 275-279.
2. Vance: #6 pp. 226-228; #7 pp. 231-234; #8 pp. 240-242.
3. Fisher: #6 pp. 258-259; #7 pp. 261-263; #8 ____.
4. Berman: #6 pp. 552-553, 559-562; #7 pp. 563-565; #8 pp. 566-568.
5. Sharron: #6 pp. 649-652; #7 pp. 664-665; #8 pp. 657-661.

II. Problems:

1. Rees: #6 pp. 267-278 ex. 1-32; #7 pp. 274-275 ex. 1-32; #8 pp. 279-280 ex. 1-32.
2. Vance: #6 pp. 227-228 ex. 1-6, 10-15, pp. 229-231 ex. 1-4, 20; #7 pp. 234-236 ex. 1-12; #8 p. 242 ex. 1-4.
3. Fisher: #6 pp. 263-264 ex. 1, 10, 12, pp. 267-278 ex. 1-4; #7 pp. 263-264 ex. 2-3, p. 268 ex. 6-7; #8 ____.
4. Berman: #6 p. 554 ex. 1-16, p. 562 ex. 1-10; #7 pp. 565-566 ex. 1-12; #8 p. 568 ex. 1-10.
5. Sharron: #6 pp. 652-653 ex. 1-20; #7 p. 666 ex. 1-6; #8 p. 661 ex. 1-22.

SELF-EVALUATION II

1. Expand the following determinants:

a) $\begin{vmatrix} 1 & -7 \\ 10 & -2 \end{vmatrix}$ b) $\begin{vmatrix} -4 & 5 \\ 6 & -9 \end{vmatrix}$ c) $\begin{vmatrix} -7 & 5 & -3 \\ 6 & -2 & -9 \\ 4 & 3 & 1 \end{vmatrix}$

2. If $A = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$, show that $\det (A^t) = \det A$.

3. Tell why each of the following are equal:

a) $\begin{vmatrix} 6 & 7 & 9 \\ -4 & -9 & -5 \\ 3 & 6 & 4 \end{vmatrix} = - \begin{vmatrix} 6 & 7 & 9 \\ 4 & 9 & 5 \\ 3 & 6 & 4 \end{vmatrix}$

b) $\begin{vmatrix} 1 & 3 & 5 \\ 7 & -5 & -10 \\ 0 & 1 & 3 \end{vmatrix} = \begin{vmatrix} 1 & 3 & 5 \\ 9 & 1 & 0 \\ 0 & 1 & 3 \end{vmatrix}$

c) $\begin{vmatrix} 1 & -10 & 3 & 6 \\ 0 & 0 & 0 & 0 \\ -4 & 5 & -11 & 3 \\ 7 & 0 & 4 & 0 \end{vmatrix} = 0$

d) $\begin{vmatrix} 1 & 2 & 0 \\ 3 & -7 & 1 \\ 6 & -5 & 0 \end{vmatrix} = \begin{vmatrix} 1 & 3 & 6 \\ 2 & -7 & -5 \\ 0 & 1 & 0 \end{vmatrix}$

4. Expand the following:

a) $\begin{vmatrix} 10 & -30 & 50 \\ -15 & 35 & 40 \\ 16 & -32 & 64 \end{vmatrix}$ b) $\begin{vmatrix} 11 & -5 & 11 & 0 \\ -6 & 2 & -6 & 19 \\ 0 & 18 & 0 & 4 \\ -12 & 8 & -12 & 16 \end{vmatrix}$

5. Determine the solution set of the following using Cramer's Rule:

a) $\begin{aligned} x + y - z &= 3 \\ 2x + 3y + z &= 10 \\ 3x - y - 7z &= 1 \end{aligned}$ b) $\begin{aligned} 4x - y + 2z &= 5 \\ 2x + y - 3z &= 7 \\ 10x - y + z &= -z \end{aligned}$

IF YOU HAVE MASTERED THE BEHAVIORAL OBJECTIVES, TAKE YOUR PROGRESS TEST.

ADVANCED STUDY II

1. Expand the following determinant:

$$\begin{vmatrix} 3 & -1 & 5 & 8 & 3 & -5 & 0 \\ -2 & 0 & 6 & -4 & 2 & 6 & -3 \\ 5 & -9 & -3 & 7 & 1 & 4 & 7 \\ 0 & 8 & 8 & 6 & 0 & 7 & 2 \\ 1 & 6 & 2 & -3 & 1 & 6 & -5 \\ 4 & -2 & 1 & 8 & -5 & -2 & 4 \\ 5 & 4 & 0 & 9 & 6 & 1 & 3 \end{vmatrix}$$

2. If the points (3, 5), (4, -3), and (-2, 3) are the three vertices of a triangle, determine the area of this triangle using determinants.

SECTION III

Behavioral Objectives

By the completion of the prescribed course of study, you will be able to:

9. Represent a vector in standard position by means of a vector diagram when given its magnitude and direction or its coordinates.
10. Determine the algebraic and/or graphical sum of two vectors and/or solve word problems relating to the sum of two vectors.
11. Determine perpendicular components of any given vector and solve problems relating to these components.
12. Solve word problems involving addition of vectors by using the Law of Cosines.
13. Compute the magnitude (norm) of any given vector.
14. Multiply a scalar times a vector and give the resultant vector.
15. Express a vector as an ordered pair when given its initial and terminal points.
16. Compute the inner (dot) product of any two given vectors.
17. Determine whether two given vectors are perpendicular or parallel and if they are parallel, whether they run in the same or opposite directions.
18. Express any given vector as the sum of the products of a scalar and a basic (unit) vector.
19. Determine the distance between two points in 3 - d space.
20. Determine the cross product of two vectors.

RESOURCES III

I. Readings:

1. Vannatta: #9 pp. 405-407; #10 pp. 29-30, 401-402, 407; #11 pp. 30, 401, 409; #12 ____; #13 pp. 403, 408; #14 pp. 403, 408; #15 pp. 402-403, 407; #16 pp. 404, 408; #17 ____; #18 pp. 404, 409; #19 p. 408; #20 pp. 409-410.

2. Vannatta (Book Two): #9 pp. 370-372; #10 pp. 372-373; #11 pp. 373-375; #12 pp. 380-382; #13 - #20 ____.

3. Berman: #9 ____; #10 pp. 390-392; #11 pp. 394-396; #12 - #20 ____.

4. Sharron: #9 - #20 ____.

5. Vance: #9 - #11, #13 - #14 pp. 158-161; #12 ____; #15-#20 ____.

6. Dolciani: #9 ____; #10 pp. 129-135, 566; #11 pp. 157-160; #12 ____; #13 pp. 137-140, 566; #14 pp. 142-144, 566; #15 pp. 125-128; #16 pp. 148-152, 566; #17 pp. 145-146, 149-150, 154-155, 566; #18 ____; #19 pp. 569-570; #20 pp. 571-575.

II. Problems:

1. Vanatta: #9 ____; #10 p. 30 ex. 4-13, pp. 404-405 ex. 1-6; #11 pp. 30, 31 ex. 16, 18; #12 ____; #13 p. 30 ex. 1-3, p. 405 ex. 11, p. 410 ex. 1-4, 15-16; #14 pp. 30-31 ex. 14-15, 17, 19; #15 p. 405 ex. 7-10; #16 p. 405 ex. 12-14, p. 410 ex. 9-13, 17; #17 ____; #18 ____; #19 p. 410 ex. 5-8; #20 ex. 14, 18.

2. Vannatta (Book Two): #9 p. 372 ex. 1-11; #10 p. 376 ex. 1-12; #11 p. 376 ex. 13-19; #12 pp. 382-383 ex. 1-6; #13-#20 ____.

3. Berman: #9 ____; #10 pp. 393-394 ex. 1-12, 1-8; #11 pp. 396-397 ex. 1-20, ex. 1-9; #12 - #20 ____.

4. Sharron: #9 ____; #10 p. 595 ex. 9-16; #11 p. 595 ex. 1-8; #12-#20 ____.

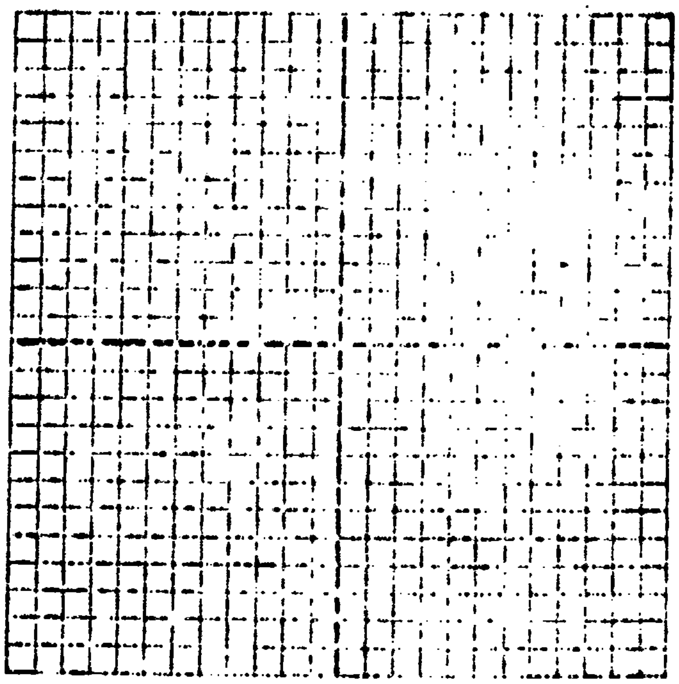
5. Vance: #9 - #11, #13 - #14 pp. 161-162 ex. 1-20; #12 ____; #15-#16 ____; #17 p. 162 ex. 26-29; #18 p. 162 ex. 23-25; #19-#20 ____.

6. Dolciani: #9 p. 570 ex. 1-8; #10 pp. 135-137 ex. 1-22, 29-34, 43-46, p. 567 ex. 1-4, 7-18; #11 pp. 160-161 ex. 13-22, 41-42, pp. 141-142 ex. 33-36; #12 ____; #13 p. 141 ex. 9-24; #14 pp. 146-147 ex. 1-12, 35-46; #15 pp. 128-129 ex. 3-32, p. 136 ex. 35, 41-42; #16 pp. 152-153 ex. 1-18, 33-45; #17 p. 147 ex. 15-20, 23-26, pp. 152-153 ex. 19-30, p. 156 ex. 1-20, p. 575 ex. 1-8; #18 p. 136 ex. 35, p. 147 ex. 27-30, p. 160-161 ex. 1-12, 29-32, 39-40; #19 pp. 570-571 ex. 9-14; #20 pp. 575-576 ex. 9-18, 23-30.

SELF-EVALUATION III

1. a) Represent the following vectors in standard position on the included diagram.

   1) $|\vec{V}| = 5, \theta = 30^0$ 2) $|\vec{V}| = 3, \theta = 145^0$ 3) $\vec{V} = (-6,3)$



   b) Draw a three dimensional diagram and sketch a graph of the given points using line segments to illustrate coordinates:

   1) (3,2,-1) 2) (-3,4,2)

2. a) If $|\vec{V_1}| = 3$, $\theta_1 = 30^0$ and $|\vec{V_2}| = 2$, $\theta = 150^0$, determine $\vec{V_1} + \vec{V_2}$ graphically.

   b) If $\vec{V_1} = (-3,4)$ and $\vec{V_2} = (-6,-5)$, then $\vec{V_1} - \vec{V_2} = ?$

   c) A plane is flying north with a speed of 500 mph. It is flying through a crosswind blowing east that has a speed of 100 mph. What is the magnitude of the plane's resultant velocity?

3. a) What are the x and y components of $\vec{V}$ if $|\vec{V}| = 7$ and $= 55^{\circ}$?

   b) If you are pulling on an object at an angle of 30° with the horizontal, what force would you have to exert to equal a force of 30 pounds directed along the horizontal?

4. Two forces of 30 and 40 pounds act on an object. If the angle between the two forces is 75°, what is the magnitude of the resultant force? Do not use the graphical method to determine the answer.

5. If $\vec{V}$ is given by the following coordinates, determine $|\vec{V}|$.

   (a) (5,-6) (b) (-3,0,5) (c) (-3,-2) (d) (-1,-3,-2)

6. If $\vec{V} = (-3,4)$ determine the following:

   a) $3\vec{V}$ (b) $-2\vec{V}$ (c) $\frac{1}{3}\vec{V}$

7. Determing $\overrightarrow{P_1P_2}$ for the following values of P_1 and P_2.

   (a) $P_1(-3,4)$, $P_2(4,-3)$ (b) $P_1(5,-6,2)$, $P_2(-1,7,-4)$

8. Determine the following inner products:

   (a) $\vec{i} \cdot \vec{j} \cdot \vec{k}$ (b) $(3,-5) \cdot (6,2)$ (c) $(-3,6,-1) \cdot (2,-5,4)$

9. Determine whether the following vectors are parallel or perpendicular or neither. If they are parallel, tell whether their direction is the same or opposite.

   (a) (3,-9), (-1,3) (b) (6,-3), (4,-2) (c) (2,3,0), (0,0,5)

   (d) (-1,3), (-2,-6) (e) (3,6), (-8,4) (d) (3,-4,6), (7,6,½)

10. Write the following vectors as the sum of the products of a scalar and a basic vector.

    (a) (-5,6) (b) (3,-2,5)

11. Determine the distance between the given points:

(a) (3,-4,6), (-7,-2,0)   (b) (-5,4,3), (6,-2,-5)

12. Determine the folloiwng cross products:

(a) (3,6) x (-5,4)   (b) (-3,-5) x (1,-2)   (c) (-5,6,3) x (7,-2,1)

IF YOU HAVE MASTERED THE BEHAVIORAL OBJECTIVES, TAKE YOUR LAP TEST.

1. a) Three men are using ropes to pull a log. The forces exerted by the men are 50, 60, and 90 pounds. If the angle between the 60 pound force and the other two is 15°, determine the magnitude of a single force that could replace them.

   b) When an object slides at a constant speed down a ramp, the force of friction opposes and is exactly equal to the component of the weight of the object parallel to the plane. If the force of friction is 68 pounds for an object with a weight of 350 pounds, what angle does the plane make with level ground?

   c) Two forces act on an object such that the angle between them is $129^{\circ}35'$. If the magnitudes of the forces are 155 and 219 pounds, what is the magnitude of the resultant force?

2. A <u>vector space</u> consists of the following:

   (a) a field F of scalars
   (b) a set V of objects called vectors
   (c) an operation, called vector addition, which associates with each pair of vectors $\vec{v}$, $\vec{u}$ in V a vector $\vec{v} + \vec{u}$, called the sum of $\vec{v}$ and $\vec{u}$ in such a way that

      1) addition is commutative, $\vec{u} + \vec{v} = \vec{v} + \vec{u}$
      2) addition is associative, $(\vec{u} + \vec{v}) + \vec{w} = \vec{u} + (\vec{v} + \vec{w})$
      3) there is a unique vector o in V, called the zero vector, such that $\vec{v} + o = \vec{v}$ for all $\vec{v}$ in V.
      4) for each vector $\vec{v}$ in V, there is a unique vector $-\vec{v}$ in V such that $\vec{v} + (-\vec{v}) = 0$.

   (d) an operation, called scalar multiplication, which associates with each scalar c in F and vector $\vec{v}$ in V a vector $c\vec{v}$ in V, called the product of c and $\vec{v}$, in such a way that:

      1) $1\vec{v} = \vec{v}$ for every $\vec{v}$ in V
      2) $(c_1c_2)\vec{v} = c_1(c_2\vec{v})$
      3) $c(\vec{v} + \vec{w}) = c\vec{v} = c\vec{w}$
      4) $(c_1 + c_2)\vec{v} = c_1\vec{v} + c_2\vec{v}$

   If F is the set of real numbers and V is the set of two dimensional vectors, prove from this definition that they form a vector space.

ADVANCED STUDY III (continued on the following page)

3. In the following, the vectors are considered to be two dimensional. Prove five:

   a) Prove: $|-\vec{v}| = |\vec{v}|$ for every vector $\vec{v}$.

   b) Prove: $|\vec{v}| \geq 0$ for every vector $\vec{v}$.

   c) Prove: $|\vec{v}| = 0$ if and only if $\vec{v} = 0$.

   d) Prove: If $\vec{v}_1 = (a,b)$ and $\vec{v} = (b,a)$, then $|\vec{v}_1| = |\vec{v}_2|$.

   e) Prove: $\vec{v}$ and $\vec{t}$ have the same direction if and only if $|\vec{v} + \vec{t}| = |\vec{v}| + |\vec{t}|$.

   f) Prove: If $\vec{v}$ and $\vec{t}$ have opposite directions, then $|\vec{v} + \vec{t}| < |\vec{v}| + |\vec{t}|$.

   g) Prove: $\vec{v}$ is perpendicular to $\vec{t}$ if and only if $|\vec{v} + \vec{t}| = |\vec{v} - \vec{t}|$.

   h) Prove: $(\vec{v} - \vec{t}) \cdot (\vec{v} + \vec{t}) = |\vec{v}|^2 - |\vec{t}|^2$.

   i) Prove: For any vectors $\vec{v}$ and $\vec{t}$, $\vec{v} \cdot \vec{t} \leq |\vec{v}|\ |\vec{t}|$.

4. a) Determine the radius and the coordinates of the sphere (if any) whose equation is given:

   1) $x^2 + y^2 + z^2 - 4x + 2y - 11 = 0$

   2) $x^2 + y^2 + z^2 - 2z + 8y - 8 = 0$

   3) $x^2 + y^2 + z^2 - 18x - 24y + 6z + 200 = 0$

   4) $x^2 + y^2 + z^2 - 2x + 4y - 6z + 14 = 0$

   b) Find an equation of the locus of all points in space equidistant from:

   1) O (o,o,o), P (10,10,2)

   2) P (1,3,4), R (-3,5,0)

5. a) Find three vectors $\vec{t}$, $\vec{v}$, and $\vec{s}$ such that:

$$(\vec{t} \times \vec{v}) \times \vec{s} \neq \vec{t} \times (\vec{v} \times \vec{s})$$

   b) Explain why the expression $(\vec{t} \cdot \vec{v}) \times \vec{s}$ has no meaning.

c) Let P, Q, and R be points in three dimensional space. Prove that the area of the triangle PQR is given by:

$$\frac{1}{2}\ (\vec{P} - \vec{Q}) \times (\vec{R} - \vec{Q})$$

d) Let P, Q, and R be points in three dimensional space. Prove that the area of the parallelogram having P, Q, and R as three consecutive vertices has area given by:

$$(P - Q) \times (R - Q)$$

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4. Dolciani, Wooton, Beckenbach, Sharron: Modern School Mathematics, Algebra and Trigonometry II (Houghton Mifflin Co., 1968).
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