The stated goals for the nine modules in this elementary math cluster, which is part of a series developed for the Adams State College Teacher Corps program, are that the intern will:

(a) understand the scope and sequence of topics in elementary mathematics,
(b) be familiar with a wide variety of activities and materials for use in a mathematics laboratory approach,
(c) know various alternative approaches to teaching given mathematics topics,
(d) utilize methods which convey understanding of concept rather than rote memorization,
(e) realize that pupil attitudes toward a subject are as important as aptitude,
(f) understand the relationship between mathematics and other elementary school curricula, and
(g) know the relevance of prevalent learning theories to the specific area of mathematics.

Each module contains a statement of the underlying rationale, the objectives for the module, enabling activities, and evaluation procedures. Module topics include set theory and operations, numbers, numeration systems, addition and subtraction, multiplication and division, geometric concepts, fractions and decimals, measurement, problem solving, and evaluation of mathematics learning.
The project presented or reported herein was performed pursuant to a Grant from the U. S. Office of Education, Department of Health, Education, and Welfare. However, the opinions expressed herein do not necessarily reflect the position or policy of the U. S. Office of Education, and no official endorsement by the U. S. Office of Education should be inferred.
Introduction

These modules are designed to enable the intern to develop a method of teaching elementary mathematics based on an activity-oriented, or mathematics laboratory, approach. The major emphasis is on involving the student in "discovering" or realizing certain mathematical concepts and facts based on participation, not rote memorization of facts or techniques.

The intern should realize that the major goal of elementary mathematics is problem-solving, not recitation of isolated facts. Unless mathematics is taught using intuitive and inductive procedures, the pupil will not know how to apply the proper basic facts to situations where they are needed.

The major resources for these modules are: TEACHING MATHEMATICS IN THE ELEMENTARY SCHOOL by Lola June May, The Free Press, 1970, and LEARNING MATHEMATICS THROUGH ACTIVITIES by S. Jeanne Kelley, Freefield Associates, Inc., 1973. The intern is encouraged to use any other materials on teaching elementary mathematics which are available to him/her through the Mathematics Laboratory or the Learning Resources Center at Adams State College.

The intern is encouraged to consult the college facilitator about any or all portions of the modules. No material, however prepared, is completely self-teaching. Whenever possible, the intern should utilize the methods learned in these modules in the on-site classrooms.
Goals

1. The intern will have a thorough understanding of the scope and sequence of topics in mathematics in the elementary grades.

2. The intern will be familiar with a wide variety of activities and manipulative materials for use in a mathematics laboratory approach to teaching mathematics.

3. The intern will know a variety of alternative approaches to teaching any given topic in elementary mathematics.

4. The intern will utilize methods which convey understanding of concepts and facts, not just rote memorization.

5. The intern will realize that pupil attitudes toward a subject are as important as aptitude in the learning of any topic.

6. The intern will understand the relationship between mathematics and other subject areas in the elementary curriculum.

7. The intern will know the relevance of the prevalent theories of learning to the specific area of mathematics.

Competency Goal Statement

Upon the completion of this series of the modules the intern will:

(1) Present a one-week teaching presentation on any concept or portion of any concept included in this series of modules. The teaching presentation should meet all of the criteria on the "Criteria Checklist" included at the end of this series of modules.
Time

Each module within this series of modules is designed to be completed within one week. The intern should pace his/her own time in order to complete these modules within the allotted time of the quarter he/she registered for the course.

Prerequisites

The intern must have completed Math 108a, Mathematics for Elementary Teachers, prior to beginning this series of modules.

Resources


TEACHING SET CONCEPTS AND OPERATIONS

Rationale:

Since the basic arithmetic operations in elementary mathematics and all geometric concepts and operations are based on set concepts and operations, the intern must be able to teach set concepts and operations to elementary pupils.

Objectives:

Know the proper grade placement for the set concepts of elements and subsets?

Know the proper grade placement for the set operations of union, intersection, complements, and Cartesian products.

Know the proper grade placement for the various set notations.

Know a wide variety of activities and manipulative materials for introducing and reviewing the set concepts and operations.

Be able to use Venn diagrams and Euler circles to illustrate set concepts and operations.

Use the correct set concepts and operations to introduce arithmetic operations and geometric concepts.

Use one-to-one, many-to-one, and one-to-many correspondences to teach students the concepts of equivalent and nonequivalent sets.

Enabling Activities:

Read and study the following materials:

Chapter 3 in HOW CHILDREN LEARN MATHEMATICS by Copeland, Macmillan, 1970.


Evaluation:

The intern will be given a written examination by the on-site instructor at the completion of this module.
TEACHING NUMBERS, NUMERALS, AND NUMERATION SYSTEMS

Rationale:

A thorough understanding of numeration systems is essential to the elementary pupil in order to apply basic arithmetic facts to higher order operations. Without a proper understanding of numbers and numeration systems, mathematics learning becomes only a disassociated series of techniques which have no meaning to the student.

Objectives:

Know how and when to use the concepts of equivalent and non-equivalent sets to teach rational counting.

Know the proper way to teach students to write the ten basic numerals.

Be able to correctly distinguish between cardinal and ordinal number concepts and numerals.

Be able to teach students to distinguish between numbers and numerals.

Use one-to-one correspondences between sets to teach students the number of elements in a set.

Use many-to-one and one-to-many correspondences between sets to teach inequalities of numbers.

Use the number line to illustrate the set of counting numbers and the set of whole numbers and relationships between numbers in each set.

Use grouping techniques such as place-value charts, abaci, and physical objects to teach place value in positional numeration systems, base ten, and others.

Introduce historically important numeration systems such as Egyptian, Roman, Babylonian, and Mayan to help students better understand the base ten numeration system, Hindu-Arabic, which we use.

Use tens, hundreds, thousands, etc., in expanded form and exponential form at the proper grade levels.

Introduce decimal notation at the proper grade level using an extended numeration system and fractions.
Know and use a variety of activities and manipulative materials to introduce and teach number concepts, numeration systems, and place value.

Enabling Activities:

Read and study the following materials:

Chapter 4 in *HOW CHILDREN LEARN MATHEMATICS* by Copeland, Macmillan, 1970.


Chapters 2 and 3 in *TEACHING ELEMENTARY SCHOOL MATHEMATICS* by Spitzer, Houghton Mifflin, 1967.

Work the exercises on pages 49-50 and 165 in *TEACHING MATHEMATICS IN THE ELEMENTARY SCHOOL* by May.

Evaluation:

The intern will be given a written examination by the on-site instructor at the completion of this module.
TEACHING ADDITION AND SUBTRACTION OF WHOLE NUMBERS

Rationale:
A proper understanding of addition and subtraction of whole numbers provides a basis for the extension of these operations to other sets of numbers and enables the teacher to relate other operations on whole numbers to these operations. The pupil must realize the inverse relationship between addition and subtraction.

Objectives:
- Introduce addition of whole numbers through the union of disjoint sets by means of physical examples.
- Know a large number of activities and manipulative materials to use to teach addition and subtraction concepts and facts.
- Use the proper notation for addition and subtraction of whole numbers.
- Develop the basic facts and families of facts in addition and subtraction by a wide variety of means.
- Know the proper grade placement for the use of addition and subtraction on single digit and multi-digit numerals.
- Create a proper balance between the development of concepts and drill in addition and subtraction.
- Know how to use place value and the principles of a positional numeration system to teach regrouping in both addition and subtraction.
- Continually relate addition and subtraction of whole numbers to physical applications.
- Use the commutative and associative properties to assist in teaching addition and subtraction.
- Introduce subtraction by relating it to set operations and to addition.
- Use the proper terminology for addends, sums, missing addends, differences, and subtraction.
- Know a wide variety of algorithms for addition and subtraction and for checking these operations.
- Introduce shortcuts in addition and subtraction only after the basic concepts have been understood by the pupils.
Utilize the number line to enhance understanding of addition and subtraction of whole numbers.

Enabling Activities:

Read and study the following materials:

Chapter 5 in HOW CHILDREN LEARN MATHEMATICS by Copeland, Macmillan, 1970.


Chapters 4 and 5 in TEACHING ELEMENTARY SCHOOL MATHEMATICS by Spitzer, Houghton Mifflin, 1967.

Work the exercises on page 88 in TEACHING MATHEMATICS IN THE ELEMENTARY SCHOOL by May.

Evaluation:

The intern will be given a written examination by the on-site instructor at the completion of this module.
Module #4

TEACHING MULTIPLICATION AND DIVISION OF WHOLE NUMBERS

Rationale:

The elementary student must understand multiplication and division of whole numbers before he can use these operations on other sets of numbers. The student must realize that multiplication can be interpreted independently of addition. A thorough understanding of place value is essential to learning and using the algorithms for multiplication and division. The elementary student must also understand the inverse relationship between multiplication and division of whole numbers so he can apply this to multiplication and division on other sets of numbers.

Objectives:

Prepare students for multiplication and division of whole numbers by a variety of physical examples.

Relate multiplication and division to the Cartesian product of sets as well as repeated addition or subtraction.

Utilize the Division Algorithm to develop the concept of division both with and without remainders.

Know a wide variety of activities and manipulative materials to introduce and drill multiplication and division.

Know the proper grade placement for the introduction of multiplication and division of single digit and multi-digit numerals.

Utilize the number line to enhance understanding of multiplication and division.

Know a large number of different algorithms for performing multiplication and division.

Develop the concept that division is related to multiplication.

Develop the basic facts and families of facts by a wide variety of activities.

Use correctly the terminology of factors, product, dividend, divisor, and remainder.

Create a proper balance between the development of concepts and drill in multiplication and division.

Develop the distributive properties of multiplication through physical models.
Develop an understanding by the students of the fact that zero may not be a divisor.

Enabling Activities:

Read and study the following materials:

Chapter 6 in *HOW CHILDREN LEARN MATHEMATICS* by Copeland, Macmillan, 1970.


Chapters 6 and 7 in *TEACHING ELEMENTARY SCHOOL MATHEMATICS* by Spitzer, Houghton Mifflin, 1967.

Work the exercises on pages 115 and 140 in *TEACHING MATHEMATICS IN THE ELEMENTARY SCHOOL* by May.

Evaluation:

The intern will be given a written examination by the on-site instructor at the completion of this module.
Rationale:

The elementary student must learn geometric concepts so that he can better understand the physical environment in which he lives. Geometric concepts are one means of introducing fractional numbers to elementary students. All measurement activities are based on an understanding of certain basic geometric concepts.

Objectives:

Know the correct names for familiar two-dimensional and three-dimensional geometric objects and use these names in the elementary classroom.

Use objects in the classroom and around the school to teach children the use of the proper names.

Know the proper grade placement of geometric concepts, terminology, and notation.

Use inductive learning activities to teach students the relationships between various geometric objects in both two and three dimensions.

Know the difference between nonmetric and metric geometric concepts and the proper grade placement of these topics.

Utilize a wide variety of activities and manipulative materials to teach geometric concepts.

Teach intuitive topological concepts such as closed curves, simple curves, simple closed curves, and interior and exterior.

Use set concepts and operations to help define curves, lines, line segments, rays, angles, polygons, polyhedra, areas, perimeter, faces, edges, vertices, and volume.

Develop intuitive concepts of congruent and similar figures, and figures of equal areas.

Enabling Activities:

Read and study the following materials:

Chapters 8, 9, and 13 in HOW CHILDREN LEARN MATHEMATICS by Copeland, Macmillan, 1970.


Chapter 12 in TEACHING ELEMENTARY SCHOOL MATHEMATICS by Spitzer, Houghton Mifflin, 1967.

Work the exercises on page 265 in TEACHING MATHEMATICS IN THE ELEMENTARY SCHOOL by May.

Evaluation:

The intern will be given a written examination by the on-site instructor at the completion of this module.
TEACHING FRACTIONAL AND DECIMAL CONCEPTS

Rationale:
Fractional and decimal concepts are essential if a student is to apply mathematics to many everyday situations. Use of money and measurement activities requires an understanding of fractional and decimal concepts. A natural extension of division of whole numbers depends on the use of both fractional and decimal concepts.

Objectives:

Develop an understanding of equal areas and congruent parts of geometric figures before introducing fractions.

Use correct notation and terminology for fractions and parts of a fractional numeral.

Use sets of discrete objects, geometric figures, and the number line to introduce fractional and decimal concepts.

Know the proper grade placement of fractional and decimal concepts.

Know a wide variety of activities and manipulative materials for teaching fractional and decimal concepts and operations.

Develop algorithms for addition, subtraction, multiplication, and division of fractional numbers from physical activities.

Develop the concepts and notation for decimals from ideas already known for fractional numbers.

Develop algorithms for addition, subtraction, multiplication, and division of decimals from algorithms previously developed for fractional numbers.

Stress understanding of concepts and algorithms rather than memorization of shortcuts or techniques.

Relate the development of decimal concepts to the child's knowledge of the monetary system.

Use geoboards and Cuisenaire rods to develop fractional concepts and operations.

Enabling Activities:
Read and study the following materials:


Chapters 9 and 10 in TEACHING ELEMENTARY SCHOOL MATHEMATICS by Spitzer, Houghton Mifflin, 1967.

Work the exercises on pages 197, 215, and 227 in TEACHING MATHEMATICS IN THE ELEMENTARY SCHOOL by May.

Evaluation:

The intern will be given a written examination by the on-site instructor at the completion of this module.
TEACHING MEASUREMENT

Rationale:

Many of the "practical" applications of mathematics for the elementary student arise out of a use of measurement. The elementary student must be able to estimate measurements as well as measure accurately with a given unit of measure. A sound knowledge of geometric concepts will enable the student to correctly distinguish between measurement of length, area, and volume. The student must understand the need for standard units of measure.

Objectives:

Develop in the students the realization of a need for measures.

Use many different arbitrary units of measure for length, temperature, area, and volume before introducing standard units.

Know the proper grade placement for measurement topics and precision of measurement.

Develop intuitively all measures before the use of specific measuring units.

When specific measurement units are employed, use predominately units from the metric system.

Use only approximate conversions between the metric system and the English system if these conversions have to be done at all.

Use correct terminology and notation in all measurement activities.

Utilize home, community, and school resources in measurement activities.

Know a wide variety of activities and manipulative materials to teach measurement.

Enabling Activities:

Read and study the following materials:

Chapters 10 and 12 in HOW CHILDREN LEARN MATHEMATICS by Copeland, Macmillan; 1970.


Chapter 13 in TEACHING ELEMENTARY SCHOOL MATHEMATICS by Spitzer, Houghton Mifflin, 1967.

Work the exercises on page 288 in TEACHING MATHEMATICS IN THE ELEMENTARY SCHOOL by May.

Evaluation:

The intern will be given a written examination by the on-site instructor at the completion of this module.
TEACHING PROBLEM SOLVING

Rationale:

All of the mathematical facts an elementary student may learn are useless unless he can apply them correctly in problem solving situations. The student must be continually exposed to physical situations when learning mathematical concepts and facts so that he can see how to apply the concepts and facts in problem solving situations.

Objectives:

When teaching any topic, use a large number of physical applications.

Have the students make up their own problems in story form from pictures or classroom situations.

Given a "story problem", set up a mathematical sentence or sentences for solving the problem.

Use a classroom store to teach problem solving with money.

Introduce ratio, proportion, and percentage using fractional and decimal concepts already developed.

Stress the need to read and reread problems before attempting solutions.

Using a variety of examples, teach students to decide what data is relevant in a problem.

Throughout the course emphasize the need to estimate answers before actual computations are carried out.

Assist the students in acquiring the ability to make "educated guesses" about problems.

Emphasize inductive reasoning rather than deductive reasoning in solution of problems.

Base all instruction on the premise that problem solving, not rote memorization, is the goal of elementary mathematics.

Enabling Activities:

Read and study the following materials:
Chapter 7 in *HOW CHILDREN LEARN MATHEMATICS* by Copeland, Macmillan, 1970.


Chapter 8 in *TEACHING ELEMENTARY SCHOOL MATHEMATICS* by Spitzer, Houghton Mifflin, 1967.

Work the exercises on pages 277 and 300 in *TEACHING MATHEMATICS IN THE ELEMENTARY SCHOOL* by May.

Evaluation:

The intern will be given a written examination by the on-site instructor at the completion of this module.
EVALUATION OF MATHEMATICS LEARNING

Rationale:

Evaluation must be an integral part of the teaching method. All evaluation must be used to help the student learn, not just for grading purposes. Evaluation must include observation, an interview, and projects, in addition to paper and pencil activities.

Objectives:

Understand that evaluation should be an extension of the learning process, not separate from it.

Utilize evaluation techniques as diagnostic aids, not just grading devices.

Be familiar with several standardized elementary mathematics tests, and know the advantages and disadvantages of such tests.

Construct pre-tests and post-tests for a variety of topics in the elementary mathematics curriculum.

Evaluate pupil progress by means of daily observation, as well as by written measures.

Understand the concept of a spiral curriculum to assist in determining proper achievement levels in a topic.

Evaluate pupil attitude as well as achievement and aptitude.

Use the results of pupil evaluation to continually improve the presentation of material.

Enabling Activities:

Read and study the following materials:

Chapters 14 and 15 in HOW CHILDREN LEARN MATHEMATICS by Copeland, Macmillan, 1970.

Chapters 14 and 15 in TEACHING ELEMENTARY SCHOOL MATHEMATICS by Spitzer, Houghton Mifflin, 1967.

Evaluation:

The intern will be given a written examination by the on-site instructor at the completion of this module.
The intern should arrange with a teacher in his/her local school a teaching presentation of at least one-week duration. This presentation should include the teaching of any concept or portion of a concept included in Modules One through Eight in this series of modules. The planning for this teaching presentation must be in written form and approved by the on-site instructor or cooperating teacher before the teaching presentation is made. The on-site instructor or cooperating teacher will use the following criteria checklist to evaluate the intern's teaching presentation. The intern should plan his/her presentation according to the criteria included on the following checklist.

This "criteria checklist" is constructed for the cooperating teachers of interns, team leaders, or on-site instructors. This checklist should be made during the time the intern is in an on-going classroom situation and must be completed by an intern's cooperating teacher, team leader, or on-site instructor and given to the college facilitator of Teaching Elementary School Math (Ed 333) before the intern is given credit and assigned a grade for the completion of this series of modules. The intern must meet the criteria of each competency listed below and the standards that are stated for minimal achievement of each competency. It is the intern's responsibility to arrange with the cooperating teacher, team leader, or on-site instructor the time for the demonstration.
of each competency listed below. The cooperating teacher, team leader, or on-site instructor will then complete and sign the criteria checklist.

I. Competency Goal Statement:

The intern will present a one-week teaching presentation on any concept or portion of any concept included in this series of modules. The teaching presentation should meet all of the criteria on the "Indicator Checklist".

a. Indicators - The cooperating teacher, team leader, or on-site instructor should complete the following checklist and rate (yes or no) the intern on each stated behavioral indicator.

"Concept Development Indicator Checklist"

<table>
<thead>
<tr>
<th>Concept Developed</th>
<th>Grade Level</th>
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1. Was the vocabulary and reading level appropriate for the grade level? ___ ___
2. Was the concept intended for the proper grade level? ___ ___
3. Was the presentation limited to the development of a single concept? ___ ___
4. Was more than one physical interpretation used? ___ ___
5. Were all prerequisite concepts and facts previously covered in the curriculum? ___ ___
6. Did the instructor ask meaningful questions which allowed the students to draw their own conclusions? ___ ___
7. Are adequate activities given for the student to reinforce the concept? ___ ___
8. Were students expected to verbalize the concept too quickly? ___ ___
9. Did the instructor try to tie the new concept to concepts already developed by the students?

10. Did the instructor encourage the students to ask questions throughout the presentation?

11. Did the instructor's attitude toward the topic and his response to student questions encourage more student participation?

b. Context - On-going classroom situation.

c. Criteria - The intern should meet all of the indicators with a rating of "yes". If the intern is given a rating of "no" on any criterion, he/she must present another one-week teaching demonstration meeting all of the criteria with a rating of "yes".

d. Procedures - The cooperating teacher, team leader, or on-site instructor uses a "yes/no" rating scale during or after the observation of the intern's presentation of this concept lesson.

e. Operationalized Statement - Given access to an on-going, regularly scheduled classroom of students, an intern will present a one-week teaching presentation on any concept or portion of any concept included in this series of modules. The teaching presentation will meet all of the criteria on the "Indicator Checklist" with a rating of "yes".

Signature (Cooperating Teacher, Team Leader, or On-Site Instructor)