The understanding of the political behavior of school boards can be advanced by conducting factor analysis of the voting records of trustees. While the application of these methods is generally appropriate, differences in subject matter highlight the problem of inferring political theory from numerical data. In this paper, several different political models for explaining trustee voting behavior are postulated—models which, in fact, have identical representations in terms of recorded votes and, hence, factor patterns. Further, actual voting records of two boards are analyzed in order to illustrate two different types of voting behavior. Explanations for these differences require additional knowledge concerning variables other than the votes themselves. Finally, there is a discussion of some of the modes of analysis currently in use, with recommendations as to the best alternatives. (Author/DR)
MODELS, ANALYSIS AND INTERPRETATION OF EDUCATION TRUSTEE VOTING BEHAVIOR

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The analysis of the voting behavior of school board members, particularly using factor analysis, is a recent extension of techniques refined by political scientists to a topic of interest to those involved in the study of educational policy making (Struby and Muskal, 1968). While the application of these methods is generally appropriate, differences in subject matter—e.g., non-partisan boards of education rather than partisan legislatures—highlight the problem of inferring political theory from numerical data.

Background

Usually, one expects factor analysis to reveal the "underlying structure" of the numerical data being analysed. However, if fundamentally different conceptual models of political behavior have identical numerical representations, then confusion may occur as to the meaning of the numbers. In this paper, several different political models for explaining trustee

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voting behavior will be postulated -- models which in fact have identical representations in terms of recorded votes and, hence, factor patterns. Further, actual voting records of two boards will be analysed in order to illustrate two different types of voting behavior. However, explanations for these differences require additional knowledge concerning variables other than the votes themselves. Finally, there is a discussion of some of the modes of analysis currently in use, with recommendations as to the best alternatives.

The various approaches to the analysis of voting behavior of members of legislatures and other political bodies are adequately treated elsewhere (e.g. MacRae, 1970 and Anderson et al, 1966), so only a brief description of a few basic elements and problems of the field will be presented here. Normally, it is assumed that the observations being studied are dichotomous votes (yes, no) though occasionally abstentions are included to form a three point scale (yes, abstain, no). Each observation or vote is then classified according to two variables: the particular motion on which the vote was cast and the particular individual who cast it. The purpose of voting analysis, then, is to analyze the patterns of votes in order to determine clusters of either individuals, motions, or both and to explain the findings. The first approach -- grouping individuals -- is often used to confirm the existence of party voting or to discover voting blocks; the second -- grouping motions -- is used to construct empirical definitions of issues; and the third -- both persons and motions grouped simultaneously -- is used to learn if voting blocks differ with the issue under consideration.

During the early phases of the present investigation, all three approaches were used. However, the school board motions, when analysed for content and by statistical methods, did not appear to define clear-cut
issues. The outcomes for the analysis of motions and of both individuals and motions simultaneously were ambiguous. In contrast, the results yielded by grouping individuals on the basis of their voting records were interpretable. Hence, it is that approach which was used and which is described here.

The major technical problem which occurs in determining voting blocks is the selection of the measure of relationship that is to be used to assess the similarity of voting records for two members. Direct clustering methods which group raw data avoid this problem (Hartigan, 1972), but were not used here due to the ambiguous results they produced when applied to the real data and the trivial results they yielded when applied to the "ideal" models which are proposed.

Research Design

Given that some measure of relationship between individuals is to be used, numerous possibilities exist; e.g., $r$, $\phi$, tau, Yule's $Q$ (the special case of gamma for a two-by-two table), percentage agreement, etc.. Having chosen a particular measure, there also are a variety of analytic techniques one may use to determine groupings: principal components, cluster analysis, etc.. One problem which occurs when focusing on the grouping of motions -- the polarity or algebraic sign (+ or -) of the voting pattern on particular motions -- does not occur when focusing on individuals since it is generally agreed that the polarity of voting patterns of individual legislators should not be tampered with.

While there are good arguments to support different choices of both measures of relationships and analytic techniques (MacRae, 1970),
it appears that sufficient justification for any choice is its ability to produce expected, interpretable results in a simulated analysis of artificial data of a known structure. In line with this viewpoint, several political models for voting behavior are presented here with hypothetical raw voting data being generated for them. The Pearson product-moment correlation coefficient $r$ -- which is equivalent to $\phi$ for dichotomous data except that it takes on both positive and negative values -- was chosen to measure the similarity of board members on the basis of their voting records so that voting blocks could be determined. The correlation matrix was then factor analyzed using the principal component method. The polarity of all votes was left unchanged.

The choice of $r$ and principal components -- two popular tools of researchers -- yields a particular type of factor pattern if used for dichotomous data which are underlain by a perfect cumulative scale. As shown by Guttman (in Stouffer et al, 1950) and MacRae (1970, pp. 150-151), this mode of analysis will produce a multi-factor solution in which the second factor is a quadratic function of the first, the third a cubic function, etc.. MacDonald (1967) suggests methods for taking these non-linear relationships into account -- either by interpreting them or removing them. Another choice is to ignore them. As will be shown, this particular inter-relationship between the form of the data and mode of analysis of considerable importance.

Models to explain the voting behavior of members of legislatures seem to be based on essentially two conceptualizations of the underlying variable(s) which determine how individuals vote. The first accepts that there exists a continuum -- often an ideological continuum -- on which a person can be placed, and that his place on it can be located by ranking the legislators on a hypothetical continuous scale. The second concep-
tualization assumes that the underlying or latent variable is discrete or categorical -- often party membership -- with members of a body voting in distinct, non-overlapping blocks. A voting behavior model for a given legislature or school board may accept either of these descriptions as correct, or incorporate both of them into a combined model with two underlying variables -- one differing by degree and the other by kind.

There is empirical justification for all three models. For example, MacRae (1970, p. 259) reports an analysis of congressmen for the 1961 United States House of Representatives in which legislators were segregated into two classes (Republican and Democratic) and arranged along an ideological continuum. The results are a clear illustration of a legislature whose members behave according to the third, combined model -- where party is the latent variable differing by kind (in this case a dichotomy) and ideology that varying by degree (with extremes at the conservative and liberal poles).

**Ideological Continuum**

The first model, which assumes an (ideological) continuum defined by four motions ABCD underlying voting behavior, implies that a given motion A will draw support from just one extreme of the continuum (Figure 1).

Insert Fig. 1 about here

In this situation, if a 1 is used to indicate support and a 0 to indicate opposition, and if the legislature under study is a three member body
$(X_1, X_2, X_3)$ having one member towards each extreme and one moderate -- a simple model, to be sure -- then motion A would produce a $1 \ 0 \ 0$ voting pattern. (If a $+$ is used to indicate support and a $-$ opposition, as is often the case in voting studies, then this would be a $+ \ - \ -$ voting pattern.) Motion B, representing a more moderate motion on the same continuum, but still left of center, might produce a $1 \ 1 \ 0$ pattern. Motion C, a moderate motion to the right of center, would probably yield a $0 \ 1 \ 1$ pattern; and motion D at the extreme right right $0 \ 0 \ 1$ pattern. The complete data and Pearson r matrix for legislators fitting this model are shown in Table 1. As we shall see, it is possible to infer the existence of the continuum from the relationships among legislators, as well as their place on the continuum.

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Insert Table 1 about here

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Often, a correlation matrix like that in Table 1b in which subjects rather than responses are correlated is designated as a Q matrix while that for response variables is termed an R matrix (Kerlinger, 1964, p. 581). However, using this common notation might cause the Pearson r matrix for subjects to be confused with the Yule's Q matrix for subjects. Therefore, the more explicit terminology is used here.

Interpreting the two-factor pattern which is produced by taking the principal components of the r matrix for the data in Table 1a is not as difficult as it may at first appear. The factor loadings on the first factor of the individual legislators replicates their rank ordering on the latent scale pictured in Figure 1. Subject $X_1$ is first, with a loading of $-1$ which indicates a scale position left of center; $X_2$ has a loading of $0$ and is located in the center; and $X_3$ has a loading of $1$ and has a position right of center. The second factor is a quadratic function of
the first -- uncorrelated with the first but fully dependent upon it.

Sometimes the linear and quadratic factors are reversed in order -- i.e., the linear factor is second, accounting for less variation than the first which is then a quadratic function of the second -- but the interpretation is the same. Guttman (in Stouffer, et. al., 1950) originally interpreted the quadratic factor as the "intensity" with which individuals behave on the linear factor. The zero point -- or point of inflexion -- of the quadratic factor locates the neutral or indifferent position on the ideological continuum identifies in the linear factor, as is suggested visually in Figure 2.

Several alternative modes of analysis for the data for the continuum model exist, though none appear as appropriate as that used here. One may, for example, transform the voting data before factor analysis by reversing the voting patterns for all motions which fall toward one end of the spectrum; i.e., by changing their polarities. The voting pattern for C (0 1 1 or - + +) would then be replaced by its opposite C' (1 0 0 or + - -), and that for D (0 0 1 or - - +) by its opposite D' (1 1 0 or + + -). The transformed data would then conform to that of perfect cumulative or Guttman scale.

Changing polarity, however, may effect the results one obtains. In the example in Table 1, for instance, the correlation matrix for legislators after changes in polarity would differ from that for the raw data and, since the correlation matrix being submitted to factor analysis would differ, so would the conclusions. This outcome would occur even if one were to use Yule's Q instead of r. MacRae notes, "If the input consists of a perfect cumulative scale, the (Yule's Q) matrix will have all entries equal to 1 and
the result will by a single factor with all loadings equal to 1 (p. 151)."
But such is not the case if the data, like that in Table 1a, includes
items of opposite polarity. In fact, for Table 1a the Yule's Q matrix
proves to be identical to the r matrix -- which is not all 1's -- and
yields two factors rather than one factor. The r matrix is also affected
by changes in polarity.

Using Yule's Q with untransformed data which includes items of
differing polarity will also yield a multifactor solution similar to
that produced for r. Comparison of factor patterns using actual school
board data suggests that r, in fact, makes greater use of the information
contained within the data than does Yule's Q, which apparently simplifies
the data's structure (Figures 3 and 4). This comparison will be dealt
with later; in any case, this conclusion is only tentative.

The very practice of altering the polarity of voting patterns on
motions appears open to criticism. First, it implies considerable prior
knowledge of voting patterns, which betrays the exploratory aspects of
most voting analyses. Second, it is a sort of tampering with the data
that may lessen the credibility of findings. And finally, it is unneces-
sary, at least when studying the relationship among legislators.

Discrete Voting Blocks

The second political model of voting behavior to be considered is
founded on the assumption that the variable underlying voting behavior is
discrete, so that all members of a body vote in identifiable blocks or fac-
tions. Assume, for example, that a board of four members is divided into
two groups. Then, the pattern of votes might appear as in Table 2.

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Insert Table 2 about here

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Paradoxically, the data for a legislative body with two separate groups factor into one dimension, rather than two, even though no underlying continuum has been assumed. However, an inferential problem does arise if the analysis is exploratory and its purpose is to discover what model of voting behavior is most appropriate. It could well be that the data displayed to illustrate this second, discrete model represents a school board or legislature which is highly polarized on an ideological continuum. One way to be more certain as to which interpretation is correct is to consider the extent of conflict on the board as measured, say, by the proportion of non-unanimous votes on record. If the polarization represented in the model occurs on only a relatively few votes, then one can infer that there is no ideological cleavage. Perhaps instead one group has a special interest in supporting a particular position on a minor issue (e.g., sending the chairman and vice-chairman to a convention). On the other hand, a high proportion of conflict votes would certainly suggest an ideological cleavage. Content analysis of motions might corroborate such an interpretation, as might analysis of voting data from an earlier period in the board's history, when polarization was not yet fully developed. If these techniques yield no acceptable explanations, members of different discrete blocks might be compared with one another on a number of other variables in order to discover one which is related to the groupings. One might expect, for example, that political or social affiliations would explain differences in voting behaviors, particularly if the motions on which
splits occur provide an opportunity to show "patronage." Of course, in provincial or federal legislatures using the parliamentary form of government in which party discipline is rigidly enforced, one would almost certainly settle upon political party as the explanatory variable.

**Combined Discrete-Continuous Model**

The third and final model presented here combines the two previous models in assuming that there are two underlying variables, one continuous and one discrete. Other combinations, such as two continuous or two discrete latent variables, or combinations of three or more variables, will not be considered. The problems posed by the three models that are presented are sufficiently difficult without facing the extreme difficulties of dealing with more complex models. At the same time, the three models are probably all that is needed to investigate most sets of voting data for boards of education.

The raw data, correlation matrices, and factor patterns for the combined discrete-continuous model would be similar to those for the continuous model alone (Table 1). Discrete groups would be revealed in one factor, while the existence of a continuum would be revealed by a quadratic relationship between two factors. In practice, sorting out the two variables from one another may prove more difficult, however, for the reasons discussed in the section dealing with the discrete model.

In summary, it is proposed that a particular political voting model, if fully specified, will provide sufficient information to predict the outcome of factor analysis of the matrix of correlation coef-
coefficients for legislators. However, the reverse is not true. A particular factor pattern does not necessarily supply sufficient information for one to infer which voting model is correct. Other data are needed. This limitation is not exceedingly restrictive in research where a model is hypothesized. Either the appropriate pattern is found, or it is not. Of course, there is a chance for error since there is not a one-to-one correspondence between models and factor patterns; both the continuous model and the combined model may produce the same pattern; the former may be hypothesized and confirmed while the latter is, in fact, correct. One would normally expect, though, that the information guiding the selection of the hypothesized model would be sufficient to limit this chance for error.

The lack of a perfect correspondence between models and factor patterns is far more bothersome for strictly exploratory research. Analysing voting patterns is then only the first step in an investigation that would probably produce several alternative explanations. These, in turn, could be tested by collecting additional data in order to discern the model most likely to be correct.

Finally, questions concerning the most appropriate methodology, and in particular whether it is better to use Yule's $Q$ or Pearson's $r$, seem resolved in favor of $r$. Unlike $Q$, the product-moment correlation coefficient satisfies the mathematical conditions imposed by factor analysis and, more important, the analysis of data based on several "ideal" models yields the expected factor patterns.
Two Ontario Boards

The remainder of this paper deals with the analysis of voting data for two Ontario boards of education. These examples illustrate actual manifestations of the types of behavior set forth in the first two models; for one, the ideological continuum interpretation appears most valid, while for the other, a discrete model fits well. In the presentation for the first board, principal components of both the \( r \) matrix and \( Q \) matrix are given, both to suggest the comparability of the two approaches in general, and the slight superiority, at least in this case, of using the Pearson product-moment correlation coefficient \( r \) instead of Yule's \( Q \).

Trustee voting data for the Toronto Board of Education collected over a period of six months provide the example of an ideological continuum. Interviews and newspaper reports suggested, before the collection of voting data, that trustees were spread on a continuum ranging from a reform position to one representing the status quo. Further, reported attempts to establish two caucuses within the board made it appear that trustees would tend to be polarized on the scale, with the greatest concentration toward the conservative end. Finally, of the 251 votes recorded (with at least a quorum voting) during the period from November 1971 to May 1972, 105 had at least two members of the board opposing the remainder; that is, 42 per cent of all votes suggested intra-board conflict. Such a high rate -- the next highest noted for the other five boards in Metropolitan Toronto was 25 per cent and two boards showed conflict whatsoever -- also implies an ideological conflict underlying the voting behavior of Toronto trustees.
The correlation matrix submitted to principal component analysis was based on 24 votes selected from the total according to two criteria: at least 80 per cent of the trustees had to have voted, and at least two had to have been in opposition. These restrictions guaranteed that the effects of missing data and of lone mavericks were minimized. Twenty-three of the twenty-four trustees were included in the analysis; the chairman -- who rarely voted -- was omitted.

The first two components of the analysis, which accounted for 63 per cent of the common variance, are plotted in Figure 3. The curvilinearity of the relationship is immediately obvious and, in line with previous discussion, can be interpreted as a factor pattern resulting from an underlying ideological continuum. Since this result was expected on the basis of prior knowledge, one can place very high confidence in this conclusion. Beneath the two-way plot is a histogram displaying the frequencies on factor one, which represents the ideological continuum. It is obvious that the board was, to a large extent, polarized because the distribution is bimodal. In addition, one can discern that the "reformers" toward the left of the scale are in the minority, and that they crowded against the scale's extreme position, thereby suggesting that their beliefs may in fact represent a "radical" position more extreme than any included on the scale. In contrast, the moderates appear normally distributed on the right half of the continuum. They do not appear to be as extreme as the reformers -- though of course the conservatives are in the majority. Perhaps out of strength in numbers comes the ability to take more moderate positions without threatening one's ability to enact or confirm basic policies.

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Insert Figure 3 about here

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If less emphasis is placed on the polarization which exists, and if the scale is divided into thirds, it is seen that the board can be said to include eight reformers, five moderates, and ten conservatives. Given this categorization, conservatives can be seen as holding a plurality, while the moderates, by supporting either end of the continuum -- i.e., the reformers or the conservatives -- as the case warrants, could decide any given issue. This latter interpretation, in line with the rationale given for the continuum model, probably most adequately reflects the Toronto board's actual behavior.

The first two components for the analysis of the Yule's Q matrix for Toronto trustees, presented in Figure 4, also display the strong curvilinearity previously noted. However, the histogram suggests that the board was more polarized than did the analysis using r. Previously, only two "conservatives" appeared in the last interval on the right, whereas using Yule's Q, seven are placed in that category. It appears that Q may be a less subtle measurement of relationships than r, and tends, in this analysis at least, to oversimplify the board's structure. Of course, it could be that this version more closely reflects reality; however, this alternative is not supported by other evidence.

The trustee voting record analysed for the Scarborough Board of Education, the second board under consideration, covered the period from October 1971 to February 1972. Of the sixty vote total, twenty were included in the analysis, each having at least eight members voting with one in opposition.
While no ideological continuum had been posited for Scarborough, there had been indications from the press and interviews that lines tended to be drawn along national political party affiliation. Hence, trustees were identified according to their membership in the Liberal, Progressive Conservative, or New Democratic parties.

Figure 5, in which the first two factors, which account for 58 per cent of the common variance, are plotted, shows evidence of an underlying discrete variable, party affiliation. This conclusion is inferred from the clustering of trustees in the two-dimensional plot rather than from their positioning at three points of a triangle as would have suggested by the discrete model. Investigation of the content of motions on which the board split suggested disagreement principally on matters of making appointments to special boards and committees, rather than on educational policy matters per se.

Conclusion

The understanding of the political behavior of school boards can be advanced by conducting factor analysis of the voting records of trustees. For confidence in one's results, this type of analysis is best suited for testing hypothetical models of behaviors such as those based on an ideological continuum, discrete voting blocks, or a combination. Exploratory analysis is complicated by the multiplicity of interpretations that can be made of the same sets of data.
Care must be taken when employing this analytic technique. The choice of the type of index to measure the similarity of voting records between trustees influences the results obtained. In particular, the Pearson correlation coefficient $r$ appears more sensitive to differences than does Yule's $\Omega$. Altering the polarity or direction of votes can also have a critical effect, and probably should not be done when relationships among voters in contrast to those among motions are being considered. Finally, one should be alert to the curvilinear relationship between the first two factors of an factor analysis that is obtained when the original subjects are arranged along a continuum, in order to ensure that the results are not given some other, less valid interpretation.


# Table 1

## Data for Continuum Model

### 1a. Raw Data Matrix

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<th></th>
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<td>B</td>
<td>C</td>
<td>D</td>
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### 1b. Pearson r Matrix for Subjects

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### 1c. Factor Pattern for r Matrix

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### Table 2

**Data for Discrete Model**

#### 2a. Raw Data Matrix

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#### 2b. Pearson r Matrix for Subjects

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#### 2b. Factor Pattern for r Matrix

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Figure 1. Ideological continuum ABCD with voters $X_1$, $X_2$, $X_3$. 
Figure 2. Graph of factor patterns
CAPTIONS

Figure 3. Graph of Factor I vs. Factor II for $r$ matrix of Toronto board members and histogram for Factor I.

Figure 4. Graph of Factor I vs. Factor II for Yule's $Q$ matrix of Toronto board members and histogram for Factor I.

Figure 5. Graph of Factor I vs. Factor II for $r$ matrix of Scarborough board members, identified as to party affiliation.
L  -  Liberal
NDP  -  New Democratic
PC  -  Progressive Conservative