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ABSTRACT

Considerable thought, research, and concern has been expanded in an effort to determine whether the assumption of a quadratic relation between a single predictor and a criterion violated the assumptions which Johnson and Neyman (1936) state for calculating regions of significance about interacting regressions. In particular, there has been special concern for the assumption of linearity. Debate has ranged from whether linearity referred to the functional relation of the criterion and predictor to whether it was in the context of a linear statistical model, if not both. This paper extends the Johnson-Neyman technique to the curvilinear case and then illustrates this extension by reanalyzing data from two previously published research studies which have considered only a linear relationship between a covariable criterion. A computer program is included in the appendix. (Author/RC)

The Research and Development Center for Teacher Education

UTR&D

The University of Texas at Austin

The Research and Development Center for Teacher Education was established on the campus of the University of Texas at Austin in 1965, to design, build and test effective products to prepare teachers for careers in the nation's schools.

A staff of more than 100 are engaged in projects ranging from basic research into effective teaching behavior, through development of special counselor training strategies, to the development, implementation and evaluation of a complete and radically different undergraduate teacher education program.

The Center's major program, the Personalized Teacher Education Program, has its roots in teacher personality research dating back to the mid-Fifties. This early research, which demonstrated how teacher's personalities and classroom behavior correlate with success in their teaching careers, has led to the development of a large group of products which help education facilities become aware of student teachers' individual needs. The program also has produced products for student teachers' use, to help them build on their strengths.

The completely modularized program is currently in field test and/or use at more than a dozen important teacher education institutions nationally.

In addition to the PTEP, the Center also supports other projects in educational evaluation, development of strategies for implementing institutional change, and in consultation techniques for helping teachers plan individualized programs for children.

The Center's work is supported by the National Institute for Education and by the University of Texas System, as well as through contract research and development programs for public agencies.

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Curvilinear Extensions to Johnson-Neyman Regions of Significance and Some Applications to Educational Research

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Consider an experiment in which there are two treatments, one criterion and one covariable, and that the researcher would like to detect a covariable by treatment interaction. In order to detect this interaction, the researcher can (1) divide the covariable into any number of discrete categories (i.e., blocks) and perform a Treatment x Blocks analysis of variance (Edwards, 1968) or (2) leave the covariable a continuous measure and test for the homogeneity of group regressions (Walker and Lev, 1953). For the homogeneity of group regressions analysis the researcher employs each covariable value as a discrete unit of measurement and thereby avoids losing information by assigning different covariable values to the same block.

Cronbach and Snow (1973) have shown that for the case in which there is a moderately strong interaction, the statistical power of the homogeneity of group regressions test is superior to blocking at the median, blocking at the 33rd and 67th percentiles, or similar configurations that may be employed in a Treatment x Blocks design. Classification schemes such as these discard power by treating dissimilar data the same, causing the risk of accepting a false null hypothesis to increase beyond that which can be expected when the homogeneity of group regressions test is applied.

When group regressions are heterogeneous, i.e., E chooses not to employ a factorial design, Johnson and Neyman's (1936) procedure can be

used to determine the region(s) of covariable values for which treatments are significantly different. These regions allow covariable scores to be used to determine the treatment for which each S is best suited or if, for any given S, there is no best treatment. This technique is used generally in the study of aptitude-treatment or trait-treatment interactions and is considered the preferred methodology for such studies (Berliner & Cahen, 1972; Cronbach & Snow, 1973).

Considerable thought has been expended in an effort to determine whether the assumption of a quadratic relation between a single predictor and a criterion violates the assumptions which Johnson and Neyman (1936) state for calculating regions of significance about interacting regressions. In particular, there has been special concern for the assumption of linearity. One may ask whether "linearity" refers to the functional relation of the criterion and predictor or whether it is in the context of a linear statistical model, if not both. The computational procedure, however, reveals no need for the former assumption of linearity, thereby providing the foundation for extending the Johnson-Neyman procedure for determining regions of significance to the curvilinear case.

Johnson and Neyman (1936, p. 73) in discussing the difference, θ, between the regression equations for two experimental groups as $\theta(x,y) = f_1(x,y) - f_2(x,y)$ where f_1 and f_2 are functions of predictor variables x and y, comment:

> The functions f_1 and f_2 may be chosen arbitrarily, according to the conditions of the particular problem, with the only theoretical restriction that both functions f_1 and f_2 must be linear with regard to the unknown constants they involve. Thus we could assume that, e.g., $f_1(x,y) = A_0 + A_1\cos xy + A_2 e^{\sin x}$ but our method would fail if it were necessary to assume $f_1(xy) = \cos A_1 x + A_2{}^y$ since here the dependence upon A_1 and A_2 is not linear.

Most crucial is the position of the coefficient of the variable term, cosxy. When placed before the variable term as a multiplicative factor, this term makes the equation consistent with what is known mathematically as a linear equation. The fitting of curvilinear regression equations defined by polynomials of the form $Y = b_0 + b_1x + b_2x^2 + \ldots + b_px^p$ is in principle no different from the fitting of multiple regression equations.

Development of Curvilinear Formulae

Group Regressions

Given groups 1 and 2 with regression equations of the form $\hat{Y} = A_1 + B_1X + C_1X^2$ and $\hat{Y}_2 = A_2 + B_2X + C_2X^2$ where the subscripts 1 and 2 refer to groups 1 and 2 respectively, the first consideration in the series of hypotheses related to the Johnson-Neyman procedure is that of homogeneity of criterion variance. The second consideration is that of heterogeneity of regression. Johnson and Jackson (1956), in their two-predictor case, provide the formula for computing the variance of the actual scores about the predicted scores and the variance of the observed scores about a regression line with a common slope in order to compute an F-ratio for homogeneity of group regressions. If the probability level associated with this F-ratio is found to be significant, then the Johnson-Neyman test for regions of significance is recommended.

The points at which the regression curves intersect, indicating the points of no difference, can also be determined. The computational forms to be employed in this series of steps is as follows.

Regions of Significance

The difference in regression curves is expressed as $(A_1 - A_2) + (B_1 - B_2)X + (C_1 - C_2)X^2 = D$. Setting $D = 0$ yields a quadratic equation $(A_1 - A_2) + (B_1 - B_2)X + (C_1 - C_2)X^2 = 0$ which can be solved. Application of the quadratic formula, $\dfrac{-B \pm \sqrt{B^2 - 4AC}}{2A}$, yields two zeroes assuming the term $B^2 - 4\,AC$ is non-negative. If this term were negative, there would be no point at which the two curves intersect. Such an occurrence is feasible as shown in Figure 1.

Insert Figures 1, 2 and 3

If $B^2 - 4AC = 0$, then Figure 2 is most likely and, if $B^2 - 4AC > 0$, then Figure 3 is likely.

To determine regions of significance there are several possibilities based on which of the situations outlined above is considered. Given Figure 1 or 2 one would most likely find a left and right boundary for the region of non-significance as indicated by the shaded portion. To determine these points along the X-axis, consider the formula:

$$\left(\frac{D^2}{P + Q}\right)\Big/ 1 \Big/ \left(\frac{S_a^{\,2}}{N - 6}\right)$$

where $S_a^{\,2}$ is the best estimate of the error variance obtained by pooling the variances about the group regression lines and $\dfrac{D^2}{P + Q}$ is the variance for the difference in regression lines where $D = (A_1 - A_2) + (B_1 - B_2)X + (C_1 - C_2)X^2$ and where $P + Q$ is a "scaling" factor used to determine the variance for the difference between the regression equations for the two groups. The value of $(P + Q)$ depends upon the values of the basic characteristics of matching the x's and y's. Where x' and y' lie near the population means of x and y, the value of $(P + Q)$ becomes small since $(P + Q)$ becomes $\dfrac{1}{N_1} + \dfrac{1}{N_2}$ when $\bar{x}_1 = x' = \bar{x}_2$ and $\bar{y}_1 = y' = \bar{y}_2$. D^2 is of

the general form $AX^4 + BX^3 + CX^2 + DX + E$ so that the equation $\left(\frac{D^2}{P+Q}\right) \Big/ \left(\frac{S_a{}^2}{N-6}\right) = F_{.05}(1,N-6)$ expressible as $D^2 - F_{.05}\,\frac{\left[(P+Q)S_a{}^2\right]}{N-6} = 0$ is a fourth order (quartic) equation. This is consistent with the illustrations presented earlier. If four distinct solutions exist then two regions of non-significance are defined as indicated in Figure 3. To find the solutions to this equation, a method suggested by <u>Standard Mathematical Tables</u> (1965) is available which requires finding a solution of a resolvent cubic equation first but which in turn can be used to obtain all four roots of the above fourth order equation. The following formulae are used in finding these values.

Given the quartic equation (1) $x^4 + ax^3 + bx^2 + cx^1 + d = 0$, a form which the equation obtained from D^2 can assume, a resolvent cubic equation of the form (2) $y^3 - by^2 + (ac - 4d)y - a^2d + 4bd - c^2 = 0$ is obtained. A root of this equation is obtained by reducing equation (2) to the form (3) $z^3 + fz + g = 0$; the "second" order term is eliminated by substituting $(z + \frac{b}{3})$ for y in (2) where $f = \frac{1}{3}\left[3(ac - 4d) - b^2\right]$ and $g = \frac{1}{27}\left[2b^3 - 9b\,(ac - 4d) + 27(-a^2d + 4bd - c^2)\right]$. Let $A = \sqrt[3]{\frac{-g}{2} + \sqrt{\frac{g^2}{4} + \frac{f^3}{27}}}$ and $B = \sqrt[3]{\frac{-g}{2} - \sqrt{\frac{g^2}{4} + \frac{f^3}{27}}}$; then $A + B = M$ is a root of equation (2) from which the roots for (1) can be obtained. Consider $R = \sqrt{\frac{a^2}{4} - b + M}$, if $R \neq 0$, $D = \sqrt{\frac{3a^2}{4} - R^2 - 2b + \frac{4ab - 8c - a^3}{4R}}$ and $E = \sqrt{\frac{3a^2}{4} - R^2 - 2b - \frac{4ab - 8c - a^3}{4R}}$. Otherwise $D = \sqrt{\frac{3a^2}{4} - 2b + 2\sqrt{M^2 - 4d}}$ and $E = \sqrt{\frac{3a^2}{4} - 2b - 2\sqrt{M^2 - 4d}}$. The four roots of the fourth order equation are then $x = \frac{-a}{4} + \frac{R}{2} \pm \frac{D}{2}$ and $x = \frac{-a}{4} - \frac{R}{2} \pm \frac{E}{2}$. These roots are real dependent upon the arguments for the square roots above being greater than or equal to zero.

Research Applications

The above computations may be simplified through the use of a computer program (Wunderlich and Borich, 1973) which, after solving the fourth order equation for determining regions of significance, plots the data points, within-group regressions and regions of significance for the case in which there are two treatments, one criterion and one predictor. Before applying the program, however, the investigator must consider the full range of calculations, both linear and nonlinear, that may be required by any given data set. These calculations are diagrammed sequentially in Figure 4.

Insert Figure 4 about here

Note that the sequence begins by testing the assumption of curvilinearity and that criterion-covariable relationships may be curvilinear for groups separately or combined. In one case the combined groups may exhibit a single underlying curvilinear relationship such that the two groups are samples from the same population, i.e., regression slopes for the groups are homogeneous. In a second case the relations between the covariable and criterion may be curvilinear for each group separately, i.e., regression slopes for the groups are heterogeneous. Only for the latter case are regions of significance tenable. Whereas the foregoing discussion addresses both homogeneity of curvilinear regressions and regions of significance, it does not cover a third opportunity to detect a curvilinear relationship. In a third case regressions that are both linear and non-intersecting (ordinal) may mask an underlying curvilinear relationship in which linear slopes of both groups form a single curvilinear trend. The continuity of separate linear slopes can take the form of a single curvilinear regression that is easily missed when the efficiency of linear

and curvilinear models are not compared at the outset of the research. This initial comparison and the sequence of steps representing the left portion of Figure 4 are calculated by the curvilinear program. When one or both within-group regressions are linear, i.e., a curvilinear model fails to significantly improve prediction, the researcher is referred to a linear program by Borich (1971) for the case in which there is one predictor or to a linear program by Borich and Wunderlich (1973) for the case in which there are two predictors.

To illustrate the above computations, a research study conducted by Hughes and published in the American Educational Research Journal (1973, 10, 21-37) was reanalyzed with the Wunderlich and Borich (1973) curvilinear program.

Background. Hughes set out to establish that differences in residual posttest achievement are a function of pupils responding to teachers' questions (a) randomly, (b) systematically and (c) in a self-selected style in which the pupil has the choice of whether or not to respond. With three schools available for experimentation, E chose a 3 x 3 factorial design with schools as the first factor and type of responding as the second. Criterion performance was established with a 222-item posttest based upon the content of the treatment, a wildlife lesson directed to seventh- and eighth-grade pupils. In addition, E collected scores on nine covariables with the foresight that these may confound posttest performance. Among these E included the most likely predictor--that of pretest achievement on the criterion instrument. E chose to remove posttest variance attributable to differences on the pretest by using as the criterion the difference between predicted and actual posttest scores (i.e., the residuals).

Hughes' data provided the opportunity to uncover nine aptitude-treatment interactions. For our illustration, however, the reanalysis was limited to one of the nine covariables, that of the pretest and to only one pair of treatments, that involving the self-selected and systematic response groups.

Results. These data, while previously reanalyzed by Borich (1974) with the assumption that covariable-criterion relationships were linear, were reanalyzed a second time assuming that relationships between covariable and criterion were curvilinear. Application of the curvilinear program to these data, however, revealed no significant improvement in prediction from that obtained from a linear model for either the systematic response or the self-selected response groups and therefore the investigator was referred to the more suitable linear program (Borich, 1971). In the interest of providing a real data illustration of the curvilinear technique, the program then was forced subsequent to the model comparisons test to continue as though curvilinear regressions had been found in one or more treatments. The program, therefore, calculated the homogeneity of group regressions test for curvilinear regressions, the test for a common intercept and regions of significance rather than "exit" as would be expected after the model comparison test for lack of a significant improvement in predictive efficiency with the curvilinear model. The resulting plot from the curvilinear analysis of these data is reported in Figure 5 and the resulting plot from the corresponding linear analysis by Borich (1974) is reported in Figure 6.

Insert Figures 5 and 6 about here

Note that for both analyses a region of significance is indicated to the left of the data mass. While the region of significance for the curvilinear plot begins at the covariable value of 14.20 (taken from the printout) and the region of significance for the linear plot at the covariable value of 13.20, both regions lie in approximately the same area and encompass approximately the same range of covariable values. These ranges represent those covariable values for which the treatments are significantly different at or beyond the .05 level. For this example the curvilinear plot lends little new information to the study and, if employed, might needlessly complicate its interpretation. Using the linear plot, we might conclude that those subjects scoring below 13.20 on the pretest should receive the self-selected response treatment while those falling above this value are likely to achieve equally well from either treatment and therefore should receive the least costly of the two treatments.

Other configurations. Lest the reader receive the impression from this example that the present data represent the only configuration of curvilinear regressions and corresponding regions of significance, other hypothetical distributions have been analyzed and plotted and are reported in an appendix to this report. A listing of the program used to generate these plots is also provided.

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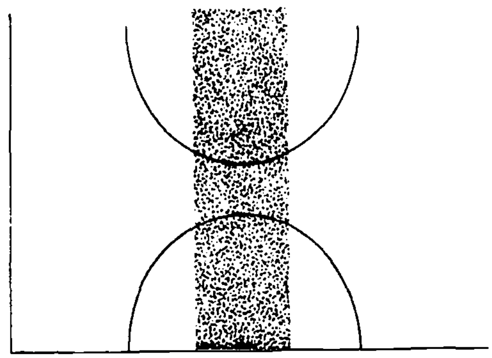


Fig 1

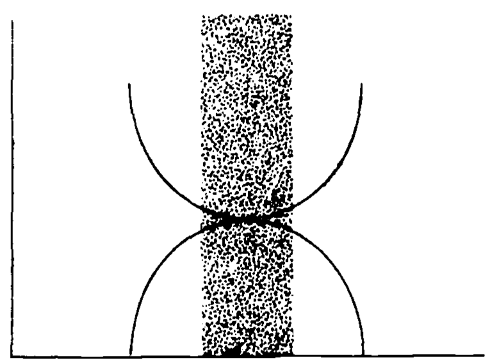


Fig 2

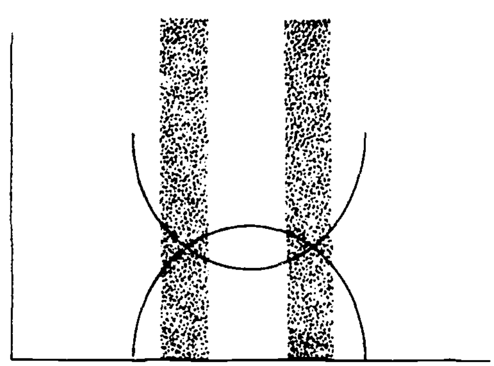


Fig 3

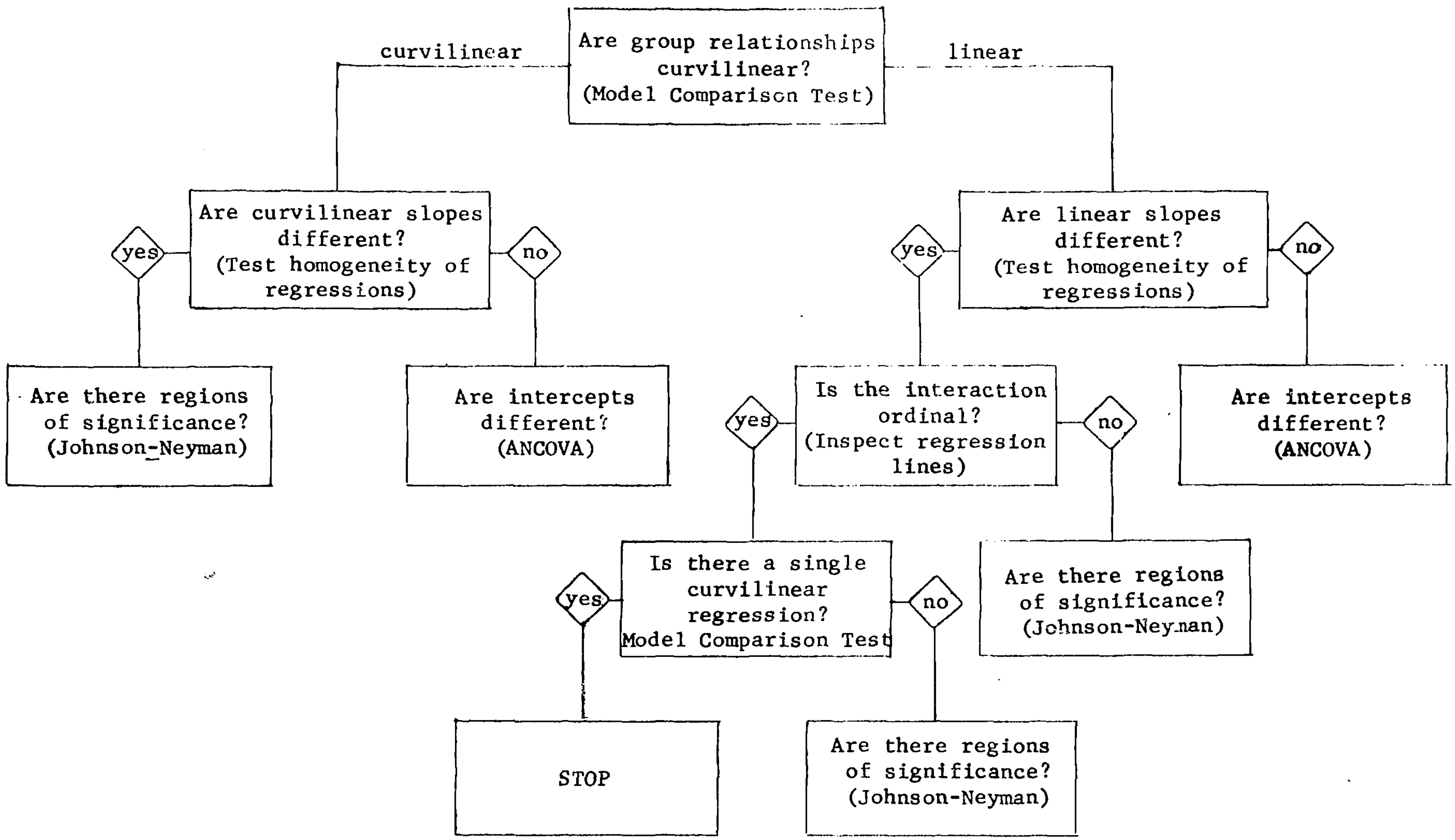

Are group relationships curvilinear? (Model Comparison Test)
curvilinear
linear
Are curvilinear slopes different? (Test homogeneity of regressions)
yes
no
Are there regions of significance? (Johnson-Neyman)
Are intercepts different? (ANCOVA)
Are linear slopes different? (Test homogeneity of regressions)
yes
no
Is the interaction ordinal? (Inspect regression lines)
yes
no
Are intercepts different? (ANCOVA)
Is there a single curvilinear regression? Model Comparison Test
yes
no
Are there regions of significance? (Johnson-Neyman)
STOP
Are there regions of significance? (Johnson-Neyman)

Fig 4

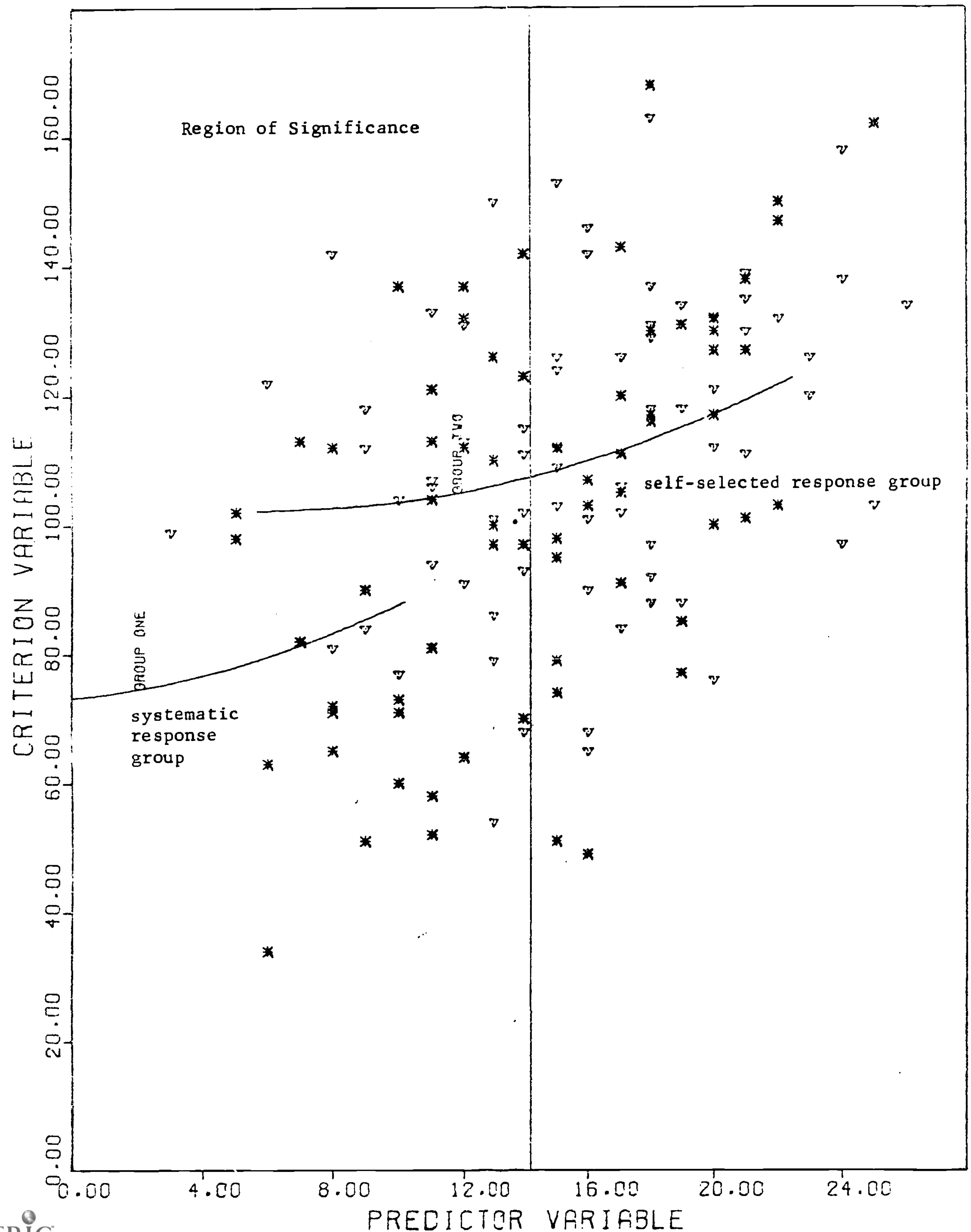

Region of Significance
GROUP TWO
self-selected response group
GROUP ONE
systematic
response
group
CRITERION VARIABLE
PREDICTOR VARIABLE
0.00
4.00
8.00
12.00
16.00
20.00
24.00
20.00
40.00
60.00
80.00
100.00
120.00
140.00
160.00

Fig 5

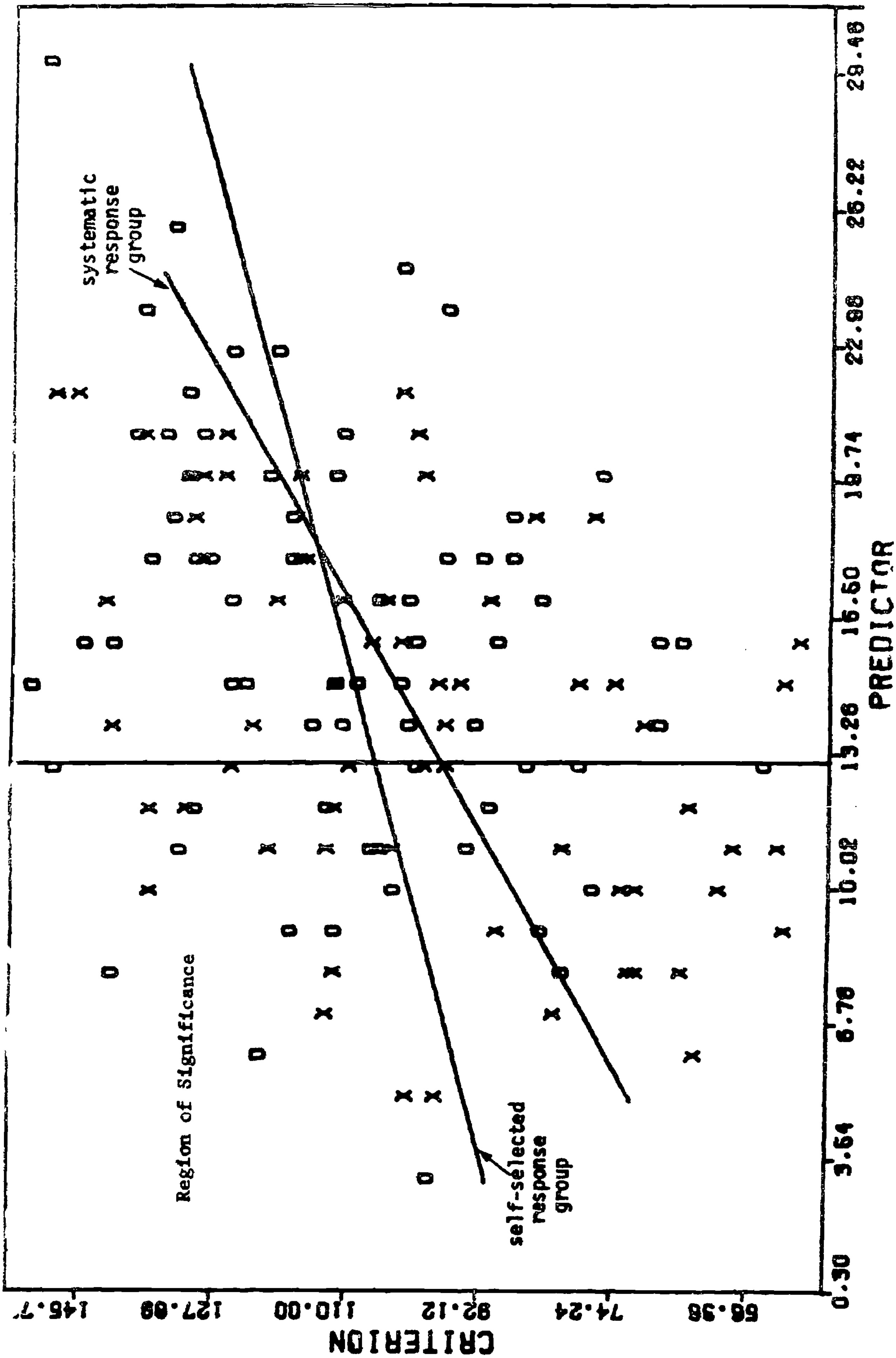

systematic response group
Region of Significance
self-selected response group
PREDICTOR
0.30
3.54
6.78
10.02
13.26
16.50
19.74
22.98
26.22
29.46
CRITERION
56.36
74.24
92.12
110.00
127.88
145.7

Fig 6

Figure Captions

Figure 1. $B^2 - 4AC < 0$.

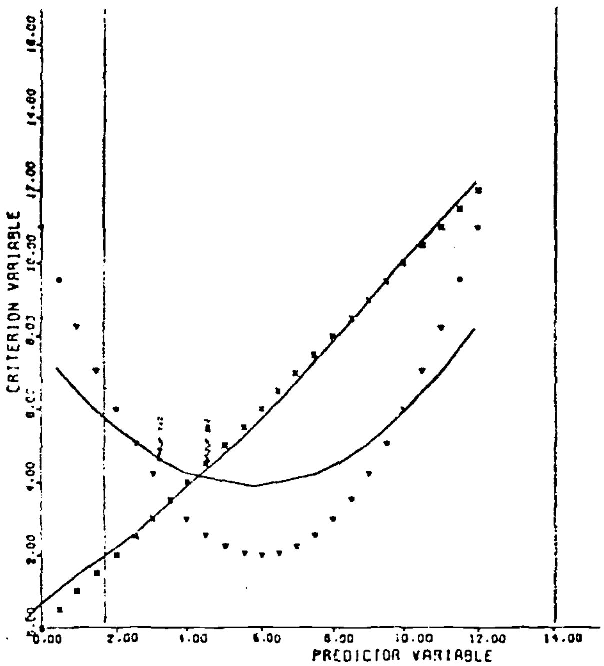
Figure 2. $B^2 - 4AC = 0$.

Figure 3. $B^2 - 4AC > 0$.

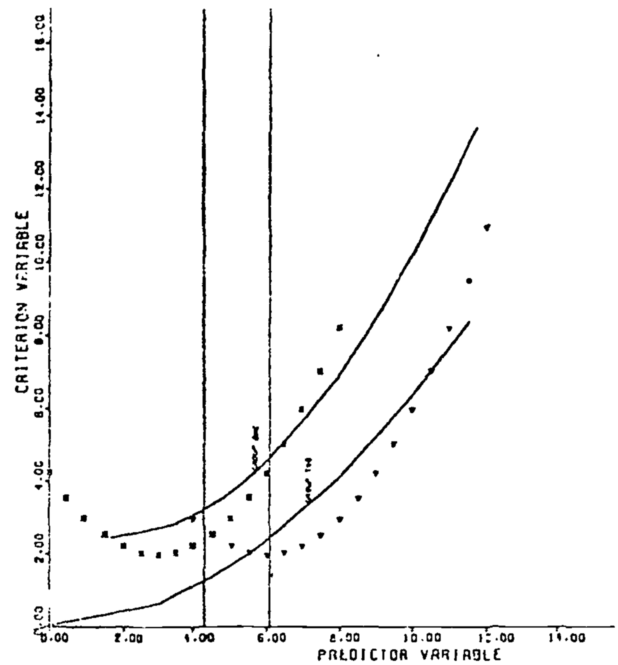
Figure 4. Sequence of linear and curvilinear calculations for ATI analyses.

Figure 5. Region of significance (< 14.21) and nonsignificance (> 14.20) for Hughes (curvilinear analysis).

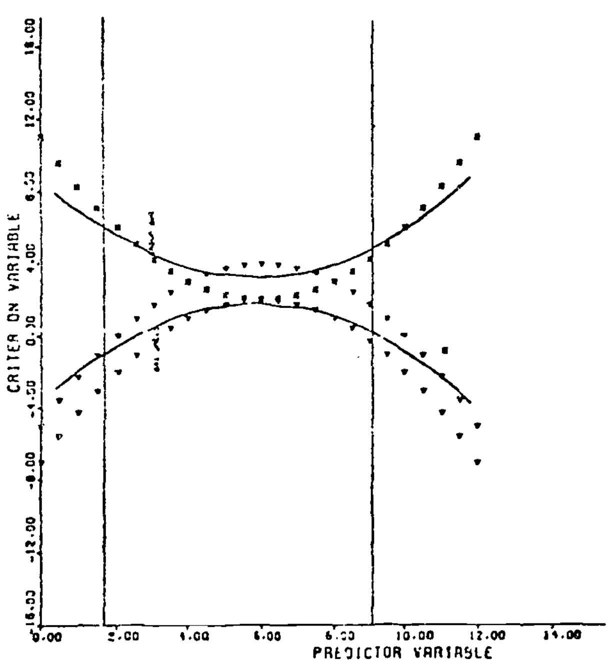
Figure 6. Region of significance (< 13.21) and nonsignificance (> 13.20) for Hughes (linear analysis).



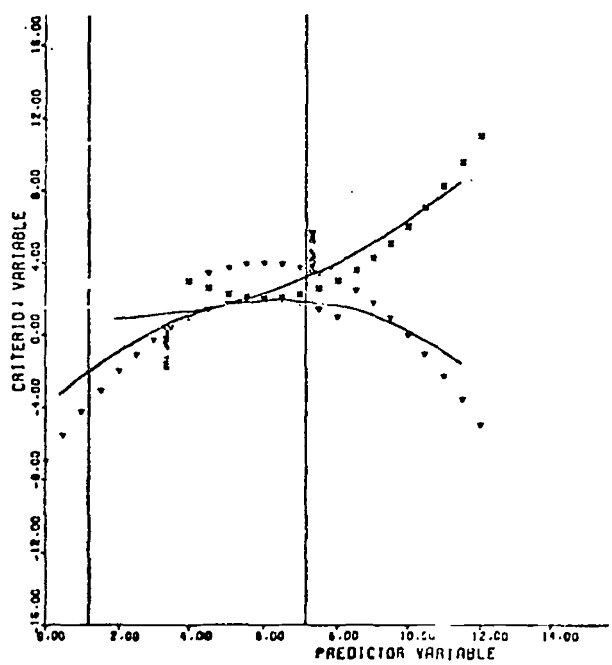
a. A positive linear regression and a moderately strong curvilinear regression with small left and right regions of significance



b. Two slightly curvilinear positive regressions with a moderately sized left and large right region of significance



c. Two moderately strong curvilinear regressions in opposite directions with a small left and a moderately sized right region of significance



d. A slightly curvilinear regression and a moderately strong curvilinear regression in opposite directions with a small left and a large right region of significance

```
      PROGRAM CURVATI(INPUT,OUTPUT,PLOT)
C     (PRELIMINARY VERSION,4/5/74)
C     CONTROL CARDS AND THE DATA ARE ARRANGED IN THE FOLLOWING ORDER
C     FIRST CARD:  COLS, 1-5 = NUMBER OF SUBJECTS IN GROUP 1
C                  COLS, 6-10 = NUMBER OF SUBJECTS IN GROUP 2
C                  COLS,15 = MISSING DATA OPTION
C     IF ZERO, ALL DATA IS INCLUDED IN THE ANALYSIS
C     IF ONE, BLANKS ARE NOT INCLUDED BUT ZEROES ARE
C     IF TWO, BLANKS AND ZEROES ARE NOT PROCESSED
C                  COLS, 16-20 = ALPHA (P) LEVEL USED TO DECIDE
C     WHETHER A LINEAR OR CURVILINEAR MODEL IS APPROPRIATE
C     SECOND CARD: COLS, 1-5 = F-VALUE ASSOCIATED WITH P, ABOVE(DF=1 AND N--3)
C     THIRD CARD:  GROUP 1 FORMAT(PREDICTOR MUST PRECEDE CRITERION)
C     INSERT GROUP 1 DATA HERE
C     FOURTH CARD:  GROUP 2 FORMAT
C     INSERT GROUP 2 DATA HERE
C     PROGRAMMED BY KEN W WUNDERLICH AND GARY D BORICH, RESEARCH AND DEVELOPMENT
C     CENTER FOR TEACHER EDUCATION, THE UNIVERSITY OF TEXAS AT AUSTIN
      DIMENSION Y1(230),X1(230),X12(230),Y2(230),X2(230),X22(230),Y(450)
     1,X(450),X02(450),FRMT1(8),FRMT2(8),A(400),B(400),C(400),NSC(10),YP
     2M(10),XX(4),PX(4),XPTLN1(35),YPTLN1(35),XPTLN2(35),YPTLN2(35),XPT(
     335),YPT(35),
     4QX1(4),QY1(4),QX2(4),QY2(4),QX3(4),QY3(4),QX4(4),QY4(4),AA(5),Z(4)
      COMPLEX Z
      READ 1,N1,N2,MISS,PBLV
    1 FORMAT (2I5,4X,I1,F5.3)
      READ 27,FSL
   27 FORMAT (F6.3)
      READ 2,(FRMT1(I),I=1,8)
    2 FORMAT (8A10)
      READ FRMT1,(X1(I),Y1(I),I=1,N1)
      CALL AMISDAT (X1,Y1,N1,MISS)
      DO 3 K=1,N1
    3 X12(K)=X1(K)*X1(K)
      CALL SUMDATA(Y1,X1,X12,Y1M,X1M,X12M,Y1SD,X1SD,X12SD,Y1SS,X1SS,X12S
     1S,RYX1,RYX12,RXX12,RSSYX1,RSSYX12,RSSXX12,N1)
      READ 2,(FRMT2(I),I=1,8)
      READ FRMT2,(X2(I),Y2(I),I=1,N2)
      CALL AMISDAT(X2,Y2,N2,MISS)
      DO 4 K=1,N2
    4 X22(K)=X2(K)*X2(K)
      CALL SUMDATA (Y2,X2,X22,Y2M,X2M,X22M,Y2SD,X2SD,X22SD,Y2SS,X2SS,X22
     1SS,RYX2,RYX22,RXX22,RSSYX2,RSSYX22,RSSXX22,N2)
      N=N1+N2
      DO 5 K=1,N1
      X(K)=X1(K)
      Y(K)=X2(K)
    5 X02(K)=X12(K)
      DO 6 K=1,N2
      X(N1+K)=X2(K)
      Y(N1+K)=Y2(K)
    6 X02(N1+K)=X22(K)
      CALL SUMDATA(Y,X,X02,YM,XM,X02M,YSD,XSD,X02SD,YSS,XSS,X02SS,RYX,RY
     1X02,RXX02,RSSYX,RSSYX02,RSSXX02,N)
      PRINT 7
    7 FORMAT (1H1)
      PRINT 8,N1,N2,N,Y1M,Y2M,YM,Y1SD,Y2SD,YSD,X1M,X2M,XM,X1SD,X2SD,XSD,
     1RYX1,RYX2,RYX
    8 FORMAT(28X,*SUMMARY STATISTICS*//,29X,*GROUP 1*8X*GROUP 2*9X*TOTAL
     1*//,5X*GROUP N*20X,I4,11X,I4,10X,I4,//,5X*CRITERION MEAN*9X,F8.3,7
     2X,F8.3,6X,F8.3,//,5X,*CRITERION ST. DEV.*5X,F8.3,7X,F8.3,6X,F8.3,/
     3/,5X,*PREDICTOR MEAN*9X,F8.3,7X,F8.3,6X,F8.3,//,5X,*PREDICTOR ST.
```

```
     4DEV,*5X,F8,3,7X,F8,3,6X,F8,3,//,5X,*CORRELATION Y AND X*6X,F6,4,
     59X,F6,4,8X,F6,4)
      AN=N
      ANDF=AN-6,0
      AN1 =N1
      AN1DF=AN1-3,0
      AN2=N2
      AN2DF=AN2-3,0
C MODEL COMPARISON
      RSQ1=(RYX1*RYX1+RYX12*RYX12-2,0*RYX1*RYX12*RXX12)/(1,0-RXX12*RXX12
     1)
      RSQ2=(RYX2*RYX2+RYX22*RYX22-2,0*RYX2*RYX22*RXX22)/(1,0-RXX22*RXX22
     1)
      R1=RYX1*RYX1
      R2=RYX2*RYX2
      VAYXX1=Y1SD*Y1SD*(1,0-RSQ1)
      VAYXX2=Y2SD*Y2SD*(1,0-RSQ2)
      VAYX1 =Y1SD*Y1SD*(1,0-RYX1*RYX1)
      VAYX2=Y2SD*Y2SD*(1,0-RYX2*RYX2)
      FGP1= (RSQ1-R1)/((1,0-RSQ1)/AN1DF)
      FGP2= (RSQ2-R2)/((1,0-RSQ2)/AN2DF)
      ANDF1=N1-2,0
      ANDF2=N2-2,0
      SLMOD1=PRBF(1,0,AN1DF,FGP1)
      SLMOD2=PRBF(1,0,AN2DF,FGP2)
      IF(SLMOD1,LE,PBLV,OR,SLMOD2,LE,PBLV)17,16
   16 PRINT 18,PBLV
   18 FORMAT(/,5X*A COMPARISON OF A LINEAR AND QUADRATIC MODEL FOR EACH
     1OF GROUPS 1 AND 2 SUGGESTS*,/,5X,*NO SIGNIFICANT DIFFERENCE AT THE
     2 *F5,3* LEVEL AND THUS USE OF A LINEAR MODEL AND */,5X,*ANALYSIS I
     3S SUGGESTED,  SEE BORICH, EDUCATIONAL AND PSYCHOLOGICAL MEASUREMEN
     4T,*,/,5X,*1971,31,P251-253,*)
      GO TO 101
   17 BX1=(Y1SD/X1SD)*(RYX1-RYX12*RXX12)/(1,0-RXX12*RXX12)
      BX21=(Y1SD/X12SD)*(RYX12-RYX1*RXX12)/(1,0-RXX12*RXX12)
      A01=Y1M-BX1*X1M-BX21*X12M
      BX2=(Y2SD/X2SD)*(RYX2-RYX22*RXX22)/(1,0-RXX22*RXX22)
      BX22=(Y2SD/X22SD)*(RYX22-RYX2*RXX22)/(1,0-RXX22*RXX22)
      A02=Y2M-BX2*X2M-BX22*X22M
      PRINT 12,A01,BX1,BX21,A02,BX2,BX22
   12 FORMAT (////,5X,*THE REGRESSION EQUATION FOR GROUP 1 IS  Y = *F8,4
     1*  +  *F8,4*  X  +  *F8,4*  XX,* //,5X,*THE REGRESSION EQUATION FO
     2R GROUP 2 IS  Y = *F8,4*  +  *F8,4*  X  +  *F8,4*  XX,*)
      PT1=PT2=0,0
      ERROR=VAYXX1+VAYXX2
      SYYW=Y1SS+Y2SS
      SXXW=X1SS+X2SS
      SXX2W=X12SS+X22SS
      RYXSSW=RSSYX1+RSSYX2
      RYX2SSW=RSSYX12+RSSYX22
      RXX2SSW=RSSXX12+RSSXX22
      VAREGRS=SYYW-((RYXSSW**2*SXX2W+SXXW*RYX2SSW**2-2,0*RXX2SSW*RYX2SSW
     1*RYXSSW)/(SXXW*SXX2W-RXX2SSW**2))
      ANO= ANDF/2,0
      FREGRES =ANO*(VAREGRS-ERROR)/ERROR
      PREGRES=PRBF(2,0,ANDF,FREGRES)
      PRINT 14,FREGRES,PREGRES
   14 FORMAT(//,5X*THE F-RATIO FOR A TEST OF HOMOGENIETY OF REGRESSION I
     1S  *F8,3,/,5X,*WITH AN ASSOCIATED PROBABILITY LEVEL OF  *F8,3)
      VINTER=YSD*YSD*((1,0-RYX*RYX-RYX02*RYX02-RXX02*RXX02+2,0*RYX*RYX02
     1*RXX02)/(1,0-RXX02*RXX02))
      ANO=N-3
```

```
      FINTER=(ANDF/3.0)*(VINTER-ERROR)/ERROR
      PINTER= PRBF(3.0,ANDF,FINTER)
      PRINT 15, FINTER,PINTER
   15 FORMAT(//,5X,*THE HYPOTHESIS OF COMMON REGRESSION CONSTANTS IS TES
     1TED BY AN F-RATIO OF *F8.3/5X,* WHICH HAS AN ASSOCIATED PROBABILIT
     2Y LEVEL OF *F8.3)
      AD=A01-A02
      BXD=BX1-BX2
      BX2D=BX21-BX22
      QUAD=BXD*BXD-4.0*BX2D*AD
      IF(QUAD.GE.0.0)20,19
   19 PRINT 21
   21 FORMAT(//,5X,*THE REGRESSION CURVES FOR GROUPS 1 AND 2 DO NOT INTE
     1RSECT.*)
      GO TO 22
   20 IF(QUAD.EQ.0.0)23,24
   23 PT1= -BXD/(2.0*BX2D)
      PRINT 25,PT1
   25 FORMAT(//,5X,*THE REGRESSION CURVES INTERSECT AT A POINT WHERE X I
     1S EQUAL TO * F6.3)
      GO TO 22
   24 PT1=(SQRT(QUAD)-BXD)/(2.0*BX2D)
      PT2=(-BXD-SQRT(QUAD))/(2.0*BX2D)
      PRINT 26,PT1,PT2
   26 FORMAT(//,5X,*THE REGRESSION CURVES INTERSECT AT THE POINTS WHERE*
     1/*X IS EQUAL TO *F6.3* AND WHERE X IS EQUAL TO *F6.3)
   22 R1=1.0/(1.0-RXX12*RXX12)
      R2=1.0/(1.0-RXX22*RXX22)
      S1=2.0*RXX12/SQRT(X1SS*X12SS)
      S2=2.0*RXX22/SQRT(X2SS*X22SS)
      FACTOR=FSL*ERROR/ANDF
      X4COEF =BX2D*BX2D- FACTOR*(R1 / X12SS + R2/X22SS)
      X3COEF =2.0*BXD*BXD2D +FACTOR*( R1*S1 + R2*S2)
      X2COEFF =BXD*BXD+2.0*AD*BX2D-FACTOR*( R1/X1SS + R2/X2SS - R1*S1*X1
     1M - R2*S2*X2M - 2.0*X12M*R1/X12SS - 2.0*X22M*R2/X22SS)
      X1COEF =2.0*AD*BXD -FACTOR*(R1*S1*X12M +R2*S2*X22M-2.0*R1*X1M/X1SS
     1-2.0*R2*X2M/X2SS)
      CONSTA= AD*AD - FACTOR *( 1.0/AN1 +1.0/AN2 +X1M*X1M*R1/X1SS + R2*
     1X2M*X2M/X2SS - R1*S1*X1M*X12M - R2*S2*X2M*X22M + R1*X12M*X12M/X12S
     2S + R2*X22M*X22M/X22SS)
      AA(1)= X4COEF
      AA(2)= X3COEF
      AA(3)= X2COEF
      AA(4)= X1COEF
      AA(5)= CONSTA
      CALL ZPOLYR (AA,4,Z,Z)
      PRINT 55,Z
   55 FORMAT (8(2X,F8.4))
      NST1=NST2=33
      CALL AMINMAX(X,N,XMIN,XMAX)
      PRINT 2000, XMIN,XMAX
 2000 FORMAT (5X,*XMIN = *F8.3*  XMAX = *F8.3)
   28 CALL PARAB(BX21,BX1,A01,XPTLN1,YPTLN1,XMIN,XMAX,AA1,NST1)
   31 CALL PARAB(BX22,BX2,A02,XPTLN2,YPTLN2,XMIN,XMAX,AA2,NST2)
   33 CALL BGNPLT
      CALL PLT(1.0,1.0,-3)
      CALL SCALE (X(1),10.0,N,1)
      CALL SCALE (Y(1), 9.0,N,1)
      QX1(3)=QX2(3)=QX3(3)=QX4(3)=XPTLN1(NST1+1)=XPTLN2(NST2+1)=X(N+1)
      QX1(4)=QX2(4)=QX3(4)=QX4(4)=XPTLN1(NST1+2)=XPTLN2(NST2+2)=X(N+2)
      QY1(3)=QY2(3)=QY3(3)=QY4(3)=YPTLN1(NST1+1)=YPTLN2(NST2+1)=Y(N+1)
      QY1(4)=QY2(4)=QY3(4)=QY4(4)=YPTLN1(NST1+2)=YPTLN2(NST2+2)=Y(N+2)
```

```
      CALL AXIS (0.0,0.0,18HPREDICTOR VARIABLE,-18,10.0,0.0,X(N+1),X(N+2
     1))
      CALL AXIS(0.0,0.0,18HCRITERION VARIABLE,18,9.0,90.0,Y(N+1),Y(N+2))
      X1(N1+1)=X2(N2+1)=X(N+1)
      X1(N1+2)=X2(N2+2)=X(N+2)
      Y1(N1+1)=Y2(N2+1)=Y(N+1)
      Y1(N1+2)=Y2(N2+2)=Y(N+2)
      DO 902 K = 1,N1
      PRINT 903,X1(K),Y1(K),K
  903 FORMAT (2X,F8.3,5X,F8.3,2X,I2)
  902 CONTINUE
      PRINT 910
      DO 904 K = 1,N2
      PRINT 905,X2(K),Y2(K),K
  905 FORMAT (2X,F8.3,5X,F8.3,2X,I2)
  904 CONTINUE
  910 FORMAT (5X,*THIS IS GROUP ONE DATA*)
      PRINT 911
  911 FORMAT(*     THIS IS GROUP TWO DATA*)
      CALL LINE(X1,Y1,N1,1,-1,11)
      CALL LINE(X2,Y2,N2,1,-1,5)
      PRINT 900,(XPTLN1(J),YPTLN1(J),J=1,NST1)
  900 FORMAT (5(2X,F8.3,2X,F8.3))
      PRINT 900,(XPTLN2(J),YPTLN2(J),J=1,NST2)
      CALL LINE(XPTLN1,YPTLN1,NST1,1,0,0)
      CALL LINE(XPTLN2,YPTLN2,NST2,1,0,0)
      SX1= (XPTLN1(6)-X(N+1))/X(N+2)
      SY1= (YPTLN1(6)-Y(N+1))/Y(N+2)
      SX2= (XPTLN2(6)-X(N+1))/X(N+2)
      SY2= (YPTLN2(6)-Y(N+1))/Y(N+2)
      IF (AA1.GE.0.0)34,35
   34 ANG1=90.0
      GO TO 36
   35 ANG1=270.0
   36 IF(AA2.GE.0.0)37,38
   37 ANG2=90.0
      GO TO 39
   38 ANG2=270.0
   39 CALL SYMBOL(SX1,SY1,.07,9HGROUP ONE,ANG1,9)
      CALL SYMBOL(SX2,SY2,.07,9HGROUP TWO,ANG2,9)
      TT=10.0*X(N+2)+X(N+1)
      TY= 9.0*Y(N+2)+Y(N+1)
      PRINT 912,TT,TY
  912 FORMAT(5X,* TT = *F8.4*  TY = *F8.4)
      QX1(1)=QX2(1)=QX3(1)=QX4(1)=X(N+1)
      QX1(2)=QX2(2)=QX3(2)=QX4(2)= X(N+1)
      QY1(1)=QY2(1)=QY3(1)=QY4(1)=Y(N+1)
      QY1(2)=QY2(2)=QY3(2)=QY4(2)= Y(N+1)
      DO 70 JL = 1,4
      PX(JL) = 0.0
      IF (AIMAG(Z(JL)) .EQ. 0.0) 71,72
   71 PX(JL) = REAL (Z(JL))
      GO TO 70
   72 PX(JL) = TT + 10.0
   70 CONTINUE
      PRINT 913,(PX(JL),JL=1,4)
  913 FORMAT (2X,4(2X,F8.3))
      IF(PX(1).GE.X(N+1).AND.PX(1).LE.TT)40,41
   40 QX1(1)=QX1(2)=PX(1)
      QY1(1)=Y(N+1)
      QY1(2)=TY
   41 IF(PX(2).GE.X(N+1).AND.PX(2).LE.TT)42,43
```

```
 42 QX2(2)=QX2(1)=PX(2)
    QY2(1)=Y(N+1)
    QY2(2)=TY
 43 IF(PX(3).GE.X(N+1).AND.PX(3).LE.TT)44,45
 44 QX3(1)=QX3(2)=PX(3)
    QY3(1)=Y(N+1)
    QY3(2)=TY
 45 IF(PX(4).GE.X(N+1).AND.PX(4).LE.TT)46,47
 46 QX4(1)=QX4(2)=PX(4)
    QY4(1)=Y(N+1)
    QY4(2)=TY
 47 CONTINUE
    PRINT 901,(QX1(I),QY1(I),I=1,2)
    PRINT 901,(QX2(I),QY2(I),I=1,2)
    PRINT 901,(QX3(I),QY3(I),I=1,2)
    PRINT 901,(QX4(I),QY4(I),I=1,2)
901 FORMAT (/,2(5X,F8.3))
    CALL LINE(QX1,QY1,2,1,0,0)
    CALL LINE(QX2,QY2,2,1,0,0)
    CALL LINE(QX3,QY3,2,1,0,0)
    CALL LINE(QX4,QY4,2,1,0,0)
    CALL ENDPLT
101 CONTINUE
    END
    SUBROUTINE AMISDAT (A,B,NS,MISDATA)
    DIMENSION A(200),B(200),ILL(200),AA(200),BB(200)
    IF (MISDATA.EQ.0)RETURN
    IJK=0
    DO 750 NQ=1,200
750 ILL(NO)=AA(NQ)=BB(NQ)=0.0
    DO 200 N=1,NS
    IF(A(N).EQ.0.0.OR.B(N).EQ.0.0)202,200
202 IF(MISDATA.EQ.2)203,204
203 IJK=IJK+1
    ILL(IJK)=N
    GO TO 200
204 IF(.NOT.A(N).OR..NOT.B(N))200,205
205 IJK=IJK+1
    ILL(IJK)=N
200 CONTINUE
    IF(ILL(1).EQ.0)222,211
211 ICOONT=0
    DO 206 JC=1,NS
    DO 207 JV=1,IJK
    IF(JC.EQ.ILL(JV))208,207
207 CONTINUE
    AA(JC-ICOONT)=A(JC)
    BB(JC-ICOONT)=B(JC)
    GO TO 206
208 ICOONT=ICOONT+1
206 CONTINUE
    NS=NS-IJK
    DO 209 JT=1,NS
    A(JT)=AA(JT)
209 B(JT)=BB(JT)
222 CONTINUE
    RETURN
    END
    SUBROUTINE SUMDATA (A,B,C,AMEAN,BMEAN,CMEAN,ASD,BSD,CSD,SSA,SSB,SS
   C,RAB,RAC,RBC,RSSAB,RSSAC,RSSBC,M)
    DIMENSION A(400),C(400),B(400)
    AM=M
```

```
      AMEAN=BMEAN=CMEAN= SSA=SSB=SSC=RAB=RAC=RBC=RSSAB=RSSBC=RSSAC=0.0
      RSAB=RSAC=RSBC = 0.0
      DO 2 J = 1,M
      AMEAN=AMEAN+A(J)
      BMEAN=BMEAN+B(J)
    2 CMEAN=CMEAN+C(J)
      AMEAN=AMEAN/AM
      BMEAN=BMEAN/AM
      CMEAN=CMEAN/AM
      DO 3 K = 1,M
      SSA=SSA+(A(K)-AMEAN)**2
      SSB=SSB+(B(K)-BMEAN)**2
      SSC=SSC+(C(K)-CMEAN)**2
      RSAB=RSAB+A(K)*B(K)
      RSSAB=RSSAB+(A(K)-AMEAN)*(B(K)-BMEAN)
      RSAC=RSAC+A(K)*C(K)
      RSSAC=RSSAC+(A(K)-AMEAN)*(C(K)-CMEAN)
      RSBC=RSBC+B(K)*C(K)
    3 RSSBC=RSSBC+(B(K)-BMEAN)*(C(K)-CMEAN)
      ASD= SQRTF(SSA/(AM-1.0))
      BSD=SQRTF(SSB/(AM-1.0))
      CSD=SQRTF(SSC/(AM-1.0))
      RAB=(RSAB/AM-AMEAN*BMEAN)/(ASD*BSD)
      RBC=(RSBC/AM-BMEAN*CMEAN)/(BSD*CSD)
      RAC=(RSAC/AM-AMEAN*CMEAN)/(ASD*CSD)
      RETURN
      END
      FUNCTION PRBF (DA,DB,FR)
      PRBF=1.0
      IF(DA*DB*FR.EQ.0.0)RETURN
      IF(FR.LT.1.0)GO TO 5
      A=DA
      B=DB
      F=FR
      GO TO 10
    5 A=DB
      B=DA
      F=1.0/FR
   10 AA=2.0/(9.0*A)
      BB=2.0/(9.0*B)
      Z=ABS(((1.0-BB)*F**0.333333-1.0+AA)/SQRT(BB*F**0.666667+AA))
      IF(B.LT.4.0)Z=Z*(1.0+0.08*Z**4/B**3)
      PRBF=0.5/(1.0+Z*(0.196854+Z*(0.115194+Z*(0.000344+Z*0.019527))))**
     14
      IF(FR.LT.1.0)PRBF=1.0-PRBF
      RETURN
      END
      SUBROUTINE AMINMAX(A,M,AMIN,AMAX)
      DIMENSION A(400)
      AMIN=A(1)
      AMAX=A(1)
      DO 2 J = 2,M
      IF (A(J).LT.AMIN)AMIN=A(J)
      IF (A(J).GT.AMAX)AMAX=A(J)
    2 CONTINUE
      RETURN
      END
      SUBROUTINE PARAB(AA,BB,CC,XPT,YPT,AMIN,AMAX,AAA,NST)
      DIMENSION XPT(33),YPT(33)
      IF(AA.EQ.0)4,3
      PRINT 5
      FORMAT(5X,*THE COEFFICIENT OF X-SQUARED IS ZERO.*)
```

```
      RETURN
    3 H=(-BB)/(AA*2.0)
      AK=(-AA)*(BB*BB-4.0*AA*CC)/(4.0*AA*AA)
      AAA=1.0/(4.0*AA)
      YPT (1)=YPT (33)=AK+6.0*AAA
      YPT (2)=YPT (32)=AK+5.5*AAA
      YPT (3)=YPT (31)=AK+5.0*AAA
      YPT (4)=YPT (30)=AK+4.5*AAA
      YPT(5)=YPT(29)=AK+4.0*AAA
      YPT (6)=YPT (28)=AK+3.5*AAA
      YPT (7)=YPT (27)=AK+3.0*AAA
      YPT (8)=YPT (26)=AK+2.5*AAA
      YPT (9)=YPT (25)=AK+2.0*AAA
      YPT(10)=YPT (24)=AK+1.5*AAA
      YPT(11)=YPT (23)=AK+1.0*AAA
      YPT(12)=YPT (22)=AK+0.83*AAA
      YPT(13)=YPT (21)=AK+0.67*AAA
      YPT(14)=YPT (20)=AK+0.50*AAA
      YPT(15)=YPT (19)=AK+0.33*AAA
      YPT(16)=YPT (18)=AK+0.17*AAA
      YPT(17)=AK
      DO 6 L=1,16
      FT= SQRT(4.0*AAA*(YPT(L)-AK))
      LN =34-L
      XPT(L)=H-FT
    6 XPT(LN)=H+FT
      XPT(17)= H
      IC=JC=0
      DO 7 K = 1,16
      LK=34-K
      IF(XPT(K).LT.AMIN)IC=IC+1
    7 IF(XPT(LK).GT.AMAX)JC=JC+1
      IF(IC.GT.0)9,10
    9 NST=33-IC
      DO 8 M=1,33
      JKL=M+IC
      XPT(M)=XPT(JKL)
    8 YPT(M)=YPT(JKL)
   10 IF (JC.GT.0)11,12
   11 NST=NST-JC
   12 CONTINUE
      RETURN
      END
      END
```