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ABSTRACT

This study examines the correspondence between a representation of subject-matter structure and a representation of the cognitive structure in students as a result of instruction. Eighty-seven eighth-grade students were assigned randomly to experimental and control treatments. The experimental students (N = 43) studied from a programmed text on probability developed by the School Mathematics Study Group (SMSG). The control group (N - 44) studied from a programmed text on factors and prime numbers. Digraphs, graphs and task analysis were used to map out the content structure. A word association test (WA) was used to measure the existing cognitive structure. The WA test, an achievement test and an attitude measure were given before, after and at a delayed post-experiment time. The results indicate that: (1) eighth-grade students were unfamiliar with the concepts of probability; (2) instruction in probability changed subjects' cognitive structure concerning concepts in probability; (3) the experimental group learned a significant portion of the structure of probability as a matter of instruction while the control group learned almost nothing; and (4) this learning of structure was retained until the retention-test time. (JP)

An Analysis of Content Structure and Cognitive Structure in the Context of a Probability Unit[1]

by

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During the past decade mathematics curricula have been revised significantly. Curriculum developers have attempted to communicate something more than algorithms and computational skills to the student (cf. Report of the Commission on Mathematics, 1959); they have attempted to communicate structures in mathematics (Report of the Cambridge Conference, 1963). Begle (1971) summarized the rationale for the change:

> ...by paying careful attention to the structure of mathematics, the way mathematical ideas fit together, rather than relying on intricate and ingenious computations, it [is] possible to solve difficult and important mathematical problems...The importance of this change of emphasis from ingenious computations to basic concepts and the structure of mathematics gradually became clear [p. 68].

In spite of this emphasis on structure in mathematics curricula, little empirical work has been done concerning the communication of a mathematical structure to students. The purpose of this study is to examine the correspondence between a representation of the structure of a subject matter (content structure) and a representation of this structure in the students' memories (cognitive structures) as a result of instruction in the subject matter.

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[1]A paper presented at the Annual Meeting of the National Council of Teachers of Mathematics, Atlantic City, April, 1974.

Definitions of Structure

In order to proceed with an investigation of the correspondence between content structure and cognitive structure, the terms structure, mathematical structure, content structure, and cognitive structure need to be defined. Shavelson (1971) defined structure as "an assemblage of identifiable elements and the relationships between those elements. Structure may be objective and real or internal and subjective [p. 1]." In a manner consistent with Shavelson's definition of structure, Begle (in preparation) defined mathematical structure as "a set of interrelated, abstract, symbolic systems." He stressed the point that mathematical structure is a combination of within system relationships and between system relationships. For the purposes of this study, then, mathematical structure is defined as the relationships between concepts within a set of abstract systems.

The structure of a subject matter as represented in instructional material is referred to as content structure. Content structure is the web of concepts and their interrelations in a body of instructional material (Shavelson, 1971, 1972). One method for representing structure in instructional material is Shavelson's procedure (Shavelson, 1971, 1972; Shavelson & Geeslin, 1973) using the theory of directed graphs (digraphs; Harary, Norman, & Cartwright, 1965). With this method key concepts are represented as points on a digraph and the relationships between concepts, as specified by syntactical and semantic features of the text, are represented by directed lines connecting points. The similarity between concepts--one aspect of structure--is represented by the distance (smallest number of lines) between points (concepts) on the digraph. The theory of directed graphs is an abstract mathematical theory of structure in which structure is defined as points and directed lines. If the correspondence between digraph theory and the empirical world is accurate, then all true statements about the digraph are also true of the empirical world. The resultant digraph is considered to be a representation of the content structure.

The structure of students' memories is referred to as cognitive structure; a "hypothetical construct referring to the organization (interrelationships) of concepts in long-term memory [Shavelson, 1971, p. 9]." Cognitive structure is examined by means of a word association technique; the student responds to a key concept by calling forth as many other related mathematical concepts as he can. By analyzing the overlap of response lists to key concepts, a measure of the relationships between key concepts in a student's memory can be obtained (Garskof & Houston, 1963; Geeslin, 1973; Johnson, 1967; Shavelson, 1971, 1972, in press).

In this study, then, students received instruction in probability (the to-be-learned structure) or in an unrelated topic in mathematics. Before and after instruction, the word association test, a measure of cognitive structure, was given. The representation of cognitive structure from the word association test was compared with the representation of content structure obtained with the digraph method. The correspondence between these two representations was interpreted as the correspondence between content structure and cognitive structure. Even though these representations of structure are not comprehensive or error free, they represent an important first step in answering crucial questions about this correspondence.

Method

Subjects

The subjects were 87 eighth grade students taken from 3 intact classes in a suburban junior high school. The principal indicated that the subjects were average and slightly above average in mathematical ability and came from varied social, economic, and ethnic backgrounds. In general, most Ss may be described as Anglo-Americans of low-middle to middle socio-economic status.

Instructional Material

The instructional material was an introductory programmed text on probability.[2] One purpose of this text was to communicate a subject matter structure, that of elementary probability, to the subjects. Probability was selected because the topic was unfamiliar to most eighth grade students, was easily placed in the normal curriculum sequence, and required few mathematical prerequisites. The programmed text format--small steps, constructed responses, and continual feedback on the correct responses--was used to minimize the chance that proctors would "teach" a structure different from that presented by the text by answering subjects' questions. It also allowed each student to proceed at his own pace. The text, divided into three sections of approximately seventy pages each, covered the following major concepts in probability: probability experiments, outcomes of experiments, equally likely outcomes, events, trial, intersection of events, probability of events, range of the probability function, independent events, and mutually exclusive events.

Representation of Content Structure

Following the digraph procedure (Shavelson & Geeslin, 1973; see also Geeslin, 1973; Shavelson, 1971, 1972), the first step is to identify the key concepts in the instructional material. Ten key concepts were selected from the probability

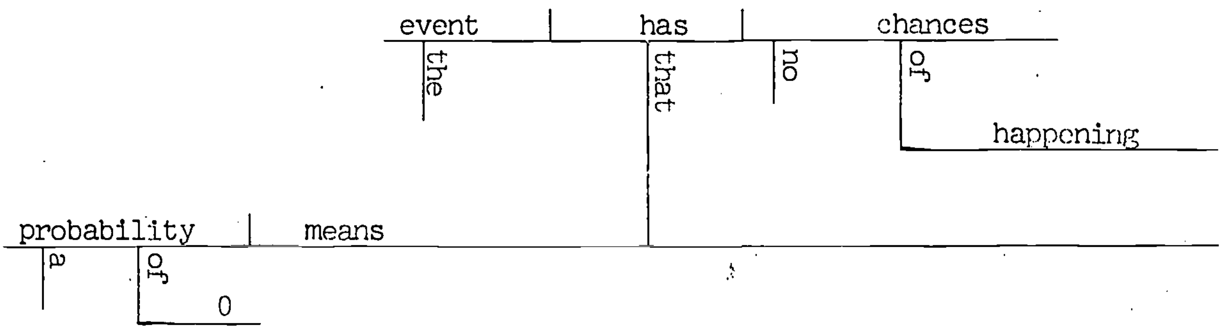
[2] The text, developed by the School Mathematics Study Group, is available from the ERIC Science, Mathematics, and Environmental Clearinghouse, Columbus, Ohio.

text: PROBABILITY, EQUALLY LIKELY, OUTCOME, EVENT, EXPERIMENT, ZERO, INTERSECTION, TRIAL, INDEPENDENT, and MUTUALLY EXCLUSIVE.

The key concepts were selected a priori as being the most important, in a mathematical sense, in the text; i.e. they were critical to the structure of probability. The text was designed specifically to teach these concepts and these concepts were thought to be crucial in the subjects' mastery of the instructional material. That is, subjects were to learn the key concepts and the interrelationships between them and thus the structure of the subject matter.

The second step is to select all sentences in the text--the unit of analysis--which contained at least two of the key concepts. Such sentences contained information on the way pairs of concepts were interrelated in the text. For example, the sentence, "A probability of 0 means that the event has no chances of happening," was selected beacause the concepts probability, 0 (zero) and event were contained in the sentence.

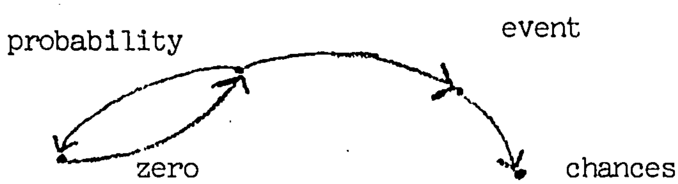
The third step was to diagram each sentence containing two or more key concepts using a parsing grammar (Warriner & Griffith, 1957). For the sentence in the example, the following diagram was obtained:



This diagram was converted to a digraph using a set of rules (Shavelson, 1971, 1972; Shavelson & Geeslin, 1973). For example, one rule is:

> A preposition is a word used to show the relation of a noun or pronoun to some other word in the sentence. A preposition specifies a relation between two points on a digraph and is represented by a line. If the preposition gives direction ("to") the relation is asymmetric; if the preposition does not specify direction ("of") the relation is symmetric.
>
> A group of words may act as a preposition: on account of, in spite of, divided by [Shavelson, 1971, p. 140].

The digraph resulting from the example is:



For each sentence in the analysis, individual digraphs were formed. These digraphs were then combined into a super-digraph as follows. The similarity between a pair of concepts was represented by the digraph with the smallest number of lines between the concept pair in the set of digraphs in which the pair was connected. Next, a "distance" matrix was formed in which each entry represented the minimum number of lines connecting any pair of key concepts. For the example, the following distance matrix was obtained:[3]

	p	e	c	z
p	0	1	2	1
e	∞	0	1	∞
c	∞	∞	0	∞
z	1	2	3	0

Where: p = probability,
e = event,
c = chances, and
z = zero.

[3] In the actual analysis, the distance matrix is computed only for the super-digraph. However, this example demonstrates the connection between the digraph and the distance matrix.

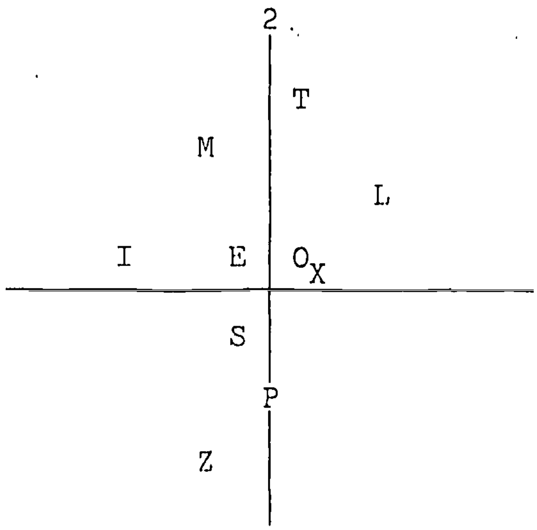
The larger the entry in the distance matrix, the more dissimilar the pair of concepts. In order to convert this "dissimilarity" matrix to a similarity matrix, each element (x) in the distance matrix was replaced by a new element (y) using the formula: $y = 1/(x+1)$. For our example we obtained:

	p	e	c	z
p	1.0	.50	.33	.50
e	0.0	1.0	.50	0.0
c	0.0	0.0	1.0	0.0
z	.50	.33	.25	1.0

In this way, the resultant super-digraph was transformed to a 10 × 10 similarity matrix (cf. Geeslin, 1973). The elements in the similarity matrix indicate the "closeness" of each pair of concepts. Note that this procedure may result in an asymmetric matrix. The similarity matrix representing content structure was examined using Kruskal's (1964) multidimensional scaling procedure. A plot of the results is shown in Figure 1. Figure 1, then was interpreted as a representation of the structure of the probability text.

Figure 1

Plot of Multidimensional Scaling Solution
Content Digraph Analysis



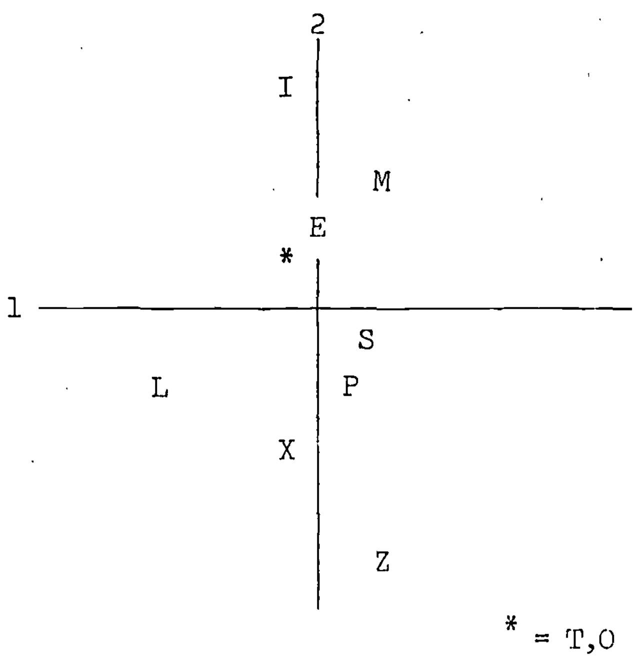
Key

P = Probability
I = Independent
E = Event
Z = Zero
L = Equally Likely
S = Intersection
T = Trial
X = Experiment
M = Mutually Exclusive
O = Outcome

A second method for representing content structure is graph theory (Harary and Norman, 1963). Graph theory may be distinguished from digraph theory in that the former ignores the direction of lines while the latter places an emphasis on directed lines. The same key concepts were used in this analysis as were used in the digraph analysis. The only change made in Shavelson's digraph procedures was to replace directed lines with non-directed lines. Thus the elements in the graph distance matrix are equal to the smallest element in each pair of corresponding cells in the digraph distance matrix. Note that the graph distance matrix will always be symmetric while this is not necessarily true of the digraph distance matrix. Obviously, if a symmetric digraph results from the digraph analysis, the structure representations by graph and digraph will be equivalent. The graph distance matrix was converted into a 10 × 10 similarity matrix and examined using multidimensional scaling. The plot of the results, shown in Figure 2, is interpreted as a second representation of the structure presented by the probability text.

Figure 2

Two-Dimensional Scaling Solution
For the Graph Analysis of Content Structure



Key

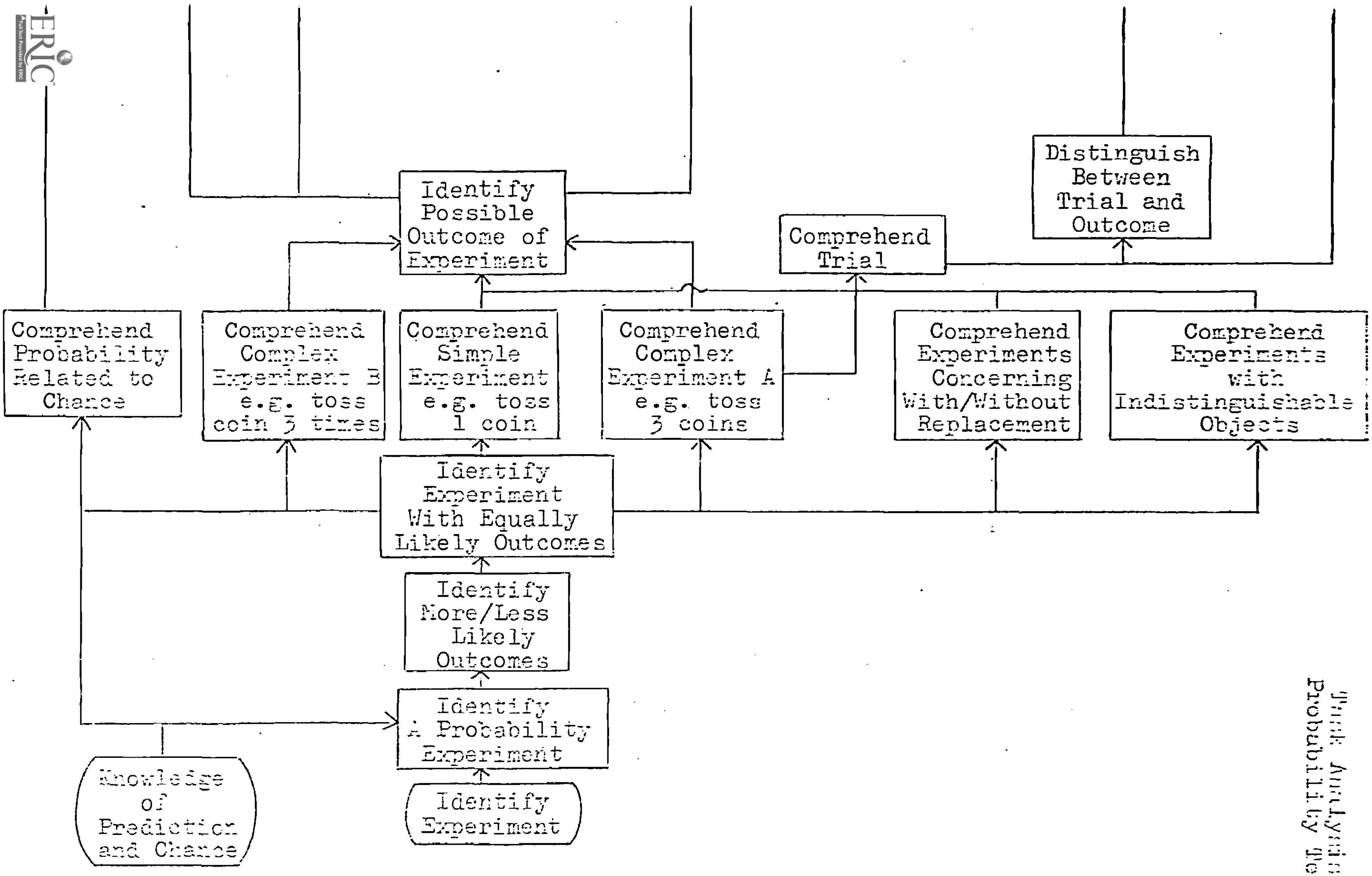
P = Probability	S = Intersection
I = Independent	T = Trial
E = Event	X = Experiment
Z = Zero	M = Mutually Exclusive
L = Equally Likely	O = Outcome

Finally, task analysis was used to map the structure of the instructional material (Gagne, 1965, 1970). Task analysis produces an alternate (to the digraph/graph analyses) structural representation. Points represent competencies and lines represent relationships between competencies. This is a psychological definition of structure and therefore different from what subject-matter experts mean when they use the term structure. However, we use task analysis in the present study to link the digraph/graph representations to a more traditional approach.

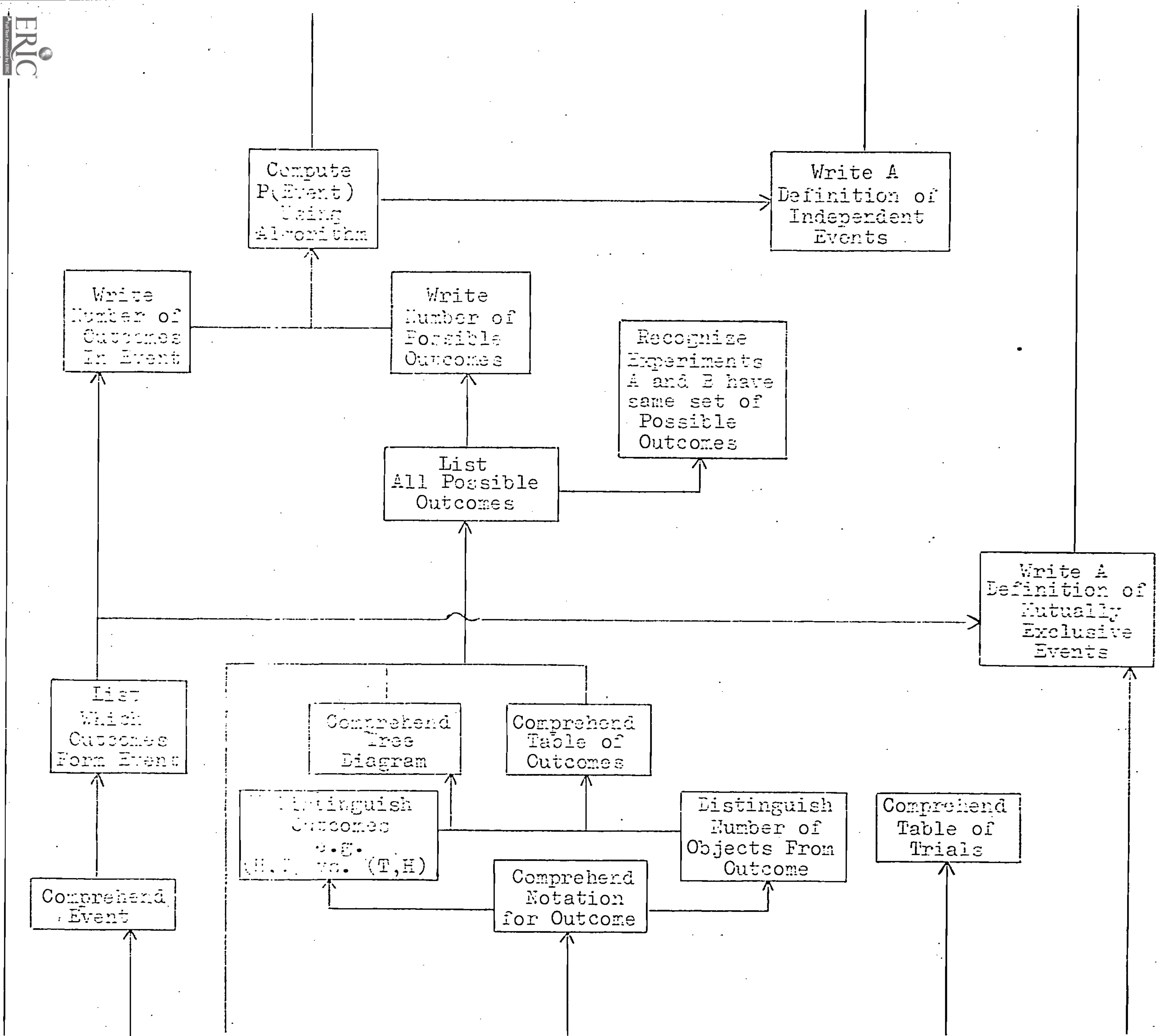
The resultant heirarchy is presented in Figure 3. The investigator was not able to determine a satisfactory method for obtaining a "distance" matrix from the resultant heirarchy. One could count "boxes" between concepts, but the boxes do not represent concepts alone but rather they represent manipulations or performances with concepts. Thus, for example, the concepts OUTCOME and EVENT appear in several boxes and one could arrive at several distances between these concepts depending on the boxes selected. Additionally, the boxes are derived in a somewhat subjective manner. A logical analysis by one author may not be the same as a logical analysis for a second author; thus causing the two authors to arrive at different distance matrices. The task analysis should be useful in interpreting the other content analyses and the analyses of the WA date, but does not appear to be a satisfactory representation of structure as we have defined it.

Figure 3

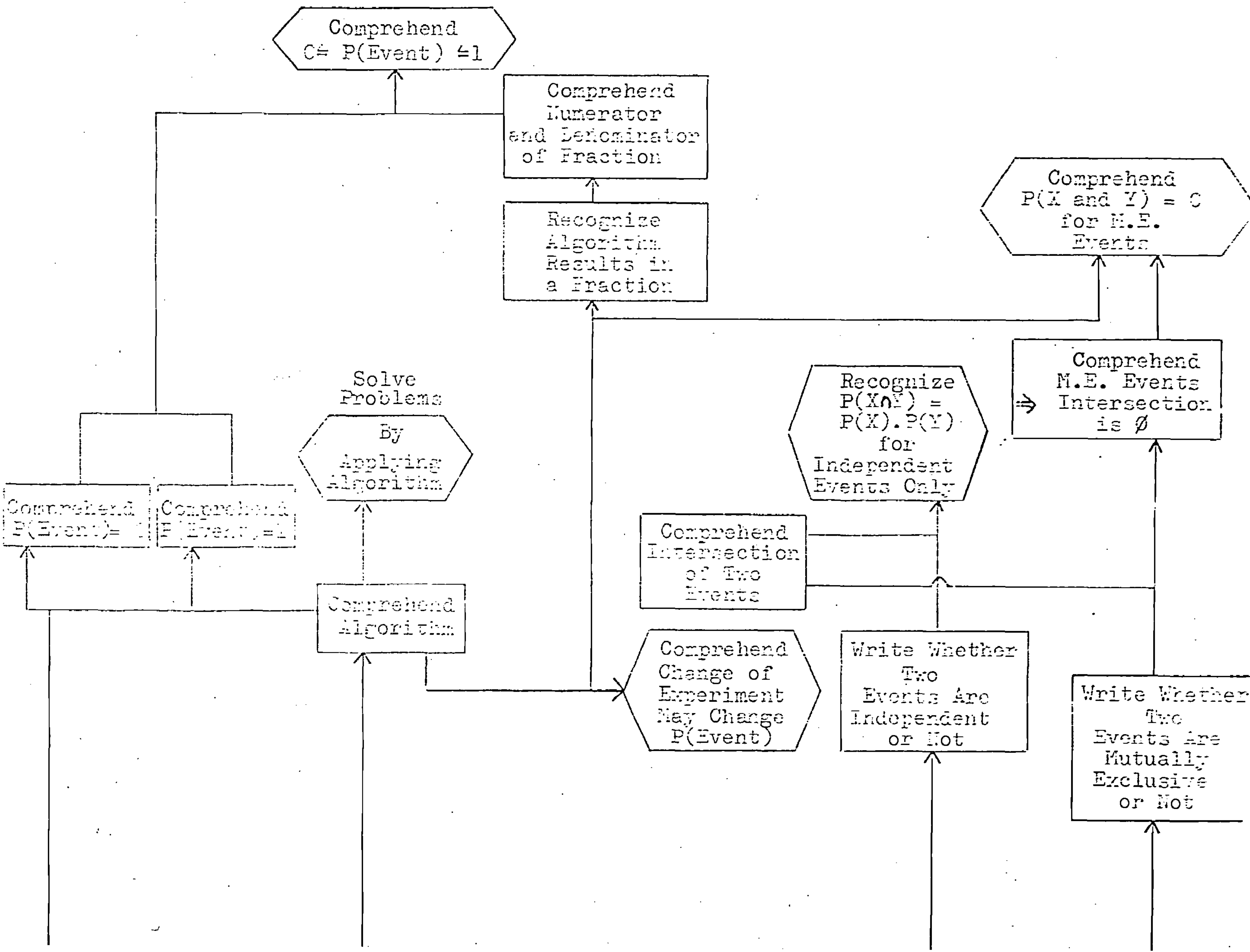
Task Analysis of Probability Text*



*The use of the word "comprehend" in the task analysis deviates somewhat from Gagne's (1970) terminology. A definition of the word comprehend and a discussion of its use is given by Romberg and Wilson (1969).



Compute P(Event) Using Algorithm
Write A Definition of Independent Events
Write Number of Outcomes In Event
Write Number of Possible Outcomes
Recognize Experiments A and B have same set of Possible Outcomes
List All Possible Outcomes
Write A Definition of Mutually Exclusive Events
List Which Outcomes Form Event
Comprehend Tree Diagram
Comprehend Table of Outcomes
Distinguish Outcomes e.g. vs. (T,H)
Distinguish Number of Objects From Outcome
Comprehend Table of Trials
Comprehend Event
Comprehend Notation for Outcome


Comprehend
0 ≤ P(Event) ≤ 1
Comprehend
Numerator
and Denominator
of Fraction
Recognize
Algorithm
Results in
a Fraction
Comprehend
P(X and Y) = 0
for M.E.
Events
Comprehend
M.E. Events
⇒ Intersection
is Ø
Solve
Problems
By
Applying
Algorithm
Recognize
P(X∩Y) =
P(X).P(Y)
for
Independent
Events Only
Comprehend
P(Event) = 0
Comprehend
P(Event) = 1
Comprehend
Intersection
of Two
Events
Comprehend
Algorithm
Comprehend
Change of
Experiment
May Change
P(Event)
Write Whether
Two
Events Are
Independent
or Not
Write Whether
Two
Events Are
Mutually
Exclusive
or Not

Representation of Cognitive Structure

Cognitive structure was investigated using a word association technique (Geeslin, 1973; Johnson, 1967, 1969; Shavelson, 1971, 1972, in press). Empirical evidence in support of the cognitive structure interpretation of WA data is provided by Deese (1962), Johnson (1967, 1969), Shavelson (1971, 1972, in press), and Shavelson & Stanton (in press).

The WA test consisted of one page of instructions and one page for each set of responses to each of the ten key concepts. Subjects were instructed to write as many other mathematical concepts related to the key concept as they could in one minute. On each response page, a key concept was printed at the top-center with the remainder of the page consisting of two columns of the key concept repeated with a line to the right of the word. Four random sequences of the stimulus words were used to prevent a possible sequence effect. A particular sequence was assigned randomly to subjects at each test administration.

The word association (WA) data were converted into a matrix of similarities between concepts by means of the relatedness coefficient (Garskof & Houston, 1963). The relatedness coefficient (RC) depends on the number of responses to a given stimulus word and the overlap between response distributions for pairs of stimulus words. The formula[4] for obtaining the RC coefficient is:

$$RC = \frac{\overline{A} \cdot \overline{B}}{(A \cdot B) - [n^P - (n-1)^P]^2}$$

[4]The RC coefficient may have a ceiling effect as suggested by Shavelson (1971). Additionally, the RC coefficient is symmetric and thus would not be able to reproduce a digraph distance matrix (asymmetric) exactly.

where:

> $\overline{A}$ and $\overline{B}$ represent the rank order of words under A which are shared in common with B and the rank order of words in B which are shared in A.
>
> $A \cdot B$ represents the rank order of words in A multiplied by the rank order of words in B.
>
> n represents all of the words in B (the longer list).
>
> P represents some fixed number greater than zero which may be determined from the shape of the probability distribution of the responses. P was set equal to 1 in this study; all portions of the S's response distribution received equal weight.

The relatedness coefficient may range from zero to one inclusive. The larger the value of the relatedness coefficient the closer the relationship between the two concepts. For example, one student responded to EVENT and EXPERIMENT on the posttest as follows:

Event	Rank	Experiment	Rank
Event	5	Experiment	5
Number	4	Event	4
Trial	3	Outcome	3
Outcome	2	Trial	2
		Probability	1

Thus $$RC = \frac{(5\ 3\ 2) \cdot \begin{pmatrix} 4 \\ 2 \\ 3 \end{pmatrix}}{(5\ 4\ 3\ 2\ 1) \cdot \begin{pmatrix} 5 \\ 4 \\ 3 \\ 2 \\ 1 \end{pmatrix} - [5^1 - (5-1)^1]^2} = 0.593$$

For each student a 10 × 10 RC matrix was formed. This was a symmetric similarity matrix. Individual or median RC matrices could then be compared to the digraph similarity matrix. The structure underlying the median RC matrix was examined using Kruskal's (1964) multidimensional scaling procedure.

Instrumentation

In addition to the WA test, three other measures were used to provide further information about the subjects: an attitude questionnaire, an achievement test on probability, and a paragraph construction task. The attitude questionnaire was the "Pro-Math Composite" scale (PY011; see Wilson, Cahen, & Begle, 1968) developed by the National Longitudinal Study of Mathematical Abilities (NLSMA). The scale was designed to measure general attitude toward mathematics. Internal consistency coefficient alpha for this scale was 0.72 for the subjects used in this study.

The main achievement test consisted of twenty-eight free-response items and seven multiple-choice items. The first thirty items tested comprehension of the material presented in the probability text. The last five items presented problems on probability in a perspective different from that used in the programmed text. Internal consistency coefficients alpha calculated from experimental subjects' data in the present study were 0.832 and 0.827 at posttest and retention test, respectively.

In addition to the thirty-five item achievement test, two ten item tests were given to the experimental subjects at the end of sections 1 and 2 of the probability text, respectively. These tests were used only to give experimental subjects a progress check and to help insure that subjects did not proceed so quickly through the programmed material that little or no learning took place.

An alternate measure of the learning of structure was the paragraph construction (PC) test. Each PC test consisted of one page of instructions and five pages for students' responses. Each response page had a concept pair printed in the upper left-hand corner. The rest of the page was left blank for S to write a paragraph explaining the mathematical relationship between the two concepts listed at the top.

The number of Ss used in the study was not sufficiently large to allow the use of all possible (45) pairs of WA stimulus words since an excessive amount of testing time would be required. Since random sampling of pairs of words would not guarantee a representation of the variety of distances (determined by the digraph analysis) between concepts, pairs of concepts were chosen with the constraint that the set of pairs reflected the variation in distances between concepts. (This is a matrix sampling problem. See Lord and Novick, 1968, pp. 236-238.) Using the further constraints that S would be presented only five pairs of concepts and that the PC test would contain all ten stimulus words from the WA test, two versions of the PC test (see Table 1) were derived.

Table 1

The Two Versions of the Paragraph Construction Test

PC Test 1	PC Test 2
experiment--zero	zero--equally likely
equally likely--mutually exclusive	trial--independent
outcome--independent	outcome--mutually exclusive
probability--event	probability--experiment
trial--intersection	event--intersection

Four random orders of the concept pairs were used on each PC test. A particular order of concept pairs was assigned randomly to S as well as assigning either PC Test A or PC Test B at random to subjects. Each S was asked to write one paragraph concerning each of five pairs of concepts; five digraphs, corresponding to the five paragraphs, were combined to form a super-digraph (one for each S). Then, for each treatment group, an element by element median was calculated and these median elements were combined to form a PC distance matrix. (It should be noted that each median entry was obtained from a different N, depending on the number of Ss who gave a response corresponding to that particular entry.) Although each S was required to discuss the relationships between only five pairs of concepts, at least some Ss in each experimental group found it necessary to include other relationships and thus no infinite elements were found in the PC distance matrices. Finally, the PC matrices were converted to a similarity matrix in the same manner as for the digraph. The structure underlying the PC similarity matrix was examined using Kruskal's (1964) multidimensional scaling procedure.

Treatment and Procedures

Subjects were assigned randomly to experimental and control treatments. Subjects in the experimental treatment read and studied the programmed text on probability theory. Subjects in the control group read and studied a programmed text on an unrelated mathematical topic, factors and prime numbers. Experimental subjects (N = 43) were never separated from control subjects (N = 44) during the experiment but subjects knew that two different programmed texts were being used.

The study was conducted over 23 calendar days in the subjects' classrooms during normal school hours near the end of the 1971-1972 academic year. With the exception of the retention test, the study was carried out during consecutive, 75-minute meetings of the classes which met every other school day. The first class meeting was devoted to orientation and pretesting. The orientation informed the subjects that they were participating in a study to find out how students learn mathematics.

The attitude questionnaire, the WA test, and the achievement test on probability were administered, in the order listed, to all subjects prior to instruction. The attitude questionnaire was given first so that tests and treatments used in the experiment would not affect subjects' responses. The WA test was administered prior to the achievement test to insure that the achievement test did not acquaint subjects with possible responses to the WA test. It was felt that neither the attitude questionnaire nor the WA test would influence subjects' responses to the achievement test. A brief discussion on using programmed instruction effectively followed the pretesting.

Each subject then read the text assigned to him. At the end of each text section, subjects in both treatments received a short review test over the particular section he had just completed. (The probability text did not have a test for Section 3, the final section.) Since instruction was self-paced not all Ss needed the entire instructional period to complete the text material; conversely, not all Ss read the entire text. However, all experimental subjects completed the second text section and most of the third section. Subjects who finished early were allowed to read, draw, or study material of their choosing as long as the material was non-mathematical.

During each phase of the study, at least one proctor was available at each session to manage materials and procedures. Proctors did not instruct subjects, but answered procedural questions, read test instructions, etcetera. The regular teacher was present to maintain discipline.

After instruction, all subjects were given the WA test, PC test, and achievement test, in the order listed. All subjects in a class were posttested at the same time[5].

On calendar day twenty-three, the WA test and achievement test were re-administered to subjects in the sequence listed. The purpose of this test administration was to measure the subjects' retention of the material. The design of the study, then, was a 2 × 3 (treatment by test occasion) with repeated measures on the latter factor.

Results and Discussion

Content Structure

The multidimensional scaling of the digraph distance matrix representing content structure (Figure 1) is consistent with our interpretation of the subject matter. Although interpretation of the multidimensional scaling solution is somewhat subjective, dimension 1 seemed to reflect mathematics as a model of the empirical experience; i.e., EXPERIMENT, OUTCOME, TRIAL, and EQUALLY LIKELY are (concrete) concepts that can be observed in the physical world. The other concepts (moving from right to left) are (abstract) mathematical concepts used to build a mathematical model of the physical world.

[5]Due to a conflicting school activity, two classes were posttested on calendar day 11 and one class was posttested on calendar day 15.

As for the second dimension, three clusters of key concepts may be identified. Cluster 1 includes the concepts of MUTUALLY EXCLUSIVE (M), INDEPENDENT (I), and EVENT (E). Cluster 2 contains the concepts of TRIAL (T), OUTCOME (O), EQUALLY LIKELY (L), and EXPERIMENT (X). Cluster 3 groups together the concepts of PROBABILITY (P), INTERSECTION (S), and ZERO (Z). These clusters may form a hierarchy of mathematical concepts with clusters 1 and 2 at one level and cluster 3 at the next superordinate level. Cluster 1 (M,I,E) represents mathematical concepts modeling cluster 2 (T,O,L,X), the physical concepts, and concepts in cluster 3 (P,S,Z) are mathematical concepts that tie together the model and the physical world.

Examining Figure 2, the graph representation of content structure, we see no essential differences from the digraph representation. TRIAL (T) and OUTCOME (O) are not distinguishable in the graph analysis indicating that we lose some information in this analysis as expected. Since no major differences were observed between the digraph and graph analysis of content structure, we will refer only to the digraph representation in the remainder of this discussion (see Geeslin, 1973 for complete results).

Cognitive Structure

Achievement test scores. Achievement test scores were used as a methodological check on students' learning of probability. If differences between treatment groups in traditional measures of learning mathematics are observed, confidence is increased in interpretations of differences observed in cognitive structures. Descriptive data from the achievement test are presented in Table 2. These data were analyzed by a 2 × 3 (treatment by test occasion) analysis of variance with repeated measures on the second factor. Results obtained were: (a) a significant treatment effect ($F = 114.92$, $df = 1/76$, $p < .01$); (b) a significant test occasion effect ($F = 86.85$, $df = 2/75$, $p < .01$); and (c) a significant interaction between effects ($F = 63.55$, $df = 2/75$, $p < .01$). The means in Table 2 indicate little difference between experimental and control subjects at pretest ($p < .01$) but large differences between groups at posttest and retention test ($p < .01$). As expected, the experimental group learned to solve significantly more problems in probability as a result of instruction than did subjects in the control group.

Table 2

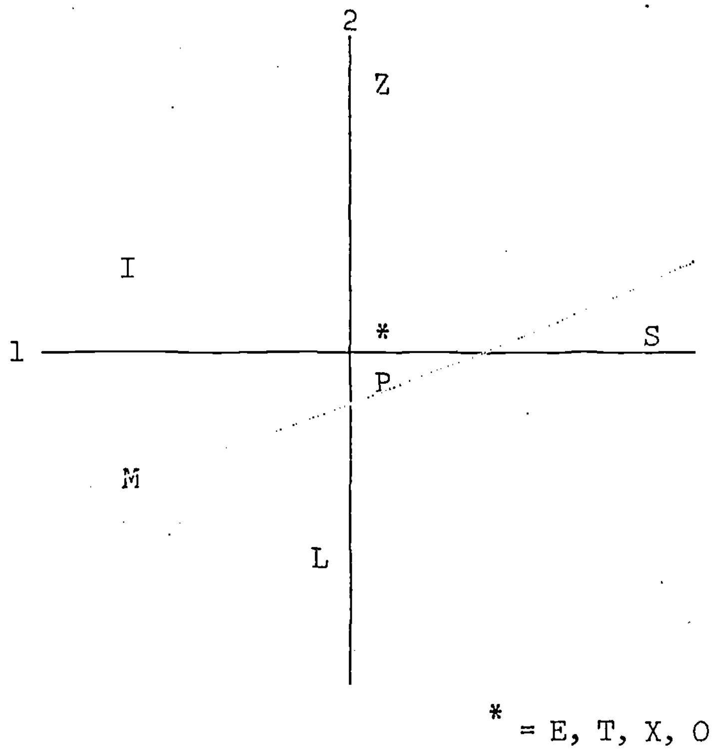
Means and Standard Deviations of Scores
on the Achievement Test for Each Treatment and Test Occasion

Treatment Group	Pretest	Posttest	Retention Test
Experimental	$\bar{X} = 3.65$ $\sigma = 2.45$ $n = 43$	$\bar{X} = 15.54$ $\sigma = 5.74$ $n = 41$	$\bar{X} = 16.21$ $\sigma = 6.32$ $n = 43$
Control	$\bar{X} = 3.00$ $\sigma = 1.90$ $n = 42$	$\bar{X} = 3.73$ $\sigma = 2.46$ $n = 40$	$\bar{X} = 4.16$ $\sigma = 3.06$ $n = 43$

Word association data. The results of the multidimensional scaling of median RC matrices at posttest and at retention test are presented in Figures 4 and 5, respectively. Pretest WA data and control group WA data were not included since the median RC matrices consisted of mostly zero elements; scaling solutions could not be obtained. These results are consistent with our hypotheses that: (a) eighth grade students were unfamiliar with the concepts of probability, and (b) that instruction in probability would change cognitive structure concerning concepts in probability.

Figure 4

Two Dimensional Scaling Solution
Experimental Subjects Posttest Median RC Matrix

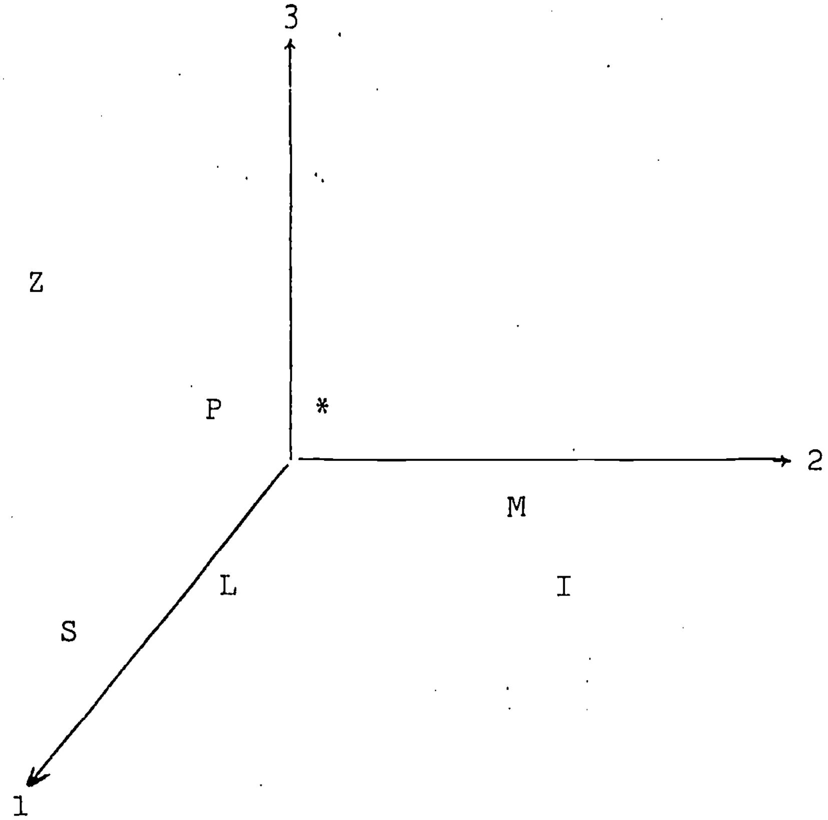


Key

P = Probability	S = Intersection
I = Independent	T = Trial
E = Event	X = Experiment
Z = Zero	M = Mutually Exclusive
L = Equally Likely	O = Outcome

Figure 5

Three Dimensional Scaling Solution
Experimental Subjects Retention Test Median RC Matrix



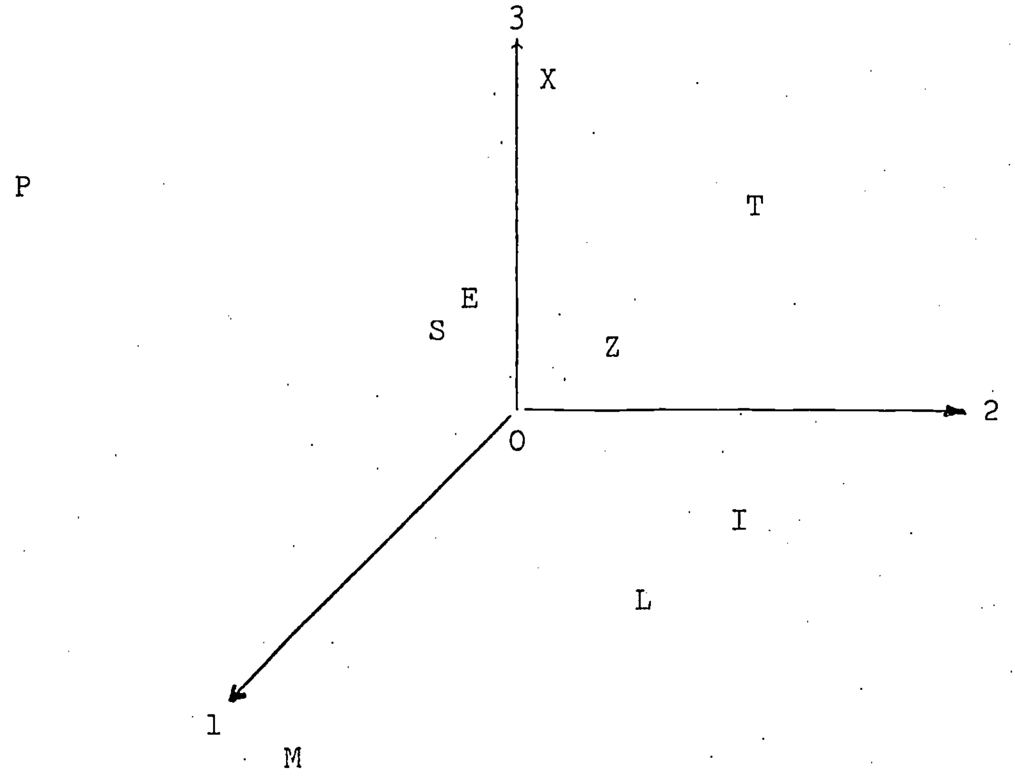
Key

P = Probability
I = Independent
E = Event
Z = Zero
L = Equally Likely
S = Intersection
T = Trial
X = Experiment
M = Mutually Exclusive
O = Outcome

Paragraph construction data. Multidimensional scaling solutions of the RC matrices are presented in Figures 6 and 7. Again the results indicate that the experimental treatment had an effect on cognitive structure concerning probability concepts. Subjects in both treatment groups found the PC task quite difficult, and many subjects in the control group made no response to the task. These factors make interpretations of the PC data tenuous.

Figure 6

Three Dimensional Scaling Solution
Experimental Subjects PC Data



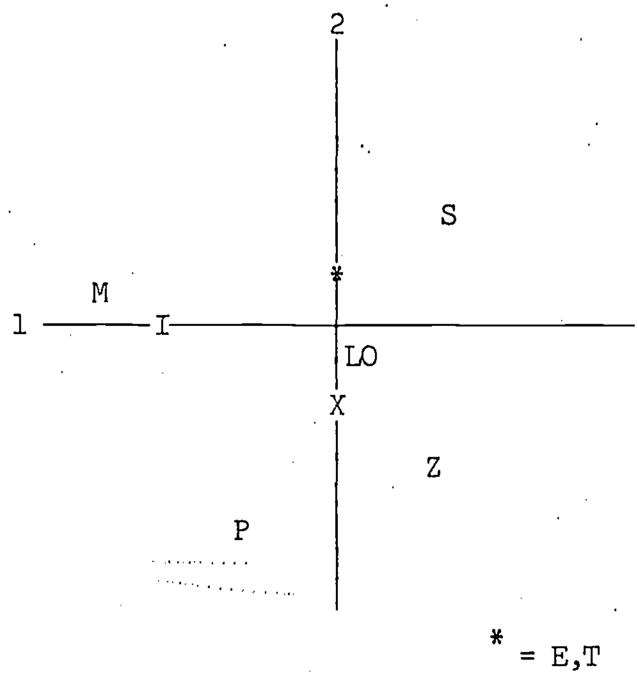
Key

P = Probability
I = Independent
E = Event
Z = Zero
L = Equally Likely

S = Intersection
T = Trial
X = Experiment
M = Mutually Exclusive
O = Outcome

Figure 7

Two Dimensional Scaling Solution
Control Subjects PC Data



Key

P = Probability	S = Intersection
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Comparison of Content Structure and Cognitive Structure

One way to compare content structure and cognitive structure is to visually examine the correspondence, or lack of it, between the multidimensional scaling solutions for the digraph and RC matrices. A strong similarity between the representations of content structure and cognitive structure was found. At posttest (Figure 4) experimental subjects grouped concepts similarly to the content structure. However, subjects appeared not to distinguish the concepts as clearly as the text. For example, E, T, X, and O were grouped together and were centered among the other concepts. Of course, E, X, and O were grouped closely in the content analysis, but a distinction between them was maintained. At retention test (Figure 5) subjects retained that portion of the structure they learned with perhaps a slight reorganization (toward correspondence with content structure). However, subjects still interrelated E, T, X and O more closely than the representation of content structure suggested they should be. In writing paragraphs, experimental subjects (see Figure 6) did distinguish E, T, X, and O; however, concepts were not tightly clustered as in the content analysis.

A second method of comparing structures is to calculate the Euclidean distance[6] between the digraph similarity matrix and the RC matrix. This provided an indication of the similarity between the two matrices. (The smaller the distance, the closer the match between the RC matrix and the digraph distance matrix[7].) For each subject, at each testing time, the Euclidean distance between his RC matrix and the digraph matrix was calculated. The correspondence

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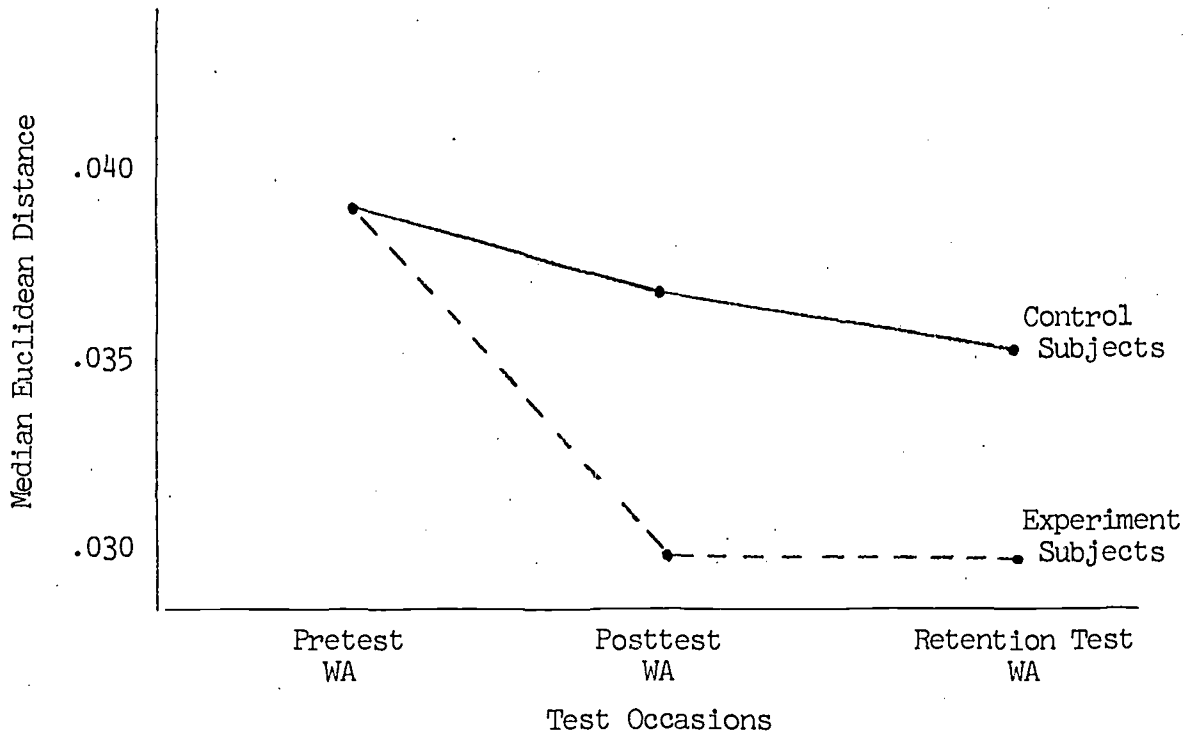
[6]The Euclidean distance is obtained by squaring each difference between corresponding elements of two matrices (e.g., a subject's RC matrix and the digraph similarity matrix), summing the squares, taking the square root of this sum, and dividing by ninety (the number of off-diagonal elements in each matrix).

[7]Since the smallest value of a RC is zero, some RC matrices consist of only off-diagonal elements that are zero. This may cause a Euclidean distance to be smaller than it should be, since it is possible to be further away from the content structure.

between content structure and cognitive structure is shown in Figure 8 the experimental and control groups. These data indicate that experimental subjects' cognitive structures correspond much more closely to the content structure following instruction than prior to instruction. Some change in control subjects' cognitive structures is noted also, but the magnitude and rate of change were not nearly of the magnitude and rate found in the experimental group.

Figure 8

Median euclidean distances betweeen cognitive structure and content structure



A nonparametric analysis of variance (Bradley, 1968) was performed on the WA Euclidean distance data at pretest and posttest. The cognitive structure of subjects in the experimental group corresponded more closely to content structure than did the cognitive structure of subjects in the control group ($p < .01$).

Scores from the attitude, achievement, and cognitive structure measures were intercorrelated to explore possible relationships among the variables. These correlations, Kendall's Tau, are presented in Table 3 for the experimental group. The correspondence variable refers to the Euclidean distance between an individual's RC matrix (cognitive structure) and the digraph similarity matrix (content structure). Perfect correspondence between achievement data and WA data would be indicated by a correlation of -1.0 since a smaller Euclidean distance score implies a closer relationship between content structure and cognitive structure.

Table 3

Rank Order Correlations (Tau) between all Measures for Subjects in the Experimental Group

	Achievement			Correspondence of Structure		
	Pre	Post	Retention	Pre	Post	Retention
Attitude	243	186	158	-181	116	042
Achievement						
Pretest		236	285	-198	-132	-144
Posttest			724	051	024	104
Retention Test				-036	-080	027
Correspondence of Structure						
Pretest					097	375
Posttest						372

The correlations indicate that scores on the attitude scale have a low correlation with scores on other variables. Scores on achievement at pretest have a low correlation with achievement posttest scores. This is consistent with the findings that subjects knew little about probability before the study but learned about probability as a result of instruction. A high correlation between scores on achievement at posttest and retention was obtained; this indicates subjects retained knowledge in accordance with their immediate learning. The correlations between variables representing the correspondence of cognitive structure with content structure showed a pattern similar to that of the achievement test scores. Low correlations between achievement and correspondence variables were obtained. This may indicate that learning to solve problems and learning of mathematical structure represent different aspects of learning. Although this finding is consistent with past studies (Shavelson, 1971, 1972, 1973) a stronger relationship was expected.

Conclusions

This study indicated that the analysis of content structure using digraph theory could be applied to a mathematics curriculum. The results of the analysis--a map of content structure--agreed with our understanding of the structure of the subject matter in probability.

The achievement test data indicated that the programmed text on probability was effective in teaching probability to eighth grade students. Compared to subjects in a control group, subjects in the experimental group learned how to solve significantly more problems as a result of instruction and retained this learning at retention test.

Furthermore, subjects in the experimental group learned a significant portion of the structure of probability as a result of instruction while the control group learned almost nothing of the structure. This learning of structure was retained until retention test time (a factor not investigated in prior studies). However, learning structure and learning to calculate solutions to problems in probability appeared to develop independently of each other.

The structure methodology used in this study appears to be applicable to many aspects of research on learning mathematical structures and might be a helpful tool in formative evaluation of mathematics curricula. That is, the data on content structure and cognitive structure seem to suggest ways to improve the text to further student learning of structure. The results on the paragraph construction task indicated students have great difficulty verbalizing mathematical relationships and more classroom practice in writing about mathematics might increase student learning too.

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