This teacher guide is part of the materials prepared for an individualized program for ninth-grade algebra and basic mathematics students. Materials written for the program are to be used with audiovisual lessons recorded on tape cassettes. For an evaluation of the program see ED 086 545. In this guide, the teacher is provided with objectives for each topic area and guided to materials written for a given topic. Three short criterion tests are included for each topic covered. The work in this package centers on linear functions and their graphs. Problems whose solutions require the use of direct or inverse variation are presented. This work was prepared under an ESEA Title III contract. (JP)
ALGEBRA I

Package #03-10

FUNCTIONS, RELATIONS, AND GRAPHS

Prepared By
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Many problems, especially those in the scientific world, require the use of two variables representing two unknowns in their solution. Some of these problems will result in the use of two equations. You will work problems of this kind in package 11, but you will gain a basic understanding in this package that will enable you to more easily solve those problems. Other problems will involve direct and inverse variation. You will learn a simple method for solving these problems in this package.

You might want to review 03-04-08 which is an introductory lesson on functions.

THE GOAL OF THIS PACKAGE:

To gain an understanding of linear functions and their graphs and to be able to solve problems whose solution requires the use of direct or inverse variation.
PACKAGE OBJECTIVES:

1. Given a function, make a bar graph, broken line graph, or a pictograph of it as directed.

2. Given an ordered pair of real numbers, plot its graph on a plane rectangular coordinate system.

3. Given a relation, plot its graph, determine if it is a function, and state its domain and range.

4. Given an open sentence in two variables, write its solution.

5. Given a linear equation in two variables, graph its solution set.

6. Given a problem which requires understanding of the meaning of the slope of a line for its solution, solve it.

7. Given a problem requiring use of the slope-intercept form of a linear equation solve it.

8. Given the information necessary to plot a line on a coordinate plane, write an equation of the line.

9. Given a problem requiring use of a direct variation or a proportion, solve it.

10. Given a problem involving a quadratic direct variation or a quadratic function, solve it.

11. Given a problem involving inverse variation, solve it.
I. U. #03-10-01

FUNCTIONS DESCRIBED BY TABLES
You will need to recall that:

A function consists of two sets, D and R, together with a rule that assigns to each element of D exactly one element of R.

OBJECTIVES:

1. When asked to write the meaning of the term "ordered pair" write "A pair of elements in which the order is important."

2. When asked to write the meaning of the term "domain of a relation" write, "The set of first elements in the ordered pairs that form the relation."

3. When asked to write the meaning of the "range of a relation" write, "The set of second elements in the ordered pairs that form the relation."

4. When asked to write the meaning of "first component", or "first coordinate" write, "The first number of an ordered pair."

5. When asked to write the meaning of "second component", or "second coordinate" write, "The second number of an ordered pair."

6. Given a function, make a bar graph, broken line graph, or a pictograph of it as directed.

ACTIVITIES:

1. Study objectives one through five and learn the meaning of the terms therein.

2. Study pages 355 - 358 in S + M. Objective 6)

3. Do the odd numbered part A written exercises on pages 358 and 359.
1. Write the meaning of the term "ordered pair".

2. Write the meaning of the term "domain of a relation".

3. Write the meaning of the term "range of a relation".

4. (A) Write the meaning of the term "first component".
    (B) Write the meaning of the term "first coordinate".

5. (A) Write the meaning of the term "second component".
    (B) Write the meaning of the term "second coordinate".

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>population</td>
<td>2000</td>
<td>2500</td>
<td>2600</td>
<td>2600</td>
<td>2500</td>
<td>2400</td>
</tr>
</tbody>
</table>

(A) Make a bar graph to represent the facts shown in the table. Scale 1/8 inch per 100 population.

(B) Make a broken line graph to represent the facts in the table. One scale mark on graph = 100 population.

(C) Make a pictograph to represent the facts in the table. Each figure represents 100 population.
1. Write the meaning of the term "ordered pair".

2. Write the meaning of the term "domain of a relation".

3. Write the meaning of the term "range of a relation".

4. (A) Write the meaning of the term "first component".
   (B) Write the meaning of the term "first coordinate".

5. (A) Write the meaning of the term "second component".
   (B) Write the meaning of the term "second coordinate".

6. (A) Make a bar graph for the facts shown in the table. (round all numbers to the nearest hundred.) Scale \( \frac{1}{2} " = 500 \) positions.

   (B) Make a line graph for the facts shown in the table. (round all numbers to the nearest hundred.) one unit on graph paper = 500 position.

   (C) Make a pictograph for the facts shown in the table. (round all numbers to the nearest hundred.) one figure = 500 positions.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>number of teaching positions available</td>
<td>5276</td>
<td>6143</td>
<td>6935</td>
<td>7550</td>
<td>8400</td>
<td>8499</td>
</tr>
</tbody>
</table>
1. Write the meaning of the term "ordered pair".

2. Write the meaning of the term "domain of a relation".

3. Write the meaning of the term "range of a relation".

4. (A) Write the meaning of the term "first component".
   (B) Write the meaning of the term "first coordinate".

5. (A) Write the meaning of the term "second component".
   (B) Write the meaning of the term "second coordinate".

6. | Name of satellite | Discoverer I | Tiros I | Courrier I | Raugu I |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Satellite's weight in pounds</td>
<td>250</td>
<td>270</td>
<td>500</td>
<td>675</td>
</tr>
</tbody>
</table>

   (A) Make a bar graph of these facts. ¼" per 50 lbs.

   (B) Make a line graph of these facts. one unit on graph paper per 50 lbs.

   (C) Make a pictograph of these facts. one figure per 50 lbs.
Answers to Criterion Tests

Test 03-10-01-01

1. An ordered pair is a pair of elements in which the ordered is important.

2. The domain of a relation is the set of first elements in the ordered pairs that form the relation.

3. The range of a relation is the set of second elements in the ordered pairs that form the relation.

4. (a) The first component of an ordered pair is the first number in the ordered pair.
   (b) The first coordinate is the first number in the ordered pair.

5. (a) The second component is the second number in an ordered pair.
   (b) The second coordinate is the second number in the ordered pair.

6. (a)
6. (b) Answers to Criterion Tests (Cont.)

![Graph showing population changes from 1920 to 1970.]

6. (c)

```
1920
1930
1940
1950
1960
1970
```

Key: □ = 100 people
Answers to Criterion Tests (Cont.)

Test 03-01-01-02

1. An ordered pair is a pair of elements in which the order is important.

2. The domain of a relation is the set of first elements in the ordered pairs that form the relation.

3. The range of a relation is the set of second elements in the ordered pairs that form the relation.

4. (a) The first component of an ordered pair is the first number in the ordered pair.

   (b) The first coordinate is the first number in the ordered pair.

5. (a) The second component is the second number in an ordered pair.

   (b) The second coordinate is the second number in the ordered pair.

6. (a)

   1965
   1966
   1967
   1968
   1969
   1970

   Positions available
6. (b)

![Graph showing positions available from 1965 to 1970]

6. (c)

<table>
<thead>
<tr>
<th>Year</th>
<th>Positions Available</th>
</tr>
</thead>
<tbody>
<tr>
<td>1965</td>
<td>☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐</td>
</tr>
<tr>
<td>1966</td>
<td>☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐</td>
</tr>
<tr>
<td>1967</td>
<td>☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐</td>
</tr>
<tr>
<td>1968</td>
<td>☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐</td>
</tr>
<tr>
<td>1969</td>
<td>☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐</td>
</tr>
<tr>
<td>1970</td>
<td>☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐</td>
</tr>
</tbody>
</table>

*key: ☐ = 500 positions*
Answers to Criterion Tests  (Cont.)

Test 03-01-01-03

1. An ordered pair is a pair of elements in which the order is important.

2. The domain of a relation is the set of first elements in the ordered pairs that form the relation.

3. The range of a relation is the set of second elements in the ordered pairs that form the relation.

4. (a) The first component of an ordered pair is the first number in the ordered pair.

   (b) The first coordinate is the first number in that ordered pair.

5. (a) The second component is the second number in an ordered pair.

   (b) The second coordinate is the second number in the ordered pair.

6. (a) 

<table>
<thead>
<tr>
<th>Discoverer I</th>
<th>Tiros I</th>
<th>Courrier I</th>
<th>Ranger I</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   0 100 200 300 400 500 600 700
   weight in lbs.
Answers to Criterion Tests  (Cont.)

Test 03-10-01-03 (Cont.)

![Graph showing weight in lbs. for Discoverer I, Tiros I, Courrier I, and Ranger I.]

Discoverer I  Tiros I  Courrier I  Ranger I

Key: 50 lbs
COORDINATES IN A PLANE
OBJECTIVES:

1. Given a plane rectangular coordinate system, identify the horizontal axis, vertical axis, and origin.

2. Given a plane rectangular coordinate system, identify the usual positive direction on the axes by circling the arrowheads that point in the positive directions.

3. Given the coordinates of a point, identify the abcissa and ordinate.

4. Given a point on a coordinate plane, write its coordinates.

5. Given a plane rectangular coordinate system, identify the first, second, third, and fourth quadrants.

6. Given the coordinates of a point, identify the quadrant containing the point.

7. Given an ordered pair of real numbers, plot its graph on a plane rectangular coordinate system.

ACTIVITIES:

1. Study pages 359 and 360, S + M, and do the odd numbered part A written exercises on page 362. (Objectives 1 - 7)
1. Identify the horizontal axis, vertical axis, and origin.

2. Identify the usual positive direction on the above coordinate system by circling the arrowheads pointing in the positive directions.

3. (3, 1) are the coordinates of a point. Identify the abscissa and the ordinate.

4. Write the coordinates of point P as an ordered pair.

5. Identify the quadrants of

6. Identify the quadrant that contains each of the following points.
   (A) (3, 2)   (B) (-2, 3)   (C) (5, -5)   (D) (-1, -4)

7. Plot the graph of the following points.
   (A) (1, 2)   (B) (-2, 3)   (C) (3, -4)   (D) (-5, 4)
Criterion Test 03-10-02-03

1. Identify the horizontal axis, vertical axis, and origin.

2. Identify the usual positive direction on the above coordinate system by circling the arrowheads pointing in the positive directions.

3. (3, 1) are the coordinates of a point. Identify the abscissa and the ordinate.

4. Identify the quadrants of point P as an ordered pair.

5. Identify the quadrants of

6. Identify the quadrant that contains each of the following points.
   (A) (4, -2)  (B) (4, 1)  (C) (-4, -1)  (D) (+4, -1)

7. Plot the graph of the following points.
   (A) (1, 1)  (B) (2, -2)  (C) (-3, 3)  (D) (-4, -4)
1. Identify the horizontal axis, vertical axis, and origin.

2. Identify the usual positive direction on the above coordinate system by circling the arrowheads pointing in the positive direction.

3. \((3, 1)\) are the coordinates of a point. Identify the abscissa and the ordinate.

4. Write the coordinates of point \(P\) as an ordered pair.

5. Identify the quadrants of

6. Identify the quadrant that contains each of the following points.

   (A) \((-4, 2)\)   (B) \((-2, -2)\)   (C) \((3, 5)\)   (D) \((3, -5)\)

7. Plot the graph of the following points.

   (A) \((0, 1)\)   (B) \((-1, 1)\)   (C) \((-2, -1)\)   (D) \((3, -1)\)
Answers to Criterion Tests

Test 03-10-02-01

1. vertical axis
   origin
   horizontal axis

2. (Arrows on coordinate system)

3. 3 is the abscissa    1 is the ordinate

4. (-2, -3)

5. II | I
     III | IV

6. (A) I  (B) II  (C) IV  (D) III

7. .  
   .  
   .  
   .  
   .  
   .  
   .  
   .  
   .  
   .  
   A
   C
Answers to Criterion Tests
(Cont.)

Test 03-10-02-02

vertical axis $\uparrow$ y

1. origin

2. (Arrows on coordinate system)

3. 3 is the abscissa and 1 is the ordinate

4. (-2, -3)

5. $\begin{array}{c|c}
      II & I \\
      III & IV \\
\end{array}$

6. (A) II  (B) III  (C) I  (D) IV

7. A B C D
Answers to Criterion Test 03-10-02-03

vertical axis \( y \) \[ \rightarrow \]

horizontal axis \( x \) \[ \rightarrow \]

1. \[ \text{origin} \]

2. (Arrows on coordinate system)

3. 3 is the abscissa  \( 1 \) is the ordinate

4. \((-2, -3)\)

5. \[\begin{array}{c|c|c|c}
   II & I & \text{III} & IV \\
\end{array}\]

6. (A) IV   (B) I   (C) III   (D) II

7. \[\begin{array}{c|c|c}
   & \bullet C & \\
   & \bullet A & \\
   & \bullet B & \\
   \bullet D & \end{array}\]
You will need to recall that:

1. A function consists of two sets, D and R, together with a rule that assigns to each member of D exactly one member of R.

2. The domain of a relation is the set of first elements in the ordered pairs that form the relation.

3. The range of a relation is the set of second elements in the ordered pairs that make up the relation.

OBJECTIVES:

1. When asked to define the term "relation" write, "A relation is any set of ordered pairs of elements".

2. Given a relation, plot its graph, determine if it is a function, and state its domain and range.

ACTIVITIES:

1. Study Objective one until you are sure that you understand what a "relation" is. (Objective 1)

2. Study pages 362 and 363, S + M, and do the odd numbered part A exercises on page 364. (Objective 2)
1. Define the term "relation".

2. Plot the relation whose ordered pairs are shown in the table. State the domain and range of the relation. Is the relation a function?

(A) 

<table>
<thead>
<tr>
<th>-1</th>
<th>-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>-3</td>
</tr>
<tr>
<td>2</td>
<td>-6</td>
</tr>
</tbody>
</table>

(B) 

<table>
<thead>
<tr>
<th>-2</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>6</td>
</tr>
<tr>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

(C) 

<table>
<thead>
<tr>
<th>-2</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>6</td>
</tr>
<tr>
<td>-1</td>
<td>5</td>
</tr>
<tr>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>

(D) 

<table>
<thead>
<tr>
<th>-2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>2</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>-1</td>
<td>2</td>
</tr>
<tr>
<td>-2</td>
<td>1</td>
</tr>
</tbody>
</table>
1. Define the term "relation".

2. Plot the relation whose ordered pairs are shown in the table. State the domain and range of the relation. Is the relation a function?

(A) \[\begin{array}{c|c}
1 & -1 \\
2 & -2 \\
1 & -3 \\
2 & -4 \\
1 & -5 \\
\end{array}\]

(B) \[\begin{array}{c|c}
0 & 0 \\
2 & 1 \\
4 & 2 \\
5 & 3 \\
6 & 5 \\
\end{array}\]

(C) \[\begin{array}{c|c}
0 & 10 \\
1 & 9 \\
2 & 6 \\
3 & 1 \\
4 & -6 \\
\end{array}\]

(D) \[\begin{array}{c|c}
0 & -1 \\
1 & 1 \\
2 & 7 \\
3 & 17 \\
-1 & 1 \\
\end{array}\]
1. Define the term "relation".

2. Plot the relation whose ordered pairs are shown in the table. State the domain and range of the relation. Is the relation a function?

(A)  
\[
\begin{array}{cc}
1 & 5 \\
2 & 4 \\
3 & 3 \\
4 & 2 \\
5 & 1 \\
\end{array}
\]

(B)  
\[
\begin{array}{cc}
5 & 1 \\
4 & 2 \\
3 & 3 \\
4 & 4 \\
5 & 5 \\
\end{array}
\]

(C)  
\[
\begin{array}{cc}
1 & 5 \\
2 & 4 \\
3 & 3 \\
4 & 4 \\
5 & 5 \\
\end{array}
\]

(D)  
\[
\begin{array}{cc}
-3 & -2 \\
0 & -1 \\
1 & 0 \\
0 & 1 \\
-3 & 2 \\
\end{array}
\]
Answers to Criterion Tests

Test 03-10-03-01

1. A relation is any set or ordered pairs of elements.

2. (A)

   Domain is \{-1, 0, 1, 2\}  Range is \{3, 0, -3, -6\}

   The relation \textit{is} a function.

(B)  

   Domain \{-2, -1, 0, 1, 2\}  Range \{7, 6, 5, 4, 3\}

   This relation \textit{is} a function.

(C)  

   Domain \{-2, -1, 0, 1\}  Range \{7, 6, 5, 4, 3\}

   This relation \textit{is not} a function.

(D)  

   Domain \{-2, -1, 0\}  Range \{1, 2\}

   This relation \textit{is} a function.
Answers to Criterion Test
Test 03-10-03-02

1. A relation is any set of ordered pairs of elements.

2. (A)

   Domain {1, 2}
   Range {-1, -2, -3, -4, -5}

   This relation is not a function.

   Domain {0, 2, 4, 5, 6}
   Range {0, 1, 2, 3, 5}

   This relation is a function.

   (B)

   Domain {0, 1, 2, 3, -1}
   Range {-1, 1, 7, -17}

   This relation is a function.

   (D)

   Domain {0, 1, 2, 3, -1}
   Range {10, 9, 6, 1, -6}

   This relation is a function.
Answers to Criterion Tests

Test 03-10-03-03

1. A relation is any set of ordered pairs of elements.

2. (A)

Domain \{1, 2, 3, 4, 5\}
Range \{5, 4, 3, 2, 1\}
This relation is a function.

(B)

Domain \{5, 4, 3\}
Range \{1, 2, 3, 4, 5\}
This relation is not a function.

(C)

Domain \{1, 2, 3, 4, 5\}
Range \{5, 4, 3\}
This relation is a function.

(D)

Domain \{-3, 0, 1\}
Range \{-2, -1, 0, 1, 2\}
This relation is not a function.
You will need to recall:

That an open sentence is a sentence (equation or inequality) which contains one or more variables. (Page 44, S + M)

OBJECTIVES:

1. When asked to give the meaning of the expression open sentences in two variables write, Equations or inequalities that involve two variables, such as \( x \) and \( y \) are called open sentences in two variables.

2. When asked to give the meaning of a solution or root of an open sentence in two variables write, A solution or root of an open sentence in two variables is an ordered pair of numbers from the replacement sets of the two variables that makes the sentence a true statement.

3. When asked to give the meaning of the solution set of an open sentence in two variables write, The set of all the solutions of an open sentence in two variables over the given replacement sets of the variables is called the solution set.

4. Given an open sentence in two variables write its solution set.

ACTIVITIES:

1. Study pages 365 and 366, S + M. (Objectives 1 - 4)

2. Do the oral exercises on page 367 to be sure you can use the terms of objectives 1 - 3. (The answers are in the teacher's edition)

3. Do the odd numbered part A written exercises on page 368 of S + M. You ought to try some of the part B and C exercises too.
Criterion Test 03-10-04-01

1. What does the term open sentence in two variables mean?

2. What do we mean when we speak of a solution or root of an open sentence in two variables?

3. What do we mean when we speak of the solution set of an open sentence in two variables?

4. Write the solution set of the following sentences given that \{\(-2, -1, 0, 1, 2\)\} is the replacement set of \(x\) and the set of the real numbers is the replacement set of \(y\).
   
   (a) \(y = -2x\)
   
   (b) \(y = x^2 + 4x\)

Criterion Test 03-10-04-02

1. What does the term open sentence in two variables mean?

2. What do we mean when we speak of a solution or root of an open sentence in two variables?

3. What do we mean when we speak of the solution set of an open sentence in two variables?

4. Write the solution set of the following sentences given that \{\(-2, -1, 0, 1, 2\)\} is the replacement set of \(x\) and the set of real numbers is the replacement set of \(y\).
   
   (a) \(y = -x - 1\)
   
   (b) \(y = x^2 - 5\)
1. What does the term open sentence in two variables mean?

2. What do we mean when we speak of a solution or root of an open sentence in two variables?

3. What do we mean when we speak of the solution set of an open sentence in two variables?

4. Write the solution set of the following sentences given that \((-2, -1, 0, 1, 2)\) is the replacement set of \(x\) and the set of the real numbers is the replacement set of \(y\).

   (a) \(y = \frac{1}{x}\)

   (b) \(y = -x^2\)
Answers to Criterion Tests

Test 03-10-04-01

1. Equations or inequalities that involve two variables such as $x$ and $y$ are called open sentences in two variables.

2. A solution or root of an open sentence in two variables is an ordered pair of numbers from the replacement sets of the two variables that makes the sentence a true statement.

3. The solution set of an open sentence in two variables is the set of all the solutions of the sentence over the given replacement sets of the variables.

4. (a) $\{(−2, 4), (−1, 2), (0, 0), (1, −2), (2, −4)\}$
   (b) $\{(−2, −4), (−1, −2), (0, 0), (1, 5), (2, 12)\}$

Test 03-10-04-02

1. Equations or inequalities that involve two variables such as $x$ and $y$ are called open sentences in two variables.

2. A solution or root of an open sentence in two variables is an ordered pair of numbers from the replacement sets of the two variables that makes the sentence a true statement.

3. The solution set of an open sentence in two variables is the set of all the solutions of the sentence over the given replacement sets of the variables.

4. (a) $\{(-2, 1), (-1, 0), (0, -1), (1, -2), (2, -3)\}$
   (b) $\{(-2, -1), (-1, -4), (0, -5), (1, -4), (2, -1)\}$
Answers to Criterion Tests (Cont.)

Test 03-10-04-03

1. Equations or inequalities that involve two variables such as $x$ and $y$ are called open sentences in two variables.

2. A solution or root of an open sentence in two variables is an ordered pair of numbers from the replacement sets of the two variables that makes the sentence a true statement.

3. The solution set of an open sentence in two variables is the set of all the solutions of the sentence over the given replacement sets of the variables.

4. (a) \{(-2, 1), (-1, -\frac{1}{2}), (0, 0), (1, 1/2), (2, 1)\}
\{(-2, -4), (-1, -1), (0, 0), (1, -1), (2, -4)\}
I. U. #03-10-05

THE GRAPH OF A LINEAR EQUATION IN TWO VARIABLES
OBJECTIVES:

1. When asked to tell what is meant by the graph of an equation write, the line or curve on a coordinate plane which is the set of all those points, and only those points, whose coordinates satisfy a given equation, is called the graph of the equation.

2. When asked to define an equation of a line write, An equation whose solution set is the coordinates of all those points and only those points which belong to a given line on a coordinate plane, is called an equation of that line.

Note: A line can have infinitely many equivalent equations, but an equation can have only one graph on a given coordinate plane.

3. When asked to define a linear equation in two variables write, In the coordinate plane, the graph of any equation equivalent to one of the form

$$Ax + By = C,$$

$x$ and $y$ are real numbers.

where $A$, $B$, and $C$ are real numbers with $A$ and $B$ not both zero is a straight line. Any such equation is called a linear equation in two variables, $x$ and $y$.

4. When asked to define a linear function write, A linear function is a function whose ordered pairs satisfy a linear equation.

5. Given a linear equation in two variables, graph its solution set.

ACTIVITIES:

1. Study pages 369 and 370, S + M. (Objective 1 - 5)

2. Do the oral exercises on page 371 to be sure that you understand the terms in objectives 1 - 4. The answers are in the teacher's edition of S + M.


4. Notice that the real numbers will be the replacement set for all open sentences in two variables from now on unless otherwise directed.
Criterion Test 03-10-05-01

1. What do we mean when we speak of the graph of an equation?
2. What do we mean when we speak of an equation of a line?
3. Define the term linear equation in two unknowns.
4. Define a linear function.
5. Graph the solution set of the following equations.
   (a) $2x - y = 3$
   (b) $5x + 3y = 0$
   (c) Write a formula for the fact that the number of feet is 16.5 times the number of rods; then graph the function.

Criterion Test 03-10-05-02

1. What do we mean when we speak of the graph of an equation?
2. What do we mean when we speak of an equation of a line?
3. Define the term linear equation in two unknowns.
4. Define a linear function.
5. Graph the solution set of the following equations.
   (a) $4x - 3y = 12$
   (b) $y = 5x - 3$
   (c) Graph the equation of the formula statement of the fact that the number of centimeters is 2.54 times the number of inches.
1. What do we mean when we speak of the graph of an equation?

2. What do we mean when we speak of an equation of a line?

3. Define the term linear equation in two unknowns.

4. Define a linear function.

5. Graph the solution set of the following equations.

   (a) \( x - 2y = 3 \)

   (b) \( 3x + y = 6 \)

   (c) Write a formula and graph the function. The weight of mercury is 13.2 times the weight of an equal volume of water.
Answers to Criterion Tests

Test 03-10-05-01

1. The line or curve on the coordinate plane which is the set of all those points, and only those points, whose coordinates satisfy a given equation, is called the graph of the equation.

2. An equation whose solution set is the coordinates of all those points and only those points which belong to a given line on a coordinate plane, is called an equation of that line.

3. In the coordinate plane, the graph of any equation equivalent to one of the form: $Ax + By = C \quad x \neq 0 \text{ or } y \neq 0$ where $A$, $B$, and $C$ are real numbers with $A$ and $B$ not both zero is a straight line. Any such equation is called a linear equation in two variables, $x$, and $y$.

4. A linear function is a function whose ordered pairs satisfy a linear equation.

5. (A) $2x - y = 3$

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$5x + 3y = 0$

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$R = 16.5F$

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<td>16.5</td>
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Answers to Criterion Tests

Test 03-10-05-02

1. The line or curve on the coordinate plane which is the set of all those points, and only those points, whose coordinates satisfy a given equation, is called the graph of the equation.

2. An equation whose solution set is the coordinates of all those points and only those points which belong to a given line on a coordinate plane, is called an equation of that line.

3. In the coordinate plane, the graph of any equation equivalent to one of the form: $Ax + By = C$ or $y 
eq 0$ where $A$, $B$, and $C$ are real numbers with $A$ and $B$ not both zero is a straight line. Any such equation is called a linear equation in two variables, $x$ and $y$.

4. A linear function is a function whose ordered pairs satisfy a linear equation.

5. (A) $4x - 3y = 12$
   \[
   \begin{array}{c|c}
   x & y \\
   \hline
   0 & -4 \\
   3 & 0 \\
   6 & 4 \\
   \end{array}
   \]

   (B) $y = 5x - 3$
   \[
   \begin{array}{c|c}
   x & y \\
   \hline
   -3 & 0 \\
   2 & 1 \\
   7 & 2 \\
   \end{array}
   \]

   (C) $c = 2.54I$
   \[
   \begin{array}{c|c}
   I & C \\
   \hline
   0 & 0 \\
   2.54 & 1 \\
   5.08 & 2 \\
   \end{array}
   \]
Answers to Criterion Tests

Test 03-10-05-03

1. The line or curve on the coordinate plane which is the set of all those points, and only those points, whose coordinates satisfy a given equation, is called the graph of the equation.

2. An equation whose solution set is the coordinates of all those points and only those points which belong to a given line on a coordinate plane, is called an equation of that line.

3. In the coordinate plane, the graph of any equation equivalent to one of the form: \(Ax + By = C\) \(x \neq 0\) or \(y \neq 0\) where \(A, B,\) and \(C\) are real numbers with \(A\) and \(B\) not both zero is a straight line. Any such equation is called a linear equation in two variables, \(x\) and \(y\).

4. A linear function is a function whose ordered pairs satisfy a linear equation.

5. (A) \(x - 2y = 3\)

   \[
   \begin{array}{c|c}
   x & y \\
   \hline
   3 & 0 \\
   5 & 1 \\
   1 & -1 \\
   \end{array}
   \]

   (B) \(3x + y = 6\)

   \[
   \begin{array}{c|c}
   x & y \\
   \hline
   0 & 6 \\
   1 & 3 \\
   2 & 0 \\
   \end{array}
   \]

   (C) \(13.2W = H\)

   \[
   \begin{array}{c|c}
   W & H \\
   \hline
   0 & 0 \\
   1 & 13.2 \\
   2 & 26.4 \\
   \end{array}
   \]
I. U. #03-10-06

SLOPE OF A LINE
OBJECTIVES:

After completion of this unit of instruction the student will be able to do the following:

1. Define writing the slope of a line in terms of ordinate and abcissa terminology.

2. Given the graph of a straight line on a coordinate system, write the formula for its slope as the following example illustrates:

\[ m = \frac{\text{difference of ordinates}}{\text{difference of abscissas}} = \frac{6 - 3}{4 - 1} = \frac{3}{3} = 1 \]

3. Determine by inspection from the graph of a straight line whether its slope is positive or negative.

4. Verbally describe the graph of a straight line if its slope is zero.

5. Explain in writing why a vertical line has no slope or its slope is undetermined.

6. Given a set of points, determine, without graphing, whether the points lie on a unique line.

7. Given a problem which requires understanding of the meaning of the slope of a line for its solution, solve it.

ACTIVITIES:

1. Study page 373, S + M. (Objective 1)

2. Study page 374, S + M. (Objectives 2, 3, 4)

3. Study page 375, S + M. (Objectives 5, 6)

4. Do the oral exercises, page 375, 376; the odd numbered part A written exercises, pages 376, 377. For a challenge, try some part B problems.
1. Define the slope of a line in terms of ordinate and abscissa terminology.

2. Write the formula for the slope of the following line.

3. Determine by inspection whether the following slopes are positive or negative.

4. Describe the graph of a straight line whose slope is zero.

5. Explain why a vertical line is said to have no slope or its slope is undetermined.

6. Determine without graphing whether the points given lie on a unique line. (Answer Yes if they do and No if they don’t.)
   (a) (-2, 3) (5, 3) (12, 3) (19, 3)
   (b) (1, 6) (4, 8) (7, 11) (11, 15)
7. Solve the following problems:

(a) Plot the points (2, 4) and (-1, -1); draw the line containing the points and determine the slope of the line from the graph.

(b) Through the point (1, 3) draw a line with a slope of 2.

(c) Determine algebraically the slope of the line which contains the points (3, -1) and (3, 4).
1. Define the slope of a line in terms of ordinate and abscissa terminology.

2. Write the formula for the slope of the following line.

\[ \text{Formula} \]

\[ \text{(6, 5)} \]
\[ \text{(2, 3)} \]

3. Determine by inspection whether the following slopes are positive or negative.

4. Describe the graph of a straight line whose slope is zero.

5. Explain why a vertical line is said to have no slope or its slope is undetermined.
6. Determine without graphing whether the points given lie on a unique line. (Answer yes if they do and no if they don't)

(a) \((-2, 2) (0, 0) (2, -2) (4, -4)\)

(b) \((-1, 3) (0, 6) (1, 3) (2, 6)\)

7. Solve the following problems:

(a) Plot the points \((2, -1)\) and \((0, 2)\); draw the line containing those points and determine its slope from the graph.

(b) Through the point \((-2, 1)\) draw a line with a slope of \(-3\).

(c) Determine algebraically the slope of the line which contains the points \((-1, -2)\) and \((2, 1)\).
1. Define the slope of a line in terms of ordinate and abscissa terminology.

2. Write the formula for the slope of the following line.

3. Determine by inspection whether the following slopes are positive or negative.

4. Describe the graph of a straight line whose slope is zero.

5. Explain why a vertical line is said to have no slope or its slope is undetermined.

6. Determine without graphing whether the points given lie on a unique line. (answer yes if they do and no if they don't)
   (a) (0, -2) (1, 0) (2, 2) (3, 4)
   (b) (0, 5) (1, 2) (2, 4) (3, 5)
7. Solve the following problems:

(a) Plot the points (-1, -2) and (-3, -3); draw the line containing those points, and determine its slope from the graph.

(b) Through the point (3, 0) draw a line whose slope is 3.

(c) Determine algebraically the slope of the line which contains the points (3, 2) and (2, 3).
Answers to Criterion Tests

Test 03-10-06-01

1. The slope of a line is the difference of the ordinates divided by the difference of the abscissas.

2. \[ m = \frac{4 - 2}{4 - 1} = \frac{2}{3} \]

3. Line B is positive, Line A is negative

4. The graph of a straight line whose slope is zero is a horizontal line, parallel to the x axis.

5. Because the difference of the abscissas is zero and division by zero is undefined.

6. (a) Yes  (b) No

7. (A)

(B) is undefined so the line is a vertical line, parallel to the y axis.
Answers to Criterion Tests

Test 03-10-06-02

1. The slope of a line is the difference of the ordinates divided by the difference of the abscissas.

2. \[ m = \frac{5 - 3}{6 - 2} = \frac{2}{4} = \frac{1}{2} \]

3. Line A is positive; line B is negative

4. The graph of a straight line whose slope is zero is a horizontal line, parallel to the x axis.

5. Because the difference of the abscissas is zero and division by zero is undefined.

6. (a) Yes          (b) No

7. (A)

   \[ m = -\frac{3}{2} \]

(B)

(C) \[ m = \frac{-2 - 1}{-3 - 2} = \frac{-3}{-5} = \frac{3}{5} \text{ with } \text{slope} = 1 \]
1. The slope of a line is the difference of the ordinates divided by the difference of the abscissas.

2. \[ m = \frac{-2 - 1}{-4 - (-1)} = \frac{-3}{-3} = 1 \]

3. Line A has a negative slope; Line B has a positive slope.

4. The graph of a straight line whose slope is zero is a horizontal line, parallel to the x axis.

5. Because the difference of the abscissas is zero and division by zero is undefined.

6. (a) Yes  (b) No

7. (A) \[ \text{slope} = 2 \]

   (B) \[ \text{slope} = \frac{2}{3} - \frac{3}{2} = \frac{-1}{1} = -1 \]
THE SLOPE - INTERCEPT FORM OF A LINEAR EQUATION
OBJECTIVES:

1. When asked to write the general form of an equation for a line which passes through the origin on a coordinate plane write, For every real number m, the graph in the coordinate plane of the equation $y = mx$ is the line that has the slope $m$ and passes through the origin.

2. When asked to define the y intercept of a line on the coordinate plane write, The y intercept of a line on the coordinate plane is the point at which the line intersects (crosses) the y axis.

3. When asked to write the general equation for the slope-intercept form of a linear equation write, For all real numbers $m$ and $b$, the graph in the coordinate plane of the equation $y = mx + b$ is the line whose slope is $m$ and whose y intercept is $b$.

4. Given a problem requiring use of the slope-intercept form of a linear equation, solve it.

ACTIVITIES:

1. Study pages 377 - 379 in S + M. (Objectives 1 - 4)

2. Do the oral exercises (answers in teachers edition) and the odd numbered written exercises on page 379. (Objective 4)
1. Write the general form of an equation for a line which passes through the origin on a coordinate plane.

2. Define the *y-intercept* of a line on the coordinate plane.

3. Write the general equation for the slope-intercept form of a linear equation.

4. Solve the following problems.
   
   (a) State the slope and *y-intercept* of the line whose equation is $3x - 2y = 4$.
   
   (b) Write a linear equation with integral coefficients whose graph has a slope of $1/2$ and a *y-intercept* of $1/2$.
   
   (c) Using only the *y-intercept* and the slope graph the equation $2x - y = 4$.

Criterion Test 03-10-07-02

1. Write the general form of an equation for a line which passes through the origin on a coordinate plane.

2. Define the *y-intercept* of a line on the coordinate plane.

3. Write the general equation for the slope-intercept form of a linear equation.

4. Solve the following problems.
   
   (a) State the slope and *y-intercept* of the line whose equation is $2x - 2y = 1$.
   
   (b) Write a linear equation whose graph has a slope of $-2$ and a *y-intercept* of $-3$.
   
   (c) Using only the *y-intercept* and the slope graph the equation $3x + y = 1$. 

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1. Write the general form of an equation for a line which passes through the origin on a coordinate plane.

2. Define the y-intercept of a line on the coordinate plane.

3. Write the general equation for the slope-intercept form of a linear equation.

4. Solve the following problems.

   (a) State the slope and y-intercept of the line whose equation is \( y - 1 = 2x \).

   (b) Write a linear equation whose graph has a slope of 5 and a y-intercept of 3.

   (c) Using only the y-intercept and slope graph the equation \( 3x + 4y = 0 \).
Answers to Criterion Tests

Test 03-10-07-01

1. For every real number \( m \), the graph in the coordinate plane of the equation \( y = mx \) is the line that has the slope \( m \) and passes through the origin.

2. The y-intercept of a line on the coordinate plane is the point at which the line intersects the y axis.

3. For all real numbers \( m \) and \( b \), the graph in the coordinate plane of the equation \( y = mx + b \) is the line whose slope is \( m \) and whose y-intercept is \( b \).

4. (a) \( 3x - 2y = 4 \), \(-2y = -3x + 4\), \( y = \frac{-3x}{-2} + \frac{4}{-2} \)

\[ y = \frac{3}{2}x - 2 \quad \text{slope} = \frac{3}{2}, \quad y\text{-intercept} = -2 \]

(b) \( y = \frac{1}{2}x + \frac{1}{2}, \quad \{2y = x + 1\} \)

(C) \[ 2x - y = 4 \\ -y = -2x + 4 \\ y = 2x - 4 \]
Answers To Criterion Tests

Test 03-10-07-02

1. For every real number $m$, the graph in the coordinate plane of the equation $y = mx$ is the line that has the slope $m$ and passes through the origin.

2. The $y$-intercept of a line on the coordinate plane is the point at which the line intersects the $y$ axis.

3. For all real numbers $m$ and $b$, the graph in the coordinate plane of the equation $y = mx + b$ is the line whose slope is $m$ and whose $y$-intercept is $b$.

4. (A) $2x - 2y = 1$, $-2y = -2x + 1$, $y = \frac{-2x}{-2} = \frac{1}{-2}$

   \[ y = x - \frac{1}{2} \]  
   Slope = 1,  \ y-intercept = -\frac{1}{2}

   (B) $y = -2x - 3$

   (C) $3x + y = 1$

   \[ y = -3x + 1 \]
1. For every real number $m$, the graph in the coordinate plane of the equation $y = mx$ is the line that has the slope $m$ and passes through the origin.

2. The $y$-intercept of a line on the coordinate plane is the point at which the line intersects the $y$ axis.

3. For all real numbers $m$ and $b$, the graph in the coordinate plane of the equation $y = mx + b$ is the line whose slope is $m$ and whose $y$-intercept is $b$.

4. (A) $y = 2x + 1$, \hspace{1cm} Slope = 2, \hspace{1cm} y$-intercept = 1

   (B) $y = 5x + 3$

   (C) \[
   \begin{align*}
   3x + 4y &= 0 \\
   4y &= -3x \\
   y &= -\frac{3}{4}x
   \end{align*}
   \]
I. U. #03-10-08

DETERMINING AN EQUATION OF A LINE
OBJECTIVES:

1. Given the information necessary to plot a line on a coordinate plane, write an equation of the line.

ACTIVITIES:

1. Study page 380 in SM and do the odd numbered part A exercises. (Objective 1)
2. Do a few of the part B exercises for reinforcement.
Criterion Test 03-10-08-01

1. (a) Find an equation of the line through point (2, 0) having the slope -3.

   (b) Find an equation of a line through the points (-1, 3) and (2, 3).

Criterion Test 03-10-08-02

1. (a) Find an equation of the line through point (3, -4) having the slope 2.

   (b) Find an equation of a line through the points (6, -5) and (-4, -2).

Criterion Test 03-10-08-03

1. (a) Find an equation of the line through point (3, 2) having the slope \( \frac{2}{3} \).

   (b) Find an equation of a line through the points (-3, 2) and (6, -5).
ANSWERS TO CRITERION TESTS

Test 03-10-08-01

1. (a) \( y = 3x + 6 \)
   (b) \( y = 3 \)

Test 03-10-08-02

1. (a) \( y = 2x - 10 \)
   (b) \( y = -\frac{3}{10}x - \frac{31}{5} \) or \( 10y = -3x -32 \)

Test 03-10-08-03

1. (a) \( y = \frac{2}{3}x \) or \( 3y = 2x \)
   (b) \( y = \frac{7}{9}x + 4\frac{1}{3} \) or \( 9y = 7x + 39 \)
DIRECT VARIATION AND PROPORTION
You will need to recall:

That a constant is a variable with just one value.

(Page 32, S + M)

OBJECTIVES:

1. When asked to describe a direct variation write, "$y = kx$ or $y = Kx$ where $K$ is a non zero constant."

2. When asked to define the "constant of proportionality" or "constant of variation" write, "In the direct variation $y = Kx$, $K$ is the constant of proportionality."

3. When asked to define a proportion write "An equality of ratios of the form $\frac{x_1}{y_1} = \frac{x_2}{y_2}$ is called a proportion."

4. When asked to define the means of a proportion write, "In the proportion $\frac{y_1}{x_1} = \frac{y_2}{x_2}$ $x_1$ and $y_2$ are the means."

5. When asked to define the extremes of a proportion write, "In the proportion $\frac{y_1}{x_1} = \frac{y_2}{x_2}$ $y_1$ and $x_2$ are the extremes."

6. When asked to state the mathematical relationship between the product of the means and the product of the extremes of a proportion write, "In any proportion, the product of the means equals the product of the extremes."

7. Given a problem requiring use of a direct variation or a proportion, solve it.

ACTIVITIES: 1. Study pages 382 - 384, S + M. (Objectives 1 - 7)

2. Do the oral exercises, page 384 and 385 in S + M. Have someone check you from a teacher's edition. (Objective 7)

3. Do the odd numbered part A written exercises on page 385. (Objective 7)
1. Describe a direct variation.

2. Define a "constant of proportionality".

3. Define a proportion.

4. Define the means of a proportion.

5. Define the extremes of a proportion.

6. State the mathematical relationship between the product of the means and the product of the extremes of a proportion.

7. Solve the following problems.

(a) Is xy = 5 a direct variation?

(b) What is the constant of proportionality in the direct variation m = \( \frac{1}{x} \)?

(c) Find the value of \( y \) in a direct variation if \( x_1 = 4, \ y_1 = 1, \ x_2 = -16, \ y_2 = ? \)

(d) Find all the values of the variable for which \( \frac{\frac{3w}{10w} + 2}{7} = \frac{2}{7} \) is true.

(e) Find the resistance of 500 feet of wire having a resistance of .0042 Ohms for each ten feet of wire if resistance varies directly with length.
Criterion Test 03-10-09-02

1. Describe a direct variation.

2. Define a "constant of proportionality".

3. Define a proportion.

4. Define the means of a proportion.

5. Define the extremes of a proportion.

6. State the mathematical relationship between the product of the means and the product of the extremes of a proportion.

7. Solve the following problems.
   (a) Is \( \frac{x}{y} = 5 \) a direct variation?
   (b) What is the constant of proportionality in the direct variation \(-7x = y\)?
   (c) Find the value for \( x_1 \) in a direct variation if
       \( y_1 = 3, \ x_2 = 16, \ y_2 = 12, \ x_1 = \_\_\_\_. \)
   (d) Find all the values of the variable for which
       \( \frac{7x - 2}{14x + 13} = \frac{6}{5} \) is true.
   (e) Twelve grams of calcium chloride can absorb 5 cubic centimeters of water. How much calcium chloride is needed to absorb 200 cubic centimeters of water?
       (The volume of water varies directly as the weight of the calcium chloride)
1. Describe a direct variation.
2. Define a "constant of proportionality".
3. Define a proportion.
4. Define the means of a proportion.
5. Define the extremes of a proportion.
6. State the mathematical relationship between the product of the means and the product of the extremes of a proportion.

7. Solve the following problems:
   (a) Is $4x = y$ a direct variation?
   (b) What is the constant of proportionality in the direct variation $-7x = y$?
   (c) Find the value of $x_2$ in a direct variation if $x_1 = 1$, $y_1 = 4$, and $y_2 = -16$, $x_2 = \underline{?}$
   (d) Find all the values of the variable for which $\frac{6x}{x + 7} = \frac{9}{5}$ is true.
   (e) The ratio of the weight of an object on Earth to its weight on Mars is 5:2. How much would a man who weighs 175 pounds on Earth weigh on Mars?
Answers to Criterion Tests

Test 03-10-09-01

1. \( \frac{y}{x} = K \) or \( y = Kx \) where \( K \) is a non-zero constant.

2. In the direct variation \( y = Kx \), \( K \) is the constant of proportionality.

3. An equality of ratios is called a proportion.

4. In the proportion \( \frac{y_1}{x_1} = \frac{y_2}{x_2} \), \( x_1 \) and \( y_2 \) are the means.

5. In the proportion \( \frac{y_1}{x_1} = \frac{y_2}{x_2} \), \( y_1 \) and \( x_2 \) are the extremes.

6. In a proportion, the product of the means equals the product of the extremes.

7. (a) No

   (b) \( \frac{1}{2} \)

   (c) \(-4\)

   (d) 4

   (e) .21 ohms
Answers to Criterion Tests  (Cont.)

Test 03-10-09-02

1. \( \frac{y}{x} = k \) or \( y = kx \) where \( k \) is a non-zero constant.

2. In the direct variation \( y = kx \), \( k \) is the constant of proportionality.

3. An equality of ratios is called a proportion.

4. In the proportion \( \frac{y_1}{x_1} = \frac{y_2}{x_2} \), \( x_1 \) and \( y_2 \) are the means.

5. In the proportion \( \frac{y_1}{x_1} = \frac{y_2}{x_2} \), \( y \) and \( x \) are the extremes.

6. In a proportion, the product of the means equals the product of the extremes.

7. (a) Yes
   (b) -7
   (c) 4
   (d) \( \frac{88}{49} \)
   (e) 480 grams
**Answers To Criterion Tests  (cont.)**

**Test 03-10-09-03**

1. \( \frac{y}{x} = K \) or \( y = Kx \) where \( K \) is a non-zero constant.

2. In the direct variation \( y = Kx \), \( K \) is the constant of proportionality.

3. An equality of ratios is called a proportion.

4. In the proportion \( \frac{y_1}{x_1} = \frac{y_2}{x_2} \), \( x_1 \) and \( y_2 \) are the means.

5. In the proportion \( \frac{y_1}{x_1} = \frac{y_2}{x_2} \), \( y_1 \) and \( x_2 \) are the extremes.

6. In a proportion, the product of the means equals the product of the extremes.

7. (a) Yes

(b) -7

(c) -4

(d) 3

(e) 70 pounds
I. U. #03-10-10

QUADRATIC FUNCTIONS
OBJECTIVES:

After completion of this instructional unit you will be able to do the following:

1. When asked to define **quadratic direct variation** write, **A quadratic direct variation is a function in the form** \( y = kx^2 \) **where** \( k \) **is a non zero constant.**

2. When asked to define a quadratic function write, **A quadratic function is a function in the form** \( y = ax^2 + bx + c \) **where** \( a \neq 0 \).

3. When asked to define a parabola write, **A parabola is the graph of an equation of the form** \( y = ax^2 + bx + c \), \( a \neq 0 \).

4. Given a function **classify** it as a quadratic function or not a quadratic function.

5. Given a quadratic function, state whether its graph opens upward or downward.

6. Given a problem involving a quadratic direct variation or a quadratic function, solve it.

ACTIVITIES:

1. Study pages 388 and 390 in S + M. (Objectives 1, 2, 3)

2. Do the oral exercises on page 391. (Objectives 4, 5)

3. Do the odd numbered Part A written exercises on page 391 and the odd numbered Part A problems on page 391 and 392. (Objectives 6, 7)

4. You will probably want to try some of the part B problems and exercises also.
1. Define a quadratic direct variation.

2. Define a quadratic function.

3. Define a parabola.

4. Which of the following functions is a quadratic function. (Write yes if it is, write no if it isn't.)
   
   (A) \( f:x \rightarrow y = 5x^2 \)
   
   (B) \( f:x \rightarrow y = 2x - 4 \)
   
   (C) \( f:x \rightarrow y = x^2 - 2x - 4 \)

5. State whether the graph of the following quadratic functions opens upward or downward.
   
   (A) \( f:x \rightarrow y = x^2 + 8 \)
   
   (B) \( f:x \rightarrow y = 8 - x^2 \)

6. Graph each equation. (Give the coordinates of each point used)
   
   (A) \( y = 2x^2 \)
   
   (B) \( y = 1 - 2x^2 \)

7. Solve the following problems.
   
   (A) The lift on an airplane wing is directly proportional to the square of the speed of the air moving over it. If the lift on the wing of a plane is 732 pounds per square foot when the plane is flying 320 miles per hour in still air, find the lift when the speed is 400 miles per hour.

   (B) A diamonds price varies as the square of its weight. If a diamond sells for $360.00 in Omaha, Nebraska and weights \( \frac{3}{8} \) carat before polishing, how much will a similar stone cost if its weight is \( \frac{1}{8} \) carat?

   (C) The distance which a freely falling body falls (neglecting air resistance) varies directly as the square of the time if falls. If an object falls 144 feet in three seconds, how far will it fall in six seconds?
1. Define a quadratic direct variation.

2. Define a quadratic function.

3. Define a parabola.

4. Which of the following functions is a quadratic function? (Write yes if it is, write no if it isn't.)
   
   (A) \( f: x \longrightarrow y = \frac{1}{3}x^2 \)
   
   (B) \( f: x \longrightarrow y = 4x + 2 \)
   
   (C) \( f: x \longrightarrow y = 3x^2 - 9 \)

5. State whether the graph of the following quadratic functions opens upward or downward.
   
   (A) \( f: x \longrightarrow y = x^2 - 8 \)
   
   (B) \( f: x \longrightarrow y = -8 - x^2 \)

6. Graph each function. (Give the coordinates of each point used.)
   
   (A) \( y = 2x^2 + 1 \)
   
   (B) \( y = -2x^2 - 1 \)

Continued on next page.
7. Solve the following problems.

(A) The lift on an airplane wing is directly proportional to the square of the speed on the air moving over it.

If the lift is 732 pounds per square foot when the air is moving over the wing at 320 M.P.H., what will be the lift when the air speed over the wing is reduced by 80 M.P.H.?

(B) A diamond's price varies as the square of its weight.

Jack and his fiancee are pricing diamonds.

If Jack finds a \( \frac{3}{8} \) carat diamond which sells for $400.00, how much should a similar diamond cost if his fiancee says that it is a one carat rock?

(C) The distance which a freely falling body falls (neglecting air resistance) varies directly as the square of the time it falls.

If an object falls 144 feet in 3 seconds how far will it fall in five seconds?
1. Define a quadratic direct variation.

2. Define a quadratic function.

3. Define a parabola.

4. Which of the following functions is a quadratic function. (Write yes if it is, write no if it isn’t.)
   
   (A) \( f:x \rightarrow y = -4x^2 \)
   
   (B) \( f:x \rightarrow y = \frac{1}{3}x - \frac{2}{3} \)
   
   (C) \( f:x \rightarrow y = 2x^2 - 2 \)

5. State whether the graph of the following quadratic functions opens upward or downward.

   (A) \( f:x \rightarrow y = 3x^2 \)
   
   (B) \( f:x \rightarrow y = -3x^2 \)

6. Graph each function. (Give the coordinates of each point used.)

   (A) \( y = x^2 \)
   
   (B) \( y = x^2 + 2x + 1 \)

Continued on next page.
7. Solve the following problems.

(A) The lift on an airplane wing is directly proportional to the square of the speed of the air moving over it.

If the lift on a certain wing is 144 pounds per square inch when the air moves over the wing at 150 M.P.H., what will be the lift if the speed is increased by 50 M.P.H.?

(B) A diamond's price varies as the square of its weight.

If today's market on first class gem quality white diamonds is $5000.00 per carat, how much is the loss if Mrs. Stu Pid loses a 1/4 carat first class gem quality stone?

(C) The distance which a freely falling body falls (Neglecting air resistance) varies directly as the square of the time it falls.

If a body falls 144 feet in 3 seconds how far will it fall in four seconds?
Answers to Criterion Tests

Test 03-09-01-01

1. A quadratic direct variation is a function in the form $y = kx^2$, where $k$ is a non-zero constant.

2. A quadratic function is a function in the form $y = ax^2 + bx + c$, where $a \neq 0$.

3. A parabola is the graph of an equation of the form $y = ax^2 + bx + c$, $a \neq 0$.

4. (a) yes
   (b) no
   (c) yes

5. (a) upward
   (b) downward

6. 

   (a) 
   (b) 

7. (a) $1,143 \frac{3}{4}$ pounds per square foot.
   (b) $40$
   (c) $576$ ft.
Answers to Criterion Tests  (Cont.)

Test 03-10-10-02

1. A quadratic direct variation is a function in the form $y = Kx^2$, where $K$ is a non zero constant.

2. A quadratic function is a function in the form $y = ax^2 + bx + c$, where $a \neq 0$.

3. A parabola is the graph of an equation of the form $y = ax^2 + bx + c$, $a \neq 0$.

4. (a) Yes  (b) No  (c) Yes

5. (a) upward  (b) downward

6. 

7. (a) $411 \frac{3}{4}$ pounds per square foot

   (b) $2,844.44$

   (c) 400 feet
Answers to Criterion Tests  (Cont.)

Test 03-10-10-03

1. A quadratic direct variation is a function in the form
\[ y = Kx^2 \], where \( K \) is a non zero constant.

2. A quadratic function is a function in the form
\[ y = ax^2 + bx + c \], where \( a \neq 0 \).

3. A parabola is the graph of an equation of the form
\[ y = ax^2 + bx + c, \ a \neq 0. \]

4. (a) Yes  (b) No  (c) Yes

5. (a) upward  (b) downward

6. 

(a)

(b)

7. (a) 256 pounds per square inch
(b) $312.50
(c) 256 feet
I. U. #03-10-11

INVERSE VARIATION
I. U. #03-10-11

INVERSE VARIATION
OBJECTIVES:

After completing this instructional unit you will be able to do the following:

1. When asked to define an inverse variation, write an inverse variation is a function in which the product of the coordinates of its ordered pairs is a non zero constant.

2. When asked to define a hyperbola, write a hyperbola is the graph of a function of the type $xy = K$ where $K$ is a non zero constant.

3. Given a hyperbola, tell whether the value of $K$ in the function it represents is negative or positive.

4. Given a formula, determine if it represents an inverse variation.

5. Given a verbal statement of an inverse variation, translate it into a formula.

6. Given an inverse variation, draw its graph.

7. Given one ordered pair of a function and one component of a second ordered pair, write the remaining component.

8. Given a problem involving inverse variation, solve it.

ACTIVITIES:

Study pages 392 - 395 in S & M.

Do oral exercises, pages 395, 396. (Answers in teacher's edition) Do some of the part A exercises page 396, and work some problems from page 397.
1. Define an inverse variation.

2. Define a hyperbola.

3. Is the value of K in the formula for the following graphs positive or negative?

4. Are the following formulae expressions of an inverse variation? (Write yes or no for your answer)
   
   (A) \( \frac{a}{b} = 10 \)  
   (B) \( ab = 10 \)

5. Translate the following statement into a mathematical formula.

   The force of attraction between two bodies is inversely proportional to the square of the distance between them.

6. Graph the following: \( xy = 6 \)

7. In these inverse variations, find the value of the indicated variable.

   \[
   \begin{align*}
   (A) & \quad x_1 = 5 \quad y_1 = 5 \quad x_2 = 10 \quad y_2 = ? \\
   (B) & \quad x_1 = ? \quad y_1 = 9 \quad x_2 = 3 \quad y_2 = 5
   \end{align*}
   \]

8. Solve: If a man weighs 200 pounds when he is 2000 miles from the center of the earth how much will he weigh when he is 4000 miles from the center of the earth. (The attraction between two bodies is inversely proportional to the square of the distance between them.)
1. Define an inverse variation.

2. Define a hyperbola.

3. Is the value of K in the formula for the following graphs positive or negative?

\[ a_1 b_1 = a_2 b_2 \]

4. Are the following formulae expressions of an inverse variation? (Write yes or no for your answer)

(A) \( \frac{a_1}{b_1} = \frac{a_2}{b_2} \)

(B) \( \frac{a_1}{b_1} = \frac{a_2}{b_2} \)

5. Translate the following statement into a mathematical formula.

The illumination on a surface is inversely proportional to the square of the distance from the light source.

6. Graph the following: \( 2xy = 8 \)

7. In these inverse variations, find the value of the indicated variable.

(A) \( a_1 = 8 \quad a_2 = 4 \quad b_1 = ? \quad b_2 = 1 \)

(B) \( a_1 = 3 \quad a_2 = 5 \quad b_1 = ? \quad b_2 = 9 \)

8. Solve: The illumination on a table which is six feet from a light is forty lumens. How much is the illumination on a surface 12 feet from the same light? (Illumination on a surface is inversely proportional to the square of the distance to the source.)
1. Define an inverse variation.

2. Define a hyperbola.

3. Is the value of \( K \) in the formula for the following graphs positive or negative?

4. Are the following formulae expressions of an inverse variation? (Write yes or no for your answer)
   
   (A) \( \frac{a_1}{a_2} = \frac{b_2}{b_1} \)
   
   (B) \( ab = 10 \)

5. Translate the following statement into a mathematical formula.
   
   The intensity of a sound is inversely proportional to the square of the distance to the source.

6. Graph the following:
   
   \( \frac{x}{4} = \frac{3}{y} \)

7. In these inverse variations, find the value of the indicated variable.
   
   (A) \( x_1 = 1 \quad x_2 = 3 \quad y_1 = 2 \quad y_2 = ? \)
   
   (B) \( x_1 = 3 \quad y_1 = 5 \quad x_2 = 4 \quad y_2 = ? \)

8. Solve: The intensity of a combo is 20 decibels at a distance of 50 feet from the stage. What is the intensity at a distance of ten feet from the stage? (The intensity of a sound is inversely proportional to the square of the distance from the source.)
Answers to Criterion Tests

Criterion Test 03-10-11-01

1. An inverse variation is a function in which the product of the coordinates of its ordered pairs is a non zero constant.

2. A hyperbola is the graph of a function of the type \( xy = k \) where \( k \) is a non zero constant.

3. (a) Positive (b) Negative

4. (a) No (b) Yes

5. \( Fd^2 = K \)

6

7. (a) \( 2\sqrt{2} \) (b) 5

8. 50 pounds
Answers to Criterion Tests  (Cont.)

Test 03-10-11-02

1. An inverse variation is a function in which the product of the coordinates of its ordered pairs is a non zero constant.

2. A hyperbola is the graph of a function of the type $xy = K$ where $K$ is a non zero constant.

3. (a) Negative  (b) Positive

4. (a) Yes  (b) No

5. $Id^2 = K$

6.

7. (a) $\frac{1}{2}$  (b) 15

8. 10 lumens
Answers to Criterion Tests  (Cont.)

Test 03-10-11-03

1. An inverse variation is a function in which the product of the coordinates of its ordered pairs is a non-zero constant.

2. A hyperbola is the graph of a function of the type \( xy = K \) where \( K \) is a non-zero constant.

3. (a) Positive  (b) Negative

4. (a) Yes  (b) Yes

5. \( Id^2 = K \)

6. [Graph of a hyperbola]

7. (a) \( \frac{2}{3} \)  (b) \( \frac{15}{4} \)

8. 5 decibels

The End
Package 03-10