A functional (as opposed to structural) theory of intellectual development is presented and used to generate specific performance models for Piagetian tasks involving the control of variables (cf. Inhelder and Piaget, 1958). On the basis of these models, it is concluded that intelligent, field independent 7- and 8-year-olds should be able to acquire the control of variables scheme, even though they have not yet acquired either conservation of weight or the combinatorial system. Preliminary data (n=52) are presented to support this conclusion. They are discussed with regard to Piaget's formal theory of intellectual development and the functional limitations of development of learning. (SDH)
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LEARNING AND INTELLECTUAL DEVELOPMENT

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Appendix A Detailed Teaching Procedure .............................. 51
As Flavell and Wohlwill (1969) have pointed out, Piaget's theory of intellectual development is predominantly a structural one: it aims at a formal description of the human organism's knowledge or competence at different points in time. What developmental psychology now needs—if it is to achieve a greater degree of predictive power—is a parallel theory which is predominantly functional, that is, one which describes the devices or mechanisms by which human knowledge is actually acquired and utilized. The purposes of the present report are as follows: (a) to outline the general functional theory of cognition which is being developed by Pascual-Leone and his co-workers, (b) to demonstrate that the theory is capable of generating detailed "performance" models for a group of Piagetian tasks, (c) to present some counter-intuitive data which are successfully predicted by these models, and (d) in the light of the data, to reconsider the nature of developmental limitations on the acquisition of specific logical structures.

I A Neo-Piagetian Theory of Development

The theory to be summarized in this section was first proposed by Pascual-Leone, in a doctoral dissertation submitted to the University of Geneva (Pascual-Leone, 1969). Since that time it has been modified and
elaborated, particularly with a view to facilitating the sort of detailed functional modeling which will be attempted in the second section.

The basic construct employed in the theory is the Piagetian notion of a scheme. Schemes are defined as the subjective units of thought, that is, as the mental blueprints which represent experience and which are responsible for producing behavior. They are classified into three main categories: figurative, operative, and executive.

**Figurative** schemes are roughly equivalent to what Miller (1956) has labelled "chunks." They are the internal representations of items of information with which a subject is familiar, or of perceptual configurations which he can recognize. Like pattern recognition devices described by Neisser (1967), they are assumed to act on a weighted set of features or cues, rather than simply to react to some "stimulus" or "input." If, for example, a subject looked at a photograph and asserted that it was a picture of his house, one would say that he did so by transforming the raw sensory input into a network of perceptual features which were already associated in his mind with a conceptual response of the order, "That is my house." More simply, one would say that he assimilated the sensory input to his (figurative) "house scheme."

**Operative** schemes correspond to what Inhelder & Plaget (1966, p. 22) have labelled "transformations," or to what computer simulators have labelled "primitive information processes" (cf. Newell, Shaw, & Simon, 1958; Klahr & Wallace, 1970). They are the internal representations of functions (rules), which can be applied to one set of figurative schemes, in order to generate a new set. If, for example, a subject were to look at two different photographs and to judge that they were of the same...
(but unknown) house, one would say that he did so by applying an operative scheme representing a "sameness" function ("If two objects are alike in all relevant aspects, they may be presumed to be the same") to the figurative schemes representing the features of each of the photographs in question, and that he generated a new figurative scheme representing the fact: "These two photographs are actually of the same house."

Finally, executive schemes correspond to what Miller, Galanter, and Pribram (1960) have labelled "plans," or what Newell and Simon (1972) have labelled "executive programs." They are the internal representations of procedures which can be applied in the face of particular problem situations, in an attempt to reach particular objectives. As such, they are to a large degree responsible for determining what figurative and operative schemes a subject activates in any particular situation. It would be unlikely, for example, that a subject would activate the figurative and operative schemes mentioned in the previous paragraph, unless the operation of comparing photographs was a planned part of some executive routine by which the subject hoped to accomplish some particular goal. The complexity of the routine could, of course, vary widely: from a simple one-step procedure in which photograph-comparison was the only important operation, to a highly sophisticated contingency plan in which photograph-comparison played only a minor role.

Although schemes are classified into three different categories, it may be seen that they are all alike in the following respects:

Al. They are all highly "active." They are not simply triggered by some input; rather, they apply on it and transform it.
A2. They are all functional units, even though they may vary in content and structural complexity.

A3. They all consist of two components, an initial set of conditions under which they can apply (i.e., a releasing component), and a subsequent set of conditions which they can generate (i.e., an effecting component).

Given this characterization of the nature of schemes, it follows from the literature on early infancy that children are born with an innate repertoire of sensory motor schemes (Piaget, 1952, 1954; Fantz, 1963; Kagan, 1971). New schemes are assumed to be acquired in a number of fashions:

B1. By the modification of an old scheme. This sort of acquisition is assumed to occur in one of two ways. (a) As the result of repeated application of an old scheme, new components can become incorporated into its releasing component. This is what is assumed to occur, for example, in the course of "perceptual differentiation" (cf. Gibson, 1969). (b) As the result of repeated application of an old scheme, new components can become incorporated into its effecting component. This is what is assumed to occur, for example, in the course of "trial-and-error learning."

B2. New schemes can also be acquired by the combination and consolidation of several old schemes. This sort of acquisition is assumed to occur in one of three ways. (a) As the result of repeated coactivation of a number of figurative schemes, the entire group can be assembled into a higher-order unit. This is what is assumed to occur, for example, in the emergence of perceptual "chunks" (cf. Miller, 1956). (b) As the result of the application of an operative scheme on a set of figurative schemes, the new figurative scheme generated by the transformation can
become a permanently fixed functional unit. This is what is assumed to occur, for example, when a subject generates and retains a new item of information from his already existing store, without ever being presented with this item directly. (c) As a result of the repeated and successful application of a series of schemes to the solution of a particular class of problems, the whole sequence can become incorporated into a higher-order functional unit which can serve in future situations of a similar sort as a more highly articulated executive scheme. This is what is presumed to occur, for example, when a subject gradually evolves a sophisticated "strategy" to deal with a particular class of problems (cf. Bruner, Goodnow, & Austin, 1956).

In the course of everyday interaction with the world, then, subjects are assumed to be constantly applying, and constantly modifying, their basic repertoire of schemes. The total set of schemes activated at any one moment is held to constitute the content of their thought. The following postulates are assumed to characterize the process of this thought (or at least that part of it which is goal-directed).

C1. In attempting to solve any problem, a subject's first step is to activate some general executive scheme. Which scheme he activates can depend on a number of factors, including the nature of the problem-constraints, the nature of the perceptual field, the nature of his past problem-solving experience, and the nature of his emotional reaction to the situation.

C2. Once a particular executive scheme is activated, it directs the activation of a sequence of figurative and operative schemes.
C3. The sequence of figurative and operative schemes is comprised of discrete "mental steps." Each of these constitutes a distinct operation (analogous to a subroutine in a computer program), in which an operative scheme applies on one or more figurative schemes, and generates a new figurative scheme.

C4. Figurative schemes which are the product of past operations are carried forward or "rehearsed," so that they can be utilized in future operations.

C5. Unless its releasing component is activated directly by the immediate perceptual input, the activation or rehearsal of any scheme requires the application of "mental effort" (cf. Kahneman, 1973). Since the amount of mental effort which can be applied at any one moment is limited, the number of schemes which can be activated in any one mental step is also limited.

C6. When a scheme (or set of schemes) which corresponds to the subject's original objective is finally generated, the executive scheme directs an appropriate terminal response and ceases to be active.

C7. If, at any time in the above process, two schemes are activated whose content is pragmatically incompatible (e.g., scheme 1: "x is bigger", scheme 2: "x is smaller") cognitive conflict ensues. Other things being equal (e.g., salience, past reinforcement, emotional involvement) cognitive conflicts are dealt with by activating all the other schemes which appear to be of relevance to the conflict, and by resolving the conflict in favor of the scheme consistent with the greatest number of other schemes. This process is assumed to be a central part of what Piaget has labelled "equilibration" (Piaget, 1970).
The above postulates are intended to characterize all directed thinking. Whether or not a particular individual actually solves a particular problem is held to depend on four additional factors.

B1. The first of these is the repertoire of schemes which the subject brings to the problem. It is assumed that this repertoire increases in complexity and accuracy with experience, according to the learning processes mentioned in B1 and B2. As a result, it varies both within and across age groups.

B2. The second of these is the maximum number of schemes which the subject's psychological system is capable of activating at any one time. This maximum mental effort or M-power is also assumed to vary both within and across age groups. Within age groups, differences are assumed to result largely from biological factors, and to be at least partially responsible for producing differences in what Spearman (1927) labelled "content-free intelligence (g)." Across age groups, differences are assumed to result largely from maturation, and to be at least partially responsible for producing differences in what Piaget labelled "operativity-level" (Piaget, 1970). For children whose cognitive development is normal, M-power is assumed to increase linearly with age, according to the following scale.

<table>
<thead>
<tr>
<th>Age</th>
<th>Developmental Substage</th>
<th>M-power</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-4</td>
<td>Early preoperations</td>
<td>e+1</td>
</tr>
<tr>
<td>5-6</td>
<td>Late preoperations</td>
<td>e+2</td>
</tr>
<tr>
<td>7-8</td>
<td>Early concrete operations</td>
<td>e+3</td>
</tr>
<tr>
<td>9-10</td>
<td>Middle concrete operations</td>
<td>e+4</td>
</tr>
<tr>
<td>11-12</td>
<td>Late concrete-early formal operations</td>
<td>e+5</td>
</tr>
</tbody>
</table>
In the above notation, the constant e refers to the mental effort (or energy, or capacity, or space) required to activate an overlearned executive scheme: the numeral refers to the maximum additional number of operative or figurative schemes which can be activated under the direction of this executive, without direct support from the immediate perceptual input.

b3. A third factor related to problem success is the subject’s tendency to utilize the full M-power which he has available. It is assumed that certain subjects are habitually low M-processors. Given the chance, they prefer to look at or respond to problems in the simplest manner possible, that is, with a set of operations involving the least possible mental effort. They are thus unlikely to do well at problems where some perceptual gestalt is presented or some simple solution pattern is suggested, yet where the most adequate response demands that this gestalt or simple pattern be broken down into a (larger and more complicated) set of sub-elements.

b4. The fourth factor which affects a subject’s chance of solving certain problems is the relative weight which he gives to cues from the perceptual field, as opposed to cues from other sources such as the task instructions, in selecting an executive scheme. In many problems, particularly those devised by psychologists, there are salient perceptual cues present which tend to activate an inappropriate executive scheme if they
are given too much attention. It is assumed that subjects vary in their tendency to give weight to such salient but misleading cues, and that individual differences in this tendency are stable across tasks and across time.

Finally, it is important to mention that the individual differences described in D3 and D4 are assumed to be highly correlated, and that together they are believed to explain the cognitive style dimension which Witkin has labelled "field dependence-independence" (Witkin, Dyk, Faterson, Goodenough & Karp, 1962). Field-dependent subjects are assumed to be habitually low M-processors who assign higher weight to perceptual cues than to cues provided by the task instructions, in situations where these two sets of cues suggest conflicting executive schemes. Field-independent subjects are assumed to be habitually high M-processors, who assign a higher weight to the task instructions than to perceptual cues in such conflicting situations.

Although this is a highly abbreviated summary of Pascual-Leone's theory, it may be seen that the framework is both comprehensive and integrative. Its formal aspects are congruent with several of the basic tenets of Piaget's theory. On the other hand, its functional aspects are congruent with many of the basic postulates of current theories of perception, attention, and cognition (cf. Neisser, 1967; Norman & Lindsay, 1972; Kahneman, 1973). While it is the formal aspect of the theory which makes it relevant to the sorts of competencies which have been studied by Piaget, it is the functional aspect (together with the emphasis on individual differences), which gives it its predictive power.
II. A Performance Model for "Control of Variables" Problems

General Procedures

Pascual-Leone has not set down an explicit set of heuristics whereby his theory may be used to generate a detailed performance model for a particular set of Piagetian tasks. The following general procedures, however, seem both workable and consistent with the specific performance models he has constructed to illustrate the utility of his general theory (Pascual-Leone, 1973). They will be followed in generating a model for tasks involving the control of variables.

E1. A general method will be postulated, analogous to an executive program, whereby a subject could consistently arrive at the correct answer to each test question. (cf. C1)

E2. This general method will be broken down into a series of steps such that only one mental transformation will be postulated in any one step (cf. C2, C3).

E3. Any scheme generated in one step, and required in a subsequent step, will be included as a "rehearsed scheme" in all intervening steps (cf. C4).

E4. The activation source for each scheme will be specified [i.e., activation by the application of mental effort, or activation directly by the visual input (cf. C5)].

E5. For each step, the minimum number of figurative and operative schemes which could execute the transformation will be specified. The reason for maximizing efficiency in this fashion is that the total number of schemes required at any step determines the youngest age at which the
problem can be solved (cf. B2), and what is of interest is a model of optimal performance.

E6. In order to assure that the model is realistic, independent evidence will be sought to determine whether or not young children could actually possess the schemes whose existence has been postulated. If no such independent evidence is discovered, the conditions under which these schemes could be constructed will be analyzed using the same procedural steps (E1-E5).

The Control of Variables Paradigm

The models which will now be presented are intended to predict children’s performance on a wide range of investigative problems requiring the control of variables. They will be derived, however, with reference to a specific problem called Bending Rods: an adapted version of a task originally designed by Inhelder and Piaget (1958). The details of the task administration will be presented later. For the moment, it is important only to mention that the task consists of three parts. In the first, children are presented with the series of rods illustrated in Figure 1; they are encouraged to discover all five of the independent variables which affect rod flexibility. In the second, they are asked to test each one of these variables individually; they receive one point for each test they conduct which is perfectly controlled. In the third, they are presented with a series of counter tests by the experimenter (five of which are uncontrolled); for every uncontrolled test to which they object (with an adequate explanation) they receive an additional point. The maximum score on the test is therefore 10.

Insert Figure 1 about here
Micromodel 1 — proof checking

The wording which is used for the counterquestion on each trial is as follows: "Would this be another fair way to prove it? Does this prove that (e.g. long) rods bend more than (e.g. short) rods?" If he were to answer this question with consistent success, it seems clear that a subject would need some general procedure for checking the explanation advanced by the experimenter, and then searching the perceptual array for any other reasonable explanation. The following executive routine could enable him to carry out such a procedure.

Step 1 - Isolate the rod which is supposed to bend more. (e.g. the long one).

Step 2 - Check to see if it really does bend more. (If so proceed, if not, respond "No, it doesn't prove that (e.g. long) rods bend more.")

Step 3 - Check to see if there is any other difference between the two rods which might be producing the greater bending. (If so, respond "No, it doesn't prove it," otherwise, respond "Yes, it does.")

In order to execute such a routine, a subject would need to activate the following figurative and operative schemes at each step.

Step 1 (Isolate the rod which is supposed to bend more)

\[ \text{Length} \]

A length operator corresponding to the rule: "If two objects differ in horizontal displacement, the one which projects the further is called the longer."

\[ \text{Chor.} \]

A figurative scheme representing the perceived difference in horizontal projection between the two rods.
In this first step, the operative scheme length would apply on the figurative scheme $\text{short, proj.}$ and generate a new scheme representing the fact that Rod A was the one which should bend more. The figurative scheme $\text{short, proj.}$ would be field-facilitated; the executive scheme and the operative scheme would not be, and would therefore require mental (M) facilitation. As a result, the M-power required for this step would be $\text{get1.}$

**Step 2** (Check to see if Rod A really does bend more)

$\text{\textgreater Rod A - more?}$ A figurative scheme representing the conclusion generated in the previous step, that Rod A (as the longer) should bend more.

$\text{\underline{$\text{\textgreater\text{quant.}$}:}}$ A quantity operator representing the rule: "If a rod projects (bend) below another, it may be said to bend more; otherwise it may not."

$\text{\underline{$\text{\textgreater\text{vert. disp.}$}:}}$ A figurative scheme representing the difference in vertical displacement of Rod B relative to Rod A (i.e. above or below it).

In this step the operative scheme $\text{\underline{$\text{\textgreater\text{quant.}$}}}$ would apply on the figurative schemes to generate the conclusion that the facts either were or were not as the experimenter had stated them. Since $\text{\underline{$\text{\textgreater\text{vert. disp.}$}}}$ would be facilitated directly by the field, the total M-power required would be $\text{get2.}$ Furthermore, since the executive scheme would continue to represent the subject's general intent—namely to check a proof of a positive relationship—the conclusion that the facts were not as described would be sufficient to terminate the routine and generate the response "No."

Assuming that this did not occur, the subject would advance to Step 3.
Step 3 (Check to see if there is any other relevant difference between
the pair of rods.)

Other dif. A difference operator corresponding to the rule: "If any
relevant difference other than ______ exists, label it."

Long-more A figurative scheme representing the conclusion generated in
the second step: that the long rod does bend more. This scheme
would in effect constitute a parameter necessary to the comple-
tion of the blank in Other dif.

Rod A A figurative scheme representing a relevant property (e.g. width)
of Rod A.

Rod B A figurative scheme representing the corresponding relevant
property of Rod B.

This final step would actually be accomplished by an iterative series
of substeps, in each of which a different relevant property of the rod
pair would be compared. Since the pretraining ensures that all the rele-
vant properties are salient, and since the apparatus is constructed such
that differences with regard to any property (when they exist) are easily
detected, the subject could run through this series of substeps with
little mental effort, simply by scanning his eyes back and forth between
the two rods, and allowing the content of his perception to be determined
by the most salient features in his perceptual field. The M-power required
would be e+3, since the executive scheme plus all four of the other schemes
would have to be activated in each substep, and since only one of the
schemes would be directly facilitated by the input at any particular time.

Given the goal inherent in the executive, the subject would object to
the proof whenever he detected a relevant difference between the
rods other than the one cited by the experimenter,
and he would accept the proof whenever he failed to find such a difference. The only time he would make an error would be when he failed to spot a relevant difference due to insufficient scanning.

Micro-model II - proof construction

The wording which is used for the first part of every trial is as follows: "Suppose I didn't believe that (e.g. long) rods bend more than (e.g. short) rods. Could you do a fair test to prove it to me?"

In order to answer this question with consistent success, a subject would need a general procedure for setting up a situation where the only relevant difference between the two rods was the one specified. The following executive routine would enable him to do this.

Step 1 - Select a long rod (visually).
Step 2 - Select a short rod for comparison.
Step 3 - Check to make sure the pair are identical in every other respect. (If not, return to Step 1 or Step 2).
Step 4 - Select equal weights.
Step 5 - Put one weight on the short rod.
Step 6 - Put the other weight on the long rod.
Step 7 - Make sure the weights are in the same position on their respective rods.

When each step in the above routine is analyzed in detail, it turns out that no step requires that a greater number of schemes be activated than Step 3. Since Step 3 in this sequence is identical to Step 3 in the proof-checking sequence, it may be concluded that the B-power required is also identical, namely e13. According to the general functional theory,
Therefore, the following conclusions may be generated. The majority of 7- and 8-year-olds should be able to pass the Trending Rule Test on their first exposure to it, provided (a) that their cognitive development is normal (cf. B2), (b) that they are high N-processors (cf. B3), (c) that they are relatively insensitive to any misleading cues the task may present (cf. B4), and (d) that they possess the required repertoire of schemes (cf. B1). It is this latter condition which must now be considered.

Is it at all reasonable to assume that children of 7 or 8 could possess a repertoire of schemes such as that which has been hypothesized?

There seems little doubt at all with regard to the figurative schemes which have been described. The familiarization period contains explicit perceptual training, and therefore serves as a check that the relevant perceptual features will in fact be part of the subjects' figurative repertoire (B1). There also seems little doubt that 7- and 8-year-olds should possess the appropriate repertoire of operative schemes. The existence of distance and quantity operators can be inferred in this age group from the work of Piaget (c.f. 1946) or from current work in developmental linguistics (cf. Harasym, Boersma, and Maguire, 1971). An operator such as other diff. can be inferred from children's success in performing tasks where they are simply asked, "Is there any other way X and Y are different?" The important question, then, is whether children of this age could be expected to have developed executive schemes of the sophisticated sort which have been postulated. If they understand that a proof is a "totally unambiguous demonstration" (i.e. one where only one independent variable is manipulated and all others are controlled), then it seems reasonable to assume that they could have constructed such executive
scheme, and that they would activate them when faced with the problem. The familiarization period would make it clear that the primary question was that made not head here, and the experimenter's wording ("Suppose I didn't believe you. Could you do a fair test to prove ... etc.") would be sufficient additional input to permit the subject to generate the appropriate routine. However, is it reasonable to assume that young children know (or could come to know) what it means "to prove something to a disbeliever." Is it reasonable to assume the existence of what is, in effect, a "control of variables" scheme?

Within the traditional Piagetian framework the answer to this question might appear obvious simply from a formal analysis of the structural properties of the scheme itself. Within the functional framework which has been advanced by Pascual-Leone, however, the question cannot be answered until one has also analyzed the nature of the experiences which might lead to the acquisition of such a scheme (according to the learning processes mentioned in II), and the demands which would be placed on a subject's functional system as a result.

III An Acquisition Model for the "Control of Variables" Scheme

Consider the following imaginary situation.

Gerry and Judy are both in the same fifth grade class. The boys in the class decide to have a relay race against the girls in the class. They win the race, but not by too much. Turning to Judy, Gerry says, "See, I told you the boys were better runners than the girls; now we've proved it." Judy replies,
"Oh come off it. You boys are not better runners. The only reason you won was because your team was wearing running shoes. You didn't prove a thing!" "Didn't prove a thing?", Gerry replies. "We won, didn't we?" "Yeah, but you can't be sure it was because you were better runners."

Suppose that a young child overheard this argument. It seems likely that he would conclude one of two things (a) Gerry is right: the boys must be faster runners (b) Judy is right: you can't be sure. He would not be able to sustain both conclusions since—in this case at least—the two are pragmatically incompatible.

The conditions that would determine which conclusion a young child formed will be discussed shortly. First, however, it is worthwhile to point out that a child who came to the second conclusion, and who had never been exposed to such a situation before, would, in effect, have actually constructed a preliminary form of the adult conception of proof. If he were to come to similar conclusions in other situations, one would expect his entire chain of reasoning to become consolidated (cf. 82(c)), and a "control of variables" or "proof" scheme to result. A child who arrived at the first conclusion repeatedly would also consolidate a notion of what it meant to "prove" something; however, the "control of variables" concept would not be an implicit part of this notion. On the contrary, the statement: "This proves that A causes B" would probably come to mean something of this order: "It is true that A causes B and you can plainly see A and B varying together in this instance."
A number of variables could be responsible for swaying a young child toward the incorrect, rather than the correct conclusion. The boys might look faster, and this salience factor might facilitate the conclusion that they must be faster runners. The child might have been taught that boys in general are faster runners, and this learning factor might also facilitate the second conclusion. Finally, the child might want to believe that boys were better runners, and this affective factor might facilitate the second conclusion. If salience, learning, and affective factors did not facilitate one conclusion or the other, however, one would expect that equilibration would ensue, and that the child would eventually favor the solution which was consistent with the greatest number of other schemes which he activated in thinking about the problem (cf. C7). Put differently, one would expect that the child would consider both the alternatives, and that he would eventually choose the one which was consistent with the greatest amount of relevant information. By this criterion, the conclusion which would be chosen would clearly be the second one, since it is consistent both with the facts pointed out by Judy, and with the facts pointed out by Gerry. However, before one can predict with certainty that a young child could actually arrive at this conclusion, one must consider the simplest chain of reasoning which could lead him to it, and the demands which this chain of reasoning would place on his S-power.

Mico-model III — acquisition of an adult "proof concept"

In order to conclude that the footrace didn't prove anything with certainty, the subject would need an executive scheme to direct his attention first to one protagonist's argument and then to the other's. The
simplified sequence of mental steps to which such an executive could give rise would be the following:

Step 1 - Conclude that Gerry's team could have won, even though they were worse runners, simply because they were wearing running shoes.

Step 2 - Conclude that Gerry's team could have won because they were better runners.

Step 3 - Conclude that, since there are two possibilities you can't be sure which one is true.

The following figurative and operative schemes would be required at each step.

**Step 1** (Conclude that Gerry's team could have won, even though they were worse runners)

*Cerry's worse* A figurative scheme representing the possibility suggested by Judy, that Gerry's team were actually somewhat worse runners.

*Shoes better* A figurative scheme representing another possibility suggested by Judy: that running shoes are a big help in a foot race.

Ycomp. A compensation operator representing the rule: "If two opposing effects oppose each other, the stronger will win."

At this step the operative scheme Ycomp. would apply on the other two schemes and generate the conclusion that--even though they were worse runners--Gerry's team could have won because of their special shoes.

**Step 2** (Conclude that Gerry's team could also have won because boys are better runners)
¿Gerry wins: A figurative scheme representing the conclusion generated in step 1.

¿Gerry's team boys: A figurative scheme representing the visible fact that Gerry's team is comprised entirely of boys.

Yecausé: A causality operator representing the rule, "If one event or fact precedes another, it may be labelled the cause."

The conclusion generated at this step would be that Gerry's team could have won simply because they were boys. Since ¿Gerry's team boys would be field-facilitated, the required E-power would be ¿1.2. Note that ¿Gerry wins boys worse would still be centrated.

Step 3 (Conclude that since there are two opposite possibilities, you can't be sure)

¿Gerry wins: A figurative scheme representing the conclusion generated in step 1.

¿Gerry wins boys worse: A figurative scheme representing the opposing conclusion generated in step 2.

Yecausity: A certainty operator representing the rule: "If there is only one possibility, you can be sure; if there are two opposing possibilities, you can't be sure."

At this final step, the operative scheme would apply on the two figurative schemes (neither of which would be field-facilitated) and generate the conclusion that you couldn't be sure whether or not the boys were better runners. Again, the required capacity would remain at ¿1.3.

From Piaget's work on causality and conservation (cf. 1930, 1952), the presence of operators such as Ycomp and Yecausé may be inferred in the
repertoire of normal 7- and 8-year-old children. The only other operator required is uncertainty, which is nothing more than an operational understanding of the meaning of "certain" or "sure." Given children's adequate use of this term in normal discourse, it also seems reasonable to assume its presence by the age of 7 or 8.

According to the general functional theory, therefore, the following three conditions should determine whether or not children could have actually followed the above thought sequence repeatedly.

1. Whether or not they had been exposed to indeterminate proof situations such as that illustrated above (cf. D1).

2. Whether or not they were cognitively normal, and old enough to have developed an M-power large enough to execute each step (cf. D2).

3. Whether or not they were field-independent: that is, whether or not they were habitually high M-processors (cf. D3), capable of resisting misleading cues (cf. D4).

If it were known that one of these conditions had not been met, then the prediction could be generated that the children in question should not have acquired the control of variables scheme. If it were known that all three of these conditions had been met, and, in addition, that children had actually gone (or been led) through a sequence of mental steps such as that itemized above, then it could be predicted that they should have acquired the control of variables scheme, that they should be able to generate the executive routines described in the first two micro-models, and that they should be able to apply these routines to problems such as bending the laws on their first exposure to them (thus passing the tests with ease).
The following section describes a preliminary test of these predictions.

Preliminary Experimental Finding

The experiment to be reported was conducted in three phases: pretesting, training, and posttesting. To eliminate the possibility of expectation effects, each of these phases was kept completely distinct. During each phase, children were interviewed by experimenters whom they had never seen before; the experimenters, in turn, were given no prior knowledge about any child.

Pretesting Phase

The main object of the pretesting phase was to select a group of 7- and 8-year-olds who were undeniably "normal" according to Piaget's scale of cognitive development, who could clearly be regarded as "high N-processors," and who would not be taken in by the misleading factor which appears to be present in the control-of-variables tasks (see footnote 9). In short, the main object was to obtain an experimental group which clearly satisfied conditions 2 and 3 above. A secondary object was to select two control groups: one which met condition 2 but not 3, and one which met condition 3 but not 2.

Criterion Measures. The measures used to assess cognitive development were conservation of substance and conservation of weight. The particular procedure employed was as follows. Each subject was shown two balls of clay, and asked to make them equal in amount (or weight, depending on the problem). After he had done this, one ball was rolled into a sausage, and the subject was asked whether or not he thought the two still contained the same amount (or weighed the same). An explanation was requested, and
answers were scored either 0 or 1, depending on their adequacy. The procedure was repeated a second time with one ball being deflected into a pancake. Again, subjects were scored either 0 or 1 depending on their answer and explanation. Finally, subjects were presented with a counter-suggestion, that is, they were given the opposite argument from the one they had advanced and were asked to react to it. Once again, their answers were scored either 0 or 1, depending on their adequacy. The exact criteria used for scoring, and a more detailed description of the standardized procedure, are available in Pascual-Leone (1969). Subjects who received 3/3 or 2/3 were classified as conservers, subjects who received 0/3 or 1/3 were classified as non-conservers.

The measure used to assess field independence was the WISC blocks. The theoretical reason for this choice was that each item demands both that a salient feature of the display (the overall pattern) be temporarily ignored, and that a series of mental transformations with a high N-demand be executed (cf. D3, D4). There were also empirical and practical reasons for the choice of the measure. In factor analyses, the WISC blocks has been shown to exhibit minimal loadings on factors defined by verbal I.Q. items or by Piagetian items without misleading cues, and to show a high loading on factors defined by the Rod and Frame Test (cf. Goodenough & Karp, 1961; Pascual-Leone, 1969; Case & Globerson, 1974). Finally, the WISC blocks has the advantage of being easy to administer and score. Subjects who scored one standard deviation above the mean on national norms were classified as field independent; subjects who scored one standard deviation below the mean were classified field dependent.
Subjects. Subjects were obtained through the cooperation of three elementary schools in the San Francisco Bay area, two of which served a high SES area, and one of which served a mixed-SES area. All students in the first three grades were tested, providing they were between the ages of 5.6 and 6.7, or 7.6 and 8.7. Three groups of subjects were then selected, according to the following criteria.

Group 1 (n=20) 8 years old, field independent, and cognitively normal by Piaget's standards (i.e. conservers on substance, non-conservers on weight).

Group 2 (n=16) 6 years old, field independent, and cognitively normal by Piagetian standards (i.e. non-conservers on both weight and substance).

Group 3 (n=16) 8 years old, field dependent, and cognitively normal by Piagetian standards (i.e. conservers on substance, non-conservers on weight).

Design. Within each group, subjects were randomly assigned to one of two conditions: instruction or no instruction. The descriptive data for each group are presented in Table 1.

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Insert Table 1 about here

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Teaching Phase

The object of the teaching phase was to provide half the subjects with the opportunity to develop and consolidate the executive schemes which were
Case

described in the second section. This was done by exposing them to a situation analogous to the footrace example, and leading them through the set of operations necessary for understanding the impossibility of being "sure" about what had produced the result; then by presenting them with a variety of similar situations in which—with the aid of the experimenter's probing questions—they could convert this newly acquired insight into a well practiced routine for setting up a fair proof or for checking the adequacy of someone else's proof.

Procedure. Teachers were provided with a set procedure, and were instructed to adhere to it rigorously. The only modifications permitted were the rephrasing of any sentence which a child did not appear to have understood, and the repetition of any question which a child did not attempt to answer. The total training period was divided into four separate sessions, each of which was conducted on a different day. The activities for each day were as follows.

On the first day, each child was presented with the set of rods and blocks illustrated in Figure 2a. After some practice in weighing rods on a balance, he was asked to see if he could figure out which of the two kinds of rods weighed more, without taking the rods and blocks apart. Due to the misleading nature of the display, all children chose an uncontrolled pair for comparison. Since the dark blocks had been weighted with concealed metal inserts, every child concluded that the aluminum rods must weigh more than the brass ones. The experimenter then removed the rods, showed the child his error, and demonstrated that if he had picked a pair where the blocks were identical, he would not have been fooled. Six more trials followed using different kinds of rods but the same blocks. Any time the

Insert Figures 2 & 3 about here
child conducted an uncontrolled test, the experimenter asked, "How do you know this pair is heavier because of the rod. Couldn't it just be because of the block?" The average duration of the first session was approximately 18 minutes.

On the second day, the first demonstration was repeated, using the altered array illustrated in Figure 2b. Then the child was shown two balls of similar appearance (a squash ball and a handball) and asked which he thought was the "bounciest." The experimenter did several tests, leaving a different factor uncontrolled each time (height of dropping, force of dropping, material of floor). Each time he asked the child if the test had been a fair one, and, if so, what it proved. The standard question (How do you know it didn't (e.g., bounce higher) because it was (e.g., thrown harder)?) was asked after any error. The child was also asked to do a few tests himself, and the standard question was repeated if any test was uncontrolled. The approximate average duration of this session was 14 minutes.

On the third day, each subject was presented with a factorial set of eight small rollers varying in material, external diameter, and internal diameter. He was given five minutes to determine which of the eight rollers would roll down an inclined plane the fastest. Then he was asked if he thought each factor "made a difference," and, if so, to do a fair test to prove it. Again, the standard question was asked after any uncontrolled test. The experimenter then did several negative countertests of his own, again asking the standard question if the subject agreed that the test was a good one. The experimenter also provided a verbal definition: "If you can't be sure it happened, then it isn't a fair proof." The approximate
average duration of this session was 11 minutes.

On the fourth day, the red and black demonstration was reviewed, using the visual array indicated in Figure 2c. Then subjects were shown an array of chips varying along the following dimensions: material, diameter, position of hole in center, and size of hole in center. They were asked to test three of these factors separately, to see if each affected the speed with which a chip would sink down a long plexiglass tube filled with water. The standard question was asked following incorrect tests, and a countertest was presented by the experimenter after each test of the child’s. Any failure to detect the uncontrolled dimension was again followed by the standard question. The approximate duration of this session was 14 minutes. The apparatus is illustrated in Figure 3.

Teachers. Two different teachers were used, one male and one female. Each was a graduate student in education, with some previous experience in teaching young children. Within each school, each teacher taught approximately half the subjects in each experimental cell.

Posttesting Phase

The first object of the posttesting phase was to determine whether subjects could transfer what they had learned to new situations, when these were presented to them by new experimenters. The second object was to determine whether subjects could retain what they had learned over a two-month period. The third and final object was to determine whether any subjects in the field independent, 8-year-old groups could be said to possess the "combinational structure" described by Inhelder & Piaget (1958).

Procedure. The first week after the teaching had been completed, each child (both instructed and non-instructed) was tested individually on
Bending Rods, and on a parallel test called Spinning Bells. Each test took about 15 minutes to administer, and each was presented on a different day. Order of presentation was counterbalanced.

During the seventh and eighth weeks after the teaching was completed, each child was retested on Bending Rods. The field independent 7- and 8-year-olds were also tested on a variation of Inhelder & Piaget's Combinations problem. The details of the test administration are summarized below.

Criterion tests. The apparatus for Bending Rods has already been described (see Figure 1). Each subject was seated facing the rod-holder "end on," with the experimenter seated beside him and the scoring sheet concealed behind the apparatus. The task was introduced as follows.

"These are supposed to be fishing rods, and these are supposed to be fishing traps. First of all, I'd like you to put some traps on this rod so that the traps are in the water, like this (demonstrating), and so that the rod is just touching the water, like this (demonstrating)." Since the rod which the experimenter selected was quite stiff, the subject had to place several traps on it before he succeeded in getting it to touch the water.

When he had finally succeeded, the experimenter asked, "Why do you think it bent more when ______ traps were on it than when only one trap was on it?"

After the subject had answered, the experimenter either asked him to suggest another reason (if he was wrong) or continued as follows, "Good, it could be that the more weight you put on a rod, the more it bends. What about this rod (Identical except for material)? Can you get it to touch the water?" When the subject had succeeded, the experimenter asked, "Why do you think it bent more easily?" Once again, the experimenter reinforced and repeated the subject's answer, if it was correct (e.g. because it is wooden), or else probed and guided the subject until he came up with another
answer. In the same fashion, the subject was led to the discovery of the remaining three variables, namely length, diameter, and "point of leverage." At the end of the pretraining, the experimenter summarized the subject's findings, using the subject's own wording as much as possible.

The subject was introduced to the testing as follows. "OK, now let's do something a bit different. Suppose that I don't believe that long rods bend more easily than short rods. What could you do to prove it to me? Show me a fair test to prove that long rods bend more than short rods." After the subject had performed an experiment, any dimensions he failed to control were noted, and the experimenter did an uncontrolled experiment, saying, "OK (placing medium weight on rods #1 and #4). Would this be another fair way to prove it? Does this prove that long rods bend more than short rods?" If the child replied that it did not, the experimenter asked, "Why not?"

The same procedure was repeated for each of the five variables: the child was asked to do a test of his own and then the experimenter presented a negative countersuggestion. The negative countersuggestions were constructed with the following constraints in mind. (1) One variable, and only one variable was uncontrolled. (2) Although one variable was uncontrolled, the observed effect never provided a clue that this was the case (e.g., the long rod did not bend less than or the same as the short rod). (3) The variable which was uncontrolled was never one which the subject had investigated in the immediately preceding trial. (4) In the course of the five trials, each of the five variables was uncontrolled once.
Finally, at two points during the testing period (Trial 1, after the negative countsuggestion, and Trial 3, before the negative countsuggestion), the experimenter also presented a positive countsuggestion. Since the purpose of these positive countsuggestions was simply to prevent the subject from developing a set to object to any test suggested by the experimenter, his response to these questions was recorded but not actually scored. No feedback was provided as to the "correctness" of any response, even if such feedback was requested.

The subject received 1 point for every test of his own which was perfectly controlled, and 1 point for every negative countsuggestion to which he objected with an adequate explanation. The criterion for adequacy was that the subject label the uncontrolled variable, and make some mention of its possible confounding effect. Since there were five independent variables to be tested, the maximum total score was therefore 10. A score of 8/10 or better was used as a conservative criterion for success; a score of 5/10 or better was used as a liberal criterion for success.

The Spinning Wheels task was formally identical to Bending Rods. The only differences were (1) in the variables investigated, and (2) in the nature of the familiarization period. In the Spinning Wheels test, the experimenter never presented subjects with a situation during the introduction where every possible variable but one was controlled.

The Combinations test which was administered is based on the one originated by Piaget & Inhelder (1951) and is described in detail by Pascual-Leone (1969). Children were told a story about alchemists, and presented with chips of five different colors (supposedly representing five different minerals). They were told that the object was to figure out every
co-hinitie which might possibly produce gold when heated in an oven. After six minutes in which manipulation of the chips was permitted, subjects were asked to write down (or dictate) all the combinations they had found.

Testers. Two posttesters were used, both of whom were female. Within each school, each tester tested approximately half of the subjects in each of the six experimental cells.

Predictions

It will be remembered that—according to the models developed in Sections II and III—the following conditions were held to be necessary and sufficient for consistent success (across problems and over time) on control of variables problems.

1. An appropriate repertoire of schemes (in particular, an appropriate executive).

2. An N-power of at least e+3.

3. A tendency to use this N-power to its fullest, and not to be distracted by any misleading cues.

Given the basic postulates of Pascual-Leone's theory, the following predictions were therefore advanced.

1. The field independent, 8-year-old, instructed group would pass both the immediate and delayed posttests involving the control of variables, even though they failed the test of combinations. This prediction was made because this group satisfied all three of the above conditions.

2. The field independent, 6-year-old, instructed group would not pass any of the immediate or delayed posttests, even though they had been exposed to the treatment. This prediction was made because this group failed to satisfy condition 2.
3. Some members of the field independent, 8-year-old, uninstructed group might pass the immediate posttests. Whatever this proportion, it would not decrease on the delayed posttests and, in fact, might well increase. This prediction was made because this group satisfied conditions 2 and 3, and its status with regard to condition 1 was unknown. On the delayed posttest, it was at least known that the group had had two occasions to construct an appropriate executive scheme, namely those occasions provided by the immediate posttests.

4. The performance of the field dependent, 8-year-old, instructed group would be intermediate between that of the other two instructed groups. This prediction was made because of the presumed failure of this group to satisfy condition 3.

Results

As may be seen from an inspection of Tables 2 and 3, the pattern of results was exactly as predicted.

A. For the instructed group:

1. Even by the most conservative criterion, eight of the ten field independent 8-year-olds passed each of the immediate posttests. This proportion showed no decrease on the delayed posttest.
2. Even by the most liberal criterion, only one of the ten field independent 6-year-olds passed either of the immediate posttests. This proportion showed no increase on the delayed posttest.

3. By either criterion the proportion of field dependent 8-year-olds who passed the immediate posttests was lower than the proportion of field independent 8-year-olds who passed. This pattern was maintained on the delayed posttest and, as is shown in Table 4, was statistically significant even when differences in verbal I.Q. were controlled through analysis of covariance.

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Insert Table 4 about here
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B. For the uninstructed groups:

1. On the immediate posttests, no group achieved a majority of successes by the conservative criterion; however, by the liberal criterion, and on the Spinning Wheels test, six of the ten field independent 8-year-olds succeeded.

2. On the delayed posttest, six of the ten field independent 8-year-olds succeeded, even by the most conservative criterion. This proportion was significantly higher than that achieved by either of the other two groups (Fisher Exact Test: \( p = .053 \) 8 yrs. FD; \( p = .026 \) 6 yrs. FI).

C. On the Combinations Test:

1. Neither of the field independent 8-year-old groups showed any evidence of a combinational procedure.
2. There was no significant difference between the instructed and uninstructed groups in the mean number of combinations generated (14.4/41: instructed; 13.9/41: uninstructed).

V. Discussion

At the beginning of this article, it was pointed out that Pascual-Leone's theory of development is not intended to replace Piaget's, but merely to make it functional. Its goal is to provide an adequate account of the mechanisms by which Piagetian competencies are acquired and utilized. Given the nature of the results which were successfully predicted, however, it seems worthwhile to raise the question of whether the two theories are really as compatible as was claimed. Can Piagetian theory accommodate itself to the fact that certain formal problems can be solved with insight before certain concrete problems: that a formal substructure can be acquired at the beginning of the period of concrete operations, and before the acquisition of the formal superstructure of which it normally forms a part (the combinational system)?

The answer appears to be that it can.

The traditional Piagetian position with regard to structural learning is quite simple. It acknowledges that such learning can occur, provided (a) that subjects already possess the structure in question, at least in some preliminary form, and (b) that the external conditions are such as to produce internal disequilibrium (cf. Piaget, 1964; Inhelder & Sinclair, 1971). Since both these conditions appear to have been satisfied in the present study, it is not difficult to explain the general form of the results from a traditional Piagetian perspective: The group which had a preliminary grasp of the structure consolidated it; the groups which had
no preliminary grasp of the structure made no progress. The only question which remains is how the field independent 7- and 8-year-olds managed to acquire a preliminary grasp of the control of variables structure to begin with. Here the answer appears to lie, as with the theory proposed by Pascual-Leone, in the distinction between an analysis which is purely structural and an analysis which also considers the operations by which structures are acquired and utilized. From a purely structural point of view, the control-of-variables scheme is indeed a formal one. As Inhelder and Piaget have pointed out (1958, p. 62), it involves the whole interpropositional combinatorial system. However, from a functional-structural point of view, the same scheme could be considered either formal or concrete, depending on the operations which were involved in its acquisition and utilization.

To appreciate how the control-of-variables scheme could be acquired by means of concrete operations, consider the footrace example once again. The understanding that the race does not constitute an unequivocal proof depends largely on the realization of one simple fact: that the boys could have won even though they were slower runners, due to the assistance they received from their special running-shoes. In turn, the realization of this fact depends on one simple mental operation: a multiplication of inverse relations (\(A \times B = AB\) where \(A\) represents the effect of the boys' bodies, \(B\) represents the effect of their shoes, and the arrows represent the magnitude and direction of these effects). Since this operation is by definition a concrete one (Piaget's Grouping VII) one would expect that repeated exposure to situations such as the footrace argument would lead to a "concrete" control of variables scheme.
To appreciate how this same scheme could be utilized to solve more complicated proof problems, consider the fact that although a subject may have to select a pair of objects in which four extraneous variables may be uncontrolled, he does not necessarily have to think of all four of these variables simultaneously. He can consider each possible pair of objects as though it were constituted by a number of sets of simple two-variable items. If the task is to establish the effect of A in the face of other possible effects BCDE, then any salient uncontrolled dimension in the potential object pair constitutes a valid (and adequately understood) reason for rejecting that pair. In exactly the same manner as he rejected the footrace as a proof, the subject who notices that (for example) D is uncontrolled, can therefore justify a rejection of the proof. He can simply reason that the positive result may not be due to the positive effect of A, but rather to the over-compensating effect of D (A
\* x D\* = AD?).

Clearly, the distinction between the structural aspects of a theory and its functional aspects is an important one. For, although Piagetian theory can explain the results obtained in the present study, it would certainly not have predicted them. In the absence of a sharp distinction between structure and function, the most obvious prediction to have made from Piagetian theory would have been just the opposite from that which was made on the basis of Pascual-Leone's theory. It would have seemed much more obvious to predict that the control of variables scheme could only be acquired by assimilation to, or in conjunction with, the global logical structure of the formal operational period. In fact—although it is by no means central to their theory—this is exactly the sort of assertion
Inhelder & Piaget have occasionally made in the past.

Whereas the stage II [concrete] subject compares any rod whatever to any other, limiting himself to a statement of the most obvious relations, the stage III [formal] subject understands that if he is to establish a given relationship, it is important to select certain pairs of rods rather than others. (Inhelder & Piaget, 1958, p. 58). Only at 14-15 years can subjects spontaneously organize and utilize [this method of verification] without error. (ibid., p. 60). ... the process of verification, based on the schema "all other things being equal," is complex and actually involves the whole interpropositional combinatorial system (ibid., p. 62).

Interviewer: What if the teacher were to demonstrate the experiment [i.e. the verification method] to the class?

Piaget: It would be completely useless. The child must discover the method for himself.

... (Hall, 1970, p. 30).

Two conclusions, therefore, may be generated on the basis of the results obtained in the present study.

1. The first is that the acquisition of any particular item of knowledge does not depend on the match between the formal structure of that knowledge and the formal structure of the knowledge which the child already
possesses. Rather, it depends upon the match between the pragmatic structure of the situation in which the child first has a chance to construct that particular item of knowledge, and the functional limitations of his thought processes at the state in his life when he first encounters such a situation.

2. The second is that, because Pascual-Leone's theory of development concentrates more heavily on functional mechanisms than does Piaget's, it is capable of generating performance models of somewhat greater predictive power.

In the present study it was shown capable of predicting the age at which a particular structure can be constructed on the basis of a particular sort of experience, as well as the kind of subject for which this is true. In other recently completed studies it has also shown capable of predicting the presence or absence (a) of horizontal décalage (cf. Pascual-Leone & Smith, 1969; Pascual-Leone, 1973), and (b) high inter-task correlations among stage-related tasks (cf. Toussaint, 1972, in press). Given the difficulties which such phenomena have traditionally presented for developmental psychology in general, and Piagetian theory in particular, the functional approach suggested by Pascual-Leone's theory seems particularly promising.
REFERENCES


Miller, G.A. The logical number seven, plus or minus two: Some limits on our capacity for processing information. Psychological Review, 1956, 63, 81-97.


FOOTNOTES

1. The pilot testing for this project and the preparation of the original proposal were supported by a grant from the Committee on Research of the University of California, Berkeley. The author is indebted to Tamar Globerson and Bart Nödy for their help in all phases of the present study; to Bob Kahn, Meg Korpi, and Wayne Haserot for their help in collecting the data; to Paul Ammon, John Biggs, John Flavell, Juan Pascual-Leone, Alan Newell, and William Rohrer for their comments on a previous draft of the manuscript; and to Bev Nash for her help in preparing it.

2. The text of this report is identical to an article which will appear in Cognitive Psychology, 1974 (Fall). The material in the appendix, however, does not appear in the article and is included for the benefit of those who might wish a more detailed understanding of the procedures.

3. Although this is the intent, the construct clearly bears an even more striking correspondence to what information theorists have labeled "working memory" (cf. Norman & Lindsay, 1972).

4. Since this scale plays a central role in the theory, it is worthwhile mentioning that considerable evidence has already been obtained in its support (cf. Parkinson, 1969; Pascual-Leone, 1970; Case, 1972, 1974; Toussaint, 1972, in press; DeAvila, 1973; Scardamalia, 1973, 1974).

5. For data which support this interpretation, see Pascual-Leone (1969), or Case & Globerson (1974).

6. It is axiomatic to Piaget's theory, for example, that knowledge may be classified as "figurative" or "operative," and that the formation of new structures depends as much on a child's general level of "operativity" as it does on his specific experience (Piaget, 1970).
7. For the balance of this paper, the symbol $p.A. (Y)$ will be used to denote an operative scheme; the symbol $\phi (\omega)$ will be used to denote a figurative scheme.

8. The necessity for the constant activation of $\phi$ long more stems from the fact that the subject's search is field-directed. Since the subject is, in effect, scanning for any salient difference whatever between the two rods, he must remember which difference, when detected, does not call for a rejection of the proof as unfair.

9. The factor which might be misleading is the criterion question itself, which explicitly draws the subject's attention to the relationship between the two variables mentioned (e.g. length and bending) and thus renders the variables not mentioned (i.e. the crucial ones) less salient.

10. Note that if subjects did not have a tendency to activate all the schemes relevant to the conflict (C7) or if they had insufficient $M$-power to "carry forward" $\phi$ Gerry wins, precisely the opposite conclusion would be generated.

11. The importance of controlling for expectations in Piagetian research is discussed by Kamii & Derman (1971).

12. Since the criterion for field dependence was a score at least one S.D. below the mean on the WISC blocks, readers in the psychometric tradition might prefer to label this group as being low in "analytic intelligence." They should note, however, that the group was still well above average in verbal intelligence as indicated by their scores on the Stanford-Binet (see Table 1).

13. A detailed account of this procedure is presented in Appendix A.

14. "Point of leverage" refers to the distance from the baseboard to the point at which a weight is placed. In Inhelder & Piaget's (1958) original procedure, crossectional shape was used as the fifth independent variable;
the change in the present experiment was introduced purely for reasons of convenience.

15. The dependent variable was the relative length of time two marbles remained on a spinning wheel. The independent variables were the size, shape, and material of the marbles, the size of the holes on the wheel (in which the marbles were placed), and the distance of these holes from the center of the wheel.

16. The performance of the high M-power, 8-year-old, uninstructed group, particularly on the second posttest, may be taken as evidence for some preliminary presence of a "control of variables" structure; the constant introduction of an opposing interpretation by the experimenter may be presumed to have produced "disequilibrium" in the instructed group.
Table 1

Mean Scores of Each Group
(and Standard Deviation)
On Each Measure Administered

<table>
<thead>
<tr>
<th>Condition</th>
<th>Age (Years; months)</th>
<th>WISC Blocks</th>
<th>Conservation Substance</th>
<th>Conservation Weight</th>
<th>Stanford-Binet I.Q.*</th>
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<tr>
<td>7-8 Field Independent</td>
<td>8; 1</td>
<td>28.9</td>
<td>3</td>
<td>.4</td>
<td>127</td>
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<tr>
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<td>(6.2)</td>
<td>(.0)</td>
<td>(.5)</td>
<td>(12.0)</td>
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<td><strong>Treatment</strong></td>
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<td>8.0</td>
<td>2.8</td>
<td>120.5</td>
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<td>(3.1)</td>
<td>(.5)</td>
<td>(.5)</td>
<td>(18.6)</td>
</tr>
<tr>
<td>5-6 Field Independent</td>
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<td>16.0</td>
<td>0</td>
<td>0</td>
<td><strong>130.1</strong></td>
</tr>
<tr>
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<td>(5.2)</td>
<td>(0)</td>
<td>(0)</td>
<td>(11.2)</td>
</tr>
<tr>
<td>7-8 Field Independent</td>
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<td>3</td>
<td>.4</td>
<td><strong>129.8</strong></td>
</tr>
<tr>
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<td>(5.0)</td>
<td>(0)</td>
<td>(.5)</td>
<td>(18.1)</td>
</tr>
<tr>
<td><strong>Control</strong></td>
<td>7-8 Field Dependent</td>
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<td>3</td>
<td><strong>117.6</strong></td>
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<td>(0)</td>
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<td>(14.9)</td>
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<td>.16</td>
<td>.16</td>
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<td>(.4)</td>
<td>(.4)</td>
<td>(12.4)</td>
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*data available for 46/52 subjects only
**cell mean within school used to estimate missing data
Table 2
Percentage of Subjects Passing the Immediate Posttests (N = 52)

(a) Bending Rods

<table>
<thead>
<tr>
<th>Subject Characteristics</th>
<th>Instructed</th>
<th></th>
<th>Uninstructed</th>
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<td>Liberal criterion (5/10)</td>
<td>Conservative criterion (8/10)</td>
<td>Liberal criterion (5/10)</td>
<td>Conservative criterion (8/10)</td>
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<tr>
<td>Field independent, 7-8</td>
<td>90</td>
<td>80</td>
<td>40</td>
<td>20</td>
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<tr>
<td>(n = 10)</td>
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<td>Field dependent, 7-8</td>
<td>50</td>
<td>13</td>
<td>25</td>
<td>25</td>
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<tr>
<td>(n = 8)</td>
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<tr>
<td>(n for treatment = 10; n for control = 6)</td>
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(b) Spinning Wheels

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<tr>
<th>Subject Characteristics</th>
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<td>Conservative criterion (8/10)</td>
<td>Liberal criterion (5/10)</td>
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<td>(n for treatment = 10; n for control = 6)</td>
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Table 3
Percentage of Subjects Passing the Two Month Delayed Posttest

| Subject Characteristics | Instructed | | | Uninstructed | | |
|-------------------------|-----------|-----------|-----------|-----------|-----------|
|                         | Liberal   | Conservative | Liberal   | Conservative |
| Field independent, 7-8  | 100       | 90         | 70        | 60         |
| Field dependent, 7-8    | 50        | 25         | 38        | 13         |
| Field independent, 5-6  | 10        | 0          | 16        | 0          |
Table 4
Analysis of Covariance for 8-year-old Groups:
Effect of I.Q. Eliminated

<table>
<thead>
<tr>
<th>Measure</th>
<th>Source</th>
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<th>$F_{(3,33)}$</th>
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<tr>
<td>Bending Rods (Immediate Posttest)</td>
<td>Field Dependence</td>
<td>28.1</td>
<td>4.17*</td>
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<tr>
<td></td>
<td>Instruction (I)</td>
<td>52.1</td>
<td>7.7**</td>
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<td></td>
<td>FD x I</td>
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<td>2.3</td>
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<td>FD</td>
<td>68.7</td>
<td>9.4**</td>
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<tr>
<td></td>
<td>I</td>
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<td>I</td>
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<td>7.2*</td>
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<td></td>
<td>FD x I</td>
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<td>.51</td>
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*p<.05
**p<.01
Day 1

E: THIS IS A BALANCE, DO YOU REMEMBER HOW IT WORKS?
S: YES.
E: O.K. CAN YOU USE IT TO TELL ME WHICH WEIGH MORE, GRAY RODS OR WHITE RODS?
   (E places 2 wood and 2 steel rods in front of S).
S: (Putting them on the balance.) THE GREY ONES WEIGH MOST.
E: GOOD, NOW HERE'S A TRICKY QUESTION. (E places the blocks in a misleading position in front of him). WHICH WEIGH MORE, ORANGE RODS OR SILVER RODS? (If S starts to take them apart, tell him he is not allowed. Make sure he also uses the balance).
S: THE SILVER.
\* E: SO YOU THINK THE SILVER RODS WEIGH MORE, LET'S SEE IF YOU'RE RIGHT. WHICH SIDE SHOULD GO DOWN?
S: This one
E: (After demonstrating). HOW DO YOU THINK I FOODED YOU?
S: 
E: THAT'S RIGHT (or if S cannot figure it out) FEEL THESE BLOCKS. I FOODED YOU BECAUSE THIS BLOCK WAS SO HEAVY THAT IT PULLED THE BALANCE DOWN (gesture) AND MADE THE SILVER ROD LOOK HEAVIER, EVEN THOUGH IT WASN'T.
E: NOW I'M GOING TO ASK YOU A TOUGH QUESTION. CAN YOU THINK OF THE WAY TO DO THE TEST, SO YOU WON'T GET FOODED BY THE BLOCKS.
   (If S says "take them apart" say "GOOD! IS THERE ANY OTHER WAY?"
   If he figures it out, praise him otherwise, read next bit).
E: YOU SHOULD PICK TWO WHERE THE BLOCKS ARE THE SAME. (Demonstrating) SEE, WHEN THL BLOCKS ARE THE SAME, (Point) IT DOESN'T FOOL YOU. THE SILVER ONE DOESN'T LOOK HEAVIER.
E: (Taking them apart and replacing them on scale). NOW PAY ATTENTION CAREFULLY AND I'LL EXPLAIN WHY THE BLOCKS HAVE TO BE THE SAME.
Step ( ) E: (Put rods on. Then ask). WHICH IS HEAVIER?

S: ORANGE.

( ) E: RIGHT. (Put some blocks on – say) SEE. WHEN THE BLOCKS ARE THE SAME THE ORANGE ONE STILL LOOKS HEAVIER. THE BLOCKS DON'T FOOL YOU.

( ) E: EVEN IF I USE THESE TWO, IT DOESN'T FOOL YOU BECAUSE THEY'RE STILL THE SAME.

( ) E: BUT WHAT HAPPENS WHEN I PUT TWO DIFFERENT ONES ON. SEE, IT DOES FOOL YOU. IT MAKES THE SILVER ONE LOOK HEAVIER, EVEN THOUGH IT ISN'T.

E: IT ALWAYS WORKS THAT WAY, IF YOU MAKE THE BLOCKS THE SAME, THEN THEY CAN'T FOOL YOU. YOU CAN TELL WHICH ROD IS HEAVIER. BUT IF YOU DON'T THEY CAN FOOL YOU.

E: NOW SHUT YOUR EYES, AND I'LL CHANGE THINGS AROUND TO SEE IF I CAN FOOL YOU AGAIN.

#2 BRASS VS. SILVER
#3 BRASS VS. COPPER
#4 BRASS VS. WOOD
#5 WOOD VS. STEEL
#6 STEEL VS. BRASS
#7 STEEL VS. COPPER

If S makes a mistake at any time, recycle to *. If S goes for one particular color at any time, extract one of them. Discontinue when 7 are finished or when 20 minutes are up and note the number of correct vs. incorrect items S obtained.
E: DO YOU REMEMBER WHAT WE DID WITH THE BLOCKS?

S: YES.

E: (If S failed). WELL, WE'RE GOING TO TRY A FEW MORE WITH THEM.

E: (If S passed). WELL, TODAY I HAVE THEM FIXED SO I'M ALMOST CERTAIN I CAN FOOL YOU. DO YOU THINK I'LL BE ABLE TO? (smiling)

S: YES OR NO

E: O.K. WELL LET'S SEE. WHICH WEIGHS MORE?

1. COPPER VS. STEEL
2. BRASS VS. STEEL
3. BRASS VS. ALUMINUM
4. STEEL VS. ALUMINUM
5. BRASS VS. WOOD

(In each of the above examples, place the underlined rod at the base in the formation shown in figure 2b. Alternate between a dark and light block at the base).

(If S picks two where the blocks are different recycle to the old explanation. If he is correct, praise him and go directly to the new example. If he picks two where the rods are the same, continue).

E: LISTEN TO MY QUESTION AND LOOK AT THE RODS. WHICH WEIGHS MORE

__________ ONES OR ___________ ONES? (If he does not realize his error after a few repetitions, ask) I SAID WHICH WEIGHS MORE

__________ ONES (pointing) OR ___________ ONES? (pointing).
IS THAT A ____________ ONE? NO. IT ISN'T. YOU HAVE TO BE CAREFUL TO ANSWER MY QUESTION. (Replace pattern as was and repeat). NOW, WHICH WEIGHS MORE ____________ ONES OR ____________ ONES?

(2) E: NOW LET'S TRY SOMETHING DIFFERENT. FIRST OF ALL, FEEL THESE TWO BALLS. (Giving them to $S$). DO THEY FEEL THE SAME?

$S$: NO

E: (Taking them back). RIGHT (or not quite) THE RUBBER IS NOT QUITE THE SAME. THE RUBBER IN THIS ONE IS HARD AND THE RUBBER IN THIS ONE IS SOFT. WHICH KIND OF RUBBER DO YOU THINK BOUNCES BETTER, HARD RUBBER OR SOFT RUBBER?

* E: O.K. I'M GOING TO DO A TEST. (Drop one on carpet, one on sponge; both from same height. Make sure the good bouncer hits the bad material. Hold them in your hands after you catch them, at the appropriate height). THERE, WAS THAT A FAIR RACE? DID THAT PROVE THAT THIS RUBBER (indicating) BOUNCES BETTER THAN THIS RUBBER?

If $S$ is correct follow (1) below, if not, follow (2) below.

(1) * E: WHY NOT?

$S$: YOU BOUNCED IT ON THE PILLOW.

E: WHAT'S WRONG WITH THAT?

$S$: ?

E: GOOD, IT'S NOT FAIR BECAUSE THE PILLOW COULD BE MAKING IT LOOK LIKE A BAD BOUNCER, EVEN THOUGH IT'S NOT. LET ME TRY AGAIN.
E: Continues from star for
(2) height uncontrolled
(3) force uncontrolled
(4) fair test

E: NOW I WANT YOU TO DO A TEST (Handing him 3 balls). SHOW ME THAT BALLS
WHICH ARE DROPPED FROM HIGH UP BOUNCE MORE THAN BALLS WHICH ARE DROPPED
FROM LOW DOWN.
(If S gets it praise him, and terminate session, if not say:

E: OH, OH, THAT WASN'T A FAIR TEST, HOW DO YOU KNOW THAT IT DIDN'T BOUNCE
LOWER BECAUSE OF THE RUBBER, NOT BECAUSE OF THE PLACE IT STARTED FROM?

(2) E: ARE YOU SURE THIS ONE BOUNCES HIGHER?

S: YES

E: YOU TRY. (removing pillow).

E: HOW COME IT BOUNCED HIGHER THIS TIME? (With prompting). BECAUSE IT
DIDN'T DROP ON THE PILLOW. YOU SEE, IT WASN'T A FAIR TEST BECAUSE THE
PILLOW WAS SLOWING THIS ONE DOWN.

Repeat test
(2) height uncontrolled
(3) force uncontrolled
(4) fair test

As soon as S gets one right, shift back to *.
E: TODAY WE'RE GOING TO BE WORKING WITH THESE ROLLERS, BUT, FIRST OF ALL, I'D LIKE YOU TO SEE IF YOU CAN FIGURE OUT WHICH ONE IS THE FASTEST. HERE, YOU TAKE THEM, ROLL THEM DOWN THE HILL, AND SEE WHICH ONE IS THE FASTEST.

(Answer any questions S may have at this point about how he is allowed to do it)

E: TELL ME, WHICH DO YOU THINK ARE FASTER, THE GREY ONES OR THE ORANGE (brown) ONES? (Wait for S to respond). AND WHICH DO YOU THINK ARE FASTER, THE BIG ONES OR THE LITTLE ONES? (Wait for S to respond). AND WHICH DO YOU THINK ARE THE FASTEST, ONES WHICH ARE FILLED WITH WAX OR ONES WHICH ARE EMPTY?

E: ARE YOU SURE THAT THE WAX ONES ARE FASTEST?

* E: SUPPOSING I DIDN'T BELIEVE YOU, HOW WOULD YOU PROVE IT TO ME?

(If S is wrong, go to (2). Otherwise continue from (1)

(1) E: GOOD, WOULD THIS BE ANOTHER FAIR WAY TO PROVE IT? WHY?

(If S is incorrect, go to (3), otherwise continue).

E: GOOD! IT WOULDN'T BE A FAIR TEST (race) TO SEE IF THE WAX WAS MAKING IT GO FASTER, BECAUSE IT COULD BE THE SIZE WHICH WAS MAKING IT GO FASTER. YOU COULDN'T BE SURE.

E: LET'S TRY THIS ONE NOW. YOU REMEMBER YOU TOLD ME THAT

(ii) BIG ONES ROLL FASTER THAN SMALL ONES.

(iii) LIGHT ONES ROLL FASTER THAN GREY ONES. (copper) (iron)

LET'S SEE YOU PROVE IT. REMEMBER! MAKE SURE IT'S A FAIR TEST.

(Continue as from * for both ii and iii. Note how many tests and how many countersuggestions S gets correct).
(2) E: WHY DO YOU THINK THIS ONE WON?

(If S says because it has wax). 

E: WELL, HOW DO YOU KNOW IT DIDN'T WIN BECAUSE IT WAS BIGGER? I WANT YOU TO PROVE THAT IT WILL STILL WIN, EVEN IF IT'S NOT BIGGER. TRY AGAIN. REMEMBER MAKE EVERYTHING ELSE THE SAME SO WE FIND OUT WHAT DIFFERENCE THE WAX MAKES.

(If S says because it has more wax and it's bigger).

E: DOES BEING BIGGER HELP IT GO FAST TOO? (Pause). THEN THAT'S NOT FAIR! MAYBE THE SIZE IS FOOLING US LIKE THE BLOCK DID ON THE BALANCE. MAYBE THE WAX ONLY LOOKS FASTER AND REALLY IT'S SLOWER. HOW COULD YOU TEST? (If necessary) MAKE THEM THE SAME SIZE.

(3) E: OH, OH, YOU LET ME FOOL YOU! THIS ISN'T REALLY A FAIR TEST AT ALL. LOOK, (pointing) HOW WOULD SOMEONE BE SURE THAT THIS ONE WENT THE FASTEST BECAUSE IT HAD WAX IN IT. IT COULD BE JUST BECAUSE IT WAS BIGGER. FOR A FAIR TEST, EVERYTHING HAS TO BE THE SAME. LET'S TRY ANOTHER; WOULD THIS BE A FAIR TEST (leave material uncontrolled - repeat explanation if S is incorrect, then go back to * Go directly back to * after congratulating him, if he is not incorrect).
F: BEFORE WE START WORKING WITH THESE TUBES OF WATER, I'D LIKE TO SEE IF
YOU REMEMBER WHAT I TAUGHT YOU WITH THE BLOCKS. COULD YOU SHOW ME
HOW YOU COULD FIND OUT WHICH WEIGH MORE: RED RODS OR BLUE RODS?

(Arrange Rods and Blocks as in Figure 2c)

(IF S starts to make a mistake say either - REMEMBER I'M ASKING YOU
ABOUT RED RODS AND BLUE RODS or REMEMBER, YOU'VE GOT TO PAY ATTENTION
TO THE BLOCKS, TOO, SO THAT THEY DON'T FOOL YOU).

(IF S fails, go through the usual explanation and give him
(2) yellow blue
(3) yellow red
then on to what follows. If S passes, go on directly to what follows.

E: GOOD, NOW DON'T FORGET, BEFORE YOU TEST ONE PART OF ONE THING (gesture)
AGAINST ONE PART OF SOMETHING ELSE (gesture), MAKE SURE THAT ALL THE
OTHER PARTS ARE EXACTLY THE SAME. MAKE SURE THERE'S NOTHING ELSE
ABOUT THEM WHICH COULD TRICK YOU INTO THINKING THE WRONG THING.
(Set up apparatus as in Figure 3)

E: SEE HOW THIS WORKS, WE DROP CHIPS DOWN LIKE THIS, SO WE CAN SEE WHICH
SINKS FASTEST. NOW FIRST OF ALL, I WANT YOU TO USE THESE ONES TO FIND
OUT WHICH SINKS FASTEST
(etc. as indicated on Figures 4, 5, and 6).

For any error haul it back up and say you said. X sinks faster than Y,
but how do you know . . . . . etc.).

For any correct response, provide a countersuggestion.
List of Figures

Figure 1. Bending Rods Apparatus.

Rod 1: wood, 13" x 2/16".
Rod 2: brass, 20" x 3/16".
Rod 3: brass, 10" x 1/16".
Rod 4: wood, 20" x 2/16".
Rod 5: wood, 16" x 5/16".
Rod 6: brass, 6" x 1/16".
Rod 7: brass, 16" x 2/16".
Rod 8: wood, 20" x 3/16".
Rod 9: brass, 10" x 1/16".
Rod 10: wood, 16" x 2/16".

Figure 2. The Layout of Rods and Blocks for Training.

Figure 3. Apparatus Used for Sinking Chips Demonstration.

Legend:

K - rubber band holding brass rod in center of tube
W - water level inside plexiglass tube
B - brass rod to insure that chip sinks to bottom without flipping on its side.
S - seal preventing water from escaping, and holding bottom end of brass rod in position.
WB - wooden blocks holding plexiglass tubes in position.
P - plunger designed to retrieve chips after they have fallen to bottom of tubes.
T - threads used to lift up plunger.

Note 1. Chips are placed on brass rod so that both threads and rods pass through their centers.

Note 2. Both rods are fitted with threads and plungers. These have been omitted from the left-hand rod in the present diagram for the sake of visual clarity.

G - gate designed to start chips down each tube simultaneously. A chip is placed over each rod and laid to rest on the starting gate as indicated on left rod. At the starting signal, the gate is jerked out in the direction of the arrow, with the result that both chips hit the water simultaneously.
List of Figures (cont.)

Figure 4. Layout of chips for trial 1. (Question: Do big ones sink faster than little ones?)

Figure 5. Layout of chips for trial 2. (Question: Does the size of the hole in the middle make any difference?)

Figure 6. Layout of chips for trial 3. (Question: Does steel sink faster than plastic?)
Figure 4

- Plastic circle
- Steel circle
- Plastic circle
- Steel circle