This teacher guide is part of the materials prepared for an individualized program for ninth-grade algebra and basic mathematics students. Materials written for the program are to be used with audiovisual lessons recorded on tape cassettes. For an evaluation of the program, see ED 086 545. In this guide, the teacher is provided with objectives for each topic area and guided to materials written for a given topic. Three short criterion tests are included for each topic covered. Techniques for solving algebraic equations are presented in this package. This work prepared under an ESEA Title III contract. (JP)
ALGEBRA I

Package # 03-04

SOLVING EQUATIONS AND PROBLEMS

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SOLVING EQUATIONS AND PROBLEMS

You have had some experience in solving simple open sentences. However, the methods you will learn in this package are more sophisticated than the methods you have been using and are based upon some of the properties of real numbers. It is very important that you understand and use the methods presented in this package. Do not make the mistake of ignoring the new methods and continuing to use the old. Eventually, the problems you are asked to solve will become so difficult that the old methods will not suffice. If you have not learned the new methods, you will not be able to solve the problem.

PACKAGE GOAL: to understand and be able to use the new methods of solving equations and problems.
PACKAGE OBJECTIVES:

1. Given an equation such as \( x + 31 = 8 \) or \( x - 5 = 13 \), show how to solve by using transformation by addition.

2. Given an equation such as \( x + 31 = 8 \), show how to solve by using transformation by subtraction.

3. Given an equation such as \( \frac{1}{3} x = 132 \) or \( 3x = 96 \), show how to solve by using transformation by multiplication.

4. Given an equation such as \( 3x = 96 \), show how to solve by using transformation by division.

5. Given an equation, show how to solve by using as many transformations as necessary.

6. Given an equation having the variable in both members, show how to solve it.

7. Given a word problem, write an equation which is the translation of the word problem, then write the solution.

8. Given a function, write the value of the function for any given member of the domain.
I. U. # 03-04-01

Transforming Equations by Addition
OBJECTIVES:

1. When asked to state the addition property of equality, you will write, "If a, b, and c are any real numbers such that a = b, then a + c = b + c and c + a = c + b".

2. Given an equation such as x + 31 = 8 or x - 5 = 13, show how to solve by using transformation by addition.

ACTIVITIES:

Study:
   S & M: pages 111 - 114 (objectives 1 and 2).

Suggested exercises:
   S & M: pages 115 - 116; ex. 1 - 25, odd (objective 2).
Criterion Test 03-04-01-01

1. State the additive property of equality.

2. Solve by using transformation by addition.  
   (Show all steps.)

   (a) \( x + 19 = 25 \)
   (b) \(-7 + x = 13\)
   (c) \(-x + 15 = 12\)

Criterion Test 03-04-01-02

1. State the additive property of equality.

2. Solve by using transformation by addition.  
   (Show all steps.)

   (a) \( x + 8 = -13 \)
   (b) \(-7 + x = -3\)
   (c) \(-x + 5 = 17\)

Criterion Test 03-04-01-03

1. State the additive property of equality.

2. Solve by using transformation by addition.  
   (Show all steps.)

   (a) \( x + 14 = 3 \)
   (b) \(-11 + x = 14\)
   (c) \(-x + 3 = 11\)
Answers to Criterion Tests

Criterion Test 03-04-01-01

1. If $a$, $b$, and $c$ are any real numbers such that $a = b$, then $a + c = b + c$ and $c + a = c + b$.

2. (a) $x + 19 = 25$
   \[ x + 19 + (-19) = 25 + (-19) \]
   \[ x = 6 \]

   (b) $-7 + x = 13$
   \[ 7 + (-7) + x = 7 + 13 \]
   \[ x = 20 \]

   (c) $-x + 15 = 12$
   \[ -x + 15 + (-15) = 12 + (-15) \]
   \[ -x = -3 \]
   \[ x = 3 \]

Criterion Test 03-04-01-02

1. If $a$, $b$, and $c$ are any real numbers such that $a = b$, then $a + c = b + c$ and $c + a = c + b$.

2. (a) $x + 8 = -13$
   \[ x + 8 + (-8) = 13 + (-8) \]
   \[ x = -21 \]

   (b) $-7 + x = -3$
   \[ 7 + (-7) + x = 7 + (-3) \]
   \[ x = 4 \]

   (c) $-x + 5 = 17$
   \[ -x + 5 + (-5) = 17 + (-5) \]
   \[ -x = 12 \]
   \[ x = -12 \]

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ANSWERS TO CRITERION TESTS:

Criterion Test 03-04-01-03

1. If a, b, and c are any real numbers such that a = b, then a + c = b + c and c + a = c + b.

2. (a) $x + 14 = 3$
   
   $x + 14 + (-14) = 3 + (-14)$
   
   $x = -11$

(b) $-11 + x = 14$
   
   $11 + (-11) + x = 11 + 14$
   
   $x = 25$

(c) $-x + 3 = 11$
   
   $-x + 3 + (-3) = 11 + (-3)$
   
   $-x = 8$
   
   $x = -8$
I. U. # 03-04-02

Transforming Equations By Subtraction
OBJECTIVES:

1. When asked to state the rule for subtraction, you will write, "For all real numbers a and b, 
   \[ a - b = a + (-b). \]

2. Given two real numbers, find their difference.

3. Given an equation such as \( x + 31 = 8 \), show how to solve by using transformation by subtraction.

ACTIVITIES:

Study:

S & M: pages 116 - 118 (pay special attention to example 4, part b, on page 118. (objectives 1, 2, and 3).

Suggested exercises:

S & M: page 121; ex. 1 - 17, odd (objective 2).
page 121; ex. 19 - 26, (objective 3).
Criterion Test 03-04-02-01

1. State the rule for subtraction.

2. Rewrite each of the following differences as a sum and simplify.
   (a) \(-32\)  (b) \(48\)  (c) \(-10\)  (d) \((x + 7) - (x - 3)\)
   \(-17\)  \(-13\)  \(5\)

3. Solve by using transformation by subtraction. (Show all steps)
   (a) \(x + 38 = 14\)
   (b) \(-17 = y + 6\)

Criterion Test 03-04-02-02

1. State the rule for subtraction.

2. Rewrite each of the following differences as a sum and simplify.
   (a) \(-17\)  (b) \(-4\)  (c) \(14\)  (d) \((x + 1) - (-2 + x)\)
   \(-11\)  \(-3\)  \(-17\)

3. Solve by using transformation by subtraction. (Show all steps)
   (a) \(x + 17 = 31\)
   (b) \(-5 = y + 9\)

Criterion Test 03-04-02-03

1. State the rule for subtraction.

2. Rewrite each of the following differences as a sum and simplify.
   (a) \(-14\)  (b) \(-8\)  (c) \(32\)  (d) \((x + 5) - (-4 + x)\)
   \(8\)  \(-11\)  \(-19\)

3. Solve by using transformation by subtraction. (Show all steps)
   (a) \(x + 10 = 20\)
   (b) \(-15 = y + 15\)
Answers:

Criterion Test 03-04-02-01

1. For all real numbers \(a\) and \(b\), \(a - b = a + (-b)\)

2. (a) -32  (b) 48  (c) -10
   \[\frac{17}{15} \quad \frac{13}{6} \quad \frac{-5}{15}\]
   (d) \((x + 7) + (-x + 3) = 10\)

3. (a) \(x + 38 = 14\)
   \(x + 38 - 38 = 14 - 38\)
   \(x = -24\)
   (b) \(-17 = y + 6\)
   \(-17 - 6 = y + 6 - 6\)
   \(-23 = y\)

Criterion Test 03-04-02-02

1. For all real numbers \(a\) and \(b\), \(a - b = a + (-b)\).

2. (a) -17  (b) -4  (c) 14
   \[\frac{11}{6} \quad \frac{-3}{7} \quad \frac{17}{31}\]
   (d) \((x + 1) + (2 - x) = 3\)

3. (a) \(x + 17 = 31\)
   \(x + 17 - 17 = 31 - 17\)
   \(x = 14\)
   (b) \(-5 = y + 9\)
   \(-5 - 9 = y + 9 - 9\)
   \(-14 = y\)
Answers (continued)

Criterion Test 03-04-02-03

1. For all real numbers a and b, \( a - b = a + (-b) \).

2. (a) \(-14\)  \hspace{1cm} (b) \(-8\)  \hspace{1cm} (c) \(32\)
     \[\frac{-8}{-22} \hspace{1cm} \frac{11}{3} \hspace{1cm} \frac{19}{51}\]
     (d) \((x + 5) + (4 - x) = 9\)

3. (a) \(x + 10 = 20\)
     \[x + 10 - 10 = 20 - 10\]
     \[x = 10\]
     (b) \(-15 = y + 15\)
     \[-15 - 15 = y + 15 - 15\]
     \[-30 = y\]
I. U. # 03-04-03

Transforming Equations By Multiplication
OBJECTIVES:

1. When asked to state the multiplication property of equality, you will write, "If a, b, and c are any real numbers such that a = b, then ac = bc and ca = cb."

2. Given an equation such as \( \frac{a}{b} = \frac{c}{d} \) or \( 3x = 96 \), show how to solve by using transformation by multiplication.

ACTIVITIES:

Study:

\[ \text{S & M: pages 123 - 123 (objectives 1 and 2).} \]

Suggested exercises:

\[ \text{S & M: page 125; ex. 1 - 17, odd (objective 2).} \]
CRITERION TESTS

Criterion Test 03-04-03-01

1. State the multiplication property of equality.

2. Solve each equation by using transformation by multiplication. (Show all work.)

   (a) \( \frac{1}{9}y = 15 \)
   (b) \( 5x = 125 \)
   (c) \( -\frac{1}{7}x = -49 \)

Criterion Test 03-04-03-02

1. State the multiplication property of equality.

2. Solve each equation by using transformation by multiplication. (Show all work)

   (a) \( \frac{1}{8}y = 25 \)
   (b) \( 4x = 24 \)
   (c) \( -\frac{1}{3}x = 39 \)

Criterion Test 03-04-03-03

1. State the multiplication property of equality.

2. Solve each equation by using transformation by multiplication.

   (a) \( \frac{1}{7}x = 32 \)
   (b) \( 6x = 54 \)
   (c) \( -\frac{1}{8}x = ? \)
ANSWERS TO CRITERION TESTS

Criterion Test 03-04-03-01

1. If $a$, $b$, and $c$ are any real numbers such that $a = b$, then $ac = bc$ and $ca = cb$.

2. (a) $\frac{1}{9} \cdot y = 15$
   
   \[
   \frac{1}{9} \cdot \frac{1}{5} \cdot 125 = \frac{25}{9} \cdot 125 = \frac{3125}{9}
   \]
   
   \[
   y = 135
   \]

Criterion Test 03-04-03-02

1. If $a$, $b$, and $c$ are any real numbers such that $a = b$, then $ac = bc$ and $ca = cb$.

2. (a) $\frac{1}{8} \cdot y = 25$
   
   \[
   \frac{1}{8} \cdot \frac{1}{4} \cdot 24 = \frac{3}{8} \cdot 24 = \frac{72}{8}
   \]
   
   \[
   y = 200
   \]

Criterion Test 03-04-03-03

1. If $a$, $b$, and $c$ are any real numbers such that $a = b$, then $ac = bc$ and $ca = cb$.

2. (a) $\frac{1}{7} \cdot x = 32$
   
   \[
   \frac{1}{7} \cdot \frac{1}{6} \cdot 54 = \frac{9}{7} \cdot 54 = \frac{486}{7}
   \]
   
   \[
   x = 224
   \]

   (c) $-\frac{1}{8} \cdot x = 72$
   
   \[
   -\frac{1}{8} \cdot \frac{1}{117} = -\frac{9}{72}
   \]
   
   \[
   x = -576
   \]
I. U. # 03-04-04

Transforming Equations By Division
OBJECTIVES:

1. When asked to state the rule for division, you will write, "For all real numbers a and all non-zero numbers b, \( a + \frac{b}{a} = a \cdot \frac{1}{b} \)."

2. Given two real numbers, find their quotient.

3. Given an equation such as \( 3x = 96 \), show how to solve by using transformation by division.

ACTIVITIES:

Study:
- S & M: pages 125 - 127 (pay special attention to part a of the example on page 127) (objectives 1, 2, and 3).
- HBW: pages 82 - 83 (objective 2).

Suggested exercises:
- S & M: page 129; ex. \( \frac{7}{17} \), odd (objective 2).
- page 129; ex. 19 - 25, odd (objective 3).
- HBW: page 83; ex. 1 - 12 (objective 2).
Criterion Test 03-04-04-01

1. State the rule for division.

2. State the value of each quotient.

   (a) $39 \div (-3)$
   (b) $0/6$
   (c) $6/-2$
   (d) $\frac{-8}{-1/2}$

3. Solve by using transformation by division. (Show all steps.)

   (a) $33x = 231$
   (b) $-7x = 35$

Criterion Test 03-04-04-02

1. State the rule for division.

2. State the value of each quotient.

   (a) $72 \div (-8)$
   (b) $8/-4$
   (c) $0/7$
   (d) $\frac{-6}{-1/3}$

3. Solve by using transformation by division. (Show all steps.)

   (a) $15x = 225$
   (b) $-5x = 35$
1. State the rule for division.

2. State the value of each quotient.
   (a) $45 \div (-9)$
   (b) $10/-2$
   (c) $0/-5$
   (d) $-14/-7$

3. Solve by using transformation by division. (Show all steps.)
   (a) $12x = 144$
   (b) $-3x = -54$
Answers:

Criterion Test 03-04-04-01

1. For all real numbers a and all non zero numbers b, $a \cdot b = a \cdot 1/b$.

2. (a) -13  (b) 0  (c) -3  (d) 16

3. (a) $33x = 231$
   $$\frac{33x}{33} = \frac{231}{33}$$
   $$x = 7$$

   (b) $-7x = 35$
   $$\frac{-7x}{-7} = \frac{35}{-7}$$
   $$x = -5$$

Criterion Test 03-04-04-02

1. For all real numbers a and all non zero real numbers b, $a + b = a + 1/b$.

2. (a) -9  (b) -2  (c) 0  (d) 18

3. (a) $15x = 225$
   $$\frac{15x}{15} = \frac{225}{15}$$
   $$x = 15$$

   (b) $-5x = 35$
   $$\frac{-5x}{-5} = \frac{35}{-5}$$
   $$x = -7$$

Criterion Test 03-04-04-03

1. For all real numbers a and all non zero numbers b, $a \cdot b = a \cdot 1/b$.

2. (a) -5  (b) -5  (c) 0  (d) 2

3. (a) $12x = 144$
   $$\frac{12x}{12} = \frac{144}{12}$$
   $$x = 12$$

   (b) $-3x = -54$
   $$\frac{-3x}{-3} = \frac{-54}{-3}$$
   $$x = 18$$
Using Several Transformations

To Solve Equations
You will need to recall:

An expression like $3x + 5x$ can be simplified by using the distributive property: $3x + 5x = (3 + 5)x = 8x$. Also, $\frac{x}{5} = \frac{1}{5} \cdot x$.

OBJECTIVE:

1. Given an equation, show how to solve it by using as many transformations as necessary.

ACTIVITIES:

Study:

S & M: pages 130 - 131 (objective 1).

Suggested exercises:

S & M: page 132; ex. 1 - 29, odd (objective 1).
1. Solve: (Show all steps)

   (a) \( 4x + 5 = 17 \)
   (b) \( \frac{x}{5} - 7 = -13 \)
   (c) \( 5x + 3x - 4x = -16 \)
   (d) \( -7 + 4x - x + 12 = -4 \)

1. Solve: (Show all steps)

   (a) \( 3x - 5 = 16 \)
   (b) \( \frac{x}{2} + 8 = -12 \)
   (c) \( 5x + 14x - 9x = 100 \)
   (d) \( 4x - 2 + 8 - 2x = -14 \)

1. Solve: (Show all steps)

   (a) \( 8x + 5 = -11 \)
   (b) \( -6 + \frac{x}{3} = 12 \)
   (c) \( 11x - 7x + 2x = 48 \)
   (d) \( 12 - 3x + 8x - 12 = -30 \)
Answers:

Criterion Test 03-04-05-01

1. (a)  $4x + 5 = 17$
   $4x + 5 - 5 = 17 - 5$
   $4x = 12$
   $4x/4 = 12/4$
   $x = 3$

(b)  $x/5 - 7 = -13$
   $x/5 - 7 + 7 = -13 + 7$
   $x/5 = -6$
   $5 \cdot x/5 = 5 \cdot (-6)$
   $x = -30$

(c)  $5x + 3x - 4x = -16$
   $4x = -16$
   $4x/4 = -16/4$
   $x = -4$

(d)  $-7 + 4x - x + 12 = -4$
   $5 + 3x = -4$
   $-5 \div 5 + 3x = -5 + (-4)$
   $3x = -9$
   $3x/3 = -9/3$
   $x = -3$

Criterion Test 03-04-05-02

1. (a)  $3x - 5 = 16$
   $3x - 5 + 5 = 16 + 5$
   $3x = 21$
   $3x/3 = 21/3$
   $x = 7$

(b)  $x/2 + 8 = -12$
   $x/2 + 8 - 8 = -12 - 8$
   $x/2 = -20$
   $2 \cdot x/2 = 2 \cdot (-20)$
   $x = -40$

(c)  $5x + 14x - 9x = 100$
   $10x = 100$
   $10x/10 = 100/10$
   $x = 10$

(d)  $4x - 2 + 8 - 2x = -14$
   $2x + 6 = -14$
   $2x + 6 - 6 = -14 - 6$
   $2x = -20$
   $2x/2 = -20/2$
   $x = -10$
Answers (continued)

Criterion Test 03-04-05-03

1. (a) \[ 8x + 5 = -11 \]
   \[ 8x + 5 - 5 = -11 - 5 \]
   \[ 8x = -16 \]
   \[ 8x/8 = -16/8 \]
   \[ x = -2 \]

   (b) \[ -6 + x/3 = 12 \]
   \[ 6 - 6 + x/3 = 12 + 6 \]
   \[ x/3 = 18 \]
   \[ 3 \cdot x/3 = 3 \cdot 18 \]
   \[ x = 54 \]

   (c) \[ 11x - 7x + 2x = 48 \]
   \[ 6x = 48 \]
   \[ 6x/6 = 48/6 \]
   \[ x = 8 \]

   (d) \[ 12 - 3x + 8x - 12 = -30 \]
   \[ 5x = -30 \]
   \[ 5x/5 = -30/5 \]
   \[ x = -6 \]
Solving Equations With the Variable in Both Members
OBJECTIVE:

1. Given an equation having the variable in both members, show how to solve it.

ACTIVITIES:

Study:
S & M: pages 138 - 141 (objective 1).

Suggested exercises:
S & M: pages 141 - 142; ex. 1 - 35, odd (red answer book shows some of the steps) (objective 1).
Criterion Test 03-04-06-01

1. Solve each equation for x. (Show steps)

(a) \(2x - 3 + 5x = x + 15\)
(b) \(11x = 8x + 42\)
(c) \(2x - 42 = -x\)
(d) \(12w - 8t = 2x\)

Criterion Test 03-04-06-02

1. Solve each equation for x. (Show steps)

(a) \(-2x + 7 = 5x - 21\)
(b) \(9x = 84 - 5x\)
(c) \(4x + 6 = 2x\)
(d) \(24a - 3x = 5x\)

Criterion Test 03-04-06-03

1. Solve each equation for x. (Show steps)

(a) \(-14 + x = 4x + 16\)
(b) \(9x - 70 = 2x\)
(c) \(54 - 2x = 7x\)
(d) \(5x - 2y = 1\)
Answers:

Criterion Test 03-04-06-01

1. (a) \[2x - 3 + 5x = x + 15\]
   \[7x - 3 = x + 15\]
   \[7x - x = 15 + 3\]
   \[6x = 18\]
   \[6x/6 = 18/6\]
   \[x = 3\]

   (b) \[11x = 8x + 42\]
   \[11x - 8x = 42\]
   \[3x = 42\]
   \[3x/3 = 42/3\]
   \[x = 14\]

   (c) \[2x - 42 = -x\]
   \[2x + x = 42\]
   \[3x = 42\]
   \[\frac{3x}{3} = \frac{42}{3}\]
   \[3x = 42\]
   \[x = 14\]

   (d) \[12w - 8t = 2x\]
   \[\frac{12w - 8t}{2} = \frac{2x}{2}\]
   \[6w - 4t = x\]

Criterion Test 03-04-06-02

1. (a) \[-2x + 7 = 5x - 21\]
   \[-2x - 5x = -21 - 7\]
   \[-7x = -28\]
   \[\frac{-7x}{-7} = \frac{-28}{-7}\]
   \[x = 4\]

   (b) \[9x = 84 - 5x\]
   \[9x + 5x = 84\]
   \[14x = 84\]
   \[\frac{14x}{14} = \frac{84}{14}\]
   \[x = 6\]

   (c) \[4x + 6 = 2x\]
   \[4x - 2x = 6\]
   \[2x = 6\]
   \[\frac{2x}{2} = \frac{6}{2}\]
   \[x = 3\]

   (d) \[24a - 3x = \frac{5x}{3}\]
   \[24a - 5x + 3x\]
   \[24a = 8x\]
   \[\frac{24a}{8} = \frac{8x}{8}\]
   \[3a = x\]

Criterion Test 03-04-06-03

1. (a) \[-14 + x = 4x + 16\]
   \[x - 4x = 16 + 14\]
   \[-3x = 30\]
   \[\frac{-3x}{-3} = \frac{30}{-3}\]
   \[x = -10\]

   (b) \[9x - 70 = 2x\]
   \[9x - 2x = 70\]
   \[7x = 70\]
   \[\frac{7x}{7} = \frac{70}{7}\]
   \[x = 10\]

   (c) \[54 - 2x = 7x\]
   \[-2x - 7x = -54\]
   \[-9x = -54\]
   \[\frac{-9x}{-9} = \frac{-54}{-9}\]
   \[x = 6\]

   (d) \[5x - 2y = 1\]
   \[5x = 2y + 1\]
   \[\frac{5x}{5} = \frac{2y + 1}{5}\]
   \[x = \frac{2y + 1}{5}\]
I. U. 0 03-06-07

Using Equations to Solve Problems
OBJECTIVE:

1. Given a word problem, write an equation which is the translation of the word problem, then write the solution.

ACTIVITIES:

Study:

S & M: pages 133 - 134 (objective 1).
       pages 142 - 143, sample problem (objective 1).

M & W: pages 130 - 131, 132 - 133, and example on pages 135.

Be sure you read each problem carefully. Do not set out to solve a problem until you thoroughly understand it. Do not try to memorise a pattern for solving a certain type of problem, but instead, learn how to translate from a word description to a numerical sentence. If you will do this you can solve any word problem that leads to a number sentence you can solve. This is a very important instructional unit because solving problems is what mathematics is all about.

Suggested exercises:

S & M: pages 135 - 137; ex. 1 - 27, odd (objective 1)
       pages 134 - 165; ex. 1 - 33, odd (objective 1)

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Criterion Test 03-04-07-01

1. Write an equation that is the translation of each problem and find the solution.

(a) Mrs. Uglynug is five times as old as her daughter, Ikk. Twenty-one years from now, she will be twice as old as Ikk. How old is Ikk?

(b) The perimeter of a rectangle is 28 feet. The length is 2 less than 3 times the width. Find the dimensions of the rectangle.

(c) The difference of two numbers is 10. Seven times the smaller number is 2 more than the larger. What are the numbers?

Criterion Test 03-04-07-02

1. Write an equation that is the translation of each problem and find the solution.

(a) Two years from now, Sylvester will be twice as old as Ferdinand. Now he is 5 years older than Ferdinand. How old is Ferdinand?

(b) Homer Brewer bought a bottle and a fancy cork for 75c. If the cork cost 5c more than the bottle, find the cost of each.

(c) The length of a rectangle is twice the width. The perimeter is 60 yards. Find the dimensions.
1. Write an equation that is the translation of each problem and find the solution.

(a) If 5 is subtracted from three times a number, the result is 10. What is the number?

(b) A six foot board is cut into two pieces, one twice as long as the other. How long are the pieces?

(c) Dilbert Dingbat weighs 12 pounds more than twice the weight of his sister, Della. What is the weight of each if their combined weight totals 180 pounds?
Answers:

Criterion Test 03-04-07-01

1. (a) let $x =$ Ikk's age now
   
   $5x =$ Mrs. Uglymug's age now
   
   $x + 21 =$ Ikk's age in 21 years
   
   $5x + 21 =$ Mrs. Uglymug's age in 21 years

   
   $5x + 21 = 2(x + 21)$
   
   $5x + 21 = 2x + 42$
   
   $5x - 2x = 42 - 21$
   
   $3x = 21$
   
   $x = 7$ Ikk is 7 years old

   (b) let $x =$ width
   
   $3x - 2 =$ length

   
   $2x + 2(3x - 2) = 28$
   
   $2x + 6x - 4 = 28$
   
   $8x = 28 + 4$
   
   $8x = 32$ The rectangle is 4 ft. by 10 ft.
   
   $x = 4$

   (c) let $x =$ smaller number
   
   $x + 10 =$ larger number

   
   $7x = 2 + x + 10$
   
   $7x - x = 12$
   
   $6x = 12$
   
   $x = 2$ the smaller number is 2
   
   $x + 10 = 12$ the larger number is 12
Answers:

Criterion Test 03-04-07-02

1. (a) let $x = \text{Ferdinand's age now}$
   $x + 5 = \text{Sylvester's age now}$
   $x + 2 = \text{Ferdinand's age two years from now}$
   $x + 7 = \text{Sylvester's age two years from now}$

   
   $x + 7 = 2(x + 2)$
   $x + 7 = 2x + 4$
   $x - 2x = 4 - 7$
   $-x = -3$
   $x = 3$

   Ferdinand is 3 years old

   (b) let $x = \text{cost of bottle}$
       $x + 5 = \text{cost of cork}$

       $x + (x + 5) = 75$
       $2x + 5 = 75$
       $2x = 70$
       $x = 35$

       The bottle cost 35¢, and the cork cost 40¢

   (c) let $x = \text{width}$
       $2x = \text{length}$

       $2(x) + 2(2x) = 60$
       $2x + 4x = 60$
       $6x = 60$
       $x = 10$

       The rectangle is 10 yards by 20 yards
Answers:

Criterion Test 03-04-07-03

1. (a) let \( x \) = the number

\[
3x - 5 = 10
\]
\[
3x = 15
\]
\[
x = 5
\]

The number is 5

(b) let \( x \) = shorter piece

\[
2x = longer piece
\]

\[
x + 2x = 6
\]
\[
3x = 6
\]
\[
x = 2
\]

The pieces are 2 ft. and 4 ft.

(c) let \( x \) = weight of Della

\[
2x + 12 = weight of Dilbert
\]

\[
x + (2x + 12) = 180
\]
\[
3x = 180 - 12
\]
\[
3x = 168
\]
\[
x = 56
\]

Della weighs 56 pounds and Dilbert weighs 124 pounds
I. U. # 03-04-08

Functions
OBJECTIVES:

1. Given a notation such as $f: x \mapsto x^2 + 1$, write the meaning of the notation.

2. Given a function and its domain, specify the range of the function.

3. Given a function, write the value of the function for any given member of the domain.

ACTIVITIES:

Study:

S & M: pages 146 - 148 (objectives 1, 2, and 3).

Suggested exercises:

S & M: page 148; ex. 1 - 6 (objective 1).
page 149; ex. 7 - 15, odd (objective 2).
page 149 - 150; ex. 17 - 35, odd (objective 3).
Criterion Test 03-04-08-01

1. Give the meaning of \( k : n + 3n^2 + 1 \)

2. Specify in roster form the range of the given function.
   (a) \( g: x + x(x + 3), \ D = \{0, 3, -7\} \)
   (b) \( h: y + y^2 - 1, \ D = \{1, 3, 5\} \)

3. Given \( f: f(x) = (x + 2)^2 \), find:
   (a) \( f(3) \)
   (b) \( f(-2) \)

Criterion Test 03-04-08-02

1. Give the meaning of \( g : x \to x^2 - 5 \)

2. Specify in roster form the range of the given function.
   (a) \( f: x \to 2x^2 + 1, \ D = \{2, 4, -6\} \)
   (b) \( h: y \to y^2 - 2, \ D = \{47, 48, 50\} \)

3. Given \( f: f(x) = x(2x-1), \) find:
   (a) \( f(-3) \)
   (b) \( f(4) \)

Criterion Test 03-04-08-03

1. Give the meaning of \( h : x \to x - 7 \)

2. Specify in roster form the range of the given function.
   (a) \( f: x \to 2x + 2, \ D = \{-5, -3, 0\} \)
   (b) \( g: y \to y^2 - y, \ D = \{-3, 4, 5\} \)

3. Given \( f: f(x) = 2x^2 + 1, \) find:
   (a) \( f(-2) \)
   (b) \( f(2) \)
Answers:

Criterion Test 03-04-08-01

1. The function \( h \) that pairs \( n \) with \( 3n^2 + 1 \).

2. (a) \( R = \{0, 18, 28\} \)
   (b) \( R = \{0, 8, 24\} \)

3. (a) 25
   (b) 0

Criterion Test 03-04-08-02

1. The function \( g \) that pairs \( x \) with \( x^2 - 5 \).

2. (a) \( R = \{9, 33, 73\} \)
   (b) \( R = \{45, 46, 48\} \)

3. (a) 21
   (b) 28

Criterion Test 03-04-08-03

1. The function \( h \) that pairs \( x \) with \( x - 7 \).

2. (a) \( R = \{-10, -4, 2\} \)
   (b) \( R = \{12, 12, 20\} \)

3. (a) 2
   (b) 9

03-04

END OF PACKAGE