DOCUMENT RESUME

ED 098 062 SE 018 191

AUTHOR Evans, Diane
TITLE Learning Activity Package, Algebra 103-104, LAPs 23-33.
INSTITUTION Ninety Six High School, S. C.
PUB DATE [73]
NOTE 190p.; See ED 069 504 for related document

EDRS PRICE MF-$0.75 HC-$9.00 PLUS POSTAGE
DESCRIPTORS *Algebra; Analytic Geometry; Curriculum; *Individualized Instruction; *Instructional Materials; Mathematics Education; Number Systems; Objectives; Probability; *Secondary School Mathematics; Teacher Developed Materials; Teaching Guides; Units of Study (Subject Fields)
IDENTIFIERS *Learning Activity Package

ABSTRACT

This set of 11 teacher-prepared Learning Activity Packages (LAPs) in intermediate algebra covers number systems; exponents and radicals; polynomials and factoring; rational expressions; coordinate geometry; relations, functions, and inequalities; quadratic equations and inequalities; Quadratic functions; systems of equations and inequalities; complex numbers; and probability. Each unit contains a rationale for the material being covered; a list of behavioral objectives; a list of resources including texts (with reading assignments and problem sets specified), tape recordings, commercial games, filmstrips, and transparencies; a problem set for student self-evaluation; suggestions for advanced study; and references. (DT)

# LEARNING ACTIVITY PACKAGE



Ninety Six High School - Ninety Six, S.C. -

## NUMBER SYSTEMS

ALGEBRA 103-104

REVIEWED BY
JR

LAP NUMBER [redacted] 1223

WRITTEN BY Evans

018 191
1/1/1
6K 88.

92172

# Instructions



I. Road Rationale

II. Read Behavioral Objectives

III. Resources

A. All work must be done in math notebook with <u>pencil only</u>.

B. Keep your notebook up to date. Your teacher may ask for it at any time (without warning).

C. Work all the Exercises in <u>at least</u> one text for each objective.

Always check your exercises (see your teacher)

IV. Self-Evaluation

A. Must be taken at completion of activities for each section.

B. Does not affect your grade in any way.

V. Advanced Study

A. To be done <u>only</u> after all previous work has been satisfactorily completed.

B. Must be approved by teacher.

VI. Progress Test and LAP Test

A. Teacher graded

B. Recycling may take place at this time if test is not satisfactory.

DO NOT LOSE YOUR LAP. If you do, you must buy another one.

## Rationale (The LAP's Purpose)

You have studied many mathematical systems in the past. When you first learned to count, you used the set of natural numbers. In your early study of arithmetic, you learned how to add, subtract, multiply, and divide. You soon found that some division and subtraction problems had no answers. To handle such situations, the set of integers was developed for closure over subtraction, and the set of rational numbers was developed for closure over division.

Irrational numbers such as $\sqrt{5}$ and $\pi$ are not included in the number sets as yet developed. The set of real numbers is the union of the rational and irrational numbers. It is the most complete number system to be developed.

In this LAP you will study the set of real numbers, its subsets, and properties.

Later, you will extend the field properties of the set of real numbers into the field of complex numbers which give meaning to numerals such as $\sqrt{-2}$.

Section I

**Behavioral Objectives**

At the completion of your prescribed course of study, you will be able to:

1. Identify the following sets: natural numbers, whole numbers, integers, rational numbers, irrational numbers, and real numbers when written

   a) in set notation

   b) as definitions

2. Determine the relationships between the sets of natural numbers, whole numbers, integers, rational numbers, irrational numbers, and real numbers.

3. Given the following sets: natural numbers, whole numbers, integers, rational numbers, irrational numbers, and real numbers, identify the properties of each:
   a) by completing the chart in Appendix I
   b) by answering True or False questions
   c) by answering multiple choice questions

4. Given any number system, state whether or not it is a field. If it is not, list the missing properties.

5. Given two or more real numbers, compute their sum, difference, product, and/or quotient.

6. Write or select from several definitions, that which defines "density" for a given number system.

## RESOURCES I

**READING AND PROBLEMS:**

**Objective 1**

Nichols, read pp. 1, 14, 21, 33, 34, Ex. 5-8 page 18; 15, 19 page 42.

Filmstrip: Rational and Irrational Numbers

**Objective 2**

Nichols, read pages 1, 4, 21, 33-34, ex. 11-13 pg. 41; 6 page 35.

**Objective 3**

Vanatta, read pages 5 - 11, 16 - 17, 29 - 37, Ex. 1-16 pages 6 - 7; 1-11 page 15; 1-10 page 26; 1-10 page 53.

Nichols, read pages 2-3, 14-17, 17-23, Ex. 1-4 page 2; 3 page 5; 3 page 17; 1, 2, 4-10 page 23; 1, 4, 5 page 35; 1, 5, 30-39 pages 37-43.

Dolciani, read pages 11-15, Ex. 1-16 page 12; 1-20 even page 16.

* Appendix I

**Objective 4**

Nichols, read pages 1, 14, 23, 24, ex. 1 page 17.

Transparencies: T510 Math, Visual Nos. 13, 14 Properties of Real Numbers

Wollensak Teaching Tapes:
C-3453 The Commutative Property
C-3454 The Associative Property
C-3455 The Distributive Property
C-3456 The Closure Property
C-3457 The Inverse Elements
C-3458 The Real Number System
C-3459 Identity Elements

**Objective 5**

Vanatta, read pages 10-11, 22-23, Ex. 5-14 pages 11-12; 1, 3, 6, 8, 9 page 25.

Nichols, read pages 30-31, Ex. 11, 12 pages 23-24; 7 page 39; 1 a, b, c, g, h, i page 32.

Dolciani, read pages 22-24, 26-29, Ex. 1-24 even page 26; 1-12 page 29.

* REQUIRED

Objective 6

Vanatta, read pages 17-18, Ex. 1, 2, 4-8 page 19; 8 page 35.

Nichols, read pages 27-28, Ex. 1, 3, 4, 5 page 28.

Dolciani, read page 256, Ex. 5-16 page 257.

ACTIVITIES:

Complete the chart in Appendix 1.

AUDIO:

For each Wollensak you use, you must secure a worksheet from your teacher, complete it, and turn in to your teacher.

(Listed on page 3 of this LAP under Objective 4.)

VIDEO:

Filmstrip: Rational and Irrational Numbers

Transparency: T-510, Math, Visual No. 8, Complex Numbers

Transparencies: T-510, Math, Visual Nos. 13, 14 Properties of Real Numbers

GAMES:

Cross Number Puzzle - Review of Fractions II

Cross Number Puzzle - Things to Know About Fractions

The Conversion Game (Bingo game on operations with real numbers)

"Propo" (Bingo game on real number system and properties)

## Self-Evaluation I

1 I. Identify the following sets of numbers. Write the name of the set in the blank.

_______________ 1. {... -2, -1, 0, 1, 2 ...}

_______________ 2. union of rationals and irrationals

_______________ 3. the natural numbers and zero

_______________ 4. the set of numbers starting with one and formed by successively adding one

_______________ 5. all numbers of the form $\frac{a}{b}$ where $a, b \in I$ and $b \neq 0$.

_______________ 6. non-repeating decimals cannot be written as $\frac{a}{b}$

2 II. True or False.

____ 7. The real numbers equal the union of the rationals and the integers.

____ 8. The real numbers are a subset of the rational numbers.

____ 9. The integers are a subset of the real numbers.

____ 10. The natural numbers are a subset of the integers.

____ 11. The intersection of the integers and the rationals is $\emptyset$.

____ 12. The rationals are a subset of the irrationals.

____ 13. The natural nos. are a subset of the whole numbers.

____ 14. The intersection of the rationals and the irrationals is the set of real numbers.

3 III. Multiple choice: for each of the following write the letter(s) for the correct answer.

____ 15. Which of these properties do not hold for the reals?

a) closure property for addition
b) associative property for multiplication
c) distributive property of multiplication over addition
d) additive identity
e) none of these

____ 16. Which of these items holds for the natural numbers?

a) multiplicative inverses
b) additive inverses
c) multiplicative identity
d) additive identity
e) none of these

BEST COPY AVAILABLE

Self-Evaluation (cont')

____ 17. Which of the following does not hold for the rationals?

a) associative property of multiplication
b) closure for addition
c) multiplicative inverses
d) commutative for multiplication
e) none of these

____ 18. Which of these holds for the irrationals?

a) multiplicative identity
b) additive identity
c) multiplicative inverses
d) closure for multiplication
e) none of these

____ 19. Which of these does not hold for the integers?

a) additive inverses
b) multiplicative inverses
c) closure for addition
d) multiplicative identity
e) none of these

____ 20. Which of these properties hold for the rationals?

a) additive inverses
b) multiplicative identity
c) additive identity
d) multiplicative inverses
e) all of these

____ 21. Which of these holds for the real numbers but does not hold for the irrational numbers?

a) multiplicative inverses
b) closure for multiplication
c) additive inverses
d) closure for addition
e) none of these

4 IV. State if each of the following is true or false. If false, state the property(s) to make the statement true.

____ 22. The set of integers is a field. ________________

____ 23. The set of real numbers is a field. ________________

____ 24. The set of irrational numbers is a field. ________________

____ 25. The set of natural numbers is a field. ________________

Self-Evaluation (cont')

5 V. **Multiple Choice:** Simplify the following choose the letter of the correct answer.

____ 26. $\frac{3}{3} + \frac{1}{2} =$ a) $\frac{3}{5}$ b) $\frac{3}{6}$ c), $\frac{7}{6}$ d) none of these

____ 27. $-3-(-6) =$ a) $-9$ b) $3$ c) $-3$ d) $-9$ e) none of these

____ 28. $\frac{-4}{3} + \frac{-2}{5} =$ a) $1$ b) $\frac{-26}{15}$ c) $\frac{-2}{15}$ d) none of these

____ 29. $\frac{3}{4} \times \frac{5}{3} =$ a) $\frac{8}{12}$ b) $\frac{8}{7}$ c) $\frac{5}{4}$ d) none of these

____ 30. $-\frac{1}{4} + \frac{3}{5} =$ a) $\frac{-5}{6}$ b) $\frac{-3}{10}$ c) $\frac{2}{10}$ d) none of these

____ 31. $(-8) + (-6) =$ a) $14$ b) $-2$ c) $-14$ d) none of these

____ 32. $-9 \times -7 =$ a) $-63$ b) $-16$ c) $63$ d) $16$ e) none of these

____ 33. $-81 \div 9 =$ a) $9$ b) $-9$ c) $3$ d) none of these

____ 34. $\frac{4}{9} =$ (a) $4.\overline{9}$ b) $.\overline{49}$ c) $.\overline{4}$ d) none of these

____ 35. $\frac{7}{25} =$ a( .28 b) .725 c) 7.25 d) none of these

6 VI. Write the definition of density.

BEST COPY AVAILABLE

# APPENDIX 1

Write an X by each property that holds for the given set.
Write a circle (0) by each property that does not hold.
Do not leave a blank.

| PROPERTIES | NATURAL | WHOLE | INTEGERS | RATIONALS | IRRATIONALS | REALS |
|---|---|---|---|---|---|---|
| Closure for + | | | | | | |
| Closure for x | | | | | | |
| Commutative for + | | | | | | |
| Commutative for x | | | | | | |
| Associative for + | | | | | | |
| Associative for x | | | | | | |
| Distributive | | | | | | |
| Additive identity | | | | | | |
| Additive inverse | | | | | | |
| Multiplicative identity | | | | | | |
| Multiplicative inverses | | | | | | |

If you have mastered your Behavioral Objectives take the LAP TEST.

## ADVANCED STUDY

1. Draw a Venn diagram showing the set of real numbers and its subsets.

2. Prove: The sum of two even numbers is an even number.

3. Prove: The product of two odd numbers is an odd number.

4. Prove: The product of an even and odd number is an even number.

5. Prove: $\forall_a \forall_b$ if $a < b$, then $a < \frac{2a+b}{3} < b$ .

6. Find a number for which $n^2-n+41$ is not a prime number.

7. Prove $\sqrt{2}$ irrational.

## References

1. Vanatta, Glen C. and Goodwin, A. Wilson, Algebra Two, A Modern Course, Charles E. Merrill Publishing Co., 1966.

2. Dolciani, Mary P., Berman, Simon L., Wooton, William, Modern Algebra Book Two,

3. Nichols, Eugene D., Modern Intermediate Algebra, Holt, Rinehart and Winston, Inc., 1965.

4. Transparency T-510 Visual No. 8, Comples Numbers

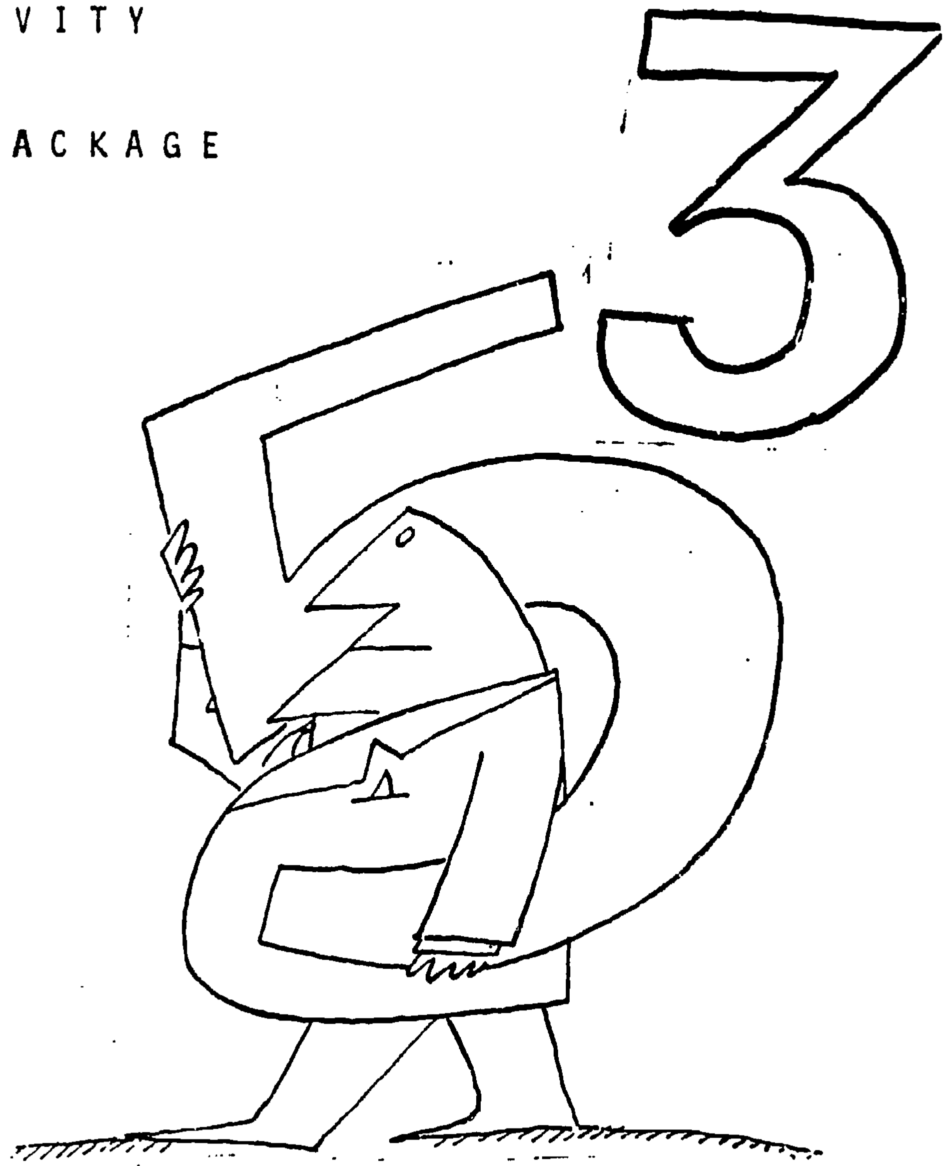
5. Transparencies T-510 Visuals 13,14 Properties of Real Numbers

6. Audio Tapes

   Wollensak Teaching Tapes: C-3453
   C-3454
   C-3455
   C-3456
   C-3457
   C-3458
   C-3459

**L**EARNING
**A**CTIVITY
**P**ACKAGE

# EXPONENTS AND RADICALS



Ninety Six High School
Ninety Six, S.C.

Algebra 103-104

REVIEWED BY

LAP NUMBER 24

WRITTEN BY Diane Evans

RATIONALE (The LAP's Purpose)

Have you ever wondered why exponents are added when powers with the same base are multiplied? Why do we subtract exponents when dividing? You may have learned how to manipulate exponents and radicals, but do you really understand what you are doing? If some of these rules have been forgotten, we will not only recall them in this LAP, but will also learn why we operate with them as we do.

We will examine operations with exponents and radicals more formally than in previous studies. Some of the topics covered will be the laws of exponents, square root, rational number exponents, radical equations, and scientific notation. Upon completion of this LAP, you should have a good foundation for future studies in mathematics.

In later LAPS, concepts involving exponents and radicals will be extended to such important topics as logarithms, rational expressions, quadratic functions, and complex numbers. Exponents and radicals are the building blocks for many topics in mathematics and in the applications of mathematical concepts to scientific studies.

Behavioral Objectives

**Upon completion of your prescribed course of study, you will be able to:**

1. Identify and apply any of the following in <u>simplifying expressions*</u> involving integral exponents.

    a. Definition of Integral Exponent

    b. Definition of Zero Exponent

    c. Definition of Negative Exponent

    d. Product of Powers Property

    e. Power of a Power Property

    f. Power of a Product Property

    g. Power of a Quotient Property

    h. Quotient of Powers Property

    i. Negative Exponent Property

    j. Order of Operations Agreements

2. Given the expression $\sqrt[n]{x}$ , identify:

    a. the radical
    b. the radical sign
    c. the radicand
    d. the index

3. Given any irrational number, state and/or identify its approximate square root.
4. Given any number, compute its square root by using the division method.
5. Given any expression involving radicals, write the <u>simplest form*</u> of any of the following

    a. any radical
    b. a product or quotient of radicals
    c. a sum or difference of radicals
    d. a product or quotient of sums or differences of radicals
    e. an expression having a radical as its denominator

6. Simplify radical expressions for which the radicand is a rational number.

*NOTE: To simplify a radical or exponential expression is to write the expression so that.

1. there is no radical in the denominator
2. each exponent of the radicand is a natural number less than the inde
3. each exponent is written as a positive exponent
4. no real number is expressed in exponential notation
5. there are no unnecessary parenthesis

APPENDIX I Algebra 103-104 LAP 24

## Part I (Objective 3)

Each number below is irrational. Replace each pair of question marks with a pair of consecutive integers to make a true statement. They will show the two integers closest to the irrational numbers.

*Example.* $? < \sqrt{20} < ?$

20 is between $4 \times 4$ and $5 \times 5$. Hence, $4 < \sqrt{20} < 5$.

1. $? < \sqrt{12} < ?$
2. $? < \sqrt{50} < ?$
3. $? < \sqrt{103} < ?$
4. $? < \sqrt[4]{10} < ?$
5. $? < \sqrt{29} < ?$
6. $? < \sqrt{79} < ?$
7. $? < \sqrt[3]{10} < ?$
8. $? < \sqrt[5]{30} < ?$

## Part II (Objective 4)

Below is an example of a square root computation. Use this example to find the square root of each of the following. (See the teacher for more explanation)

Example: Find $\sqrt{24384}$

```
                  1    5     6    .1    5
                √ 2   43    84   .00   00
20                1
25              ----------------------
300               1   43
306               1   25
3120            ----------------------
3121                  18    84
31220                 18    36
21225           ----------------------
                            48    00
                            31    21
                ----------------------------
                            16    79    00
                            15    61    25
                            ----------------
```

Thus

$\sqrt{24384} \approx 156.2$

Exercise:
Compute the following:

1) $\sqrt{2138}$
2) $\sqrt{39}$
3) $\sqrt{1841}$
4) $\sqrt{16291}$
5) $\sqrt{46852}$

Resources

Objective 1

Vanatta, read pp. 8, 243-247, Ex. 1-24 page 244; 1-10 page 247: 24-28 page 27.

Dolciani, read pages 117-118, 153-154, 29; Ex. 1-28 even oral page 120, 1-20 odd written pages 120-121; 1-18 even page 154; 17-24 page 156.

Nichols, read pages 44-45, Ex. 1-3, 5-7 pages 45-47.

Payne, read pages 57-58, 72-76, 348-345 353-354; Ex. 1-48, even pages 58-59; 1-12 page 73; 1-35 even pages 76-77; 1-9 page 349; 1-50 even page 350; 1-21 even pages 354-355.

Pearson, read pages 390-395, Ex. 1-8 pages 391-392; 1-4 page 395.

<u>Introduction to Exponents (Programmed)</u> #1A frames 1-14, 22-26, 39: #1B frames 105-177: #1d frames 80-96: #1e 118-128: #1h frames 142-152: #1i frames 178-196: #1g frames 197-204.

Objective 2

Nichols, read pages 47-48, Ex. 4, page 50.

Objective 3

Payne, read pages 5,6; Ex. 19-20 page 6 and Appendix I, part I.

Objective 4

Appendix I, part II.

Objective 5

Vanatta, read pages 248-250, 252-254, 256-257; Ex. 1-9 at top and bottom p. 251; 1-8, 10-15 top page 255; 1-9, 12-16 bottom pages 255-256; 1-18 odd page 253; 20,21, 25-27 top p. 255; 1-8 page 257.

Dolciani, read pages 258-260, 263-266; Ex. 1-14 even page 261; 15-40 odd page 261; 1-12, 16, 19 page 264; 1-12, 36 pages 266-267.

Nichols, read pages 47-48, Ex. 1-30 page 48.

Payne, read pages 5-10, 12-15, 17-18; Ex. 1-18 page 6; 1-36 even page 9; 21-48 even, 61-70, 79-82 page 11; 1-26 even, 36-55 even page 13; 1-20 odd, 23-28, 39-42 pages 15-16; 31-45 odd pages 18-19.

Objective 6

Vanatta, read pages 248-250, Ex. 13, 14, 15, 17, 18, 21 top page 251; 10, 13-17 bottom page 251.

Payne, read page 17, Ex. 9-24 page 18.

## Self-Evaluation I

1 I. Give a quanified statement for each of the following.

1. The defintion of the zero exponent.
2. The definition of the negative exponent.
3. Product of powers property.
4. Powers of a power property.
5. Power of a product property.
6. Power of a quotient property.
7. Quotient of powers property.

II. Write in simplest form without negative exponents:

8. $\left(\dfrac{-2s^2}{3r^2s^4}\right)^{-2}$

9. $\dfrac{a^{-2}\cdot b^3}{a^5\cdot b^{-3}} =$

10. $2 \cdot 3 + 7 \cdot 4 \div 2 - 9 =$

11. $\dfrac{(x^2y)^3}{x^3y^2}$

12. $\dfrac{(6a)^0}{6a^0}$

13. $\dfrac{b^{-2}}{a^{-5}c^{-6}}$

14. $\dfrac{(2x^2y^3)^4}{8x^8y^{14}}$

2 III. For $\sqrt[n]{x}$, complete the following.

15. $\sqrt[n]{x}$ is called ________________

16. $\sqrt{\phantom{x}}$ is called ________________

17. For $\sqrt[n]{x}$, the x is called ________________

18. For $\sqrt[n]{x}$, the n is called ________________

5 **IV. Write the simplest name for each of the following.**

19. $\sqrt[3]{54a^3b^2c^4} =$

20. $\sqrt{128}$

21. $\sqrt[3]{50} \cdot \sqrt[3]{5}$

22. $\sqrt[4]{a^3b^2} \cdot \sqrt[4]{a^3b} =$

23. $10\sqrt{40} \div 5\sqrt{10} =$

24. $\sqrt[5]{64x^7} + \sqrt{2x^4} =$

25. $\sqrt{48} + \sqrt{12} =$

26. $3\sqrt{18} + \sqrt{200} =$

27. $\sqrt[3]{54} - \sqrt[3]{16} =$

28. $5\sqrt{27} - \sqrt{12} =$

29. $\dfrac{1 + \sqrt{2}}{1 - \sqrt{2}} =$

30. $\dfrac{\sqrt{5} + \sqrt{3}}{\sqrt{5} - \sqrt{3}}$

6 **V. Simplify the following.**

31. $\sqrt[3]{\dfrac{8}{216}}$

32. $\sqrt{\dfrac{18}{49}}$

33. $\sqrt{\dfrac{3}{2}}$

34. $\sqrt[3]{\dfrac{8}{2a^2}}$

35. $\sqrt[5]{\dfrac{3x^6}{32}}$

3 VI. Give the approximate square root of each of the following.

36. $\sqrt{20}$ 37. $\sqrt{138}$ 38. $\sqrt{70}$

4 VII. Compute the square root of the following.

39. $\sqrt{2,834}$ 40. $\sqrt{12,348}$

IF YOU HAVE MASTERED ALL THE BEHAVIORAL OBJECTIVES, TAKE THE PROGRESS TEST.

## SECTION 2

### Behavioral Objectives

Upon completing your prescribed course of study, you will be able to:

7. Given a radical expression, write the simplest equivalent expression using fractional exponents.

8. Given an expression with fractional exponents, write the simplest equivalent radical expression.

9. Given a real number raised to the power of any rational number, write the simplest name for the number.

10. Given an expression of the form $\sqrt[n]{b^m}$, write it in simplest form using the theorem, $\sqrt[n]{b^m} = (\sqrt[n]{b})^m$.

11. Given any expression involving rational exponents, simplify indices, write their product in simplest radical form.

12. Given two or more radical expressions with different indices, write their product in simplest radical form.

13. Given a radical equation, find its solution set.

14. Given a real number, write it in scientific notation.

Resources



Objective 7

Dolciani, read pages 333-334, Ex. 1-6 page 335.
Payne, read pages 355-356, Ex. 49-62 page 357
Nichols, read page 49, Ex. 7 page 50.

Objective 8

Vanatta, read pages 245-246, Ex. 25-30 page 248.
Payne, read pages 355-356, Ex. 33-44 page 357.
Nichols, read page 49, Ex. 8 page 51.

Objective 9

Vanatta, read pages 245-246, Ex. 22-30 page 247; 7-15, 24 page 248.
Dolciani, read pages 334-335, Ex. 3-10 page 334; 15-20 page 334.
Payne, read pages 355-356, Ex. 1-20 even page 357.

Objective 10

Payne, read page 356, Ex. 12, 15, 19, 20 page 357.
Introduction to Exponents frames 262-271

Objective 11

Vanatta, read pages 245-246; 13-21 page 247.
Wooton, 41-46 page 401
Pearson, read page 405, Ex. 6-12 page 406
Nichols, read pages 49-50, Ex. 13 page 51

Objective 12

Vanatta, read page 254, Ex. 3, 9, 16, 17, 18 top page 255;
10, 17-21, 23 page 256.
Pearson, read pages 396-397, Ex. 1-5 page 398.
Wooton, read pages 399-400, Ex. 7-10 page 401

Objective 13

Vanatta, read pages 258-260, Ex. 1-9 page 260
Nichols, read pages 51-52, Ex. 1-5 pages 52-53
Wooton, read pages 402-403, 334-336, Ex. 27-36 page 332;
1-36 pages 336-337.

Objective 14

Nichols, read page 53, Ex. 1,2,3, page 54.
Wooton, read pages 412-413, Ex. 1-38 pages 413-414.
Payne, read page 351, Ex. 1-16 page 352.
Introduction to Exponents frames 51-79

## Self-Evaluation II

7 I. For each of the following write the simplest equivalent expression using fractional exponents.

______________ 1. $\sqrt{x}$

______________ 2. $\sqrt[3]{m^2n}$

______________ 3. $\sqrt[4]{y^5}$

______________ 4. $\sqrt[3]{2x^2y^3}$

8 II. For each of the following write the simplest equivalent expression using radicals.

______________ 5. $(4ab)^{\frac{1}{3}}$

______________ 6. $7^{\frac{2}{5}}$

______________ 7. $(3x)^{\frac{1}{2}}$

______________ 8. $a^{\frac{3}{4}}$

9 III. Write the simplest name for the following.

______________ 9. $25^{\frac{3}{2}}$

______________ 10. $16^{\frac{1}{4}}\ 8^{\frac{1}{3}}\ 9^{\frac{1}{2}}$

______________ 11. $\dfrac{5^0\ 16^{\frac{3}{4}}}{8^{\frac{1}{3}}}$

______________ 12. $16^{-\frac{3}{2}}$

______________ 13. $81^{\frac{3}{4}}$

10 IV. Write the following in simplest form.

______________ 14. $\sqrt[3]{27^2}$

__________ 15. $\sqrt[4]{81^2}$

__________ 16. $\sqrt[3]{8^4}$

__________ 17. $\sqrt[5]{32^4}$

11 V. Write the following in simplest form.

__________ 18. $a^{\frac{2}{3}}\ a^{\frac{3}{4}}$

__________ 19. $y^{\frac{1}{4}} + y^{-\frac{3}{4}}$

__________ 20. $(64x^4)^{\frac{1}{8}}$

__________ 21. $(-8x^6)^{\frac{1}{3}}$

__________ 22. $x^{\frac{2}{3}}y^{\frac{2}{3}}$

12 VI. Write the following in simplest radical form.

__________ 23. $(5\sqrt{2})(2\sqrt[3]{2})$

__________ 24. $(4\sqrt[4]{2})(\sqrt[6]{4})$

__________ 25. $\sqrt{2} \cdot \sqrt[3]{3}$

__________ 26. $\sqrt[3]{2} \cdot \sqrt[4]{2}$

13 VII. Solve the following radical equations.

__________ 27. $\sqrt{x} - 3 = 1$

__________ 28. $4\sqrt{x} + 1 = 25$

__________ 29. $\sqrt{2x + 1} = \sqrt{4x - 23}$

_______________ 30. $9 - \sqrt{x + 2} = 5$

_______________ 31. $3\sqrt{x} - 1 = \sqrt{x} + 1$

14 VIII. Write each of the following in scientific notation.

_______________ 32. 2.61

_______________ 33. 2,000,600

_______________ 34. .002712

_______________ 35. $30.61 \times 10^{-2}$

_______________ 36. $.00027 \times 10^{-6}$

IF YOU HAVE MASTERED ALL THE BEHAVIORAL OBJECTIVES, TAKE THE LAP TEST.

BEST COPY AVAILABLE

## ADVANCED STUDY

I. The following are in depth problems. To get advanced study credit, you must do at least one set and 80%of that set.

1. Dolciani, Modern Algebra. p. 156, numbers 29-38.
2. Dolciani, Modern Algebra. p. 261, numbers 43-50.
3. Dolciani, Modern Algebra. p. 265, numbers 21,22,24
   p. 267, numbers 27-30
4. Vanatta, Algebra Two. p. 252, numbers 23-30.
5. Vanatta, Algebra Two. p. 255, numbers 28-30
   p. 256, number 24
   p. 257, numbers 11,12
6. Vanatta, Algebra Two. p. 260, numbers 13-15
   p. 262, numbers 21,22,39,40

II. Develop a game involving exponents.

III. Develop a set of rules to adapt the Equation game to operating with exponents.

IV. Can YOU do this??

The earth gravitational pull on an object is directly proportional to its mass (m) and inversely proportioned to the square of the distance of the object from the center of the earth ($d^{-2}$)

Mathematically this may be written:

$$F = kmd^{-2}$$

If an object experiences a force (weight) of 120 lbs. at the earth's surface ($4 \times 10^3$ miles from the center of the earth), what pull or force would the same object experience at $8 \times 10^3$ miles from the center of the earth.

V. EXTRA FOR EXPERTS - Dolciani, Algebra One, pp. 276-278.

VI. Modern School Mathematics, p. 332 nos. 21-26, page 234 nos. 29-36.

REFERENCES

Vanatta (Abbreviation)
Vanatta, Glen D., and Goodwin, Wilson A., Algebra Two, A Modern Course, Charles E. Merrill Publishing Co., 1966.

Dolciani (abbreviation)
Dolciani, Mary P., Berman, Simon L., and Wooton, William, Modern Algebra and Trigonometry, Book Two, Houghton Mifflin Co., 1965.

Nichols (abbreviation)
Nichols, Eugene D., Modern Intermediate Algebra, Holt, Rinehart and Winston, Inc., 1965.

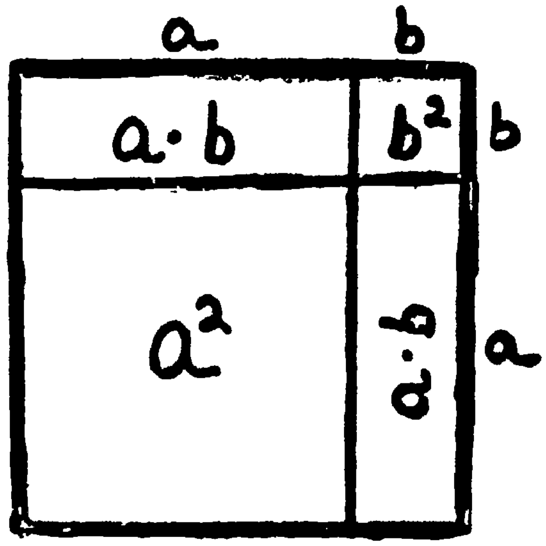
Pearson (abbreviation)
Pearson, Helen R. and Allen, Frank B., Modern Algebra, A Logical Approach, Book Two, Ginn and Company, 1966.

Payne (abbreviation)
Payne, Joseph N., Zamboni, Floyd F., Lankford, Jr., Frances G., Algebra Two, Harcourt, Brace and World, 1969.

Wooton (abbreviation)
Dolciani, Mary P., Wooton, William, Beckenback, Edwin F., Sharron, Sidney, Modern School Mathematics, Algebra 2, Houghton Mifflin Company, 1968.

Odom, Mary Margaret, Introduction to Exponents, A Programmed Unit, Holt, Rinehart and Winston, Inc., 1964.

# LEARNING ACTIVITY PACKAGE




$$(a+b)^2 = a^2 + 2ab + b^2$$

## POLYNOMIALS AND FACTORING

Ninety Six High School - Ninety Six, S.C.

Algebra 104

REVIEWED BY J. Ritchie

LAP NUMBER 1625

WRITTEN BY Diane Evans

## RATIONALE (The LAP's Purpose)

We will now extend our concept of a mathematical system to include the set of polynomials. You will learn how to add, subtract, multiply, divide, and factor polynomials. The field properties are used extensively in applying these operations on polynomials.

You probably will recognize many of these skills from your previous work in Algebra. They are presented again only in more depth because of thier importance when working with rational expressions, polynomial functions, and solving equations which will be developed in your future study.

UNIT I

Behavioral Objectives

After completion of this unit of study, you will be able to:

1. Given an algebraic expression, state whether or not it is a polynomial.*

2. Given a polynomial:

   a. State the degree of the polynomial.

   b. State if it is a polynomial over the integers, rationals or reals. *

   c. State if it is monomial, binomial, trinomial or perfect square trinomial.

   d. State if it is in two or more variables.

3. Given a polynomial and replacements for the variables, determine the value of the polynomial.

4. Given any two polynomials, name the polynomial in its simplified form which represents their:

   a. sum

   b. difference

   c. product **

   d. quotient (where the polynomial of higher degree is divided by the polynomial of lesser degree).

5. Given any polynomial, name its additive inverse.

6. Given any polynomial, factor it over the integers if possible. **

7. Given any polynomial that is factorable over the rationals, write it as a factorization over the integers times some rational number. (The examples and exercises in Appendix 3 are highly recommended for all students as a review of factorization of polynomials.)

8. Given any binomial to any power, write and/or identify its expansion using Pascal's triangle.

* Check definition in Appendix 1.

** Apply the full patterns listed in Appendix 2 where applicable.

BEST COPY AVAILABLE

OBJECTIVE #1

[illegible] page [illegible]
Payne, read pages 49-50, Ex. 1-20 page [illegible]
Pearson, read page 148-149, Ex. 1-7 page [illegible]

OBJECTIVE #2

Nichols, read pp. 59-62, Ex. 1-4 page 62; 4-6 page 93.
Pearson, read pages 153-154, Ex. 1-5 page 155.

OBJECTIVE #3

Nichols, read p. 63, Ex. 1-18 page 63; 1 a-h page 9[illegible]
Vanatta, read pages 56-58, Ex. 41-46 page 60.
Payne, read pages 56-57, Ex. 21-26 page 77.
Pearson, read pages 148-149, Ex. 8-9 page 150.

OBJECTIVE #4

Nichols, read pp. 64-68, 70-78, Ex. 1, 2 page 65; 2, 3 page 67; 1-3 pages 69-70; 1-2 pages 74-75; 1-10 page 78.
Vanatta, read pages 61-68, 73, Ex. 1-4, 6, 8, 17, 20, 21, 30 pages 62-63; [illegible], 14, 16, 20, 21, 24, 27, 29 page 65; 1, 2, 5, 8, 11, 16, 19 pages 68-69; 1, 3, 4, 9, 11, 12. 16, 20, 21, 25 page 73.
Payne, read pages 52-53, 57-58, 72-76, Ex. 1-7, 11, 13-17, 23-28 pages 54-55; 1-29 odd page 59; 1-12 page 73, 1-35 even page 76.
Wooten, read pages 58-59, 251-254, 256-258, 272-275; Ex. 5-38 even pages 60-61; 1-26 even pp. 258-259; 1-20 add page 276;

OBJECTIVE #5

Nichols, read pages 65-66, Ex. 1 page 67; 2 page 91.

OBJECTIVE #6

Nichols, read pages 78-90, Ex. 1, 2 page 82; 1-2 pages 84-85; 1-2 pages 86-87; 1-2 page 88; 2 page 90.
Vanatta, read pages 74-79, Ex. 5, 6, 8, 11, 13. 15, 16, [illegible], 25, 27, 34, 42 page 76; 1, 2, 8, 10, 12, 14. 16, 17, 18, [illegible], 21, 23, 25 page 79; 1, 5, 7, 13, 16, 18 page [illegible].
Payne, read pages 65, 68-70, 80-81, Ex. 1-27 even page 66; 1-29 odd page 69; 1-30 even page 71; 33-58 odd page 72 [illegible] pages 81-82.
Pearson, read pages 165-166, 172-184; Ex. 1-5 page [illegible]; 1-[illegible] even pages 17[illegible]-175; 1-20 odd page 176; 1-3 page 17[illegible] [illegible]

OBJECTIVE #7

Nichols, read p. 69, Ex. 1-8 page 90.
Pearson, Ex. 1-20 even page 182.

OBJECTIVE #8

Pearson, read pages 740-745, Ex. 1, page 74[illegible]
Wooton, read pages 614-616, Ex. 1-6 page 61[illegible]

## APPENDIX

**Behavioral Objective 1**

I. **Definition:** A polynomial in the variable x is the set of all symbols $a_0 + a_1x + a_2x^2 \dots + a_nx^n$ where n can be any non negative integer and the coefficients $a_0, a_1, \dots a_n$ are members of specified set.

Some examples of polynomials in one variable are:

1. $4 - 3x + 6x^2$ which can be written $6x^2 - 3x + 4$. The specified set is the integers. We, therefore, say this is a <u>polynomial over the integers</u>.

2. $\frac{1}{2} + 4x + \frac{2x^2}{5} - 5x^3 + 4x^4$

   This is a polynomial over the ____________________

3. $5y^2 - 3y$

   This is a polynomial over the ____________________

4. $\frac{1}{2}$

   This is a polynomial over the ____________________

5. $2x$

   This is a polynomial over the ____________________

Refer to page 60 of the Nichols text for the definition of a polynomial in n variables. In this definition, the symbols $x_1, x_2, x_3, \dots\dots x_n$ represent n different variables such as x, y, z, w, etc.

The following are examples of expressions which are not polynomials:

1) $\frac{4}{x}$ 2) $5xy^{-1}$ 3) $4x^{\frac{3}{4}}y$ 4) $\sqrt{x} + 2x^{-3}$

**QUESTION:** Is the number "0" a polynomial? Careful, does it conform to the definition. ?

**ANSWER:** Yes, it certainly does. Think about it.......!

The following are examples of different types of [illegible] will work in in Appendix 3. Study these examples and then complete Appendix 3.

I. Apply the following patterns where applicable in computing the product of two binomials:

a. $(x + y)^2 = x^2 + 2xy + y^2$

b. $(x-y)^2 = x^2 - 2xy + y^2$

c. $(x + y)(x - y) = x^2 - y^2$

(d) $(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

(e) $(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$

II. Use the following patterns where applicable to factor polynomials over the integers:

a. $acx^2 + (bc + ad)x + bd = (ax + b)(cx + d)$

b. the distributive property to remove common factors

c. grouping-by-pairs

d. $x^2 + 2(xy) + y^2 = (x + y)(x + y)$

e. $x^2 - a^2 = (x+a)(x-a)$

f. $x^3 + a^3 = (x + a)(x^2 - ax + a^2)$

g. $x^3 - a^3 = (x - a)(x^2 + ax + a^2)$

h. $a^5 + b^5 = (a+b)(a^4 - a^3b + a^2b^2 - ab^3 + b^4)$

i. $a^5 - b^5 = (a-b)(a^4 + a^3b + a^2b^2 + ab^3 + b^4)$

# PRACTICE SHEET ON FACTORING

## I: REMOVING MONOMIAL FACTORS

**EXAMPLE:**

$2x^2y + 4x^3y^2 + 2xy^3 =$

$2xy(x + 2x^2y + y^2)$

* **EXERCISES:**

1. $3ax + 6a^2xy$
2. $16x^2 - 12xy + 8xy$

## II: TRINOMIALS OF FORM $ax^2 + bx + c$

**EXAMPLE:**

$2x^2 + 5x + 3 =$

$(2x + 3)(x + 1)$

**EXERCISES:**

1. $a^2 - 2a - 15$
2. $10x^2 - 11x - 6$
3. $9a^2 - 24a + 16$

## III. THE DIFFERENCE OF TWO SQUARES

**EXAMPLE:**

a. $25x^2 - 1 =$

$(5x - 1)(5x + 1)$

b. $t^2 - b^2 =$

$(t - b)(t + b)$

c. $(a + b)^2 - 49 =$

$(a + b)^2 - 7^2 =$

$\left[(a + b) + 7\right]\left[(a + b) - 7\right] =$

$(a + b + 7)(a + b - 7)$

d. $16t^4 - 1 =$

$(4t^2)^2 - 1^2 =$

$(4t^2 - 1)(4t^2 + 1) =$

$\left[(2t)^2 - 1^2\right]\left[4t^2 + 1\right] =$

$(2t - 1)(2t + 1)(4t^2 + 1)$

## III: THE DIFFERENCE OF TWO SQUARES (continued)

EXERCISES:

1. $x^2 - y^2$
2. $4a^2 - 25b^2$
3. $9x^2 - 49$
4. $x^2 - (y + z)^2$
5. $(2a - b)^2 - 25$
6. $(x^2 + 2x + 1) - y^2$
7. $9x^2 - 12xy + 4y^2 - 16$
8. $x^2 + y^2 + 2xy - 9$
9. $x^4 - y^4$
10. $16 - a^4b^4$
11. $81 - a^4$
12. $b^4 - 16a^4$

## IV: FOUR TERMS - COMMON FACTORS IN EACH PAIR

EXAMPLE:

$xy + x + y^2 + y =$

$(xy + x) + (y^2 + y) =$

$x(y + 1) + y(y + 1) =$

$(x + y)\ (y + 1)$

EXERCISES:

1. $xy - y + 2x - 2$
2. $rs + 2r + 3s + 6$
3. $2xy + y - 6x - 3$
4. $5a + 3ab - 3b - 5$
5. $3ax + 5bx - 3ay - 5by$

## V: TRINOMIALS OF THE FORM $ax^2 + bxy + cy^2$ AND TRINOMIAL SQUARES

EXAMPLE:

a. $4a^2 + 12ab + 9b^2 =$

$(2a + 3b)\ (2a + 3b) =$

$(2a + 3b)^2$

b. $6x^2 + 11xy + 4y^2 =$

$(2x + y)\ (3x + 4y)$

EXERCISES:

1. $9x^2 + 24xy + 16y^2$
2. $4t^2 + 4t + 1$
3. $4k^2 + 16km + 16m^2$
4. $36m^2 + 84pm + 49p^2$
5. $6x^2 - xy - 12y^2$
6. $20t^2 - 9tq - 20q^2$

## VI: SUM OF CUBES

**EXAMPLE:**

a. $a^3 + b^3$

$(a + b)(a^2 - ab + b^2)$

b. $t^3 + 8 =$

$t^3 + 2^3 =$

$(t + 2)(t^2 - 2t + 4)$

**EXERCISES:**

1. $x^3 + 125$
2. $27x^3 + 1$
3. $64 + y^3$
4. $x^6 + y^6$
5. $27x^3 + 64y^3$

## VII: DIFFERENCE OF CUBES

**EXAMPLE:**

a. $a^3 - b^3 =$

$(a - b)(a^2 + ab + b^2)$

b. $x^3 - 27 =$

$x^3 - 3^3 =$

$(x - 3)(x^2 + 3x + 9)$

**EXERCISES:**

1. $64 - 27c^3$
2. $x^3 - 125$
3. $x^6 - y^6$
4. $343x^3 - 8y^3$
5. $8x^3 - 125y^3$

## VIII. OTHER TYPES - COMBINATIONS - FACTOR USING THE DIFFERENT METHODS

a. COMPLETE FACTORING

EXERCISES:

1. $ma^2 - mb^2$
2. $a^4 - 1$

b. COMMON BINOMIAL FACTORS

EXERCISES:

1. $3(a - b) - 4x(a - b)$
2. $(x^2 - y^2) - 5(x + y)$

c. GROUPING

1. $ax + ay - bx - by$
2. $a^3 - a - a^2 + 1$

d. POLYNOMIALS OR DIFFERENCE OF SQUARES

1. $4a^2 + 9b^2 - 25c^2 - 12ab$
2. $25c^2 - 4a^2 - 9b^2 - 12ab$

## SELF-EVALUATION

1 I. Identify which of the following is not a polynomial. Write YES or NO. If NO, explain your answer.

1. $5$
2. $x^2 + 2x + 1$
3. $\frac{x + 1}{x}$
4. $\frac{x^2}{5} - 2x + \frac{1}{6}$
5. $\frac{5}{x} + 7$

2 II. Consider the following polynomials:

6. $2xy$
7. $\frac{1}{5} x y^2 + 3x^2 + 2$
8. $\sqrt{2}\ x - x^2y$
9. $x^2 + 2xy + y^2$
10. $\sqrt{3}\ x^2 + 7x^3 + 3xy + 7$

a. Give the degree of each of the above polynomials.

b. Which of the above polynomials are over the reals, over the rationals, over the integers ?

c. Which of the above are monomials, binomials, trinomials, perfect square trinomials ?

III. Given the polynomials and replacements below, compute the value of the polynomial.

11. $x^2 - x;\ x = 2$
12. $2(xy - 3);\ x = -3$ and $y = 1$
13. $(\frac{1}{3}xy^2) \cdot (x - y) + 7;\ x = y$
14. $xy + \frac{c}{d};\quad x = 1,\ y = 2,\ c = -1,\ d = 4$

IV. Perform the indicated operations.

15. $(as^3 + bs^2 - s) + (-as^3 + n - 4)$

16. $(2-x) - (x^2 + 3x + 5)$

17. $(4y^2 + y + 3) \cdot (y-2)$

18. $(5s^2 - 16s + 3) \div (s - 3)$

19. $(2a - 3)^2$

20. $(a + 2)^2$

21. $(a + b) \cdot (a - b)$

22. $(x^5 + x^3 - 7x^2 + 2x) \div (3 + 4x - 6x^3) =$

23. $(19a^2 + 26ab - 5ac) - (16ac + 15a^2 - 7ab) =$

24. $(2x^2 - 3x + 5)(3x^2 + 4x - 2) =$

25. $(6x^3 - 19x^2 + 21x - 10) \div (2x - 5)$

26. $(3x^2 - 26)(3x^2 + 2y)$

V. True or False?

27. $-(2y)^2 = -4y^2$

28. $-(2x^2 - 3x + 4) = -2x^2 - 3x - 4$

29. $-(x-y) = y - x$

30. $-(-4a^2 - 4ab + b^2 - 9) = 4a^2 + 4ab - b^2 + 9$

VI. Factor over the integers if possible.

31. $3x^2 + 17x + 10$

32. $a^2 + 4a + 4$

33. $33x^3 - 121x^2$

34. $xy + 3y - 2x - 6$

35. $a^2 - b^2$

36. $8x^3 - a^3$

37. $x^3 + 27a^3$

38. $-x^2 + 1$

39. $x^2 - y^2 - 6x + 9$

40. $x^4 - 16$

41. $x^6 - y^6$

42. $a^{10} - b^{10}$

43. $x^4 - 4ax^3 - 3a^2x^2$

VII. Factor over the integers with a rational monomial factor.

44. $y^2 - y + \frac{1}{4}$

45. $\frac{1}{4}x^2 + \frac{1}{3}x + \frac{1}{9}$

46. $x^2 - \frac{1}{4}$

VIII. Expand the following using Pascal's triangle.

47) $(a - b)^9$

48) $(x - 2y)^3$

49) $(2x + y)^5$

50) $(2a - 3b)^6$

IF YOU HAVE SATISFACTORILY COMPLETED YOUR WORK, TAKE THE LAP TEST. CONSULT YOUR TEACHER FIRST.

ADVANCED STUDY

1. Research and learn to use synthetic division. Demonstrate your knowledge by completing either of the following groups of exercises:

   a. Vanatta, Algebra Two, pp. 71-72, numbers 1,3,6,8, and 10.

   b. Dolciani, Modern Algebra, Book Two, p. 523, numbers 1,2,3,4,6.

2. Dolicani, Modern Algebra Two, p. 145 numbers 23-26.

# REFERENCES

Vanatta (abbreviation)

Vanatta, Glen D., Algebra Two: A Modern Course, Charles E. Merrill Publishing Co., 1966.

Dolciani (abbreviation)

Dolciani, Mary P., Berman, Simon L., and Wooton, William, Modern Algebra, Book Two, Houghton Mifflin Co., 1965.

Nichols (abbreviation)

Nichols, Eugene D., Modern Intermediate Algebra, Holt, Rinehart and Winston, Inc., 1965.

Wooton, (abbreviation)

Dolciani, Mary P., Wooton, William, Beckenbach, Edwin F., Sharron, Sidney, Modern School Mathematics, Algebra 2, Houghton Mifflin Co., 1968.

Payne (abbreviation)

Payne, Joseph N., Zamboni, Floyd F., Lankford, Jr., Francis G., Algebra Two, Harcourt, Brace and World, 1969.

Pearson (abbreviation)

Pearson, Helen R., and Allen, [illegible] Modern Algebra: A Logical A[illegible] Ginn and Company, [illegible].

Equations by Layman Allen

# LEARNING ACTIVITY PACKAGE

$$2x - \frac{5}{2x - \frac{5}{2x}}$$

## RATIONAL EXPRESSIONS

Ninety Six High School - Ninety Six, S.C.

ALGEBRA 104

LAP NUMBER 26

WRITTEN BY Diane Evans

REVIEWED BY J. S. Ritchie

## Acknowledgement

The administration and staff of Ninety Six High School gratefully acknowledges the assistance provided by the staff of Nova High School, Fort Lauderdale, Florida. We are especially indebted to Mr. Lawrence G. Insel and Mr. Laurence R. Wantuck of Nova's Math Department for permitting us to use much material developed by them, some of which has been reproduced in its original form.

## RATIONALE (The LAP'S Purpose)

In the past you have learned to solve equations. You have also learned that equations are not always in a form which is easy to work. Many times it is necessary to simplify equations to get them in a workable form. This LAP is essential to your future work in solving equations.

In this LAP "algebraic fractions" will also be reviewed along with the study of the set of rational expressions under addition and multiplication. The treatment of the operations is based on the notion of the quantification of variables over the set of real numbers and the resulting availability of field properties. Addition and multiplication of rational expressions is done first by strict application of definitions so that the underlying principles, when short cuts are used, will be understood. The concluding section will make use of rational expressions in problem solving.

Behavioral Objectives

After the completion of your prescribed course of study, you will be able to:

1. Given an expression, determine whether or not it is a rational expression.
2. Given a rational expression, express it in simplified form; i.e., express it so that the numerator and denominator have no common factors.
3. Given a rational expression (or a sum, difference, product or quotient of two rational expressions), find all replacements for the variables for which the expression(s) is undefined.
4. Given two or more rational expressions, determine the least common denominator for these expressions.
5. Given a rational expression, find
   a) its multiplicative inverse (if it exists)
   b) its additive inverse
6. Given two rational expressions, express any of the following in simplified form:
   a) the product of the two expressions
   b) the quotient of the two expressions
   c) the sum of the two expressions
   d) the difference of the two expressions
7. Given a complex rational expression, write its simplified form.
8. Solve equations involving rational expressions.
9. Solve any given word problem involving rational expressions.

# RESOURCES

Objective 1

Nichols, read pages 95-96, Ex. 1-9 oral page 96. Payne, read pages 95-96, Ex. 1-12 page 97.

Objective 2

Vanatta, read pages 81-83, Ex. 1-18 even pages 83-84.
Dolciani, read pages 158-160, Ex. 1,3,5,11,15,20,22,26, 29-32 pages 160-161
Nichols, read pages 96-98, Ex. 1-9 page 98
Wooton, read pages 277-279, Ex. 1-36 pages 279-280
Payne, read pages 98-99, 102-103. Ex. 1-12 page 100; 1-10 page 102; 1-39 even page 103
Pearson, read page 187, Ex. 20 page 53; 1 (a-f) page 191; 28a page 196

Objective 3

Vanatta, read page 21, Ex. 2 page 22
Dolciani, read pages 157-158, Ex. 1-24 odd page 158
Payne, read page 96, Ex. 13-33 page 97
Pearson, Ex. 12 page 53.

Objective 4

Nichols, read pages 98-106, Ex. 1 (a-j), 2 (a-j) pages 104-105

Objective 5 -- Appendix I

Objective 6

Vanatta, Parts a, b; read pages 87-90, Ex. 2,3,4,8,9,11,13,14, page 80; 2,4,6,7,8,10 page 90; 5,9,11,12 page 91
parts c,d; read pages 81-85, Ex. 2,3,5,13,14,16,19,22,23 pages 86-87; 15,16 page 96
Dolciani, parts a,b; read pages 161-162, Ex. 3-6, 8,11,15,16, 19-21, 23,24,27,28 pages 162-163; 19,32 page 169
Parts c,d; read pages 164-165, Ex. 5,8,10,11,12,15,18,25,26,28,31, 32,35,38,42 pages 166-167.
Nichols, read pages 98-106, Ex. 1 (a-j) pages 99-100; 1(a-j), 2 (a-j) page 102; 1(a-j), 2 (a-j) pages 107-108
Payne, part a ; read page 104, Ex. 1-29 page 105
Part b; read page 106, Ex. 1-24 page 107
Parts C,d; read pages 110-111, Ex. 1-40 even page 112.
Pearson, parts a,b; read pages 187-189, Ex. 2(a-h) page 191
Parts c,d; read pages 187-189, Ex. 4 (a-h) page 192; 28 (b,c) page 196.

Objective 7

Vanatta, read pages 91-92, Ex. 1,3,4,9,11 pages 92-93; 18 page 96
Dolciani, read pages 167-168, Ex. 1,7,9,10,13,14,25,26 pages 168-169
Nichols, read pages 108-109, Ex. 1-9 page 109
Wooton, read pages 284-286, Ex. 1-62 odd page 286.
Payne, read page 113, Ex. 1-20 page 114

Pearson, read pages 188-189, Ex. 3 (a-f) page 191; 28 (d-f) page 196

Objective 8

Dolciani, read pages 169-170, Ex. 2,5,6,15,17, 18 page 171

Objective 9

Vanatta, Ex. 1,2,5,6,8,9,12,14,16,17 pages 138-139; 2-4 page 141
Dolciani, read pages 173-174, Ex. 1, 3-9, 13, 16 pages 174-176
Wollensak C-3808 Reading Written problems.

## SELF-EVALUATION

OBJ.

1 I. Which of the following are rational expressions? Circle the number by each rational expression.

1. $\frac{1}{\sqrt{x}} + 2$

2. $\frac{1}{\sqrt{2}} \cdot x^2 + 2$

3. $\frac{1}{\sqrt{x^2 + 2}}$

4. $2$

5. $\frac{y(y + 3)}{2}$

6. $\frac{\sqrt{x}}{\sqrt{x}} + 2 \quad (x \neq 0)$

7. $\frac{x^2 + y^2}{z^2}$

8. $\frac{\frac{3}{7}}{x}$

2 II. Simplify the following rational expressions:

9. $\frac{2}{4}$

10. $\frac{x^2}{xy}$

11. $\frac{x^2 - z^2}{yx + yz}$

12. $\frac{r^2 - s^2}{r^2 + 3rs + 2s^2}$

13. $\frac{x^2 + xy - 2x}{x^2 + 2xy + y^2 - 4}$

14. $\frac{u^3 - v^3}{u^2 + uv + v^2}$

3 **III.** Decide on the replacements for the variables for which the following expressions are undefined:

15. $\dfrac{3}{x + 2}$

16. $\dfrac{t}{2 - t}$

17. $\dfrac{5 + x}{3y - 4}$

18. $\dfrac{12x + 2}{x^2 - 4}$

19. $\dfrac{3t + 3}{4t - t^2 - 4}$

4 **IV.** What is the Least Common Denominator of each pair of expressions?

20. $\dfrac{x}{y}$ ; $\dfrac{x + 2}{y \cdot 2}$

21. $\dfrac{2}{x^2 - y^2}$ ; $\dfrac{6}{(x - y)}$

22. $\dfrac{2x}{x^4 - 16}$ ; $\dfrac{3y}{(x^2 + 4)(x - 2)}$

23. $\dfrac{2}{xyz}$ ; $\dfrac{5}{rxy}$

24. $\dfrac{6x + 2}{x\ (x - y)}$ ; $\dfrac{3x - y}{x^3 - y^3}$

5 **V.** Find the multiplicative and additive inverse of each of the following expressions; then simplify.

25. $\frac{1}{2}$

26. $\frac{x}{y}$

27. $\frac{x}{x^2}$

28. $\frac{9}{\frac{x}{4}}$

29. $y - x$

30. $\frac{x}{x + 1}$

31. $\frac{y}{-x}$

32. $\frac{-c}{-d}$

6 **VI.** Perform the indicated operation.

33. $\frac{x}{y} + \frac{x + 2}{2y}$

34. $\frac{x}{y} - \frac{(x + 2)}{2y}$

35. $\frac{3s^2 + sr}{9s^2 - r^2} + \frac{s - r}{r - 3s}$

36. $\frac{y^2}{4x} \cdot \frac{x}{y}$

37. $\frac{2x}{x^2+3x+2} - \frac{x}{x^2-1}$

38. $\frac{3x+4}{x^2-16} + \frac{x-3}{x^2+8x+16}$

39. $\frac{y^2}{4x} \div \frac{y}{x}$

40. $\frac{8x^3 + 27}{3x^2 - 3} \cdot \frac{x^4 - 1}{2x^2 - x - 6}$

41. $\frac{4x^2 - 6x + 9}{2x - 3} \div \frac{8x^3 + 27}{4x^2 - 12x + 9}$

42. $\frac{5x-x^2}{x^2+8x+12} \cdot \frac{x^2-2x-8}{x^2-4x-5}$

43. $\frac{xy+x}{5y} \div \frac{x^2y+x^2}{3y}$

7 VII. Simplify the following complex rational expressions:

44. $$\frac{\frac{4x}{y^2} + \frac{x}{y}}{y - \frac{y^2 - xy}{y}}$$

46. $$\frac{\frac{2}{x} + \frac{3}{y}}{\frac{x}{y} + \frac{y}{x}}$$

45. $$\frac{t - u - \frac{t^2 + tu}{t}}{\frac{3u}{t} - \frac{u}{t^2}}$$

47. $$\frac{\frac{1}{a} + \frac{1}{b}}{\frac{1}{a} - \frac{1}{b}}$$

8 VIII. Solve the following:

48. $\frac{2x}{3} + 8 = 0$

49. $\frac{2x}{3} - \frac{2x+1}{5} = \frac{1}{3}$

50. $\frac{3-2x}{8} - \frac{x-3}{6} - 1 = \frac{x+4}{3} + \frac{1}{24}$

9 IX. Solve the following.

51. The average of two numbers is 15. Find the numbers if the smaller is two-thirds of the larger.

52. One card-sorter can process a deck of punched cards in 30 minutes, while another can sort the deck in 45 minutes. How long would it take the two sorters together to process the cards?

53. A solution of silver nitrate in water is 12% silver nitrate. How many ounces of the compound must be added to 23 ounces of this solution to produce a 20% solution?

54. Three men receive together \$1285 from a business venture. If A's share is \$25 more than $\frac{5}{6}$ of B's share, and C's share is $\frac{4}{15}$ of B's share, find the amount of money each should receive.

55. The length of a rectangle is two feet longer than its width. Find the width if the perimeter of the rectangle is 144 feet.

GRADE YOUR OWN TEST. If you have satisfactorily completed your work, you may take the LAP TEST. CONSULT YOUR TEACHER FIRST.

## Appendix I

Objective 5

Example: The additive inverse of $\frac{x}{y}$ is $\frac{-x}{y}$.

The multiplicative inverse of $\frac{x}{x-1}$ is $\frac{x-1}{x}$.

Give the additive inverse and multiplicative inverse of the following:

1. $\frac{3}{4}$
2. $\frac{k}{l}$
3. $\frac{m}{n^2}$
4. $\frac{x}{x^2}$
5. $\frac{3}{\frac{x}{4}}$
6. $y - x$
7. $\frac{y}{-x}$
8. $\frac{-k}{-d}$
9. $-\frac{-3}{-4}$
10. $\frac{7}{\frac{y}{2}}$

## ADVANCED STUDY

1. Make up a game using rational expressions.

At least 80% of any set of the following problems MUST BE COMPLETED for credit.

2. Allendoerfer, <u>Fundamentals of Freshman Mathematics</u>, Ex. 1 -20 p. 74.
3. Allendoerfer, Ex. 17-20 p. 78, 15-22 p. 80.
4. Allendoerfer, Ex. 10-20 p. 80.
5. Dolciani, Modern Algebra Two, Ex. 26, 31, 33, 34, p. 177, and 9, 10, 15, 19 p. 182.

# REFERENCES


Vanatta (abbreviation)

Vanatta, Glen D., and Goodwin, Wilson A., Algebra Two, A Modern Course, Charles E. Merrill Publishing Co., 1966.

Dolciani (abbreviation)

Dolciani, Mary P., Berman, Simon L., and Wooton, William, Modern Algebra, Book Two, Houghton Mifflin Co., 1965.

Nichols (abbreviation)

Nichols, Eugene D., Modern Intermediate Algebra, Holt, Rinehart and Winston, Inc., 1965.

Pearson (abbreviation)

Pearson, Helen R., and Allen, Frank B., Modern Algebra: A Logical Approach, Book Two, Ginn and Company, 1966.

Payne (abbreviation)

Payne, Joseph N. Zamboni, FLoyd F., Lankford, Jr., Francis G., Algebra Two, Harcourt, Brace and World, 1969.

Wooton (abbreviation)

Dolciani, Mary P., Wooton, William, Beckenbach, Edwin F., Sharron, Sidney, Modern School Mathematics, Algebra 2, Houghton Mifflin Company, 1968.

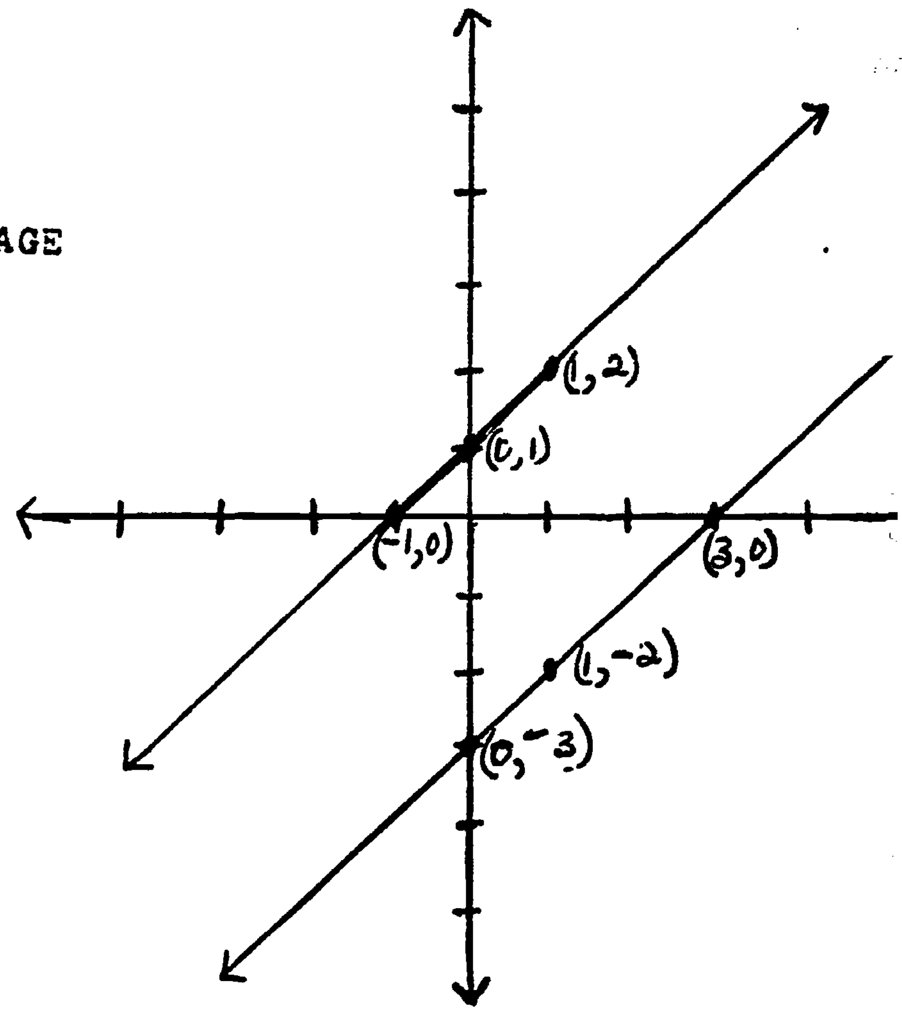
Allendoefer, C.B., Oakley, C.O., Fundamentals of Freshmen Mathematics, McGraw-Hill Book Company, Inc., 1959

Wollensak Teaching Tape c-3809 Reading Written Problems.

**L**EARNING

**A**CTIVITY

**P**ACKAGE




# INTRODUCTION TO COORDINATE GEOMETRY

Ninety Six High School - Ninety Six, S.C. -

ALGEBRA 103-104

LAP NUMBER 27

REVIEWED BY [signature]

WRITTEN BY Mrs. Deane Evans

11773

## RATIONALE (The LAP's Purpose)

In 1600 European mathematicians worked with two branches of mathematics - geometry and algebra. However, there was no link between these two branches. René Descartes, a French philosopher and mathematician, provided the connection in his Géometrie, published in 1637, by devising a scheme for locating points by using numbers. From this idea the whole subject of analytic geometry or coordinate geometry has developed.

In this LAP you will begin an introduction to coordinate geometry. You will study the most basic coordinate figure, the straight line. You will also investigate the concepts of slope, intercepts, distance, midpoint, parallelism, and perpendicularity.

SECTION 1

BEST COPY AVAILABLE

Behavioral OBjectives

At the completion of your prescribed course of study, you will be able to:

1. Identify or define the following:

   a. cartesian coordinate system
   b. Descartes
   c. abscissa
   d. ordinate
   e. origin

2. Given a coordinate system for a line:

   a. Find the coordinate of any given point.
   b. Given two points, find the coordinate of any point of the segment joining the two points.
   c. Given the coordinates of two points, determine the distance between them.

3. Given a coordinate system for a plane:

   a. Given a point, identify its coordinates.
   b. Given an ordered pair, graph the corresponding point.
   c. Given a point, identify the quadrant or the axis which contains the point.
   d. Given the lengths of two sides of a right traingle, use the Pythagorean Theorem to find the length of the third side.
   e. Given a geometric figure, one or more of whose sides lie along a horizontal or a vertical line, find lengths of segments or coordinates of points for this figure.

4. Given the coordinates of two points in a plane:

   a. Use distance formula to determine the distance between them.
   b. Find the coordinates of the midpoint of the segment joining them.
   c. Find the slope of the line containing them.

5. Given the slope of a line or sufficient information to determine this slope, decide whether:

   a. the line "rises to the right".
   b. the line "falls to the right".
   c. the line is horizontal
   d. the line is vertical

# RESOURCES 1

Obj. 1: Vanatta, Algebra Two, read p. 115, Ex. Define the terms in Obj. One.
Dolciani, Modern Algebra, read p. 81, Ex. Define the terms in Obj. One.

Obj. 2: Nichols, read pp. 115-117, 122, ex. 1, 2 page 116; 1-4 pages 117-118, 1-6 page 122.
Wooton, read pp. 154-155, 160, ex. 5-8 page 159.
Payne, read p. 136, ex. 1-22, pp. 138-139.
Pearson, read p. 204, ex. 1-20, p. 205.

Obj. 3: Vanatta, (a) ___: (b) p. 115-116, ex. 1-4 page 117: (c)-(e) ___.
Nichols, read pp. 118-119, ex. 1-9 pages 119-121.
Wooton, read pp. 155-157, 160-161, ex. 9-24 page 159.

Pearson, read pp. 205-210, ex. 1-12 pages 210-211.
Wollensak Tape C-3852 Graphing Linear Functions
Games: Graphing Pictures
An Ordered Pair Code

Obj.s 4,5: Vanatta (#4a) read pp. 152-153, ex. 1, 2, 4 page 154:
(b) read pp. 154-155, ex. 1-3 pages 155-156:
(c) read pp. 142-143, ex. 1-12 odd page 145.
(#5) ___.

Dolciani, (#4a,b) read page 294, ex. 1-6 page 295:
(c) read pages 84-88, Ex. 1-12 even page 89:
(#5) ___.

Nichols, #4,5 read pages 122-126, 129-133, ex. 1-14 even pages 122-123; 1-8 pages 124-125; 1-14 even page 131; 1-14 even page 133.

Wooton, #4,5 read pages 168-172, 433-437, ex. 1-18 bottom page 173; 1-30 pages 437-438.

Payne, #4,5 read pages 140-146, 148-151, ex. 1-14 page 143; 1-12 pages 146-147;18-23 page 153.

Pearson, #4,5 read pages 212-218, ex. 1-4, 12, 13, pages 216-217; 1-14 pages 218-219.

Wollensak C-3854 The Slope of a Line

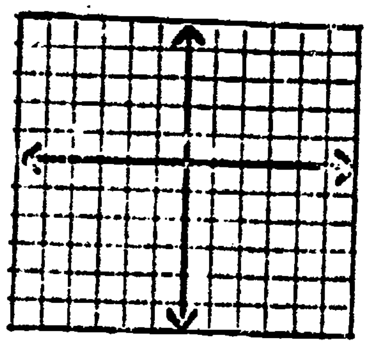
## SELF-EVALUATION 1

Obj:

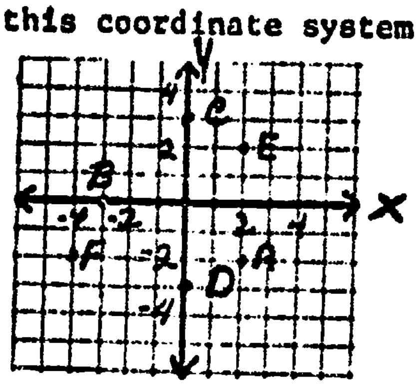
1 I. a. Define Cartesian coordinate system.

b. For whom is the Cartesian coordinate system named?

c. On the following coordinate plane, label (1) the abscissa, (2) the ordinate, (3) the origin.



2 II. Use this coordinate system to answer the following questions.




(1) Give the coordinate for each of the following points.

A ____________ E ______________

B ____________ F ______________

C ____________

D ____________

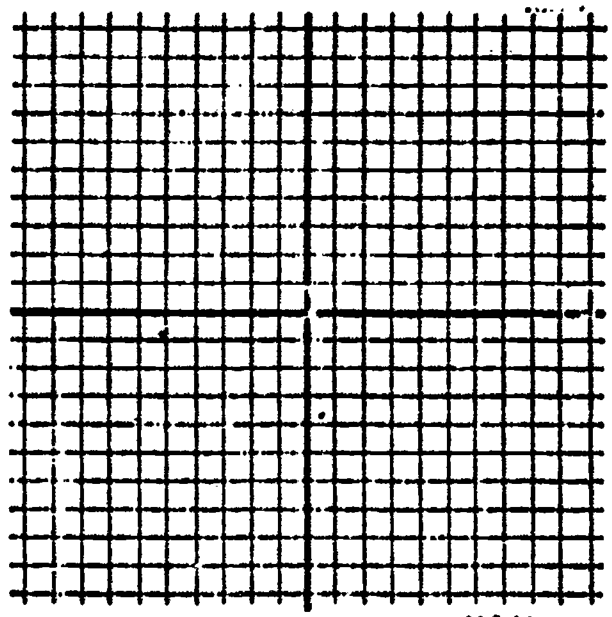
(2) Give the distance between the following pairs of points.

A and E ________________

C and D ________________

3b III. (1) Use the following graph and plot these points.

A (2,4)
B (-6,-4)
C (-1,-1)
D (0,5)
E (-3,4)
F (4,-2)
G (5,0)



3c (2) Identify the quadrant or axis which contains each of the following points.

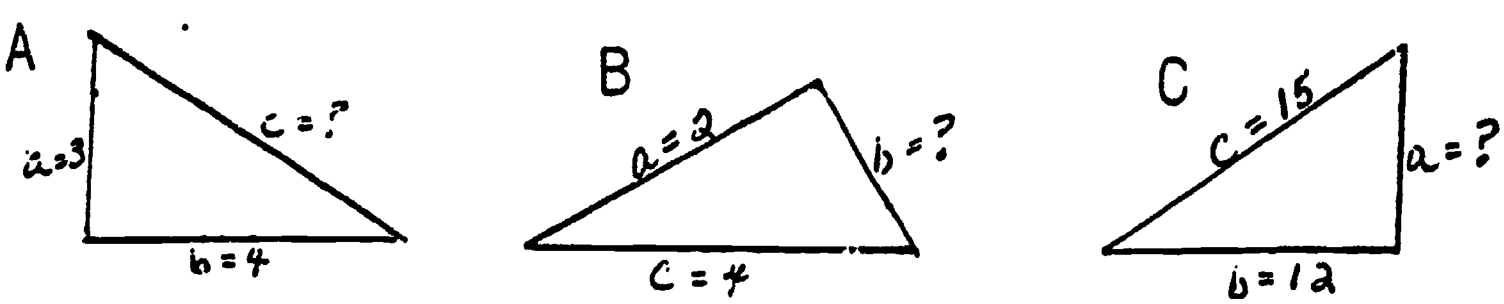
_______ A (4,-1) _______ E (-6,8)

_______ B (2,8) _______ F (7, -1)

_______ C (-7,-2) _______ G (0,-3)

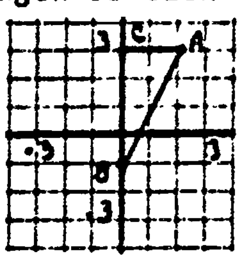
_______ D (-3,0) _______ H (-1,-4)

3d (3) Find the length of the third side in each of the following triangles.




C = __________ B = __________ A = __________

(4) Find the length of each of the sides of the following triangle.

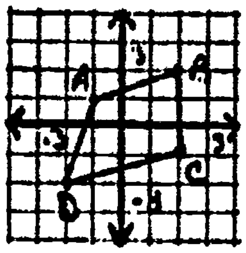



AC = __________

CB = __________

AB = __________

(5) Write the coordinates for the points of the vertices of the following figure.



A ______________

B ______________

C ______________

D ______________

4 IV. For each of the following pairs of points, find:

(a) the distance between them
(b) the coordinate of the midpoint of the segment joining each pair
(c) the slope of the line containing each pair

| | DISTANCE | MIDPOINT | SLOPE |
|---|---|---|---|
| (1) (4,0) (0,-3) | ________ | ________ | ______ |
| (2) (0,5) (-2,-2) | ________ | ________ | ______ |
| (3) (4,3) (8,7) | ________ | ________ | ______ |
| (4) (-2,8) (5,-3) | ________ | ________ | ______ |
| (5) (8,-2) (-3,9) | ________ | ________ | ______ |

5 V. Given two ordered pairs: (a) determine the slope of the line joining them, and (b) determine if the line: (1) rises to the right
(2) falls to the right
(3) is horizontal
(4) is vertical

| | A. SLOPE | B. DIRECTION |
|---|---|---|
| 1. (8,2) (3,7) | | |
| 2. (-2,-5) (-4,-9) | | |
| 3. (2,7) (9,7) | | |
| 4. -5,3) (-5,-4) | | |
| 5. (8,0) (0,-2) | | |

IF YOU HAVE SATISFACTORILY COMPLETED YOUR WORK ON SECTION 1, CONSULT YOUR TEACHER. THEN TAKE THE PROGRESS TEST ON SECTION 1.

## SECTION 2

### Behavioral Objectives

At the completion of your prescribed course of study, you will be able to:

6. Given an equation of a line:
   a. write an equivalent equation in slope intercept form.
   b. determine the slope of the line.
   c. determine the y - intercept of the line.
   d. sketch and/or identify the graph of the line.

7. Write the equation of a given line, when given any one of the following:
   a. the slope of the line and the y - intercept of the line.
   b. the coordinates of a point on the line and the slope of the line.
   c. the coordinates of two points on the line.

8. Given the coordinates of a point or information sufficient to find such coordinates, write:
   a. the equation of the horizontal line containing this point.
   b. the equation of the vertical line containing the point.

## RESOURCES 2

Obj. 6: Vanatta, read pages 146-148, ex. page 148, work any 10 problems.
Nichols, read page 144, ex. 2 a - j page 144.
Wooton, read pp. 161-162, 175-177, ex. 1-12 oral page 162, 5-18 page 172.
Payne, read pages 162-165, ex. 22-27 page 166.
Pearson, read pages 161-162, 175-177, ex. 1-7 page 222, 4 page 227.
Wollensak Tape C-3852 Graphing Linear Functions
C-3855 Slope Intercept Form
Transparency 3M The Straight Line
Games: Graphing Pictures
An Equation Code

Obj. 7: Dolciani, 7a, read pages 90-93, ex. 19-26 page 93: 7b read pages 90-93, ex. 1-8 page 93: 7c ___.
Nichols, 7a read pages 142-143, ex. 1 page 144: 7b read pp. 142-143, ex. even numbers bottom page 143: 7c 2, 4, 6, 8 top page 143.
Wooton, read pages 175-177, Ex. 1-6 and 13-18 orals page 178, 1-12 written page 178, 13-24 page 179.
Payne, read pages 162-165, ex. 7a ___: 7b, 9-17 page 166: 7c, 1-8 page 165, 48 page 167.
Pearson, read pages 224-226, Ex. 1 page 228, 1-17 pages 233-236.

Obj. 8: Nichols read pp. 139-140, Ex. 2a,c,e,g,i page 140.
Payne, read pp. 162-165, ex. 1-55 even pages 165-167.
Pearson, read pp. 223-226, Ex. 1-4 pages 226-227.

## SELF-EVALUATION 2

Obj.

6 I. Rewrite the following in slope-intercept form, state the slope and y - intercept, and graph each (use graph paper on next page).

(1) $2x + 3y = -6$ __________ $m =$ ______ $b =$ ______

(2) $2y = -4x + 8$ __________ $m =$ ______ $b =$ ______

(3) $y = -x$ __________ $m =$ ______ $b =$ ______

(4) $-18x - 6y = 18$ __________ $m =$ ______ $b =$ ______

(5) $3x = 6y - 12$ __________ $m =$ ______ $b =$ ______

7 II. In each problem below, use the given information to write the equation for a line.

(1) $m = 5$, $b = 2$ __________

(2) $m = -2$, $P_1(-3,4)$ __________

(3) $P_1(1,-1)$, $P_2(-1,-1)$ __________

(4) $m = -9$, $b = 0$ __________

(5) $m = \frac{4}{5}$, $P_1(3,-1)$ __________

(6) $P_1(2,3)$, $P_2(-1,-4)$ __________

(7) $m = -\frac{2}{3}$, $b = 3$ __________

(8) $m = 5$, $P_1(-2,-4)$ __________

(9) $P_1(4,-3)$, $P_2(-6,2)$ __________

8 III. A) Write the equation for: (1) the vertical line, and (2) the horizontal line through (-2,3).

(1) vertical __________

(2) horizontal __________

IV. Multiple Choice.

9 ____ 1. If a line is vertical and passes through the point (-2,-3) then its equation is:

(a) $x = -3$
(b) $y = -3$
(c) $x = -2$
(d) $y = -2$
(e) none of these

## SELF-EVALUATION 2 (cont')

Graph Paper

9 ____ 2. If a line is horizontal and passes through the point (a,b), then its equation is:

a) $x = a$
b) $x = b$
c) $y = a$
d) $y = b$
e) none of these

Questions 3 - 5 refer to the line with equation $3x - y = 2$.

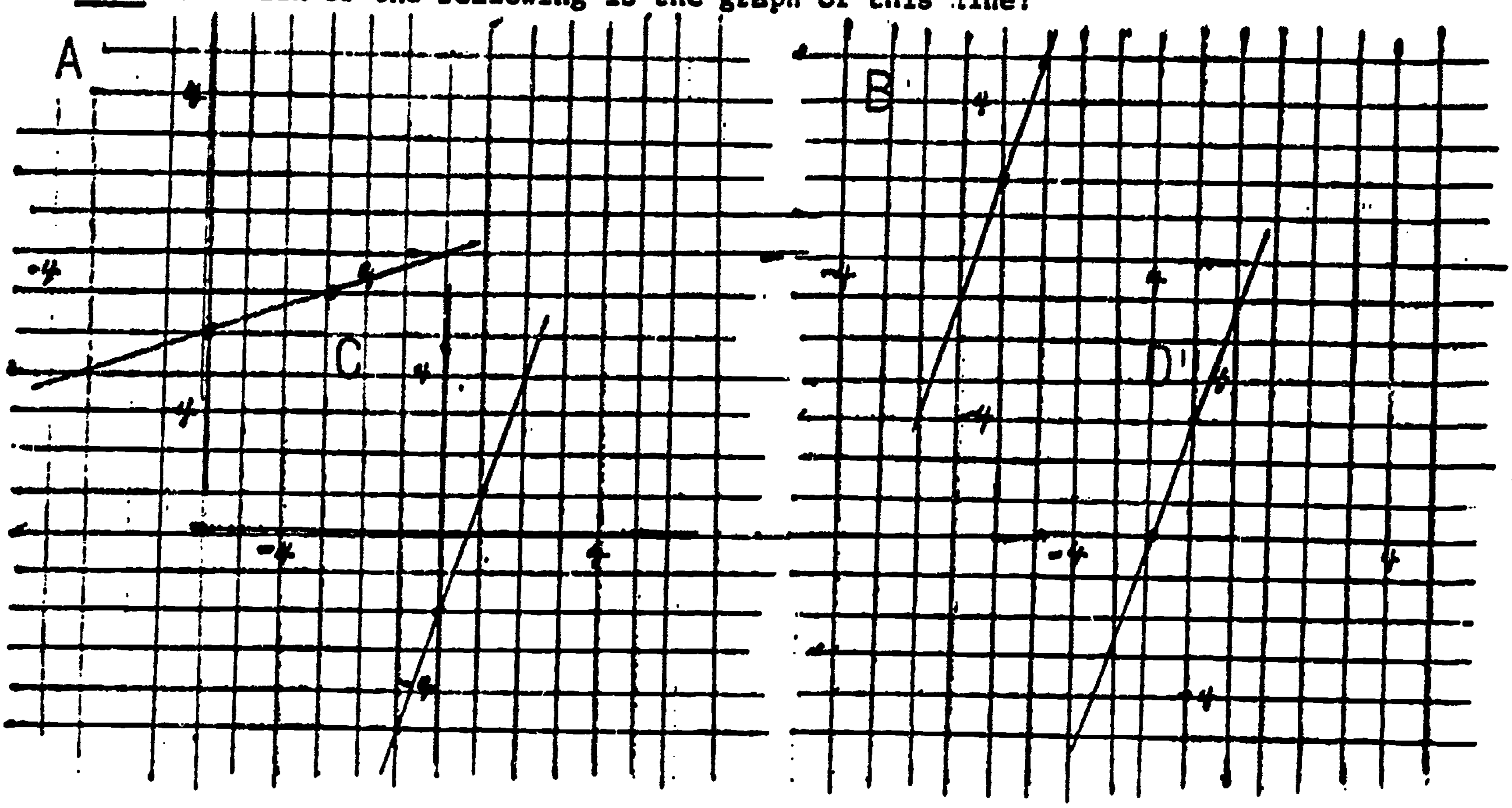
6 ____ 3. The slope - intercept form of the equation of this line is

(a) $-y = -3x + 2$
(b) $y = 3x + 2$
(c) $y = 3x - 2$
(d) $3x + -y = 2$
(e) $y = 3(x - 2)$

6 ____ 4. The slope and y - intercept of this line are:

(a) -3, (0,2)
(b) -3, (0, -2)
(c) 3, (0,2)
(d) 3, (0,-2)
(e) none of these

6 ____ 5. Which of the following is the graph of this line?




(e) none of these

7 V. Choose the correct equation in Column B for each item in Column A:

COLUMN A

COLUMN B

____ 1. The line contains (1,1) and (2,2)

A. $2x + y = -2$

| | | |
|---|---|---|
| ____ 2. | The line contains (-2,-3) and has slope ½. | B. $x - 2y = 4$ |
| ____ 3. | The line has slope -2 and y - intercept (0,-2). | C. $y = x$ |
| ____ 4. | The line contains (1,-1) and has slope -1. | D. $x + y = 0$ |
| ____ 5. | The line has slope ½ and contains (2,-1). | E. none of these |

WHEN YOU HAVE COMPLETED YOUR RESOURCES AND SELF-EVALUATION, CONSULT YOUR TEACHER. IF YOU HAVE DONE SATISFACTORY WORK, YOU MAY TAKE YOUR PROGRESS TEST ON SECTION 2.

SECTION 3



**Behavioral Objectives**

At the completion of your prescribed course of study, you will be able to:

9. Given the slopes of two lines or sufficient information for finding slopes, determine if,

   a. the two lines are parallel
   b. the two lines are perpendicular
   c. the two lines are neither parallel nor perpendicular

10. Given the coordinates of points on two lines, such that certain of the coordinates are variables, determine the value of the missing numbers when the lines are:

    a. parallel
    b. perpendicular

11. Given the equation of a line and a point not on the line, write and/or identify the equation of a line through the given point and parallel and/or perpendicular to the given line.

RESOURCES 2

Obj. 9: Vanatta, read pp. 150-151, Ex. 3,4 p. 152.
Nichols, read pp. 133-135, Ex. 1-3 p. 136.
Wooton, read pp. 175-177, 439-440, Ex. 1-8 and 14-18 page 441, 43-44 page 179.
Payne, read pages 155-156, Ex. 1-16 pages 157-158.

Obj. 10: Nichols, read pp. 133-135, Ex. 4-9 page 136.
Wooton, read pp. 175-177, 439-440, Ex. 41-42 page 179.

Obj. 11: Vanatta, read pp.150-151, Ex. 6-7 p. 152.
Wooton, read pp. 175-177, 439-440, Ex. 25-36 page 179, 9-12 page 441.

## SELF-EVALUATION 3

9 **I.** Given the following pairs of slopes, determine if the lines with these slopes are parallel, perpendicular, or neither.

__________ 1. $\frac{2}{5}$, $\frac{5}{2}$

__________ 2. $\frac{1}{2}$, $\frac{3}{6}$

__________ 3. $\frac{1}{3}$, $-3$

__________ 4. $-5$, $\frac{-10}{2}$

__________ 5. $\frac{4}{3}$, $\frac{-3}{4}$

__________ 6. $-6$, $6$

9 **II.** Given the following pairs of linear equations, determine if their graphs are parallel, perpendicular, or neither.

__________ 7. $y = \frac{7}{4}x - 1$
$y = \frac{7}{4}x + 6$

__________ 8. $3x - 6y = 9$
$2x + y = 4$

__________ 9. $2y = 3x - 12$
$2x + 3y = 3$

__________ 10. $7x + y = 7$
$2y = -14x + 4$

10 **III.** __________ 11. Determine x such that the line through $P_1(x,3)$ and $P_2(-2,1)$ is parallel to the line through $P_3(5,-2)$ and $P_4(1,4)$.

__________ 12. Determine x such that the line through $A(x,3)$ and $B(-2,1)$ is perpendicular to the line through $C(5,-2)$, and $D(1,4)$.

__________ 13. Determine m such that $y = mx + 5$ is perpendicular to $y = 2x + 5$.

__________ 14. Determine C such that $Cx + y = -2$ is parallel to $2x + y = 6$.

## SELF-EVALUATION 3 (cont')

11 IV. Write the equation of a line parallel to each given line and through the given points.

__________ 1. (2,3) $y = -3x + 1$

__________ 2. (-3,2) $4x - y = 6$

__________ 3. (4,1) $y = \frac{-3}{4}x + 6$

__________ 4. (-2,-3) $5x + 2y = 3$

11 V. Write the equation of a line perpendicular to each given line & through the given points.

__________ 1. (3,1) $y = -x + 2$

__________ 2. (-1,3) $2x - y = 4$

__________ 3. (-3,-2) $3x - 2y = 4$

__________ 4. (7,-2) $4x - 3y = 5$

IF YOU HAVE SATISFACTORILY COMPLETED YOUR WORK, CONSULT YOUR TEACHER. THEN TAKE THE LAP TEST.

## ADVANCED STUDY

I. Nichols, read pages 137-138, Ex. work any 4 of 1-11, pp. 138-139.

II. Wooton, Ex. 19, 20 page 441.

III. Nichols, Ex. 7, 8 page 129.

IV. Nichols, Ex. 5, 7, 8 page 125.

V. Vanatta, work any 5 of the following: page 152 nos. 8, 9, 10; page 154 nos. 3, 6, 10; page 156 no. 5

VI. Work any 4 of the following:

1. Determine an equation of the line satisfying the stated conditions.

   a. Through (-3,2) and parallel to the line joining (2,3) and (1,-2).

   b. With x - intercept 2 and y - intercept 3.

   c. Through (b,-2b) with slope $\frac{2}{b}$ .

2. Show that the figure whose vertices are (2,1), (4,2), (5,2), and (7,3) is a parallelogram.

3. If a line has x - intercept a ($a \neq 0$) and y - intercept b ($b \neq 0$), show that an equation of the line (called the <u>intercept form</u> of the equation) is:

$$\frac{x}{a} + \frac{y}{b} = 1$$

4. If $(x_1, y_1)$ and $(x_2, y_2)$ are two points of a line and if $x_1 \neq x_2$, then an equation of the line (called the <u>two-point form</u> of the equation) is:

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) .$$

5. Show that if the graphs of the linear equations $A_1x + B_1y = C_1$ and $A_2x + B_2y = C_2$ are parallel, then $A_1B_2 = A_2B_1$.

6. Show that if $A_1B_2 = A_2B_1$, then the graphs of the linear equations $A_1x + B_1y = C_1$ and $A_2x + B_2y = C_2$ are either the same line or parallel lines.

## REFERENCES


Vanatta, Glen D., and Goodwin, Wilson A., Algebra Two, A Modern Course, Charles E. Merrill Publishing Co., 1966.

Dolciani, Mary P., Berman, Simon L., and Wooton, William, Modern Algebra and Trigonometry, Book Two, Houghton Mifflin Co., 1965.

Nichols, Eugene D., Modern Intermediate Algebra, Holt, Rinehart and Winston, Inc., 1965.

Pearson, Helen R. and Allen, Frank B., Modern Algebra, A Logical Approach, Book Two, Ginn and Company, 1966.

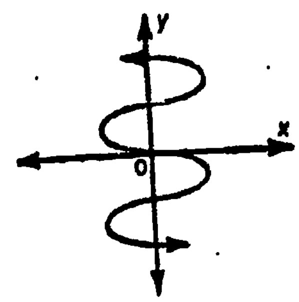
Payne, Joseph N., Zambone, Floyd F., Lankford, Jr., Frances G., Algebra Two, Harcourt, Brace and World, 1969.

Dolciani, Mary P., Wooton, William, Beckenback, Edwin F., Sharron, Sidney, Modern School Mathematics, Algebra 2, Houghton Mifflin Company, 1968.

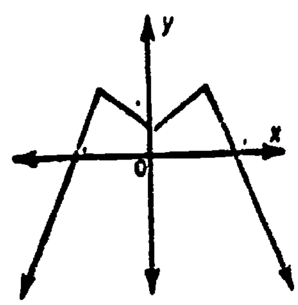
Wollensak Teaching Tapes C-3852
C-3855
C-3854

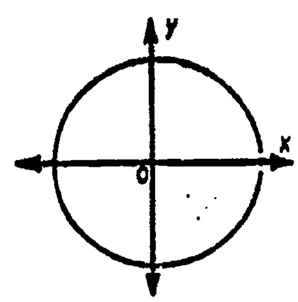
3M Transparencies - The Straight Line

# LEARNING ACTIVITY PACKAGE




# RELATIONS, FUNCTIONS, AND INEQUALITIES




Ninety Six High School
Ninety Six, S.C.

Algebra 103-104

REVIEWED BY J. Ritchie

LAP NUMBER 28

WRITTEN BY Diane Evans

11873

# RATIONALE

In mathematics, the concept of a function is very important and extremely useful. It appears in almost every branch of the subject. The concept in mathematics, however, has a slightly different meaning than in ordinary language. We use the word function to denote a certain specific type of correspondence between the elements of two sets. But previous to any discussion on functions one must initially be concerned with the idea of a relation.

In this LAP emphasis is placed on review and extension of concepts which are basic to the study of relations, functions, and inequalities. The basic definitions important to the study of functions will be studied more formally than in previous units. You will have enough experience with graphing and analysis of graphs so that you should be able to transfer your knowledge of functions to future studies in mathematics and to situations in other academic fields or occupational endeavors.

BEST COPY AVAILABLE

## SECTION 1

**BEHAVIORAL OBJECTIVES:**

At the completion of your prescribed course of study, you will be able to:

1. Given a pair of sets:

    a. Determine the product set (A x B)

    b. Construct the graph of the determined product set.

2. Given a set of ordered pairs and a product set, determine whether or not that set of ordered pairs in a subset of that product set.

3. Write and/or identify the definition of these terms: relation, domain, range, and function.

4. Given a relation as a set of ordered pairs, name its range and domain.

5. Write and/or identify any or all of the five ways to express a relation.

6. Given a relation, designate it by

    (a) a statement

    (b) an equation

    (c) the roster method (ordered pairs)

    (d) constructing a table

    (e) displaying its graph

    and name its domain and range. Appendix I will be completed and turned in to the teacher.

7. Given a relation, determine its inverse.

# RESOURCES I

## Objective 1

Nichols, read pp. 155-157, Ex. 1-16 pp. 157-158.

Pearson, read pp. 31-34, Ex. 1-8 pp. 34-35.

## Objective 2

Nichols, read pp. 158-160, Ex. 2 page 160.

## Objective 3

Vanatta, read pp. 99-103, Ex. write the definitions of the words in this goal.

Dolciani, MA, read pp. 203-204, Ex. ___.

Filmstrip: Relations and Functions

Transparency: 3M - Functions

## Objective 4

Vanatta, Read pp. 99-103, Ex. 10 page 105.

Dolciani, MA, read pp. 203-204, Ex. 1-8 oral page 205.

Nichols, read pp. 158-160, Ex. 2 page 160.

Wooton, MSM, read pp. 149-151, Ex. 1-10 oral page 152.

Payne, read pp. 196-198, Ex. 1-6 page 197; 1-11, 18-22 pages 198-199.

## Objective 5

Vanatta, read pp. 99-103, Ex. write the five ways to express a relation.

Dolciani, MA, read pp. 203-204, Ex. ___.

## Objective 6

Vanatta, read pp. 99-103, Ex. 1-5 pp. 103-105.

Nichols, read pp. 160-164, Ex. 1-3 pp. 163-164.

Wooton, MSM, read pp. 154-158, Ex. 1-24 pp. 158-159.

Payne, read pp. 194-195, Ex. 1-6, 11, 12, 16-20 page 195.

Wollensak teaching tapes - C-3852: Graphing Linear Functions

C-3855: Slope Intercept Form

Objective 7

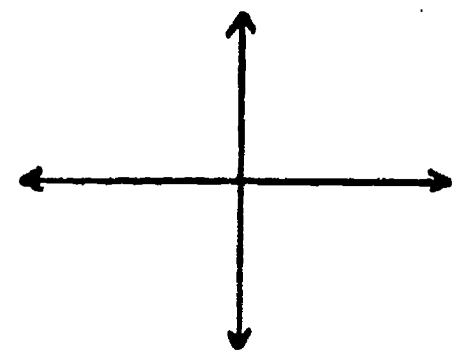
Nichols, read pp. 164-169, Ex. 1-3 page 168.

Wooton, MSM, read pp. 404-407, Ex. 1-7 (state if inverse and draw graphs) page 407.

Payne, read pp. 220-222, Ex. 1-5 page 222.

Pearson, read pp. 293-299, Ex. 1-7 pp. 300-301; 3 a - h page 306.

## SELF-EVALUATION 1



Objective 1 I. Given $A = \{2, 3, 5\}$ and $B \{3, 5\}$.

1. Find $A \times B$.

2. Graph $A \times B$.

2 II. Given $N = \{1, 2, 3, 4, \ldots\}$. Consider $N \times N$. Which of the sets below is a relation in $N \times N$?

3. $\{(0, 1), (1, 0), (2, 3)\}$

4. $\{(1, 1), (1, 2), (1, 3)\}$

5. $\{(\frac{1}{2}, 2), (2, \frac{1}{2}), (5, 5), (6, 1), (7, 7)\}$

6. $\{(-2, 2), (-6, 2), (5, 5), (6, 6)\}$

4 III. Identify the domain and range of each of the sets in Part II. Assume they are relations in $R \times R$.

7. $D =$

   $R =$

8. $D =$

   $R =$

9. $D =$

   $R =$

10. $D =$

    $R =$

3 IV. Write the definitions of the following words:

1. relation

2. domain

3. range

4. function

5 V. List the 5 ways to express a relation.

1.

2.

3.

4.

5.

6 VI.A.GIVEN: the relation $3x + 1 = y$,

1. write it in words

2. write 5 ordered pairs

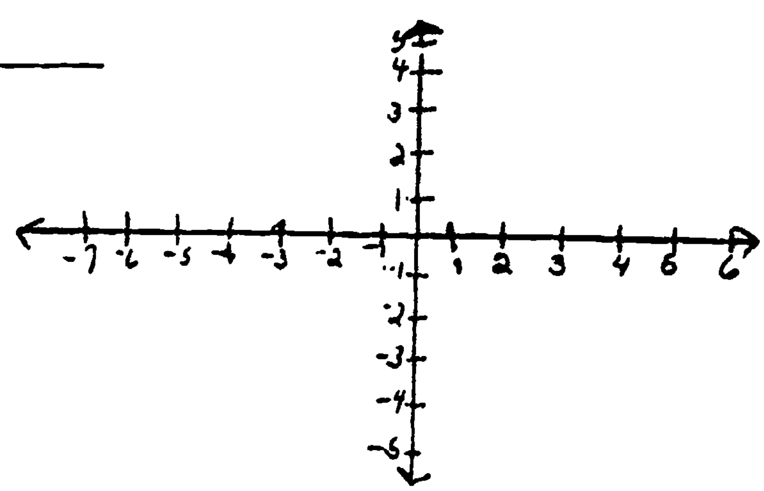
3. make a table using these ordered pairs

| | |
|---|---|
| x | |
| y | |

4. graph the ordered pairs

5. write the domain ______________

range ______________



6 B. Given the relation: (3,9)(-1,-3)(2,6)(0,0)(-3,0)

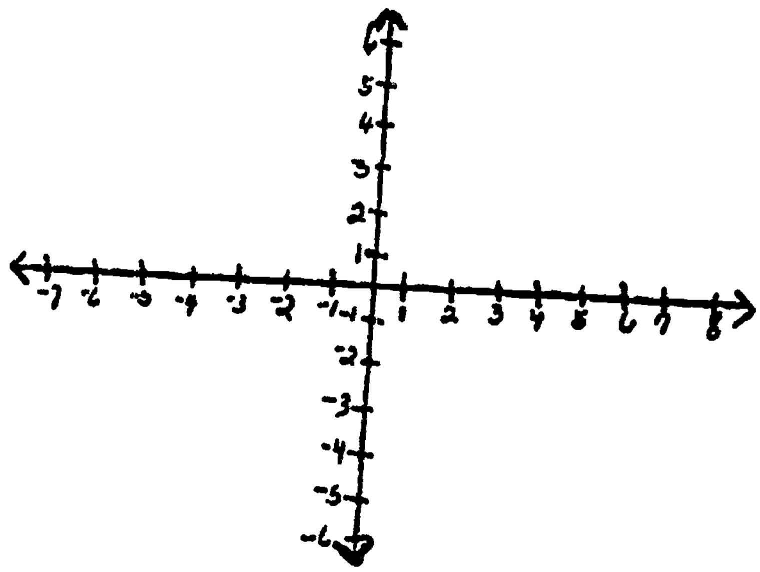
1. Write an equation ______________________________

2. Write the relation in words ______________________________

3. Construct a table

| x | y |
|---|---|
| | |

4. Graph the relation



5. Write the domain __________

range __________

4 VII. Write the domain and the range of each of the following:

DOMAIN RANGE

1. (8,1)(7,2)(-3,1)(7,-6)

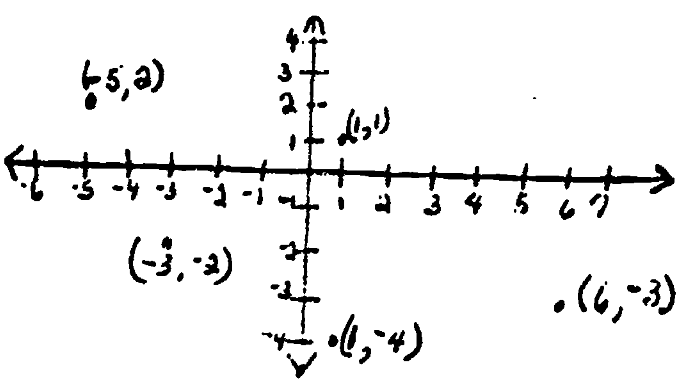
2. $y = x^2$

3.

| x | y |
|---|---|
| -8 | 5 |
| 0 | 2 |
| -6 | 7 |
| 2 | 5 |

4. y is equal to twice x

5.



7 VIII. Write the inverse of the following relations:

(a) $\{(2,3), (4,4), (6,1)\}$ ______________________

(b) $x + 2 = y$ ______________________

(c) $y = x^2$ ______________________

(d) $\{(-1,4), (5,-7), (2,-3), (4,-6)\}$ ______________________

If you have satisfactorily completed your work, take the PROGRESS TEST.

## SECTION 2

BEHAVIORAL OBJECTIVES:

At the completion of your prescribed course of study, you will be able to:

8. Given a relation, decide if that relation is a function or not.
9. Given any function, determine the value of the function for any given number.
10. Given a function, name its inverse and decide if its inverse is itself a function.
11. Apply the vertical line test to determine if a relation is a function.
12. Given two functions f and g over the reals and a real number "a" determine the following:
    (a) $a \cdot f(x)$
    (b) $f(a \cdot x)$
    (c) $f(x) + g(x)$
    (d) $f(x) \cdot g(x)$
    (e) $f(g(x))$
    (f) $g(f(x))$
    (g) $f(a)$
13. Given a function f, be able to identify it as:
    (a) a constant function
    (b) the identity function
    (c) the greatest integer function
    (d) a linear function
    (e) a non-linear function
14. Given a proportion function:
    (a) Identify it as a direct proportion function or as an inverse proportion function.
    (b) Find its constant of proportionality (constant of variation)
15. Construct the graph of an inequality of degree 1 (i.e. a relation which is a subset of the product set R x R).

## Objective 8

Vanatta, read 105-106, Ex. 3, 5, 7, 8 page 107.

Dolciani, Modern Algebra, read pages 207-208. Ex. 1-24 even pages 208-209.

Nichols, read pages 169-173, Ex. 1 page 171.

Payne, read pp. 199-201, 220-222, Ex. 1-3 (checkpoint) and 1-9 page 101; 11 page 206; 1-5 page 222.

Wooton, MSM, read pages 154-157, Ex. 5-16 pages 157-158; 13-16 page 153.

Pearson, read pages 273-275, Ex. 1, 2, 5 page 276.

Filmstrip: Relations and Functions

Transparencies 3M: Functions

## Objective 9

Vanatta, read pages 108-110, Ex. 1-4, 12-15, 20, 26, 28 pages 110-111.

Dolciani, MA, read pages 207-208, Ex. 1-16 page 209.

Payne, read pages 199-201, Ex. 1-5 page 201, 16-21 page 203.

Wooton, MSM, read pages 149-151, Ex. 11-18 oral page 152, 1-8 written page 152.

Pearson, read pp. 277-280, Ex. 1-3 pages 280-281.

## Objective 10

Vanatta, read pages 112-113, Ex. 1, 3-8 page 114; 9 page 126.

Nichols, read pp. 169-173, Ex. 3 page 173; 2, 3 page 268.

Payne, read pages 220-222, Ex. 1-20 even page 223; 28-40 pages 224-225; 8-17 and 21-23 page 226.

Pearson, read pp. 293-299, Ex. 1-7 page 300.

## Objective 11

Nichols, read pages 169-173, Ex. 2 pages 172-173.

Objective 12

Nichols, read pp. 173-176, Ex. 1-7 pages 175-176.

Wooton, MSM, read pp. 152-153, Ex. 1-34 pages 152-153.

Payne, read pp. 215-217, Ex. 1-25 pages 218-219.

Pearson, read pp. 277-280, 288-291, 316-319, Ex. 1-10 pages 280-281; 1-7 page 292; 1-10 pages 319-321.

Objective 13

Nichols, read pp. 176-179, Ex. 1-7 page 178.

Wooton, MSM, read pp. 183-183, 187-188, Ex. 1-24 pages 184-185; 1-15 page 188.

Payne, read pp. 206-208, Ex. 1-46 pp. 209-212.

Pearson, read pp. 302-305, Ex. 1-2 page 305.

Objective 14

Nichols, read pp. 179-185, Ex. 1-5 pages 185-186.

Wooton, MSM, read pp. 180-183, Ex. 1-24 pages 184-185.

Payne, read pp. 206-208, Ex. 1-46 pages 209-212.

Pearson, read pp. 307-308, 310, Ex. 1-14 pages 308-309; 1-13 page 3111.

Filmstrip: Direct Variation

Objective 15

Vanatta, read pp. 121-123, Ex. 2, 3, 6, 7 page 124.

Nichols, read pp. 186-190, Ex. 1-2 pages 189-190.

Wooton, MSM, read pp. 164-167, Ex. 1-28 page 167; 1-16 pages 188-189.

Payne, read 441-442, Ex. 1-14 page 442.

Pearson, read pp. 314-315, Ex. 1-7 pages 315-316.

Wollensak teaching tape, C-3806: Inequality and Equality Sentences

Filmstrip: Graphs of Inequalities in One Variable

BEST COPY AVAILABLE

## SELF-EVALUATION 2

Obj.

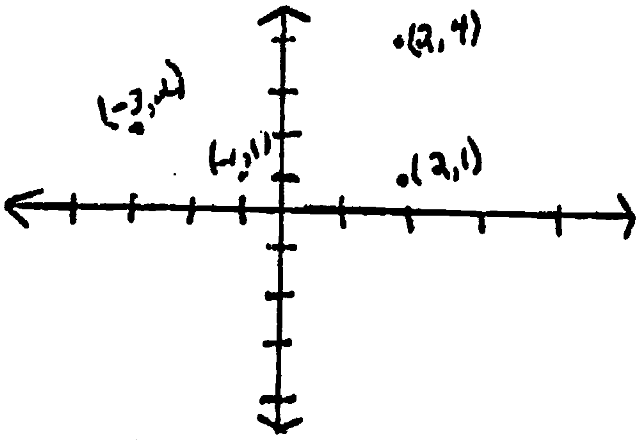
8 I. Determine if each of the following is a function. Write F for function if it is a function. If it is not a function, write R for relation only.

_____ 1. (4,1)(6,8)(2,1)(-4,3)

_____ 2.

| x | -6 | -2 | -4 | -6 |
|---|---|---|---|---|
| y | 8 | 1 | 3 | 7 |

_____ 3.




_____ 4. y = x + 1

_____ 5. (3,-3)(4,-4)(5,-6)(7,-8)(3,-9)

_____ 6.

| x | y |
|---|---|
| 0 | 7 |
| 7 | 0 |
| 3 | 5 |
| 9 | 4 |
| 5 | 3 |

9 II. For each of the following functions, find the value indicated.

_______________ 1. Find f(2) for $f(x) = 6x + 1$

_______________ 2. Find f(-3) for $f(x) = 2x - 1$

_______________ 3. Find f(0) for $f(x) = \frac{x + 1}{6x}$

_______________ 4. Find f(30) for $f(x) = x^2 - x$

_______________ 5. Find f(-10) for $f(x) = \frac{8 - 2x}{4}$

10 III. Write the inverse of each of the following. State whether the inverse is a function or only a relation. Circle F or R.

_______________ F or R 1. $y = 7x + 6$

_______________ F or R 2.

| x | -6 | -6 | -6 | -6 | -6 |
|---|---|---|---|---|---|
| y | 4 | 2 | 8 | 9 | 7 |

________________ F or R 3. $y = x$

________________ F or R 4. $y = -6 - 2x$

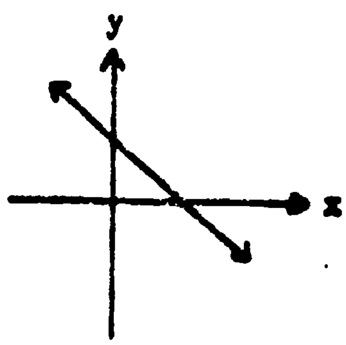
________________ F or R 5. (4,7)(-3,2)(9,8)(2,-4)(4,8)

________________ F or R 6.

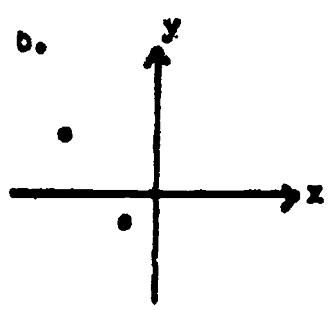
| x | -2 | -4 | 0 | 10 | -3 | -2 |
|---|---|---|---|---|---|---|
| y | 7 | 3 | 10 | 0 | 8 | 11 |

11 IV. Which of the following is not the graph of a function? (Circle the correct answer)

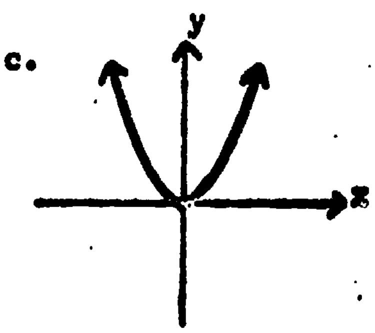
a.



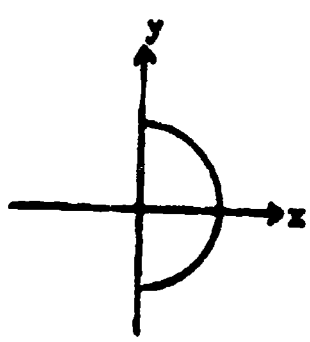

b.



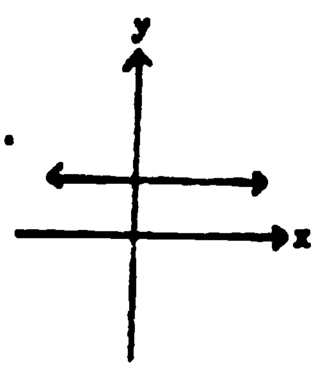

c.




d.




e.




12 V. Given $f(x) = x^2$ and $g(x) = x + 2$, find the following:

1) $f(x) \cdot g(x) =$

2) $f(g(x)) =$

3) $g(f(x)) =$

4) $2 \cdot g(x) =$

5) $g(2x) =$

6) $f(x) + g(x) =$

7) $f(100) =$

13 VI. Classify the functions below as linear, non-linear constant, greatest integer, or identity. A function may have two such classifications.

______________ 1. $f(x) = x^2$

______________ 2. $f(x) = a$ where a is a real number

______________ 3. $f(x) = 2x$

______________ 4. $f(x) = [x]$

______________ 5. $f(x) = x$

14 VII. Identify each of the following functions as direct or inverse proportional functions; then find the constant of proportionality.

__________ 1. $f(x) = \frac{2x}{4}$

__________ 2. $f(x) = \frac{\frac{1}{2}}{x}$

13 VIII. Which of the following when in R x R, are not linear functions?

a. a direct proportion function

b. the greatest integer function

c. the identity function

d. the function defined by $y = 2x + 3$

13,14 IX. Choose from Column B a graph of the type of function given in Column A.

COLUMN A

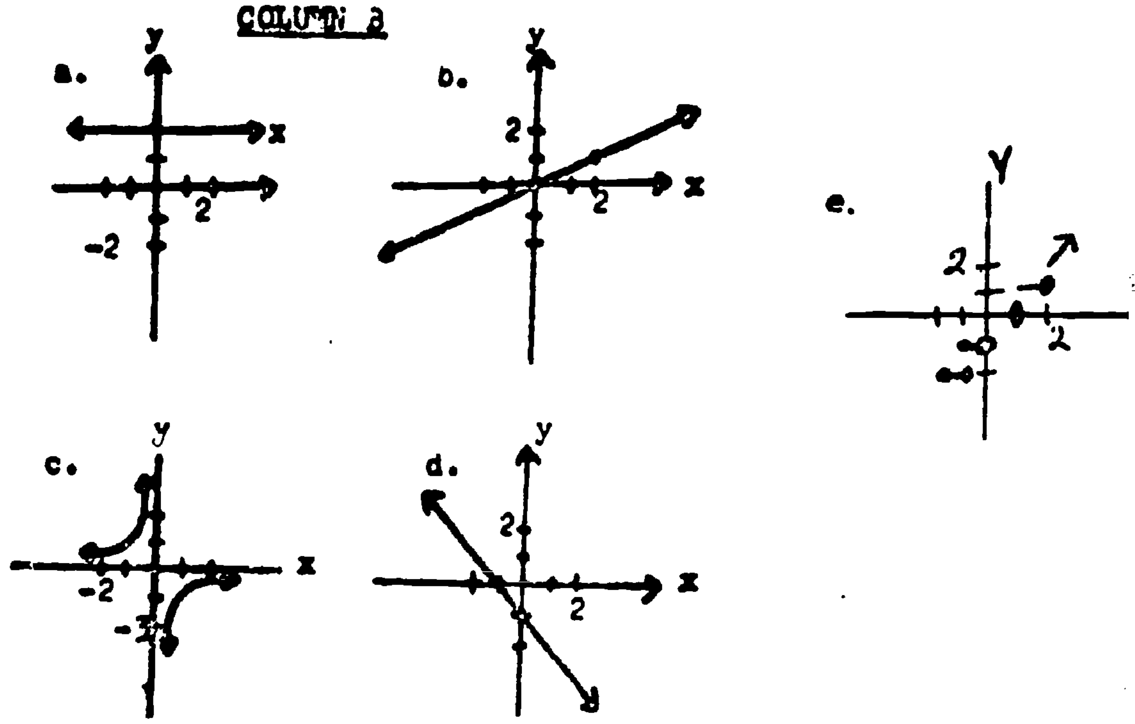
_____ 32. Inverse proportion function

_____ 33. Constant function

_____ 34. Direct proportion function

_____ 35. Greatest integer function

COLUMN B



## SELF-EVALUATION 2 (cont')

15 X. Graph the following on the graph paper included. (next page)

(1) $y > 3x + 1$

(2) $y \geq -2x$

(3) $y < -4x + 1$

(4) $y \leq -x - 3$

If you have satisfactorily completed your work, take the LAP test. CONSULT YOUR TEACHER FIRST.

BEST COPY AVAILABLE

## APPENDIX 1

I. Given the relation: 5 more than twice x is equal to y

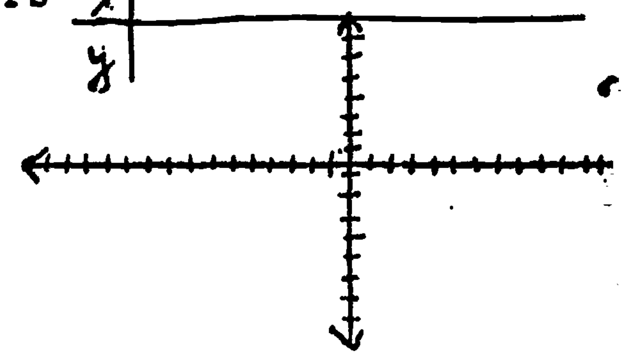
a. write an equation ____________________

b. write 5 ordered pairs ____________________

c. construct a table using the 5 ordered pairs

| x | |
|---|---|
| y | |

d. graph the ordered pairs



II. Given the relation: (4, 8) (5, 10) (2, 4) (-1, -2) (-2, -4)

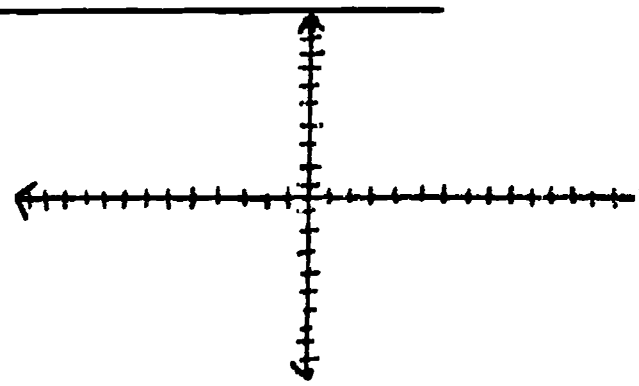
a. write an equation ____________________

b. write the relation in words ____________________

c. construct a table

| x | y |
|---|---|
| | |

d. graph the relation



III. Given the relation:

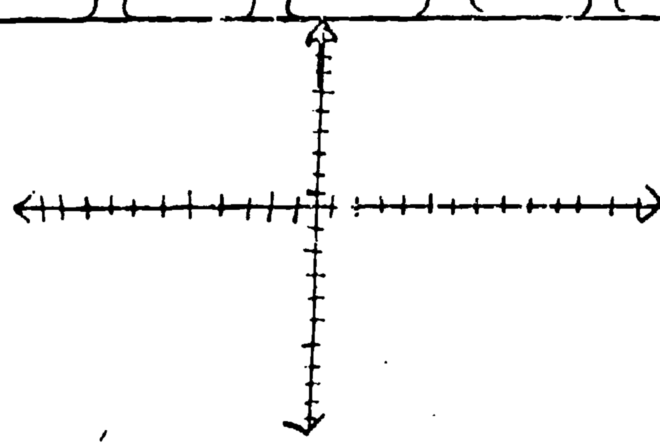
| x | y |
|---|---|
| -2 | 0 |
| -1 | 1 |
| 2 | 4 |
| 3 | 5 |
| 6 | 8 |

a. write an equation ____________________

b. write the relation in words ____________________

c. write the table in ordered pairs ( ) ( ) ( ) ( ) (

d. graph the relation



## ADVANCED STUDY

I. Newton's Law of Universal Gravitation is expressed as follows:

$F_{(grav)} = \frac{G\,M\,m}{R^2}$ where $F_{(grav)}$ is the attractive force in newtons between two masses (M) and (m). These masses are expressed in kilograms. R is the distance between the two centers of mass and is expressed in meters. G is the proportionality constant with a value of

$$G = 6.67 \times 10^{-11}\ N \cdot m^2/kg^2.$$

Find the force (F) that the earth with mass $M = 5.98 \times 10^{24}$ kg exerts on a body of mass m = 10 kg located at its surface. Radius of earth

$$R = 6.38 \times 10^{6} \text{ meters}$$

II. Payne, page 225, no. 41

III. Dolciani, page 210, nos. 31-40

IV. Dolciani, read p. 218, Ex. 1-16 any 6 page 219

V. Vanatta, page 121 no. 8, page 112 no. 30, page 104 no. 8.

## REFERENCES

Nichols (abbreviation)

Nichols, Eugene, D., MODERN INTERMEDIATE ALGEBRA, Holt, Rinehart and Winston, Inc., 1965.

Pearson (Abbreviation)

Pearson, Helen R., and Allen, Frank B., MODERN ALGEBRA: A LOGICAL APPROACH, Book 2, Ginn and Company, 1966.

Payne (abbreviation)

Payne, Joseph N., Zamboni, Floyd F., Lankford, Jr., Francis, Algebra Two, Harcourt, Brace and World, Inc., 1969.

Wooton (Abbreviation)

Dolciani, Mary P., Wooton, William, Beckenback, Edwin F., Sharron, Sidney, MODERN SCHOOL MATHEMATICS, ALGEBRA 2, Houghton Mifflin Company, 1968.

Dolciani (abbreviation)

Dolciani, Mary P., Berman, Simon L., Freilich, Julius MODERN ALGEBRA, Book 2, Houghton Mifflin Co., 1965.

Vanatta (abbreviation)

Vanatta, Glen D., Goodwin, A. Wilson, Algebra Two, A Modern Course, Charles E. Merrill Publishing Co., 1966.

Wollensak teaching tapes: C-3806 Inequality and Equality Sentences

C-3852 Graphing Linear Functions

C-3855 Slope-Intercept Form

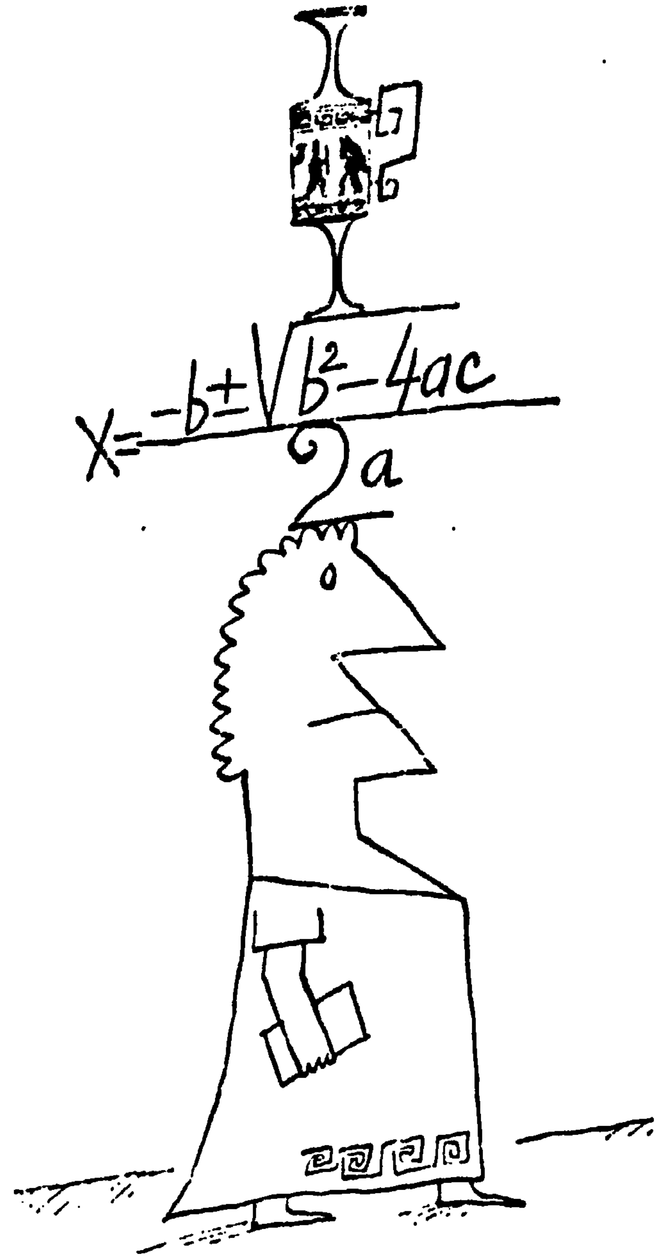
Filmstrips: 1. Relations and Functions

2. Direct Variation

3. Graph of Inequalities in One Variable

Transparencies 3M - Functions

# LEARNING ACTIVITY PACKAGE




## QUADRATIC EQUATIONS AND INEQUALITIES

Ninety Six High School - Ninety Six, S.C.

Algebra 103-104

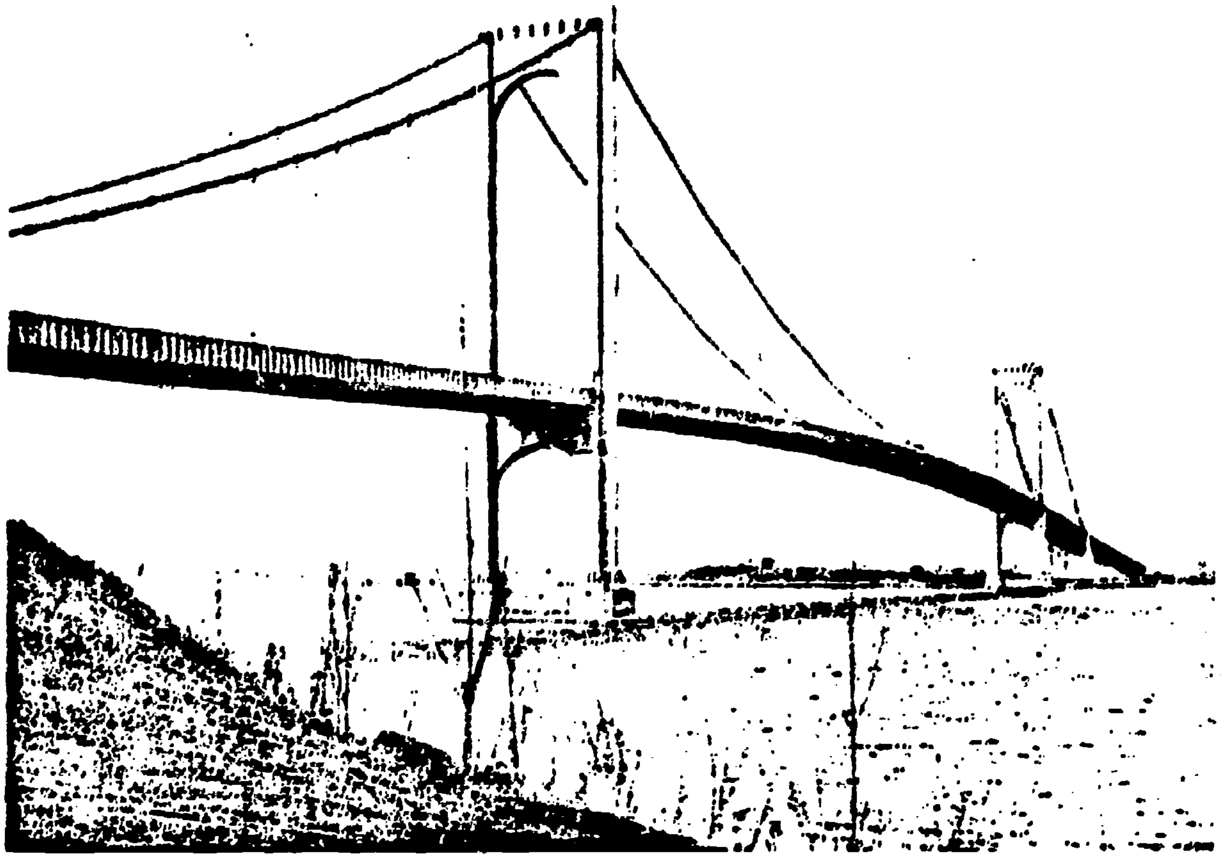
REVIEWED BY

LAP NUMBER 29

WRITTEN BY Diane Evans

## RATIONALE





In the preceding LAP, you studied linear functions which were defined by linear equations in two variables. Unfortunately, nature was not so kind, so in your study of science you will need a working knowledge of all forms of quadratic equations and inequalities. For example, the cable of the bridge in the picture above forms a parabolic curve and can be reduced to a quadratic equation.

In this LAP we will study all quadratic equations and inequalities and some other kinds of equations which are expressible as quadratic equations and inequalities.

## SECTION 1

### Behavioral Objectives

At the completion of your prescribed course of study, you will be able to:

1. Define quadratic equation or quadratic function.
2. Sketch or identify the graph of a quadratic function.
3. Given any factorable quadratic function, determine its roots.
4. Given any quadratic function that is not factorable, determine its roots in simplest form by completing the square.
5. State and/or identify the quadratic formula.
6. Given the equation $Ax^2 + Bx + C = 0$, derive the quadratic formula.
7. Given any equation that is not factorable, determine its solutions by substituting into the quadratic formula.

### RESOURCES

Obj 1

Vanatta, read p. 163, write the definition of quadratic equation.

Nichols, read p. 221, write the definition of quadratic equation,

Dolciani, read p. 220, write the definition.

Payne, read p. 251, write the definition.

Pearson, read p. 337, write the definition.

Obj. 2

Vanatta, read pp. 163-164, ex. 1-10 even page 164.

Dolciani, read p. 220, ex. 5-10 page 234.

Payne, read pp. 251-253, ex. 1-6 page 153.

Pearson, read pp. 337-340, ex. 1 a b, 2, 3 b c e (draw graphs only) p. 340.

Obj. 3

Vanatta read pp. 165-166, Ex. 2, 5, 6, 7, 10, 12, 14, 17, 18 page 167.

Dolciani, ____, ex. 1-8 even, 15, 18, 22, 28 page 136.

Wooton, read pp. 265-268, ex. 1-26 every fourth problem.

Pearson, read pp. 176-177, ex. 3 page 178.

Obj. 4

Vanatta, read pp. 167-170, ex. 1, 3, 7, 10, 14 page 172.

Dolciani, read pp. 268-270, 1, 3, 5, 6, 8 written pages 270-271.

Wooton, read pp. 337-340, ex. 1-8 page 340.

Payne, read pp. 257-259, ex. 1-10 even p. 259; 19-32 odd p. 260.

Pearson, read pp. 355-357, ex. 1-12 even page 183 (solve by completing the square)

Obj. 5

Vanatta, read p. 170, state the quadratic formula.

Dolciani, read p. 269, state the quadratic formula.

Nichols, read pp. 224-225, state the quadratic formula.

Wooton, read p. 339, state the formula.

Payne, read pp. 260-261, state the formula.

Obj. 6

Exercise for all books: Derive the quadratic formula.

Vanatta, read pp. 269-170, ex. above.

Dolciani, read p. 268, ex. above.

Nichols, read pp. 224-225, ex. above.

Payne, read p. 260, ex. above.

S-M Transparency 6M - The Quadratic Formula

Obj. 7

Vanatta, read pp. 170-171, ex. 2, 4, 5, 8, 11 p. 172 solve by using quadratic formula.

Dolciani, read pp. 268-270, ex. 10, 13, 15, 18, 20 written p. 270.

Nichols, read pp. 224-226, ex. 2 every other letter page 227.

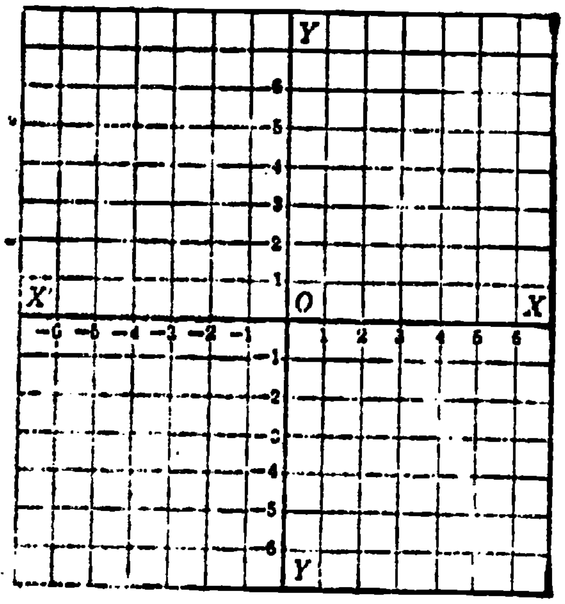
Wooton, read pp. 337-340, ex. 9-20 even page 341.
Payne, read pp. 260-261, ex. 1-20 odd page 262.
Pearson, read pp. 355-357, ex. 1, a,b,c,e,g, 3 a,b,c,d page 358.

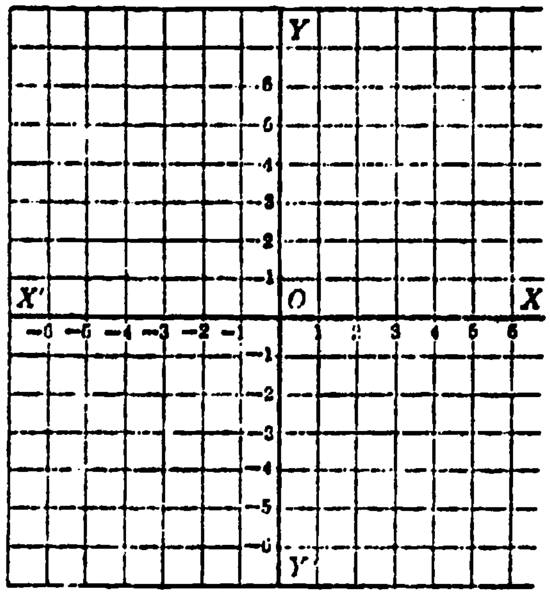
## SELF-EVALUATION 1

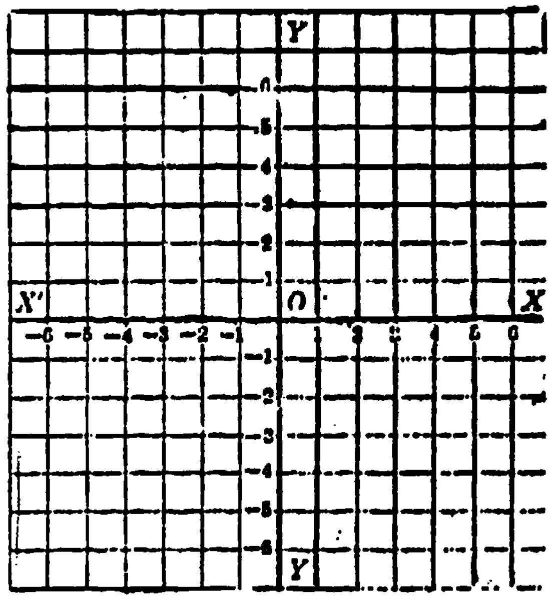
I. Define: quadratic equation.

II. Graph the following.

(A) $x^2 - 3x - 4 = y$ (b) $y = 3x^2 + 17x + 20$ (c) $y = -2x^2 - 3x + 2$







III. Solve the following by factoring.

__________ 1. $x^2-3x+2=0$

__________ 3. $x^2+4bx-12b^2=0$

__________ 2. $6x^2-5x+1=0$

__________ 4. $\frac{x^2-12}{3} = \frac{x^2-4}{4}$

IV. Find the roots of each of the following by completing the square. Show your work.

__________ 1. $x^2+11x+24=0$

__________ 2. $x + \frac{1}{x-1} = \frac{9}{2}$

__________ 3. $x^2+5x-7=0$

__________ 4. $3x^2+8x+2=0$

V. State the quadratic formula.

VI. Derive the quadratic formula. Begin with $Ax^2 + Bx + C = 0$.

VII. Find the roots of the following by substituting into the quadratic formula. SHOW YOUR WORK!

_____________ (1) $3r^2+r-1=0$ _____________ (2) $x^2-2x-1=0$

_____________ (3) $6x^2+10x+3=0$ _____________ (4) $5x^2+x+1=0$

_____________ 5. $\frac{1}{2} - \frac{7}{6}x = x^2$

If you have satisfactorily completed your work, you may take the progress test. Consult your teacher first.

## SECTION 2

Behavioral Objectives

At the completion of your prescribed course of study, you will be able to:

8. Write and/or identify the definition of imaginary numbers.

9. Given any square root you will be able to determine whether it is imaginary or real. If it is real, you will be able to determine if it is rational or irrational.

10. Given a quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$:

    a. Find the sum of the roots of the equation.

    b. Find the product of the roots of the equation.

11. Given a quadratic equation of the form $ax^2 + bx + c = 0$, $a \neq 0$:

    a. Name the discriminant of the equation.

    b. Specify the number of real roots of the equation.

    c. Give the nature of the roots by determining:

        1. If the roots are real or imaginary.

        2. If the roots are real determine if they are rational or irrational.

        3. If the roots are equal or unequal.

        4. How many times the graph will touch the x axis.

12. Write a quadratic equation $ax^2 + bx + c = 0$:

    a. Having a given set $\{r, s\}$ as its solution set.

    b. When given the sum and product of its roots.

    c. When certain coefficients are unknown and sufficient information about the roots of the equation is given to find these coefficients.

13. Given a fractional equation:

    a. Write the corresponding quadratic equation. (assuming one exixts)

    b. Find the solution set of the quadratic equation.

BEST COPY AVAILABLE

Obj. 8

Vanatta, read page 166, ex. write the definition of imaginary numbers.

Payne, read pp. 24-25, ex. write the definition of imaginary numbers.

Obj. 9

Vanatta, read pp. 166, 30-32, ex. Appendix I.

Payne, read pp. 24-25, ex. 1-27 even page 26.

Filmstrip: Rational and Irrational Numbers.



Obj. 10

Vanatta, read pp. 172-173, ex. 1-10 even pages 173-174.

Dolciani, read page 273, ex. 1-15 odd (oral) page 274.

Nichols, read pp. 228, ex. 2 pages 229-230.

Payne, read pp. 267-268, ex. 1-6 page 268.

Wooton, read pp. 343-344, ex. 1-9 page 344.

Pearson, read pp. 359-362, ex. 2 page 362.

Obj. 11

Vanatta, read p. 175-177, ex. write the discriminant of the equation $ax^2 + bx + c = 0$; ex. 1-10 page 177.

Dolciani, read pp. 275-278, ex. write the discriminant of $ax^2 + bx + c = 0$; 1-10 page 278.

Payne, read pp. 264-266, ex. 1-10 page 266.

Wooton, read pp. 374-376, ex. 1-14 page 377.

Obj. 12

Vanatta, read pp. 172-173, ex. 11, 14, 16, 18, 20 page 173.

* Dolciani, read pp. 273-274, 16-23 even page 274 top; ex. 1, 2, 6, 13, 15, 16, 18, 21, 23 bottom pages 274-275.

Nichols, read pp. 228-229, ex. 1, 3-6 pages 229-230.

Payne, read pp. 246-271, ex. 1-14 pp. 266-267; 1-14 p. 269; 1-18 p. 271.

Wooton, read pp. 343-344, ex. 16-23 p. 345; 1-14 even, 19-24 pages 345-346.

Obj. 13

Nichols, read pp. 230-232, ex. 1, 2 pages 232-233

Pearson, read pp. 365, ex. 4, 6 pages 368-369.

* required

# SELF-EVALUATION 2

Obj.

8 I. Write the definition of imaginary numbers.

9 II. State whether each of the following is real or imaginary. If real, state if it is rational or irrational.



__________ 1. $\sqrt{18}$

__________ 2. $\sqrt{-4}$

__________ 3. $\sqrt{24}$

__________ 4. $\sqrt{25}$

__________ 5. $\sqrt{-100}$

10 III. Write the sum of the roots of the following:

__________ 6. $y^2 - 2y - 9 = 0$

__________ 7. $2x^2 + 3x = 0$

__________ 8. $2x^2 - 8x - 1 = 0$

__________ 9. $3x^2 + 15x + 2 = 0$

10 IV. Write the product of the roots of the following:

__________ 10. $y^2 - 2y - 9 = 0$

__________ 11. $2x^2 + 3x = 0$

__________ 12. $2x^2 - 8x - 1 = 0$

__________ 13. $3x^2 + 15x + 2 = 0$

11 V. In each of the following equations:

(a) determine the value of the discriminant.
(b) specify the numberof real roots of the equation.
(c) determine if the roots are real or imaginary.
(d) if the roots are real, determine if they are rational or irrational.
(e) state if the roots are equal or unequal.
(f) state how many times the graph touches the x-axis.

14. $x^2 + 4x + 3 = 0$

15. $x^2 - 5x + 7 = 0$

16. $3x^2 - 2x = 4$

17. $-5x^2 - x - 3 = 0$

18. $-3x^2 + 4x + 1 = 0$

19. $3x^2 - 4x + \frac{4}{3} = 0$

12a VI. Write an equation for each of the following solution sets.

______________ 20. $\{5, -7\}$

______________ 21. $\{0, \frac{1}{3}\}$

______________ 22. $\{-\frac{1}{7}, \frac{-1}{9}\}$

______________ 23. $\{\sqrt{2} + \sqrt{3}, \sqrt{2} - \sqrt{3}\}$

______________ 24. $\left\{\frac{-1 + \sqrt{2}}{2}, \frac{-1 - \sqrt{2}}{2}\right\}$

12b VII. Given the following sum and product of roots, write an equation.

______________ 25. sum = -7 product = 3

______________ 26. sum = 9 product = -2

______________ 27. sum = $\sqrt{3}$ product = 0

__________ 28. sum = 0          product = -7

12c VIII. Find the real values to satisfy the conditions given.

__________ 29. For what value(s) of "a" will the sum of the roots be 8?

$$x^2 - (a^2 - 2a)x + 3 = 0$$

__________ 30. For what value(s) of "b" will the equation $2x^2 + 4x + (2 - b - b^2) = 0$ have exactly one root?

13 IX. For each of the following fractional equations:

(a) Write the corresponding quadratic equation (assuming one exists).
(b) Find the solution set of the quadratic equation.

31. $\frac{1}{x} + \frac{x - 1}{x(x + 2)} = \frac{-x}{x + 2}$

32. $x - 4 = \frac{-1}{x}$

If you have satisfactorily completed your work, you may take the PROGRESS TEST. Consult your teacher first.

## SECTION 3

### Behavioral Objectives

At the completion of your prescribed course of study, you will be able to:

14. Given a radical equation:
    a. Write the corresponding quadratic equation. (assuming one exists)
    b. Find the solution set of the quadratic equation.
    c. Find the solution set of the radical equation.
    d. State whether the two equations are equivalent.
    e. Name the roots of the quadratic equation which are not permissible roots of the radical equation.

15. Given any quadratic inequality:
    a. Find the solution set of the inequality.
    b. Graph the solution set on a number line.

16. Given a word problem solvable by means of a quadratic equation:
    a. Translate the problem into a quadratic equation.
    b. Solve the problem.

17. Given a word problem solvable by means of a fractional equation:
    a. Translate the problem into a quadratic equation.
    b. Solve the problem.

Obj. 14

Work one set of problems.

Vanatta, read pp. 258-260, ex. 1-14 page 260.

Nichols, read pp. 233-235, ex. 1, 2, pages 235-236.

Payne, read pp. 360-363, ex. 1-26 even pages 361-362; 1-25 even pp. 363-364.

Pearson, read pp. 370-371, ex. 1-3 page 371.

Wooton, read pp. 334-336, ex. 1-36 even page 336.

Dolciani, read pp. 281-282, ex. 1, 3, 13, 15, 17, 24, 26 pages 282-283.

Obj. 15

Vanatta, read pp. 205-206, ex. 1-8 page 208.

Nichols, read pp. 239-243, ex. 1-3 pages 243-245.

Payne, read pp. 276-278, ex. 3-8 (bottom) page 278.

Wooton, read pp. 362-363, ex. 1-16 even page 364.

Dolciani, read pp. 279-280, ex. 1-8 page 280.

Obj. 16

Vanatta, read pp. 179-181, ex. 1, 2, 4, 7, 15, 19 pages 181-183; 13, 14, page 211.

Nichols, read pp. 221-226, ex. 4-12 pages 227.

Wooton, read pp. 337-340, ex. 1, 2, 7 page 342; 1, 2, 4, 7, 13 pages 269-270.

Dolciani. ____, ex. 32, 38 page 201; 1-4, 8-13 pages 137-138.

Goal 17

Vanatta, read pp. 179-181, ex. 5, 6 page 182; no. 18 page 160.

Nichols, read pp. 230-232, ex. 1, 2 pages 232-233.

Dolciani, read pp. 178-179, ex. 10, 19, page 182.

## SELF-EVALUATION 3

Obj.

14 I. For each of the following radical equations:

a. Write the corresponding quadratic equation (assuming one exists).
b. Find the solution set of the quadratic equation.
c. Find the solution set of the radical equation.
d. State whether the two equations are equivalent.
e. Name the roots of the quadratic equation which are not permissible roots of the radical equation.

(1) $x - 3 = \sqrt{2x - 3}$

(2) $\sqrt{3x + 2} = 3\sqrt{x} - \sqrt{2}$

(3) $5x - \sqrt{2x + 1} = 4x + 1$

(4) $\sqrt{x + 4} + \sqrt{x - 3} = 7$

15 II. Solve the following inequalities and graph their solution sets on the real number line.

(5) $3x^2 - 5x - 4 < 0$

(6) $2x^2 + 5x < 3$

(7) $x + 3x \leq 10$

(8) $3x - x^2 \geq 0$

16 III. Write the equation and solve the following word problems.

(9) Find the 2 consecutive positive integers whose product is 756.

(10) Find the length of a side of a square if the length of a diagonal is 5 units greater than the length of a side.

(11) The length of a rectangle is 4 feet more than twice the width. If the area of the rectangle is 30 square feet, find the length and width.

(12) The rug in a bedroom is 9 feet by 12 feet. If the area of the rug is 154 sq. feet, how wide is the strip of bare floor around the rug if the bare strip is of uniform width?

17 IV. Write the equation and solve the following.

(13) A certain integer increased by 4 times its reciprocal equals $8\frac{1}{2}$. Find the number.

(15) Jim can pick a bushel of apples in 25 minutes. Sam can pick a bushel in 15 minutes. How long will it take the boys to pick a bushel together?

If you have satisfactorily completed your work, you may take the LAP test. Consult your teacher first.

# APPENDIX I

State whether each of the following is real or imaginary. If real, state if it is rational or irrational. Write each in simplest form.

1. $\sqrt{144}$

2. $\sqrt{-81}$

3. $\sqrt{24}$

4. $\sqrt{-100}$

5. $\sqrt{70}$

6. $\sqrt{36}$

7. $\sqrt{-1}$

8. $\sqrt{-126}$

9. $\sqrt{25}$

10. $\sqrt{-25}$

ADVANCED STUDY

1. Wooton, page 342 nos. 49, 50
2. Payne p. 264 nos. 55-62
3. Dolciani, p. 271 nos. 49, 50
4. Wooton, page 346, nos. 26-28
5. Wooton, page 342 nos. 49, 50
6. Pearson, page 363, nos. 9-14
7. Dolciani, pages 283-284, nos. 43-46 any 3 problems.

# REFERENCES



Vanatta (abbreviation)

Vanatta, Glen D., and Goodwin, Wilson A., Algebra Two, A Modern Course, Charles E. Merrill Publishing Co., 1966.

Dolciani (abbreviation)

Dolciani, Mary P., Berman, Simon L., and Wooton, William, Modern Algebra, Book Two, Houghton Mifflin Co., 1965.

Nichols (abbreviation)

Nichols, Eugene D., Modern Intermediate Algebra, Holt, Rinehart and Winston, Inc., 1965.

Pearson (abbreviation)

Pearson, Helen R., and Allen, Frank B., Modern Algebra: A Logical Approach, Book Two, Ginn and Company, 1966.

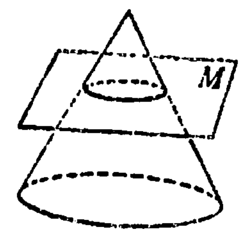
Payne (abbreviation)

Payne, Joseph N. Zamboni, FLoyd F., Lankford, Jr., Francis G., Algebra Two, Harcourt, Brace and World, 1969.

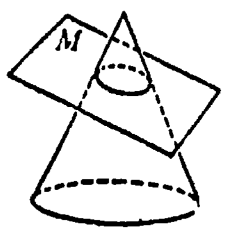
Wooton (abbreviation)

Dolciani, Mary P., Wooton, William, Beckenbach, Edwin F., Sharron, Sidney, Modern School Mathematics, Algebra 2, Houghton Mifflin Company, 1968.

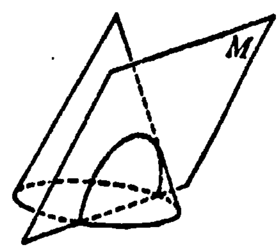
# LEARNING ACTIVITY PACKAGE



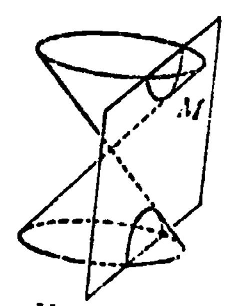

Circle




Ellipse




Parabola




Hyperbola

Ninety Six High School - Ninety Six, S.C. -

## QUADRATIC FUNCTIONS

Algebra 103-104

REVIEWED BY

LAP NUMBER 30

WRITTEN BY Diane Evans

41073 4

# RATIONALE

In preceding LAPs you studied functions in general, and more specifically, the straight line. Recall that Descartes is credited with originating the Cartesian coordinate system. In the concept of coordinates, Descartes gave mathematicians a new way to look at mathematical information. Not only did he show that first degree, or linear, equations can be graphed as straight lines, but he also showed that all second degree, or quadratic equations can be graphed to become circles, ellipses, parabolas, or hyperbolas. These quadratic functions are collectively referred to as conic sections.

Conics appear frequently in nature and in numerous applications; for example, the orbits of planets about the sun are ellipses. The supporting cables of a suspension bridge form a parabola. The hyperbola appears as the edges of the shadow cast on a wall by a lampshade. In this LAP we will investigate the graphs of these quadratic functions in some detail. In addition, we will study quadratic inequalities.

**Behavioral Objectives**

At the completion of your prescribed course of study, you will be able to:

1. Given a relation in R x R, determine whether or not it is a quadratic function.
2. Write and/or identify the definition of <u>conic section</u>.
3. List and/or identify the four conic sections.
4. Describe and/or identify the descriptions of the following terms as they relate to a cone:
   a. element
   b. axis
   c. circle
   d. ellipse
   e. parabola
   f. hyperbola
5. Write and/or identify:
   a. the definition of a circle
   b. the standard form of the equation of a circle with radius r and center at the origin.
   c. The standard form of the equation of a circle with radius r and center (h,k).
6. Given the equation of a circle, write and/or identify:
   a. the center
   b. the radius
   c. the graph of the circle
7. Given the center and radius of a circle,
   a. graph and/or identify the curve
   b. write and/or identify the equation in standard form

## RESOURCES 1

### Objective 1

* Nichols, read pp. 195-199, Ex. 1 a-d page 199-200.

  Pearson, read pp. 337-339, Ex. 2 page 340.

### Objectives 2, 3, 4

Nichols, read pp. 312-313, Exercise Appendix I.

Wooton, read pp. 456-457, Exercise Appendix I.

Dolciani, read pp. 330-331, Ex. Appendix I.

Vanatta, read pp. 183-185, Ex. Appendix I.

Pearson, read pp. 697, Ex. Appendix I.

Payne, read page 417, Ex. Appendix I.

### Objective 5

Exercise for all books: Write the definition and equations in Objective 5.

* Vanatta, read pp. 191-192, Ex. above.

  Dolciani, read pp. 300-302, Ex. above.

  Wooton, read pp. 442-443, Ex. above.

  Payne, read pp. 418-419, Ex. above.

  3M Transparency: <u>Circle</u>

### Objectives 6, 7

* Vanatta, read pp. 191-193, Ex. 1, 3, 5, 6, 8, 10 page 194; 11-16 page 194.

  Dolciani, read pp. 300-302, Ex. 11-16 page 302; 1-4, 9, 10 pp.300-302 (graph and write equations).

  Wooton, read pp. 442-443, Ex. 13-17, 20 page 443; 1-8 page 443 (graph and write equations).

  Payne, read pp. 418-419, Ex. 1-6 page 419; 3, 4, 6, 9-12 page 420.

  3M Transparency: <u>Circle</u>

* required

## SELF-EVALUATION 1

I. True or False.

__________ 1. A conic section is the set of points determined by a plane intersecting a cone.

__________ 2. An axis is a straight line that lies wholly within the surface of a cone.

__________ 3. A parabola is the section of a cone formed by a plane that is perpendicular to one element.

__________ 4. A hyperbola is the section of a cone formed by a plane that intersects the cone so that the plane is parallel to one element.

__________ 5. An element is a line that joins the vertex of a cone with the center of the circle that is its base.

__________ 6. An ellipse is the section of a cone formed by a plane that cuts completely through the cone perpendicular to the axis. A circle is a special kind of an ellipse.

II. Which of the following are quadratic functions?

____ 7. $x^2 + y = 1$

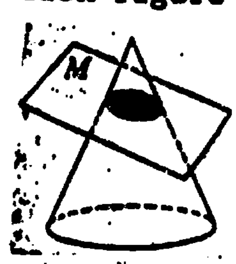
____ 8. $y = x + 2$

____ 9. $3x - 2y^2 = 7$

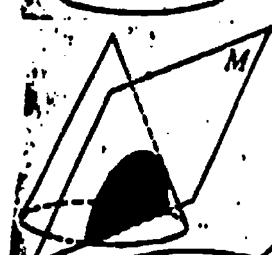
____ 10. $y = \frac{1}{2}x^2 - 3$

III. Match each figure on the left with its name on the right.

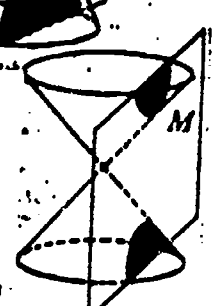
____ 11.



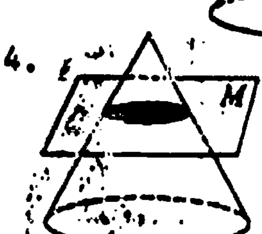

____ 12.




____ 13.




____ 14.




A. ellipse

B. circle

C. hyperbola

D. parabola

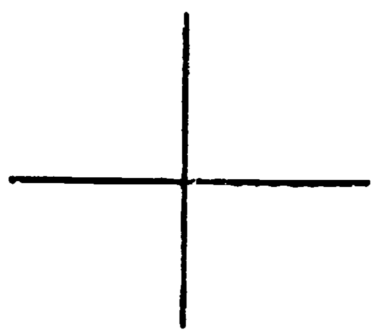
**IV. 15. A. Define circle.**

B. Write the equation of the circle with radius r and center (0,0).

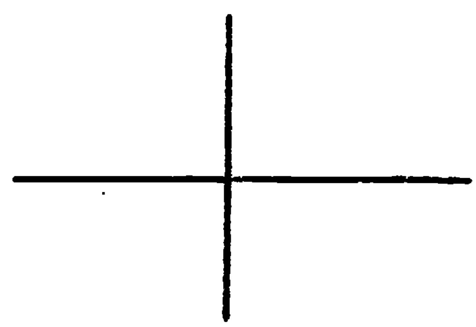
C. Write the equation of the circle with radius r and center (h,k).

V. Give the center and radius of the following circles and graph each.

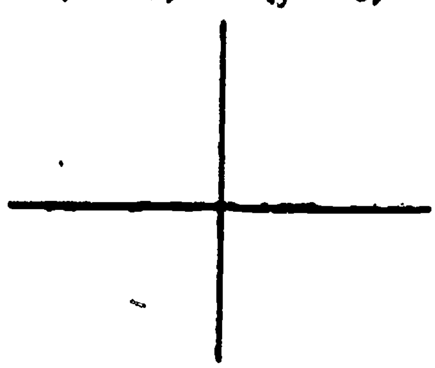
16. $x^2 + y^2 = 36$



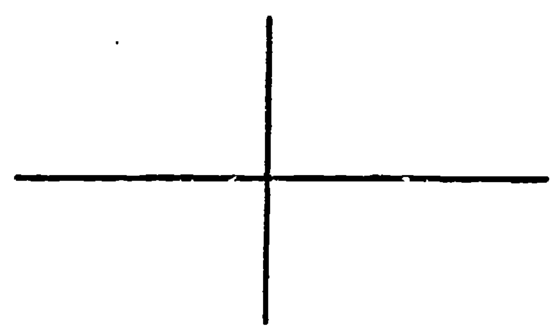
18. $x^2 + y^2 + 12x + 11 = 0$



17. $(x + 5)^2 + (y - 8)^2 = 4$

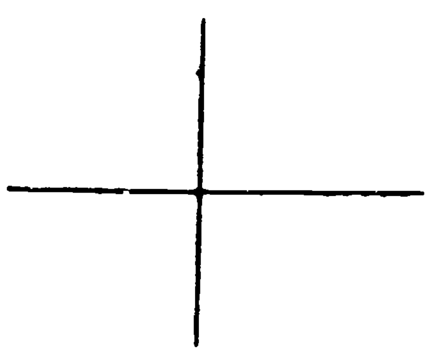


19. $x^2 + y^2 - 10x + 4y + 20 = 0$

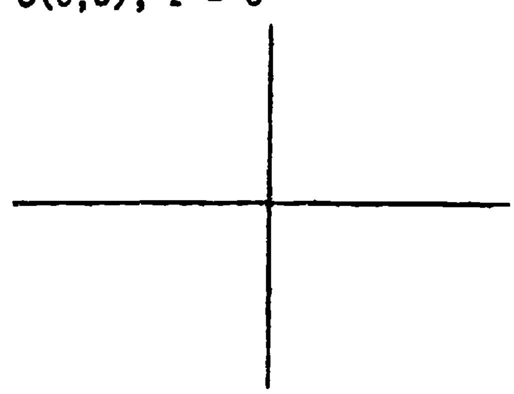


VI. For each given center and radius (1) graph the curve, and (2) write the equation in standard form.

20. C(2,1), r = 3

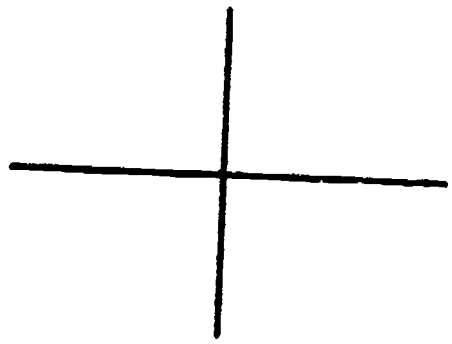


21. C(0,0), r = 6

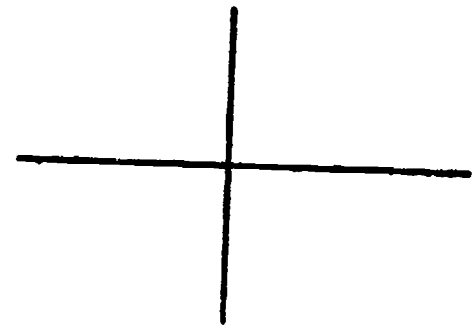


## SELF-EVALUATION 1 (cont')

22. $C(4,-2)$, $r = \sqrt{7}$



23. $C(-8,-1)$, $r = 3\sqrt{2}$



If you have satisfactorily completed your work, take the Progress Test. Consult your teacher first.

## SECTION 2

Behavioral Objectives

At the completion of your prescribed course of study, you will be able to:

8. Write and/or identify the definitions of the following terms:

   a. parabola

   b. axis of symmetry

   c. the value of p

   d. focus (F)

   e. directrix

   f. vertex (V)

9. Write and/or identify a description of the equations $x^2 = 4py$ and $y^2 = 4px$.

10. Given any equation of the form $x^2 = 4py$ and/or $y^2 = 4px$;

   A. determine the value of p

   B. determine the focus

   C. determine the equation of the directrix

   D. graph the curve, focus, and directrix

11. Given a focus and an equation of a directrix,

   a. graph the curve

   b. write the equation of the parabola

12. Given an equation of the form $(y - k)^2 = 4p(x - h)$ or $(x - h)^2 = 4p(y - k)$, determine the vertex, focus, directrix, and sketch the graph.

Objective 8

Exercise for all texts: Write the definitions of the terms in Objective 8.

Vanatta, read pp. 146, 196, Ex. above.

Nichols, read p. 146, Ex. above.

Wooton, read pp. 444-445, Ex. above.

Payne, read pp. 425-426, Ex. above.

3M Transparency: Parabola

Objective 9

Vanatta, read p. 188, Ex. describe the equations $x^2 = 4py$ and $y^2 = 4px$

Objective 10

Vanatta, read pp. 188-190, Ex. 1, 2, 5, 7, 9, 10 pages 190-191.

3M Transparency: Parabola

Objective 11

* Vanatta, read pp. 188-190, Ex. 11-16 page 191.
* Dolciani, read page 306, Ex. 15-18 page 306.

Objective 12

Vanatta, read pp. 203-204, Ex. 4, 7, 12, 13 page 204.

Wooton, read pp. 444-448, Ex. 1-6 page 448.

* required

## SELF-EVALUATION 2



OBJ.

8 I. Define the following:

a. parabola

b. axis of symmetry

c. the value of p

d. focus (F)

e. directrix

f. vertex (V)

9 II. Describe the graph of each of the following:

1. $x^2 = 4py$
2. $y^2 = 4px$

10 III. Write the value of p for each of these.

__________ 1. $x^2 = -16y$

__________ 2. $y^2 = 100x$

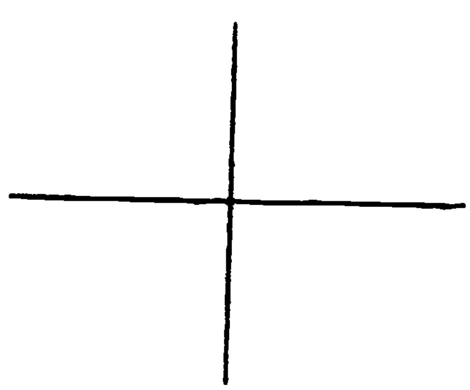
__________ 3. $x^2 = -6y$

__________ 4. $y^2 = -2x$

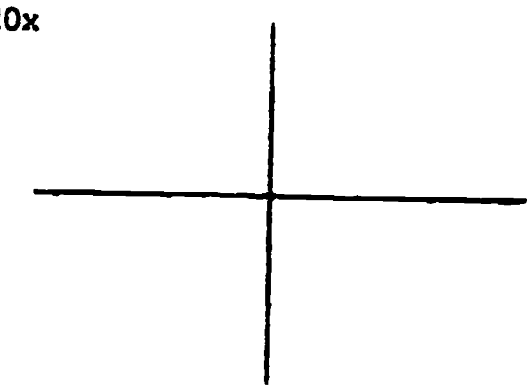
__________ 5. $x^2 = 10y$

10 IV. For each of the following: (1) determine the value of p, (2) determine the focus, (3) determine the equation of the directrix, (4) locate two points other than the vertex and graph the curve, focus, and directrix.

1. $x^2 = 16y$

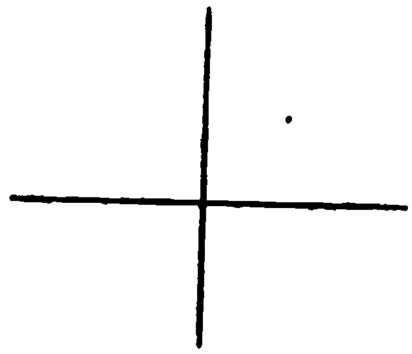


2. $y^2 = -20x$

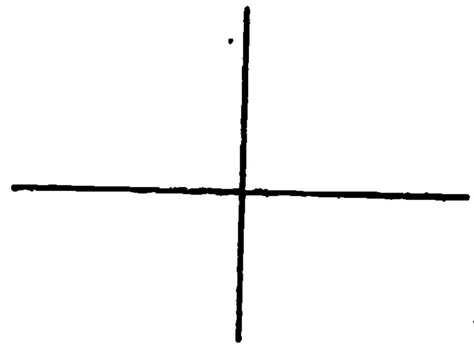


3. $y^2 = 2x$

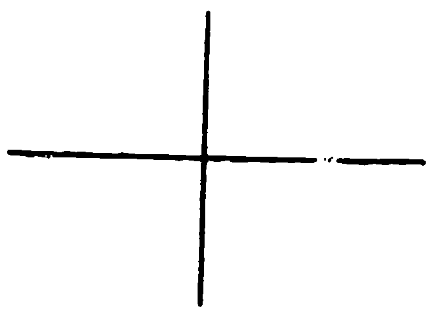


4. $x^2 = -8y$

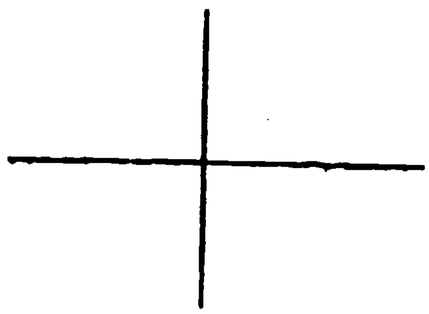


11 V. Given the following foci and directrices, graph each curve formed by them.

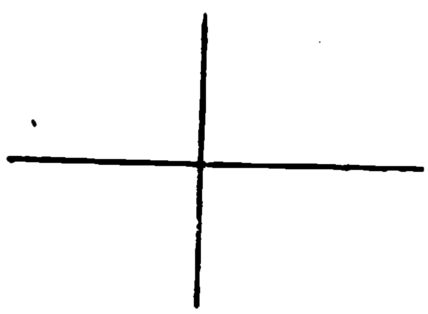
1. F (2,0) $x = -2$



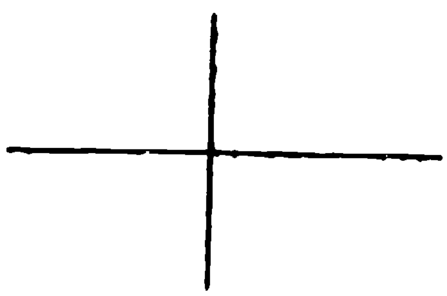
2. F (0,-4) $y = 4$



3. F $(-\frac{3}{2},0)$ $x - \frac{3}{2} = 0$



4. F $(0,\frac{1}{3})$ $y + \frac{1}{3} = 0$



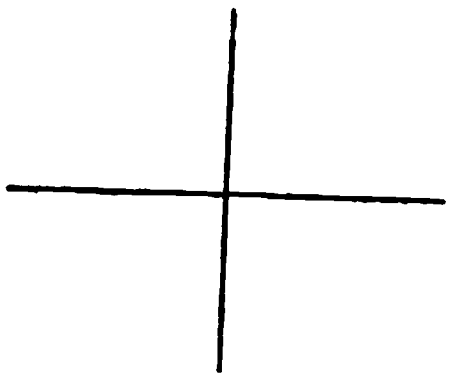
11 VI. Write an equation for each parabola in example VI above.

1. ______________________ 2. ______________________

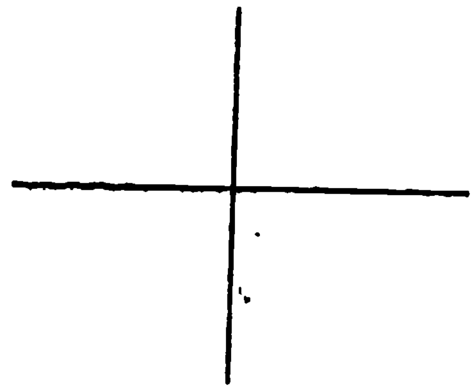
3. ______________________ 4. ______________________

12 VII. For each of the following (1) give the vertex, (2) give the focus, (3) give the directrix, (4) plot the graph, vertex, focus, and directrix.

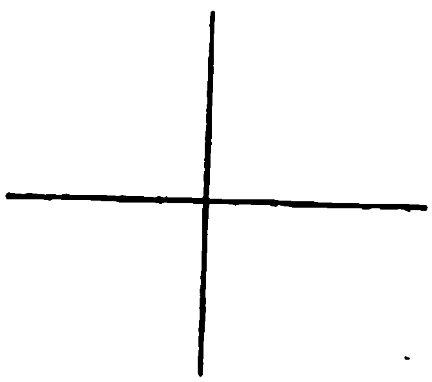
1. $(y - 2)^2 = 16(x + 2)$



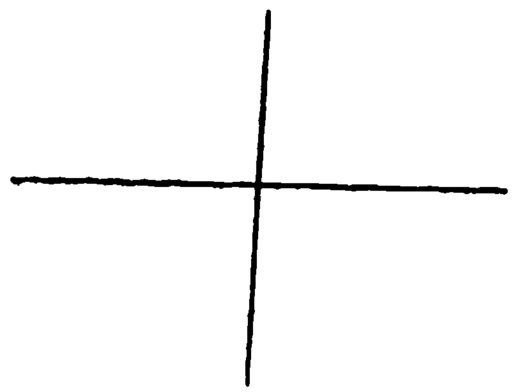
2. $(x + 3)^2 = -8(y - 1)$



3. $(y + 2)^2 = -2(x + 1)$



4. $x^2 + 8x + 8y + 8 = 0$



If you have satisfactorily completed your work, take the PROGRESS TEST. Consult your teacher first.

# SECTION 3

## Behavioral Objectives

At the completion of your prescribed course of study, you will be able to:

13. Write and/or identify the definitions of the following terms:
    - A. ellipse
    - B. foci
    - C. vertices
    - D. major axis, length of major axis
    - E. minor axis, length of minor axis
14. Given a drawing of an ellipse, identify the following parts:
    - A. major axis
    - B. minor axis
    - C. foci
    - D. vertices
    - E. center
15. Write and/or identify the standard form of both an ellipse with its center at the origin and major axis on the x-axis and an ellipse with its center at the origin and major axis on the y-axis.
16. Given any equation of an ellipse, determine by looking at the equation if the major and minor axes are on x or y and/or determine the length.
17. Given any equation of an ellipse, determine
    - A. the semi-major (a) and semi-minor (b) axes
    - B. the foci
    - C. graph the ellipse, plot the foci and vertices
18. Given the major and/or minor axes, the length of the semi-major (a) and semi-minor (b) axes and the center at the origin, determine the equation of the ellipse.

19. Given the vertices and foci,

    A. determine the equation of the curve

    B. sketch the curve

20. Given the equation of an ellipse whose center is not at the origin, determine

    A. the center

    B. the foci

    C. whether the major axis is parallel to the x or y axis

    D. sketch the graph, plot the center, vertices, and foci

## RESOURCES

<u>Objectives 13, 14, 15, 16</u>

Exercise for all texts: Appendix 2 parts I-IV

Vanatta, read pp. 194-196, Ex. above.

Payne, read pp. 421-423, Ex. above.

Wooton, read pp. 449-452, Ex. above.

3M Transparency: The Ellipse

<u>Objective 17</u>

* Appendix II part V

Vanatta, read pp. 195-197, Ex. 1, 3, 4, 7, 9 page 197.

Dolciani, read ___, Ex. 1, 2, 6, 7 page 308; plot foci and vertices.

Wooton, read pp. 449-452, Ex. 1-8 even page 452.

3M Transparency: The Ellipse.

<u>Objective 18</u>

Vanatta, read pp. 194-197, Ex. 11, 12 page 198.

<u>Objective 19</u>

Vanatta, read pp. 194-197, Ex. 17, 18 page 198.

<u>Objective 20</u> (work all exercises)

Dolciani, read ___, Ex. 23, 24 page 309.
Vanatta, read pp. 203-204, Ex. 2, 8, 10, 15 page 204.
Pearson, ___ Ex. 7 b, e, g page 691.

* required

OBJ.
13 I. Define the following terms:

a. ellipse

b. foci

c. vertices

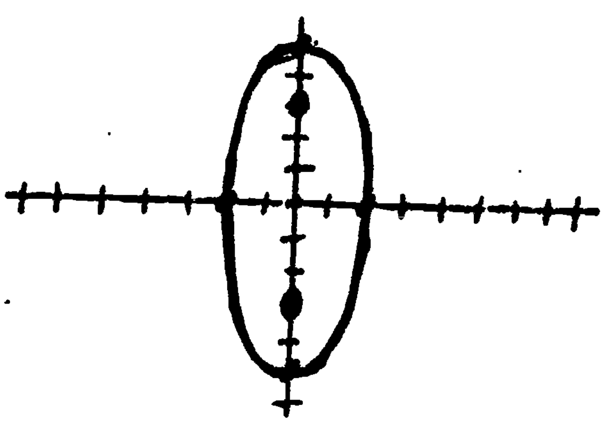
d. major axis

e. length of major axis

f. minor axis

g. center

14 II. Using the following graph identify these parts: (a) major axis, (b) minor axis, (c) foci, (d) vertices, (e) center.



15 III. 1. Write the equation of the ellipse with center at the origin and major axis on x-axis.

2. Write the equation of the ellipse with center at the origin and major axis on y-axis.

BEST COPY AVAILABLE



16 IV. Determine in each of the following if the major axis is on x or y and give its length.

_______________ 1. $\frac{x^2}{16} + \frac{y^2}{9} = 1$

_______________ 2. $\frac{x^2}{100} + \frac{y^2}{64} = 1$

_______________ 3. $\frac{x^2}{121} + \frac{y^2}{144} = 1$

_______________ 4. $\frac{x^2}{9} + \frac{y^2}{49} = 1$

16 V. Determine in each of the examples in problem V if the minor axis is on x or y and give its length.

_______________ 1. _______________ 3.

_______________ 2. _______________ 4.

17 VI. In each of these find the value of a (the semi-major axis) and the value of b (the semi-minor axis).

_______________ 1. $\frac{x^2}{25} + \frac{y^2}{9} = 1$ _______________ 3. $\frac{x^2}{81} + \frac{y^2}{121} = 1$

_______________ 2. $\frac{x^2}{100} + \frac{y^2}{36} = 1$ _______________ 4. $\frac{x^2}{16} + \frac{y^2}{49} = 1$

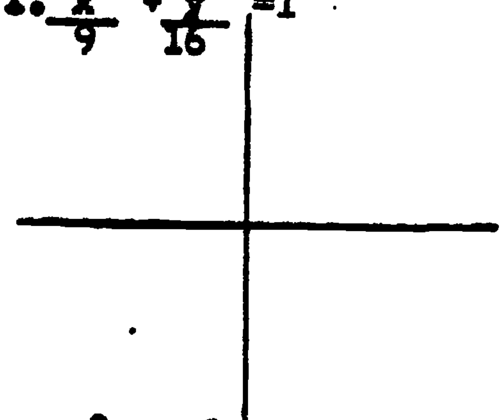
17 VII. Find the foci for each of the following.

_______________ 1. $\frac{x^2}{16} + \frac{y^2}{9}$ _______________ 3. $\frac{x^2}{100} + \frac{y^2}{64} = 1$

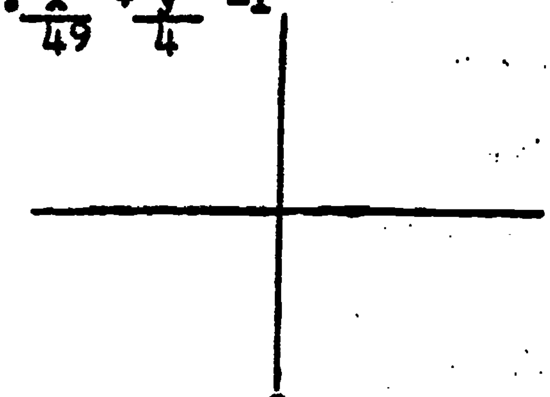
_______________ 2. $\frac{x^2}{36} + \frac{y^2}{4} = 1$ _______________ 4. $\frac{x^2}{9} + \frac{y^2}{81} = 1$

17 VIII. Graph the following ellipses, plot the foci, and vertices of each.

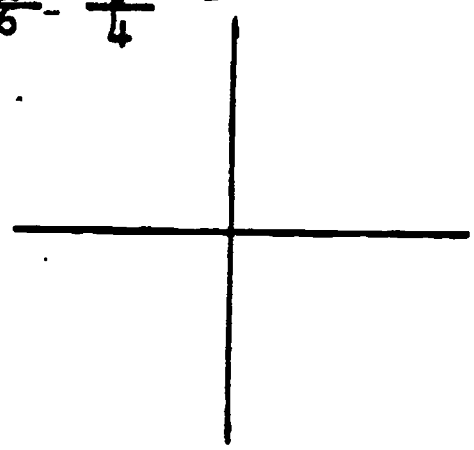
1. $\frac{x^2}{9} + \frac{y^2}{16} = 1$



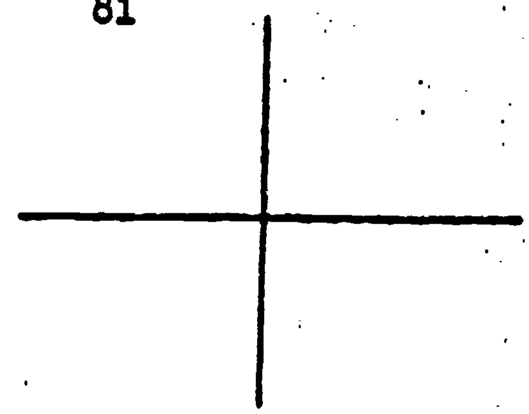
2. $\frac{x^2}{49} + \frac{y^2}{4} = 1$



3. $\frac{x^2}{36} + \frac{y^2}{4} = 1$



4. $\frac{x^2}{9} + \frac{y^2}{81} = 1$



18 IX. Given the following centers and values of a and b, write an equation for each ellipse.

________________ 1. $a = 10$
$b = 3$
C(4,1)
Major axis parallel to x

________________ 2. $a = 4$
$b = 2$
C(-1,3)
Major axis parallel to y

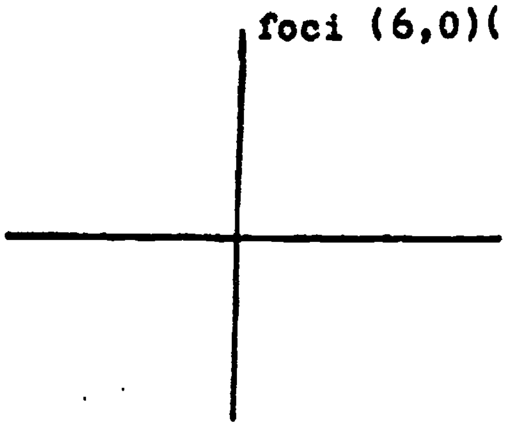
________________ 3. $a = 12$
$b = 9$
C(0,7)
Major axis parallel to x

________________ 4. $a = 12$
$b = 8$
C(3,-2)
Major axis parallel to y

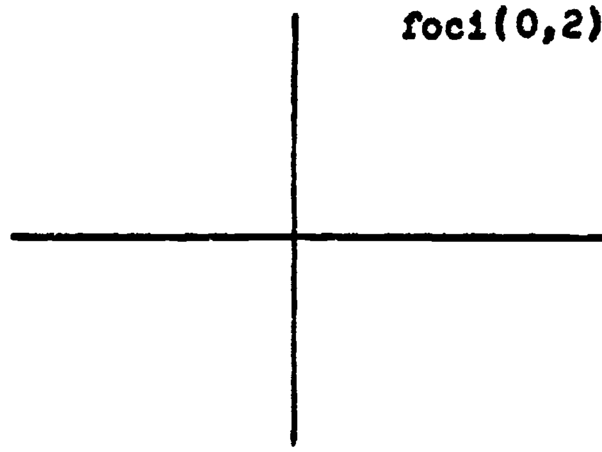
 BEST COPY AVAILABLE

19 **X. Given the following vertices and foci, write an equation for each and graph the curve, centers are at (0,0).**

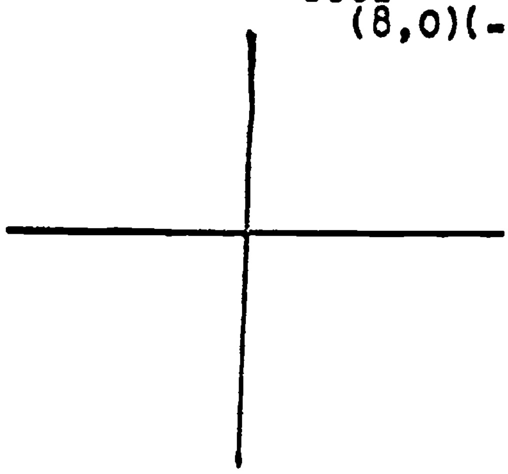
____________ 1. vertices(8,0)(-8,0)
foci (6,0)(-6,0)



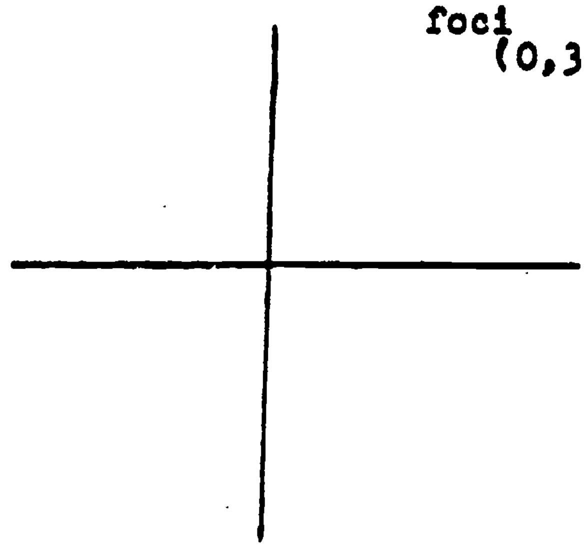
____________ 2. vertices (0,4)
(0,-4)
foci(0,2)(0,-2)



____________ 3. vertices
(10,0)(-10,0)
foci
(8,0)(-8,0)

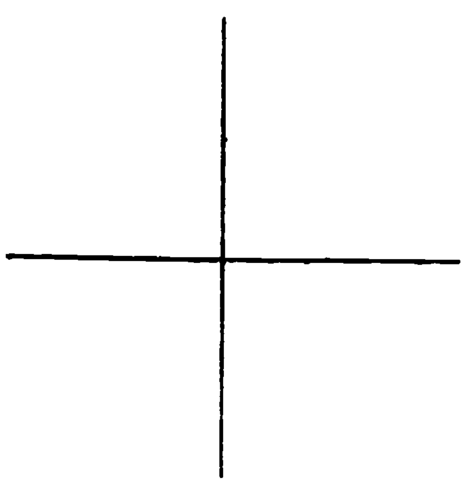


____________ 4. vertices (0,5
(0,-
foci
(0,3)(0,-3)

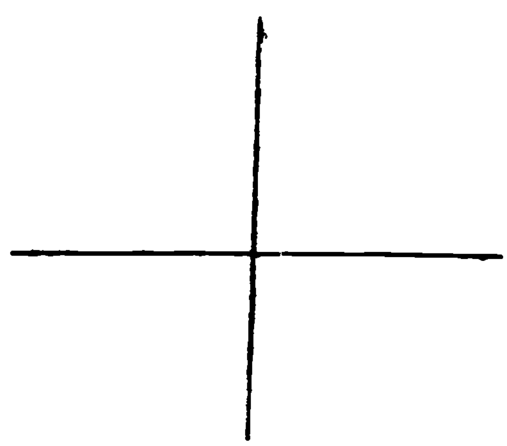


20 XI. For each of the following give (1) the center, (2) the foci, (3) tell if the major axis is parallel to x or y, (4) sketch the curve and plot the center, vertices, and foci.

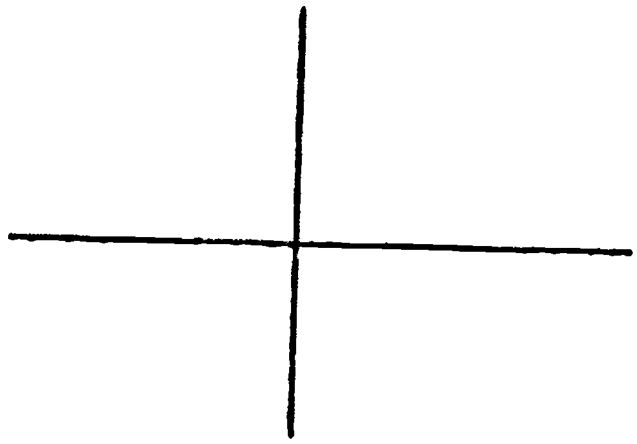
1. $\frac{(x+1)^2}{16} + \frac{(y-2)^2}{25} = 1$



2. $\frac{(x-3)^2}{100} + \frac{(y+4)^2}{36} = 1$



3. $25x^2 + 9y^2 - 100x - 36y - 89 = 0$



If you have satisfactorily completed your work, take the PROGRESS TEST. Consult your teacher first.

Behavioral Objectives

At the completion of your prescribed course of study, you will be able to:

21. Write and/or identify the definition of
    A. hyperbola
    B. transverse axis and its length
    C. conjugate axis and its length
22. Given a drawing of a hyperbola, identify the following parts:
    A. transverse axis
    B. asymptotes
    C. conjugate axis
    D. foci
    E. vertices
    F. center
23. Identify the standard form of
    A. the ellipse whose center is at the origin and transverse axis on x
    B. the ellipse whose center is at the origin and transverse axis on y
24. Given the equation of a hyperbola, determine
    A. the length of the transverse axis, the length of the conjugate axis, draw the asymptotes, and sketch the curve
    B. the coordinates of the foci and plot them on the graph
25. Determine the equation of a hyperbola when given
    A. the transverse axis and the length of a and b
    B. the foci and vertices
26. Given any equation of a hyperbola whose center is not the origin, determine
    A. the center

B. the length of the transverse and conjugate axes

C. the vertices

D. plot the asymptotes

E. the foci

F. draw and/or identify the sketch, plot the center, vertices, and foci

## RESOURCES

OBJ. 21, 22, 23

Exercise for all resources: Appendix 3

* Vanatta, read pp. 198-201, Ex. above.

Payne, read pp. 427-430, Ex. above.

Wooton, read pp. 453-457, Ex. above.

3M Transparencies: The Hyperbola

OBJ. 24

* Vanatta, read pp. 198-201, Ex. 1, 2, 4, 7, 9 page 202.

Dolciani, read pp. 311-312, Ex. 1, 2, 4, 7, 10 pages 311-312 (follow directions in Obj. 24).

Payne, read pp. 427-340, Ex. 3-8 page 430 (follow directions in Obj. 24).

Wooton, read pp. 453-457, Ex. 1-10 even page 457 (follow directions in Obj. 24).

3M Transparencies: The Hyperbola.

OBJ. 25

* Vanatta, read page 201, Ex. 11-14 page 202.

Payne, read pp. 427-430, Ex. 9-12 page 430.

3M Transparencies: The Hyperbola.

OBJ. 26

* Vanatta, read pp. 203-204, Ex. 3, 6, 11, 14 page 204.

  Wooton, read ___, Ex. 21, 22 page 458.

  Pearson, read ___, Ex. 7 page 695, (follow directions in Obj. 26).

  3M Transparencies: The Hyperbola.

* required

## SELF-EVALUATION



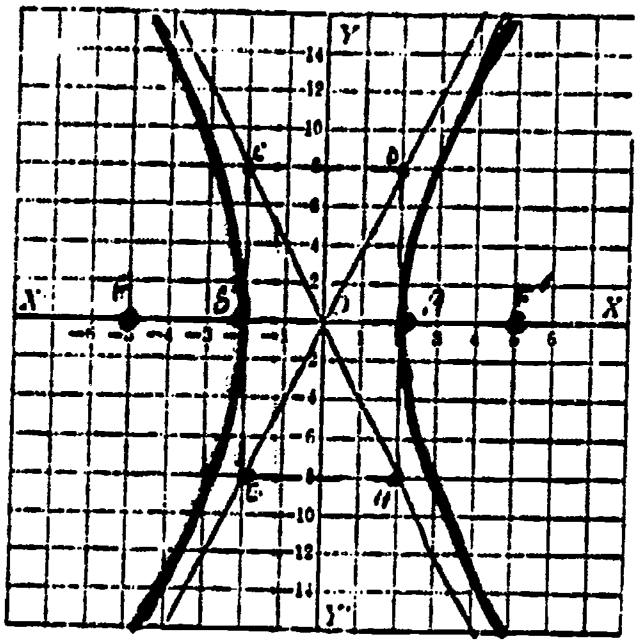
OBJ.

21 I. Write the definition of hyperbola.

21 II. a. Write the definition of transverse axis, give its length.

b. Write the definition of conjugate axis, give its length.

22 III. Identify these parts of the following graph: (1) transverse axis (2) asymptotes (3) conjugate axis (4) foci (5) vertices and (6) center.



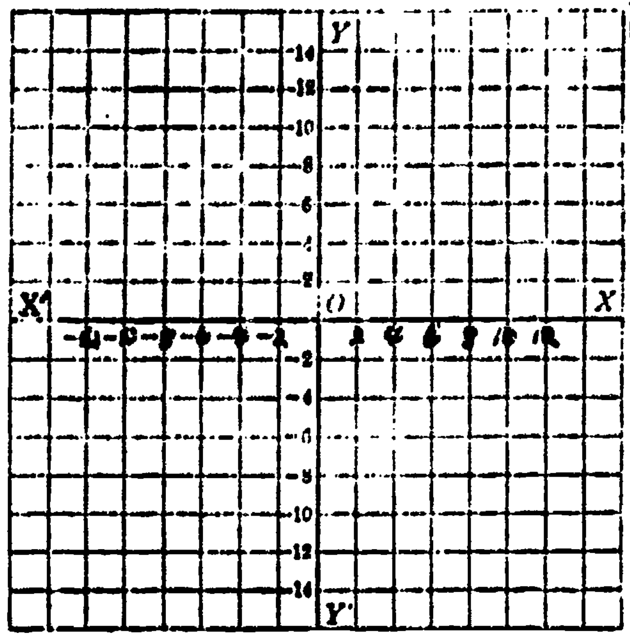

23 IV. (1) Write the equation of the ellipse whose center is at the origin and transverse axis on x.

(2) Write the equation of the ellipse whose center is at the origin and transverse axis on y.

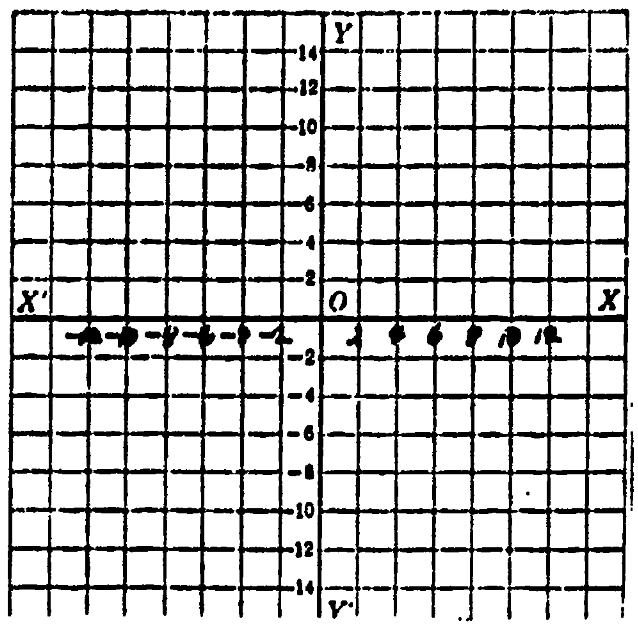
24 V. For each of the following (1) determine the length of the transverse axis and conjugate axis, (2) draw the asymptotes, (3) sketch the curve, (4) determine the foci and plot them on the graph.

1. $\frac{x^2}{100} - \frac{y^2}{64} = 1$

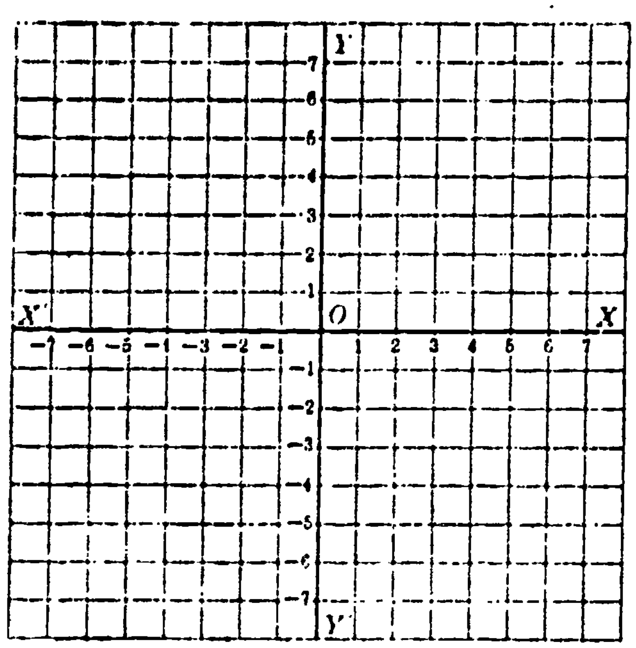
(graph is on the following page)



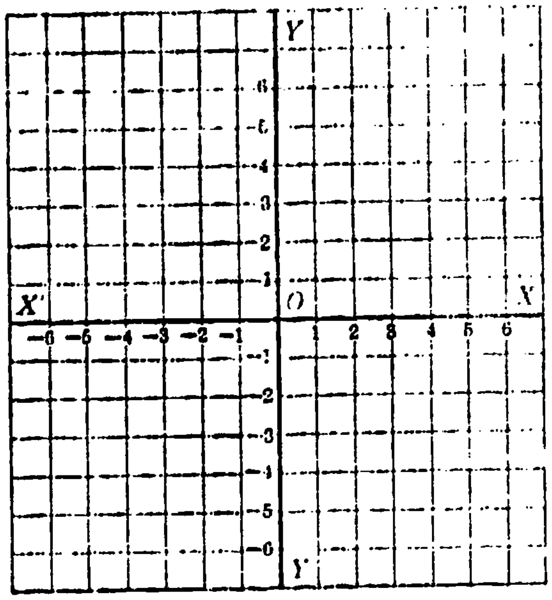

2. $\frac{y^2}{49} - \frac{x^2}{36} = 1$




3. $\frac{y^2}{1} - \frac{x^2}{16} = 1$



4. $\frac{x^2}{1} - \frac{y^2}{4} = 1$




25 **VI.** Given the following values of a and b and transverse axis, write an equation for each.

1. a = 2, b = 3, transverse axis on x ______________________
2. a = 4, b = 1, transverse axis on y ______________________
3. a = 2, b = 7, transverse axis on x ______________________
4. a = 9, b = 12, transverse axis on y ______________________

25 **VII.** Given the following foci and vertices, write an equation for each hyperbola.

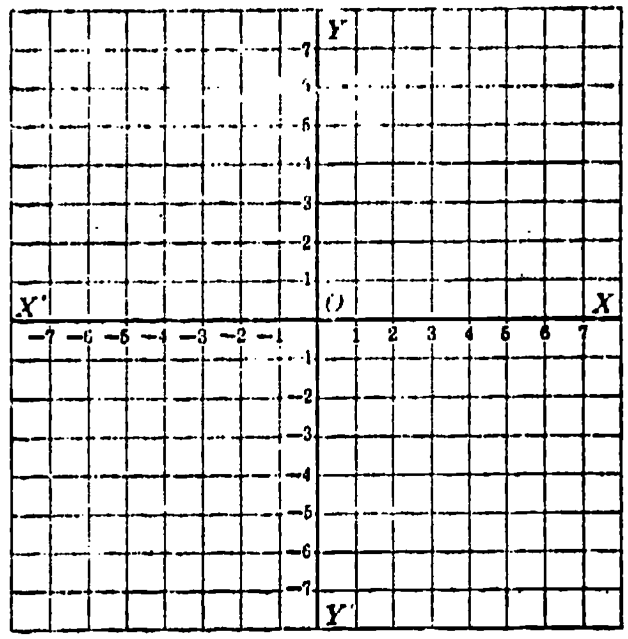
1. F(12,0)(-12,0)
   V(8,0)(-8,0) ______________________

2. F(0,6)(0,-6)
   V(0,3)(0,-3) ______________________

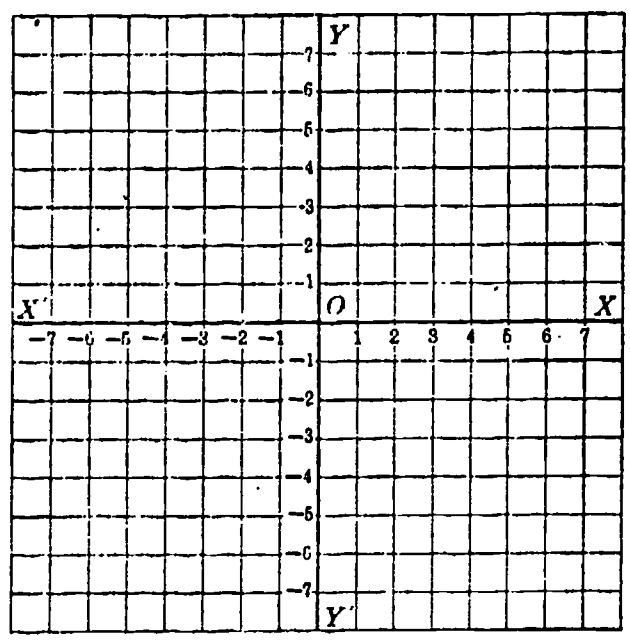
3. F(0,2)(0,-2)
   V(0,1)(0,-1) ______________________

26 VIII. For each of the following (1) give the center, (2) give the length of the transverse and conjugate axes, (3) vertices, (4) foci, (5) plot the asymptotes, plot the curve, center, vertices and foci.

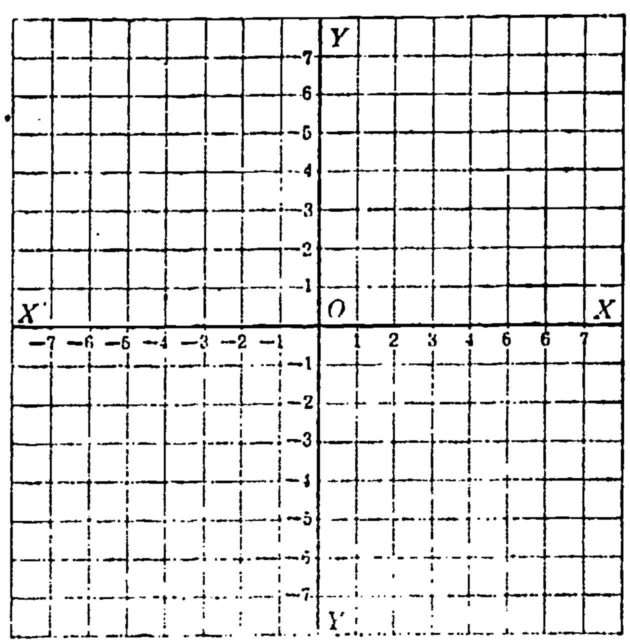
(1) $\frac{(x-1)^2}{4} - \frac{(y+2)^2}{16} = 1$




(2) $\frac{(y+3)^2}{64} - \frac{(x-5)^2}{36} = 1$




(3) $25x^2 - y^2 + 150x + 125 = 0$




If you have satisfactorily completed Your work, take the PROGRESS TEST. Consult your teacher first.

SECTION 5



Behavioral Objectives

At the completion of your prescribed course of study, you will be able to:

27. Given any quadratic inequality, determine its graph.

28. Given any word problem, determine its equation and determine its solution.

RESOURCES

Obj. 27 (both are required)

Vanatta, read pp. 205-208, Ex. 9-12, 16 page 208.

Dolciani, read ___, Ex. 10, 12 page 309.

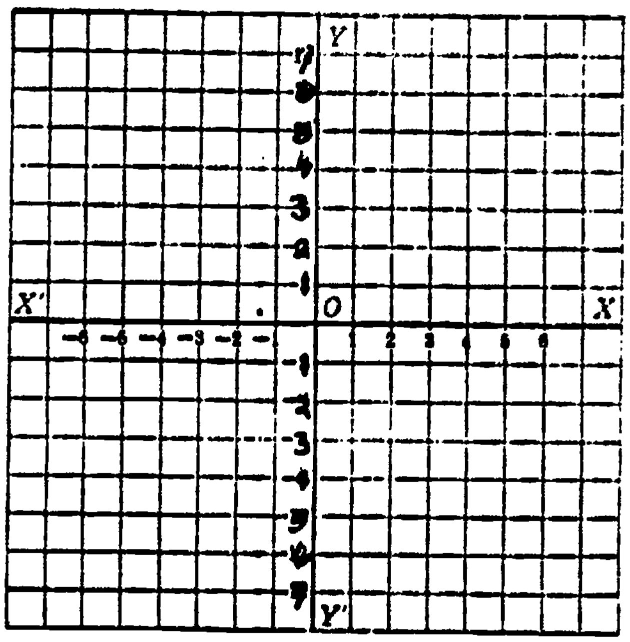
Obj. 28

Vanatta, read pp. 179-181, Ex. 1-4, 6-8, 16, 19 pages 181-182; 13-14 page 211

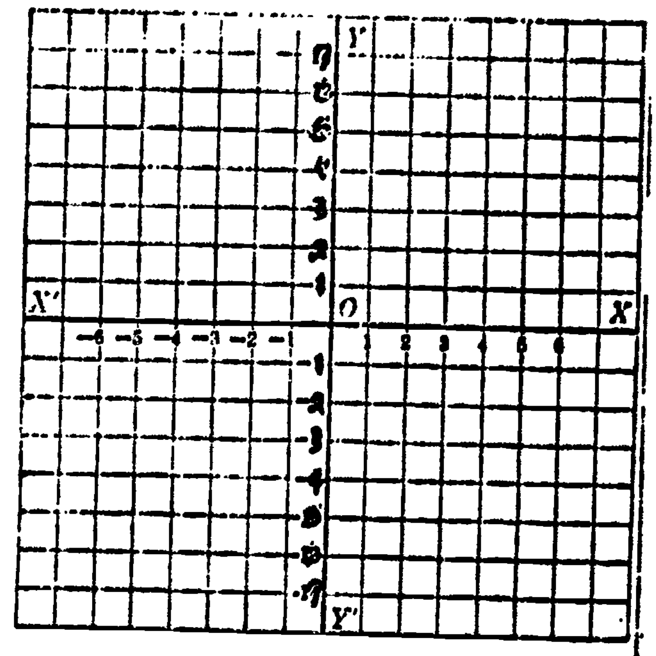
OBJ.

27 I. GRAPH the following inequalities.

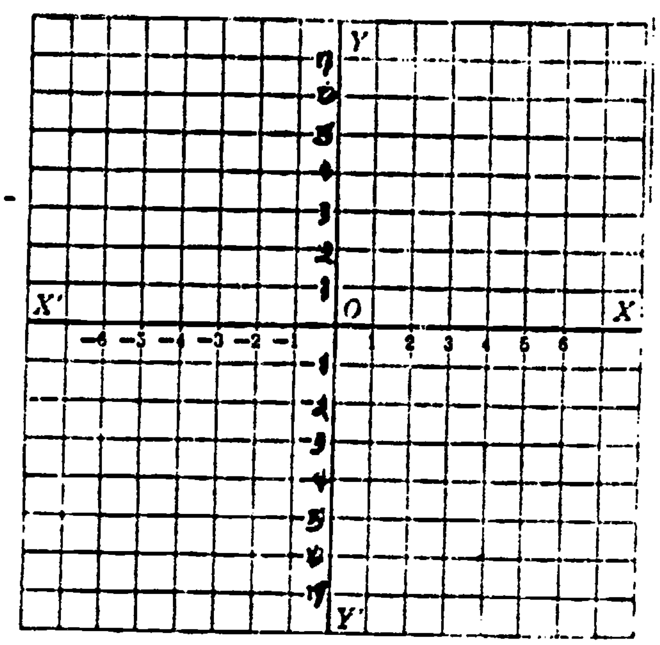
(1) $x^2 + y^2 < 36$



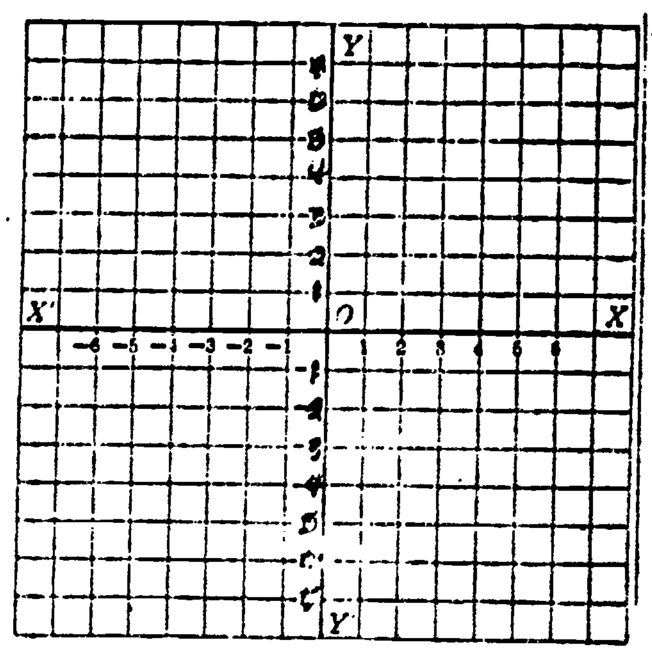

(2) $9x^2 + 25y^2 > 225$




(3) $9y^2 - x^2 < 9$




(4) $x^2 \geq 16y$



28 II. Solve the following problems.

1. Three times the square of a positive integer, decreased by twice the product of the number and the next smaller integer, is 143. Find the number.

2. Find two consecutive integers such that, if twice the larger is added to three times the square of the smaller, the sum will be 58.

3. The base of a triangle is 4 feet less than the altitude, and the area of the triangle is 48 square feet. Find the length of the base.

4. A rectangular lot is surrounded on all sides by a driveway 5 yards wide. The lot is twice as long as it is wide. If the area of the lot and driveway together is 6600 square yards, find the dimensions of the lot.

5. One leg of a given right triangle exceeds the other by 2 feet. If the hypotenuse is 10 feet, find the legs of the triangle.

6. A field of tomatoes contains 3825 plants. The number of plants in each row is 5 less than twice the number of rows. Find the number of plants in each row.

If you have satisfactorily completed your work, take the Progress test. Consult your teacher first. After progress test 5, take the LAP TEST.

## ADVANCED STUDY

I. Work any 4 of the following:

Vanatta, page 194 nos. 17-20.

page 211 no. 5; page 214 no. 40.

Dolciani, page 326 no. 11.

II. Work any 4.

Vanatta page 198 nos. 15, 16, 19, 20

page 214 no. 41

Dolciani page 326 no. 13.

III. Work any 4 from no. 1 and one from no. 2.

(1) Vanatta page 202 nos. 15-20

page 212 no. 18, page 214 no. 43

Dolciani page 326 no. 12, 14

(2) Dolciani page 312 no. 19, 20

IV. Work any one of these three.

(1) Graph (A) $xy = 36$
(B) $xy = -10$

(2) Vanatta page 205 nos. 16, 19, 20

(3) Write a report on LORAN, a system of navigation. Tell how it uses the concept of hyperbola (at least 500 words).

V. Work any 5 of the following.

Vanatta page 212, no. 20; page 214 no. 44, 45.

Dolciani, page 237, nos. 22, 25, 27, 28

page 309, nos. 13, 14.

## APPENDIX I

I. Write the definition of conic section.



II. Name the four conic sections.

(1)

(2)

(3)

(4)

III. Write a description or definition of each of the following terms as they relate to a cone.

1. element
2. axis
3. circle
4. ellipse
5. parabola
6. hyperbola

## APPENDIX 2

I. Define the following:

a. ellipse

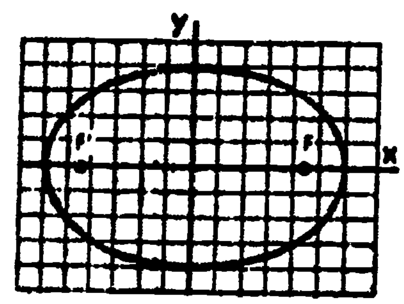
b. foci

c. vertices

d. major axis

e. minor axis

II. Identify the following parts of the ellipse in the figure: (a) major axis, (b) minor axis, (c) foci, (d) vertices (e) center.




III. Describe the ellipses with the following equations:

(a) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ where $a > b$

(b) $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$ where $a > b$

IV. In each of the following, determine (1) if the major axis is on x or y and give its length, and (2) if the minor axis is on x or y and give its length.

(a) $\frac{x^2}{16} + \frac{y^2}{9} = 1$

(b) $\frac{x^2}{25} + \frac{y^2}{100} = 1$

(c) $\frac{x^2}{49} + \frac{y^2}{81} = 1$

(d) $\frac{x^2}{49} + \frac{y^2}{9} = 1$

V. In each equation in IV, determine the value of a and b.

I. Define the following:

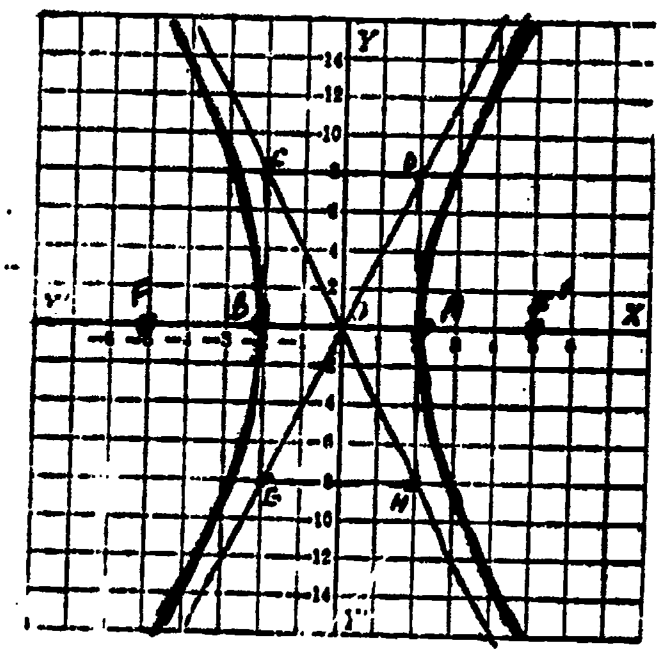
a. hyperbola

b. transverse axis - give its length

c. conjugate axis - give its length

II. Identify the parts of the following graphs

a. conjugate axis
b. transverse axis
c. asymptotes
d. foci
e. vertices
f. center




III. a. Write the equation of the ellipse whose center is at the origin and transverse axis on the x axis.

b. Write the equation of the ellipse whose center is at the origin and transverse axis on the y axis.

## REFERENCES



Vanatta, Glen D., and Goodwin, Wilson A., Algebra Two, A Modern Course, Charles E. Merrill Publishing Co., 1966.

Dolciani, Mary P., Berman, Simon L., and Wooton, William, Modern Algebra and Trigonometry, Book Two, Houghton Mifflin Co., 1965.

Nichols, Eugene D., Modern Intermediate Algebra, Holt, Rinehart and Winston, Inc., 1965.

Pearson, Helen R. and Allen, Frank B., Modern Algebra, A Logical Approach, Book Two, Ginn and Company, 1966.

Payne, Joseph N., Zamboni, Floyd F., Lankford, Jr., Frances G., Algebra Two, Harcourt, Brace and World, 1969.

Dolciani, Mary P., Wooton, William, Beckenback, Edwin F., Sharron, Sidney, Modern School Mathematics, Algebra 2, Houghton Mifflin Company, 1968.

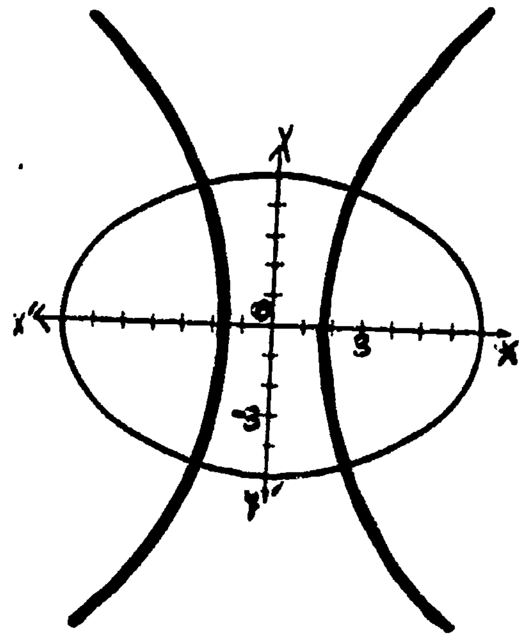
3M Transparencies - The Circle

The Parabola

The Ellipse

The Hyperbola

# LEARNING ACTIVITY PACKAGE




## SYSTEMS OF EQUATIONS AND INEQUALITIES

Ninety Six High School - Ninety Six, S.C. -

Algebra 103-104

REVIEWED BY

LAP NUMBER 31

WRITTEN BY Diane Evans

## RATIONALE

Many problems in mathematics result in mathematical models involving more than one sentence, yet the problem requires a single solution. Two beams of light across the sky might each be described as the graph of a linear equation. If they were to cross one another, you might then have a point of intersection, the single solution.

This LAP should help you gain an understanding of systems of equations or of inequalities. The use of previously learned concepts about functions will be used in developing methods for solving these systems. Equations of conic sections will also be continued and studied with systems of first and second-degree equations so that you should be able to solve these systems, and relate the solutions to their graphs.

SECTION 1



Behavioral Objectives

At the completion of your prescribed course of study, you will be able to:

1. Given an equation, or system of linear equations, and ordered pairs of real numb... determine if the ordered pairs satisfy the equation or system of equations.

2. Given a system of two linear equations:
   a. graph the system R X R
   b. find the solution set of the system

3. From a system of linear equations, determine if the system is independent, inconsistent, dependent, or consistent when given:
   a. the equations of the system
   b. the graph of the equations of the system
   c. the solution set of the system
   d. the number of ordered pairs of the solution set of the system

4. Given two systems of equations, determine if they are equivalent.

5. Given a system of two linear equations, rewrite each equation in the form $Ax + By + C = 0$.

6. Given a system of two linear equations, solve the system by any one of the following methods:
   a. the comparison method
   b. the substitution method
   c. the addition method

Behavioral Objectives (cont')

7. Given a word problem to be solved by the use of a system of two linear equations:

   a. write a system of equations which fits the problem.

   b. solve the resulting system.

   c. give the solution of the original problem.

## RESOURCES

**Objective 1**

Nichols, read pp. 289, 290-294, Ex. 1, 3, 5, 7 page 290; 1, 3, 4 page 292.

**Objective 2, 3**

Vanatta, read pp. 215-218, Ex. 1, 3, 4, 9, 10, 13 page 219.

Dolciani, read pp. 95, Ex. 1, 2, 5, 6, 15 page 95.

Nichols, read pp. 290-294, Ex. 1-3, 5, 7, 8 page 294.

Payne, read pp. 291-293, 303-306, Ex. 1-10 even page 294; 1-4, 6, 11-13 pages 305-306.

* Appendix I

**Objectives 4, 5**

Nichols, read pp. 295, Ex. 1 a, b, 2 a, c, e pages 295-296.

**Objective 6**

Vanatta, read pp. 219-222, Exercises

6(a) [for explanation read Nichols 296-300] Ex. 6,8,9 solve by comparison.

6(b) Ex. 1, 4, 9, 10 solve by substitution

6(c) 2, 3, 5, 7 solve by addition

Dolciani, read pp. 95-98, Exercises

6(a) 1, 2, [illegible] page [illegible] (see Nichols reading for explanation [illegible] by comparison.

* required

6(b) 3, 4, 7, [illegible] 10 pages 99-100 solve by substitution

6(c) 6, 11, 13, 14, 17, pages 99-100 solve by addition

Nichols, read pp. 296-300, Exercises

6(a) 1-7, 9-11 pages 297-298 solve by comparison

6(b) 1-11 odd, 12 page 299 solve by substitution

6(c) 1 a,c,e,g,j,k,l; 2 a, c, e, g, h, j pages 300-301 solve by addition.

Payne, read pp. 294-301, 309-310, Exercises

6(a) 2,3,4,7,8 page 298-299 (solve by comparison)

6(b) 2,4,5,6,8 page 300 (solve by substitution)

6(c) 1-12 even pages 298-299 (solve by addition)

Objective 7 (all problems are required)

* Nichols, read pp. 296-300, Ex. 3 a,c,e,g,i,k pages 301-302.

Payne, read pp. 294-301, 309-310, ex. 27, 28 pages 306.

Wooton, read pp. 210-212, Ex. 3-5, 9, 12, pages 213-214.

Dolciani, read p. 102, Ex. 6, 11, 12, 13 page 103.

* required

Obj.

1 **I.** Match each equation or system of equations with the ordered pair that belongs to its solution set.

| | |
|---|---|
| ____ 1. $4y - x = 1$ | A. $(4,1)$ |
| ____ 2. $y = 2x + 3$ | B. $(-5,11)$ |
| $y = x - 1$ | C. $(7,2)$ |
| ____ 3. $3x - 4y = 8$ | D. $(-4,-5)$ |
| ____ 4. $x + y = 6$ | |
| $3x + 3y = 18$ | |

2 **II.** Graph the following systems. (Use the graph paper that follows the self-evaluation.) Determine if each system is consistent or inconsistent, dependent or independent.

5. $x + y = 7$

   $3x - 2y = 6$

6. $3x - 2y = 2$

   $6x - 4y = -8$

7. $x - 4y = 24$

   $4x + y = 2$

**III.** True or False.

3 ____________ 8. The system $2(x - y) = 5$ and $4x - 4y = 10$ is dependent and inconsistent.

____________ 9. If a system's solution set is the set of all ordered pairs, then the system is independent.

____________ 10. The solution set of the system $x = 4$ and $y = 2$ is the set containing the ordered pair $(4,2)$.

____________ 11. Parallel lines are independent and inconsistent.

____________ 12. A system of equations that has only one point in its solution set is dependent and inconsistent.

4 ____________ 13. The system $\begin{cases} 3 + y = 4 + 2x \\ \text{[illegible]} \end{cases}$ is equivalent to $\begin{cases} y = 2x \\ y - 2x = -1 \end{cases}$.

5 ____________ 14. [illegible]

4 __________ 15. The system $\begin{cases} x + 1 = y + 2 \\ 3(x + 1) = 2(y - 1) \end{cases}$ is equivalent to $\begin{cases} x - y = 1 \\ x + 1 = y - 1 \end{cases}$

5 __________ 16. Standard form for $\frac{3}{x + 2} = \frac{4}{y - 6}$ is $3y - 18 = 4x + 8$.

6 IV. Solve the following by the specified method.

COMPARISON

17. $3x + y = 9$
    $x - y = 16$

18. $7x + 7y = 14$
    $3y = x + 3$

SUBSTITUTION

19. $3x + 2y = 6$
    $x - 2y = 4$

20. $y = 7x + 2$
    $2x - 4y = 5$

ADDITION

21. $8x + 2y = 1$
    $2x + 3y = 4$

22. $2x - 3y = 4$
    $3x + 2y = 5$

7 V. Write an equation for each of the following and solve.

23. The sum of two numbers is 59. If the sum of 12 and twice the first number is 4 more than the second number, what are the numbers?

24. A collection of nickels and dimes has a total value of \$2.40 and contains 35 coins. How many of each kind of coin are there in the collection?

25. The perimeter of a rectangle is 44 inches. If its length is decreased twice its width, the result is 17. Find the length and width.

If you have satisfactorily completed your work, take the PROGRESS TEST. Consult your teacher first.

## SECTION 2

Behavioral Objectives

At the completion of your prescribed course of study, you will be able to:

8. Given a system of a linear equation and a second-degree equation, or a system of two-second degree equations:
   a. graph the equation or equations of the system
   b. solve the system algebraically

9. Given a <u>word problem</u> to be solved by a system of one linear and one second-degree equation, or two second-degree equations:
   a. write a system of equations which fit the problem.
   b. solve the resulting system.
   c. give the solution set of the original problem.

10. Graph in R X R any of the following systems:
   a. a system of two linear inequalities.
   b. a system of one linear inequality and one-second degree inequality.
   c. a system of two second-degree inequalities.

11. Given a graph of a system of inequalities as listed in Objective 10, identify the solution set of that system.

## RESOURCES

<u>Objective 8</u>

Vanatta, read pp. 231-237; Exercises
Solve by graphing: 1, 4, 5, 9 page 234 and 1, 4, 6, 7 page 237.
Solve algebraically: 3, 6, 8 page 234 and 2, 3, 7 page 237.

Dolciani, read pp. 320-321, Exercises
Solve by graphing: 3,5, 7,9 page 321, and 5, 14, 15 page 325.
Solve algebraically: 1, 4, 6, 9 page 321, and 4,6,14,15 page 325.

Nichols, read pp. 312-322, Exercises 1-5 page 314; 1 a,c,e,g,i,k, m,o,q and 2 a,c,e,g,i pages 316-317; 1-23 odd page 321.

Wooton, read pp. 463-469, Exercises
Solve by graphing: 2,6,7,9,13 page 464;
Solve by algebra: 1,3,8,9 pages 466-467.

Pearson, read pp. 687-697, 698-705, Exercises
Solve by graphing 3,4,7,8,9 page 699;
Solve by algebra 1,4,7,8 page 699.

Objective 9 (all problems are required)

Nichols, read pp. 312-322, Ex. 2 a, c, d, page 317; 2 a, c, e page 323.

Dolciani, read pp. 320-321, Ex. 1,5,8 page 322; 34 page 325.

Wooton, read pp. 466-469, Ex. 1, 2, 4, 5 pages 467-468.

Objectives 10, 11

* Vanatta, read pp. 229-230 and 238-239, Ex. 1, 3, 6, 7 page 230; 2, 3, 6, 8 page 239.

Nichols, read pp. 324-327, Ex. 1 a,c,i,k,m,o,q, 2 a,c, 3 d, 4 a,c, 5 a,c,e pages 327-328.

Wooton, read pp. 216-219, Ex. 19, 21, 22 page 465.

Pearson, read pp. 642-644, 706-708, Ex. 5 EOL, 6f, 8 d,e,f page 645; 1 a,b,d, 2 a,b,d,e page 708.

## SELF-EVALUATION 2

**Objective**

8 **I. Solve the following by graphing. (Use the graph paper provided)**

1. $x^2 + y^2 = 36$
   $x - y = 5$

2. $y^2 = 4x$
   $\frac{x^2}{4} + \frac{y^2}{9} = 1$

3. $4x^2 - 9y^2 = 36$
   $\frac{x^2}{9} + \frac{y^2}{4} = 1$

4. $xy = 8$
   $x + y = 4$

8 **II. Solve algebraically. Show your work.**

5. $x^2 + 4y^2 = 32$
   $x - 2y = -8$

6. $x^2 + y^2 = 16$
   $2y = x - 10$

7. $x^2 = 12y$
   $x = y + 1$

8. $\frac{x^2}{4} + \frac{y^2}{9} = 1$
   $\frac{x^2}{4} - \frac{y^2}{9} = 1$

10,11 **III. Solve the following by graphing. (Use the graph paper provided).**

9. $2x + y < 4$
   $x - 3y > -6$

10. $x^2 + y^2 \leq 16$
    $2x - y > 1$

11. $x^2 + y^2 < 9$
    $\frac{x^2}{4} - \frac{y^2}{9} \geq 1$

12. $\frac{x^2}{9} - \frac{y^2}{16} < 1$
    $x^2 + y^2 = 49$

13. $x^2 \geq 16y$

$3x - y \leq 2$

14. $x + y \leq 2$

$\frac{x^2}{9} + \frac{y^2}{25} > 1$

9 IV. Write an equation for each of the following and solve. Show your work.

15. The preimeter of a rectangle is 26 inches. Its area is 12 square inches. Find the dimensions of the rectangle.

16. The product of two numbers is 8. The sum of their reciprocals is $\frac{3}{4}$. What are the numbers?

17. The area of a right triangle is 24 sq. in. The measure of the hypotenuse is 10 in. Find the measure of the two legs.

18. Find two numbers such that the square of their sum is 20 more than the square of their difference, and the difference of their squares is 24.

19. Find two numbers whose difference is 2 and whose product is 2.

20. Find two numbers such that the sum of their squares is 170 and the difference of their squares is 72.

If you have satisfactorily completed your work take the LAP TEST. Consult your teacher first.

Objective

3c I. Given the following solutions to the systems of linear equations, determine if each system is inconsistent, consistent, dependent, or independent.

a. (3,8) in common

b. no points in common

c. all points in common

d. (-3,2) in common

e. equations are parallel

f. equations have same graph

3d II. Given the following numbers of ordered pairs of solution sets of linear equations, determine if each system is inconsistent, consistent, dependent, or independent.

a. no points are in the solution set

b. one point is in the solution set

c. an infinite number of points are in the solution set

## ADVANCED STUDY

I. Dolciani, read pp. 100A to 101, ex. 1-6, 12, 13, 14, 15 page 101.

II. Work any one of the following:

1. Nichols, read pp. 302-310, ex. 1, 2 page 306; 1, 2a,c,d,e,h,j,l,m,o pages 310-311.
2. Payne, read pp. 327-329, 334-336, ex. 1-10 page 329; 21 page 337.
3. Wooton, read pp. 649-652, 657-661, ex. 1-12, 21-26 pages 652-653; 1-8 page 661.

Dolciani, ex. 11-14 page 326.

III. Wooton, read pp. 223-226, ex. 1-20 oral page 226.

IV. Vanatta, read pp. 224-227, ex. 1-4, 6 pages 227-228.

# REFERENCES


Vanatta (abbreviation)

Vanatta, Glen D., and Goodwin, Wilson A., Algebra Two, A Modern Course, Charles E. Merrill Publishing Co., 1966.

Dolciani (abbreviation)

Dolciani, Mary P., Berman, Simon L., and Wooton, William, Modern Algebra, Book Two, Houghton Mifflin Co., 1965.

Nichols (abbreviation)

Nichols, Eugene D., Modern Intermediate Algebra, Holt, Rinehart and Winston, Inc., 1965.

Pearson (abbreviation)

Pearson, Helen R., and Allen, Frank B., Modern Algebra: A Logical Approach, Book Two, Ginn and Company, 1966.

Payne (abbreviation)

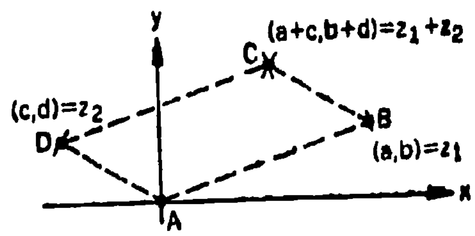
Payne, Joseph N. Zamboni, FLoyd F., Lankford, Jr., Francis G., Algebra Two, Harcourt, Brace and World, 1969.

Wooton (abbreviation)

Dolciani, Mary P., Wooton, William, Beckenbach, Edwin F., Sharron, Sidney, Modern School Mathematics, Algebra 2, Houghton Mifflin Company, 1968.

**L**EARNING
**A**CTIVITY
**P**ACKAGE






# COMPLEX NUMBERS

Ninety Six High School - Ninety Six, S.C.

Algebra 103-104

REVIEWED BY

LAP NUMBER 32

WRITTEN BY Diane Evans

# RATIONALE

Some polynomial equations have no real roots.

For example. $x^2 + 2 = 0$ or $x^2 = -2$

There is no real number which, when squared, will be negative. Therefore, we construct a number system which contains roots for all equations. This system, which we call the complex number system contains the real numbers as a subset; satisfies the eleven field postulates; and contains a solution for all polynomial equations with real coefficients.

This LAP presents the complex numbers as an extension of the real numbers and defines them as ordered pairs of real numbers. Some reasons for the ordered pair approach are:

(1) Graphs of complex numbers seem more natural.

(2) It leads easily and naturally to De Moivre's Theorem.

(3) It can be used later to show the connection between vectors, complex numbers, polar coordinates, and points in a plane.

The standard notation, $a + bi$, is introduced so that students will be able to compute with this form as well as with the ordered pair form.

SECTION 1 **BEST COPY AVAILABLE**

## Behavioral Objectives

After having completed your prescribed course of study, you will be able to:

1. Define imaginary number, complex number, pure imaginary number.
2. Simplify any imaginary number.
3. Given the sets of numbers natural, whole, integer, rational, irrational, real, and complex, determine if relationships among these sets are true or false.
4. Given two complex numbers in ordered pair form, determine if they are equal.
5. Given two equal ordered pairs in which one or more of the components are variables, determine the value of each variable according to the definition of equality of complex numbers.
6. Given two complex numbers in order pair form, give the ordered pair that names:
   a. their sum
   b. the additive inverse of either number
   c. their difference
7. Determine if the following properties apply to the set of complex numbers and/or identify examples of each:
   a. closure under addition
   b. commutative for addition
   c. associative for addition
   d. additive identity
   e. additive inverses
8. Given two complex numbers in ordered pair form, give the ordered pair that names:
   a. their product
   b. the multiplicative inverse of either number provided the number is not zero
   c. their quotient, if it is defined
   d. the $n^{th}$ power of either
9. Determine if the following properties apply to the set of complex numbers and/or identify examples of each:
   a. closure for multiplication
   b. commutative for multiplication
   c. associative for multiplication
   d. the multiplicative identity
   e. multiplicative inverses
   f. distributive

**Behavioral Objectives (cont')**

10. Given a number system, determine whether or not it is a field. If it is not, list the missing properties.

11. Given any complex number, express it in the form a + bi where a and b are real numbers.

12. Given two equal complex numbers in which one or more of the components are variables, determine the value of each variable according to the definition of equality of complex numbers.

13. Given two complex numbers, expressed in standard form, compute:

    a. their sum
    b. the additive inverse of either number
    c. their difference
    d. their product
    e. the multiplicative inverse, provided the number is nonzero
    f. their quotient, if it is defined

14. Evaluate any expression of the form $i^a$ where a is a natural number.

## SECTION 1

### RESOURCES

Objective 1

Vanatta, read pages 350-353, Ex. -define the terms in the objective.

Objective 2

Vanatta, read pages 350-352, Ex. 1-14 page 352.

Objective 3

Vanatta, read pages 350-353, Ex. - Appendix I.*

Objective 4, 5

Nichols, read pages 251-252; Ex. 1 a, c, d, e, g, i, j; 2 pages 252-253.

Objective 6, 7

Nichols, read pages 253-258; Ex. 1-26 even pages 253-254; 1-4 page 255; 1-3 page 256; 1, 2, 3 a, c, e-q, 4 page 258.

*Appendix II - Part I

Objective 8, 9

Nichols, read pages 258-266; Ex. 1, 2 page 259; 1, 4, 5, 6 pages 260-261; 1, 2 pages 263-264; 1, 2 a, c, e-t, 3, 4 a, b pages 266-267.

*Appendix II - Part II

Objective 10

Nichols, read pages 267-268; Ex. 1-5 page 268; 1-24 even pages 268-269.

*Appendix III

Objectives 11, 12, 13, 14

Nichols, read pages 269-270; Ex. 1 a, c, e, 2, 3 a, c, e, 4 a, c, e, 5, 6, 7, 8, 9 a, c, e pages 270-273.

Payne, read pages 24-41, Ex. 1-33 even page 26; 1-32 even pages 28-29; 1-16 pages 30-31; 1-27 even pages 33-34; 1-25 even page 36; 1-15 page 38; 11-31 page 41.

Wooton, read pages 364-374; Ex. 1-24 page 367; 1-50 odd pages 368-369; 1-42 even pages 373-374.

Pearson, read pages 591-603; Ex. 1-7 pages 593-594; 1-13 pages 597-598; 1-6 pages 601-602; 1-4, 6-10 page 604.

* required

## SELF-EVALUATION 1



OBJ.

1 I. WRITE THE DEFINITION OF THE FOLLOWING:

a. imaginary number

b. complex number

c. pure imaginary number

2 II. SIMPLIFY THE FOLLOWING:

______ a) $\sqrt{-9}$

______ b) $-\sqrt{-3}$

______ c) $-\sqrt{-25}$

______ d) $-\sqrt{-9a^2}$

3 III. FOR THE FOLLOWING USE N = NATURALS, W = WHOLES, I = INTEGERS, Q = RATIONALS, T = IRRATIONALS, R = REALS, and C = COMPLEX TO ANSWER TRUE OR FALSE.

______ a) $N \subset T$

______ b) $T \subset C$

______ c) $R \subset C$

______ d) $W \subset R$

______ e) $Q \subset T$

______ f) $T \subset C$

______ g) $I \subset Q$

4,5 IV. FOR WHAT VALUE OF x AND y IS EACH OF THE FOLLOWING STATEMENTS TRUE WHEN $x, y \in$ Reals?

______ a) $(x, -3) = (2, y)$

______ b) $(-x, 6) = (2, 2y)$

______ c) $(2 + \frac{3}{x}, 4) = (5, \frac{y}{4} + 3)$

______ d) $(3 - m, n - 3) = (6, 6)$

6 V. COMPUTE.

______ a) $(3, -4) + (6, -6)$

________ b) $(a, -b) + (-a, b)$

________ c) $(3, 2) - (4, 9)$

________ d) $(c, 0) - (d, 0)$

________ e) $(2, -4) + (8, 3)$

6 VI. GIVEN THE ADDITIVE INVERSE OF THE FOLLOWING.

________ a) $(3, 4)$

________ b) $(-2, -6)$

________ c) $(-2, 2)$

________ d) $(8, -10)$

________ e) $(0, 0)$

8 VII. COMPUTE.

________ a) $(7, 4) \cdot (3, -4)$

________ b) $(-2, 6) \cdot (3, -1)$

________ c) $(6, 0) \cdot (0, 5)$

________ d) $(1, 0) \cdot (b, 0)$

________ e) $(-10, 7) \cdot (2, -3)$

________ f) $(4, 3) \div (2, 3)$

________ g) $(10, 2) \div (2, 3)$

________ h) $(a, b) \div (-c, -d)$

________ i) $(2, 8) \div (8, 8)$

8 VIII. GIVE THE MULTIPLICATIVE INVERSE OF THE FOLLOWING:

________ a) $(3, -5)$

________ b) $(-\frac{8}{7}, 0)$

________ c) $(5, 0)$

________ d) $(0, t)$

8 IX. EVALUATE AND EXPRESS THE RESULT AS AN ORDERED PAIR.

__________ a) $(0, 1)^4$

__________ b) $(0, 1)^7$

__________ c) $(0, 1)^2$

11 X. EXPRESS THE FOLLOWING COMPLEX NUMBERS IN STANDARD FORM. ($a = bi$, where $a, b \in$ Reals.

________ a) (3, 5)

________ b) (0, -2)

________ c) (a, b)

12 XI. FOR WHAT VALUE OF x and y IS EACH OF THE FOLLOWING STATEMENTS TRUE WHEN $x, y \in$ Reals?

__________ a) $x + 3i = 5 + yi$

__________ b) $x + yi = 4i$

__________ c) $x + yi = 7$

13 XII. EXPRESS THE FOLLOWING IN THE FORM $a + bi$ when $a, b \in$ Reals.

________ a) $(3\sqrt{2} + 5i) + (\sqrt{2} - 7i)$

________ b) $(-5 - 3i) - (2 - 6i)$

________ c) $(\sqrt{3} - \sqrt{2}\, i)(\sqrt{3} + \sqrt{2}\, i)$

________ d) $(2 + 3i) \div (5 + 4i)$

________ e) $(-5 - 3i) - (2 - 6i)$

________ f) $-(6 - 5i)$

________ g) $\dfrac{i}{7 + 3i}$

________ h) $(3 + 5i)(2 + 4i)$

________ i) $(8 + 2i) + (6 + 4i)$

________ j) $(5 + 6i) \div (2 + 3i)$

14 XIII. EXPRESS THE FOLLOWING IN THE FORM $a + bi$ where $a, b \in$ Reals.

________ a) $i^2$

_________ b) $i^3$

_________ c) $i^4$

_________ d) $i^5$

7, 9 XIV. NAME THE PROPERTY ILLUSTRATED BY EACH OF THE FOLLOWING AND STATE WHETHER OR NOT EACH HOLDS FOR THE COMPLEX NUMBERS. $z_1$, $z_2$, and $z_3$ ARE THREE COMPLEX NUMBERS.

_________ a) $z_1 + z_2 = z_2 + z_1$

_________ b) $z_1 + (0, 0) = z_1$

_________ c) $z_3 \cdot \frac{1}{z_3} = (1, 0)$

_________ d) $z_1 + (z_2 + z_3) = (z_1 + z_2) + z_3$

_________ e) $z_1 \cdot (z_2 + z_3) = (z_1 \cdot z_2) + (z_1 \cdot z_3)$

_________ f) $z_1 + z_2 = (0, 0)$

10 XV. STATE IF EACH OF THE FOLLOWING IS TRUE OR FALSE. IF FALSE, STATE THE PROPERTY(S) TO MAKE THE STATEMENT TRUE.

_______ a) The set of integers is a field. _______________

_______ b) The set of real numbers is a field. _______________

_______ c) The set of irrational numbers is a field. _______________

_______ d) The set of natural numbers is a field. _______________

_______ e) The set of complex numbers is a field. _______________

IF YOU HAVE SATISFACTORILY COMPLETED YOUR WORK, TAKE THE PROGRESS TEST. CONSULT YOUR TEACHER FIRST.

## Behavioral Objectives

After having completed your prescribed course of study, you will be able to:

15. Compute the absolute value of a complex number.

16. Determine the conjugate of a complex number.

17. Given a pair of complex numbers:

    a. Plot the complex numbers in the coordinate plane.
    b. Plot the additive inverse of either complex number in the coordinate plane.
    c. Plot the conjugate of either complex number in the coordinate plane.
    d. Draw on the plane a geometric interpretation of the sum of the two complex numbers.
    e. Draw on the plane a geometric interpretation of the difference of the two complex numbers.

18. Given an expression involving square roots of real numbers, write the expression in simplified form so that the radicand is a positive integer.

19. Given any quadratic equation over the real numbers, determine its solution set in the complex numbers.

20. Given a set of two complex numbers, write a quadratic equation with real coefficients having this set as its solution set in the field of complex numbers.

## RESOURCES

### Objectives 15, 16

Nichols, read pages 273-274; Ex. 1-4 pages 273-274; 1-5 pages 274-275.
Payne, read pages 38-40, 43-44; Ex. 1-10 page 41; 1-19 page 45.
Pearson, read pages 603-604, 610-615, 617-619; Ex. 5 a-n, 11, 12, 13 pages 604-605; 3 page 615; 8, 11 page 616; 1-5, 7, 9 pages 619-620.

### Objective 17

*Nichols, read pages 275-278; Ex. 1 a, c, e, 2 page 276; 3 a-e pp. 277-278; 1-3 pp.278-279.

### Objective 18

Nichols, read pages 279-280; Ex. 1-2 pages 280-281.
Payne, read pages 24-29, Ex. 1-33 even pages 28-29.
Wooton, read pages 365-369; Ex. 1-24 page 367; 1-50 even pp. 368-369.
Pearson, read pp. 605-608; Ex. 1, 2 page 608.

### Objectives 19, 20

Nichols, read page 281-283; Ex. 1, 2 page 284.
Payne, read pp. 254-255, 269-271; Ex. 3, 8, 12, 16, 25-30, 40-47 p. 256; 1-18 page 271.
Wooton, read pages 374-376; Ex. 1-26 page 377.
Pearson, read pages 605-608; Ex. 3-5, 7, 8 page 609.

## SELF-EVALUATION 2

OBJ.
16, 16 I. FIND THE ABSOLUTE VALUE AND CONJUGATE OF THE FOLLOWING COMPLEX NUMBERS.

a) $10 + 3i$

b) $-3i$

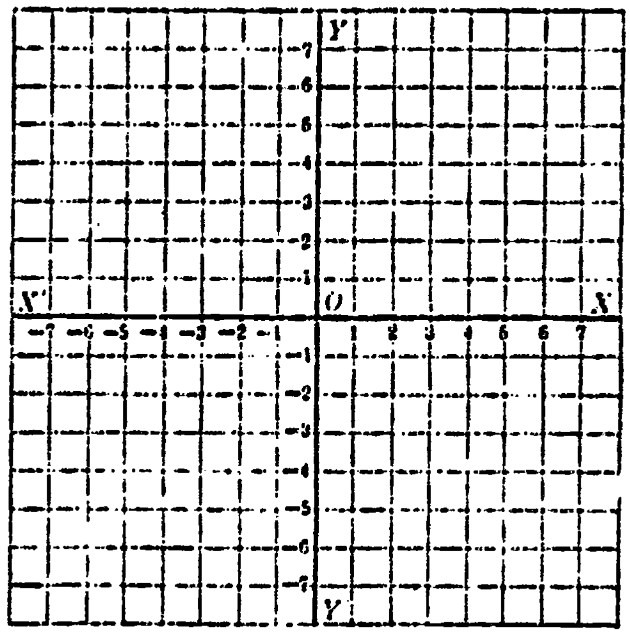
c) $9$

d) $5 - 2i$

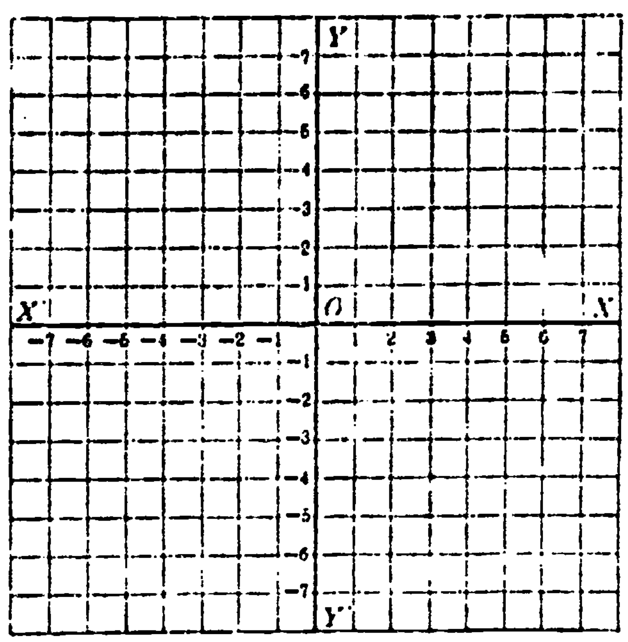
e) $-7 - i$

17 II. A. ON THE GRAPH BELOW, PLOT THE FOLLOWING COMPLEX NUMBERS.

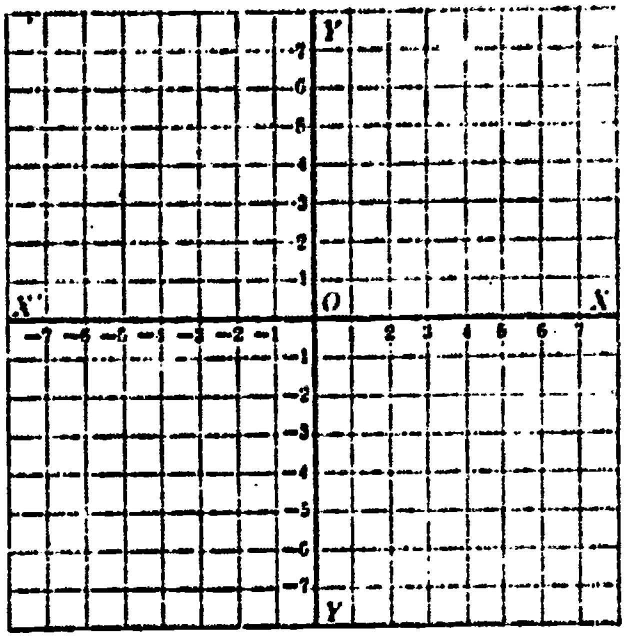
1) $3 - 4i$
2) $2 + 6i$
3) $5i$
4) $-6$




B. PLOT THE ADDITIVE INVERSE OF EACH COMPLEX NUMBER IN EXERCISE A.

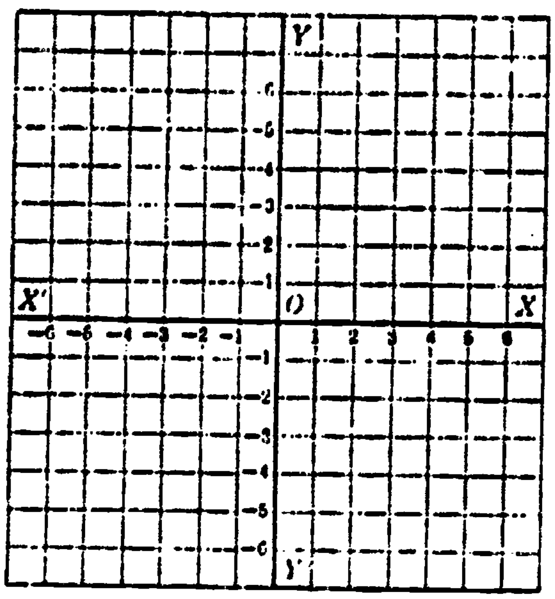


C. PLOT THE CONJUGATE OF EACH COMPLEX NUMBER IN EXERCISE A.

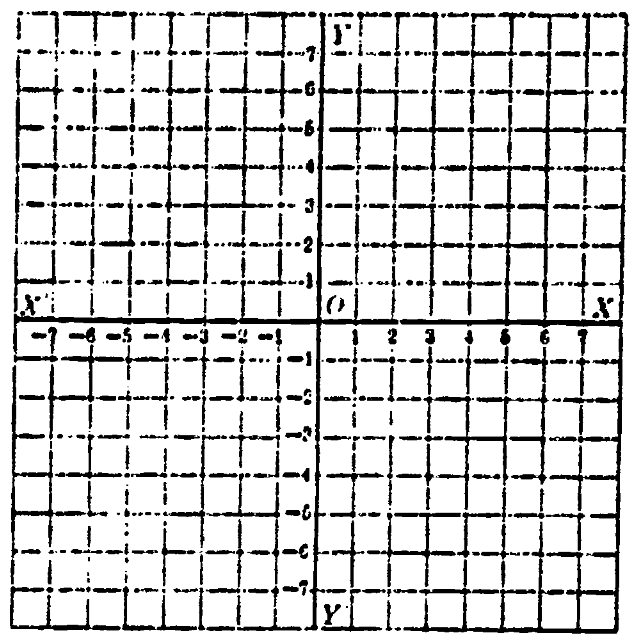



D. CONSTRUCT A GRAPH FOR EACH OF THE FOLLOWING:

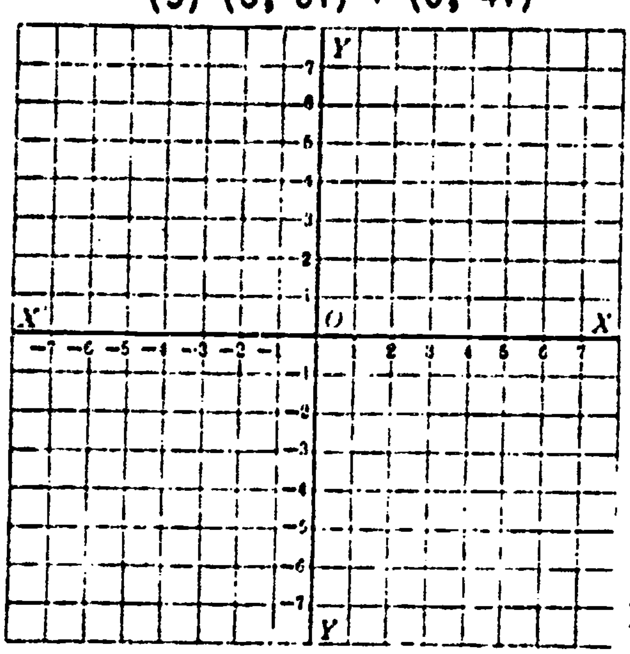
(1) (-2, 4)+ (3, 2)



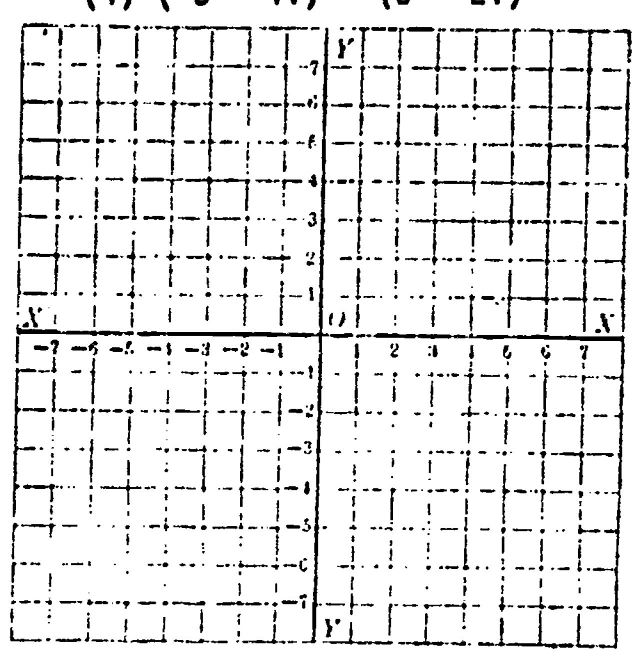

(2) (-2 + 6i) - (4 - 2i)




(3) (0, 6i) + (0, 4i)




(4) (-3 - 4i) - (5 - 2i)



## SELF-EVALUATION 2 (cont')

18 III. SIMPLIFY EACH OF THE FOLLOWING:

A) $\sqrt{-32} + \sqrt{-8}$

B) $\sqrt{-32} - \sqrt{-8}$

C) $-\sqrt{-3} \cdot \sqrt{7}$

D) $\sqrt{-7} + \sqrt{-28}$

E) $\sqrt{-3} \cdot \sqrt{-7}$

F) $-\sqrt{-32} - \sqrt{-8}$

19 IV. FIND THE SOLUTION SET OVER THE COMPLEX NUMBERS.

A. $x^2 + 9 = 0$

B. $\frac{1}{2}x^2 + 10 = 4x$

C. $3x^2 + 12 = 0$

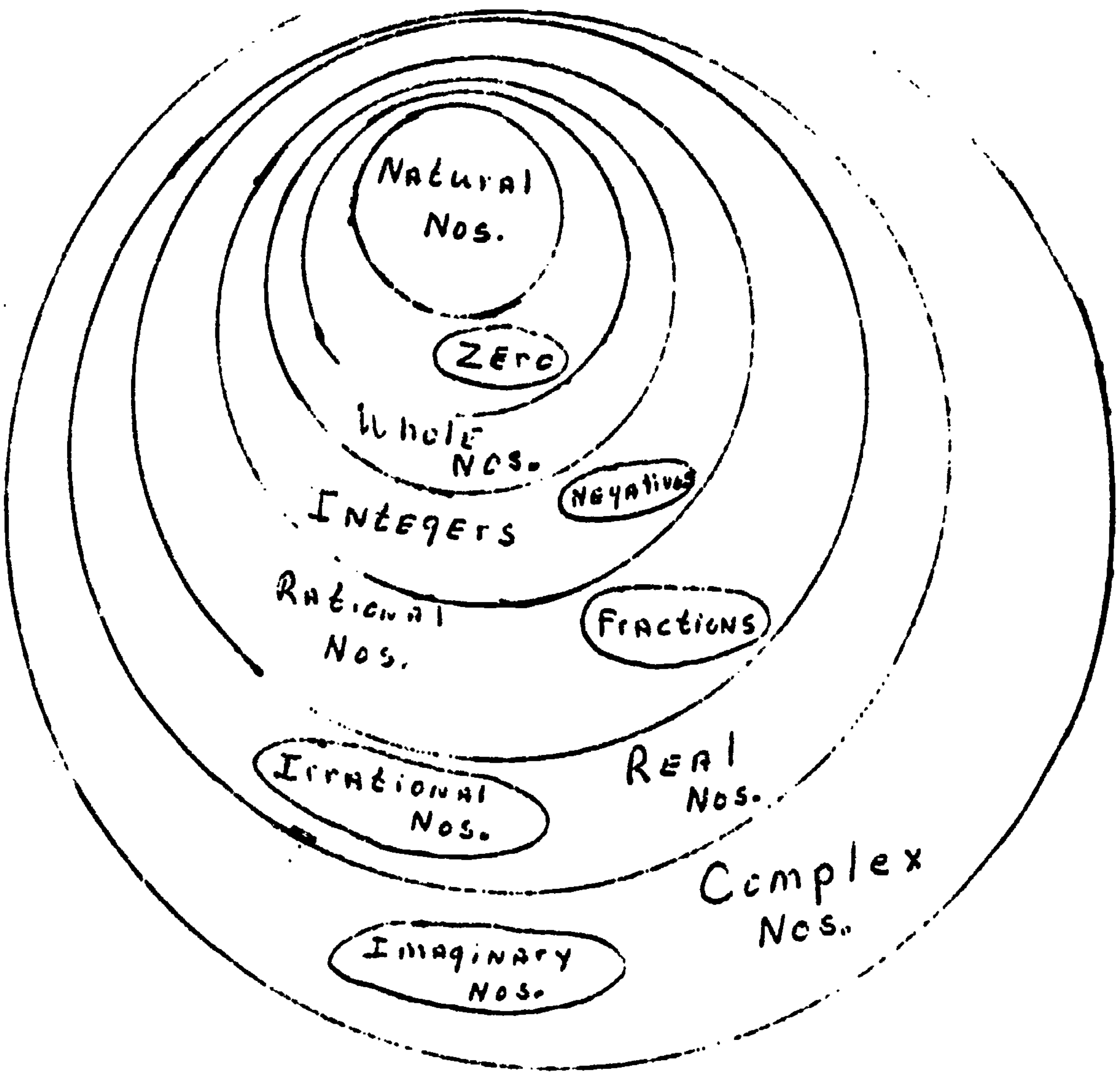
20 V. FIND A QUADRATIC EQUATION OVER THE REAL NUMBERS HAVING THE GIVEN SET AS ITS SOLUTION SET.

A) $\{-2 + 5i, -2 - 5i\}$

B) $\{i, -i\}$

IF YOU HAVE SATISFACTORILY COMPLETED YOUR WORK, TAKE THE LAP TEST. CONSULT YOUR TEACHER FIRST.

Using the following Venn Diagram of the set of complex numbers, determine if the given statements are true or false.




_______ 1. $Q \subset C$

_______ 2. $T \subset N$

_______ 3. $N \subset W$

_______ 4. $R \subset C$

_______ 5. $N \subset C$

_______ 6. $W \subset I$

_______ 7. $Q \subset R$

_______ 8. $N \subset Q$

_______ 9. $W \subset C$

_______ 10. $T \subset C$

_______ 11. $W \subset T$

_______ 12. $W \subset Q$

_______ 13. $R \subset N$

_______ 14. $I \subset C$

_______ 15. $C \subset T$

# APPENDIX II

## PART I

### Objective 4

Determine if each of the following properties apply to the set of complex numbers. Give an example for each.

a) closure for addition

b) commutative for addition

c) associative for addition

d) additive identity

e) additive inverse

## PART II

### Objective 6

Determine if each of the following properties apply to the set of complex numbers. Give an example for each.

a) closure for multiplication

b) commutative for multiplication

c) associative for multiplication

d) multiplicative identity

e) multiplicative inverses

f) distributive

BEST COPY AVAILABLE

APPENDIX III

Write an X by each property that holds for the given set.
Write a circle (0) by each property that does not hold.
Do not leave a blank.

| PROPERTIES | NATURAL | WHOLE | INTEGERS | RATIONALS | IRRATIONALS | REALS | Complex |
|---|---|---|---|---|---|---|---|
| Closure for + | | | | | | | |
| Closure for x | | | | | | | |
| Commutative for + | | | | | | | |
| Commutative for x | | | | | | | |
| Associative for + | | | | | | | |
| Associative for x | | | | | | | |
| Distributive | | | | | | | |
| Additive identity | | | | | | | |
| Additive inverse | | | | | | | |
| Multiplicative identity | | | | | | | |
| Multiplicative inverses | | | | | | | |

## REFERENCES


1. Vanatta, Glen D., Algebra Two: A Modern Course, Charles E. Merrill Publishing Co., 1966.

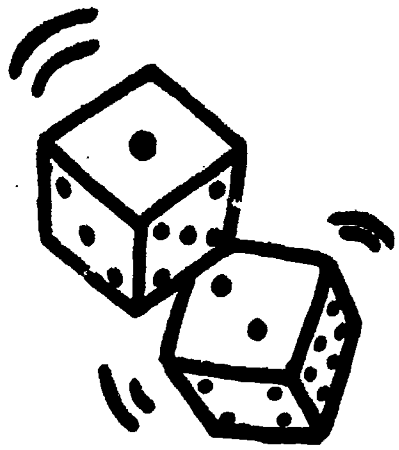
2. Dolciani, Mary P., Berman, Simon L., and Wooton, William, Modern Algebra, Book Two, Houghton Mifflin Co., 1965

3. Nichols, Eugene D., Modern Intermediate Algebra, Holt, Rinehart, and Winston, Inc., 1965.

4. Dolciani, Mary P., Wooton, William, Beckenbach, Edwin F., Sharron, Sidney, Modern School Mathematics, Algebra 2, Houghton Mifflin Co., 1968.

5. Payne, Joseph N., Zamboni, Floyd F., Lankford, Jr., Francis G., Algebra Two, Harcourt, Brace and World, 1969.

6. Pearson, Helen R., and Allen, Frank B., Modern Algebra: A Logical Approach, Book Two, Ginn and Company, 1966.



LEARNING
ACTIVITY
PACKAGE

# INTRODUCTION TO PROBABILITY

Ninety Six High School – Ninety Six, S.C.

Algebra 83-84

REVIEWED BY

LAP NUMBER 33

WRITTEN BY Mrs. Diane Evans

31273

# RATIONALE

## P R O B A B I L I T Y

Mathematics has long been closely associated with the physical sciences. Recently, there have been important applications of mathematics to the social and biological sciences, and application to all sciences have increasingly involved probability and statistics. The increasing significance of probability and statistics throughout our society makes it more and more important that people gain knowledge of probability and statistics.

This LAP takes a brief introductory look at probability. You will find this study of probability not only fun in itself, but also the basis for beginning the study of statistics.

NOTE: This LAP does not have any tests. There are three assignments that you are to complete and turn in to the teacher. You will be graded on these assignments.

## SAMPLE SPACES

So far in your study of mathematics, the concept of "set" has played an important role. A particular kind of set very important in the study of probability, consists of a collection of all the possible outcomes of an experiment. This set is called a sample space.

Example 1: Suppose you are asked to toss two coins, a dime and a quarter. In how many ways can they land?

Solution: Here is a list of all the possible outcomes of the toss:

| Dime | Quarter | |
|---|---|---|
| H | H | Both heads |
| H | T | Dime, heads; quarter, tails |
| T | H | Dime, tails; quarter, heads |
| T | T | Both tails |

The sample space for the above experiment may be written {HH, HT, TH, TT}. In this set, the first letter in each pair represents the outcome of the toss of the dime, the second letter represents the quarter.

Example 2: Consider the experiment where three different coins are tossed: a dime, quarter, and a penny. The list of all possible outcomes appears as follows:

| Dime | Quarter | Penny |
|---|---|---|
| H | H | H |
| H | H | T |
| H | T | H |
| H | T | T |
| T | H | H |
| T | H | T |
| T | T | H |
| T | T | T |

Again, the list of all possible outcomes or the sample space is as follows: {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}.

<u>Example 3</u>: Suppose you are asked to toss a die. What is the sample space of this experiment?

Assuming the die is conventional and your toss is legitimate, it can land, with only six faces up. Therefore, the sample space for this experiment is the following set: {1, 2, 3, 4, 5, 6}.

<u>Example 4</u>: Consider the problem of tossing a pair of dice. What is the sample space of this experiment?

To clarify the experiment in our minds, let us call one die, die (1) and the other die, die (2). Thus an outcome of a three on die (1) and a four on die (2) is different from an outcome of a four on die (1) and a three on die (2). In determining the sample space of this experiment, the following chart may be helpful:

| | Outcomes of Die (2) | | | | | |
|---|---|---|---|---|---|---|
| Outcomes of Die (1) | 1 | 2 | 3 | 4 | 5 | 6 |
| 1 | (1,1) | (1,2) | (1,3) | (1,4) | (1,5) | (1,6) |
| 2 | (2,1) | (2,2) | (2,3) | (2,4) | (2,5) | (2,6) |
| 3 | (3,1) | (3,2) | (3,3) | (3,4) | (3,5) | (3,6) |
| 4 | (4,1) | (4,2) | (4,3) | (4,4) | (4,5) | (4,6) |
| 5 | (5,1) | (5,2) | (5,3) | (5,4) | (5,5) | (5,6) |
| 6 | (6,1) | (6,2) | (6,3) | (6,4) | (6,6) | (6,6) |

The outcomes for die (2) are listed on top of the chart, while the outcomes for die (1) are listed along the left. The entries of the table are to be interpreted as follows:

(2,3): 2 on die (1) and 3 on die (2)

(3,2): 3 on die (1) and 2 on die (2)



## ASSIGNMENT ONE

Answer the following questions and TURN THEM IN to the teacher.

I. List a sample space for each of the following experiments:

1. A die is tossed, then a coin is tossed.
2. A three-digit numeral is to be formed from the digits 2, 4, and 5, with no digit to be used more than once in a numeral.
3. A box contains four balls: one white, one red, one blue, and one green. You are to draw one ball from the box, replace it and then draw a second ball.
4. Repeat exercise 3, except this time you draw two balls from the box, one at a time, without replacement.
5. Four dimes are tossed.

II. Using the sample space for die (1) and die (2) on Page 3, answer these questions:

6. How many of the outcomes show both the dice with the same outcome? For example: (1,1) etc.
7. How many outcomes have the sum of 7?
8. List the outcomes when die (1) is 1.
9. List the outcomes when die (2) is 1.
10. Using the results of problems 8 and 9, list the intersection and union of the two sets of outcomes.

### PROBABILITY

The word "chance" is often used in conversation. We may say, "we take a chance on a raffle," "the chances are the Gamecocks will win," "the chances are we will not get a test today."

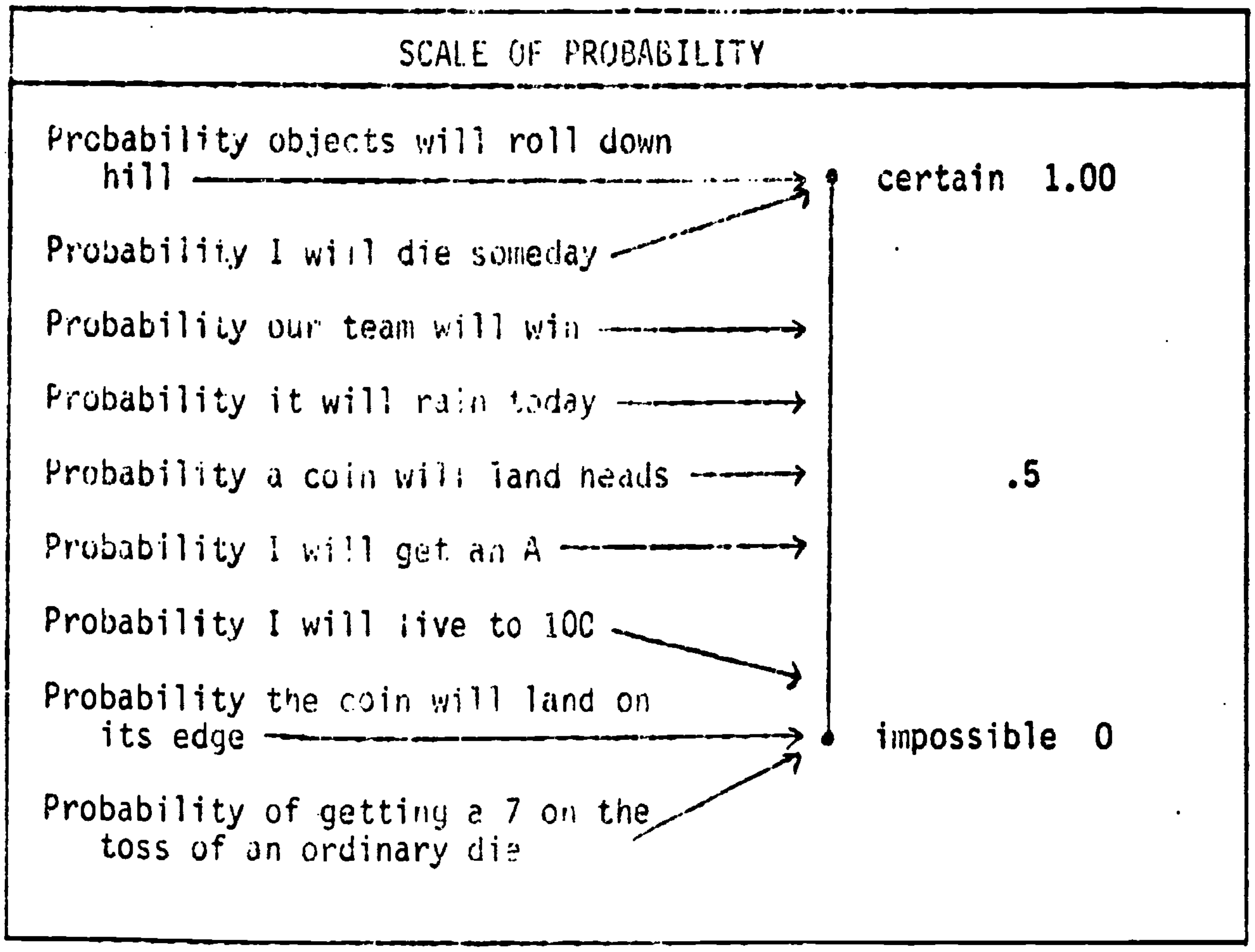
Sometimes we use numbers when talking about "chance." For example:

"The chances are 2 to 1 out team will win."

"The chances are 50-50 that a coin will come up tails."

In this section we will give the notion of chance a definite mathematical meaning. We will do this by introducing the concept of probability.

Let's consider an appropriate scale for probability. Study the chart below which shows a scale starting at 0 and extending to 1, with .5 representing the half-way point.




The top end represents an event we know will happen for certain. The bottom of the scale represents an event that is an impossibility. For example: gettina a 7 on a die that has only six faces numbered 1 to 6 has a probability of zero. Any event which is impossible to occur will always have a probability of zero. Some events are certainties. For example: the probability that I will die some day is a certainty. Any event which is a certainty will have a probability of one. Some events are easy to place on the scale; others, however, cause us to wonder.

We need to determine how we can arrive at the measure of the probability of a real event when it's neither a certainty nor an impossibility.

## EQUALLY LIKELY EVENTS

Consider the experiment of tossing a coin. Since there are two possible outcomes, a sample space for this experiment consists of just these two outcomes: {H, T}.

It seems reasonable to assume that heads and tails both have an equal opportunity to occur. When each element in the sample space has an equal chance of occurring, we say they are equally likely to happen.

Definition: Probability of an event

If an experiment can result in N different outcomes, and if each of these outcomes is equally likely, and if m of these outcomes corresponds to event A, the probability of event A, written P(A) is:

$$P(A) = \frac{m}{N}$$

In other words:

$$\text{The probability of } A = \frac{\text{the number of favorable outcomes}}{\text{Total possible outcomes}}$$

Example 1: Let's consider again the experiment of flipping a coin, N = 2, since there are two different outcomes, heads or tails. Therefore, the probability of the event heads is ½ and the probability of the event tails is ½; written:

$$P(H) = \tfrac{1}{2} \qquad P(T) = \tfrac{1}{2}$$

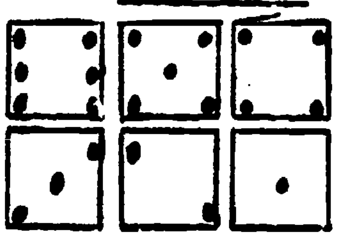
Consider the following experiment. Do you think this experiment verifies the result of example one?

> During World War II, one mathematician in an internment camp tossed a coin 1,000 times and then repeated this experiment over and over. He kept a record of the results. In ten sets of 1,000 tosses, he found that the number of heads was: 502,

511, 497, 529, 504, 476, 507, 528, 504, and 529. Notice how the numbers of heads clustered around 500 (one-half of 1,000) althouth none of the numbers was precisely 500. It is important to examine the results of such experiments to find if they do agree with our definition.

Spin the penny: Do you think the results will be the same if you <u>spin</u> a penny as they are when you toss a penny?
Take a <u>brand</u> new penny--be sure it is new--and spin it so that it rotates for a large number of turns. Observe the results. Repeat the spin 50 times and record the number of heads. Do the results approximate ½?

<u>Example 2:</u> Consider tossing a die. The six faces are marked by dots as shown. There are six possible equally likely outcomes. Our sample space looks like this: {1, 2, 3, 4, 5, 6}. Therefore, applying our definition



$$P(1) = \frac{1}{6} \quad P(2) = \frac{1}{6} \quad P(3) = \frac{1}{6}$$

$$P(4) = \frac{1}{6} \quad P(5) = \frac{1}{6} \quad P(6) = \frac{1}{6}$$

Since 7 is not a member of the sample space, P(7) = 0.

Every element in the sample space is called an <u>elementary event</u>.

Notice, each outcome of the experiment corresponds to a single element in the sample space.

<u>Example 3:</u> We toss a die again as in example one. This time we are concerned as to whether the outcome is an even number or an odd number. What is the probability the outcome is even?

<u>Solution:</u> The event "outcome is even," consists of the three elementary events {2, 4, 6}. Since three of the six equally likely events

result in even numbers, we say-- P(even) = $\frac{3}{6}$ or ½.

<u>Example 4:</u> A class of 30 students selects its class president in the following way. The name of each student is written on a slip of paper and placed in a box. The slips are mixed up by shaking the box. Without looking, a person draws a slip. The person named on the slip is appointed class president.

<u>Questions:</u>

(A) How many elements are in the sample space?

(B) What is the probability of a particular individual being chosen president?



<u>Solution:</u> For A: Since there are 30 students in the class, there are 30 elements in the sample space (or 30 possible outcomes).

For B: Since there are 30 different outcomes, P(A particular person is chosen) = $\frac{1}{30}$.

<u>Example 5:</u> In a bag there are 7 balls, 3 balls are red and 4 balls are green.

a) What is the probability of picking a red ball on the first draw?

b) What is the probability of picking a green ball on the second draw, given that a red ball was picked on the first draw?

c) What is the probability of picking a red ball on the third draw, given that two green balls have been picked on the first and second draws?

<u>Solution:</u> a) Sincere there are seven balls and three of them correspond to choosing a red ball, the probability of choosing a red ball is $\frac{3}{7}$.

b) Since a red ball was drawn on the first draw, we have six balls left, two red balls and four green balls. Therefore, the probability of choosing a green ball is now $\frac{4}{6}$ or $\frac{2}{3}$.

c) Since two green balls have been drawn, there are five balls left, three red balls and two green balls. Therefore, the probability of choosing a red ball is $\frac{3}{5}$.

## ASSIGNMENT TWO

Answer the following questions and turn them in to the teacher.

1. In a bag there are six white balls and one red ball. What is the probability of picking a red ball on the first draw?

2. In a bag there are six white balls and two red balls. What is the probability that you will pick a white ball on the first draw?

3. What is the probability you will get a three when you toss a die?

4. What is the probability you will not get a three when you toss a die?

5. A pupil is required to read a play, a short story, and a poem. What is the probability that he reads the poem first (assuming that each is equally likely to be in the book he picked up first)?

6. If the batting average of the star shortstop is .289, what is the probability he will get a hit the next time he is at bat?

7. If you have one dime and seven pennies in your pocket, what is the probability that the coin you take out at random will be a penny? A dime? A nickel?

## ASSIGNMENT THREE

Answer the following questions and TURN THEM IN TO THE TEACHER.

1. The probability of getting a head when tossing a coin is ½. Toss a coin 30 times. Give the number of times heads come up.

2. Suppose you toss a red die and a green die at the same time,

   a. List the sample space.
   b. Give the probability of rolling a 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12.
   c. What is the probability of rolling a 1 when rolling two dice?
   d. Roll two dice 30 times and record the results to show how close to true probability each result is.

3. If you choose a card from a regular bridge deck,

   a. What is the probability you will choose a 10?

b. What is the probability you will choose a heart?
c. What is the probability you will choose a 10 of hearts?

Use the Time-Life book Mathematics, pages 134-147, to answer 4 and 5.

4. When playing poker, what is the probability of getting a Royal Flush (10, Jack, Queen, King, Ace, of the same suit)?

5. What is the probability of a family having 13 boys and no girls?

6. Read one of the following and write a report (at least 50 words).

   A) Chapter 3 from Probability and Statistics for Everyone by Irving Adler.

   B) Any chapter that interests you from How to Take a Chance by Darrell Huff and Irving Geis.

## REFERENCES

How to Take a Chance, Darrell Huff and Irving Geis, W. W. Norton and Co., Inc., 1959.

Probability and Statistics for Every Man, Irving Adler, The John Day Co., 1963.

Mathematics (510), David Bergamini, by the editors of Time-Life Inc., 1963.