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ABSTRACT

This document is one of a series of eight Teacher Education Modules developed by Adams State College Teacher Corps Program. As a result of completion of this module, which is itself a cluster of nine mathematics learning modules, the elementary education student will: a) gain a knowledge and understanding of the concept of elementary school mathematics; b) have a deeper understanding of the specific topics than would be expected of an elementary student; c) realize and appreciate the logical development of material from precise definitions based on intuitive concepts to facts derived from these concepts; d) increase his confidence in his own mathematical ability; e) see mathematics as a body of interdependent knowledge; and f) realize that mathematics proficiency in the mechanical processes of mathematics is not sufficient for the present-day student. Each module contains a statement of the underlying rationale; a statement of the objectives of the module; enabling activities, which are generally taken from other reference works; and procedures for evaluation. The topics of the modules include a) logic; b) set concepts; c) whole numbers and counting; d) operations on sets; e) binary operations; f) addition, subtraction, and inequalities of whole numbers; g) multiplication and division and the distributive properties of operations on the set of whole numbers; h) fractions; and i) geometric concepts. (HMD)

ADAMS STATE COLLEGE

MATHEMATICS FOR ELEMENTARY TEACHERS

(MATH 108a)

Prepared for the

Adams State College

Teacher Corps Program

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TEACHER CORPS PROGRAM

CYCLE 8

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Introduction

This series of modules is based on current mathematics curricula in the elementary grades. The material is presented, not at a level for elementary pupils, but for college students who are preparing to teach in the elementary grades. The person using the modules should continually consult recent elementary mathematics textbooks to see where the topics fit into the curriculum.

Since mathematics is a logical development of certain fundamental ideas, the modules must be studied in the following order: (1) Logic (2) Set Concepts (3) Whole Numbers and Counting (4) Operations on Sets (5) Binary Operations (6) Addition, Subtraction and Inequalities of Whole Numbers (7) Multiplication, Division of Whole Numbers and the Distributive Properties of Operations on the Whole Numbers (8) Fractions (9) Geometric Concepts.

The principal reference for the modules is PRINCIPLES OF ARITHMETIC AND GEOMETRY by Carl B. Allendoerfer, MacMillan, 1971. Additional references are given to broaden the intern's understanding of the topics. Each of the references contains many exercises, with some answers provided, which the student should work to gain proficiency in the use of the concepts introduced.

The intern using the modules should not hesitate to call upon the college facilitator for assistance with any part of any module. No material, however prepared, is completely self-teaching. Some interaction with others is usually necessary to fully integrate the concepts and methods presented. Each module has a set of exercises whose purpose is to assist the intern to better understand the material. There are three examinations covering various groups of modules, and a final examination is also provided. Mathematics cannot, and should not, be taught as a succession of isolated concepts, facts, and processes. These must be seen as interlocking ideas and procedures. The purpose of the final examination is to help facilitate the bringing together of all of these ideas.

Goals

1. The intern will gain a knowledge of an understanding of the content of elementary school mathematics.
2. The intern will have a deeper understanding of the specific topics than is expected of an elementary student.
3. The logical development of the material from precise definitions based on intuitive concepts to facts derived from these concepts and definitions should be realized and appreciated by the intern.
4. Successful completion of each module should assist the intern to increase his confidence in his own abilities in mathematics.
5. The intern should begin to see that mathematics is not just a cornucopia of isolated facts, but is a body of interdependent knowledge.
6. The use of logic in set concepts and operation and then the use of these set concepts and operations in the development of skills in arithmetic and geometry should improve the intern's understanding of the interrelationships in all of mathematics.
7. Proficiency in the mechanical processes of arithmetic is not sufficient for the present-day student, and the development of the ideas in the modules should convey this fact.
8. Above all, the attitude of the intern studying these modules should be one of excitement and anticipation. Mathematics is not the most important subject in the curriculum, but neither should it be the dullest or most oppressive in the curriculum.

Time

Each module within this series of modules is designed to be completed within one week. The intern should pace his/her own time in order to complete these modules within the allotted time of the quarter he/she registered for Math 108a.

Prerequisites

This series of modules represents and introductory course in mathematics for elementary teachers. The modules are arranged in a logical order relative to the logical development of certain fundamental ideas in mathematics. Therefore, each module within this series is a prerequisite for the succeeding module.

Resources

Allendoerfer, Carl B. Principles of Arithmetic and Geometry. New York: The Macmillan Co., 1971.

Copeland. Mathematics and the Elementary Teacher. 2nd ed. Saunders, 1972.

Kelley, John L. and Richert, Donald. Elementary Mathematics for Teachers. San Francisco: Holden-Day, 1970.

Peterson, John A. and Hashisaki, Joseph. Theory of Arithmetic. 3rd ed. New York: John Wiley and Sons, Inc., 1971.

Wheeler, Ruric E. Modern Mathematics: An Elementary Approach. 2nd ed. Belmont, California: Brooks/Cole Publishing Co., 1970.

LOGIC

Rationale:

A thorough understanding of the meaning of "and", "or", "not", and "if, then" is essential to the definition and use of other concepts in the elementary mathematics program. The differences between proofs, examples, and counterexamples must be understood by the prospective elementary teacher.

Objectives:

Define a proposition and an open sentence.
Distinguish given statements as propositions or open sentences or neither.
Convert open sentences into propositions by:
(a) substitution for the variable or variables;
(b) prefixing "For all x".
Find the truth value of propositions of the form: "For all x, p_x", and use counterexamples to prove that such propositions are false (when they are indeed false.)
Form conjunctions of open sentences and determine the truth value of the result when values are substituted for the variables.
Form disjunctions of open sentences and determine the truth value of the result when values are substituted for the variables.
Form the negations of simple open sentences by using:
(a) "It is false that" (b) "Not ... " (c) insertion of "not" in a suitable place.
Find the truth value of the negation of an open sentence when a value is substituted for x.
Define what is meant by the logical equivalence of two open sentences.
Determine when two open sentences are logically equivalent.

Enabling Activities:

Read and study the following materials:
Chapter 2 in PRINCIPLES OF ARITHMETIC AND GEOMETRY by Allendoerfer, Macmillan, 1971:
Chapter 4 in MATHEMATICS AND THE ELEMENTARY TEACHER by Copeland, second edition, Saunders, 1972:
Chapter 7 in ELEMENTARY MATHEMATICS FOR TEACHERS by Kelley and Richert, Holden-Day, 1970
Chapter 1 in MODERN MATHEMATICS: AN ELEMENTARY APPROACH by Wheeler, second edition, Brooks/Cole, 1970
Complete "Handout # 1".

Evaluation:

The intern will be given a problem solving, written examination by the college facilitator or on-site instructor at the completion of Modules I, II and III. The intern should use the objectives of each module as a study guide for the examination.

Work the following exercises:

1. Given that p is a true proposition and q is a false proposition, find the truth value of:

   a. p and q
   b. not p
   c. p or q
   d. q and (not p)
   e. (not p) or (not q)
   f. not (q or p)
   g. if p, then q
   h. if (not p), then q
   i. if q, then p
   j. if (not q), then (not p)
   k. p if and only if q
   l. (not p) if and only if q
   m. ((not q) or p) and p
   n. if p, then ((not p) or q)
   o. (not p) if (q or (not p))
   p. if (p or q), then (p and q)

2. Write a negation of each of the following:

   a. $x + 3 = 7$ and $2x - 1 \neq 5$
   b. $x + 5 < 4$ or $x + 1 = 3$
   c. The moon is blue or the sun is red.
   d. All men are mortal.
   e. Some men are sane.

3. Fill in each of these truth tables:

p	not p
T	
F	

p	q	p or q
T	T	
T	F	
F	T	
F	F	

p	q	p and q
T	T	
T	F	
F	T	
F	F	

p	q	if p, then q
T	T	
T	F	
F	T	
F	F	

SET CONCEPTS

Rationale:

The set concepts of element, subset, and proper subset are the foundations of the development of both ordinal and cardinal number concepts in mathematics. These set concepts are also used in all areas of mathematics including arithmetic, geometry, fractions, rational numbers, real numbers, and algebra.

Objectives:

Write unambiguous descriptions of sets.
Describe a set by listing its elements and using proper notation.
Translate between set notation and ordinary English in giving descriptions of sets.
Apply the concept and notation of "element of a set."
Define the empty set and write it in standard notation.
Write sets defined in terms of properties in "set-builder" notation.
Translate between set-builder notation and ordinary English.
Define the universal set and hypothesize reasonable guesses as to the universal set implied when it is not explicitly stated.
Use set-builder notation to describe sets (a) which have two properties in common, (b) which have either or both of two properties.
Recognize the equality or inequality of sets.
Recognize subsets and proper subsets, make use of correct notations for these, and distinguish them from elements of a set.
List all of the subsets of a given finite set.
Compute the number of subsets of a given finite set.

Enabling Activities:

Read and study the following materials:
Chapter 3 in PRINCIPLES OF ARITHMETIC AND GEOMETRY by Allendoerfer, Macmillan, 1971
Chapter 4 in MATHEMATICS AND THE ELEMENTARY TEACHER by Copeland, second edition, Saunders, 1972
Chapter 1 in ELEMENTARY MATHEMATICS FOR TEACHERS by Kelley and Richert, Holden-Day, 1970
Chapter 2 in THEORY OF ARITHMETIC by Peterson and Hashisaki, third edition, Wiley, 1971
Chapter 2 in MODERN MATHEMATICS: AN ELEMENTARY APPROACH by Wheeler, second edition, 1970
Complete "Handout #2".

Evaluation:

The intern will be given a problem solving, written examination by the college facilitator or on-site instructor at the completion of Modules I, II and III. The intern should use the objectives of each module as a study guide for the examination.

Handout #2
Module #2

Work the following exercises:

1. Tell which of the following are unambiguous descriptions of sets:

   a. the set of all fat people
   b. the set of people whose weight is more than 250 pounds
   c. the set of students at ASC in 1972 whose GPA was 3.5 or higher
   d. the set of good books

2. List the elements of each of these sets:

   a. $\{x \mid x \text{ is a whole number and } x < 7\}$
   b. $\{x \mid x \text{ is a counting number and } 2 \leq x \leq 10\}$
   c. $\{x \mid x \text{ is the state capital of Colorado}\}$
   d. $\{x \mid x \text{ is a whole number and } x \text{ is less than 4 and more than } 3\}$

3. Use set builder notation to describe each of these sets:

   a. $\{a,e,i,o,u\}$
   b. $\{4,5,6,7,8\}$
   c. $\{0,2,4,6,\ldots\}$
   d. $\{\text{red, white, blue}\}$

4. Let $S = \{0,1,2,3,4,5,6\}$ and tell whether each of the following is true or false:

   a. $3 \in S$
   b. $5 \subseteq S$
   c. $\emptyset \subset S$
   d. $\{1,4,6\} \subset S$
   e. $\{0\} = \emptyset$
   f. $\{2\} \in S$
   g. $\{1,3,5,6,2,4\} = S$
   h. $\{0,2,4,6,8\} \subseteq S$

5. Write all of the subsets of $\{x,y,z,w\}$

WHOLE NUMBERS AND COUNTING

Rationale:

Since the whole numbers form the basis for the majority of the work in mathematics in the elementary grades, the prospective elementary teacher must understand fully the connections between the number concepts and physical applications of numbers. The counting procedures based on one-to-one correspondences and properties of one-to-one correspondences and ordination are particularly relevant today due to the research of Piaget and others. Many teachers do not realize that these fundamental concepts must be continually tied to physical objects and applications. They must learn how to present these ideas in many different ways so that each child can integrate the concepts into his own experiences. Just memorizing a sequence of sounds and symbols is not sufficient for the child to understand and use the number concepts.

Objectives:

Recognize one-to-one correspondences between sets, which may be finite or infinite.
Set up one-to-one correspondences between pairs of sets when this is possible, and recognize when this is impossible. Sets may be finite or infinite.
Given a one-to-one correspondence between two sets, find other such correspondences.
Determine when two sets are equivalent.
Distinguish between equivalence of sets and equality of sets.
Recognize the reflexive property of set equivalence.
Recognize the symmetrical property of set equivalence.
Recognize the transitive property of set equivalence.
Distinguish between finite and infinite sets.
Establish the nonequivalence of pairs of finite sets.
Define and recognize an equivalence relation.
Define the cardinal number of a finite set.
Apply the definition of the cardinal number of a finite set to find such cardinal numbers.
Describe the properties of the cardinal number of a finite set.
Distinguish the cardinal number of a set by using the number line.
Distinguish the cardinal number of a finite set by using an intermediate set.

Enabling Activities:

Read and study the following materials:
Chapters 4 and 5 in PRINCIPLES OF ARITHMETIC AND GEOMETRY by Allendoerfer, Macmillan, 1971
Chapters 2 and 4 in MATHEMATICS AND THE ELEMENTARY TEACHER by Copeland, second edition, Saunders, 1972
Chapter 1 in ELEMENTARY MATHEMATICS FOR TEACHERS by Kelley and Richert, Holden-Day, 1970
Chapter 3 in THEORY OF ARITHMETIC by Peterson and Hashisaki, third edition, Wiley, 1971
Chapter 2 and 3 in MODERN MATHEMATICS: AN ELEMENTARY APPROACH by Wheeler, second edition, Brooks/Cole, 1970
Complete "Handout #3".

Evaluation:

At the completion of this module the intern will be given a problem solving, written examination. The examination will cover Modules I, II, and III. The on-site instructor or college facilitator will administer this examination. Make arrangements for the time and place.

Work the following exercises:

1. Tell which of the following pairs of sets may be placed in a one-to-one correspondence:

   a. $\{0,2,4,6\}$ $\{a,b,c\}$
   b. $\{1,3,5\}$ $\{x,y,z\}$
   c. $\{0,2,4,6,8,\ldots\}$ $\{1,3,5,7,\ldots\}$
   d. $\{1,2,3,4,\ldots\}$ $\{0,1,2,3,\ldots\}$
   e. $\{x,y,z,w,a\}$ $\{a,w,z,y,x\}$

2. Give an example of two sets which are equivalent but not equal.

3. Give an example to illustrate that the equivalence of sets is:

   a. reflexive
   b. symmetric
   c. transitive

4. Define an equivalence relation.

5. For each of the following relations, tell whether they are reflexive, symmetric, or transitive:

   a. "less than" defined on the set of whole numbers,
   b. "is perpendicular to" defined on the set of lines in a plane,
   c. "is a subset of" defined on the set of subsets of a set.

6. Give the cardinal number of each of these sets:

   a. $\{a,b,c\}$
   b. $\{0,1,2,5\}$
   c. $\{\ \}$
   d. $\{1,2,3,4\}$
   e. $\{0,1,2,3,\ldots,10\}$

7. Suppose that A and B are sets and $A \sim B$ and $n(A)=7$. What is $n(B)$?

8. Suppose that A and B are sets and $n(A) = 10$ and $n(B) = 10$. Is $A = B$?

OPERATIONS ON SETS

Rationale:

The operations of union, intersection, complementation, and Cartesian products of sets are the operations performed on physical and semiabstract elements and sets which later lead to the arithmetic operations of addition, subtraction, multiplication, and division of whole numbers as well as operations and properties in geometry, probability, and other areas of mathematics. The elementary teacher must not only be able to perform these operations on sets, but he must also understand why they are essential to the arithmetic operations of whole numbers which are the major content areas in elementary mathematics.

Objectives:

Find the intersection of two sets.
Use the term "disjoint sets" properly.
Describe the intersection of sets in set-builder notation.
Define intersection and union of sets.
Distinguish intersection and union using Venn diagrams.
Complete the truth tables for intersection and union.
Define multiple unions and intersections.
Define and prove the commutative properties of set intersection and union.
Define the absolute complement of a set.
Distinguish the absolute complement of A.
Prove that $(A \cup B)' = A' \cap B'$ and $(A \cap B)' = A' \cup B'$
Define the complement of B relative to A.
Find the complement of B relative to A.
Describe the concept of an ordered pair.
Define the Cartesian product of two sets.
Translate $A \times \emptyset$, $\emptyset \times A$, and $\emptyset \times \emptyset$ in simpler form.
Show that AxB is not commutative.
Recognize in general that AxBxC, (AxB)xC, and Ax(BxC) are not equal.
Recognize that for all sets $A \times B \times C \sim (A \times B) \times C \sim A \times (B \times C)$.

Enabling Activities:

Read and study the following materials:
Chapters 6 and 7 in PRINCIPLES OF ARITHMETIC AND GEOMETRY by Allendoerfer, Macmillan, 1971
Chapter 4 in MATHEMATICS AND THE ELEMENTARY TEACHER by Copeland, second edition, Saunders, 1972
Chapter 2 in ELEMENTARY MATHEMATICS FOR TEACHERS by Kelley and Richert, Holden-Day, 1970
Chapter 2 in THEORY OF ARITHMETIC by Peterson and Hashisaki, third edition, Wiley, 1971
Chapter 2 in MODERN MATHEMATICS: AN ELEMENTARY APPROACH by Wheeler, second edition, 1970
Complete "Handout #4".

Evaluation:

The intern will take a problem solving, written examination on the objectives of this module at the completion of this module and modules #5 and #6. Use the objectives of these modules as a study guide to prepare for the examination.

Work the following exercises:

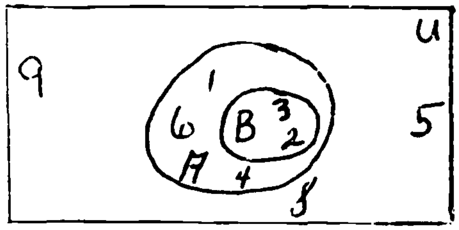
1. Let $U = \{0,1,2,3,\ldots,10\}$, $A = \{1,5,6,7\}$, $B = \{2,4,6\}$
$C = \{5,6,9\}$ find:

a. $A \cup B$	e. $A \cap C$	i. $(A \cup B)'$
b. $A \cap B$	f. $A \cap B \cap C$	j. $A' \cup B'$
c. A'	g. $A \cup B \cup C$	k. $A' \cap B'$
d. $B \cap C$	h. $(A \cap B)'$	l. $(A \cup B \cup C)'$

Draw a Venn diagram to represent the sets U,A,B,C

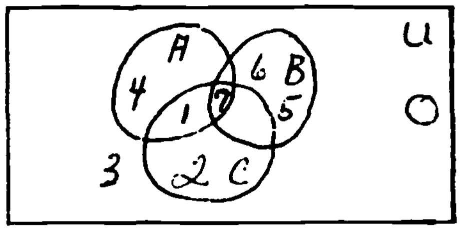
2. From the Venn diagram on the right, find:

a. U	e. $A \cup B$
b. A	f. $A \setminus B$
c. B	g. A'
d. $A \cap B$	h. B'



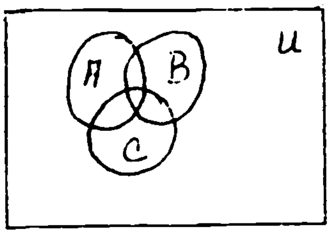
3. From the Venn diagram on the right, find:

a. U	f. C'
b. $A \cap B$	g. A'
c. $A \cap B \cap C$	h. $B \cap C$
d. $C \cap A$	i. $(B \cup C)'$
e. $A \cup B$	

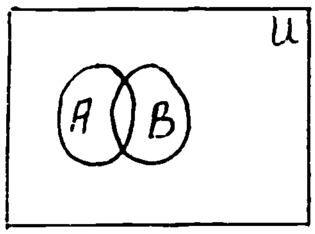


4. In the accompanying Venn diagrams, shade the regions mentioned:

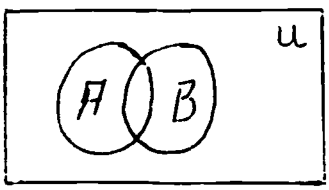
a. $(A \cap B) \cup C$



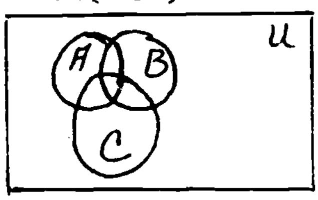
b. $(A \cup B)'$



c. $(A' \cap B')'$



d. $A \cap (B \cup C)$



5. Let $A = \{3,4,5\}$ and $B = \{a,b\}$, find:

a. $A \times B$	c. $A \times \emptyset$
b. $B \times A$	d. $B \times B$

6. If $A \times B = \{(2,7), (2,5), (2,3), (5,7), (5,5), (5,3), (6,7), (6,5), (6,3), (0,7), (0,5), (0,3)\}$
find: a. A ; b. B ; c. $B \times A$; d. $\emptyset \times B$

BINARY OPERATIONS

Rationale:

The study of general binary operations enable the prospective elementary teacher to see which properties are important in the study of all operations in mathematics. It is not sufficient to study properties only with respect to addition, subtraction, multiplication, and division, but in a much more general setting so that the true meanings of the properties can be realized and understood.

Objectives:

Describe the meaning of the terms "unary", "binary", and "ternary".
Define a <u>binary operation</u> on a set.
Recognize which operations of sets and arithmetic are and are not binary operations on given sets S.
Define a <u>commutative binary operation</u>.
Recognize which binary operations of sets and arithmetic are and are not commutative.
State the associative property of a binary operation.
Recognize which binary operations of sets and arithmetic are and are not associative.
Assign meaning to the expression a*b*c when * is an associative binary operation.

Enabling Activities:

Read and study the following materials:
Chapter 8 in PRINCIPLES OF ARITHMETIC AND GEOMETRY by Allendoerfer, Macmillan, 1971
Chapter 7 in ELEMENTARY MATHEMATICS FOR TEACHERS by Kelley and Richert, Holden-Day, 1970
Chapter 4 in THEORY OF ARITHMETIC by Peterson and Hashisaki, third edition, Wiley, 1971
Chapter 2 in MODERN MATHEMATICS: AN ELEMENTARY APPROACH by Wheeler, second edition, Brooks/Cole, 1970
Complete "Handout # 5".

Evaluation:

A problem solving, written examination will be given for this module at the completion of this module and module #6. Use the objectives of this module as a study guide to prepare for the examination.

Work the following exercises:

1. Let * be the operation on W, the set of whole numbers, defined by: a*b = a+b+2

   a. Is * a binary operation on W?
   b. 2*3=
   c. 3*4=
   d. 6*1=
   e. 3*2=
   f. (5*2)*3=
   g. 5*(2*3)=
   h. Is * commutative?
   i. Is * associative?

2. Let # be an operation defined on W by a#b = ab - 3

   a. Is # a binary operation on W?
   b. 3#4=
   c. 1#6=
   d. 5#4=
   e. 4#5=
   f. (4#3)#2=
   g. 4#(3#2)=
   h. Is # commutative?
   i. Is # associative?

3. Let $ be an operation defined on W by a$b = ab + a

   a. Is $ a binary operation on W?
   b. 4$2=
   c. 2$4=
   d. 3$5=
   e. (3$2)$4=
   f. 3$(2$4)=
   g. Is $ commutative?
   h. Is $ associative?

4. Let * be the operation defined on S = {1,2,3} given by this table:

*	1	2	3
1	2	3	1
2	3	1	2
3	1	2	3

   a. Is * a binary operation on S?
   b. 2*3=
   c. 3*2=
   d. 1*2=
   e. 1*3=
   f. 3*(2*1)=
   g. (3*2)*1=
   h. Is * commutative?
   i. Is * associative?
   j. Does * have an identity element?

5. Let $ be an operation defined on A = {a,b,c,d} by the table:

$	a	b	c	d
a	b	d	c	a
b	x	a	c	d
c	a	d	c	b
d	d	a	x	c

   a. Is $ a binary operation on A?
   b. b$a=
   c. a$c=
   d. c$d=
   e. a$(b$c)=
   f. (a$b)$c=
   g. Is $ commutative?
   h. Is $ associative?
   i. Does $ have an identity element?

ADDITION, SUBTRACTION, AND INEQUALITIES OF WHOLE NUMBERS

Rationale:

The major thrust in the lower elementary grades in mathematics is the understanding of the operations of addition and subtraction of whole numbers. The relation of these operations to the set concepts of union, intersection, and complementation must be continually stressed as well as innumerable physical applications. Mere memorization of number facts is not enough. Problem solving techniques can only be learned through applications, not memorization; and problem solving techniques are the main reason for studying these operations on whole numbers. The development of inequalities from addition and subtraction and set concepts is essential.

Objectives:

Define the sum of two whole numbers.
Given whole numbers a and b, find sets A and B such that $n(A)=a$, $n(B)=b$, and $A \cap B = \emptyset$.
Interpret the meaning of the statement "The addition of whole numbers is uniquely defined."
Verify that addition is a binary operation on the set of whole numbers.
Recognize the meaning of "the addition of whole numbers is commutative" and prove that this is true.
Recognize the meaning of the associative property of the addition of whole numbers, and base this upon the corresponding property of set union.
State the generalized commutative property for the addition of whole numbers, and prove special cases of it.
Distinguish between the addition of whole numbers and the union of sets.
Translate into a mathematical statement the old saying "If equals are added to equals, the sums are equal."
State the cancellation law for the addition of whole numbers.
Verify the formula

$$n(A) + n(B) = n(A \cup B) + n(A \cap B)$$

Use the above formula to solve certain counting problems.
Define the inequalities $a < b$, $a > b$, $a \leq b$, and $a \geq b$.
Recognize the properties of inequalities.
Define the difference $a-b$.
Recognize the properties of subtraction.
Add and subtract using the number line.

Enabling Activities:

Read and study the following materials:

Chapters 9 and 10 in PRINCIPLES OF ARITHMETIC AND GEOMETRY by Allendoerfer, Macmillan, 1971
Chapter 6 in MATHEMATICS AND THE ELEMENTARY TEACHER by Copeland, second edition, Saunders, 1972
Chapter 2 in ELEMENTARY MATHEMATICS FOR TEACHERS by Kelley and Richert, Holden-Day, 1970
Chapter 4 in THEORY OF ARITHMETIC by Peterson and Hashisaki, third edition, Wiley, 1971
Chapter 3 in MODERN MATHEMATICS: AN ELEMENTARY APPROACH by Wheeler, second edition, Brooks/Cole, 1970
Complete "Handout #6".

Evaluation:

The on-site instructor or college facilitator will administer a problem solving, written examination for Modules #4, #5, and #6. Make your own arrangements for time and place to take the examination.

Work the following exercises:

1. Tell how you would use the definition of addition to find 3+5.

2. If $A = \{1,2,3,4\}$ and $B = \{4,5,6\}$, find $n(A)$, $n(B)$, $n(A \cup B)$, $n(A \cap B)$, and $n(A) + n(B)$. Is $n(A) + n(B) = n(A \cup B)$?

3. For each of the following, tell which property of addition justifies it:

   a. $7+4=4+7$
   b. $(8+6)+3=8+(6+3)$
   c. $(9+7)+5=5+(9+7)$
   d. If $8+x=8+5$, then $x=5$.
   e. If $x=3$, then $x+4=3+4$.
   f. $8+5+2=2+5+8$

4. Suppose in a class of 30 students that all of them like music or art, 20 of them like music and 15 like both music and art. How many like art?

5. Tell what, if anything, is wrong with the statement, "9 oranges plus 5 oranges is 14 oranges."

6. Use the <u>definition</u> of $<$ to prove that $6 < 9$.

7. Use the definition of the difference to prove that $10-6=4$.

8. If $a < b$ and $b < c$, is $a < c$?

9. If $b < a$ and $a > c$, is $c < b$?

10. If $a-b=8$, what is $(a+c) - (b+c)$?

11. If $a-c = b-c$, is $a=b$?

MULTIPLICATION AND DIVISION OF WHOLE NUMBERS AND THE DISTRIBUTIVE PROPERTIES OF OPERATIONS ON THE WHOLE NUMBERS

Rationale:

The operations of multiplication and division of whole numbers must be developed from a variety of sources. Defining multiplication only as repeated addition is misleading and insufficient. The use of the Cartesian product of sets and geometric concepts of area are very helpful in developing the proper concept of multiplication and division. The distributive properties are helpful in simplifying computations and expressions in mathematics, and allow students to perform more than one operation in problem solving situations.

Objectives:

Define the <u>product</u> of two whole numbers.
Recognize what is meant by the statement "The product of two whole numbers is uniquely defined."
State why multiplication is a binary operation on the set of whole numbers.
State and prove the commutative property of the multiplication of whole numbers.
State the associative property for the multiplication of whole numbers and describe its relation to set concepts.
State the generalized commutative property of the multiplication of whole numbers.
Apply the properties of multiplication by being able to give reasons justifying equations involving commutative and associative properties.
State the cancellation law for the multiplication of whole numbers.
Prove the following special theorems: (a) $a \cdot 1 = a$ (b) $a \cdot 0 = 0$ and (c) if $ab = 0$, then at least one of and b is 0.
State the conclusions derived from the following hypotheses:

1. If a b and $c \neq 0$, then what is the relation between ac and bc?
2. If a c and $c=0$, then what is the relation between ac and bc?
3. If $c=0$, can ac be less than bc?
4. If ac bc, then what is the relation between a and b?

Define the <u>quotient</u> of two whole numbers.
Recognize the facts concerning division by zero.
State the conclusions drawn from the following hypotheses:

1. If a, b, and c are whole numbers and $c \neq 0$, and if $a=b$ and a c and b c are defined, then what is the relation between a c and b c?
2. If a, b, and c are whole numbers with $c \neq 0$, and if a c = b c, then what is the relation between a and b?
3. If a c and b c are both defined and (a c) (b c), then what is the relation between a and b?
4. If a b, $c \neq 0$, and c a and c b are both defined, then what is the relation between c b and c a?

Recognize the meaning of division with remainder.
State the distributive properties of Cartesian product over union and intersection and illustrate them by examples.
State the distributive property of multiplication over addition.
Apply the distributive property to remove parentheses and remove common factors.
Show that multiplication of whole numbers can be regarded as repeated addition.
State the meaning of the distributive property of * over $.
Show that multiplication is distributive over subtraction.
Show that addition is not distributive over multiplication.
State and derive the distributive properties of division over addition and subtraction.
State which distributive properties of arithmetic and sets are true.
Determine or deduce whether or not one nonstandard operation is distributive over another.
Use the correct conventions of mathematics to evaluate expressions both with and without parentheses.
Complete "Handout #7".

Enabling Activities:

Read and study the following materials:
Chapters 11 and 12 in PRINCIPLES OF ARITHMETIC AND GEOMETRY by Allendoerfer, Macmillan, 1971
Chapters 7 and 8 in MATHEMATICS AND THE ELEMENTARY TEACHER by Copeland, second edition, Saunders, 1972
Chapter 2 in ELEMENTARY MATHEMATICS FOR TEACHERS by Kelley and Richert, Holden-Day, 1970
Chapter 4 in THEORY OF ARITHMETIC by Peterson and Hashisaki, third edition, Wiley, 1971
Chapter 3 in MODERN MATHEMATICS: AN ELEMENTARY APPROACH by Wheeler, second edition, Brooks/Cole, 1970

Evaluation:

A problem solving, written examination will be given for this module at the completion of Modules #8 and #9. Use the objectives of this module as a study guide in preparation for the examination.

Work the following exercises:

1. Classify each of the following as true or false:

a. $8 \cdot 0 = 8$	c. $(16+1) \cdot 0 = 17$	e. $(ab) \cdot (0) = 0$
b. $7+0 = 0$	d. $(16+1)+0 = 17$	f. $(1+1) \cdot 1 = 1$

2. Evaluate:

a. $(2+3) \cdot 4 =$	c. $2+(3 \cdot 4) =$	e. $(2 \cdot 3)+4 =$
b. $(2+3)+4 =$	d. $2 \cdot (3+4) =$	f. $2 \cdot (3 \cdot 4) =$

3. Demonstrate a pair of whole numbers which have the property that:

a. their product is 1	c. their product is 8
b. their product is 0	d. their product is 7

4. Let R be the set $\{1,2,3,4,5\}$. Is this set closed with respect to:

   a. addition? b. multiplication?

5. Let S = $\{1\}$. Is it closed with respect to:

   a. multiplication? b. addition?

6. Let S = 0 . Is this set closed with respect to:

   a. multiplication? b. addition?

7. Complete the following sentences to make them true statements:

   a. $(10 \div 2)$ = ____ because 10 = ____ · ____.

   b. $(0 \div 7)$ = ____ because 0 = ____ · ____.

   c. $(\_\_ \div 3)$ = 4 because __ = 3 · ____.

   d. $(18 \div \_\_)$ = 3 because __ = ____ · ____.

8. Solve for x in each of the following equations:

a. $x + 6 = 11$	d. $5x = 30$	g. $2x = 16$
b. $3x = 12$	e. $x + 4 = 4$	h. $17 + x = 20$
c. $5 + x = 12$	f. $x + 7 = 9$	i. $6x = 0$

9. Construct an example to show that division is not associative.

10. Fill in the blanks to demonstrate the distributive property:

   a. ____________ = $(8) \cdot (2) + (8) \cdot (7)$

   b. $(3)(6) + (4)(6)$ = ____________

   c. (9)(____________) = ________(5) + ________(6)

   d. ____________(6) = (4)________ + (5)________

11. Rename the following using the distributive property:

    a. $2(3+4)$
    b. $5a + 3a$
    c. $a(b+c)$
    d. $(4 \cdot 2) + (4 \cdot 3)$
    e. $(2 \cdot 6) + (3 \cdot 6)$

12. Compute the following expressions in two different ways:

    a. $6(5-2)$
    b. $(7-4)9$
    c. $(6+8) \div 2$
    d. $(12-9) \div 3$

FRACTIONS

Rationale:

The development of fraction concepts are dependent upon set concepts and geometric concepts in addition to counting techniques. A wide variety of applications are essential to the development of useful fractional concepts. An understanding of equivalent fractions is necessary for the later development of operations with fractions and rational numbers.

Objectives:

Represent portions of objects by means of fractions.
Represent points on the number line with fractions.
Multiply a fraction by a natural number.
State the definition of equality for fractions.
Define the unit, a prime number, and a composite number.
Distinguish whether a number is the unit, a prime, or a composite number.
Obtain the prime decomposition of natural numbers.
Find the G.C.D. and L.C.M. of pairs of natural numbers.
Define and apply the concept of relatively prime numbers.
Define equivalent fractions and obtain their properties.
Recognize the meaning of the statement: "The fraction a/b is in lowest terms.", and apply this concept.
Solve equations of the type $ax = b$ where a and b are whole numbers with $a \neq 0$.

Enabling Activities:

Read and study the following materials:
Chapter 22 in PRINCIPLES OF ARITHMETIC AND GEOMETRY by Allendoerfer, Macmillan, 1971
Chapter 12 in MATHEMATICS AND THE ELEMENTARY TEACHER by Copeland, second edition, Saunders, 1972
Chapter 5 in ELEMENTARY MATHEMATICS FOR TEACHERS by Kelley and Richert, Holden-Day, 1970
Chapter 6 in THEORY OF ARITHMETIC by Peterson and Hashisaki, third edition, Wiley, 1971
Chapter 7 in MODERN MATHEMATICS: AN ELEMENTARY APPROACH by Wheeler, second edition, Brooks/Cole, 1970
Complete "Handout #8".

Evaluation:

A problem-solving, written examination will be given for this module at the completion of Modules #8 and #9. Use the objectives of this module as a study guide in preparation for the examination. The on-site instructor or college facilitator will administer this examination. Make arrangements for the time and place.

GEOMETRIC CONCEPTS

Rationale:

Piaget has recognized that the early development of geometrical concepts, both two and three-dimensional, are natural for the small child. The prospective elementary teacher must be aware of these concepts and how to present them in many physical situations. This can be one of the most exciting, interesting, and challenging experiences for a teacher. It no longer can be postponed until the secondary level of education. Geometric concepts are also useful in introducing other mathematical concepts such as fractions and multiplication.

Objectives:

Recognize the abstract character of points, line, and planes, and approximate portions of these with physical objects.
Describe the five regular polyhedra giving the number of vertices, edges, and faces.
State the incidence properties of points and lines in the plane.
Identify half-lines and rays.
Describe the separation properties of a plane by a line.
Recognize closed curves, simple curves, and simple closed curves.
Describe the Jordan Curve Theorem.
Define polygon, regular polygon, and polygonal region.
Define an angle and a straight angle in the plane.
Define the interior of an angle.
Describe the incidence properties of points, lines, and planes in space.
Describe the separation properties of a plane in space.
Define a sphere and a polyhedron.

Enabling Activities:

Read and study the following materials:
Chapter 28 in PRINCIPLES OF ARITHMETIC AND GEOMETRY by Allendoerfer, Macmillan, 1971
Chapters 3 and 9 in MATHEMATICS AND THE ELEMENTARY TEACHER by Copeland, second edition, Saunders, 1972
Chapter 8 in THEORY OF ARITHMETIC by Peterson and Hashisaki, third edition, Wiley, 1971
Chapter 12 in MODERN MATHEMATICS: AN ELEMENTARY APPROACH by Wheeler, second edition, Brooks/Cole, 1970
Complete "Handout #9".

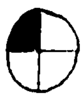
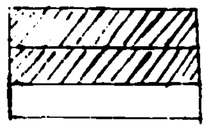
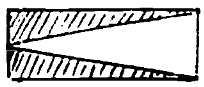
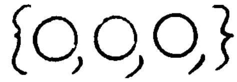
Evaluation:

A problem-solving, written examination will be given for modules #7, #8, and #9 at the completion of this module. Use the objectives of these modules as a study guide in preparation for the examination.

A final problem-solving, written examination will also be given at the completion of all the modules in this series. The final examination will be given by the college facilitator. The intern should make arrangements with the college facilitator for the time and place of the final examination.

Work the following exercises:

1. Represent the shaded regions and the unshaded regions in each of the following by means of a fraction:

   a. 
   b. 
   c. 
   d. 
   e. 
   f. 

2. Tell whether each of the following is the unit, a prime, or a composite number:

a. 37	d. 1	g. 36
b. 8	e. 2	h. 100
c. 3	f. 9	i. 81

3. Find the prime decomposition of each of the following:

a. 27	c. 150	e. 120
b. 48	d. 6	f. 1342

4. Find the G.C.D. (greatest common divisor) and L.C.M. (least common multiple) of each of the following pairs of natural numbers:

a. 36 and 27	d. 8 and 9
b. 17 and 7	e. 12 and 18
c. 100 and 25	f. 15 and 45

5. Write each of the following fractions in lowest terms:

a. 12/15	c. 63/49	e. 9/9
b. 24/100	d. 21/28	f. 36/24

6. Find a fracion x to satisfy each of these equations:

a. $5x = 7$	d. $9x = 5$
b. $2x = 3$	e. $8x = 1$
c. $11x = 17$	f. $4x = 6$

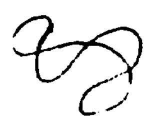
Work the following exercises:

1. Fill in the following table for the regular polyhedra:

Name	number of faces	number of vertices	number of edges	shape of faces
tetrahedron				
hexahedron				
octahedron				
dodecahedron				
icosahedron				

2. Identify each of the following curves as a closed curve, simple curve, simple closed curve, or none of these:

a.


b.


c.


d.


e.


f.


3. Sketch an angle and shade the interior of the angle.

4. Tell the difference between a line, a half-line, and a ray, and illustrate each with a sketch.

5. Identify each of the points a,b,c,d,e as being on either the interior or exterior of this simple closed curve: