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ABSTRACT

A set of nine teacher-prepared Learning Activity Packages (LAPs) in geometry, these units cover the topics of proof; geometric inequalities; perpendicular lines and planes in space; parallel lines in a plane, and parallel lines and planes; polygonal regions and areas; similarity; plane coordinate geometry; circles and spheres; and characterization and construction. The units each include a rationale for the material being covered, a list of behavioral objectives, a list of resources which indicate reading assignments from texts and which specify problem sets for the students to complete, a student self-evaluation sheet, suggestions for advanced study, and references. (DT)

LEARNING ACTIVITY PACKAGE

A CLOSER LOOK AT PROOF

Ninety Six High School - Ninety Six, S.C. -

Geometry 114

LAP NUMBER 37

WRITTEN BY Bill Holland

REVIEWED BY

21673 2½

RATIONALE

This LAP introduces a variety of important, but rather difficult aspects of a proof. Most important of these is the indirect proof, for such a proof is vital to the acceptance of existence and uniqueness theorems. Such a proof is presented in two forms; the conventional two column form and also in paragraph form.

Here you will also learn to recognize a characterization theorem. Also, for the first time, you will be introduced to the use of auxiliary sets as an important technique in proofs.

Behavioral Objectives

By the completion of the prescribed course of study, you will be able to:

1. Given a set of hypotheses, state a valid conclusion.

2. Given a statement which calls for an indirect proof:

   a. Write a supposition for this indirect proof.
   b. Identify a resulting conclusion.
   c. Identify the known fact which is a contradiction to the conclusion.

3. Given a statement, determine if it is a statement of existence, uniqueness, existence and uniqueness, or a characterization statement.

4. Given a relationship existing between points, lines, and segments in a plane; answer true-false, completion or multiple choice type questions concerning theorems, corollaries and definitions relative to perpendicularity:

   a. Theorem: In a given plane, through a given point of a line, there is one and only one line perpendicular to the given line.

   b. The Perpendicular Bisector Theorem.

   c. Corollary: Given a segment and a line in a plane, if two points of the line are each equidistant from the end points of the segment, then the line is the perpendicular bisector of the segment.

   d. Theorem: Through a given external point there is at least one line perpendicular to a given line.

   e. Theorem: Through a given external point there is at most one line perpendicular to a given line.

   f. Corollary: No triangle has two right angles.

   g. Definition of a Right Triangle.

   h. Definition of Equidistant.

5. Given a drawing of a triangle and necessary information:

   a. Determine if it is a right triangle.
   b. Identify the sides which are legs.
   c. Identify the side which is the hypotenuse.

RESOURCES I

I. Readings:

1. Moise: #1, #2 pp. 153-154; #3 pp. 157-160; #4, #5 pp. 161-167.

2. Jurgensen: #1, #2 pp. 87-91, 163-165, 569-570, 575; #3 pp. 104-106; #4, #5 pp. 132-134, 168.

3. Anderson: #1, #2 pp. 190-193, 219; #3 pp. 194-197, 200-201; #4, #5 pp. 200-206.

4. Lewis: #1, #2 pp. 224-229; #3 pp. 255-257; #4-#5 pp. 172-190.

5. Nichols: #1, #2 pp. 82-85; #3 pp. 163-165; #4, #5 pp. 170-171, 128, 132.

II. Problems:

1. Moise: #1, #2 pp. 155-156 ex. 1-4, 7-11; #3 p. 161 ex. 1-7, p. 173 ex. 4; #4, #5 pp. 167-178 ex. 1-3, 7-10.

2. Jurgensen: #1, #2 pp. 89-90 ex. 7-44, pp. 92-93 ex. 1-16, p. 165 ex. 1-16; #3 pp. 106-107 ex. 1-4; #4, #5 pp. 134-135 ex. 1-20, pp. 168-169 ex. 1-12.

3. Anderson: #1, #2 pp. 193-194 ex. 1-8; #3 p. 198 ex. 1-7, pp. 198-199 ex. 1-4; #4, #5 p. 203 ex. 1-10, p. 204 ex. 3-5, p. 208 ex. 1, 3-6.

4. Lewis: #1, #2 ____; #3 p. 258 ex. 1; #4, #5 ____.

5. Nichols: #1, #2 p. 86 ex. 1; #3 ____; #4, #5 pp. 128-129 ex. 1-9.

SELF-EVALUATION I

1. For each of the following pairs of hypotheses, state a conclusion that follows logically from them.

   a) If Jack bought a car, it would use up all his savings.

      Jack still has his savings.

   b) If the house is white, it has green blinds.

      The house has blue blinds.

2. Assume that the following statements are true.

   All children eat candy.
   Mary Martin is an adult.
   John Jones does not eat candy.
   Nancy Newton eats candy.

   Which of the following is a valid conclusion?

   a) Nancy Newton is a child.

   b) John Jones is not a child.

   c) Mary Martin does not eat candy.

   d) None of the above

3. Assume that you are going to try to prove each statement below by the indirect method.

   a. If a triangle has no two angles congruent, then it is not isosceles.

      1. What is the supposition?

      2. Identify a resulting contradiction.

b. In a plane, there is at most one line perpendicular to a given line at a given point of the line.

   1. What is the supposition?

   2. Identify a resulting contradiction.
      (Hint: Angle construction postulate)

4. Which of the following does not imply uniqueness:

   a) A plane containing three noncollinear points R, S, T.

   b) A plane containing a given angle ∠DEF.

   c) A plane containing a segment $\overline{RS}$ and its midpoint.

   d) A bisector of a given angle ∠DEF.

5. For each of the following statements, indicate whether it states existence, uniqueness, both, or none.

   a) If M, N, and P are collinear points, there is exactly one plane containing all three points.

   b) If L is a line and P is a point on L, there is at least one line containing P and perpendicular to L.

   c) If E is a plane, and P is a point on E, there is at most one line through P and perpendicular to E.

6. Answer true or false:

   _______ a) In a given plane, the perpendicular bisector of a segment is the line which is perpendicular to the segment.

   _______ b) "For every two points there is one line that contains both points" is a statement of uniqueness.

   _______ c) "Given a line and a point not on the line, there is exactly one plane containing both of them," is a statement of uniqueness and existence.

   _______ d) The perpendicular bisector of a segment, in a plane, is the set of all points of the plane that are equidistant from the segment.

______ e) Through an external point there is at most one line perpendicular to a given line.

______ f) No triangle has two right angles.

______ g) The longest side of any triangle is called the hypotenuse.

______ h) In a right triangle the legs are the sides adjacent to the right angle.

______ i) Every segment has exactly one midpoint.

______ j) Every angle has exactly one bisector.

______ k) If M is between B and C, and A is any point not on $\overleftrightarrow{BC}$, then B is in the interior of $\angle AMC$.

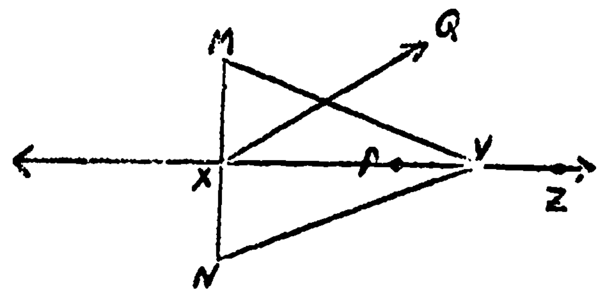
______ l) Proving that "there is exactly one" means proving both existence and uniqueness.

______ m) In our development of geometry, S.A.S. is used as a postulate to prove A.S.A. and S.S.S. as theorems.

______ n) In a plane, there are at most two perpendiculars to a line at a point of that line.

7. Given: X, P, Y, Z are collinear points, X is between M and N and all points are coplanar:

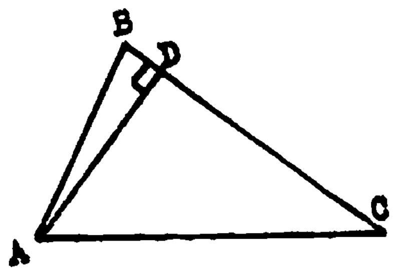
From COLUMN B indicate which reason supports each of the statements in COLUMN A.



COLUMN A	COLUMN B
a. If MX = XN and MY = YN, then $\overleftrightarrow{XZ}$ is the perpendicular bisector of $\overline{MN}$.	1. Given a line in a plane and a point on the line, there is exactly one line in the given plane perpendicular to the line at the given point.
b. If $\overleftrightarrow{XZ} \perp \overline{MN}$, then $\overleftrightarrow{QX}$ is not perpendicular to $\overline{MN}$.	2. Definition of perpendicular bisector of a segment.
c. If MX = XN and $\overline{MX} \perp \overleftrightarrow{QX}$, then $\overleftrightarrow{QX}$ is the perpendiuclar bisector of $\overline{MN}$.	3. Perpendicular Bisector Theorem.
d. If $\overleftrightarrow{XY} \perp \overline{MN}$, then $\overline{MY}$ is not perpendicular to $\overline{MN}$.	4. Through a point not on a given line, there is exactly one line perpendicular to the given line.
e. If $\overleftrightarrow{XY}$ is the perpendicular bisector of MN, then PM = PN.	5. None of these.

SELF-EVALUATION I (cont')

8. Given the following drawing and information as indicated, match the items in COLUMN B which is the best choice to the statement in COLUMN A. You may use an answer more than once.



COLUMN A	COLUMN B
a. The hypotenuse of △ACD	1. ∠BDA
b. The hypotenuse of △ADB	2. $\overline{AD}$
c. A leg of △ADB	3. $\overline{AB}$
d. A leg of △ACD	4. ∠BAD
e. An acute angle.	5. $\overline{AC}$

ADVANCED STUDY I

I.

1. Assume the following statements are true:

   Litmus paper is a paper impregnated with a chemical which changes color when in contact with an acid or an alkali base.

   Acids cause blue litmus paper to turn red.

   Alkalis cause red litmus paper to turn blue.

   Vinegar causes blue litmus paper to turn red.

   Which of the following is a valid conclusion?

   (a) Vinegar is an alkali.

   (b) Vinegar is an acid.

   (c) Vinegar is neither alkaline or acidic.

   (d) Vinegar can be either an alkaline or acidic.

   (e) None of the above.

2. Assume the following statements are true:

   A particle with a negative electric charge is attracted to the positive side of an electric field.

   A particle with a positive electric charge is attracted to the negative side of an electric field.

   A neutron enters an electric field and is not attracted to either side.

   Which of the following is a valid conclusion?

   (a) The neutron is a positively charged particle.

   (b) The neutron is a negatively charged particle.

   (c) The neutron has no electric charge.

   (d) None of the above.

3. Assume the following statements are true:

   A permanent magnet is attracted to iron.

   A permanent magnet is not attracted to aluminum.

   A permanent magnet is not attracted to a nail.

   Which of the following is a valid conclusion?

   (a) The nail is made of iron.

   (b) The nail is made of aluminum.

   (c) The nail is not made of iron.

   (d) The nail is not made of aluminum.

   (e) None of these.

4. If the distance between the origin of a light wave and an observer decreases rapidly, the observer sees the light at a higher frequency.

   If the distance between the origin and the observer increases rapidly the observer sees the light at a lower frequency.

   As seen from earth, the light from a distant star is observed to decrease in frequency and then increase in frequency.

   Which of the following is a valid conclusion?

   The distance between the earth and the star is:

   (a) increasing

   (b) decreasing

   (c) increasing then decreasing

   (d) decreasing then increasing

   (e) remaining the same

ADVANCED STUDY I (cont')

II. Prove that the S.A.S. was the only congruence postulate we needed, i. e. prove that the A.S.A. and S.S.S. postulates may be derived from the S.A.S. postulate.

III. a) Write the contrapositive of the following sentences:

1) If X is not an even integer, then X is an odd integer.
2) If two lines are parallel, then they do not intersect.
3) If two angles are supplementary, then the sum of their measures equals 180.
4) If a triangle is not isosceles, then it is not equilateral.
5) If $x^2 = 9$, then $x = 3$.
6) If two angles are congruent, then their measures are equal.

b) Prove the following by using an indirect proof to show that the contrapositive is true: If the square of an integer is not odd, the integer is not odd.

SECTION II

Behavioral Objectives

By the completion of the prescribed course of study, you will be able to:

6. Given an auxiliary set to be introduced in a figure, justify its introduction with any of the following:

   a. The Line Postulate
   b. The Point Plotting Theorem
   c. The Plane and Space Postulate
   d. Existence and Uniqueness Theorems
   e. The Angle Construction Postulate
   f. The Angle Bisector Theorem
   g. Theorems of Perpendicularity

7. Given a statement to be proved, complete such a proof as a direct or indirect proof as required.

RESOURCES II

I. Readings:

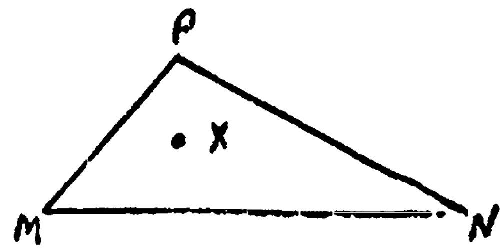
1. Moise: #6, #7 pp. 169-172.
2. Jurgensen: #6, #7 pp. 123-125, 132-135.
3. Anderson: #6, #7 pp. 209-211.
4. Lewis: #6, #7 pp. 172-186.
5. Nichols: #6, #7 pp. 44-57, 82-91.

II. Problems:

1. Moise: #6, #7 p. 156 ex. 5, 6, p. 167 ex. 4-6, pp. 173-174 ex. 1-3, 5-8.
2. Jurgensen: #6, #7 p. 166 ex. 1-4, p. 577 ex. 25-27, p. 135 ex. 1-18, p. 145 ex. 2.
3. Anderson: #6, #7 p. 194 ex. 9-13, p. 204 ex. 6-8, p. 208 ex. 1-4, p. 208 ex. 2, p. 211 ex. 1-13, pp. 212-213 ex. 1-9, pp. 218-219 ex. 5-8.
4. Geometry: #6, #7 pp. 230-231 ex. 1-4, p. 233 ex. 1-4, p. 258 ex. 2-6, pp. 176-177 ex. 1-8, pp. 187-189 ex. 2, 3, 9-11, 14.
5. Nichols: #6, #7 p. 86 ex. 2-3, p. 96 ex. 1-4, 12, 15, p. 165 ex. 1-2.

SELF-EVALUATION II

1. For the given drawing, which of the following auxiliary sets could be introduced with validity. Answer yes or no for each item, and justify each yes answer.



   a) The median $\overline{PQ}$

   b) $\overrightarrow{MX} \perp \overline{PN}$

   c) $\overleftrightarrow{XY}$, the perpendicular bisector of $\overline{PN}$.

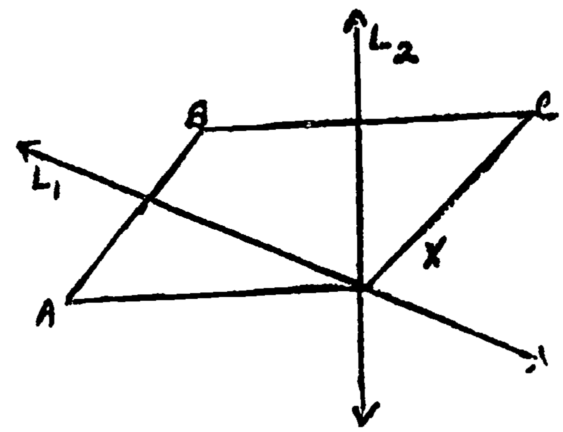
   d) $\overleftrightarrow{YZ} \perp \overline{PN}$

   e) The bisector $\overrightarrow{PX}$ of $\angle MPN$.

2. Prove the following using an indirect proof written in paragraph form: $\sqrt{17} > 4$

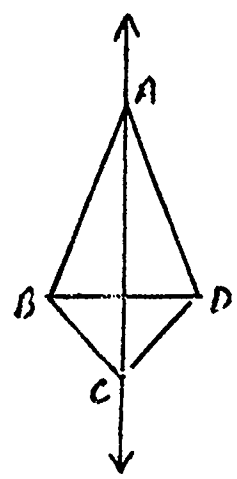
3. Prove the following using an indirect proof written in two column form: If $m\angle A + m\angle B \neq 180$, then $\angle A$ and $\angle B$ are not supplementary.

4. Prove the following:



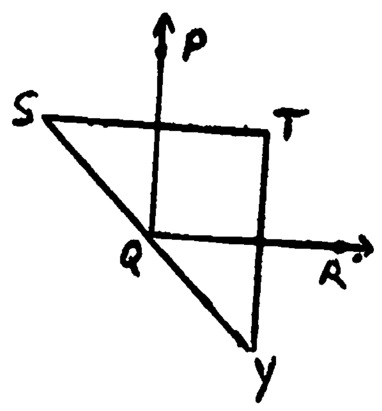
   Given: L_1 the perpendicular bisector of $\overline{AB}$
   L_2 the perpendicular bisector of $\overline{BC}$
   L_1 and L_2 intersect at X
   Prove: AX = CX

5. Prove the following:



   Given: $\angle ABD \cong \angle ADB$
   $\angle CBD \cong \angle CDB$
   Prove: $\overleftrightarrow{AC}$ is the perpendicular bisector of $\overline{BD}$.

6. Prove the following:



Given: $\overleftrightarrow{QP}$ is the perpendicular bisector of $\overline{ST}$

$\overleftrightarrow{QR}$ is the perpendicular bisector of $\overline{TY}$

PROVE: QS = QY

7. For the following statement, draw a figure, list the "given", list "to prove" and then write an indirect proof in two column form: If two angles of a triangle are not congruent, then the sides opposite these angles are not congruent.

If you have mastered the Behavioral Objectives, take the Progress Test.

ADVANCED STUDY II

1. Use an indirect proof written in paragraph form to prove $\sqrt{5}$ is not rational.

2. a. Given $\triangle ABC$ and $\triangle DEF$ are two congruent acute scalene triangles. Prove that corresponding altitudes of these two triangles are congruent.

   b. If $\square ABCD$ is a quadrilateral such that one diagonal of this quadrilateral bisects two angles of the quadrilateral, prove that it also bisects the other diagonal.

   c. If $\triangle XYZ$ is an isosceles triangle with $XZ = YZ$ and the bisectors of the base angles X and Y bisect each other at E, prove that $\overleftrightarrow{ZE}$ is perpendicular to $\overline{XY}$.

   d. Prove the following using an indirect proof written in paragraph form: Given: $a > o$, $b > o$, Prove: $a + b \neq \sqrt{a^2 + b^2}$

BIBLIOGRAPHY

I. Textbooks:

1. Moise, Downs: Geometry (Addison-Wesley Publishing Co., Inc., 1967).
2. Jurgensen, Donnelly, Dolciani: Modern Geometry (Houghton Mifflin Co., 1965).
3. Anderson, Garon, Gremillion: School Mathematics Geometry (Houghton Mifflin Co., 1969).
4. Nichols, Palmer, Schact: Modern Geometry (Holt, Rinehart and Winston, Inc., 1968).
5. Lewis: Geometry, A Contemporary Course (D. Van Nostrand Co., Inc., 1968)

LEARNING ACTIVITY PACKAGE

GEOMETRIC INEQUALITIES

Geometry 114

REVIEWED BY ______________

LAP NUMBER 38

WRITTEN BY Bill Holland

RATIONALE

You are familiar with relations "greater than" and "less than" in the system of real numbers. Since we have introduced the system of real numbers in our geometry in connection with segment measures and angle measures, it seems reasonable that these relations can have a geometric interpretation. You will now study conditions under which we can say one segment is longer than another (has greater length) or one angle is larger than another (has a greater measure).

Theorems of geometric inequality are of a different nature and should present a greater challenge than those of previous units. For proofs of inequalities we should, first start by making reasonable conjectures about statements that ought to be true, second, we should express our conjectures in good mathematical language. However, the second conjecture may be more difficult.

There are two types of inequality properties to be considered involving either a single triangle or a pair of triangles.

SECTION I

Behavioral Objectives

By the completion of the prescribed cou e of stu ,, you wi l be able to:

1. Given two line segments or angles, identify the order property which justifies the measure of one being less than, equal to, or greater than the measure of the other.

2. Given a triangle:

   a. Identify the exterior angles of the triangle.
   b. Identify the remote interior angles to a given exterior angle.
   c. State the order relation between part (a) and part (b).

3. Given two congruent triangles, with airs of c rresponding parts congruent, state if they can b proved congruent by:

   a. Side-Angle-Angle Theorem
   b. Hypotenuse-Leg Theorem

4. Given any triangle, determine the order relation between the:

   a. angles when given the measure of the sides.
   b. sides when given the measure of the angles.

5. Given a statement containing a hypothesis and a conclusion:

   a. State its converse.
   b. Identify if its converse is true.

6. Given a line and a point not on the line, define the shortest segment from the point to the line.

7. Given any three numbers, determine if they could be the lengths of the sides of a triangle.

8. Given two sides of a triangle congruent, respectively, to two sides of a second triangle, answer correctly questions concerning the third pair of sides using:

   a. The Hinge Theorem
   b. Converse of the Hinge Theorem

9. Given a drawing properly labeled of a triangle and its altitudes, name the segments representing the altitudes of the triangles.

10. Given a drawing or a statement, answer correctly true-false, multiple choice, or matching questions involving:

    a. Geometric inequalities
    b. Theorems of inequality of real numbers
    c. Definition of exterior and interior angles

Behavioral Objectives (cont')

d. Exterior Angle Theorem
e. If a triangle has one right angle, then the other angles are acute.
f. S.A.A. Theorem
g. Hypotenuse-Leg Theorem
h. Inequalities in a Triangle Theorem
i. Converse of a Theorem
j. Definition of distance from a point to a line
k. First Minimum Theorem
l. Triangle Inequality Theorem
m. The Hinge Theorem and its converse
n. Definition of altitude of a triangle
o. Theorem: The sum of the measures of any two angles of a triangle is less than 180.

RESOURCES I

I. READINGS:

1. Moise: #1, #2 pp. 183-191; #3 pp. 192-193; #4 pp. 195-196; #5 - #7 pp. 198, 200-201; #8 pp. 203-204; #9 pp. 206-207; #10 (a-c, o) pp. 183-191, (f-g) pp. 192-193, (m) pp. 195-196, (h-l) pp. 198, 200-201, (m) pp. 203-204, (n) pp. 206-207.

2. Jurgensen: #1, #2 pp. 58-59, 100-101; #3 pp. 194, 198-199; #4 ____; #5 - #7 p. 168; #8 ____; #9 p. 59; #10 (a-e, o) pp. 58-59, 100-101, (f-g) pp. 194, 198-199, (h-l) p. 168, (m) ____, (n) p. 242.

3. Anderson: #1, #2 pp. 221-227; #3 pp. 229-231; #4 pp. 234-235; #5-#7 pp. 238-242; #8 pp. 245-246; #9 p. 242; #10 (a-e,o) pp. 221-227, (f-g) pp. 229-231, (m) pp. 34-235, (h-l) pp. 238-242, (m) pp. 245-246, (n) p. 242.

4. Lewis: #1, #2 pp. 218-221; #3 pp. 1)-161; #4 pp. 564-566; #5-#7 pp. 517-522; #8 pp. 567-569; #9 p. 147; #10 (a-e,o) pp. 218-221, (f-g) pp. 160-161, (m) pp. 564-566, (h-l) pp. 517-522, (m) pp. 567-569, (n) p. 147.

5. Nichols: #1, #2 pp. 20-21, 131, 179-190; #3 pp. 154-156, 159-160; #4 pp. 195-197; #5- #7 pp. 193-194; #8 pp. 196-197; #9 pp. 166-167; #10 (a-e,o) pp. 20-21, 131, 179-190, (f-g) pp. 154-156, 159-160, (m) pp. 195-197, (h-l) pp. 193-194, (n) pp. 166-167.

II. PROBLEMS:

1. Moise: #1, #2 pp. 183-185 ex. 1-10, pp. 186-187 ex. 1, 3, 5, pp. 190-191 ex. 1-3, 5, 7-8; #3 p. 194 ex. 1; #4 pp. 196-197 ex. 1-2, 4, 6-7, 10; #5 - #7 p. 199 ex. 1-6, pp. 201-202 ex. 1-2, 5-7; #8 p. 205 ex. 5; #9 p. 207 ex. 1-2; #10 Appendix I.

2. Jurgensen: #1, #2 p. 101 ex. 1-12, p. 102 ex. 1-14; #3 p. 195 ex. 1-16, pp. 199-200 ex. 1-16; #4 ____; #5 - #7 pp. 169-170 ex. 5-12; #8 ____; #9 pp. 59-60 ex. 1-10; #10 Appendix I.

3. Anderson: #1, #2 pp. 223-224 ex. 1-2, 5-9, p. 227 ex. 1-13, p. 228 ex. 1-2, 4; #3 pp. 231-232 ex. 1-4; #4 p. 236 ex. 1-2, 5-7; #5- #7 p. 240 ex. 1-6, p. 243 ex. 1-5; #8 p. [illegible] ex. 1-9; #9 p. 243 ex. 1-2; #10 Appendix I

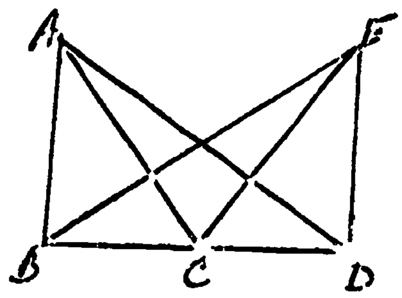
4. Lewis: #1, #2 pp. 222-223 ex. 2-9; #3 ____; #4 ____; #5-#7 pp. 523-524 ex. 1, 3, 6; #8 ____; #9 ____; #10 Appendix I.

5. Nichols: #1, #2 p. 192 ex. 1-5, 11; #3 pp. 156-157 ex. 1, 14, p. 161 ex. 1; #4 p. 198 ex. 9-10; #5-#7 p. 73 ex. 6, p. 195 ex. 6, 9, 14; #8 pp. 200-201 ex. 1-3; #9 p. 167 ex. 1-4; #10 Appendix I.

SELF-EVALUATION I

1. Consider the given figure and name the property that each of the following statements is an example of:

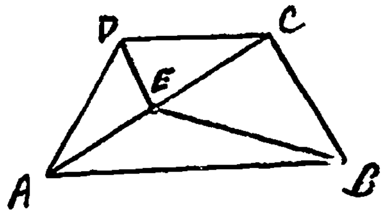
Given: C is the midpoint of $\overline{BD}$.
$\angle ABD$ and $\angle EDB$ are right angles.



a) $BC = CD$
b) In $\triangle ACB$ if $AB \neq BC$, and $AB \not< BC$, then $AB > BC$.
c) If $m\angle ABD = m\angle ABE + m\angle EBD$, then $m\angle ABD > m\angle ABE$.
d) If $ED > AB$, and $AB > BC$, then $ED > BC$.
e) If $AD = EB$, then $AD + BC = EB + BC$
f) If $AB < ED$, then $ED > AB$.
g) If $AB^2 + BC^2 = AC^2$, than $BC^2 < AC^2$
h) If $m\angle ACB \not< m\angle ECD$, then $m\angle ACB \geq m\angle ECD$.

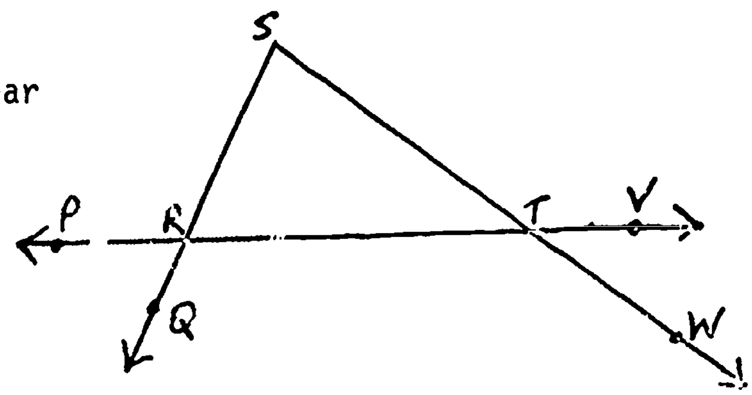
2. Using the included diagram, name each of the following:

a) An exterior angle of $\triangle AEB$
b) An exterior angle of which $\angle EDC$ and $\angle ECD$ are remote interior angles.
c) A triangle of which $\angle AEB$ is an exterior angle.
d) Two remote interior angles of which $\angle CED$ is an exterior angle.



3. From the given figure identify each of the following:

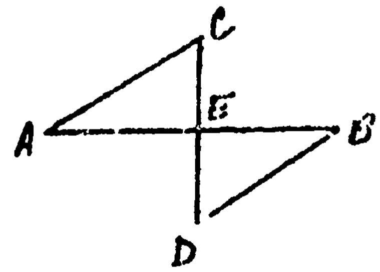
a) the exterior angle(s) for remote interior $\angle SRT$
b) the interior angle froming a linear pair with $\angle WTR$
c) the remote interior angle(s) to $\angle STV$



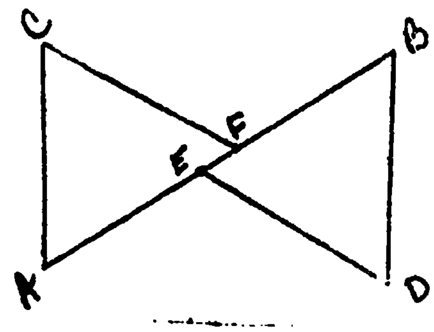
4. From the choices given, select what is most appropriate in proving the following triangles congruent:

   a) S.A.A. Theorem
   b) Hypotenuse-Leg Theorem
   c) A.S.A. Theorem
   d) None of these.

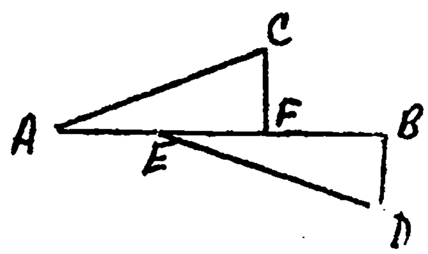
   1. Given: $\overleftrightarrow{CD}$ the $\perp$ bisector of $\overline{AB}$
      AC = BD



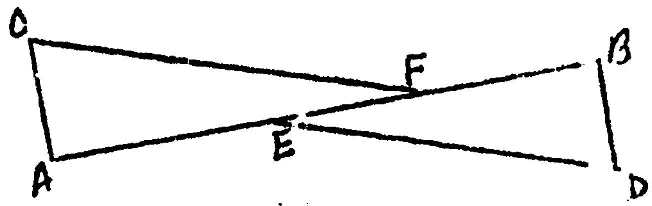
   2. Given: AE = FF
      m ∠C = m ∠D
      m ∠A = m ∠B



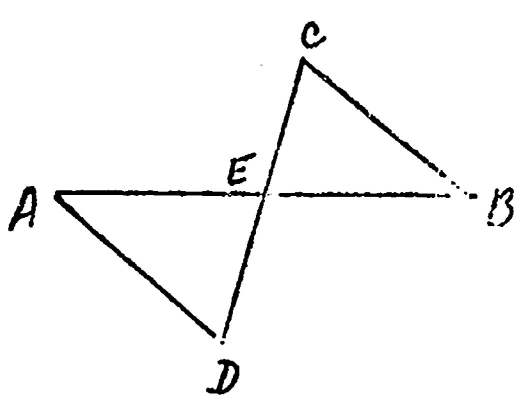
   3. Given: $\overline{CF} \perp \overline{AB}$, $\overline{BD} \perp \overline{AB}$
      AE = FB, m ∠A = m ∠BED



   4. Given: $\overline{CA} \perp \overline{AB}$, $\overline{BD} \perp \overline{AB}$
      CF = ED, AC = BD



   5. Given $\overleftrightarrow{AB}$ the $\perp$ bisector of $\overline{CD}$
      m ∠B = m ∠A



5. Decide which of the following is true or false. If false, tell why.

____ a) Given ΔABC, if $AB > BC$, then $\angle C > \angle B$.

____ b) Given ΔABC, if $\angle A < \angle B$, then AC BC.

____ c) Given ΔRAT, if $AT > RT$, then $\angle R > \angle A$.

____ d) Given ΔRAT, if $\angle T < \angle R$, then $AR < AT$.

____ e) Given ΔPDQ, if $\angle Q > \angle P$, then $PQ > DQ$.

____ f) Given ΔPDQ, if $\angle P > \angle Q$ and $\angle Q > \angle D$, then $DQ > PQ$.

____ g) Given ΔZOT, if $ZT < OZ$ and $OZ < OT$, then $\angle Z < \angle T$.

____ h) Given ΔABC, then $AB < AC$ or $AB = AC$, or $AB > AC$.

____ i) Given ΔCAT, then $\angle C > \angle A$ or $\angle C = \angle A$ or $\angle A < \angle C$.

____ j) Given ΔCTF, if $CF > FT$, then $\angle T > \angle C$.

6. For each of the following statements, write a converse which is a true statement. If the converse is not true, write none.

____ a) If two angles are complementary, then each is an acute angle.

____ b) If two triangles are congruent, the corresponding sides are congruent.

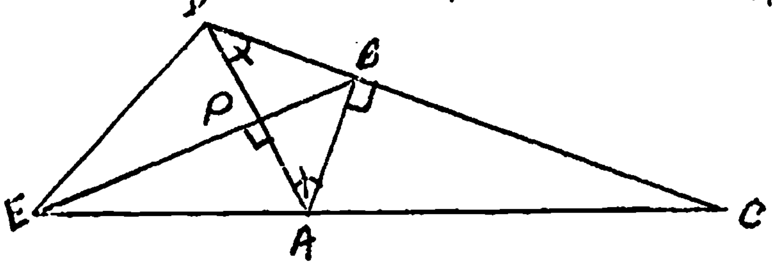
____ c) Every equilateral triangle is equiangular.

____ d) If two angles are right angles, then they are congruent.

____ e) If two angles are complementary, each of them is an acute angle.

____ f) If two sides of a triangle are congruent, then the angles opposite those sides are congruent.

7. From the diagram, match the statement in Column A with the appropriate theorem or postulate which applies from Column B.



COLUMN A

____ a) $AB + CB > AC$
____ b) $AD > AB$
____ c) $ED + AD < EA + 2AD + BD$
____ d) $EP < ED$
____ e) $EP + PB < EA + AC$

COLUMN B

1. The shortest segment joining a point to a line is the perpendicular segment.
2. The distance between a line and a point on the line is zero.
3. The sum of the lengths of any two sides of a triangle is greater than the length of the third side.
4. The hypotenuse of a right triangle is the side opposite the right angle.
5. None of these

8. Which of the following sets of numbers could be the lengths of the sides of a triangle?

____ (a) {5, 12, 16}

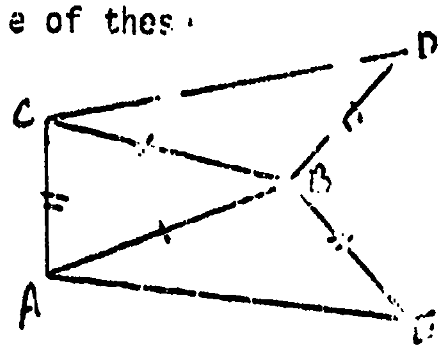
____ (b) {5, 10, 16}

____ (c) {5, 9, 14}

____ (d) {5, 5, 6}

9. For the following diagram, answer each question as:

(a) < (b) = (c) > (d) none of these



1) if ∠CBD > ∠ABE, then CD ? AE.

2) if BE < AB, then ∠CAB ? ∠ABE

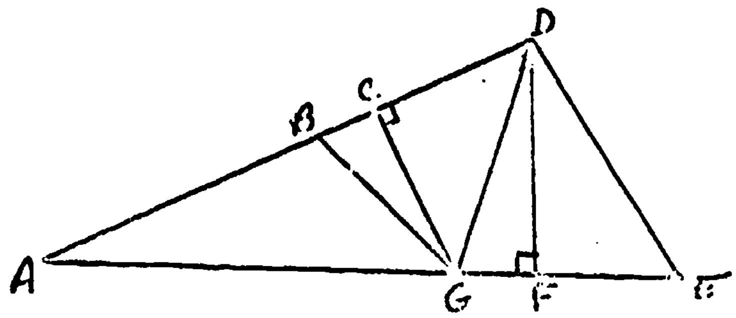
3) if BA < CD, then ∠CBD ? ∠ACB

4) if CD = EA, then ∠ABE ? ∠CBD

5) if ∠CAB < ∠ABE, then CB ? AE.

10. From the drawing as shown, determine the segment which is the altitude of each triangle indicated.

a) △GAD
b) △DAE
c) △BAG
d) △EDG
e) △BGD



11. Answer true or false:

____ a) An exterior angle of a triangle is greater than each interior angle of a triangle.

____ b) A median of a triangle is perpendicular to the side of the triangle to which it is drawn.

____ c) An altitude of a triangle can be a perpendicular segment from a vertex to a point in the exterior of the triangle.

____ d) If two sides of a triangle are of unequal length, then two angles of the triangle are of unequal measure

____ e) The distance of a point on a line to the line is said to be zero.

____ f) If $\overline{EB} > \overline{AG}$, then EB > AG.

____ g) A triangle can be formed having sides of lengths 89, 106, and 17.

____ h) If a triangle has one right angle, then its other angles are acute.

IF YOU HAVE MASTERED THE BEHAVIORAL OBJECTIVES, TAKE YOUR PROGRESS TEST.

ADVANCED STUDY I

1. a) Prove: If two legs of one right triangle are congruent to two legs of another right triangle, the triangles are congruent.

   b) Prove: If the hypotenuse and one acute angle of a right triangle are congruent to the hypotenuse and an acute angle of another right triangle, the triangles are congruent.

   c) If the leg and an acute angle of one right triangle are congruent to a leg and an acute angle of another right triangle, then the triangles are congruent.

2. Prove: If $x < y$ and $a < 0$, then $ax > ay$.

3. Given ΔXYZ with $XY = m$, $YZ = n$, and $XZ = 1$. Prove that $|1 - n| < m$. In words, what does the above proof say?

4. Determine whether the following method could be used to trisect an angle: Take $\angle XYZ$. Choose a point M on $\overrightarrow{YX}$ and a point N on $\overrightarrow{YZ}$ such that $YM = YN$. Draw $\overline{MN}$ and pick points A and B on $\overline{MN}$ such that $MA = \frac{1}{3} MN$, $AB = \frac{1}{3} MN$, and $BN = \frac{1}{3} MN$. Then draw $\overrightarrow{YA}$ and $\overrightarrow{YB}$. Then $\angle XYZ$ is now trisected.

SECTION II

Behavioral Objectives

By the completion of the prescribed course of study, you will be able to:

11. Given a statement to be proved, use the appropriate definitions, theorems, postulates, or corollaries to complete a proof for the statement or to write a complete proof for the statement.

RESOURCES II

I. READINGS:

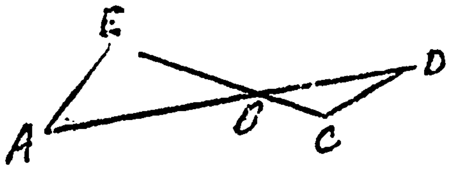
Same as in Section I for all texts.

II. PROBLEMS:

1. Moise: #11 pp. 186-187 ex. 2-4, 6-7; pp. 190-191 ex. 4, 6, 9-10, p. 194 ex. 2-8, p. 197 ex. 3, 5, 8-9, p. 202 ex. 3-4, 8, p. 205 ex. 1-4, 6-9, pp. 207-208 ex. 3-8.
2. Jurgensen: #11 p. 114 ex. 3-4, 12, pp. 196-197 ex. 4-6, 8, 18, p. 200 ex. 3.
3. Anderson: #11 p. 224 ex. 10-14, pp. 232-233 ex. 1-5, 7-10, pp. 236-237 ex. 3-4, 8-13, p. 240 ex. 7-8, pp. 243-244 ex. 1-12, pp. 247-248 ex. 1-11.
4. Lewis: #11 p. 222 ex. 1, pp. 162-163 ex. 1, 2, 5, 7, 9, p. 572 ex. 1-14.
5. Nichols: #11 p. 192 ex. 6-7, 9-10, pp. 161-162 ex. 2-13, pp. 197-198 ex. 1-2, 5-8, pp. 194-195 ex. 2-3, 7-8, 10-13, p. 168 ex. 5-12.

SELF-EVALUATION II

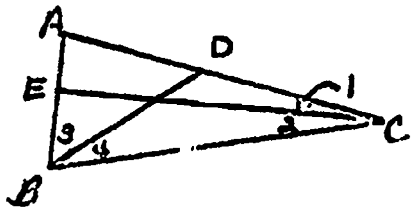
1. Prove the following:



Given: $\angle E > \angle A$
$\angle C > \angle D$

Prove: $AD > EC$

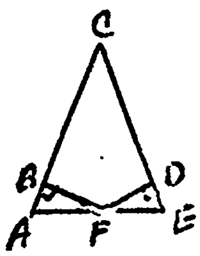
2. Prove the following:



Given: $\angle 1 < \angle 4$
$\angle 2 < \angle 3$

Prove: $AB \neq AC$

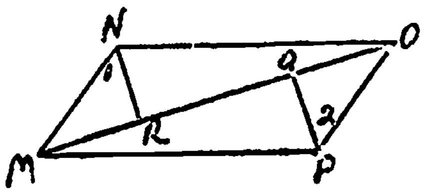
3. Prove the following:



Given: $CA = CE$, F is the midpoint of AE, $\overline{FB} \perp \overline{AC}$, $\overline{FD} \perp \overline{CE}$

Prove: $BA = DE$

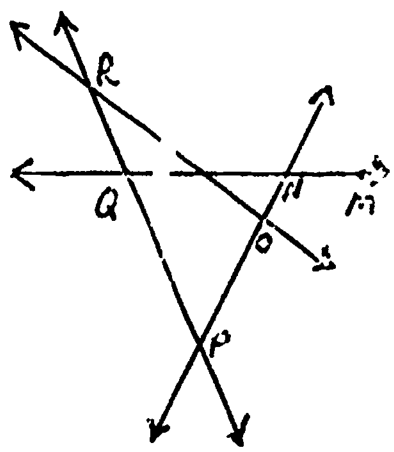
4. Prove the following:



Given: $\overline{NR}$ and $\overline{QP} \perp \overline{MO}$
$\overline{MQ} \cong \overline{RO}$
$\overline{MN} \cong \overline{OP}$

Prove: $\angle 1 \cong \angle 2$

5. Prove the following:



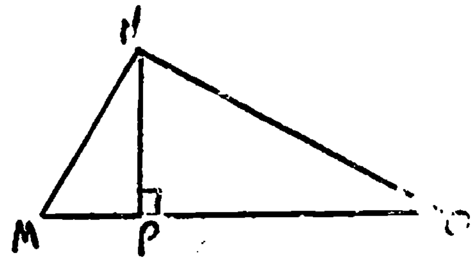
Given: the plane figure as shown

Prove: $\angle PNM > \angle QRO$

6. Prove the following:

   Given: $\overline{NP} \perp \overline{MO}$
   $MP < PO$

   Prove: $NM < NO$



7. Write a complete proof for the following statement: If an altitude of a triangle bisects an angle of the triangle, then the triangle is isosceles.

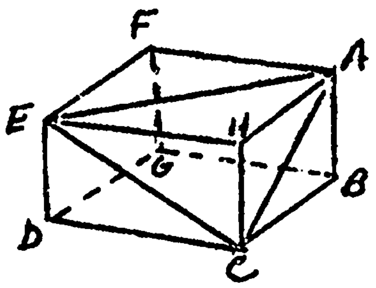
IF YOU HAVE MASTERED THE BEHAVIORAL OBJECTIVES, TAKE YOUR PROGRESS TEST. AFTER THE PROGRESS TEST A LAP TEST IS SCHEDULED.

ADVANCED STUDY II

1. Use an indirect proof to show that the base angles of an isosceles triangle are acute.

2. Let $\overleftrightarrow{MN}$ intersect $\overleftrightarrow{XY}$ at A, between X and Y. Perpendiculars from X and Y to $\overleftrightarrow{MN}$ meet $\overleftrightarrow{MN}$ at B and C respectively. Prove that B and C are not on the same side of $\overleftrightarrow{XY}$.

3. Prove the following:



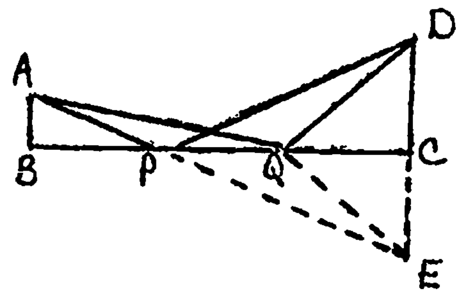
Given: $\overline{ED} \perp \overline{DC}$, $\overline{ED} \cong \overline{AB}$
$\overline{DC} \cong \overline{BC}$, $\angle AEC > \angle EAC$

Prove: $\overline{AB}$ is not perpendicular to $\overline{BC}$

4. Prove: The triangle formed by joining the midpoints of the sides of an equilateral triangle is also equilateral.

5. In isosceles $\triangle XYZ$, $\overline{XY} \cong \overline{YZ}$. Base $\overline{XZ}$ is extended to point A. The bisectors of $\angle XYZ$ and $\angle XYA$ intersect at B. Justify in any manner whatsoever that $m\ \angle BZA \neq 30$.

6. When light is reflected from the plane mirror $\overline{BC}$ from point A to point D, it will take the path from A to P to D. Show that this path is shorter than any other path such as from A to Q to D.



1. Name all the theorems and postulates you know to prove triangles congruent.

2. Draw an example to show S.S.A. could never be a congruence statement.

3. Draw a triangle whose sides are 2", 4½",and 3½". Draw the angle bisectors of this triangle, the medians of this triangle, and the altitudes of this triangle.

4. Answer the following true or false.

_____ a) The converse of a false statement is false.

_____ b) If $7 < 16$ and $3 < y$, then $16 + y > 10$.

_____ c) A triangle has only three exterior angles.

_____ d) A right triangle can have at most one obtuse angle.

_____ e) In ΔRST if $\angle RST > \angle TSR$, then $ST > SR$.

_____ f) The converse of "If two angles are congruent, they have the same measure is "Two angles have the same measure if they are congruent."

_____ g) In ΔMNO since $\angle M + \angle N < 180$ and $\angle N + \angle O < 180$, then $\angle M + \angle N < \angle N + \angle O$.

_____ h) A right triangle has only one altitude.

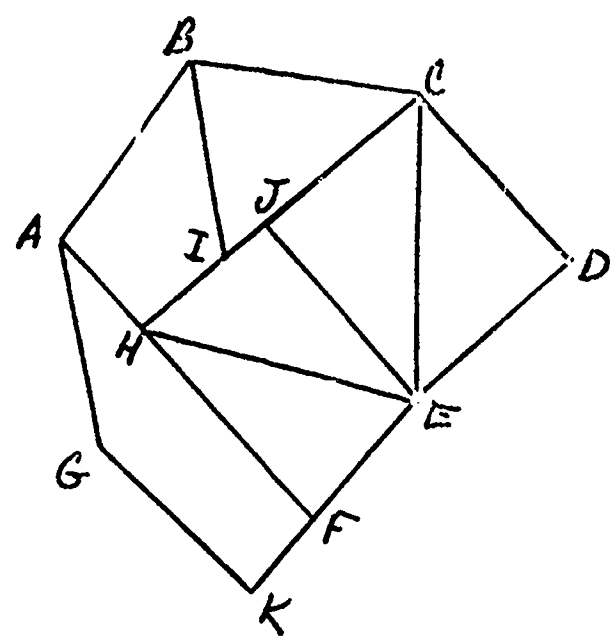
_____ i) In ΔRST and ΔXYZ if $\overline{RS} \cong \overline{XY}$, $\overline{ST} \cong \overline{YZ}$, and $\angle Z > \angle T$, then $RS > XY$.

_____ j) An exterior angle of a triangle forms a linear pair with a remote interior angle of the triangle.

5. Answer the following questions about the figure which is given:

a) Name an exterior angle of ΔHEF

b) Name two angles whose measures are less than the measure of $\angle BIH$.



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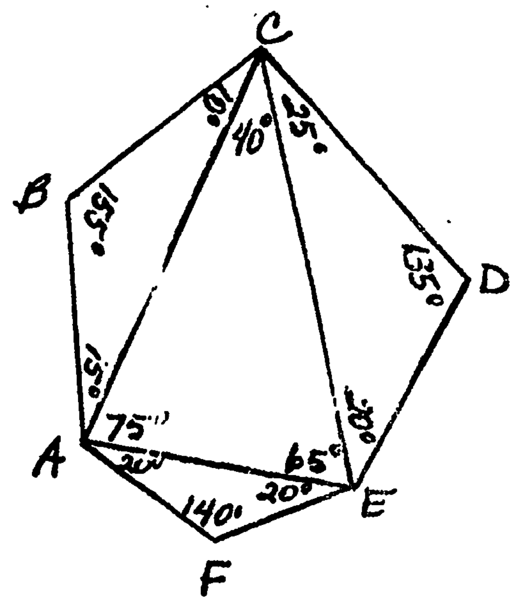
c) Name a remote interior angle of $\angle EJC$.

d) Is $\angle ABI$ an exterior angle of ΔBIC? Explain.

6. Answer the following about the given figure:



a) BC ? AC

b) AF ? CE

c) CD ? DE

d) CD ? CE

e) AE ? CA

f) Can you say BC > DE? Explain.

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1. Moise, Downs: Geometry (Addison Wesley Publishing Co., 1967)

2. Jurgensen, Donnelly, Dolciani: Modern Geometry, Structure and Meth d (Hought n Mifflin Co., 1965)

3. Anderson, Garon, Gremillion: School Mathematics Geometry (Houghton Mifflin Co., 1969)

4. Lewis: Geometry, A Contem ary Course (D. Van Nostrand Co., Inc., 1968)

5. Nichols, Palmer, Schacht: Modern Geometry (Holt, Rinehart, and Winston, Inc., 1968)

LEARNING ACTIVITY PACKAGE

PERPENDICULAR LINES AND PLANES IN SPACE

Ninety Six High School - Ninety Six, S.C. -

Geometry 114

REVIEWED BY ____________

LAP NUMBER 39

WRITTEN BY Bill Holland

RATIONALE

In previous units you considered perpendiculars in a plane. However, this would not adequately describe the workd in which we live. Our environment is filled with perpendicularity of lines and planes in space. Look around the very room you are in! We live in a three dimensional world, therefore, we must extend our concept of perpendicularity and concern ourselves with geometric relationships as they exist in space.

The drawings you will use, although drawn upon paper, are intended to represent a three dimensional figure, and you will need to develop some skill in determining the spatial relationship intended.

Additional postulates will not be necessary, for all the theorems in this unit are developed from postulates previously introduced. For a very lengthy proof of a theorem, a Lemma, known as a helping theorem, will be introduced in order to shorten such a proof.

SECTION I

Behavioral Objectives

By the completion of the prescribed course of study, you will be able to:

1. Given a plane and a line (or lines) not in the plane, demonstrate that the line is perpendicular to the plane, using any one of the following:
   a. Definition of perpendicularity between lines and planes.
   b. Theorem: If B and C are equidistant from P and Q, then every point between P and C is equidistant from P and Q.
   c. Corollary: In a plane, line L is the perpendicular bisector of $\overline{AB}$ if two points of L are equidistant from A and B.
   d. Theorem: If a line is perpendicular to each of two intersecting lines at their points of intersection, then it is perpendicular to the plane that contains them.
   e. Theorem: Through a given point of a given line, there passes a plane perpendicular to the given line.
   f. Theorem: If a line and plane are perpendicular, then the plane contains every line perpendicular to the given line at its point of intersection with the plane.
   g. Theorem: Through a given point of a given line, there is only one plane perpendicular to the given line.

2. Given information about points in a plane and a segment intersecting the plane, use the appropriate theorem to:
   a. Identify if the plane is the perpendicular bisecting plane of the segment.
   b. Identify the congruent segments.
   c. Identify the congruent triangles.

3. Given information about a plane, and lines not in the plane, use the appropriate theorem to verify whether or not the lines are coplanar.

4. Given a plane and a line perpendicular to the plane, use appropriate theorems to identify uniqueness and existence.

5. Answer correctly Multiple Choice, True-False, or Completion Type questions relating to the following definitions and theorems of perpendicularity of lines and planes in space:
   a. Definitions and theorems stated in Objective I.
   b. The Perpendicular Bisecting Plane Theorem.
   c. Theorem: Two lines perpendicular to the same plane are coplanar.
   d. Existence and uniqueness theorems for perpendicularity of lines and planes.
   e. Definition of distance from a point to a plane.
   f. The Second Minimum Theorem: The shortest segment to a plane from an external point is the perpendicular segment.

RESOURCES I

I. READINGS:

1. Moise: #1 (a) p. 213, #1 (b-d), #5 (a) pp. 215-216, #1 (e-g), #2, #3, #5 (a-b) pp. 218-220, #4, #5 (c-f) pp. 222-225.

2. Jurgensen: #1 (a) pp. 133-134, #1 (b-g) ____; #2-#5 ____.

3. Anderson: #1 (a) pp. 255-256, #1 (b-d), #5 (a) p. 256, pp. 259-261; #1 (e-g) #2, #3, #5 (a-b) p. 267, pp. 269-275; #4, #5 (c-f) pp. 263-264, pp. 266-267.

4. Lewis: #1 (a-d), #5 (a) pp. 207-210; #1 (e-g), #2, #3, #5 (a-b) p. 214; #4, #5 (c-f) pp. 255-258.

5. Nichols: #1 (a) p. 57; #1 (b-d), #5 (a) pp. 172-173; #1 (e-g), #2, #3, #5 (a-b) pp. 62-63; #4, #5 (c-f) pp. 170-171, pp. 193-194.

II. PROBLEMS:

1. Moise: #1 (a) pp. 213-214 ex. 1-7; #1 (b-d), #5 (a) p. 217 ex. 2, 4-8; #1 (e-g), #2, #3, #5 (a-b) pp. 220-222 ex. 1-6, 9-11; #4, #5 (c-f) ____.

2. Jurgensen: #1 (a) pp. 134-135 ex. 1-20; #1 (b-g) ____; #2-#5 ____.

3. Anderson: #1 (a) p. 257 ex. 1-9; #1 (b-d), #5 (a) p. 258 ex. 1-6, p. 261 ex. 1-9 pp. 261-262 ex. 1-5; #1 (e-g), #2, #3, #5 (a-b) p. 268 ex. 1-8, p. 268 ex. 1-4, p. 271 ex. 1-5, pp. 271-272 ex. 1, 3-4, 7, 9, p. 274 ex. 1-5, p. 274 ex. 2-3, pp. 275-276 ex. 1-3; #4, #5 (c-f) pp. 264-265 ex. 1-5, p. 265 ex. 1.

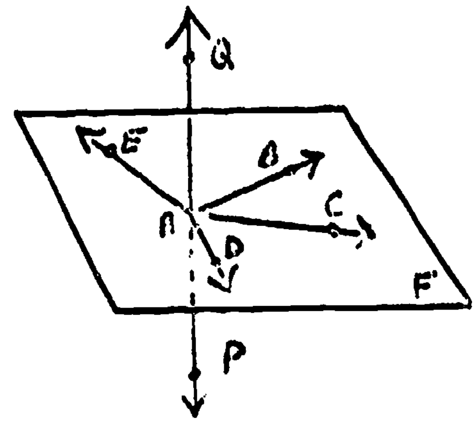
4. Lewis: #1 (a) pp. 210-211 ex. 1-5; #1 (e-g), #2, #3, #5 (a-b) pp. 214-215 ex. 1-8; #4, #5 (c-f) p. 258 ex. 1.

5. Nichols: #1 (a) p. 58 ex. 1-4; #1 (b-d), #5 (a) ____; #1 (e-g), #2, #3, #5 (a-b) p. 63 ex. 1; #4, #5 (c-f) ____.

SELF-EVALUATION I

1. For the given diagram answer each of the following questions as:

   (a) definitely yes (b) definitely no (c) not necessarily



____ (a) If $\overline{QA} \perp \overline{AD}$ is $\overline{QA} \perp F$?

____ (b) If $\overline{QA} \perp \overline{AD}$ and $\overline{AB} \perp \overline{AD}$, is $\overline{AQ} \perp F$?

____ (c) If $\overline{QA} \not\perp F$, and $\overline{AC} \perp \overline{QA}$ is $\overline{AQ} \perp \overline{AE}$?

____ (d) If $\overline{QA} \perp F$, is $\overline{QA} \perp \overline{AB}$?

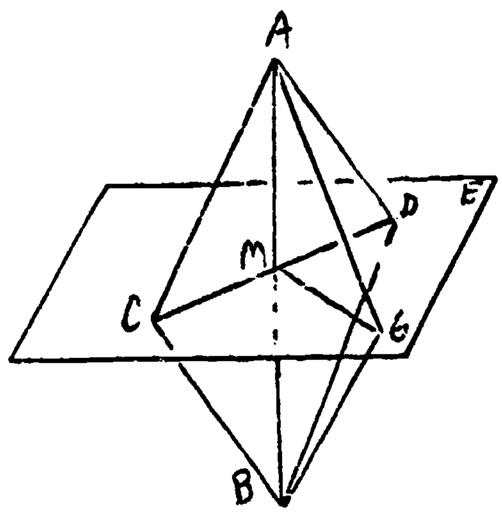
____ (e) If $\overline{QA} \perp \overline{AB}$ and $\overline{QA} \perp \overline{AD}$, is $\overline{QA} \perp F$?

____ (f) If $\overline{QA} \not\perp F$, is $\overline{QA} \perp \overline{AC}$?

____ (g) If $\overline{QA} \perp \overline{AB}$ and $\overline{QA} \perp \overline{AC}$, is $\overline{QA} \perp \overline{AD}$?

2. Complete the following statements using the given diagram:

   Given: Plane E is the perpendicular bisecting plane of $\overline{AB}$.



a) AC = ?

b) DB = ?

c) AG = ?

d) m $\angle$ADM = ?

e) $\triangle$ADM $\cong$?

3. Using the drawing in exercise 2, answer the following true or false, and justify your answer:

Given: Plane E containing points C, M, D, and G.

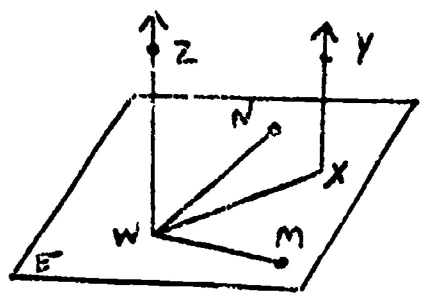
_________ a) If DA = DB, GA = GB, and CA = CB, then E is the perpendicular bisecting plane of $\overline{AB}$.

_________ b) If E is the perpendicular bisecting plane of $\overline{AB}$, then AM < AD.

_________ c) If E is the perpendicular bisecting plane of $\overline{AB}$, then $\overline{GM} \perp \overline{AB}$.

_________ d) If $\angle AGB \cong \angle ACB \cong \angle ADB$, then E is the perpendicular bisecting plane of $\overline{AB}$.

4. Answer the statements relating to the given figure as true or false and justify your answer:



Given: Points W, M, and X are contained in plane E.

_______ a) If $\overrightarrow{WZ} \perp \overline{WX}$ and $\overline{WM} \perp \overline{WZ}$, then $\overrightarrow{WZ} \perp E$.

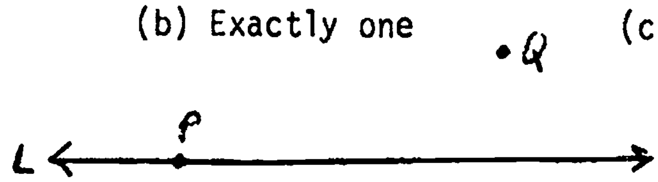
_______ b) If $\overline{WZ} \perp E$ and $\overline{WN} \perp \overrightarrow{WZ}$, then WN is contained in E.

_______ c) If $\overrightarrow{WZ} \perp \overline{WX}$ and $\overrightarrow{XY} \perp E$, then $\overline{WZ}$ and $\overrightarrow{XY}$ are coplanar.

_______ d) If $\overrightarrow{WZ} \perp E$ and $\overrightarrow{XY} \perp E$, then $\overrightarrow{WZ}$ and $\overrightarrow{XY}$ are coplanar.

5. Given: Line L containing point P, and point Q is not on L. Answer the following as:

(a) None (b) Exactly one (c) More than one



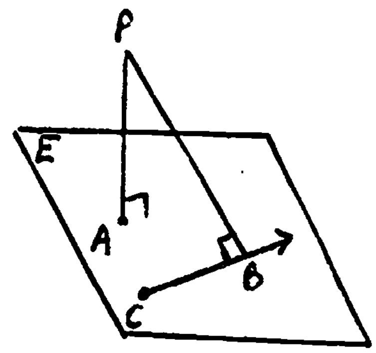
____ 1) How many planes perpendicular to L contain P.

____ 2) How many lines perpendicular to L contain P.

____ 3) How many planes perpendicular to L contain Q?

____ 4) How many lines perpendicular to L contain Q?

6. For each of the following statements about the given figure, answer true or false, and justify your choice.



Given: A, B, and C are noncollinear points in E, point P is not contained in E, $\overleftrightarrow{PA} \perp E$ and $\overleftrightarrow{PB} \perp \overleftrightarrow{BC}$

_______ a) PA = PB

_______ b) PA > PB

_______ c) PA < PB

_______ d) PB < PC

ADVANCED STUDY I

1. You have learned in previous math courses how to represent any ordered pair (x, y) on a graph. We represent points in space as ordered triples (x, y, z). Represent the following points on a spatial diagram:

   (a) (3, 6, -4) (c) (0, 6, -3)

   (b) (-2, 5, 3) (d) (5, -1, 4)

2. In space a line runs through two points whose coordinates are (3, 6, -4) and (-2, 5, 3). Find two points on a line perpendicular to this line.

SECTION II

Behavioral Objectives:

By the completion of the prescribed course of study, you will be able to:

6. Given a hypothesis, and a conclusion to be proved, use appropriate definitions, postulates, and theorems to complete and/or write such a proof.

RESOURCES II

I. READINGS:

Same in all texts as for Section I.

II. Problems:

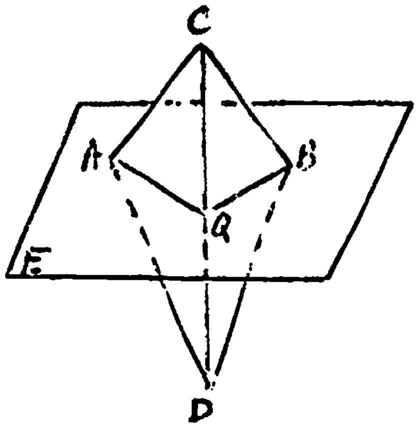
1. Moise: #6 pp. 214-215 ex. 8-10, pp. 217-218 ex. 1, 3, 9-11, p. 221 ex. 7-8, pp. 224-225 ex. 1-3.
2. Jurgensen: #6 ____.
3. Anderson: #6 pp. 258-259 ex. 7-10, p. 262 ex. 6-8, p. 269 ex. 5-8, p. 272 ex. 2, 5-6, 8, 10, p. 274 ex. 1, 4-6, p. 265 ex. 2-7.
4. Lewis: #6 pp. 211-212 ex. 1-5, 7-10.
5. Nichols: #6 p. 174 ex. 4-5, 7-9, p. 195 ex. 10-11.

SELF-EVALUATION II

1. Prove the following:

   Given: $\triangle AQC \cong \triangle AQD$,
   $\triangle BQC \cong \triangle BQD$
   Points A, B, Q in plane E are collinear

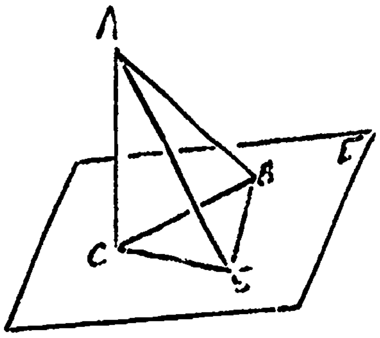
   Prove: E is the perpendicular bisecting plane of $\overline{CD}$.



2. Prove the following:

   Given: $\overline{AC} \perp E$, $AC > CS$
   $\overline{SB} \perp \overline{CB}$, $\overline{SB} \perp \overline{AB}$

   Prove: $AB > BS$

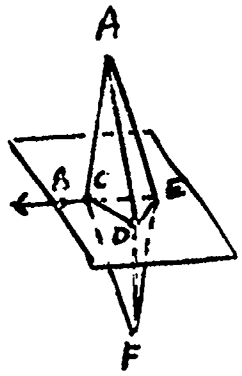


3. State and prove the Second Minimum Theorem.

4. Prove the following:

   Given: $CA = CF$, $EA = EF$,
   $B - C - E$.

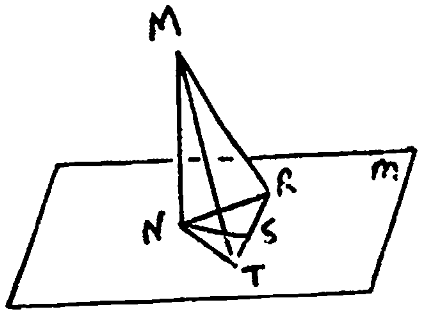
   Prove: B is contained in the perpendicular bisecting plane of $\overline{AF}$.



5. Prove the following:

   Given: $\overleftrightarrow{MN} \perp m$
   $\overleftrightarrow{NS}$ is the perpendicular bisector of $\overline{RT}$.

   Prove: $MT = MR$



IF YOU HAVE MASTERED THE BEHAVIORAL OBJECTIVES, TAKE YOUR LAP TEST.

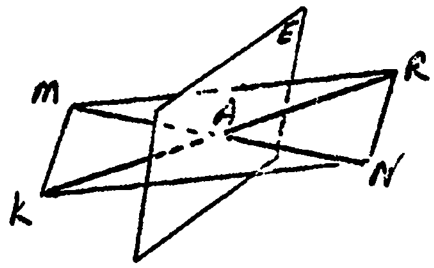
ADVANCED STUDY II

1. Prove the following: If L_1 is a line intersecting plane E at a point A, there is at least one line L_2 in E such that $L_1 \perp L_2$.

2. Prove the following:

   Given: The figure in which E is the perpendicular bisecting plane of $\overline{MN}$ at A. R is on the same side of E as N, K is on the same side of E as M such that A is between K and R, $\overline{RN} \perp \overline{MN}$, and $\overline{KM} \perp \overline{MN}$.

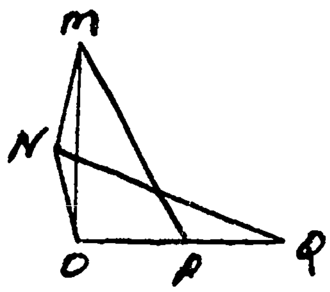
   Prove: $\overline{MK}$ and $\overline{RN}$ are coplanar.
   MR = KN



3. Prove the following:

   Given: $\overline{MN} \perp \overline{NQ}$, $\overline{MN} \perp \overline{NO}$
   $\overline{NO} \perp \overline{OQ}$, P is between O and Q

   Prove: $\overline{MO} \perp \overline{OQ}$

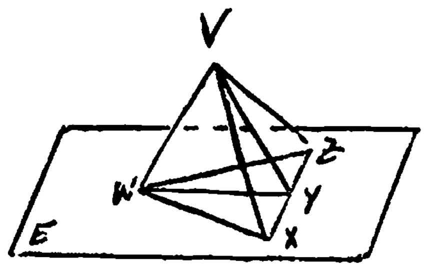


4. Prove: There cannot exist four rays $\overrightarrow{MW}$, $\overrightarrow{MX}$, $\overrightarrow{MY}$ and $\overrightarrow{MZ}$ each of which is perpendicular to the other three.

5. Prove the following:

   Given: $\triangle WXZ$ is in plane E,
   V is not in E,
   $\triangle VZX \cong \triangle WZX$

   Prove: $\angle WVY \cong \angle VWY$



BIBLIOGRAPHY

I. TEXTBOOKS:

1. Moise, Downs: Geometry (Addison-Wesley Publishing Co., 1967).
2. Jurgensen, Donnelly, Dolciani: Modern Geometry, Structure and Method (Houghton Mifflin Co., 1965).
3. Anderson, Garon, Gremillion: School Mathematics Geometry (Houghton Mifflin Co., 1969).
4. Nichols, Palmer, Schacht: Modern Geometry (Holt, Rinehart and Winston, Inc., 1968).
5. Lewis: Geometry, A Contemporary Course (D. Van Nostrand Co., Inc., 1968).

LEARNING ACTIVITY PACKAGE

BEST COPY AVAILABLE

PARALLEL LINES IN A PLANE

PARALLEL LINES AND PLANES

Ninety Six High School - Ninety Six, S.C. -

Geometry 114

REVIEWED BY AR

LAP NUMBER 40

WRITTEN BY Bill Holland

RATIONALE

PARALLEL LINES IN A PLANE

We have already studied many situations involving intersecting lines. As you look about your classroom you can find numerous representations of skew lines (lines that are not in the same plane and do not intersect.) In this LAP we will devote our study to the idea of parallelism, a concept which is not easy to "pin down."

It is easy to show in Euclidean Geometry through a point not on a given line there is at least one line which is parallel to the given line. For over two thousand years mathematicians have tried to prove that this line is unique but invariably failed. And only in recent years have they recognized the fact that it is impossible to prove. Thus, we introduce probably the most significant property of Euclidean Geometry, namely the "Parallel Postulate." This postulate enables us to prove a vast array of familiar theorems. For example, the much needed theorem, "The sum of the measure of the angles of a triangle equals 180°."

You might at this time be interested in reading the development of other geometries, called Non-Euclidean Geometries based on the following assumptions:

(1) There exist at least two parallels to a given line through an external point (Lobachevskian Geometry).

(2) There are no parallel lines (Riemannian Geometry).

In addition, this LAP will extend the definitions and theorems related to parallel lines in a plane to a three dimensional world of parallel lines and planes in space.

Some of the definitions and theorems studied in this LAP are used in the discussion of spheres. The section on projection of figures in a plane is not necessary for the development or study of the remaining LAPs in this course of study.

SECTION I

Behavioral Objectives

By the completion of the prescribed course of study, you will be able to:

1. Given a series of statements or figures relating to the following topics identify:

   a. Parallel lines, rays, and segments
   b. Intersecting lines
   c. Skew lines
   d. Transversal with respect to two or more lines in a plane
   e. Alternate interior angles
   f. Interior angles on the same side of the transversal

2. Apply the following theorems on parallel lines and alternate interior angles in evaluating given relationships and in writing proofs: *

   a. Theorem 9-1
   b. Theorem 9-2
   c. Theorem 9-3
   d. Theorem 9-4
   e. Theorem 9-5
   f. Theorem: If two lines are cut by a transversal and a pair of interior angles which contain points on the same side of the transversal are supplementary, the lines are parallel.

3. Apply the definition of corresponding angles in evaluating relationships pertaining to them.

4. Apply the following theorems on corresponding angles and parallel lines in evaluating given relationships and writing proofs: *

   a. Theorem 9-6
   b. Theorem 9-7

5. Apply the following postulates and theorems in evaluating relationships pertaining to them: *

   a. The Parallel Postulate
   b. Theorem 9-8
   c. Theorem 9-9
   d. Theorem 9-10
   e. Theorem 9-11
   f. Theorem 9-12

6. Given a hypothesis and a conclusion, write the required proof involving any of the theorems or postulates listed in Objective 5.

* See appendix for complete statements of Postulates, Theorems, and Corollaries.

RESOURCES I

I. READINGS:

1. Moise: #1, #2 pp. 229-233; #3, #4 pp. 236-237; #5, #6 pp. 238 239.

2. Jurgensen: #1-#4 pp. 153-160; #5, #6 pp. 166-173.

3. Anderson: #1, #2 pp. 281-287; #3, #4 pp. 287-288; #5, #6 pp. 291-293.

4. Lewis: #1, #2 pp. 233-236; #3, #4 pp. 239-241; #5, #6 pp. 245-250.

5. Nichols: #1-#4 pp. 98-106; #5, #6 pp. 107-108.

II. PROBLEMS:

1. Moise: #1, #2 pp. 234-235 ex. 1-10; #3, #4 p. 237 ex. 1-4; #5, #6 pp. 240-241 ex. 1-13.

2. Jurgensen: #1-#4 p. 160 ex. 1-2, pp. 161-163 ex. 1-28 (odd numbers); #5-#6 pp. 174-175 ex. 1-26 (odd numbers).

3. Anderson: #1, #2 p. 283 ex. 1-7, pp. 284-285 ex. 1-10; #3, #4 pp. 288-289 ex. 1-8, pp. 289-290 ex. 1-15; #5, #6 pp. 293-294 ex. 1-8, pp. 294-296 ex. 1-19 (odd numbers).

4. Lewis: #1, #2 pp. 237-238 ex. 1-5; #3, #4 pp. 241-244 ex. 1-20 (odd numbers); #5, #6 pp. 251-254 ex. 1-18 (odd numbers).

5. Nichols: #1-#4 pp. 103-104 ex. 1-3, pp. 106-107 ex. 1-8; #5, #6 pp. 109-110 ex. 1-9.

SELF-EVALUATION I

1. Mark the following true or false:

_______ a. Two lines are either parallel or they intersect.

_______ b. Any two parallel rays are coplanar.

_______ c. Any two lines that do not intersect are parallel.

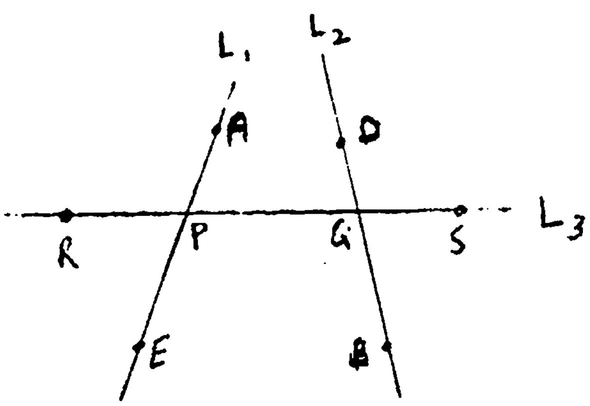
_______ d. No two parallel segments intersect.

_______ e. A ray that does not intersect a line is parallel to the line.

_______ f. Any two non-parallel lines are skew lines.

_______ g. Two lines perpendicular to the same line are parallel to each other.

2. L_3 is a transversal of lines L_1 and L_2. Complete the following sentences:



a. ∠APQ and ∠PQB are a pair of _______________ angles.

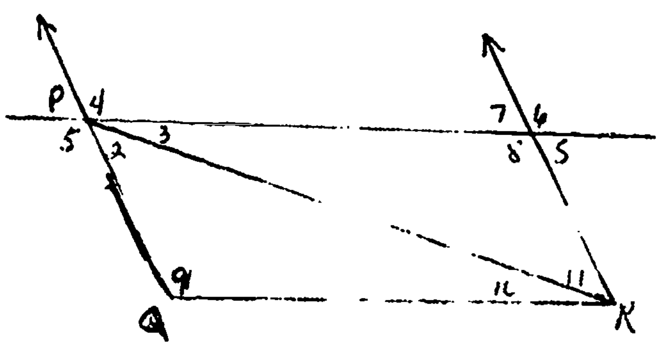
b. ∠EPQ and __________ are a pair of alternate interior angles.

c. ∠RPE and ∠PQB are a pair of _______________ angles.

d. ∠RPA and ________ are a pair of corresponding angles.

3. In the given figure, name the segments, if any, that are parallel if:

a. ∠5 ≅ ∠9
b. ∠2 ≅ ∠10
c. ∠3 ≅ ∠10
d. ∠4 ≅ ∠6
e. ∠2 ≅ ∠11
f. ∠5 ≅ ∠7
g. ∠5 ≅ ∠8



4. Given $L_1 \parallel L_2$ and $m\angle 1 = 55$. Find the measures of the following angles:

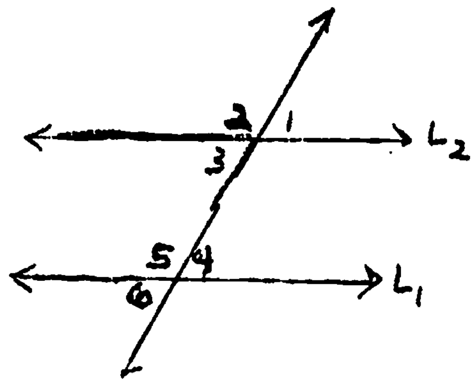
   a. $\angle 2$

   b. $\angle 5$

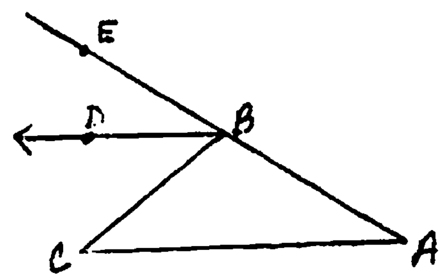
   c. $\angle 6$

   d. $\angle 4$

   e. $\angle 3$

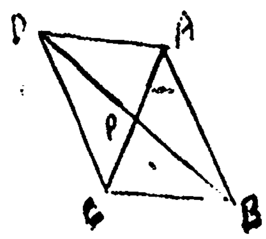


5. Given: $\overrightarrow{BD}$ bisects $\angle EBC$, $\overleftrightarrow{BD} \parallel \overleftrightarrow{AC}$
   Prove: $AB = BC$



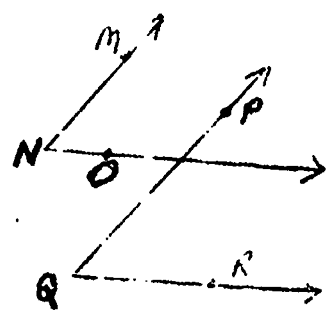
6. Prove the following:

   Given: $AP = PC$, $DP = PB$
   Prove: $\overline{AD} \parallel \overline{BC}$



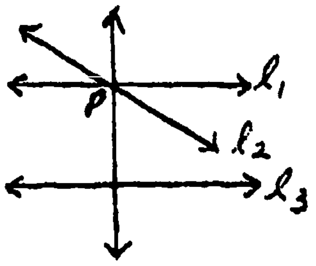
7. Prove the following: If two parallel lines are cut by a transversal, then the bisectors of a pair of alternate interior angles are parallel.

8. Prove the following:
   Given $\angle MNO$ and $\angle PQR$ are coplanar
   $\overline{NM} \parallel \overline{QP}$, $\overline{NO} \parallel \overline{QR}$
   PROVE: $\angle MNO \cong \angle PQR$



IF YOU HAVE MASTERED ALL THE BEHAVIORAL OBJECTIVES, TAKE PROGRESS TEST I.

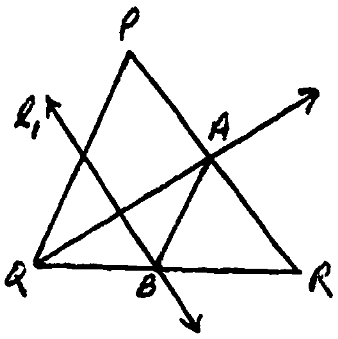
1. Prove that if the theorem "If two parallel lines are cut by a transversal, then alternate interior angles are congruent" is accepted as a postulate, then the Parallel Postulate can be proved as a theorem; i.e. prove the following:



Given: $l_1 \parallel l_2$, l_3 and l_2 intersect at P

Prove: l_2 is not parallel to l_3

2. a. Prove the following:



Given: $\overrightarrow{QA}$ bisects $\angle PQR$
$\overrightarrow{QA}$ intersects $\overline{PR}$ at A
l_1 is the perpendicular bisector of $\overline{QA}$

Prove: $\overline{QP} \parallel \overline{BA}$

b. Prove the following by using the indirect method of proof: If two lines are parallel respectively to two intersecting lines, then the first two lines must intersect each other.

3. In Euclidean Geometry we have a postulate that states through a point external to a line there is exactly one line parallel to the given line. There is another form of geometry that asserts that in the situation given above there are two lines parallel to the given line. Also, in Euclidean Geometry we say parallel lines never intersect. There is another form of geometry that asserts that parallel lines intersect at infinity.

You are to prepare a paper at least two pages in length that gives a brief description of the life of one of the "discovers" of one of the forms of geometry listed above, that gives a description of the form of geometry he discovered, and finally that lists some of the theorems and proofs of this geometry.

SECTION II

Behavioral Objectives:

By the completion of the prescribed course of study, you will be able to:

7. Apply the following theorems involving the angles of a triangle in evaluating given relationships and in writing proofs: *

   a. Theorem 9-13
   b. Corollary 9-13.1
   c. Corollary 9-13.2
   d. Corollary 9-13.3

8. Given a drawing of a quadrilateral properly labeled, name or identify relationships pertaining to the following:

   a. The angles, sides, and vertices of the quadrilateral.
   b. The pairs of opposite angles.
   c. The pairs of consecutive angles.
   d. The pairs of opposite sides.
   e. The pairs of consecutive sides.
   f. The diagonals.
   g. The convexity of a quadrilateral.

9. Given sufficient information pertaining to a parallelogram, name or identify relationships pertaining to the following:

   a. The pairs of parallel sides.
   b. The pairs of congruent sides.
   c. The pairs of congruent sides.
   d. The pairs of supplementary angles.
   e. The diagonals.
   f. The measure of each angle.
   g. The distance between two parallel lines.

* See appendix for complete statements of Postulates, Theorems, and Corollaries.

10. Given a hypothesis and a conclusion, write the required proof involving properties * and definitions of a parallelogram and trapezoid. ****

11. Apply or identify the properties of the segment joining the midpoints of two sides of a triangle. **

12. Apply the definitions and properties of the following quadrilaterals in evaluating relationships pertaining to them.

    a. square
    b. rectangle
    c. rhombus
    d. parallelogram
    e. trapezoid

* See Appendix for Theorems (9-14 thru 9-21).
** See Appendix for Theorems (9-22).
**** See Appendix for methods of proof.

Behavioral Objectives (cont')

13. Given a property of a quadrilateral decide which of the following quadrilaterals has that property:

    a. parallelogram
    b. rectangle
    c. rhombus
    d. square
    e. trapezoid

14. Given a series of figures or statements about quadrilaterals, decide which statements or figures are sufficient to determine the quadrilateral to be a(n):

    a. parallelogram
    b. rectangle
    c. rhombus
    d. square
    e. trapezoid
    f. isosceles trapezoid

15. Given a hypothesis and a conclusion, write the required proof involving the definitions and properties * of a rhombus, square, or rectangle: ***, ****

16. Given a right triangle and the necessary information, apply the special properties of a right triangle to find:

    a. lengths of segments
    b. measures of angles

17. Given a hypothesis and conclusion, write the required proof involving the following theorems: *

    a. Theorem 9-26
    b. Theorem 9-27
    c. Theorem 9-28
    d. Converse of Theorem 9-26

18. Apply the theorems involving the relationship between segments intercepted on a transversal by a series of parallel lines in evaluating given relationships and in writing proofs. *

* See Appendix for statement of theorems.
*** See Appendix for Theorems (9-23 thru 9-25).
**** See Appendix for methods of proof.

I. READINGS:

1. Moise: #7 pp. 242-243; #8-#10 pp. 245-249; #11-#15 pp. 251-252; #16-#17 pp. 254-255; #18 pp. 256-258.
2. Jurgensen: #7 pp. 176-177; #8-#15 ____; #16-#17 pp. 176-177; #18 ____.
3. Anderson: #7 pp. 296-297; #8-#10 pp. 300-303, 306; #11-#15 pp. 306-308, 310-311; #16-#17 pp. 399-400; #18 pp. 314-315.
4. Lewis: #7 pp. 300-303; #8-#10 pp. 258-262; #11-#15 pp. 262-263; #16-#17 ____; #18 pp. 324-325.
5. Nichols: #7 pp. 129-132; #8-#10 pp. 137, 381-294; #11-#15 pp. 295-296; #16-#17 pp. 236-238; #18 pp. 297-299.

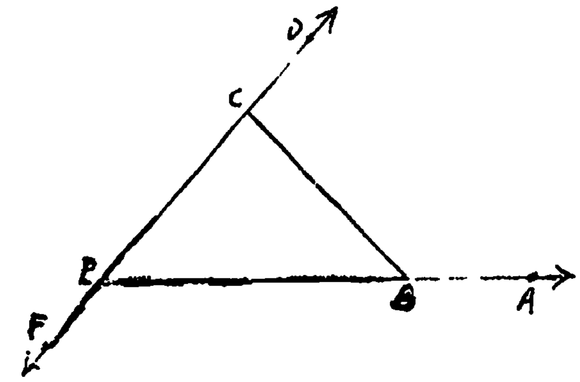
II. PROBLEMS:

1. Moise: #7 pp. 243-245 ex. 1-13; #8-#10 pp. 249-251 ex. 1-6, 8-18; #11-#15 pp. 252-253 ex. 1-7, 10, 11; #16-#17 pp. 255-256 ex. 1-9; #18 pp. 259-260 ex. 1-5, 9 (a-d).
2. Jurgensen: #7 pp. 178-180 ex. 1-27 (odd numbers); #8-#15 ____; #16-#17 pp. 178-180 ex. 1-27 (odd numbers); #18 ____.
3. Anderson: #7 p. 298 ex. 1-7; pp. 298-299 ex. 1-16; #8-#10 pp. 303-304 ex. 1-6, pp. 304-305 ex. 1-14; #11-#15 pp. 308-310 ex. 1-15 (odd numbers), pp. 311-312 ex. 1-16 (even numbers); #16-#17 pp. 401-403 ex. 1-22 (even numbers); #18 p. 316 ex. 1-7.
4. Lewis: #7 pp. 303-308 ex. A(1-12), B (1-15 odd numbers), #8-#10 pp. 265-265 ex. 1-9 pp. 270-271 ex. 1-8; #11-#15 p. 276 ex. 1-10 (even numbers); #16-#17 ____; #18 p. 326 ex. 1-6.
5. Nichols: #7 pp. 132-133 ex. 1-10; #8-#10 p. 288 ex. 1-12, p. 290 ex. 1-16 (even numbers), p. 293 ex. 1-12, pp. 137-138 ex. 1-3; #11-#15 pp. 296-297 ex. 1-12; #16-#17 p. 239 ex. 1-5, 8; #18 p. 313 ex. 1-2, 3a, 9, p. 315 ex. 1, 3.

SELF-EVALUATION II

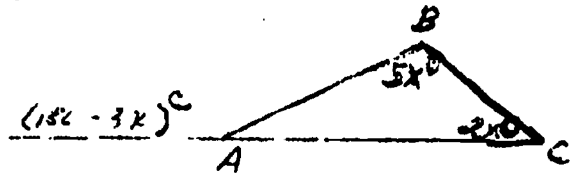
1. If m ∠CEB = 40 and m ∠ECB = 58, find

   a) m ∠CBE
   b) m ∠ABC
   c) m ∠DCB
   d) m ∠FEB



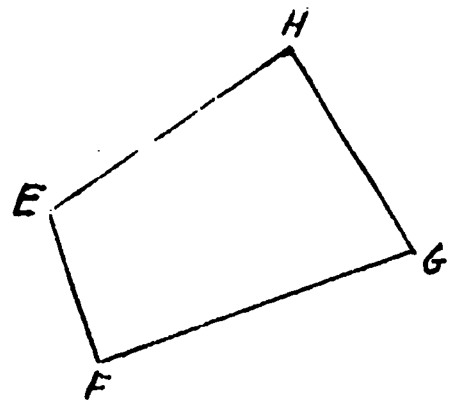
2. Find x and the measures of each of the angles of △ABC.

   a. x = ?
   b. m ∠BAC = ?
   c. m ∠B = ?
   d. m ∠C = ?



3. Given the quadrilateral as shown:

   a. Name it by its vertices.
   b. Name the side opposite $\overline{HE}$.
   c. Name the angle opposite ∠G.
   d. Name its diagonals.
   e. Name the angles consecutive to ∠E.
   f. Do $\overline{EG}$ and $\overline{FH}$ intersect?

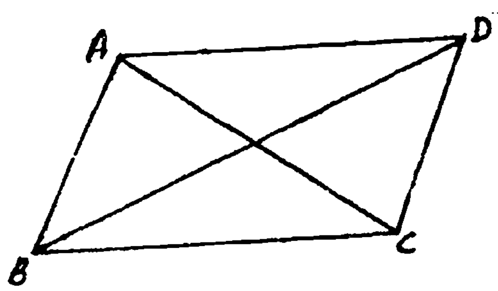


4. Is the following information about a quadrilateral sufficient to prove it a parallelogram:

   a. Its diagonals are congruent.
   b. A pair of its opposite sides are congruent.
   c. A pair of consecutive sides are congruent.
   d. A pair of consecutive sides are congruent and perpendicular.
   e. The diagonals are perpendicular.
   f. Each diagonal bisects two angles.
   g. A pair of consecutive angles are supplementary.

5. ABCD is a parallelogram. Complete the following statements:

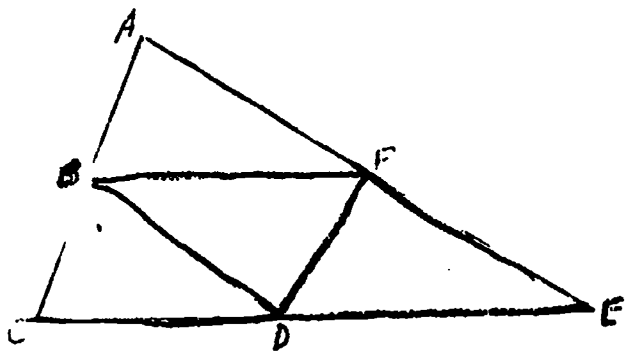
   a. $\overline{AB}$ ≅ ?
   b. ? ≅ $\overline{AD}$
   c. △ABC ≅ △
   d. $\overline{BD}$ is a ______ of ▱ ABCD
   e. ∠ABC ≅ ?; ∠DCB ≅ ?
   f. Name two angles that are supplementary to ∠BAD.



6. Write a proof of the following:

   Given: ABCD is a trapezoid, DA = CB and $\overline{DC} \parallel \overline{AB}$
   PROVE: ∠A ≅ ∠B

7. AC = 5, CE = 8, AE = 9, B, D, and F are the midpoints of $\overline{AC}$, $\overline{CE}$, and $\overline{AE}$ respectively. Find the lengths of the sides of ΔBDF.



8. If each of the following is always true, mark it true. Otherwise, mark it false:

_______ a. The diagonals of a square are perpendicular to each other.

_______ b. A square is a parallelogram.

_______ c. If the diagonals of a quadrilateral are perpendicular, it is a rhombus.

_______ d. A quadrilateral with three right angles is a rectangle.

_______ e. The diagonals of a square are congruent and perpendicular.

_______ f. Each pair of opposite angles of a trapezoid are congruent.

_______ g. A rhombus is an equiangular quadrilateral.

_______ h. The diagonals of a rhombus are perpendicular and bisect each other.

_______ i. The median of a trapezoid bisects both diagonals.

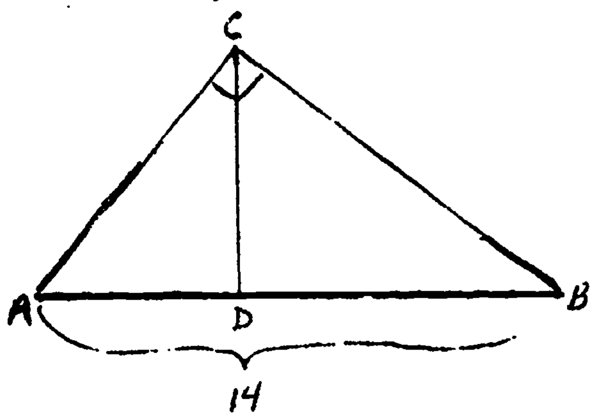
9. Write on your paper these names of quadrilaterals: parallelogram, rhombus, rectangle, square. After each name write the number of every statement below shich applies to it

a. Each two opposite sides are parallel.
b. Each two opposite angles are congruent.
c. Each two opposite sides are congruent.
d. Diagonals have equal lengths.
e. Diagonals bisect each other.
f. Diagonals are perpendicular.
g. All sides are congruent.
h. All angles are congruent.
i. All angles are bisected by the diagonals.

10. Write a proof in good form for the following implication:

If the diagonals of a quadrilateral bisect each other and are perpendicular, then the quadrilateral is a rhombus.

11. For $\triangle ABC$ with angle measures as shown, $BC = 7\sqrt{3}$ and $AB = 14$, find each of the following:

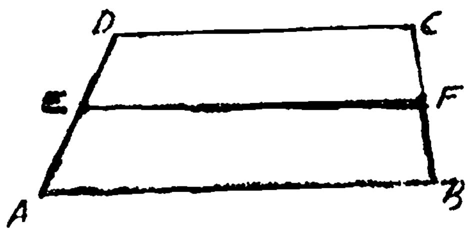


a. m $\angle A$
b. AC
c. m $\angle ACD$
d. AD
e. CD

12. Write a proof in good form of the following:

In a triangle, if a median is half as long as the side which it bisects, then the triangle is a right triangle and the side is its hypotenuse.

13. ▭ABCD is a trapezoid with $\overline{AB} \parallel \overline{DC}$. $\overline{EF}$ is the median:



a. If AB = 12 and DC = 7 then EF = ?
b. If CD = 6 and EF = 14 then AB = ?
c. If AB = 27 and EF = 18 then DC = ?

IF YOU HAVE MASTERED ALL THE BEHAVIORAL OBJECTIVES, TAKE PROGRESS TEST II.

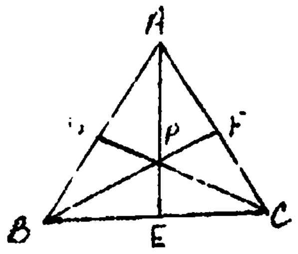
ADVANCED STUDY II

1. a. What is the sum of the measures of the exterior angles of a triangle? Give a proof to justify your answer.

   b. What is the sum of the measures of the interior angles of any quadrilateral? Give a proof to justify your answer.

   c. What is the sum of the measures of the exterior angles of any quadrilateral? Give a proof to justify your answer.

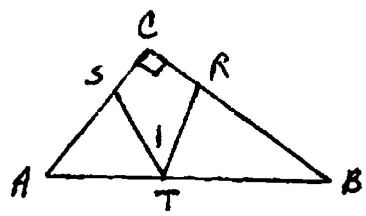
2. Prove the following:



Given: D is the midpoint of $\overline{AB}$
E is the midpoint of $\overline{BC}$
F is the midpoint of $\overline{AC}$

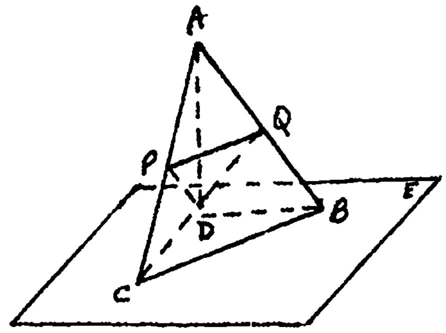
Prove: AE, BF and CD have a point P in common such that $AP = \frac{2}{3} AE$, $BP = \frac{2}{3} BF$, $CP = \frac{2}{3} CD$.

3. a. Prove the following:



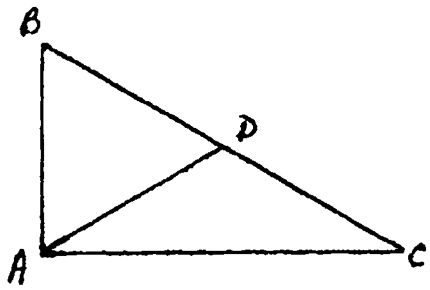
Given: $\angle C$ is a right angle
$AS = AT$, $BR = BT$
Prove: $m \angle 1 = 45$

   b. Prove the following:



Given: $\triangle ABC$ is equilateral
$\overline{AD} \perp E$, P and Q are the midpoints of $\overline{AC}$ and $\overline{AB}$
Prove: $\triangle PDQ$ is equilateral

   c. Prove the following:



Given: $\triangle DAC$ is isosceles with $\overline{DA} \cong \overline{DC}$, $\overline{DB} \cong DC$
Prove: $\angle BAC$ is a right angle

4. a. In the convex quadrilateral ABCD, $\overline{AD}$ is the shortest side and $\overline{BC}$ is the longest side. Prove that $\angle D > \angle B$.

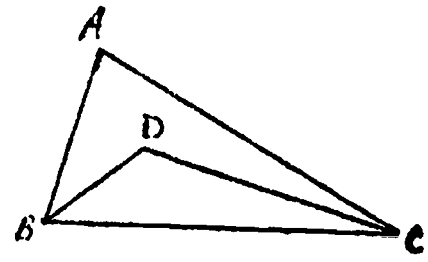
   b. Prove that the sum of the lengths of the perpendiculars drawn from any point on the base of an isosceles triangle to the legs is equal to the length of the altitude drawn to either leg.

   c. Prove that the sum of the lengths of the perpendiculars drawn from any point in the interior of an equilateral triangle to the three sides is equal to the length of an altitude.

5. a. Prove the following:

   Given: $\overrightarrow{BD}$ is the bisector of $\angle ABC$
   $\overrightarrow{CD}$ is the bisector of $\angle ACB$

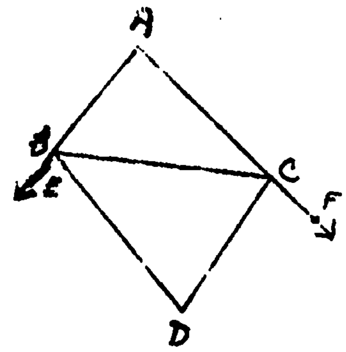
   Prove: $m\angle D = 90 + \frac{1}{2} m\angle A$



b. Prove the following:

   Given: $\overrightarrow{BD}$ is the bisector of the exterior angle $\angle EBC$
   $\overrightarrow{CD}$ is the bisector of the exterior angle $\angle FCB$

   Prove: $m\angle D = 90 - \frac{1}{2} m\angle A$



c. Prove the following: The altitudes to sides $\overline{AB}$ and $\overline{AC}$ in acute $\triangle ABC$ intersect at point E. Prove $m\angle BEC = m\angle B + m\angle C$.

SECTION III

Behavioral Objectives:

By the completion of the prescribed course of study, you will be able to:

19. Given a description or drawing of lines and planes, determine if relationships of parallelism or perpendicularity hold by applying any of the following:

    a. The definition of parallel planes or lines.
    b. The definition of a line parallel to a plane.
    c. If a plane intersects two parallel planes, then it intersects them in two parallel lines.
    d. If a line is perpendicular to one of two parallel planes, it is perpendicular to the other.
    e. Two planes perpendicular to the same line are parallel.
    f. If each of two planes is parallel to a third plane, then they are parallel to each other.
    g. Two lines perpendicular to the same plane are parallel.
    h. A plane perpendicular to one of two parallel lines is perpendicular to the other.
    i. If each of two lines is parallel to a third line, then they are parallel to each other.
    j. Parallel planes are everywhere equidistant.

20. Given a description or drawing of lines and planes, apply the theorems and definitions listed above to identify the following:

    a. Parallel lines
    b. Perpendicular lines
    c. Parallel planes
    d. Lines perpendicular to planes
    e. Right angles or triangles
    f. Congruent segments
    g. Congruent triangles
    h. Congruent angles
    i. Lengths of segments
    j. Measures of angles

21. Given a statement or drawing, tell if this statement is always ture, sometimes but not always true, or never true, using any of the definitions or theorems listed in Objective #1.

22. Given a hypothesis and a conclusion, use any of the definitions or theorems of Objective #1 to write a proof of the conclusion if the conclusion is valid, or to disprove the conclusion if the conclusion is invalid.

23. Given a drawing or statement, determine the relationship described by applying the following definitions or theorems listed below:

    a. The definition of a dihedral angle and its parts (edge, side or face, interior, exterior).

Behavioral Objectives (cont')

b. The definition of a plane angle.
c. The measure of a dihedral angle.
d. The definition of a right dihedral angle.
e. Two planes are perpendicular if they contain a right dihedral angle.
f. Vertical dihedral angles are congruent.
g. Theorem 10-7: If a line is perpendicular to a plane, then every plane containing the line is perpendicular to the given plane.
h. Theorem 10-8: If two planes are perpendicular, then any line in one of them, perpendicular to their line of intersection, is perpendicular to the other plane.
i. Theorem: If two parallel planes are intersected by a third plane, the alternate interior dihedral angles are congruent.
j. Theorem: If two intersecting planes are each perpendicular to a third plane, their intersection is perpendicular to the third plane.

24. Given a hypothesis and a conclusion, use any of the definitions and theorems of Objective #1 and statements a-e and g-1 of Objective #5 to write a proof of this conclusion.

RESOURCES III

I. READINGS:

1. Moise: #19-#22 pp. 269-273; #23-#24 pp. 275-278.
2. Jurgensen: #19-#22 pp. 153-155; #23-#24 ____.
3. Anderson: #19-#22 pp. 329-332, 334-338; #23-#24 pp. 340-344.
4. Lewis: #19-#22 pp. 279-285; #23-#24 pp. 289-294.
5. Nichols: #19-#22 pp. 114-115; #23-#24 pp. 60-62.

II. PROBLEMS:

1. Moise: #19-#22 pp. 273-275 ex. 1-8, 10, 11, p. 279 ex. 6; #23-#24 pp. 278-280 ex. 1-5, 7, 9, 10, 12.
2. Jurgensen: #19-#22 p. 156 ex. 1-18, p. 170 ex. 20-21, 23-24; #23-#24 ____.
3. Anderson: #19-#22 p. 332 ex. 1-7, pp. 338-339 ex. 1-13; #23-#24 p. 341 ex. 1-8, p. 342 ex. 1-10, p. 345 ex. 1-2, 5.
4. Lewis: #19-#22 pp. 285-287 ex. 1-8, p. 288 ex. 1-2; #23-#24 p. 295 ex. 1-2, 4, 8, p. 296 ex. 6, 12.
5. Nichols: #19-#22 p. 116 ex. 1-14; #23-#24 p. 63 ex. 1-5.

SELF-EVALUATION II

1. Answer the following true or false:

   ________ a. Two planes are parallel if their intersection with another plane is two parallel lines.

   ________ b. Two planes perpendicular to the same line are parallel.

   ________ c. If each of two planes is parallel to a line, the planes are parallel to each other.

   ________ d. Two lines are parallel if they have no points in common.

   ________ e. Two lines parallel to the same plane are parallel to each other.

   ________ f. If each of two intersecting planes is perpendicular to a third plane, their line of intersection is perpendicular to the third plane.

   ________ g. If a line not contained in a plane is perpendicular to a line in the plane, then it is perpendicular to the plane.

   ________ h. At a point on a line, there are infinitely many lines perpendicular to the line.

   ________ i. Through a point outside a plane there is exactly one line perpendicular to the plane.

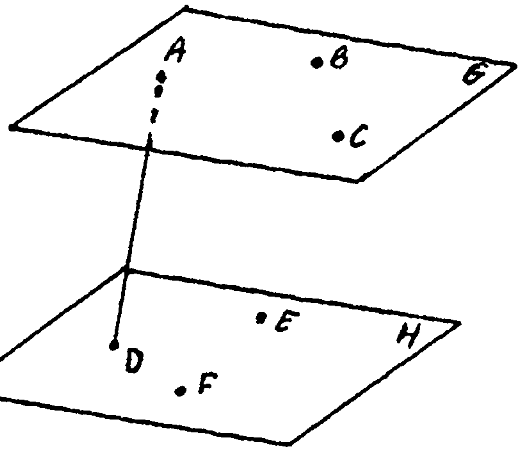
   ________ j. If a plane E is perpendicular to $\overleftrightarrow{AB}$ and $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$, then $E \perp \overleftrightarrow{CD}$.

   ________ k. A plane perpendicular to one of two perpendicular planes is never perpendicular to the other plane.

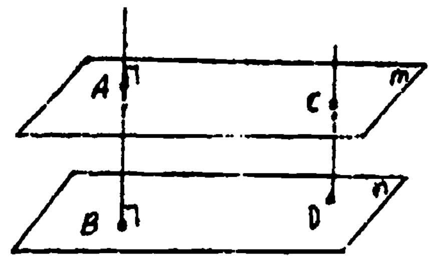
   ________ l. If a line is not perpendicular to a plane, then each plane containing this line is not perpnedicular to the plane.

2. Plane G contains points A, B, C and plane A contains points D, E, F such that $\overleftrightarrow{AD} \perp G$, $\overleftrightarrow{AD} \perp H$, and AB = DF. Which of the following statements must be true:

   a. AF = BD
   b. G || H
   c. $\overline{BC} \parallel \overline{EF}$
   d. $\overline{AC} \perp \overline{AD}$
   e. $\overline{AF}$ and $\overline{BD}$ bisect each other
   f. $\triangle ABC \cong \triangle DFE$
   g. $\angle AFD \cong \angle DBA$
   h. $\overline{AC} \parallel \overline{DF}$



3. For the following statements, refer to the figure at the right and justify each statement with a theorem or definition:



Given: $\overleftrightarrow{AB} \perp m$
$\overleftrightarrow{AB} \perp n$
$\overleftrightarrow{CD} \parallel \overleftrightarrow{AB}$

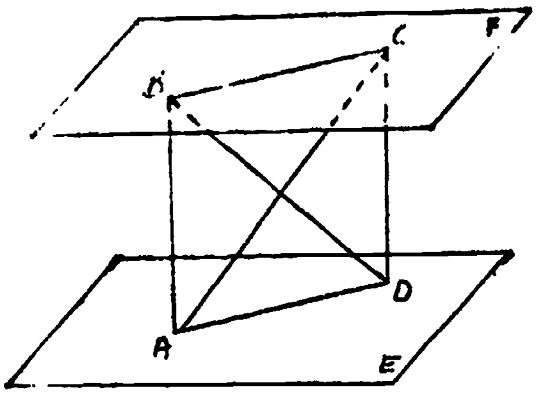
a. $m \parallel n$

b. $\overleftrightarrow{AC} \parallel \overleftrightarrow{BD}$

c. ABDC is a parallelogram

d. $\overline{CD} \cong \overline{AB}$

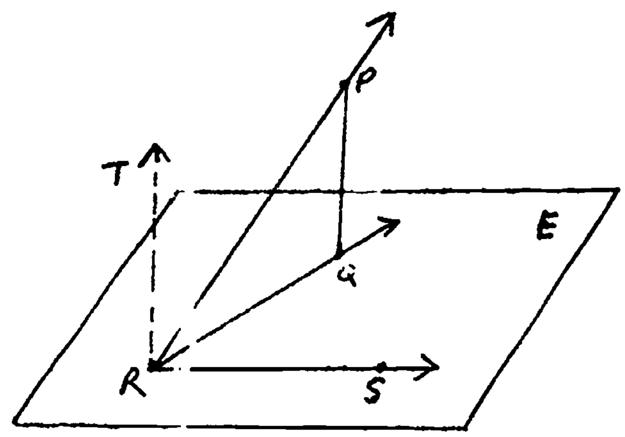
4. Write a proof in good form for the following:



Given: $E \parallel F$, $\overleftrightarrow{AB} \perp E$ at A
$\overleftrightarrow{CD} \perp E$ at D

Prove: AC = BD

5. Prove the following:



Given: $\overrightarrow{RS}$ is in plane E
$\angle PRS$ is a right angle
$\overleftrightarrow{PQ} \perp E$ at Q

Prove: $\angle QRS$ is a right angle

6. Answer the following true or false:

________ a. The intersection of a plane with the faces of a dihedral angle is a plane angle of the dihedral angle.

________ b. Vertical dihedral angles are congruent.

________ c. Each side of a dihedral angle contains the common edge.

________ d. Two planes perpendicular to the same plane are parallel to each other.

_______ e. All plane angles of the same dihedral angle are congruent.

7. Complete the following sentences:

   a. If two dihedral angles are right dihedral angles, then they are ______________________.

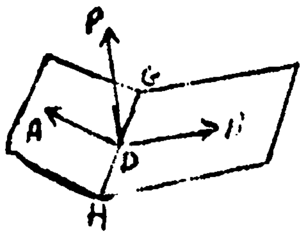
   b. If two dihedral angles are congruent, the ______________ of these two dihedral angles will be congruent.

   c. If two planes intersect to form congruent adjacent dihedral angles, then the planes are ___________________.

8. Prove the following:

   Given: $\angle ADB$ is a plane angle of dihedral angle $\angle A - GH - B$, P is a point of plane ADB

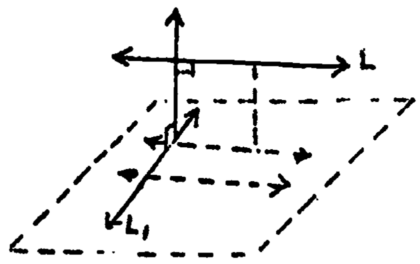
   Prove: $\overline{PD} \perp \overline{GH}$



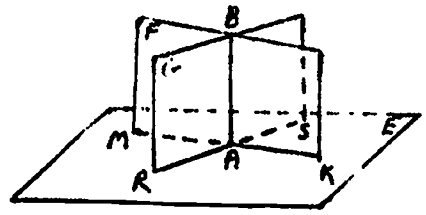
IF YOU HAVE MASTERED ALL THE BEHAVIORAL OBJECTIVES, TAKE YOUR LAP TEST.

ADVANCED STUDY III

1. a. Prove the following using the given figure: There is one and only one line which is perpendicular to each of two given skew lines.



   b. Prove the following: If two intersecting planes are each perpendicular to a third plane, their intersection is perpendicular to the third plane.



   c. Prove the following: If three planes E_1, E_2, and E_3 intersect in the three lines L_{12}, L_{23}, and L_{13}, then either the three lines intersect in a common point or each line is parallel to the other two lines.

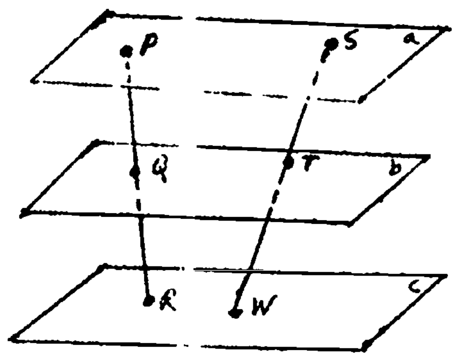
2. a. Prove the following: If a given line is parallel to a given plane, then the intersection of any plane containing this line with the given plane must be parallel to the given line.

   b. Prove the following: If each of two intersecting lines is parallel to a given plane, then the plane determined by these lines is parallel to the given plane.

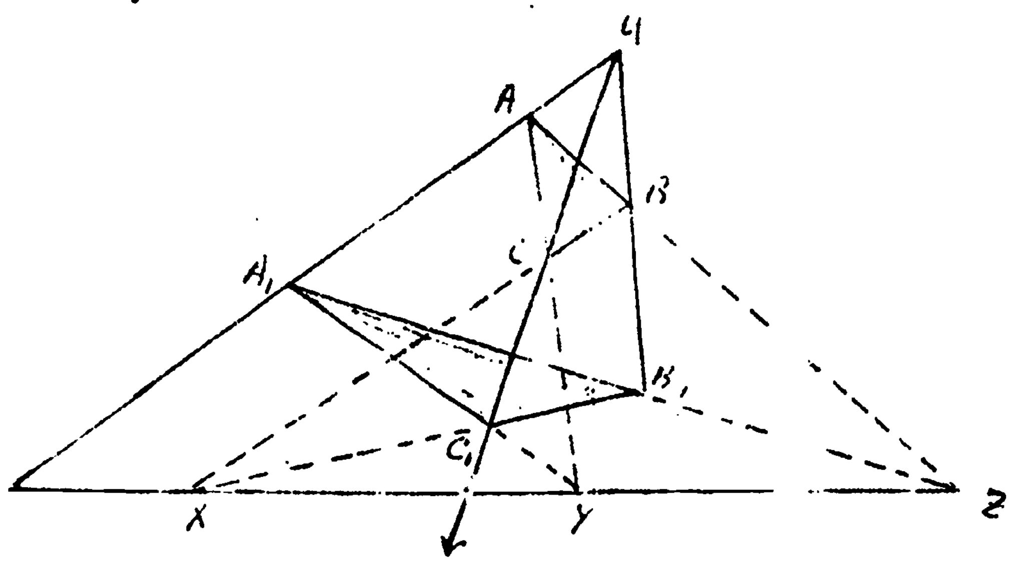
   c. Prove the following:

   Given: $a \parallel b$, $c \parallel b$
   $\overline{ST} \cong \overline{TW}$

   Prove: $\overline{PQ} \cong \overline{QR}$



3. Given a correspondence between two disjoint triangles that lie in non-parallel planes. If the three lines joining corresponding vertices intersect at a common point, and if the lines containing corresponding sides intersect, then the three points of intersection are collinear.

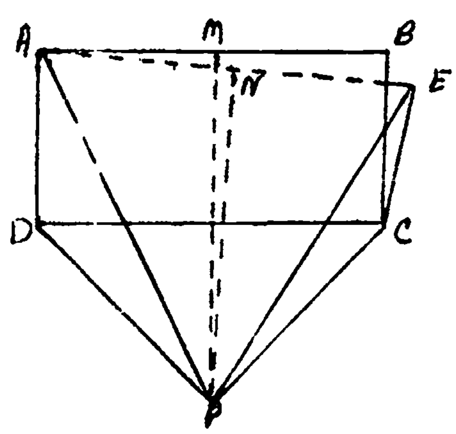


RESTATEMENT:

Given: A correspondence $ABC \leftrightarrow A_1B_1C_1$ between $\triangle ABC$ and $\triangle A_1B_1C_1$ which lie in non-parallel planes. $\overleftrightarrow{AA_1}$, $\overleftrightarrow{BB_1}$, and $\overleftrightarrow{CC_1}$ intersect at point U. $\overleftrightarrow{CB}$ intersects $\overleftrightarrow{C_1B_1}$ at point X; $\overleftrightarrow{CA}$ intersects $\overleftrightarrow{C_1A_1}$ at point Y; and $\overleftrightarrow{AB}$ intersects $\overleftrightarrow{A_1B_1}$ at point Z.
Prove: X, Y, and Z are collinear.

4. In the following we are going to prove that the measure of a right angle is equal to the measure of an obtuse angle. You are to write the indicated proof. After you have finished if you think there is a fallacy in the proof (the drawing or the argument) you are to explain what it is. If you do not think there is a fallacy, you are to explain how such a contradiction could exist.

Given: ABCD is a rectangle
$\overleftrightarrow{PM}$ is the $\perp$ bisector of $\overline{AB}$
$\overleftrightarrow{PN}$ is the $\perp$ bisector of $\overline{AE}$
$\overline{EC} \cong \overline{AD}$
Prove: $\angle ECD \cong \angle ADC$



SECTION IV

(OPTIONAL: Consult your teacher before doing any work on this section)

Behavioral Objectives:

By the completion of the prescribed course of study, you will be able to:

25. Given a drawing or statement evaluate relationships described by using the definitions and theorems listed below:
    a. Definition of the projection of a point into a plane.
    b. Definition of the projection of a line into a plane.
    c. Definition of the projection at A, where A is any set of points in space into plane E.
    d. Theorem: If a line and a plane are not perpendicular, then the projection of the line into the plane is a line.
26. Given a hypothesis and a conclusion, write the required proof involving any of the theorems or definitions listed in Objective #25.

RESOURCES IV

I. READINGS:

1. Moise: #25-#26 pp. 281-284.
2. Jurgensen: #25-#26 p. 259, pp. 278-279.

II. PROBLEMS:

1. Moise: #25-#26 p. 284 ex. 1-8.
2. Jurgensen: #25-#26 p. 264 ex. 17-18, p. 279 ex. 1-5, p. 280 ex. 9-16.

SELF-EVALUATION IV

1. Answer the following true or false:

   ________ a. The projection of a line into a plane is always a line.

   ________ b. For each acute angle there is a plane such that the projection of the acute angle into the plane is an obtuse angle.

   ________ c. The length of the projection of a segment into a plane is always less than the length of the segment.

   ________ d. If plane M is perpendicular to plane N and ΔABC lies in plane M, then the projection of ΔABC into plane N is a line segment.

   ________ e. The projection of a point is always a point.

   ________ f. The projection of a segment is always a segment.

   ________ g. The projection of an angle can be a segment.

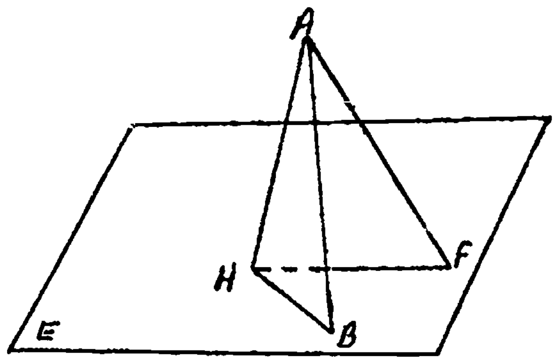
   ________ h. The projection of two skew lines can be two parallel lines.

   ________ i. The projection of a right angle can be a right angle.

2. Prove the following:

   Given: H is the projection of A into plane E, $\overline{HB}$ is the projection of $\overline{AB}$ into E, $\overline{HF}$ is the projection of $\overline{AF}$ into E, AF = AB

   Prove: HF = HB



IF YOU HAVE MASTERED THE BEHAVIORAL OBJECTIVES, CONSULT YOUR TEACHER.

APPENDIX

THEOREMS, COROLLARIES, AND POSTULATES

Part I

Theorem 9-1: Two parallel lines lie in exactly one plane.

Theorem 9-2: Two lines in a plane are parallel if they are both perpendicular to the same line.

Theorem 9-3: Let L be a line and let P be a point not on L. Then there is at least one line through P, parallel to L.

Theorem 9-4: If two lines are cut by a transversal, and one pair of alternate interior angles are congruent, then the other pair of alternate interior angles are also congruent.

Theorem 9-5: The AIP Theorem. Given two lines cut by a transversal. If a pair of alternate interior angles are congruent, then the lines are parallel.

Theorem 9-6: Given two lines cut by a transversal. If a pair of corresponding angles are congruent, then a pair of alternate interior angles are congruent.

Theorem 9-7: Given two lines cut by a transversal. If a pair of corresponding angles are congruent, then the lines are parallel.

Parallel Postulate: Through a given external point there is only one parallel to a given line.

Theorem 9-8: The PAI Theorem. If two parallel lines are cut by a transversal, then alternate interior angles are congruent.

Theorem 9-9: If two parallel lines are cut by a transversal, each pair of corresponding angles are congruent.

Theorem 9-10: If two parallel lines are cut by a transversal, the interior angles on the same side of the transversal are supplementary.

Theorem 9-11: In a plane, if two lines are each parallel to a third line, then they are parallel to each other.

Theorem 9-12: In a plane, if a line is perpendicular to one of two parallel lines, it is perpendicular to the other.

Theorem 9-13: For every triangle, the sum of the measures of the angles is 180.

Corollary 9-13.1: Given a correspondence between two triangles. If two pairs of corresponding angles are congruent, then the third pair of corresponding angles are also congruent.

Corollary 9-13.2: The acute angles of a right triangle are complementary.

Corollary 9-13.3: For any triangle, the measure of an exterior angle is the sum of the measures of the two remote interior angles.

Theorem 9-14: Each diagonal separates a parallelogram into two congruent triangles.

Theorem 9-15: In a parallelogram, any two opposite sides are congruent.

Theorem 9-16: In a parallelogram, any two opposite angles are congruent.

Theorem 9-17: In a parallelogram, any two consecutive angles are supplementary.

Theorem 9-18: The diagonals of a parallelogram bisect each other.

Theorem 9-19: Given a quadrilateral in which both pairs of opposite sides are congruent. Then the quadrilateral is a parallelogram.

Theorem 9-20: If two sides of a quadrilateral are parallel and congruent, then the quadrilateral is a parallelogram.

Theorem 9-21: If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

Theorem 9-22: The segment between the mid-points of two sides of a triangle is parallel to the third side and half as long.

Theorem 9-23: If a parallelogram has one right angle, then it has four right angles and the parallelogram is a rectangle.

Theorem 9-24: In a rhombus, the diagonals are perpendicular to one another.

Theorem 9-25: If the diagonals of a quadrilateral bisect each other and are perpendicular, then the quadrilateral is a rhombus.

Theorem 9-26: The median to the hypotenuse of a right triangle is half as long as the hypotenuse.

Theorem 9-27: The <u>30-60-90 Triangle Theorem</u>. If an acute angle of a right triangle has measure 30, then the opposite side is half as long as the hypotenuse.

Theorem 9-28: If one leg of a right triangle is half as long as the hypotenuse, then the opposite angle has measure 30.

Theorem 9-29: If three parallel lines intercept congruent segments on one transversal T, then they intercept congruent segments on every transversal T; which is parallel to T.

Theorem 9-30: If three parallel lines intercept congruent segments on one transversal, then they intercept congruent segments on any other transversal.

Part II

1. A quadrilateral may be established to be a parallelogram by proving any of the following:

    (a) Both pairs of opposite sides are parallel.

    (b) Both pairs of opposite sides are congruent.

    (c) Both pairs of opposite angles are congruent.

    (d) A pair of opposite sides is parallel and congruent.

    (e) The diagonals bisect each other.

2. A quadrilateral may be established to be a rectangle, rhombus or square by proving any of the following:

    (a) Rectangle:

        (1) By proving the quadrilateral is a parallelogram with a right angle.

        (2) By proving the quadrilateral is a parallelogram with congruent diagonals.

        (3) By proving the diagonals of the quadrilateral are congruent and bisect each other.

    (b) Rhombus:

        (1) By proving the quadrilateral is a parallelogram with four congruent sides.

        (2) By proving the quadrilateral is a parallelogram with perpendicular diagonals.

        (3) By proving the diagonals are perpendicular and bisect each other.

    (c) Square:

        (1) By proving the quadrilateral is a rectangle with four congruent sides.

        (2) By proving the quadrilateral is both a rhombus and a rectangle.

        (3) By proving the diagonals of the quadrilateral are congruent, perpendicular and bisect each other.

BIBLIOGRAPHY

I. TEXTBOOKS:

1. Moise, Downs: Geometry (Addison Wesley Publishing Co., 1967).

2. Jurgensen, Donnelly, Dolciani: Modern Geometry, Structure and Method (Houghton Mifflin Co., 1965).

3. Anderson, Garon, Gremillion: School Mathematics Geometry (Houghton Mifflin Co., 1969).

4. Lewis: Geometry, A Contemporary Course (D. Van Nostrand Co., Inc., 1968).

5. Nichols, Palmer, Schacht: Modern Geometry (Holt, Rinehart, and Winston, Inc., 1968).

LEARNING ACTIVITY PACKAGE

POLYGONAL REGIONS AND AREAS

Ninety Six High School - Ninety Six, S.C. -

Geometry 114

LAP NUMBER 41

WRITTEN BY Bill Holland

REVIEWED BY

31473

2½

RATIONALE

In previous mathematics courses you learned to compute the areas of some polygonal regions by applying a rule or a formula. As a result, you have some familiarity with the word "area", although you may have some difficulty defining it.

In this LAP, area is developed as a relation between a geometric region and a unique number. The measure of area is introduced in much the same way as the measures of distance and angle; by means of postulates. The well known Pythagorean Theorem will be proved, and special numerical relationships for the lengths of sides of special right triangles will be established.

The Pythagorean Theorem, and the theorems pertaining to 30 - 60 - 90 right triangles are used extensively in many of the following LAPs.

NOTE: Polygonal is pronounced - po • lyg´ • o • nal

Behavioral Objectives:

By the completion of the prescribed course of study, you will be able to:

1. Given an appropriate drawing of a polygon, use the necessary definitions and postulates to identify:

   a. a polygon
   b. a polygonal region
   c. a union of triangular regions
   d. the area

2. Given the drawing of a geometric figure, and sufficient information, use appropriate theorems and postulates to compute the area, altitude, or base of:

   a. a square
   b. a rectangle

3. Apply the following definitions, theorems, or postulates in evaluating given relationships and writing proofs:

   a. Definition of a Polygonal Region
   b. The Area Postulate
   c. The Congruence Postulate
   d. The Area Addition Postulate
   e. The Unit Postulate
   f. Theorems for finding area of a square or a rectangle

4. Given the drawing of a geometric figure, and sufficient information, use appropriate postulates and theorems to compute the area, altitude, or base of:

   a. any triangle
   b. a trapezoid
   c. a parallelogram

5. Given two triangles, compute the ratio of their areas when the following is known:

   a. The triangles are congruent
   b. The triangles have the same base and the same altitude
   c. The ratioes of the bases and the ratio of the altitudes of the two triangles

6. Apply the following theorems in evaluating given relationships and writing proofs:

   a. Theorems for finding the area of a triangle, trapezoid, and a parallelogram.
   b. <u>Theorem 11-6 (Moise)</u> - If two triangles have the same base and altitude, they have the same area.
   c. <u>Theorem 11-7 (Moise)</u> - If two triangles have the same altitude, then the ratio of their areas is equal to the ratio of their bases.

SECTION I

RESOURCES

I. READINGS:

1. Moise: #1 - #3 pp. 291-296; #4-#6 pp. 298-301.

2. Jurgensen: #1-#3 pp. 471-472; #4-#6 pp. 474-480, 482-483.

3. Anderson: #1-#3 pp. 377-383; #4-#6 pp. 385-388, 390-392.

4. Lewis: #1-#3 pp. 580-582; #4-#6 pp. 584-588.

5. Nichols: #1-#3 pp. 316-318; #4-#6 pp. 320-323.

II. PROBLEMS:

1. Moise: #1-#3 pp. 296-297 ex. 1-14; #4-#6 pp. 302-305 ex. 1-23.

2. Jurgensen: #1-#3 pp. 473-474 ex. 1-13; #4-#6 pp. 477-478 ex. 1-21, p. 481 ex. 1-19 (odd numbers), p. 482 ex. 23-28, 31-32, pp. 483-484 ex. 1-12 (odd numbers).

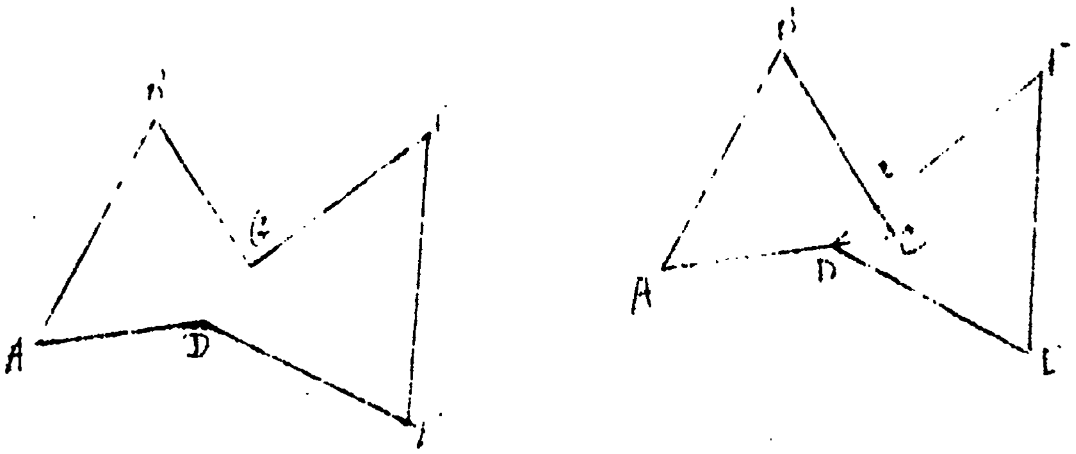
3. Anderson: #1-#3 p. 384 ex. 1-15; #4-#6 pp. 389-390 ex. 1-17, pp. 392-393 ex. 1-9, 14-15.

5. Lewis: #1-#3 pp. 583-584 ex. 1-15; #4-#6 pp. 588-589 ex. 1-22 (even numbers), pp. 590-591 ex. 1-10, 16, 18.

6. Nichols: #1-#3 pp. 318-320 ex. 1-12; #4-#6 p. 323 ex. 1-6, 9, 11, 15-16.

SELF-EVALUATION

1. The figure ABGFED at the left below results when triangles ABC and DEF intersect as shown on the right:

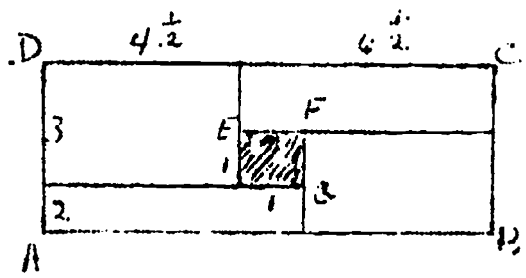


   a) Is the union of $\triangle ABC$ and $\triangle DEF$ the same as ABGFED?

   b) If the area of $\triangle ABC$ is 12 and the area of $\triangle DEF$ is 15, can you find the area of ABGFED? Why?

   c) Is ABGFED a polygonal region?

2. The figure below consists of four rectangles and a square hole, one unit on a side:



   Determine the area of the four rectangles.

3. Answer the following true or false:

_______ a) A triangle is a polygonal region.

_______ b) For every real number A there corresponds some polygonal region R that has area A.

_______ c) Every polygonal region has a unique area.

_______ d) The union of two polygonal regions has an area equal to the sum of the areas of each region.

_______ e) The interior of a square is a polygonal region.

_______ f) If the side of one square is double the side of another square, then the area of the first square is four times the area of the second square.

_______ g) If you double the altitude of a rectangle and leave the base the same then the area of the second rectangle would be twice the first.

_______ h) The area of a square $2\frac{1}{2}$ inches on a side is equal to $4\frac{1}{4}$ square inches.

_______ i) The area of a rectangle 50 ft. long and $16\frac{1}{2}$ ft. wide is 825 sq. ft.

_______ j) If the area of a square is 50 sq. ft. then each side is equal to $5\sqrt{2}$ ft.

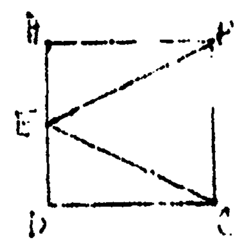
_______ k) If the altitude of a rectangle is [illegible] in. and the area is 75 sq. in., then the base of the rectangle is 15 in. long.

_______ l) If two triangles are congruent, then the triangular regions have equal areas.

4. Prove the following:

Given: ABCD is a square
EA = ED

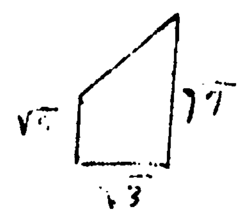
Prove: a $\triangle$ABE = a $\triangle$DCE



5. a) Determine the area of each trapezoid pictured.

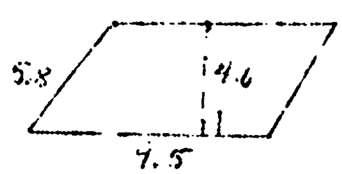
1)

2)

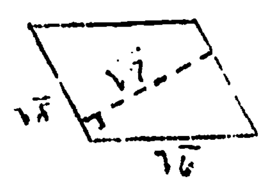


b) Determine the area of each parallelogram:

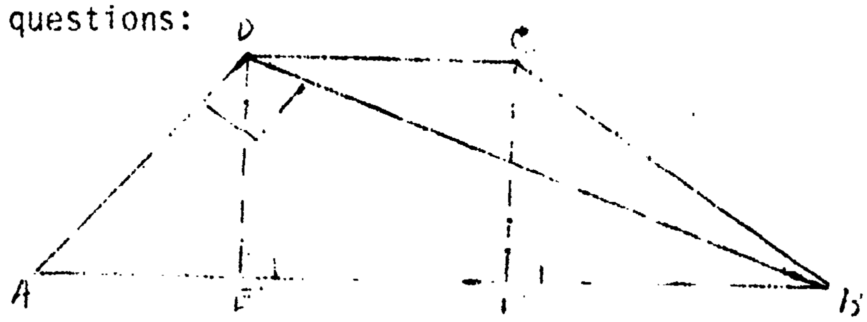
1)



2)



c) Using the information given and the figure below, answer the following questions:



1) AB = 12, DC = 6, DE = 4. Find a ABCD.

2) AB = 10, DC = 6, a ABCD = 64. Find DE.

3) AB = 30, DE = 8, BD = 18. Find AD (without using the Pythagorean Theorem).

4) EF = FB, BF = 5½, DE = 8. Find a CFED.

5) DE = 7 and the median of ABCD is 14. Find a ABCD.

6. Complete the following:

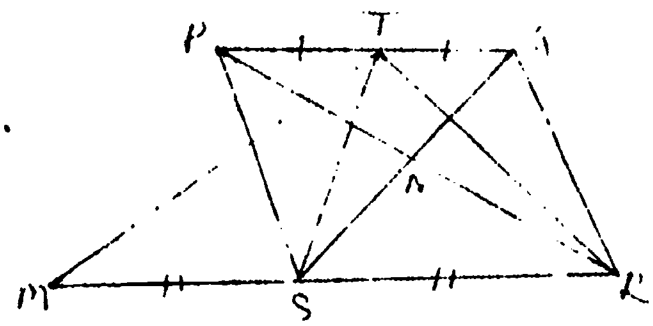
a) If two triangles have equal altitudes, then the ratio of their ______________ is equal to the ratio of their bases.

b) If two triangles have equal altitudes and equal bases, then they have equal ______________.

c) If two triangles have equal altitudes and their bases are 10 in. and 12 in., respectively, then the ratio of their areas is equal to ______________.

d) Is it true that two triangle having equal areas have equal altitudes and equal bases? ______________

7. a) In the figure PQRS is a parallelogram with PT = TQ and MS = SR. In the following, compare the areas of the two figures listed.



1) ΔPSQ and ΔTSQ

2) ΔSTR and ΔSPR

3) ΔMTR and ΔSTR

b) Find the area of a triangle with altitude of 10½ inches and base 16 inches.

c) The area of a triangle is 72. If one side is 12, what is the altitude to that side?

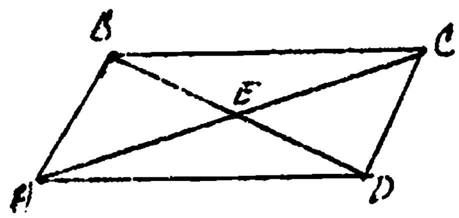
d) Find the area of a trapezoid with altitude of 8 inches and with bases of $14\frac{1}{2}$ inches and 16 inches.

e) The area of a parallelogram is 816 sq. ft. If the width is 10 feet and the length is 34 feet, what is the altitude?

8. Prove the following:

Given: ABCD is a parallelogram with diagonals $\overline{AC}$ and $\overline{BD}$.

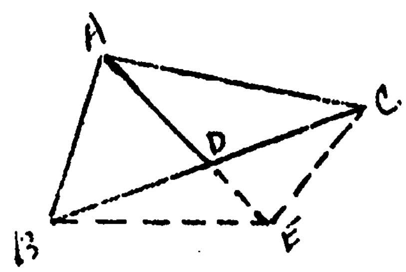
Prove: a $\triangle AED = \frac{1}{4}$ a $\square ABCD$



9. Prove the following:

Given: Median $\overline{AD}$ of $\triangle ABC$ was extended to point E.

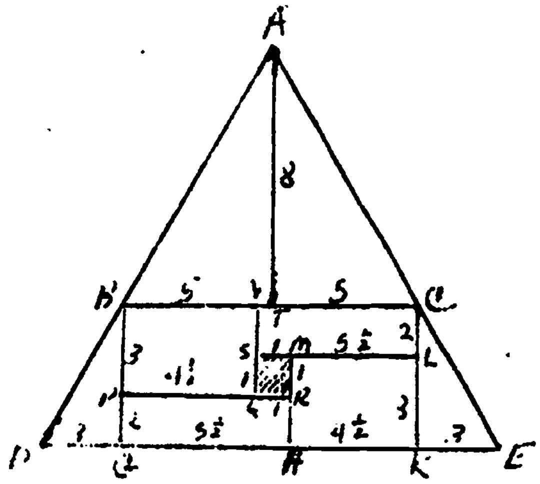
Prove: a $\triangle ABE$ = a $\triangle ACE$.



IF YOU HAVE MASTERED THE BEHAVIORAL OBJECTIVES, TAKE THE PROGRESS TEST.

ADVANCED STUDY I

1. For the following diagram:

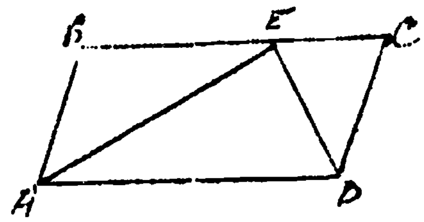


   a) Find the sum of the areas of the eight regions - four right triangles and four rectangles (Don't count the hole).

   b) Find the base, DE and the altitude from A to $\overline{DE}$. Find one-half the product of these two numbers.

   c) Can you explain why the results of part (a) and part (b) are the same in spite of the hole?

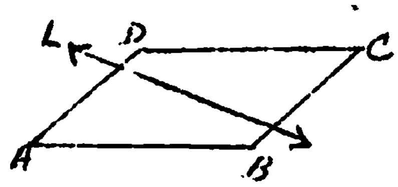
2. Prove the following:



   Given: ABCD is a parallelogram

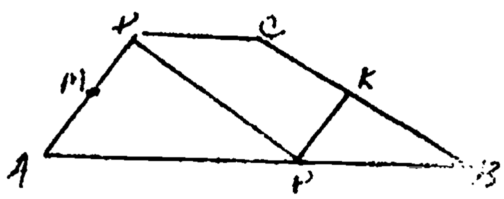
   Prove: $a\,\triangle AED = \frac{1}{2}\,a\,\square ABCD$

3. Prove the following:



   If a line separates a parallelogram into two regions of equal area, then the line passes through the point of intersection of the diagonals.

4. Prove the following:



   Given: $\square ABCD$ is a trapezoid with $\overline{AB} \parallel \overline{CD}$
   M and K are mid-points of $\overline{AD}$ and $\overline{BC}$ respectively
   $\overline{PK} \parallel \overline{AD}$

   Prove: $a\,\triangle APD = a\,\square PBCD = \frac{1}{2}\,a\,\square ABCD$

5. Prove the following:

   Given: $\triangle MNO$ with A, B, and C the midpoints of $\overline{MN}$, $\overline{NO}$, and $\overline{MO}$ respectively.

   Prove: $a\,\square MABC = \frac{1}{2}\,a\,\triangle ABC$

SECTION II **BEST COPY AVAILABLE**

Behavioral Objectives

By the completion of the prescribed course of study, you will be able to:

7. Given a right triangle and a measure of two of its sides, apply the Pythagorean Theorem to compute the measure of the third side.

8. Given the measures of the sides of a triangle, apply the converse of the Pythagorean Theorem to determine if it is a right triangle.

9. Supply a proof of the Pythagorean Theorem and its converse and/or apply these theorems in evaluating given relationships and in writing proofs.

10. Given the measures of a side of a triangle, compute the measure of the other two sides or its area when the triangle is:

    a. an isosceles right triangle
    b. a 30 - 60 - 90 triangle
    c. an equilateral triangle

11. Prove and/or apply the following theorems in evaluating given relationships and in writing proofs:

    a. Isosceles Right Triangle Theorem
    b. Converse of the Isosceles Right Triangle Theorem
    c. The 30 - 60 - 90 Triangle Theorem

RESOURCES

I. READINGS:

1. Moise: #7 - #9 pp. 306-307; #10-#11 pp. 312-313.
2. Jurgensen: #7 - #9 pp. 265-267; #10 - #11 pp. 269-271.
3. Anderson: #7 - #9 pp. 394-396; #10 - #11 pp. 399-400.
4. Lewis: #7 - #9 pp. 365-366; #10 - #11 ____.
5. Nichols: #7 - #9 pp. 230-234, 236-238; #10 - #11 pp. 238-239.

II. PROBLEMS:

1. Moise: #7 - #9 pp. 308-310 ex. 1-6, 7 (a), 9-18; #10 - #11 pp. 313-315 ex. 1-20.
2. Jurgensen: #7 - #9 pp. 267-279 ex. 1-32 (even numbers); #10 - #11 pp. 272-273 ex. 1-21.
3. Anderson: #7 - #9 pp. 396-397 ex. 1-13, 17-19; #10 - #11 p. 401 ex. 1-9, 11-14, 17, p. 407 ex. 1-16.
4. Lewis: #7 - #9 pp. 366-367 ex. 1-12, p. 369 ex. 1-5; #10 - #11 p. 370 ex. 10 - 12, p. 372 ex. 16.
5. Nichols: #7 - #9 p. 235 ex. 1-8, p. 239 ex. 1-5, 11; #10 - #11 p. 239 ex. 1-5.

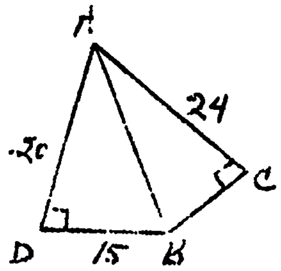
SELF-EVALUATION II

1. a) The hypotenuse of a right triangle is 8 in. long and one leg has length of 4 in. What is the length of the other leg?

   b) How long must a tent rope be to reach from the top of a 12 foot pole to a point on the ground which is 16 feet from the foot of the pole?

   c) A boat travels south 24 miles then east 6 miles, and then north 16 miles. How far is it from its starting point?

   d) For the figure at the right, find AB and CB.



2. Which of the following sets of three numbers could not represent the sides of a right triangle?

   (a) {5, 6, 7}
   (b) {3, 4, 5}
   (c) {5, 12, 13}
   (d) {7, 8, 6}

3. a) The hypotenuse of a right triangle is 17 and one leg is 15. Find the area of the triangle.

   b) In $\triangle ABC$, $\angle C$ is a right angle, AC = 30, and BC = 25. Find

      1) a $\triangle ABC$ 2) AB

   c) The hypotenuse of a triangle is 10 and another side is 8. Find the area of the triangle.

4. a) If an altitude of an equilateral triangle is 18 in. long, how long is one side of the triangle?

   b) What is the area of the isosceles triangle whose congruent sides have lengths of 20 in. each and whose base angles have measures of:

      1) 30   2) 45

   c) The area of an equilateral triangle is $9\sqrt{3}$. Find its side and its altitude.

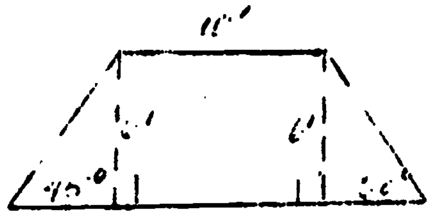
   d) Find the two legs of right $\triangle ABC$ with $\angle C$ a right angle, m $\angle A$ = 30 and AB = 20.

5. a) Prove the Theorem: The area of an equilateral triangle with the side S is given by $\frac{S^2}{4}\sqrt{3}$.

   b) The area of a square is 64. How long is a diagonal of the square?

   c) The diagonal of a square is $5\sqrt{2}$. What is the perimeter of the square?

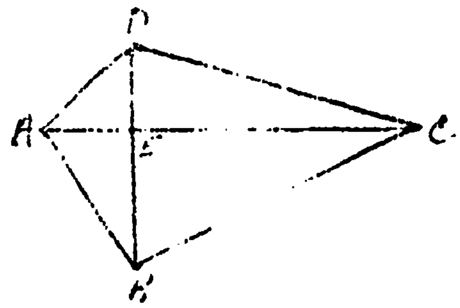
d) Find the area of the given trapezoid:



6. Prove the following:

Given: $\overleftrightarrow{DB} \perp \overleftrightarrow{AC}$

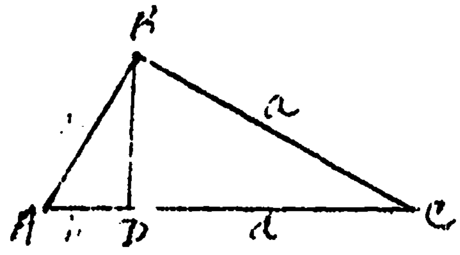
Prove: $(AD)^2 + (BC)^2 +$
$(AB)^2 + (DC)^2$



IF YOU HAVE MASTERED THE BEHAVIORAL OBJECTIVES, TAKE YOUR LAP TEST.

ADVANCE STUDY II

1. Prove the following:

   Given: $\triangle ABC$ with altitude $\overline{BD}$

   Prove: $c^2 = a^2 + (b + d)^2 - 2d(b + d)$

2. Prove the following: In a triangle, two sides have lengths a and b. The altitude to the third side separates that side into segments of length c and d respectively. Prove: $(a + b)(a - b) = (c + d)(c - d)$

3. A helicopter pilot makes the following trip. He goes 30 miles north, 40 miles east and 2 miles straight up. How far is he from his starting point?

BIBLIOGRAPHY

1. Moise, Downs: Geometry (Addison-Wesley Publishing Co., 1967).
2. Jurgensen, Donnelly, Dolciani: Modern Geometry, Structure and Method (Houghton Mifflin Co., 1965).
3. Anderson, Garon, Gremillion: School Mathematics Geometry (Houghton Mifflin Co., 1969).
4. Lewis: Geometry, A Contemporary Course (D. Van Nostrand Co., Inc., 1968).
5. Nichols, Palmer, Schacht: Modern Geometry (Holt, Rinehart, and Winston, Inc., 1968).

LEARNING ACTIVITY PACKAGE

SIMILARITY

Ninety Six High School
Ninety Six, S.C.

GEOMETRY 114

REVIEWED BY ____________

LAP NUMBER 42

WRITTEN BY Bill Holland

39.3 3

RATIONALE

As you recall, a congruence is a correspondence between the sides and angles of a pair of triangles. A similarity is also a correspondence between a pair of triangles, but instead of the measures of the sides being equal, the ratio of their measures must be equal. Also, the corresponding angles must be congruent - as they are in a congruence.

A proportion is merely a simple algebraic equation. You will be expected to rely heavily on your algebraic knowledge of fractional equations.

Similarity is extremely helpful in trigonometry. In this LAP, you will also study the three trigonometric ratioes - sine, cosine, and tangent and apply them to determine the measures of the angles and sides of a right triangle.

Section I

BEHAVIORAL OBJECTIVES:

By the completion of the prescribed course of study, you will be able to:

1. Given two sequences of positive numbers, determine if they are proportional.

2. Given any pair of positive real numbers, compute their:

   a) Arithmetic mean (average)
   b) Geometric mean

3. Given two similar triangles and sufficient information:

   a) Name the corresponding angles which are congruent.
   b) Name the corresponding sides which are proportional.
   c) Compute the lengths of specified sides when given the lengths of other sides.

4. Given a line parallel to one side of a triangle intersecting the other two sides:

   a) Determine the segments which are proportional.
   b) Compute the lengths of specified segments when given the lengths of the other ones.

5. Be able to apply the following theorems in evaluating or proving relationships between segments and sides of a given triangle:

   a) Theorem: If a line intersects two sides of a triangle and cuts off segments proportional to these two sides, then it is parallel to the third side.

   b) Theorem: The bisector of an angle of a triangle separates the opposite side into segments whose lengths are proportional to the lengths of the adjacent sides.

   c) If three or more parallels are each cut by two transversals, the intercepted segments on the two transversals are proportional.

6. Given a correspondence between two triangles and sufficient information, determine if the correspondence is a similarity by one of the following reasons and evaluate specific relations pertaining to them:

   a) Three pairs of corresponding angles are congruent.
   b) Two pairs of corresponding angles are congruent.
   c) If a line parallel to one side of the triangle intersects the the other two sides in distinct points, then it cuts off a triangle similar to the given triangle.

7. Prove a correspondence between two triangles is a similarity when the following is given and evaluate specific relationships pertaining to them:

a) A correspondence between two triangles, two pairs of corresponding sides are proportional, and the included angles are congruent.

b) A correspondence between two triangles and the sides are proportional.

8. Given any right triangles and an altitude to the hypotenuse:

   a) Name proportionalities between segments.
   b) Name the similar triangles.
   c) Compute the length of any specified segment when sufficient information is given.
   d) Prove any given implication relating to the above.

9. Given any pair of similar triangles and sufficient information:

   a) Compute the ratio of any pair of corresponding sides when given the ratio of their areas.

   b) Compute the ratio of the areas of two triangles when given the ratio of any pair of corresponding sides.

   c) Given sufficient information, use an appropriate proportion and compute the length of sides.

   d) Given sufficient information pertaining to any of the above, evaluate specified relationships and prove any given implication relating to them.

RESOURCES I

I. Readings:

1. Moise: # 1, 2 pps. 321-323; # 3 pps. 326-328; # 4, 5 pps. 330-331; # 6 pps. 336-337; # 7 pps. 341-343; #8 pps. 346-347; # 9 pps. 349-350.

2. Jurgensen: # 1, 2 pps. 229-232, 234-235; # 3 pps. 238-241; # 4, 5 pps. 251-254; # 6 pps. 244-246; # 7 ___ ; #8 pps. 258-262; # 9 pps. 484-485.

3. Anderson: # 1, 2 pps. 409-414; # 3 pps. 416-418; # 4, 5 pps. 420-422; # 6 pps. 425-426; # 7 pps. 429-432; # 8 pps. 435-436; # 9 pps. 438-439.

4. Lewis: # 1, 2 pps. 329-331; # 3 pps. 342-344; # 4, 5 pps. 333-338; # 6, 7 pps. 345-348; # 8 pps. 360-363; # 9 pps. 594-595.

5. Nichols: # 1, 2 pps. 202-206; # 3 pps. 207-210; # 4, 5 pps. 211-212; # 6 pps. 214-216; # 7 pps. 217-218; # 8 pps. 224-225; # 9 p. 324

II. Problems:

1. Moise: # 1, 2 pps. 324-325 exs. 1-13; # 3 pps. 328-329 exs. 1-10; # 4, 5 pps. 332-334 exs. 1-13; # 6 pps. 338-339 exs. 1-10; # 7 pps. 344-346 exs. 1-12; # 8 pps. 348-349 exs. 1-6; # 9 p. 351 exs. 1-10

2. Jurgensen: # 1, 2 pps. 232-233 exs. 1-14, pps. 236-237 exs. 1-12; # 3 pps. 242-243 exs. 1-22; # 4, 5 pps. 255-258 exs. 1-25; # 6 pps. 247-250 exs. 1-31; # 7____ ; #8 p. 263 exs. 1-4; # 9 p. 486 exs. 1-20

3. Anderson: # 1,2 pps. 414-415 exs. 1-12; # 3 pps. 418-420 exs. 1-16; # 4, 5 pps. 423-425 exs. 1-15; # 6 pps. 427-429 exs. 1-17; # 7 pps. 433-434 exs. 1-12; # 8 p. 436 exs. 1-5, pps. 436-437 exs. 1-7; # 9 p. 439 exs. 1-4, p. 440 exs. 1-11.

4. Lewis: # 1, 2 pps. 332-333 exs. 1-8; # 3 pps. 348-350 exs. 1-8; # 4, 5 pps. 338-340 exs. 1-6, pps. 340-341 exs. 1-8; # 6, 7 pps. 350-351 exs. 1-10; # 8 pps. 363-364 exs. 1-5, pps. 364-365 exs. 1-4, 6; # 9 pps. 599-600 exs. 1-2, 5-12, 16

5. Nichols: # 1, 2 pps. 205-206 exs. 1-13, 15, 17; # 3 pps. 210-211 exs. 1-7; # 4, 5 pps. 213-214 exs. 1-9; # 6 pps. 216-217 exs. 1-12; # 7 pps. 218-221 exs. 1-14; # 8 pps. 225-226 exs. 1-5, pps. 227-228 exs. 3-5, 7; # 9 p. 324 exs. 6, 12

SELF EVALUATION

1. Are any two pairs of the following sequences proportional:

   a) 3, 7, 12   b) 9, 21, 36   c) $\frac{5}{2}$, $\frac{35}{6}$, 10

2. Which of the following is a true proportion:

   a) $(3 + 1) : (4 + 1) = 3 : 4$ b) $(2 \cdot 3 + 3) : (2 \cdot 4 + 4) = 3 : 4$

   c) $2^3 : 2^4 = 3 : 4$ d) $(3 - 3) : (4 - 4) = 1 : 1$

3. If $\frac{2}{3} = \frac{11}{x + 3}$, then x is equal __?__.

4. For each of the following proportions, solve for x:

   a) $\frac{3}{x} = \frac{6}{8}$ b) $\frac{2x}{3y} = \frac{4a}{5b}$ c) $\frac{2}{3} = \frac{11}{x}$

5. Complete each statement:

   a) If $3a = 2x$, then $\frac{a}{x}$ = ___, and $\frac{a}{2}$ = ____.

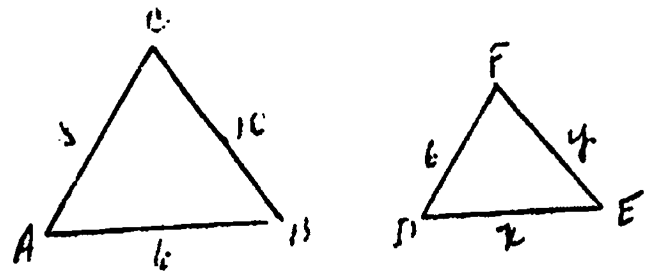
   b) If $7b = 4a$, then $\frac{a}{b}$ = ____, and $\frac{b}{a}$ = ____.

6. Find the geometric mean and the arithmetic mean of the following pairs:

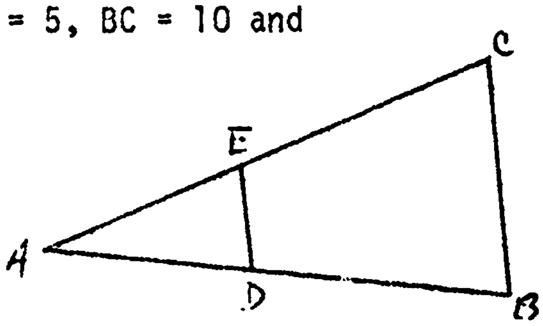
   a) 6 and 12   b) $\sqrt{3}$ and $\sqrt{12}$

   c) 8 and 10   d) $6\sqrt{2}$ and $3\sqrt{2}$

7. Given $\triangle ABC \sim \triangle DEF$ and lengths of sides are as marked. Find x and y.



8. In the figure $\triangle ABC \sim \triangle ADE$. If AD = 3, AE = 5, BC = 10 and AB = 12, find AC and DE.

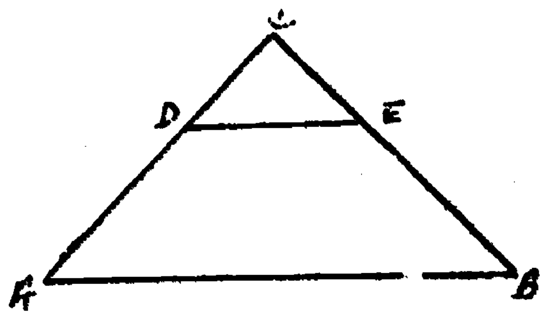


Self Evaluation (cont.)

9. If $\Delta SXY \sim \Delta SRT$, the ratio of their altitudes is the same as the ratio of:

   a) their preimeter
   b) their area
   c) the measures of their corresponding angles
   d) none of the above.

10. In the figure, $\overline{DE} \parallel \overline{AB}$:



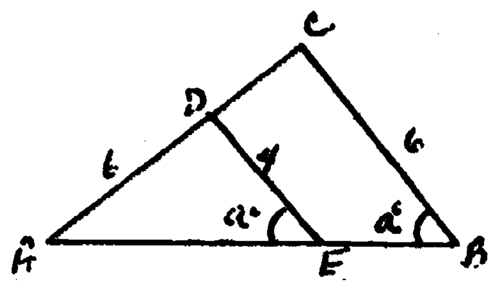
   a) If AC = 12, CD = 4, CE = 8, find BC.

   b) If AD = 6, BE = 10, CD = 4, find CE.

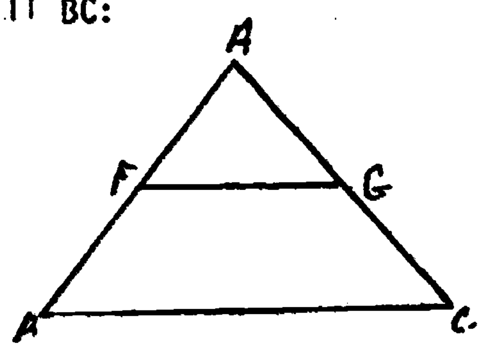
   c) If BC = 22, EB = 6, CD = 8, find AC.

   d) If AC = 15, CE = 6, BC = 18, find AD.

11. Given the figure with AD = 6, ED = 4, and BC = 6. Find DC and AC.



12. Which of the following sets of data make $\overline{FG} \parallel \overline{BC}$:



   a) AB = 14, AF = 6, AC = 7, AG = 3

   b) AF = 6, FB = 5, AG = 9, GC = 8

   c) AC = 21, GC = 9, AB = 14, AF = 5

   d) AB = 24, AC = 6, AF = 8, GC = 4

13. Given a correspondence $ABC \leftrightarrow DEF$ between two triangles. Which of the following cases are sufficient to show that the correspondence is a similarity:

   a) $\angle A \cong \angle D$, $\angle B \cong \angle E$

   b) Both triangles are equilateral

   c) Both triangles are isosceles and $m\angle A = m\angle D$

   d) $m\angle C = m\angle F = 90$, and AB = DE

   e) $m\angle A = 40$, $m\angle B = 60$, $m\angle E = 60$, $m\angle F = 80$.

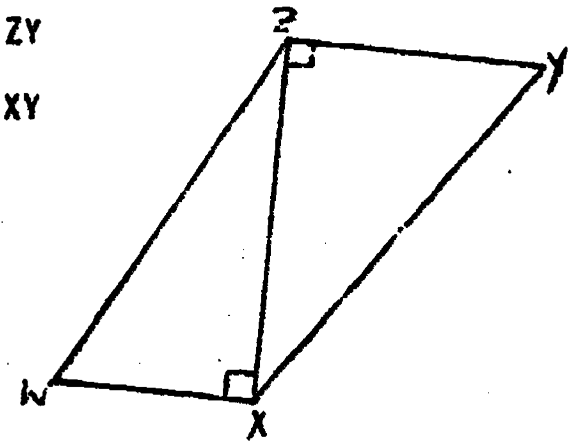
14. The given diagram shows the union of two right triangles with leg $\overline{ZX}$ in common, and $m\angle W = m\angle XYZ$. ZX is the geometric mean of:

a) WZ and WX

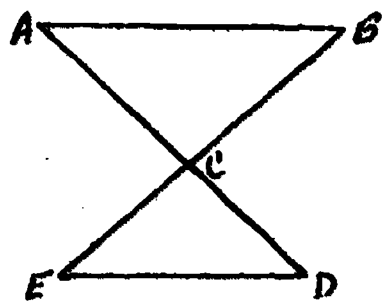
b) WX and ZY

c) WZ and ZY

d) WX and XY



15. Given the figure with $AC \cdot CE = BC \cdot CD$ and $\overline{AD}$ and $\overline{BE}$ intersecting at C Prove: $\triangle ABC \sim \triangle DEC$

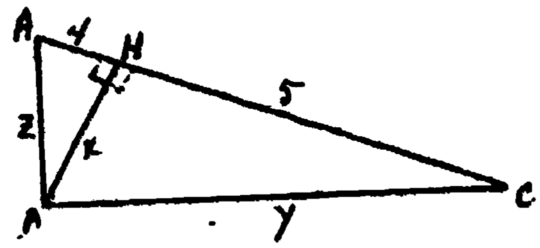


16. If 2, 5, 6 are the lengths of the sides of one triangle and $7\frac{1}{2}$, 9, 3 are the lengths of the sides of another triangle, are the triangles similar?

17. In the figure $\overline{AB} \perp \overline{BC}$, $\overline{BH} \perp \overline{AC}$, and the lengths of the segments are as shown.

a) Name the pairs of similar triangles

b) Find x, y, and z



18. Given two similar triangles in which the ratio of a pair of corresponding sides is $\frac{2}{3}$. What is the ratio of the area?

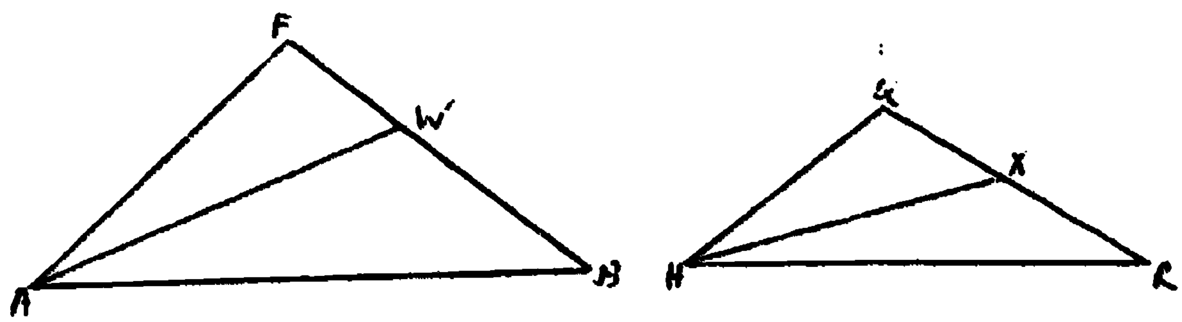
19. If the ratio of the areas of two similar triangles is $\frac{1}{3}$, what is the ratio of a pair of corresponding altitudes?

20. The areas of two similar triangles are 225 sq. in. and 36 sq. in. Find the base of the smaller triangle if the base of the larger is 20 inches.

21. The areas of two similar triangles are 144 and 81. If a side of the former is 6, what is the corresponding side of the latter?

22. How long must a side of an equilateral triangle be in order that its area shall be twice that of an equilateral triangle whose side is 10?

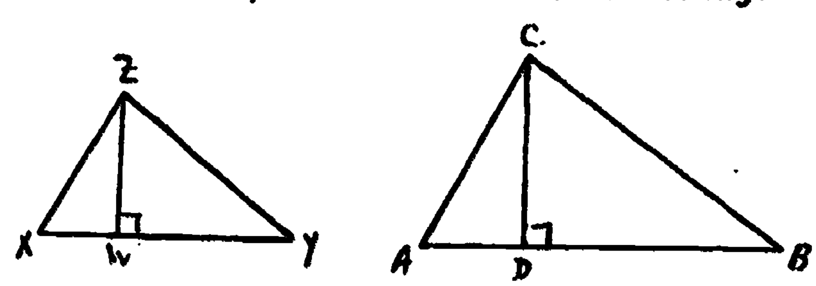
23. Prove the following theorem: In similar triangles corresponding medians have the same ratio as corresponding sides:



Given: $\triangle ABF \sim \triangle HRQ$ with $\overline{AW}$ and $\overline{HX}$ median of $\triangle ABF$ and $\triangle HRQ$ respectively.

Prove: $\frac{AW}{HX} = \frac{AF}{HQ} = \frac{FB}{QR} = \frac{AB}{HR}$

24. Given: $\triangle ABC \sim \triangle XYZ$. Complete each of the following:



a) If AB = 5 and XY = 3, then $\frac{a\ \triangle ABC}{a\ \triangle XYZ} = ?$

b) If AC = 7 and XZ = 4, then $\frac{a\ \triangle XYZ}{a\ \triangle ABC} = ?$

c) If a $\triangle ABC = 36$ and a $\triangle XYZ = 25$, then $\frac{BC}{YZ} = ?$

d) If $\frac{a\ \triangle XYZ}{a\ \triangle ABC} = \frac{9}{25}$, then $\frac{CD}{ZW} = ?$

If you have mastered the Behavioral Objectives, take your Progress Test.

ADVANCED STUDY I

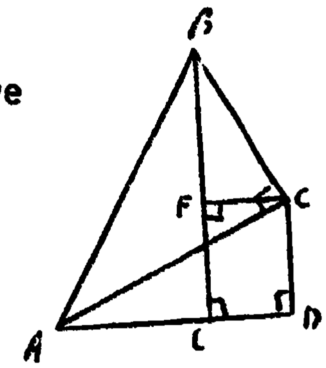
1. Explain how two triangles can have 5 parts (sides and angles) of one congruent to 5 parts of the other triangle and still not be congruent. Draw a figure to illustrate your explanation.

2. Prove the following: The geometric mean of two positive numbers is always less than the arithmetic mean i.e.
Show
$\sqrt{ab} < \frac{1}{2}(a + b)$. (Hint: use an indirect proof.)

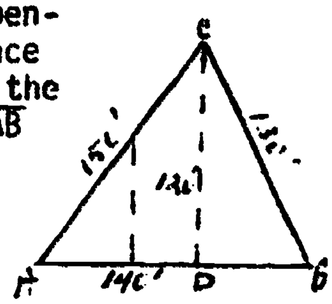
3. Prove the following: Given $\triangle ABC$ with $AB > AC$. The bisectors of the interior and exterior angles at A intersect $\overleftrightarrow{BC}$ at points D and E, respectively. Prove

$$\frac{\sqrt{AD^2 + AE^2}}{CD} - \frac{\sqrt{AD^2 + AE^2}}{BD} = 2$$

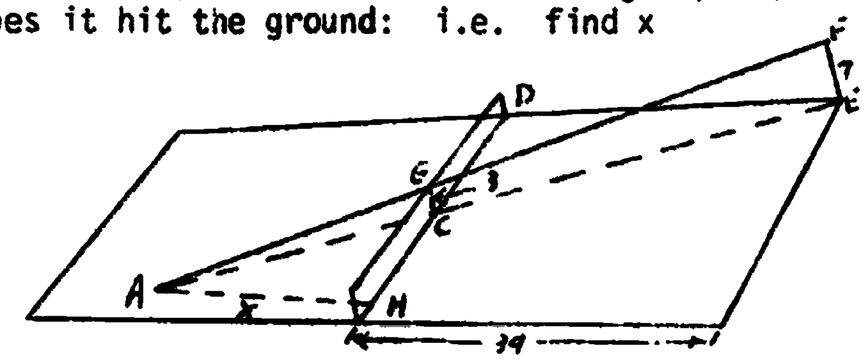
4. Given the figure with perpendiculars as marked. Prove $\triangle BFC \sim \triangle ADC$



5. A triangular lot has sides with lengths 130 ft., 140 ft., and 150 ft. as indicated in the figure. The length of the perpendicular from a corner to the 140 ft. side is 120 ft. A fence is to be erected perpendicular to the 140 ft. side so that the area of the lot is equally divided. How far from A along $\overline{AB}$ should this perpendicular be drawn?



6. A tennis ball is served from a height of 7 ft. and just clears a net 3 ft. high. If the ball is served from the baseline, which is 39 ft. behind the net, and travels in a straight path, how far from the net does it hit the ground: i.e. find x



Section II

BEHAVIORAL OBJECTIVES:

By the completion of the prescribed course of study, you will be able to:

10. Determine the trigonometric ratioes of an angle when that angle is a part of:

    a) a right triangle
    b) an isosceles triangle

11. Determine the measure of the parts of a triangle when given a trigonometric ratio of one angle of that triangle.

12. Determine the trigonometric ratioes of 30°, 45°, and 60° without the use of tables.

13. Use the table of trigonometric ratioes to give the decimal form of any of the trigonometric ratioes of any angle and vice versa.

14. Determine the measure of the parts of a triangle using the definition of the trigonometric ratioes and the table of trigonometric ratioes.

RESOURCES II

I. Readings:

1. Moise: # 10-# 12 pps. 353-355; # 13 - # 14 pps. 357-358

2. Jurgensen: # 10- # 12 pps. 293-295, 298 - 299; # 13 - # 14 p. 296, p. 303

3. Anderson: # 10 - # 12, pps. 623-627; # 13- # 14 pps. 630-633

4. Lewis: # 10 - # 14 pps. 650-662

5. Nichols: # 10, 11 pps. 240-242; # 12 p. 243; #13-#14 pps. 245-247

II. Problems:

1. Moise: # 10 - # 12 pps. 355-356 exs. 1-14; # 13 - # 14 pps. 359-361 exs. 1-11, 15-18

2. Jurgensen: # 10- # 12 pps. 295-296 exs. 1-2, # 13 - # 14 pps. 296-297 exs. 1-10, 15-18, pps. 302-303 exs. 1-10, 19-20, p. 304 exs. 1-7, 9-17

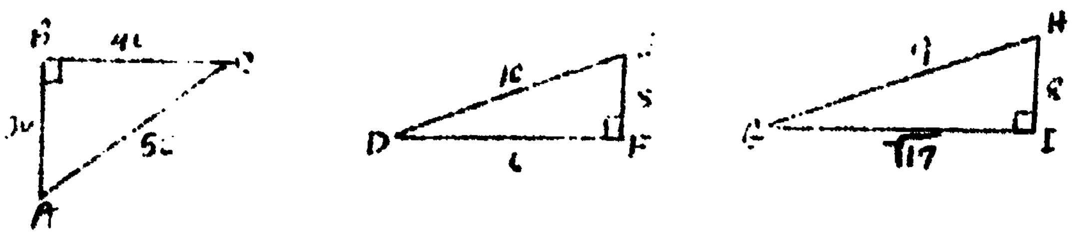
3. Anderson: # 10 - # 12 _____ ; # 13- # 14 pps. 634-635 exs. 1, 8-14

4. Lewis: # 10- # 14 pps. 655-657 exs. 1-7, pps. 657-658 exs. 1-5, pps. 662-665 exs. 1-7, pps. 665-666 exs. 1-10

5. Nichols. # 10, 11 p. 242 exs. 1-4; #12 p. 244 exs. 1-5; # 13, 14 p. 245 exs. 1-2, pps. 248-249 exs. 1-23.

SELF EVALUATION II

1. Determine the following trigonometric ratioes from the given triangles:



a) sin ∠A	d) tan ∠D	g) tan ∠H
b) tan ∠E	e) sin ∠H	h) cos ∠A
c) cos ∠G	f) cos ∠E	i) sin ∠C

2. a) If sin ∠X = $\frac{7}{10}$, then tan ∠X = ?

   b) If △ABC is an isosceles triangle with AB = BC = 16 and AC = 12, then cos∠A = ?

   c) In △PQR, PQ = 16, PR = 30 and sin∠P =.25. What is a △PQR?

3. Answer the following without use of tables:

   a) sin 30^o = ? b) tan 45^o = ? c) cos 60^o = ?

4. Use the table of trigonometric ratioes to give the decimal form of:

a) sin 29^o	c) tan 89^o	e) tan 13^o
b) cos 37^o	d) cos 50^o	f) sin 69^o

5. Determine m∠A to the nearest degree given that:

a) tan∠A = .625	c) sin∠A = .342	e) tan∠A = 16.625
b) cos∠A = .191	d) cos∠A = .489	f) sin∠A = .770

6. a) Determine the measure of the smaller angle of a 7 - 24 -- 25 triangle

   b) If cos∠A = .6, find tan∠A and sin∠A.

7. In△MNO, MN = 18, MO = 10 and m∠M = 73. What is the length of the altitude to MO. What is a△MNO?

8. A side of a rhombus is 20 inches while one of its diagonals is 16 inches. What is the measure of the largest angle of the rhombus?

9. What is the angle of elevation of the sun when a tree casts a shadow that is twice as long as the tree?

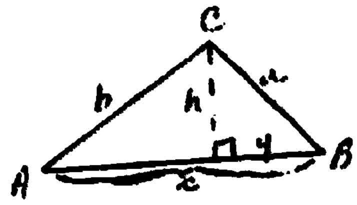
10. A jet plane takes off from an airport and climbs steadily at an angle of 8^o until it reaches an altitude of 28,200 ft. What is its ground distance from the airport?

If you have mastered the Behavioral Objectives, take your LAP Test.

ADVANCED STUDY II

1. Prove the following:

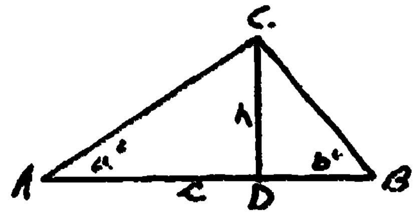
   Given: $\triangle ABC$ with $\angle A$ acute
   Prove: $a^2 = b^2 + c^2 - 2bc \cos \angle A$



2. In $\triangle ABC$, $\overline{CD}$ is the altitude to $\overline{AB}$, and $AB = c$

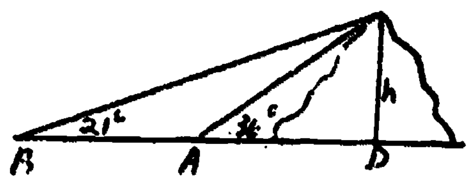
   a) Show that the altitude h is given by the formula

   $$h = c \cdot \frac{\tan a^{\circ} \cdot \tan b^{\circ}}{\tan a^{\circ} + \tan b^{\circ}}$$



   b) Compute h given that c = 68, a = 35, and b = 45

3. To find the height of a mountain peak two points, A and B, were located on a plain in line with the peak and the angles of elevation were measured from each point. The angle at A was 36° and the angle at B was 21°. The distance from A to B was 570 ft. How high is the peak above the level of the plain?



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2. Jurgensen, Donnelly, Dolciani: Modern Geometry, Structure and Method (Houghton Mifflin Co. , 1965)

3. Anderson, Garon, Gremillion: School Mathematics Geometry (Houghton Mifflin Co., 1969,

4. Lewis: Geometry, A Contemporary Course (D. Van Nostrand, Inc. 1968)

5. Nichols, Palmer, Schacht: Modern Geometry (Holt, Rinehart, and Winston, Inc. 1968)

** Drawings by: Ginny Holland

LEARNING ACTIVITY PACKAGE

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PLANE

COORDINATE

GEOMETRY

Ninety Six High School · Ninety Six, S.C.

GEOMETRY 114

REVIEWED BY

LAP NUMBER 43

WRITTEN BY Bill Holland

41073 3½

RATIONALE

A recent innovation in the study of geometry was the introduction of Coordinate Geometry. The development of this form of geometry was a major break-through in mathematical thought, and was first introduced in the seventeenth century by Réne Déscartes.

Coordinate Geometry points out the complete logical equivalence of the familiar Euclidean Geometry with what you have previously studied in algebra. Many of the concepts studied in the previous LAPs of this course are necessary for the study of Coordinate Geometry. The number scale is the most obvious of them all. The idea of plane separation, and the theory of parallels to justify the rectangular network used for graphs are others. Similarity is used in establishing the constant slope of a line. The distance formula is derived by the use of the Pythagorean Theorem. These are a few of the concepts you must be familiar with in order to study Coordinate Geometry.

The remaining LAPs in this course are not necessarily dependent on Coordinate Geometry. However, you will apply some of the concepts in the study of the graph of a circle which is taken up in LAP 44, and the use of characterization of Coordinate Geometry in LAP 45.

Generally speaking, you will find this LAP a good introduction to your future work in Analytic Geometry.

Section I

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BEHAVIORAL OBJECTIVES:

By the completion of the prescribed course of study, you will be able to:

1. Given a coordinate system and a point or an ordered pair:

   a) denote what ordered pair of real numbers corresponds to a given point.
   b) denote what point corresponds to a given ordered pair of real numbers.
   c) denote the projection of any given point into the x-axis or into the y-axis.
   d) identify the quadrant or axis containing the point or ordered pair.

2. Given the coordinates of the vertices of a polygon and sufficient information pertaining to them, compute the area and perimeter of the polygon.

3. Given the coordinates of any two points in a particular line:

   a) compute the slope of that line.
   b) determine if the line is horizontal, vertical or oblique.

4. Given the equation or sufficient information about the coordinates of any two lines in a coordinate system, determine:

   a) when the lines are parallel
   b) when the lines are perpendicular

5. Given the corrdinates of any two points in a coordinate system, compute the distance between them.

6. Given the corrdinates of the vertices of a polygon and a statement to be proved, use a cordinate system to verify such a statement.

7. Given the coordinates of any two or more points in a coordinate system, use the appropriate formula to:

   a) name the coordinates of the mid-point of the segment determined by any two points.
   b) find the coordinates of any required point between any two given points.

Resources

I. Readings:

1. Moise: # 1 pps. 371-374; # 2 pps. 378-380; # 3 pps. 383-386; # 4 pps. 389-391; # 5,6 pps. 392-394; # 7 pps. 396-399

2. Jurgensen: # 1 pps. 393 -397, 399-400; # 2___; # 3 pps. 412-414; # 4 pps. 415-417; # 5,6 pps. 404-405; # 7 pps. 410-411

3. Anderson: # 1 pps. 451-453, 455-457; # 2 pps. 451-453; # 3 pps. 459-463; # 4 pps. 465-467; # 5, 6 pps. 470-471; # 7 pps. 472-474

4. Lewis: # 1 pps. 375-380; # 2___; # 3,4 pps. 393-403; # 5,6 pps. 382-385; # 7 pps. 386-389

5. Nichols: # 1 pps. 254-256; # 2___; # 3 pps. 266-269; # 4 pps. 270-272; # 5,6 pps. 259-261; # 7 p. 262

II. Problems:

1. Moise: # 1 pps. 374-376 Answer mentally exs. 1-9, work exs. 10-12, 14-15; # 2 p. 381 exs. 1-12; # 3 pps. 387-388 exs. 1-12, 14; # 4 pps. 391-392 exs. 1-15; # 5,6 pps. 394-395 exs. 1-11; # 7 pps. 399-400 exs. 1-13.

2. Jurgensen: # 1 pps. 398-399 exs. 1-6, p. 401 exs. 1-10; # 2___; # 3 p. 415 exs. 1-16; # 4 p. 418 exs. 1-10; # 5,6 pps. 405-406 exs. 1-10, 11, 13, 14, 16, 18, 19; # 7 pps. 411-412 exs. 1-8, 9, 11, 13, 14, 15, 17

3. Anderson: # 1 pps. 453-454 exs. 1-10, pps. 458-459 exs. 1-14; # 2 pps. 453-454 exs. 1-10; # 3 pps. 464-465 exs. 1-15; # 4 pps. 467-469 exs. 1-14; # 5, 6 p. 471 exs. 1-20; # 7 p. 474 exs. 1-16

4. Lewis: # 1 pps. 380-382 exs. 1(a,d,g,j,m), 2 (a,c,e), 4, 5(a,c), 8, 11, 13; # 2___; # 3 pps. 403-404 exs. 1-7; # 4 pps. 403-405 exs. 7-18; # 5,6 pps. 390-391 exs. 1-6, 9(a), 22; # 7 pps. 390-391 exs. 7 (a,c,e,h) 8, 10, 12-14, 16, 19

5. Nichols: # 1 pps. 256-257 exs. 1-8; # 2___; # 3 pps. 269-270 exs. 1-5, 10; # 4 pps. 272-273 exs. 1-5; # 5,6 p. 261 exs. 1-8; # 7 pps. 262-263 exs. 1(a,c,e-k), 2 - 12

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Unit Evaluation I

1. a) What name is given to the projection of the point (5,0) into the y axis?

   b) State the number of the quadrant in which each of the following points is located.

      1. (3,3) 2. (6,-2) 3. (-2,8)

   c) What are the coordinates of a point on the x-axis if the distance from the point to the y-axis is 4?

   d) What is the y-coordinate of the point (7,-3)?

   e) What must be true of a point which does not lie in any quadrant?

   f) If s is a negative number and r is a positive number, in what quadrant will each of the following points lie:

      1. (s,r) 3. (-s,r) 5. (-s, -r)

      2. (s, -r) 4. (r, s) 6. (-r, s)

2. Find the perimeter and area of a square with vertices (4,4), (-4,4) (-4,-4), (4,-4) (Do not use the distance formula).

3. Determine the slopes of the line segments between the following pairs of points:

   a. (0,0) and (5,3) c. (-2,2) and (3,-4)

   b. (1,4) and (4,8) d. (-2,-3) and (-2,3)

4. If a square is to be placed with two of its sides along the x and y-axes, what are the slopes of each of its diagonals?

5. If scalene $\triangle ABC$ is placed with $\overline{AB}$ along the x-axis which of the following lines has no slope?

   a) $\overline{AB}$ c) the altitude to $\overline{AB}$

   b) the median to $\overline{AB}$ d) the angle bisector of $\angle C$.

6. The vertices of a triangle are the points A(2,3), B(5,-4), and C(-1,8). Find the slope of each side.

7. Answer the following true or false:

   a) If a segment is horizontal, then its slope is 0.______________

   b) If a segment is vertical, then its slope is not defined.____________

   c) The slope of the x-axis is 0.___________

   d) If a segment "rises" from left to right, its slope is positive._______

8. Given L_1, L_2, and L_3 are coplanar, with $L_1 \parallel L_3$, and $L_2 \perp L_3$.
If the slope of L_1 is $\frac{3}{4}$

a) the slope of L_2 is____________.

b) the slope of L_3 is ____________.

9. Given A(3,-2), B(-2,4), C(0,0), and D(a,6)

a) If $\overline{CD} \parallel \overline{AB}$, a = ?

b) If $\overline{CD} \perp \overline{AB}$, a = ?

10. Four points A(3,6), B(8,2), C(5,9), and D(6,-1) taken in pairs determine six segments. Which segments are parallel?

11. a) What values of q will make the line containing points (q,3) and (-2,1) parallel to the line through (5,q) and (1,0)?

b) What values of q will make the lines perpendicular?

12. a) Given: M(-6,-1), N(-1,2), and P(2,-3)

1) MN = ? 2) MP = ? 3) NP = ?

b) Which of the following would be true for ΔMNP:

1) It is a scalene right Δ.

2) It is an isosceles right Δ.

3) It is an equilateral Δ.

4) It is not a right triangle.

13. The vertices of quadrilateral MNPQ are:

M(-a,-b), N(a,-b), P(a,b), Q(-a,b)

a) MP = ? b) NP = ? c) NQ = ?

14. The coordinates of the vertices of a trapezoid are (-2,3), (0,7), (3,7), and (9,3). What is the length of the median of the trapezoid? (The median of a trapezoid is the segment joining midpoints of its non-parallel sides)

15. One end point of a segment is (13,19). The midpoint of the segment is (-9,30). Determine the x and y coordinates of the other endpoint.

16. What are the coordinates of the two points that trisect the segment having end points (4,-2) and (13,3)?

If you have mastered the Behavioral Objectives, take your Progress Test.

Advanced Study I

1. a) Plot the following points on a three dimensional graph:

   1. (1,0,-6) 4. (-2,6,-3)
   2. (-4,2,0) 5. (5,-3,4)
   3. (-,-2,5) 6. (3,7,-5)

   b) Determine the distance between the following pairs of points:

   1. (2,-6,3) and (-3,5,-2) 3. (3,2,-4) and (1,-3,2)
   2. (-5,2,0) and (4,6,3) 4. (-2,2,-2) and (1,-1,1)

   c) Prove that the triangle with vertices A(2,0,8), B(8,-4,6) and C(-4,-2,4) is isosceles.

   d) Show that ΔABC is a right triangle if its vertices are A(2,4,1) B(11,-8,1) and C(2,4,21).

   e) If the vertices of quadrilateral ABCD are A(3,2,5), B(1,1,1), C(4,0,3), and D(6,1,7), show that the opposite sides of quadrilateral ABCD are congruent.

   f) Is the quadrilateral in part (e) a parallelogram? Explain.

   g) Determine the coordinates of the midpoint of the diagonals of the quadrilateral given in part (e).

   h) Determine the coordinates of the two points which trisect the segment with the endpoints (3,-6,9) and (-3,5,-7).

2. The vertices of ΔMNO are M(0,0), N(6,4), and O(8,2).

   a) Determine the coordinates of the points which trisect each median of ΔMNO.

   b) What conjecture can you make based on the points you determined in part (a)?

Section II

BEHAVIORAL OBJECTIVES:

By the completion of the prescribed course of study, you will be able to:

8. Given any previously proved theorem involving polygons, demonstrate such a proof using a coordinate system.

9. Given any linear equation or inequality, draw or identify its graph on a coordinate plane.

10. Given a line and the coordinates of any two points, or a slope and the coordinates of any one point, write an equation of the line:

    a) in point - slope form

    b) in slope intercept form

    c) in the form $Ax + By + C = 0$

11. Given any form of an equation of a line:

    a) identify the slope

    b) determine the coordinates of any two points of the line.

    c) determine the y-intercept

Resources II

I. Readings:

1. Moise: # 8 pps. 402-404; # 9 pps. 406-408; # 10,11 pps. 410-414

2. Jurgensen: # 8 pps. 437-440, 452-453; # 9 pps. 402-403; # 10,11 pps. 419-422, 424-425

3. Anderson: # 8 pps. 475-477; # 9 pps. 479-482; #10, 11 pps. 484-487

4. Lewis: # 8 pps. 423-424; # 9 pps. 425-433; # 10, 11 pps. 414-418

5. Nichols: # 8 pps. 263-265, 273-275; # 9 - # 11 pps. 404-408

II. Problems:

1. Moise: # 8 pps. 405-406 exs. 1-8, 10; # 9 pps. 408-410 exs. 1-10; # 10, 11 pps. 414-416 exs. 1-10

2. Jurgensen: # 8 pps. 441-442 exs. 1-8, 10, pps. 453-454 exs. 1-6, pps. 455-456 exs. 9-12; # 9 p. 404 exs. 1-20, p. 427 exs. 29-36; # 10, 11 pps. 422-423 exs. 1-24, 26, 28, 30, p. 426 exs. 1-20.

3. Anderson: # 8 pps. 478-479 exs. 1-8, 11; # 9 p. 483 exs. 1-13, 15-16; # 10, 11 p. 487 exs. 1-9, 11, 12, 14

4. Lewis: # 8 pps. 424-425 exs. 1-13; # 9 pps. 433-434 exs. 1-4; # 10, 11 pps. 418-419 exs. 1-6, 8-10, 13

5. Nichols: # 8 p. 265 exs. 1-6, p. 176 exs. 1-14; # 9 pps. 408-409 exs. 5, 11, 14, 15; # 10, 11 pps. 408-409 exs. 6-10, 13.

Self Evaluation II

1. Use a coordinate system to prove that the diagonals of a square:

   a) have equal lengths
   b) bisect each other
   c) are perpendicular to each other

2. Sketch the graph of the following:

   a) $x + 4 = 0$
   b) $3x + 4y = 0$ and $X \leq 0$

   c) Sketch the intersection of the graphs of the following conditions:

      1) $x > -1$ 2) $y > -6$ 3) $x \leq 5$ 4) $y \leq 2$

3. Find linear equations of which the following lines are the graphs. Express each equation in

   a) slope intercept form
   b) the form $Ax + By + C = 0$

      1) the line through (1,2) with slope 3
      2) the line through (1,0) and (0,1)
      3) the line with slope 2 and y-intercept -4
      4) the horizontal line through (-5,-3)

4. Write the equation of the line through (6,-1) that is perpendicular to the line $4y = 2x + 1$.

5. If $\triangle MNO$ has vertices M(6,-2), N(-2,6) and O(2,10), what is the coordinate of the intersection of $\overline{MN}$ and the altitude to $\overline{MN}$.

6. For each of the equations below, determine the slope, the y-intercept and give the coordinates of two points of the line:

   a) $\frac{1}{3}(y + 3) = x - 1$

   b) $y - 6 = 4(x - 2)$

   c) $x - y = 3$

If you have mastered the Behavioral Objectives, take your LAP Test.

Advanced Study

1. Prove the following using the methods of coordinate geometry:

   The four diagonals of a rectangular solid are congruent and intersect at a common midpoint.

2. In a three dimensional coordinate system sketch a graph of the following conditions.

   a) $y = 5$ c) $x = 2$ and $z = 3$ e) $|x| = 4$

   b) $x = -2$ d) $y = 1$ and $z = 2$

3. Sketch a graph of the following conditions:

   a) $y < |x|$ c) $|y| = |x|$

   b) $x = |y|$ d) $|x| + |y| = 1$

4. Given right triangle ABC with right angle at A whose coordinates are $(-2,4)$. The hypotenuse $\overline{BC}$ goes through $(1,-2)$ and has slope $\frac{1}{4}$. What is a $\triangle ABC$?

5. a) In a three dimensional coordinate system $3x + 2y + 6z = 12$ is the equation of a plane which intersects each axis. What are the coordinates of the intercepts?

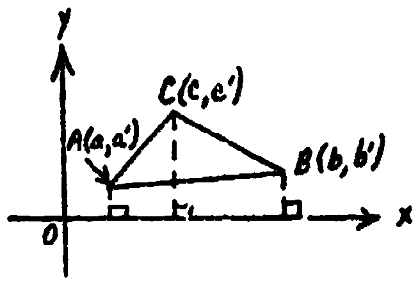
   b) Write an equation of the plane determined by the three points $(12,0,0)$, $(0,4,0)$, and $(0,0,-3)$.

   c) Sketch a three dimensional graph of the equation given in part (a).

6. Given: $\triangle ABC$ with vertices $A(a,a^1)$, $B(b,b^1)$ and $C(c, c^1)$

   Such that $0 < a < c < b$ and $0 < a^1 < b^1 < c^1$

   Prove: a $\triangle ABC = \frac{1}{2}\left[a(b^1 - c^1) + b(c^1 - a^1) + c(a^1 - b^1)\right]$



7. Two lines l_1 and l_2 intersect at P and have respective equations

   $A_1 x + B_1 y + C_1 = 0$, $A_2 x + B_2 y + C_2 = 0$

   Prove that for eacn real number k,

   $(A_1 x + B_1 y + C_1) + k(A_2 x + B_2 y + C_2) = 0$

   is an equation of a line through P.

BIBLIOGRAPHY

1. Moise, Downs: Geometry (Addison - Wesley Publishing Co., Inc., 1964)
2. Jurgensen, Donnelly, Dolciani: Modern Geometry (Houghton Mifflin Co., 1965)
3. Nichols, Palmer, Schacht, Spencer: Modern Geometry (Hotl, Rinehart and Winston, Inc., 1968)
4. Anderson, Garon, Gremillion: School Mathematics Geometry (Houghton Mifflin Co., 1969)
5. Lewis: Geometry, A Contemporary Course (E. Van Nostrand Co., Inc. 1968)

LEARNING ACTIVITY PACKAGE

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CIRCLES AND SPHERES

Ninety Six High School

Ninety Six, S.C.

GEOMETRY 114

REVIEWED BY

LAP NUMBER 44

WRITTEN BY Bill Holland

CIRCLES AND SPHERES

RATIONALE

The terms "circle" and "sphere" must be familiar to you. However, what will be new to you will be the study of common properties of circles and spheres relative to intersecting lines and planes. You will state and prove the fundamental theorems on the intersection of line and circle with great precision. For the fundamental theorems on circles, there is a corresponding section concerning spheres.

You will have to rely on your algebraic skills to deal with degree measure of circular arcs and related properties of angles, chords, secants and tangents.

Finally, as an extension of a previous IAP on Coordinate Geometry, you will characterize a circle in the coordinate plane with an equation.

Our universe is a collection of spheres. In the age of interglobal exploration, a basic knowledge of spheres and circles is a necessary factor in better understanding of some most significant events past and future.

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BEHAVIORAL OBJECTIVES:

By the completion [illegible] will be able to:

1. Apply the following definitions or theorems in a proof or in evaluating relationships pertaining to [illegible]:

   a) Definition of
      1) a circle or sphere
      2) a radius
      3) a diameter
      4) concentric circles
      5) a chord
      6) a secant

   b) *Theorem 44-1

   c) Definition of a great circle of a sphere

2. Apply the following definitions or theorems in a proof or in evaluating relationships pertaining to tangent lines to a circle:

   a) Definition of
      1) interior of a circle
      2) exterior of a circle

   b) Definition of a tangent to a circle

   c) *Theorem 44-2 and converse *Theorem 44-3

   d) Definition of
      1) internally tangent circles
      2) externally tangent circles

3. Apply the following definitions, theorems, and corollaries in a proof or in evaluating relationships pertaining to perpendicular segments in a circle, equidistant chords, and a line intersecting a circle:

   a) * Theorems 44-4 and converse * Theorem 44-5
   b) * Theorems 44-6 and *corollary 44-6.1
   c) Definition of congruent circles
   d) * Theorem 44-7 and converse * Theorem 44-8
   e) *Theorem 44-9: "The Line Circle Theorem"

4. Apply the following definitions and theorems in a proof or in evaluating relationships pertaining to tangent planes to spheres, a plane intersecting a sphere and a chord of a sphere:

   a) Definitions of
      1) interior and exterior of a sphere
      2) a tangent plane to a sphere
      3) a point of tangency
   b) *Theorem 44-10 and converse * Theorem 44-11
   c) *Theorem 44-12, *Theorem 44-13, * Theorem 44-14

* See appendix

I. Readings:

1. Moise: # 1 pps. [illegible] # 2 pps. [illegible]-431; # 4 pps. 434-437

2. Jurgensen: # 1 pps. 57-[illegible], [illegible]; # 2 pps. 57, 355-361; # 3 pps. 364, 372; # 4 p. 60

3. Anderson: # 1 pps. 501-502; # 2 pps. 503-504, 506-507, 510, 532-534; # 3 pps. 507-511; # 4 pps. 503-504, 513-515

4. Lewis: # 1 pps. 450-451, 477-478; # 2 pps. 467-472; # 3 pps. 460-462; # 4 pps. 478-480

5. Nichols: # 1 pps. 340-342; # 2 pps. 348-351, 353-357; # 3 pps. 342-344, 346; # 4 pps. 358-363

II. Problems:

1. Moise: # 1 pps. 423-425 exs. 1-10; # 2 pps. 427-429 exs. 1-15; # 3 pps. 432-433 exs. 1-14; # 4 pps. 437-435 exs. 1-12

2. Jurgensen: # 1 pps. [illegible] exs. 1, 3-5, 8-10, p. 61 exs. 1-3, 10; # 2 p. 59 exs. 2, [illegible], pps. [illegible]-364 exs. 7-22; # 3 pps. 374-376 exs. 5-6, 8, 24-25; # 4 pps. [illegible] exs. [illegible]

3. Anderson: # 1 pps. 505-506 exs. 1-9; # 2 p. 509 exs. 1-6, pps. 511-512 exs. 1, 7-8, 13-17; p. 536 exs. 1,4,7-8; # 3 p. 510 exs. 7-15, pps. 511-513 exs. 2-6, 10-12, 14-16, 18-22; # 4 pps. 517-518 exs. 1-24

4. Lewis: # 1 pps. 459-460 exs. 1-14, p. 482 exs. 1-4; # 2 pps. 472-476 exs. 1-25, p. 477 exs. 1-10; # 3 pps. 463-465 exs. 1-16, p. 466 exs. 1-9; # 4 p. 481 exs. 1-5

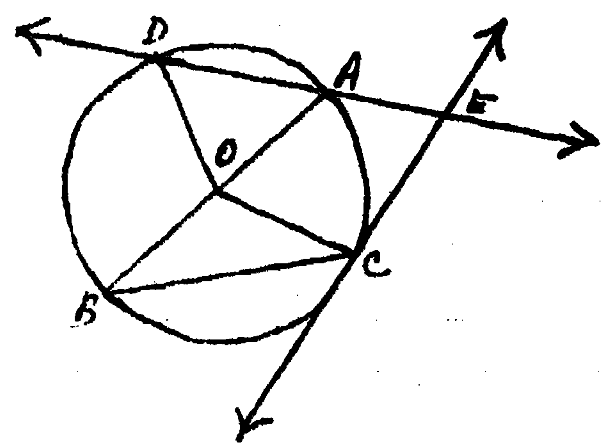
5. Nichols: # 1 pps. 344-345 exs. 1-[illegible]; # 2 pps. 352-353 exs. 1-14, p. 365 exs. 2-7; # 3 pps, 345 exs. 5-12, pps. 347-348 exs. 1-12, pps. 364-365 exs. 1-3; # 4 pps. 360-361 exs. 1-[illegible]

1. Choose from Column B the appropriate term for each description in Column A.

COLUMN A	COLUMN B
a) The set of all points equidistant from a given point.	1) a radius
b) A circle having the same center and radius as a sphere.	2) a diameter
c) A chord containing the center of a circle.	3) a circle
d) A line intersecting a circle in exactly two points.	4) a secant
e) A segment half as long as a diameter of a circle.	5) a sphere
	6) a great circle

2. Given the circle with center O. Match each item in Column A with an appropriate item from Column B.

COLUMN A	COLUMN B
a) $\overline{DA}$	1) a radius
b) $\overline{AB}$	2) a diameter
c) $\overline{OD}$	3) a chord
d) $\overleftrightarrow{ED}$	4) a tangent
e) $\overline{OA}$	5) a secant



3. a) Which of the following is not true of two tangent circles:

   1) They must be coplanar.
   2) They have a point in common.
   3) They both intersect a line at the same point.
   4) It is possible that their radii be unequal.

   b) Two internally tangent circles have how many common tangent lines?

   c) Two externally tangent circles have how many common tangent lines?

   d) If two coplanar circles intersect in two points, then how many common tangent lines do they have?

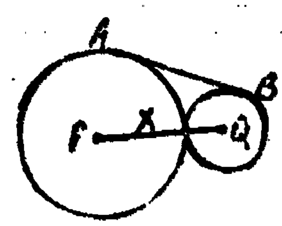
e) If P is in the interior of a circle with center A and radius 10, which of the following is not true?

1) P is in the plane of circle A
2) PA < 10
3) PA > 10

f) How many common tangents to a circle are there from a point in its exterior?

4. a) A chord of a circle with radius 20 has a length of 16. Find its distance from the center.

b) In the given figure circle P has radius 8 and circle Q has radius 5. Find AB if the circles are tangent at X.



c) The distance of a point A from the center of a circle of radius 9 is 18. A line through A is tangent to the circle at B. Find AB.

5. Answer the following true or false:

a) If two circles are congruent, then they have congruent diameters.

b) In congruent circles, chords equidistant from the respective centers are congruent.

c) In a circle any radius that intersects a chord is perpendicular to the chord.

d) If a chord of a circle is perpendicular to a radius, then it bisects the radius.

e) In a circle if chord C_1 is further from the center than chord C_2, then chord C_1 is the longer chord.

f) Every perpendicular bisector of a chord of a circle contains the center of the circle.

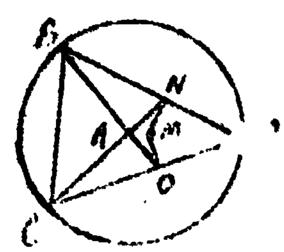
6. For each item in Column A choose from Column B all possibilities for the intersection of the sets given in the line:

Self Evaluation (con't.)

COLUMN A	COLUMN B
a) a line and a circle	1) 0
b) two distinct circles	2) 1 point
c) two distinct spheres	3) 2 points
d) a plane and a sphere	4) 1 circle
e) a line and a sphere	5) 2 circles

7. Compute the following: Two spheres S and R intersect in a circle. The plane of this circle is at a distance of 6 from the center of S and 4 from the center of R. If the radius of the circle is 3, what is the radius of R and of S?

8. Prove the following:

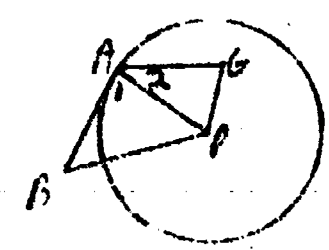


Given: $\overline{MN} \perp \overline{NB}$, $\overline{MO} \perp \overline{OC}$

M is the center of the circle
NC = OB, NB = OC

Prove: MN = MO

9. Prove the following:



Given: AB = AP = AG

Prove: $m\angle 1 > m\angle 2$

10. Prove the following: $\overline{AB}$ is a chord of sphere S with center P and radius r that does not contain the center. Prove AB < 2r. Draw a figure, list the "given" and "to prove" and write a two column proof.

If you have mastered the Behavioral Objectives, take your Progress Test.

[illegible]

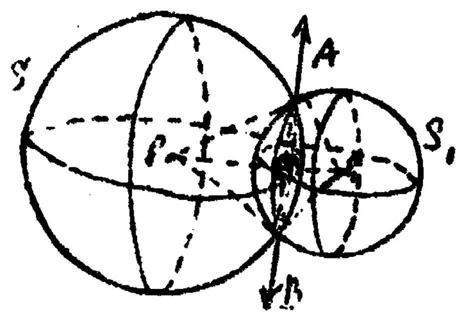
1. Prove the following: If A, [illegible], and [illegible] are any three noncollinear points, then they are contained in one circle.

2. Given: C_1 and C_2 are two pulleys with radii 22in. and 8in. respectively. If the distance between their centers is 50 in., what is the length of a belt wrapped around them? (Hint: Draw figure)

3. In the figure P and P_1 are the centers of spheres S and S_1. A and B are two points of intersection of the two spheres. $\overleftrightarrow{AB}$ and $\overleftrightarrow{PP_1}$ intersect at M_1. $\overleftrightarrow{PA}$ is tangent to S_1 at A.

   a) Describe the intersection of spheres S and S_1

   b) If the radius of S is 12 and PA = AB, find the radius of S_1 and the distance between the centers of the spheres.



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BEHAVIORAL OBJECTIVES.

By the completion of the prescribed course of study, you will be able to:

5. Apply the following definitions and theorems in a proof or in evaluating relationships pertaining to arcs of circles:

   a) Definition of
      1) central angle
      2) major arc
      3) minor arc
      4) semicircle
      5) degree measure of an arc

   b) * Theorem 44-15: "Arc Addition Theorem"

6. Apply the following definitions, theorems and corollaries in a proof or in evaluating relationships pertaining to inscribed angles and intercepted arcs, and the measure of an inscribed angle:

   a) Definition of
      1) an inscribed angle in an arc
      2) an angle intercepting an arc

   b) * Theorem 44-16, * Corollary 44-16.1 and * Corollary 44-16.2

   c) Definition of a quadrilateral

      1) inscribed in a circle
      2) circumscribed about a circle

7. Apply the following definitions and theorems in a proof or in evaluating relationships pertaining to congruent arcs and congruent chords, and intercepted arcs formed by secants and tangents:

   a) Definition of congruent arcs

   b) * Theorem 44-17 and Converse * Theorem 44-18

   c) * Theorem 44-19

8. Apply the following definitions and theorems in a proof or in evaluating relationships pertaining to secant and tangent segments and the power of a point with respect to a circle:

   a) Definition of a tangent segment

   b) * Theorem 44-20

   c) Definition of a secant segment

   d) * Theorem 44-21 "The Power of a Point Theorem,"

      * Theorem 44-22, and * Theorem 44-23

(* See Appendix)

I. Readings:

1. Moise: # 5 pps. 43[illegible]-441; # 6 pps. 442-445; # 7 pps. 448-450; # 8 pps. 453-456

2. Jurgensen: # 5 pps. 364-366; # 6 pps. 361, 372-373, 376-379; # 7 pps. 366-367, 369-372, 380, 384-385, # 8 pps. 388-390

3. Anderson: # 5 pps. 518-521; # 6 pps. 523-525; # 7 pps. 527-529, 533-534; # 8 pps. 537-540

4. Lewis: # 5 pps. 451-456, 459-460; # 6 pps. 482-487, 491-493; # 7 pps. 487-489; # 8 pps. 498-502

5. Nichols: # 5 pps. 369-372; # 6 pps. 377-380; # 7 pps. 373-375, 383-384, 387-388; # 8 pps. 390-391

II. Problems:

1. Moise: # 5 pps. 441-442 exs. 1-8; # 6 pps. 446-448 exs. 1-17; # 7 pps. 450-453 exs. 1-22, 25, 26; # 8 pps. 457-461 exs. 1-26, 28, 31, 32

2. Jurgensen: # 5 pps. 368-369 exs. 1, 2, 4-8, 11, 12; # 6 pps. 362-363 exs. 1-6, p. 369 exs. 13-16, p. 374 exs. 3-4, pps. 381-384 exs. 1-8, 24-26, 33-36; # 7 pps. 368-369 exs. 3, 9, 10, 17, 18, pps. 374-376 exs. 1, 2, 5-25, pps. 382-383 exs. 9-23, 27-32, pps. 386-388 exs. 1-36; #8 pps. 391-393 exs. 1-26

3. Anderson: # 5 p. 522 exs. 1-9; # 6 pps. 525-526 exs. 1-18; # 7 pps. 530-531 exs. 1-20 pps. 536-537 exs. 5-7, 9-19; # 8 pps. 540-542 exs. 1-17

4. Lewis: # 5 pps. 455-457 exs. 1-15, pps. 459-460 exs. 1-14; #6 pps. 489-490 exs. 1-2, p. 494 exs. 1, 5, 6, p. 495 exs. 10-11, p. 497 ex. 20, p. 498 exs. 1-5; # 7 pps. 689-691 exs. 1-10; # 8 pps. 502-505 exs. 1-13, pps. 505-508 exs. 1-17

5. Nichols: # 5 pps. 372-373 exs. 1-12; # 6 pps. 380-382 exs. 1-25; # 7 pps. 375-376 exs. 1-18, pps. 385-387 exs. 1-14, pps. 388-389 exs. 1-14; #8 pps. 392-394 exs. 1-21

SELF EVALUATION II

1. Answer the following true or false:

   a) The midpoint of an arc is the point of the arc which bisects the arc.

   b) If the measure of an [illegible] is [illegible], then the measure of the central angle intercepting the same arc is n.

   c) If the measure of a major arc is 180 +n, then the measure of its corresponding minor arc is 180-n.

   d) An angle inscribed in a semicircle is a right angle.

   e) If inscribed quadrilateral ABCD has m ∠A equal to 3 times the measure of ∠C, then the measure of ∠A is 135.

   f) If an inscribed angle and a central angle of a circle intercept the same arc, then they are congruent.

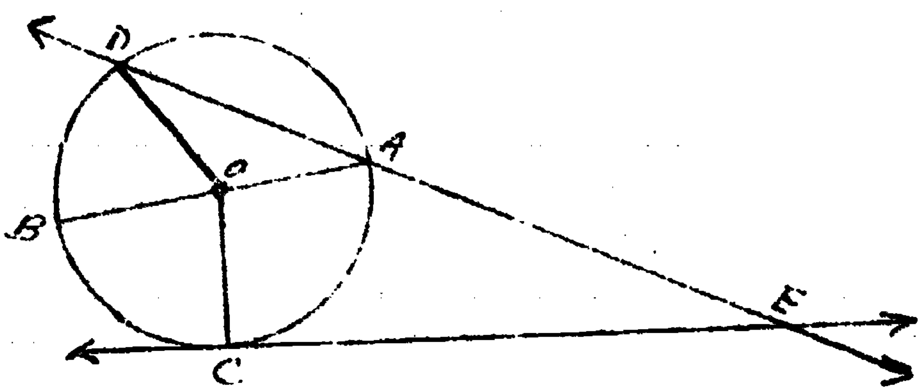
   g) Two angles which are inscribed in congruent arcs are congruent.

   h) If an angle intercepts an arc, then the arc is in the interior of the angle except for its end points.

   i) An acute angle would be inscribed in a minor arc.

   j) A quadrilateral circumscribed about a circle has each of its sides tangent to the circle.

2. Answer the following questions about the given figure:



In the figure m ∠ DOB = 100, m $\widehat{AC}$ =70 and O is the center of the circle. Find each of the following:

a) m $\widehat{DB}$

b) m ∠DOA

c) m $\widehat{BC}$

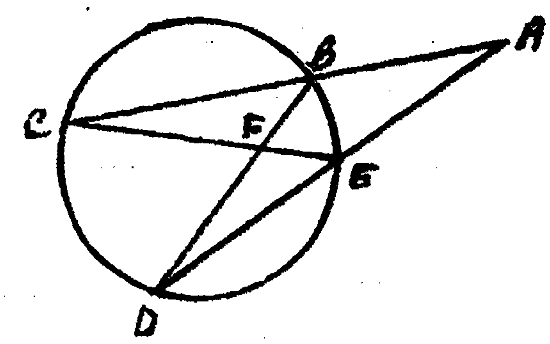
d) m $\widehat{BAC}$

e) m $\widehat{AOC}$

3. Use the figure given in the previous problem to match the appropriate item from Column A with Column B:

COLUMN A	COLUMN B
a) $\angle AOD$	1) an inscribed angle
b) $\angle DAB$	2) a central angle
c) $\overset{\frown}{BD}$	3) a major arc
d) $\overset{\frown}{BDA}$	4) a minor arc
e) $\overset{\frown}{BAC}$	5) a semicircle
f) $\overset{\frown}{AC}$	

4. For the given figure answer the following:



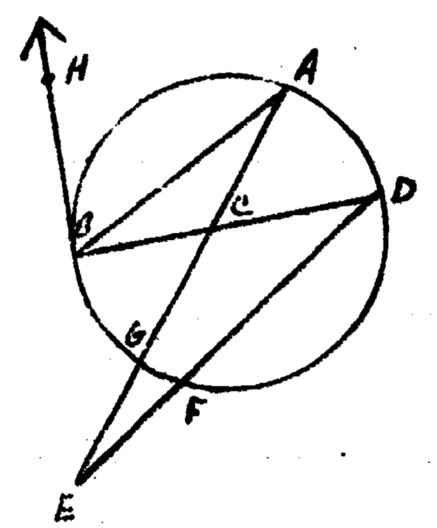
a) If $m\angle A = 25$ and $m\overset{\frown}{CD} = 115$, find $m\overset{\frown}{BE}$

b) If $m\overset{\frown}{CD} = 90$, $m\overset{\frown}{BE} =$ [illegible], find $m\angle A$ and $m\angle BFE$

c) If $m\angle BCE = 15$, and $m\angle CFD = 46$, find $m\overset{\frown}{CD}$

d) If $m\overset{\frown}{BE} = 18$, $m\angle A = 32$, find $m\angle BFC$

5. Answer the following about the given figure:



a) $\angle ABD$ is inscribed in which arc?

b) $\angle HBD$ intercepts which arc?

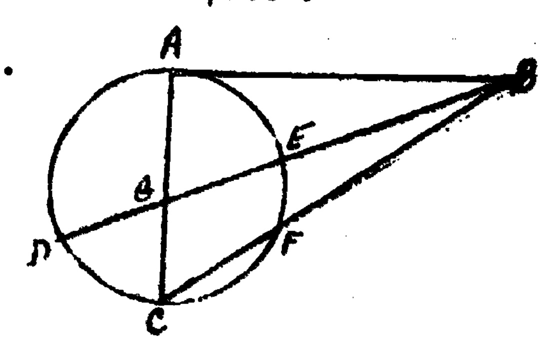
c) $\angle HBD$ is inscribed in which arc?

d) $\angle E$ intercepts what pair of arcs?

6. Compute the following: Two tangent segments to a circle from an external point determine a 70° angle. Find the measure of the two arcs intercepted by the circle.

7. Prove the following: In a circle, if two arcs are congruent, then their corresponding chords are equidistant from the center of the circle. Draw a figure, list the "given" and "to prove" and write a two column proof.

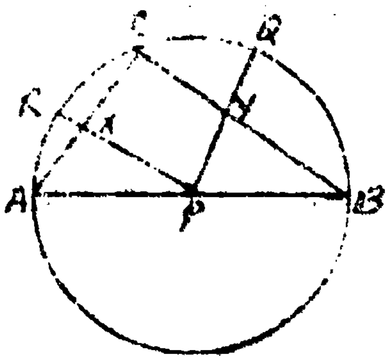
8. For the given figure, $\overleftrightarrow{BA}$ is tangent to the circle at A.



a) If AB = 5, DE = 7, find BD
b) If BE = 4, DE = 12, BF = 6, find CF
c) If BC = 18, DE = 19, DF = 6, find BE
d) If AG = 4, AC = 12, GE = 3, find DG
e) If AC = 15, DG = 9 and GE = 4 find AG and GC.

Self Evaluation (cont.)

9. Prove the following:



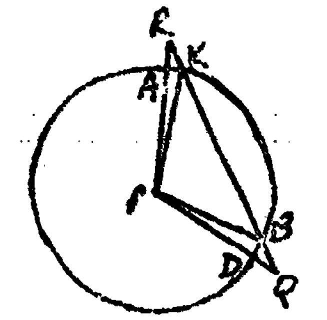
Given: P is the center of the circle

$\overline{PR}$ bisects $\overline{AC}$ at X

$\overline{PQ}$ bisects $\overline{BC}$ at Y

Prove: $\overline{RP} \perp \overline{QP}$

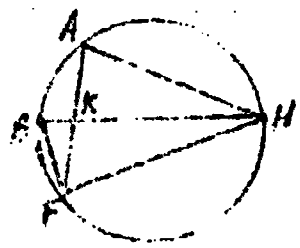
10. Prove the following:



Given: Circle C with center P $m\,\widehat{AB} = m\,\widehat{KD}$, AR = DQ

Prove: RK = BQ

11. Prove the following:



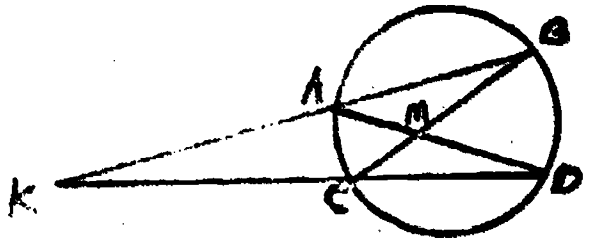
Given: $m\,\widehat{AB} = m\,\widehat{BF}$

Prove: $\triangle AHK \sim \triangle BKF$

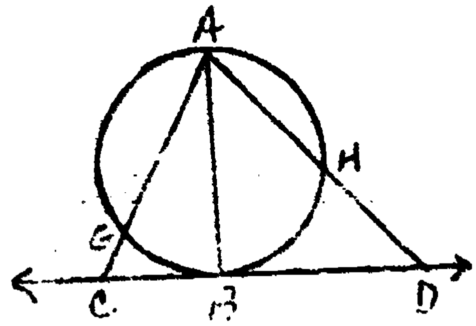
12. Prove the following: $\overleftrightarrow{MN}$, $\overleftrightarrow{NO}$, and $\overleftrightarrow{OQ}$ are tangent to circle C at A, B, and D respectively. Prove NA + OD = NO. Draw a figure, list the "given" and to "prove" and write a two column proof.

If you have mastered the Behavioral Objectives, take your Progress Test.

1. In the figure, [illegible], find m $\overset{\frown}{AC}$ and m $\angle K$.



2. In the figure, $\overline{AB}$ is a diameter and $\overleftrightarrow{CD}$ is the tangent at B. Prove that $AC \cdot AG = AD \cdot AH$.



3. One of the first facts that a student of astronomy learns is that the latitude of a position on the earth is the same as the angle of Polaris (the North Star) above the horizon observed from that position. Show why this is true by proving the following theorem. The physical situation is described by the following symbolism:

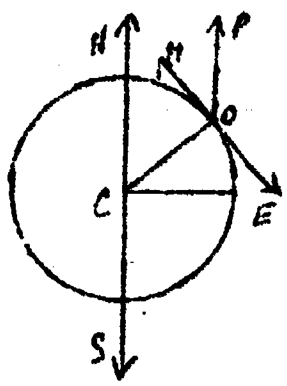
$\overleftrightarrow{NS}$ is the earth's axis, the circle is a meridian, C is the center, E is the equator, O is the observer, $\overleftrightarrow{OH}$ is the horizon and m $\angle POH$ is the elevation of Polaris.

Given: The circle with center C

Radius $\overline{CE} \perp \overleftrightarrow{NS}$, $\overleftrightarrow{OH}$ is tangent at O

$\overrightarrow{OP} \parallel \overleftrightarrow{NS}$

Prove: m $\overset{\frown}{OE}$ = m $\angle POH$



4. Two noncongruent circles intersect in two points X and Y. A secant through X intersects the larger circle at A and the smaller circle at B. A secant through Y intersects the larger circle at C and the smaller circle at D. Prove that $\overline{AC} \parallel \overline{BD}$.

5. On the bridge of a ship at sea, the captain asked the new, young officer standing next to him to determine the distance to the horizon. The officer took pencil and paper, and in a few moments came up with an answer. On the paper he had written the formula $d = \frac{5}{4}\sqrt{h}$. Show that this formula is a good approximation of the distance, in miles, to the horizon, if h is the height, in feet of the observor above the water.
(Assume the radius of the earth to be 4,000 miles) If the bridge was 88ft. above the water, what was the distance to the horizon?

Section III

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BEHAVIORAL OBJECTIVES:

By the completion [illegible], you will be able to:

9. Apply the following [illegible] in evaluating relationships pertaining to circles [illegible]

   a) *Theorem 44-24
   b) *Theorem 44-25
   c) *Theorem 44-26

(* See Appendix)

RESOURCES III

I. Readings:

1. Moise: # 9 pps. 461-465
2. Jurgensen: # 9 pps. [illegible]-456
3. Anderson: # 9 pps. 54[illegible]-545
4. Lewis. # 9 pps. 44[illegible]-445
5. Nichols: # 9 pps. 41[illegible]-412

II. Problems:

1. Moise: # 9 pps. 465-468 exs. 1-25, 27-2[illegible]
2. Jurgensen: # 9 pps. 456-458 exs. 1-30
3. Anderson: # 9 pps. 546-548 exs. 1-21
4. Lewis: # 9 pps. 445-447 exs. 1-14
5. Nichols: # 9 pps. 413-414 exs. 1-7

Self Evaluation [illegible]

1. a) On separate axes, draw [illegible] of the circle described by each of the following equations:

   1) $x^2 + y^2 = 1$[illegible]   3) [illegible] $= 4$

   2) $(x - 2)^2 + (y - 3)$ [illegible]   4) [illegible] $+ 16 = 0$

   b) Give the radius of each of the circles of part (a).

   c) Give the coordinates of the center of each circle of part (a).

   d) Is the point $(3, \sqrt{7})$ on the circle of part (a) # 1?

   e) Is the point (2,5) in the interior, exterior, or on the circle of part (a) # 2.

2. a) Are the points [illegible], $\sqrt{?}$) and $(-\sqrt{3}, -1)$ on the graph of $x^2 + y^2 = 4$? Justify your answer.

   b) Write the equation of the circle with center (7,9) which is tangent to a line 2 units above the x axis.

3. Given that A(12,5) and B(0, -13) are two points of the circle $x^2 + y^2 = 169$, determine the distance from the center of the circle to $\overline{AB}$.

4. Write the equation of each of the following circles in the form $(x - a)^2 + (y - b)^2 = r^2$

   a) The circle with center (0,0) and radius 7.

   b) The circle with center (-1,0) and radius $\sqrt{11}$.

   c) The circle concentric to the circle with equation $x^2 + (y + 5)^2 = 12$ and with radius 4.

   d) The circle having the points (0,-2) and (6,6) as the endpoints of a diameter.

   e) The circle with radius 4 that is tangent to both of the lines $x = 4$ and $y = 4$.

If you have mastered the Behavioral Objectives, take your LAP Test.

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[illegible]

1. If A(5,-2), B(-6, [illegible]
   a) What is the center of the circle containing them?
   b) What is the radius of the circle?
   c) Write an equation for the circle.

2. Given the circle whose equation is $x^2 + y^2 = 36$. For what values of a is the point (a, a + 4) in the interior of the circle?

3. Show that the two circles whose equations are $x^2 + y^2 = 16$ and $x + y - 16x - 12y = 0$ are externally tangent. What are the coordinates of the point of tangency?

4. Given the circle whose equation is $x^2 + y^2 + 16x + 12y = 125$:
   a) Find the equation of the circle with radius 5. Which is internally tangent to the given circle at (4,3).
   b) Find the equation of their common tangents.

5. Find the equation of the circle which is tangent to all four of the circles characterized by these four equations.
   a) $x^2 + y^2 + 10x = 0$
   b) $x^2 + y^2 - 10x = 0$
   c) $x^2 + y^2 + 10y = 0$
   d) $x^2 + y^2 - 10y = 0$

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APPENDIX

THEOREMS, COROLLARIES, AND POSTULATES

Theorem 44-1: The intersection of a sphere with a plane through its center is a circle with the same center and the same radius.

Theorem 44-2: A line perpendicular to a radius at its outer end is tangent to the circle.

Theorem 44-3: Every tangent to a circle is perpendicular to the radius drawn to the point of contact.

Theorem 44-4: The perpendicular from the center of a circle to a chord bisects the chord.

Theorem 44-5: The segment from the center of a circle to the mid-point of a chord is perpendicular to the chord.

Theorem 44-6: In the plane of a circle, the perpendicular bisector of a chord passes through the center.

Corollary 44-6.1: No circle contains three collinear points.

Theorem 44-7: In the same circle or in congruent circles, chords equidistant from the center are congruent.

Theorem 44-8: In the same circle or in congruent circles, any two congruent chords are equidistant from the center.

Theorem 44-9: If a line intersects the interior of a circle, then it intersects the circle in two and only two points.

Theorem 44-10: A plane perpendicular to a radius at its outer end is tangent to the sphere.

Theorem 44-11: Every tangent plane to a sphere is perpendicular to the radius drawn to the point of contact.

Theorem 44-12: If a plane intersects the interior of a sphere, then the intersection of the plane and the sphere is a circle. The center of this circle is the foot of the perpendicular from the center of the sphere to the plane.

Theorem 44-13: The perpendicular from the center of a sphere to a chord bisects the chord.

Theorem 44-14: The segment from the center of a sphere to the mid-point of a chord is perpendicular to the chord.

Theorem 44-15. [illegible]

[illegible]

$m\angle C$. [illegible]

Theorem 44-16: The measure of an inscribed angle is half the measure of its intercepted arc.

Corollary 44-16.1: Any angle inscribed in a semicircle is a right angle.

Corollary 44-16.2: Every two angles inscribed in the same arc are congruent. Again this is obvious: they intercept the same arc.

Theorem 44-17. In the same circle or in congruent circles, if two chords are congruent, then so are the corresponding minor arcs.

Theorem 44-18: In the same circle or in congruent circles, if two arcs are congruent, then so are the corresponding chords.

Theorem 44-19: Given an angle with its vertex on a circle, formed by a secant ray and a tangent ray. The measure of the angle is half the measure of the intercepted arc.

Theorem 44-20: The two tangent segments to a circle from a point of the exterior are congruent and determine congruent angles with the segment from the exterior point to the center.

Theorem 44-21: Given a circle C, and a point Q of its exterior. Let L_1 be a secant line through Q, intersecting C in points R and S; and let L_2 be another secant line through Q, intersecting C in points U and T. Then

$$QR \cdot QS = QU \cdot QT.$$

Theorem 44-22: Given a tangent segment QT to a circle, and a secant line through Q, intersecting the circle in points R and S. Then

$$QR \cdot QS = QT^2.$$

Theorem 44-23: Let RS and TU be chords of the same circle, intersecting at Q. Then

$$QR \cdot QS = QU \cdot QT.$$

Theorem 44-24: The graph of the equation

$$(x - a)^2 + (y - b)^2 = r^2$$

Theorem 44-25: [illegible] equation of the form

$$x^2 + y^2 + [illegible] = 0.$$

Theorem 44-26: The graph of the equation

$$x^2 + y^2 + Dx + Ey + F = 0$$

is (1) a circle, (2) a point, or (3) the empty set.

BIBLIOGRAPHY

1. Moise, Downs: Geometry (Addison Wesley Publishing Co., 1967)

2. Jurgensen, Donnelly, Dolciani: Modern Geometry, Structure and Method (Houghton Mifflin Co., 1965)

3. Anderson, Garon, Gremillion: School Mathematics Geometry (Houghton Mifflin Co., 1969)

4. Lewis: Geometry, A Contemporary Course (D. Van Nostrand Co., Inc., 1968)

5. Nichols, Palmer, Schacht: Modern Geometry (Holt, Rinehart, and Winston, Inc., 1968)

LEARNING
ACTIVITY
PACKAGE

CHARACTERIZATION AND CONSTRUCTION

Ninety Six High School
Ninety Six, S.C.

GEOMETRY 114

REVIEWED BY

LAP NUMBER 45

WRITTEN BY Bill Holland

RATIONALE

In your previous units of work you were not necessarily concerned with the accuracy of your drawings of geometric figures. You merely sketched the figure in such a manner it appeared to satisfy the given conditions. In this unit of work you will learn how to construct figures with a higher degree of accuracy by means of a compass and straight-edge. You will learn to characterize a set of points by a word description or a sketch. When the set of points is in the coordinate plane, you will characterize it with a graph and an equation or inequality. Thus, your knowledge of equations of lines and circles from previous chapters will be useful here. Characterizations of sets of points are used extensively in coordinate geometry. The graph of every equation is a characterization of that set of points. We characterize a set by specifying a condition which is satisfied by all elements of the set, but no other elements.

You will study the properties of the various points of concurrency for a triangle as well as learning how to construct them.

Besides the aesthetic value of construction and characterization much of this can be applied to the study of polygons and coordinate geometry.

SECTION I

BEHAVIORAL OBJECTIVES:

By the completion of the prescribed course of study, you will be able to:

1. Given sufficient information about a set of points:

   a) Sketch the set of points.
   b) Characterize the set of points with a word description.
   c) Name the geometric set these points represent.

2. Give sufficient information about a set of points:

   a) Graph the set of points in the coordinate plane.
   b) Characterize the set of points with an equation or inequality.

3. Given sufficient information, evaluate relationships pertaining to the following:

   a) The intersection of the perpendicular bisectors of the sides of a triangle (circumcenter)
   b) The intersection of the altitudes of a triangle (orthocenter)
   c) The intersection of the angle bisectors of a triangle (incenter)

4. Given the centroid of a triangle and sufficient information pertaining to it:

   a) Find its distance from a vertex of the triangle.
   b) Find the distance between the centroid of a triangle and a side of the triangle.
   c) Find the length of a median of a triangle.
   d) Find the coordinates of the centroid.

I. Readings:

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1. Moise: # 1 pps. 475-476; # 2 pps. 479-480; # 3 pps. 481-483, 485-487; 503 # 4 pps. 489-490

2. Jurgensen: # 1 pps. 425-427, 429-431; # 2 pps. 450-451, 455-456; # 3___; # 4 ____

3. Anderson: # 1 _____; # 2 pps. 479-482; # 3 pps. 557-559, 562-564; # 4 pps. 562-563

4. Lewis: # 1 pps. 434-437; # 2 pps. 425-433; # 3 pps. 536-537; # 4 pps. 277-278, 392

5. Nichols: # 1 pps. 414-417; # 2 pps. 404-408, 420-421; # 3 pps. 423-428, # 4 p. 428

II. Problems:

1. Moise: # 1 pps. 476-479 exs. 1-9, 11-15, 19, 21, 23, 24, 26; # 2 pps. 480-481 exs. 1-8; # 3 p. 484 exs. 2-9, pps. 487-488 exs. 1-8; # 4 pps. 490-491 exs. 1-6

2. Jurgensen: # 1 pps. 427-429 exs. 1-12, 15, 17, 18, 19, 23, p. 431 exs. 1, 2, 4, 6, 8, 10, p. 438 exs. 17-22; # 2 p. 451 exs. 1-12, pps. 456-457 exs. 1, 3, 6, 8, 9, 11, 13, 16, 23, pps. 483-484 exs. 1,4,5,7,8,9,11,13,16,18; # 3 _____; # 4_____

3. Anderson: # 1_____; # 2 _____; # 3 p. 560 exs. 1-6, pps. 560-561 exs. 1,2, 4,7,12; # 4 p. 564 exs. 1-3, p. 565 exs. 1-2, p. 566 exs. 9, 12, 13

4. Lewis: # 1 pps. 437-440 exs. 1-3, 5-9, 11, p. 417 exs. 1-2, p. 418 exs. 4, 5, 8,9,11,12; # 2 p. 433 exs. 2-4; # 3 pps. 537-539 exs. A(1-12, 14), B(1,3,5,7,9,11); # 4 p. 392 exs. 19, 20

5. Nichols: # 1 pps. 417-418 exs. 1-12; # 2 pps. 408-409, exs. 1-4, p. 422 exs. 1,2,4,6,7,12; # 3 p. 429 exs. 5,7,8,11, pps. 430-431, exs. 21, 23, p. 432 ex. 3; # 4 p. 431 ex. 21, p. 432 ex. 3

SELF EVALUATION I

1. Describe as precisely as possible the following sets:

   a) The set of centers of all circles in space which have a given radius r and pass through a given point p.

   b) The set of the midpoints of all chords 16 inches long of a circle O with radius of 12 inches.

   c) The set of points at a given distance from a segment.

   d) The set of points in a plane equidistant from three non-collinear points.

   e) The set of points equidistant from two intersecting planes.

   f) The set of points which consist of the vertex angles of all isosceles triangles having $\overline{AB}$ as base.

2. Sketch the graphs of the following:

   a) $\{(x,y) \mid x = 5\}$

   b) $\{(x,y) \mid x + y = 2\}$

   c) $\{(x,y) \mid y \leq 2\}$

   d) $\{(x,y) \mid x = y \text{ and } y = 3\}$

   e) $\{(x,y) \mid x^2 + y^2 = 16 \text{ and } x = 2\}$

   f) $\{(x,y) \mid (x - 2)^2 + y^2 < 9\}$

   g) $\{(x,y) \mid |x| > 2\}$

3. Sketch and describe with an equation the following sets:

   a) The set of all points P(x,y) which are equidistant from A(3,2) and B(6,-2).

   b) The set of all points in the coordinate plane at a distance of 3 from (-2,3).

   c) The set of all points greater than 5 units from the origin.

   d) The set of all points at a distance 2 from the equation x = -1.

4. a) The point of concurrency of the altitudes of a triangle is the ________ .

   b) The point of concurrency of the medians of a triangle is the ________ .

   c) The ________ can be a vertex of a triangle.

   d) In a (n) ________ the orthocenter coincides with the point of concurrency of the perpendicular bisectors of its sides.

   e) The ____________ is the center of the inscribed circle.

   f) The ____________ is the center of the circumscribed circle.

5. Answer the following true or false:

a) If the centroid, orthocenter, circumcenter and the incenter of a triangle are all the same point the triangle is equilateral.

b) The length of the radius of the circle inscribed in an equlateral triangle is two-thirds of the length of the altitude of the triangle.

c) In a right triangle the distance from the vertex of the right angle to the point of intersection of the medians is one-third the length of the hypotenuse.

d) Three lines are concurrent if they have one point of intersection.

e) All angle bisectors of a triangle intersect at a point called the orthocenter.

f) The centroid of a triangle may be in the exterior of the triangle.

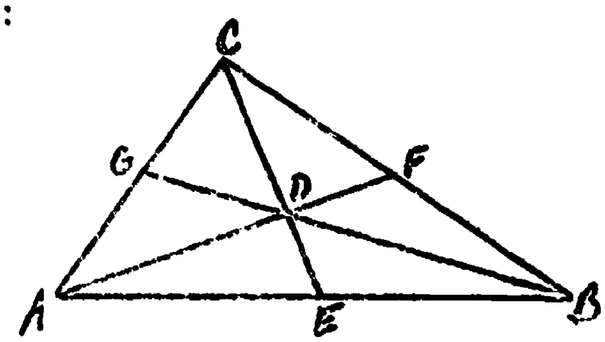
___ g) The area of a square inscribed in a circle with a radius measuring 4 inches is 32.

h) A circle tangent to each side of a triangle is called the inscribed circle.

i) The incenter of a triangle is equidistant from the vertices of the triangle.

j) The centroid of a triangle is equidistant from the sides of the triangle.

6. Given $\triangle ABC$, with medians $\overline{AF}$, $\overline{BG}$, and $\overline{CD}$ intersecting at D as shown in the figure. Complete the following statements:



a) If AF = 15, what is AD?

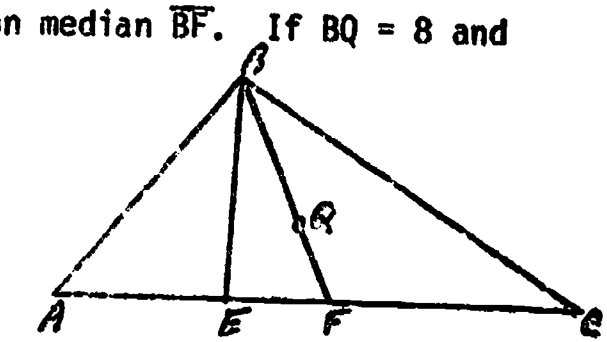
b) If GD = 3, what is BD?

c) If AD = 18, what is DF?

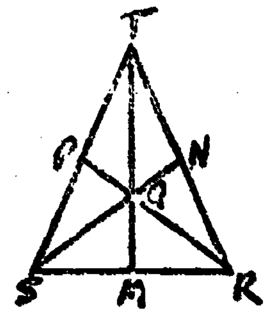
d) If CD = 8, what is CE?

e) If C = (0,6), B = (4,0) and A = (-4,0), give the coordinates of D.

7. In $\triangle ABC$, $\overline{BE}$ is an altitude, and centroid Q is on median $\overline{BF}$. If BQ = 8 and EF = 5, what is BE?



8. Prove the following:

   Given: $\overline{RP}$, $\overline{SN}$, and $\overline{TM}$ are medians with RP = SN

   Prove: QP = QN

If you have mastered the Behavioral Objectives, take your Progress Test.

1. a) Make a sketch and describe by an equation the set of all points $P(x,y)$ which are twice as far from $(8,0)$ as from $(2,0)$.

   b) Sketch the following set: $\{(x,y) \mid -1 \leq x \leq 5 \text{ and } 0 \leq y \leq 4\}$

   c) Sketch the following set:

$$\{(x,y) \mid (x - 3)^2 + y^2 = 25 \text{ or } (x + y)^2 + y^2 = 52\}$$

2. Given PQR with vertices $P(-6,0)$, $Q(2,)$ and $R(0,6)$.

   a) Find the distance between the centroid and the point of concurrency of the perpendicular bisectors of the sides.

   b) Find the coordinates of the orthocenter and the distance from the orthocenter to the centroid.

3. The following instructions were found on an old map:

   "Start from the crossing of King's Road and Queen's Road. Proceed due north on King's Road and find a large pine tree and then a maple tree. Return to the crossroads. Due west on Queen's Road there is an elm, and due east on Queen's Road there is a spruce. One magical point is the intersection of the elm-pine line with the maple-spruce line. The other magical point is the intersection of the spruce-pine line with the elm-maple line. The treasure lies where the line through the two magical points meets Queen's Road."

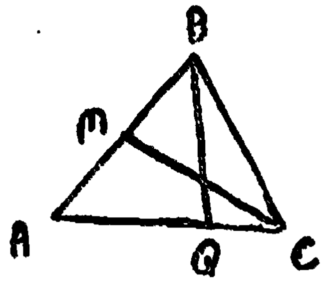
   A search party found the elm 4 miles from the crossing, the spruce 2 miles from the crossing and the pine 3 miles from the crossing, but could find no trace of the maple. Nevertheless, they were able to locate the treasure from the instructions. Show how they could do this.

   One member of the party remarked on how fortunate they were to have found the pine still standing. The leader laughed and said, "We didn't need the pine tree." Show that he was right.

4. Prove the following:

   Given: $\overline{CM}$ bisects $\overline{AB}$ at M, $\overline{BQ}$ bisects $\overline{CM}$ at P

   Prove: $AQ = 2 \cdot QC$



5. Given any segment $\overline{BC}$ and line l parallel to $\overline{BC}$. We shall give directions for bisecting BC with only a straightedge. Choose any point Q in the half plane H with edge l. Draw $\overline{BQ}$ intersecting l at A. Draw CQ intersecting l at D. Draw $\overline{BD}$ and $\overline{AC}$ intersecting at P. Finally, draw $\overleftrightarrow{QP}$ intersecting $\overline{BC}$ at M. $\overline{BC}$ is now bisected.

   a) Follow the above instructions to besect $\overline{BC}$.

   b) Prove that M is the midpoint of $\overline{BC}$.

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BEHAVIORAL OBJECTIVES

By the completion of the ... course of study, you will be able to:

5. Using a straightedge and a compass, correctly:

   a) Copy a given segment
   b) Bisect a given angle
   c) Copy a given angle
   d) Copy a given triangle

6. Using a straightedge and compass, correctly:

   a) Construct a line parallel to a given line through a given external point.

   b) Divide a given segment into a specified number of congruent segments.

   c) Construct the perpendicular bisector of a given segment.

   d) Construct a line perpendicular to a given line through a given point on the line.

   e) Construct a line perpendicular to a given line through a given external point.

   f) Construct an angle of measure 15x where x is an element of the set

      {1,2,3,4,5,6,7,8,9,10} .

   g) Construct a triangle when given sufficient information by the S.S.S., S.A.S. or A.S.A. method.

   h) Construct a quadrilateral when given sufficient information.

7. Using a straightedge and compass, correctly:

   a) Construct the altitudes of a triangle.

   b) Construct the medians of a triangle.

   c) Construct the angle bisectos of a triangle.

   d) Construct the perpendicular bisectors of the sides of a triangle.

   e) Inscribe a circle in a triangle.

   f) Circumscribe a circle about a triangle.

RESTRICTED [illegible]

I. Readings:

1. Moise: # 5 pps. [illegible]; # 7 pps. 413-415, 417-419

2. Jurgensen: # 5 pps. [illegible], 421; # 7 pps. 413-415, 417-419

3. Anderson: # 5 pps. 567-570, # 6 pps. 570-572, 574-575, # 7 pps. 576-577

4. Lewis: # 5 pps. 539-541; # 6 pps. 542-547, 550-554; # 7 pps. 547-549

5. Nichols: # 5 pps. 92-95, # 6 pps. 93-95, 299-301, 361; # 7 pps. 362, 423-428

II. Problems:

1. Moise: # 5 p. 497 exs. 1-5,7, # 6 pps. 500-501 exs. 1-12, 15; # 7 p. 504 exs. 1-5, 8-11

2. Jurgensen. # 5 pps. 411-412 exs. 1, 4-6, 8, 9, 12, 13; p. 437 exs. 1-4, p. 438 exs. 13-16; # 6 pps. 416-417 exs. 1-5, 11, 13, 19, p. 423 exs. 1-2, p. 437 exs. 5-8; # 7 p. 416 exs. 21-25, p. 419 exs. 1-6, p. 420 exs. 1, 3,5,7,9,13,14,17, 19-21, p. 438 exs. 9-12

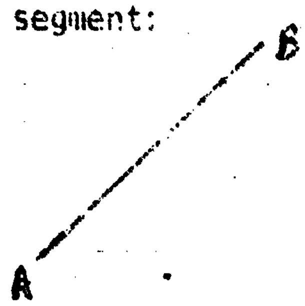
3. Anderson: # 5 pps. 569-570 exs. 1-4, 6,7,10,12; # 6 pps. 573-574 exs. 1-3, 5,8,10,12,14,18; # 7 p. 577 exs. 1-3, p. 577-578 exs. 1-7

4. Lewis: # 5 p. 545 exs. 2,3,8,9; # 6 pps. 545-546 exs. 1,4,5,6,7,10,15, 17,18, pps. 554-557 exs. 1-7, 9,13,14; # 7 p. 546 exs. 12-14

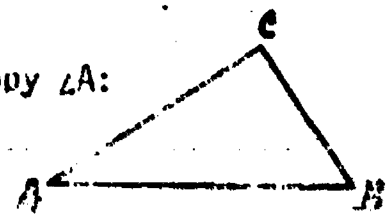
5. Nichols: # 5 p. 95 exs. 2-4; # 6 pps. 301-302 exs. 1,2,4,6,9,12,14; # 7 p. 362 exs. 2,8, pps. 429-431 exs. 9,10,22,25

1. Do each of the following constructions below using straight edge and compass:

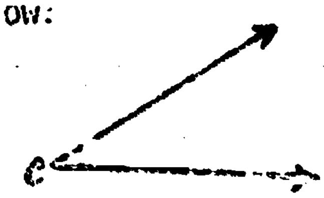
   a) Copy this segment:



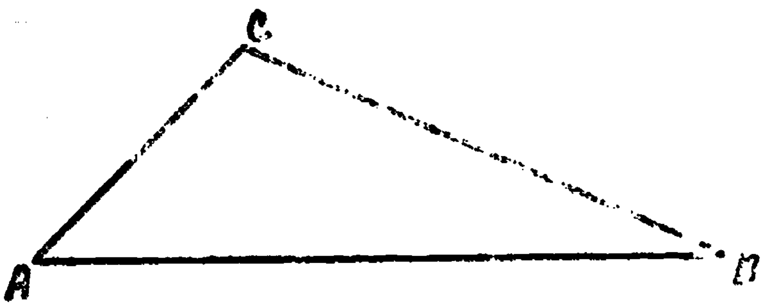
   b) Copy ∠A:



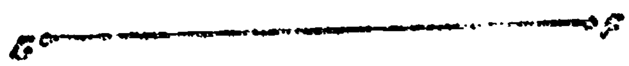
   c) Bisect ∠C in the figure below:



2. Copy this triangle using the A.S.A. method.

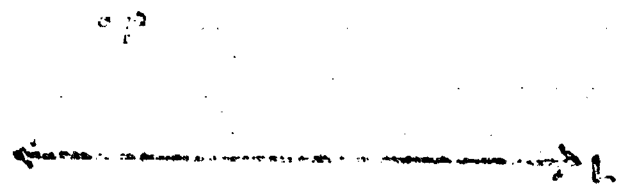


3. Divide this segment into 3 congruent parts.

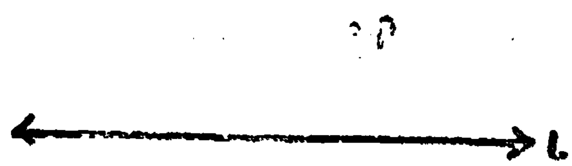


4. Construct a line parallel to line L through point P.

SELF EVALUATION (cont.)



5. Construct a line perpendicular to line l through point P:

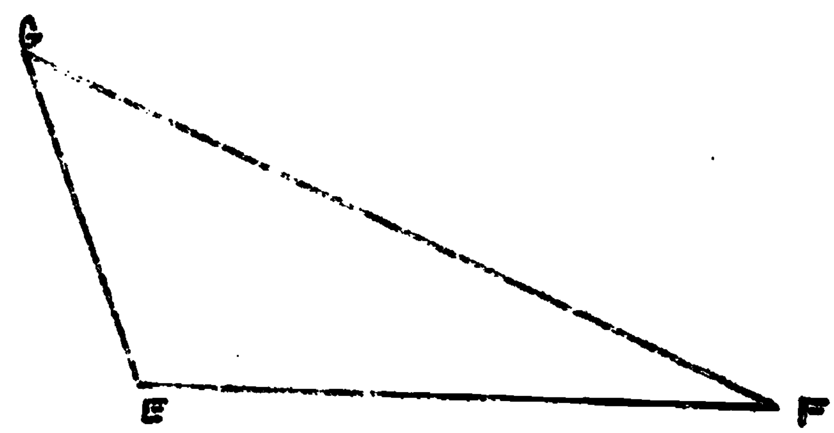


6. Construct an isosceles right triangle:

7. Construct a 60^{0} angle:

8. Given a circle with center O and a point P on the circle. Construct a line tangent to the circle at P.

9. Inscribe a circle in the triangle:



If you have mastered the Behavioral Objectives, take your LAP Test.

BIBLIOGRAPHY

1. Moise, Downs: Geometry (Addison Wesley Publishing Co., Inc., 1964).
2. Jurgensen, Donnelly, Dolciani: Modern Geometry, Structure and Method (Houghton Mifflin Co., 1969).
3. Nichols, Palmer, Schacht: Modern Geometry (Holt, Rinehart and Winston, Inc., 1968).
4. Lewis: Geometry, A Contemporary Course (E. Van Nostrand Co., Inc., 1968).
5. Anderson, Garon, Gremillion: School Mathematics Geometry (Houghton, Mifflin, Co., 1969).