DOCUMENT RESUME

ED 087 699 SP 007 547

AUTHOR Wilmoth, James Noel
TITLE Deductive Reasoning in Introductory Educational Research: A Survey of Theoretical- and Cognitive-Content from Selected Logic References.
PUB DATE Nov 73
NOTE 105p.

EDRS PRICE MF-$0.65 HC-$6.58
DESCRIPTORS *Deductive Methods; *Educational Research; *Logic; Logical Thinking; *Research Methodology; *Theories

ABSTRACT

This document, which consists of three parts and a summary, surveys a) the structure of theoretical systems, b) the functions of theoretical systems, and c) the formal logical methods of the theoretical scientist in order to find approaches to educational research. Emphasis throughout is placed on the discrimination of concepts-by-intuition from concepts-by-postulation. Part I is a discussion of the parts of a theoretical system including primitives, concepts, postulates, thoerems, and hypotheses. Part II indicates that the primary functions of a theoretical system are to predict what kinds of data are needed to solve a problem and whether the data obtained are reasonable. In part II formal logical methods of the theoretical scientist are stated, including a) method of definition, b) method of hypothesis, c) method of espistemic correlation, and d) method for verifying theories. Also described in this section are three methods of proof a) sentence reasoning, b) indirect proof, and c) class reasoning. The author concludes the summary with two approaches to educational research. Tables and a three-page bibliography are included. (PD)

DEDUCTIVE REASONING IN INTRODUCTORY EDUCATIONAL RESEARCH

A SURVEY OF THEORETICAL- AND COGNITIVE-CONTENT

FROM SELECTED LOGIC REFERENCES

James Noel Wilmoth
Foundations of Education Department
School of Education
Auburn University

November 1973

CONTENTS

ILLUSTRATIONS

TABLES

I. INTRODUCTION

As a science matures its methods become more complex. Concerning this phenomenon Northrop writes as follows:

> . . . in the passage of a science from its natural history stage of its development to that of deductively formulated theory a fundamentally new type of scientific concept [concept-by-postulation] appears which entails, in turn, a radically different type of scientific method. This means that the kind of scientific training which is adequate for the natural history stage is necessary but quite insufficient when the science passes to deductively formulated theory.[1]

A new problem moreover seems to reflect the developmental stages of its parent science. Having initiated inquiry through specification of a particular problematic situation, the researcher ideally sequences attention to his problem through an analysis of problem stage, followed by a natural history stage, and finally through a deductively formulated theory stage. These usually are not self-contained, independent stages of inquiry but are mutually interactive stages. As information is gained for one stage, it may affect or modify what already has been done or will be done in another stage.

As the descriptive component of a problem area approaches some vaguely defined boundary around diminishing returns, it may be found that descriptive concepts become inadequate to account for all

[1]Northrop, Logic of the Sciences and the Humanities, pp. 135-36.

accumulatable, apprehendable data and their relations. The investigation may thereby evolve into a more sophisticated, deductively formulated theoretical stage capable of accounting for unobservable entities and factors as well as directly inspectable data. In this stage, in fact, any adequate deductively formulated theory for a problematic situation should account for all the associated natural history data in all its diversity and complexity as well as accounting for hypothetical relations and/or unobservable concepts.

A basic problem in coordinating educational knowledge in support of an investigation is in the demarcation of concepts used to describe its raw data (concepts-by-inspection) from concepts which enter into its deductively formulated theory stage (concepts-by-postulation). In the physical sciences this problem may be illustrated with meanings associated with the concept "sulfur."

Immediately apprehended sulfur is known by the senses as being yellow (a described fact), as burning with a disagreeable odor, as being a viscous liquid when melted, as having textural qualities ranging from a yellow powder to a yellow fibrous solid, et cetera. Theoretical sulfur is a type of atomic structure having a specific number and arrangement of sub-atomic particles, a specific pairing of outer electrons, a specific set of energy levels, a specific location in the periodic chart of the atoms, and specific chemical properties.

In education, "intelligence," in the Thurstone sense, is immediately apprehendable through certain of its component constructs (operational definitions). The verbal component for intelligence is known from word combinations either read or heard; the number component registers as sensory receptions of results of computational efforts;

and appropriate apprehendable considerations apply to word fluency, spatial ability, reasoning, memory and perception.

With each educational problem there should be an associated theory, maybe no more sophisticated than an intuitive hunch, otherwise the problem-solver would not know how to approach the finding of an answer nor whether what he finds is, in fact, an answer. Usually the theory that accommodates the problem is exposed for examination. In education this exposure is commonly in narrative form while in the physical sciences it may be shown as an equation or an equation with an amplified explanation.

For an estimation problem the researcher usually wishes to pursue its solution(s) with methods that will allow him to place confidence intervals around some most probable answer(s). The theory supporting this type of study may be simply that a most probable approximation for the actual answer to the problem does exist and can be found with available methods.

II. STRUCTURE IN THEORETICAL SYSTEMS

PRIMITIVES

Initial concepts of a deductive theory may be thought of as needing no definition; that is, as being "primitives." For these concepts, meanings are prescribed in the contexts reflected in the theories they support, particularly their relational contexts with other primitives.

CONCEPTS

Concepts in a theoretical structure are either primitives or defined. As already noted primitive concepts are those undefined terms whose meanings are tied to contexts in which they are used. One does not attempt to define primitive concepts as such an effort would launch him on a path of infinite regress or in a circle doubling back onto his initial primitives.

Again, it should be emphasized that meanings for primitive concepts are tied to contexts in which they are used rather than to definitions. The other primitive concepts together with the relations uniting them in a theoretical context prescribe the meaning (syntactic only) that a particular primitive term should have. Within an assignment of meaning the term labeling a concept may be associated with a set of data, a particular datum, or a set of postulates which it represents.

All concepts in theorems are either primitive concepts or concepts-by-postulation rooted in the primitives with the method of definition.

The method of definition is used in theory synthesis for providing meanings for its defined concepts. Defined concepts are labels arbitrarily provided for all concepts that are used in a theory and that are not primitives. Primitives, and/or other defined concepts are employed with the method of definition to convey a meaning for each newly defined concept. It should be emphasized by restatement that the method of definition involves nothing more than the specification of a concept in terms of other concepts: the latter being either primitive, undefined concepts, other defined concepts, or combinations of the foregoing.

In the natural history stage of inquiry one attempts to effectively transform observed fact into described fact and attempts to conceptually classify described fact according to logical considerations for class inclusion. A class will usually be conceptualized, that is, labeled with a concept. Concepts for public, common-sense objects or for their qualitative sensed properties or for the introspective feelings generated by problematic situations should not be confused with concepts-by-postulation associated with generation of deductively formulated theory.

The former are concepts from the natural history stage and are called concepts-by-intuition (concepts-by-inspection or concepts-by-induction). They are further classified as either concepts-by-sensation or concepts-by-introspection. Concepts used with deductively formulated theory on the other hand are grossly labeled concepts-by-postulation which are subclassified by Northrop as either logical-concepts-by-intuition, concepts-by-perception, concepts-by-imagination, or concepts-by-intellection.[1]

Concepts-by-intuition in the natural history stage are restricted entirely to that which is immediately apprehendable. If the immediately apprehendable becomes a part of a deductively formulated theory, then all concepts-by-intuition that are postulationally employed become logical-concepts-by-intuition. This new status in theory usually occurs as the theoretical component of an investigation employs primitives that are directly apprehendable concepts-by-sensation or concepts-by-introspection. However, not all primitives need to be logical-concepts-by-intuition.

Logical-concepts-by-intuition, as primitives in a theory, provide a natural transition from naturalistic observations to deductively formulated theory. They are concepts-by-intuition with respect to their content and are logical with respect to their inclusion as a primitive in a postulational system.

As a theory matures, it may require more deductively fertile scientific objects and relations than logical-concepts-by-intuition can provide.[2] The scientist may satisfy his requirements through creatively postulating objects and relations with meanings more and more divergent from meanings associated with the directly apprehendable. He thereby may choose, for some or all of his primitives, to introduce undefined concepts whose initial existence and possible immortality are

---

[1]Northrop, Logic of the Sciences and the Humanities, Chapter V passim.

[2]Ibid., p. 59.

functions of postulation only. This course of theoretical concept synthesis results in either concepts-by-perception, concepts-by-imagination, or concepts-by-intellection in order of increasing abstraction. Concepts-by-perception are defined in terms of concepts-by-imagination and concepts-by-intellection. Concepts-by-imagination and concepts-by-intellection are the basic scientific concepts representing subtle, usually unobservable, deductively fertile, scientific objects and relations.

Requirements for deductive fertility necessitating extensions to concepts-to-postulation seem to parallel an increasing need to account for observations that are very gross, very complex, or very crude. In the physical sciences consider the many manifestations of energy for which Einstein's comprehensive equation demonstrated overlays or in the social sciences consider the outcomes of Thurstone's or Guilford's analysis of intelligence with the deductively formulated theories attendant thereto.

The physical scientist with his advanced stage of theory development seems inclined to postulate unobservable scientific objects or phenomena such as electrons or electromagnetic waves while the social scientist, perhaps on a less abstract level, is more inclined to postulate motivations, behaviors, common factors, stimulus-response chains, instructional domains, and so forth, for entities in problematic situations. The justifications for such postulations are functions of their theoretical necessities for resolutions of problems initiating inquiry.

Each object and relational phenomenon designated by a concept-by-postulation within a theory should have objective properties that are wholly defined by theoretical hypotheses of the theory in which the concept is embedded. One cannot find their meanings by observing anything in the objective world; however, the search for such meanings may lead the investigator to study consequences deduced from his hypotheses--consequences from which the hypotheses may be subjected to empirical and experimental tests. Such tests, moreover, change as the deductively formulated theories employing abstract concepts-by-postulation change. One uses different tests for intelligence depending on whether Thurstone's or Guilford's deductively formulated theories provide the frame of reference for his testing. This is true even though the same words may be used as concepts-by-postulation in two or more theories. Each test, for which results are positive, only indirectly confirms the theoretical (hypothetical) objects or relations giving rise to that test.

Ambiguity in the meaning of abstract concepts-by-postulation could occur anytime two or more theories employ an identical label for objects or relational phenomena while ascribing different meanings for these identical labels with their contextual variations. Perhaps Guilford should have coined a new term instead of "intelligence"--a new term that would immediately distinguish what he denoted from what all other investigators who have used "intelligence" had in mind. While it is often ambiguous, it is nevertheless common practice to select some well-known term, as Guilford and others did, with common meaning closest to one's technical meaning which is known only from contextual use in its associated deductively formulated theory. Were a technical terminology of specific concept labels to arise chances for ambiguity would be minimized but the accompanying proliferation of technical vocabulary would probably require expertise, and perhaps technical training, for communication by layman and professional alike. The wider audience reached by labeling concepts-by-postulation with common terms, particularly for information having relevance for the layman, seems to have served as a focal consideration for educators selecting the common term approach. However, this practice has often misled laymen as they have not always critically examined the lengthy theoretical contexts required to accurately convey the associated, specific, unique meanings that the terms should carry.

CONCEPTS-BY-POSTULATION

Concepts-by-postulation have connotations with respect only to the deductive theories prescribing their meanings in terms of primitives and other concepts-by-postulation used in defining each of them by the method of definition. With the method of definition all primitive concepts basic to a defined concept thru postulation are relationally united in some specific manner depending upon the dictates of the theoretician's imagination; his creativity; and his experiential, theoretical, and empirical background. The particular concepts in a theory which may be relationally united may generate a formidable number of combinations quite independent of whether particular combinations do or do not exist as directly observable phenomena.

By this procedure many of the fundamental concepts-by-postulation for the physical sciences have been first known in theory then provisionally verified under the qualifications attached to assumptions and postulates of the theories giving birth to them; for example, electrons, ions, wave lengths, molecules, valence, volts, and so forth. In education, some examples are objectives, distribution functions, significance, discovery method, conditions on learning, intelligence, grades, and other like abstractions.

Meanings for concepts-by-postulation have been classified by Northrop as being monistic or pluralistic.[1] (Note Figure 1 for examples.) A concept with a monistic meaning has postulated for it a single property, a single continuum, or a single attribute; a concept with a pluralistic meaning associates several properties, continua, or attributes. Each such property, continuum, or attribute may be thought of as a factor uniting a set of theoretical or observed facts. This term, "factor," may be used interchangeably with dimension, variable, result or effect, quality, essential, element, component, criterion, or relation.

POSTULATES

Postulates are collections of assumptions and defined concepts in a deductively formulated theory. A postulate is usually considered to be unprovable as there is an absence of lower-order terms from which the postulate can be deduced in some logical argument which would be necessary for a proof to exist. With postulates, however, one should be able to construct arguments leading to conclusions (theorems). These arguments would use the same types of mental activities that are

[1]Northrop, Logic of the Sciences and the Humanities, pp. 94-95.

Figure 1. A classification of concepts-by-postulation enlarged from Northrop's _The Logic of the Sciences and the Humanities_, pages 94-95. The classification is in serial order according to increasing abstraction from the aesthetic continuum.

A CLASSIFICATION OF CONCEPTS-BY-POSTULATION

I. Logical-concepts-by-intuition are concepts that simultaneously designate apprehendable fragments of the undifferentiated aesthetic continuum and have postulational (immortal, universal) status in a specific deductively formulated theory.

   A. A monistic example is the "Universal Mover" in Aristotle's metaphysics. Another example would be pencil "marks" answering the questions on a test.

   B. Some pluralistic examples are Whitehead's "eternal objects," Santayana's "essences," Aristotle's "ideas," and the very common educational phenomenon of "teaching."

II. Concepts-by-perception are postulated concepts used to connote theoretical factors which are partly sensed and partly imagined.

   A. Monistic examples are the public "space" of daily life and "reading," when done orally.

   B. Some pluralistic examples are "other persons," "tables," "chairs," "classrooms," the "back side of the moon" on the basis of a "front" side which we can see.

III. Concepts-by-imagination are postulated concepts connoting postulated entities or relations which can be imagined but not sensed.

   A. Monistic examples are the "ether" concept of classical, prerelativistic field physics, and "adding" as a mathematical operation.

   B. Some pluralistic examples are "atoms" and "molecules" of classical particle physics, the kinetic-molecular theory, and the kinetic theory of heat and gasses, "logic," "intelligence," and "discipline" of education.

IV. Concepts-by-intellection are postulated concepts connoting entities or relations which can neither be imagined nor sensed.

   A. Some monistic examples are the "space-time continuum" of Einstein's field physics, Einstein's "tensor equation," for gravitation, and "I.Q.," and "socio-economic level" in the social sciences.

   B. Some pluralistic examples are Plato's "atomic ratio's," "correlation ratio's," and "statistically significant differences."

familiar to those who have had experience in proving mathematical theorems--mental activities which most college and university students may have developed prior to entering graduate school.

Postulates are statements (assumptions) prescribing relationships between primitive terms. <u>All other concepts</u> in deductively formulated theory <u>have roots tracing back to the primitive ones in the postulates</u> through the methods of definition and postulation that give existence to these higher level concepts.

Concepts within the postulates may be coined words to represent objects or relations that are unobservable and whose existence is attributable only to that same type of fertility in the theoretician's intellect which creates hunches, guesses, or suppositions (postulates) with potential to account for (explain) his problematic situation in the context of his apprehendable world.

A postulate may be schematically represented as <u>ABC . . . R . . . XYZ</u> where A, B, C, . . . Q, S, . . . X, Y, and Z are primitive or undefined concepts and R is a specified relation between them. Underlining indicates a proposition.

Postulates are also known synonymously in the literature as axioms, assumptions, hypotheses, et cetera. In every case these terms refer to carefully stated contextual statements containing primitives with logical connectives between/among them.

Postulates within theories may be conclusions advanced in other research, each such conclusion being not an absolute but a tested probability. In any case, postulates are formalized assumptions (working conjectures) uniting basic component variables and relations within some specific theoretical substructure.

THEOREMS

Theorems (consequences) are propositions formed from postulates with the formal logical relation known as "formal implication." Theorems seem to arise from postulates by application of rules of formal logic which will be examined in detail in a later section. All propositions in a deductively formulated theory other than its postulates are theorems provable from its postulates, each postulate being usually assumed true pending the establishment of relationships with the apprehendable continuum which may permit its empirical testing with direct and indirect procedures.

The development of theorems from postulates may be represented as in Figure 2. In that representation, A, B, . . . are primitive concepts which are propositionized (underlined) with relational connectives, R_1 . . . R_n. These postulates are operated upon with the method of definition to produce defined concepts-by-postulation. From the postulates and defined concepts logical implication produces theorems upon which the method of definition may be used to establish still more defined concepts that may be incorporated recursively with the more basic postulates and defined concepts to enrich the initial set of theorems.

The interaction in theory of postulates, concepts, and theorems is obvious. Unless the researcher examines the complete theory, he can obtain neither a precise denotation nor a feeling for the connotative range of terms used. The same terms, removed from their theoretical context, will probably mean whatever one wants them to mean: what popular usage ascribes to them or what one's intuition suggests.

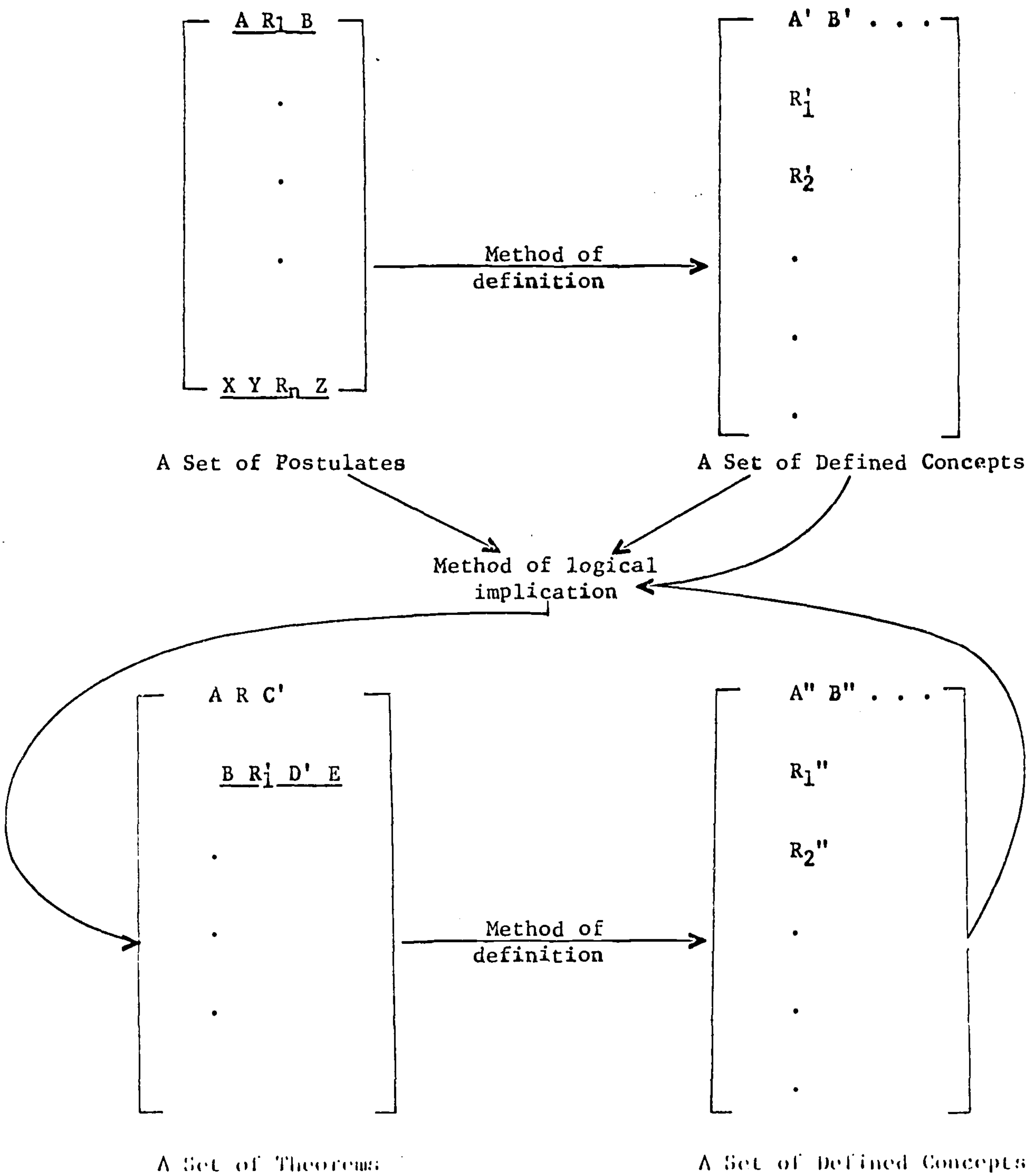


Figure 2. Forming theorems from postulates. A, B, C, . . . , P, Q, S, T, . . . , X, Y, Z are primitive or undefined concepts. R_i are the relations uniting concepts in propositions. Each proposition is underlined. Priming indexes defined concepts.

HYPOTHESES

The term "hypothesis" is used to connote two dependent expressions in research. One expression is speculative, the second is statistical. (Figure 3 lists other meanings for the term "hypothesis.") The two expressions are dependent in the sense that the hypotheses tested statistically have their roots in the speculations associated with the hypothetically inferred, deductively formulated theory stage of an investigation. The researcher apparently is led to statistically correlate two variables, test the equality of two or more sample means, and so forth, by considerations arising from hypothesizing object and relational possibilities which are interwoven into his theoretical network.

When the theoretician develops hypotheses, he should draw upon his full range of perceptions, imaginations, and abstractions (particularly mathematical abstractions) relevant to his problematic situation. Ideally, out of these he should speculate or guess then formalize <u>all</u> hunches by <u>hypothesizing</u> what are usually non-apprehendable objects and relations which can serve as primitives or in postulates from which theorems can be deduced. Each such speculation represents a hypothesis concerning the problematic situation--a hypothesis that is not conformable by inspection of anything. Inspection is important however for testing <u>statistical</u> hypotheses which are links of theory (some of its theorems) with the apprehendable, empirical, pragmatic world. Theoretical hypotheses are provisionally confirmed if data confirm predictions in the statistical hypotheses.

Connotations for the Term "Hypothesis"

"Hypotheses are suggested problem solutions which are expressed as generalizations or propositions." (Van Dalen, Understanding Educational Research, p. 146.)

. . . .

"A hypothesis is a provisional or possible explanation which accounts for the factors, events, or conditions that the research worker is trying to understand." (Van Dalen, Understanding Educational Research, p. 154.)

. . . .

"A hypothesis suggests that an antecedent condition (independent variable) is related to the occurrence of another condition, event, or effect (dependent variable)." (Van Dalen, Understanding Educational Research, p. 243.)

A hypothesis is a logical explanation for the probable solution to a problem.

A hypothesis is a prediction

"A good hypothesis should: (1) bring together, in a simple manner a given number of instances and interpret the connection between them; (2) be fruitful--fertile--that is, suggestive of significant deductions which are testable; (3) be comprehensible and not inconsistent." (Gephart and Ingle, Educational Research: Selected Readings, p. 47.)

. . . .

"A hypothesis consists of elements expressed in an orderly system of relationships which seek to explain a condition that has not yet been verified by facts." (Van Dalen, "The Role of Hypotheses in Educational Research," in Educational Research: Selected Readings, p. 150.)

Functions of hypotheses as "hypothesis" is used in the literature:

1. To state assumptions.
2. To present explanations.
3. To predict observations creating relevance for facts.
4. To suggest sources of variance affecting research design.
5. To provide logical frameworks for conclusions.
6. To stimulate formulation of new explanations (hypotheses).
7. To imply consequences.
8. To propose what exists (entities and relations, perhaps unobservable).

A "null hypothesis" is a relation (ususally mathematical) that is to be statistically tested.

A "research hypothesis" is a relation (usually mathematical) that is provisionally accepted if the null hypothesis is shown to be probably false.

Figure 3. Some attributes giving meaning to the term "hypothesis."

Hypothesized objects have theoretical existence in postulates denoted by concepts-by-imagination and concepts-by-intellection; hypothesized relations have existence in concepts by either perception, imagination, or intellection. The usual goal of the researcher is to modify, adjust, or otherwise alter some of his concepts-by-imagination and concepts-by-intellection such that they become fundamental for explaining perceptions, sensations, or introspections generated by that which is apprehendable.

THEORIES

A theory is a <u>propositional system</u> of coordinated postulates (axioms, concepts, assumptions), theorems, and logical relationships. Generally, propositions in a theoretical system are related as premises and conclusions in arguments that vary in complexity with conclusions of lower order arguments often serving as premises for more complex arguments. Formal implication ties antecedent propositions to consequent propositions throughout the system.

Arguments are sets of statements each of whose truth values are either (1) individually determined from associated evidence in the empirical (aesthetic) world, (2) individually assigned under some assignment function, or (3) assumed as true because of definitional considerations or status as a primitive. When postulational statements of an argument each have a truth value of <u>true</u> from one of the three considerations above then theorems can be proved by means of the logical relation of formal implication. It may happen, however, that the truth value of some of the postulational statements is unknown in which case the researcher attempts to indirectly demonstrate their <u>provisional</u>

truth by verifying, with non-logical methods, the truth of one or more theorems logically derived from the postulates. This process produces provisional truth only as there may be other theories that also could lead to that which was verified.

Proof of a theory should not be confused with its truth. Proof is a relation between propositions, a relation between those that serve as postulates (premises) and those which are theorems (conclusions). Truth, on the other hand, is a relation between a proposition and the factual state of affairs. Truth considerations apply to both types of statements, both the postulates and the theorems. Proof is a purely formal relation while truth is a purely empirical one; thus, the methods of proof are defined in the discipline of formal logic and the methods of truth in the discipline of the empirical sciences.

Some postulates, perhaps all of them, in a theory are accepted as true without empirical testing; these are called assumptions. Those same untested postulates may be used to prove theorems. Proof of theorems with postulates whose truth value of "true" has been assumed should not be confused with the question of postulate truth or falsity determined by empirical methods. Indeed proof of theorems is an activity which may be undertaken without knowledge of the truth value for any propositions of arguments in their deductive theory and without verification of anything in immediate apprehension identical with what the propositions denote. Proof and truth are concepts for activities which may be approached independently.

Some concepts (by postulation) may be introduced for theory building with contextual meanings for unobservable entities and relations--meanings that are believed to account for apprehendable facts. Such theories are given credibility as their predicted consequences lead to empirical verifications (positive epistemic correlations). The set of empirical verifications emerging from such a theory constitute strong indirect evidence for the probable (not unqualified) existence of the postulated, unobservable entities and relations embedded within the theory. An example of this procedure is reflected in Mendeleef's theory of periodicity for atomic structure which has since led to many discoveries in physical science, each discovery constituting indirect evidence for the postulated (theoretical) structural relations in atoms. The theory of operant conditioning in psychology serves as an example from the social sciences, in which case linkages are postulated between stimuli and responses such that selective reinforcement will condition or establish particular response sets which are triggered by presentation of particular stimuli.

That which is directly apprehended is roughly the same in any educational investigation. How the apprehended is analyzed, interpreted, or correlated among its differentiations is usually different from one study to another. It is precisely these differences which should be foci of concern as the researcher reviews the theoretical and empirical literature concerning his problem and from this literature constructs or accepts a logically consistent theory against which his investigation can be validated. These decision processes for supportive theories seem to require extensive skills of formal logic.

DEDUCTIVELY FORMULATED THEORIES

Deductively formulated theory may be constructed either from (1) logical-concepts-by-intuition; (2) concepts-by-perception, -imagination, or -intellection; or (3) combinations of the preceding types of concepts. A theory is called an _abstractive, deductively formulated_ theory if it uses logical-concepts-by-intuition only; it is called an _hypothetically inferred, deductively formulated_ theory in all cases in which a primitive is assumed that is not directly linked to the aesthetic continuum through a concept-by-intuition (which may be either a concept-by-sensation or a concept-by-introspection).

ABSTRACTIVE, DEDUCTIVELY FORMULATED THEORIES

Postulates of abstractive, deductively formulated theory are "logical-concepts-by-intuition" which are a class of "concept-by-postulation." These are concepts that most unambiguously link the aesthetic continuum to the theoretic continuum. Their content is specified with respect to their aesthetic properties; their logical properties are specified by deductive contexts of the theory using them.

Because the concepts out of which it is constructed are logical-concepts-by-intuition, abstractive, deductively formulated theory is directly, empirically _verifiable_ by verifying the truth of its postulates (assuming the theory is logically consistent).

Deductively formulated theory in its abstractive variety suggests a situation of hope in its psychological, subjective sense. The hypothetically inferred variety which will be examined next offers prediction rather than hope--prediction under conservation laws for the system's factors. With hypothetically inferred, deductively formulated theory, future states of experimental systems may be logically deduced from knowledge of its present state and of the environmental (treatment) differences known to be deliberately or naturally introduced to effect its future state.

HYPOTHETICALLY INFERRED, DEDUCTIVELY FORMULATED THEORIES

The basic assumptions of hypothetically inferred, deductively formulated theory relate to either concepts-by-imagination or concepts-by-intellection. Concepts-by-imagination may manifest themselves in theories tied to common sense objects or to non-word-based models. Concepts-by-imagination also serve to establish concepts-by-perception thereby reflecting inherrent connection of theory with the world of imported meanings.

Concepts-by-intellection are the most abstract concept-by-postulation employed for theory structure. Although they are neither imagined nor sensed, they can serve as fundamental entities that account for human sense perceptions thus they can have ties with concepts-by-perception. "Electron" is a concept-by-intellection used to account for deflections in galvanometers which can be perceived. Intelligence is a concept-by-intellection sometimes used to account for performance (a concept-by-perception) on a test.

If problems initiating inquiry are embedded in hypothetically inferred, deductively formulated theories, then they may be solved only through the indirect approach of deducing theorems from assumed concepts-by-postulation and checking some or all of the theorems empirically. These solutions may not appeal solely to directly inspectable entities and relations, but they can appeal to the directly inspectable through epistemic correlations with it. A later section focuses on epistemic correlations.

There is a risk of committing the formal logical fallacy known as the "fallacy of the hypothetical syllogism" if premises are assumed to be true after a theory is indirectly affirmed by testing its deduced consequences (its theorems). The logical relation under test is "logical implication" which has unqualified truth value only under conditions of denial of the consequent or affirmation of the antecedent. The section on formal logical methods of the theoretical scientist presents a more complete consideration of this phenomenon.

Each theory seems to have at least three distinguishing attributes:

1. <u>A specific theoretical content</u> reflecting the disciplines used for explaining why predicted apprehendable data, with relations among those data, give credibility to the theory.

2. <u>Specific</u> species of <u>theoretical epistemic correlates</u> reflecting those entities and/or relations within the theory that have an association with the immediately apprehendable. In abstractive, deductively formulated theory the immediately apprehendable correlates are logical-concepts-by-inspection. In hypothetically inferred, deductively formulated theory the hypothesized correlates are either concepts-by-imagination or concepts-by-intellection. (Concepts-by-perception as theoretical epistemic correlates have roots in concepts-by-imagination and concepts-by-intellection.)

3. <u>Specific</u> species of <u>apprehendable epistemic correlates</u> reflecting those entities and/or relations within the observable problematic situation that have an association with deductively formulated theory. These apprehendable correlates may be either factual or normative. If factual then one fact contrary to what was predicted will refute the theory. If normative then facts are used to establish norms and standards for attributes of the organizations, situations, etc., which were concerns of the deductive theory and were measured. Also normative measurements may be used for comparing the actual state of affairs with that which was defined to be "ideal" in the theory.

Cattell has suggested four standards against which a theory may be evaluated. He calls them "Dimensions of Theory Structure."[1] Note Figure 4.

In the first dimension, "F," a judgment is needed concerning the extent to which the theory is <u>formulated</u> as a strict model, that is, is associated with some analogy having fully defined properties. This model might be a paradigm; a mathematical equation or graph; a physical entity such as a mechanical device, a chemical procedure, or an electrical device; or a symbolic representation such as a verbal description, a picture, a sketch, or a diagram. Ratings for the "extent of formulation as a model" dimension may assume positions on a continuum of abstractions extending from "word-based" to "strictly qualified as a model." In all cases in which theories are modeled, the models should be servants of theoreticians for interpreting data, theoreticians should not be servants of the model.

A second dimension, "L," to be assessed is the theory's "degree of dependence on directly inspectable versus theoretical properties." This dimension translates into what has been referred to elsewhere in this paper as "abstractive" versus "hypothetically-inferred" deductively-formulated theory. Cattell suggests rating on the continuum connecting "xenoid" (strange) with "yielded locally." It is this dimension that seems to be responsible for some errors in meaning associated with theory interpretation. If there are only remote connections with the theory implied in epistemic correlations, then it may be difficult to

---

[1]Cattell, "Principles of Design and Analysis," p. 50.

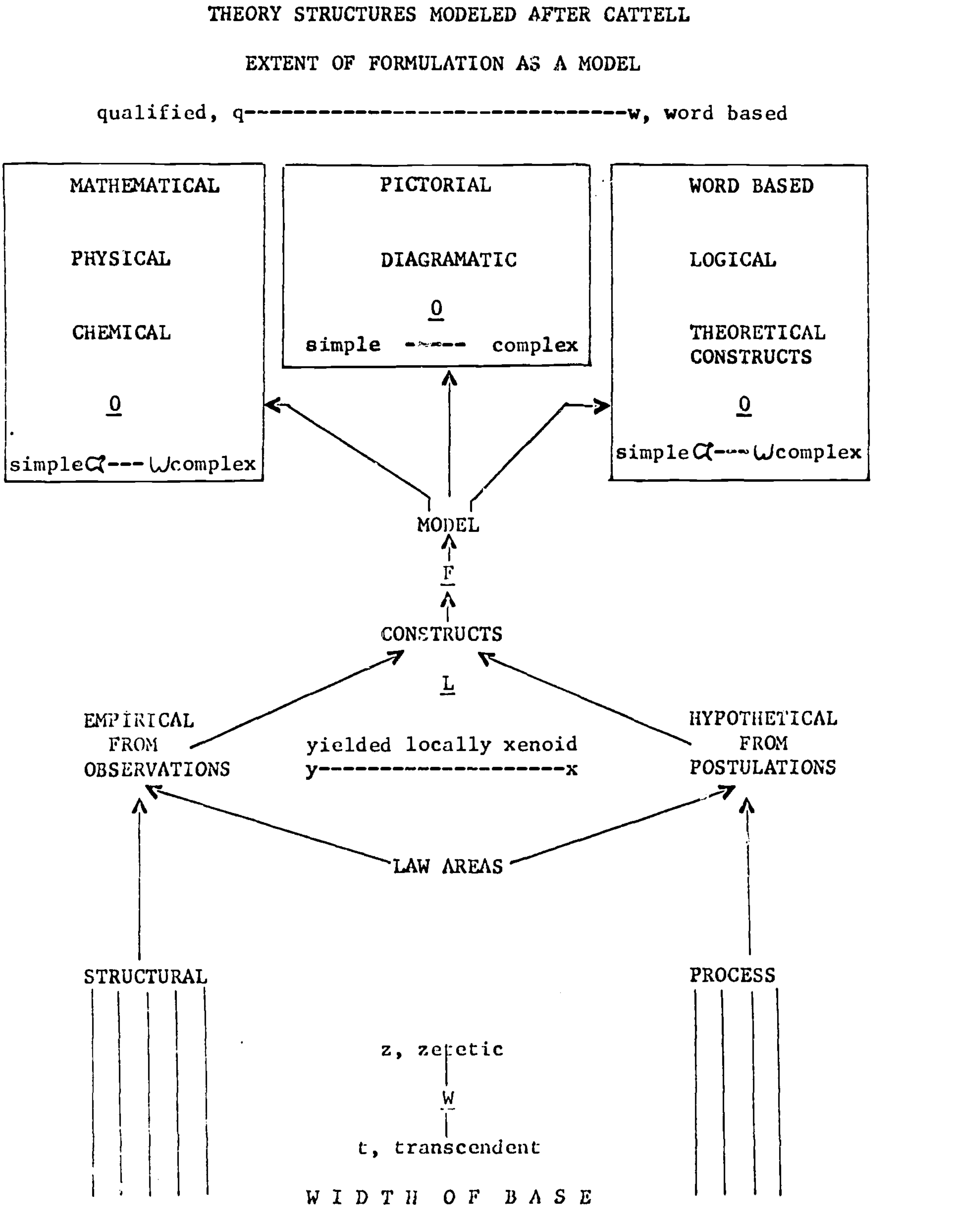


Figure 4. A model for the structure of theory adapted from Cattell's "Principle of Design and Analysis," page 50.

rationalize relationships between perceived and postulated entities. Also, as a model is applied to a new situation that imports it, it may be difficult to fit the new context without distorting either the model itself or the context it was intended to serve.

For a third dimension, "O," of theory quality, one judges the "order of complexity of theoretical relations." These may vary on a continuum from "simple" to "complex." A simpler relation might be the "less than" relation between two entities. A more complex relation might be that of "cannonical correlation" between two sets of entities. Cattell neither defines "simple" nor "complex."

The fourth qualitative dimension, "W," suggested by Cattell for assessing theories is associated with the "number of empirical constructs or postulated entities that are connected by inductive and deductive methods." Cattell refers to this as the "width of base" dimension which may vary from "zetetic" (seeking all proof) to "transcendent" (tied to little). Polar numbers associated with terminal points of this dimension were not stated.

Scientific investigations seem to be characterized by the foregoing theoretical attributes. In each, the type of theory proposed by the investigator seems to be a function of the complexity of his thinking about his problem. If the explanation leading to the solution to the problem arises from apprehendable considerations only, then the theory is referred to as abstractive, deductively formulated theory; if even one non-apprehendable object, relation, or entity is postulated, then the theory is considered to be hypothetically inferred, deductively formulated theory.

LOGICAL OPERATORS

A sentence is a statement (proposition) that has a truth value (either true or false) associated with it. It is a basic unit of deductive reasoning being connected to other sentences with logical operators or prefixes. The operators to be considered in this section are the conditional, biconditional, conjunction, disjunction, and negajunction; the single prefix to be considered is the denial. Attention will also be given to class reasoning.

In conditioned reasoning one sentence is a condition which, if true, is sufficient for the truth of some consequent. The conditional sentence, is called the antecedent. If the consequent is known to be false, then the antecedent is necessarily false too.

Sentence is used throughout this section on "Logical Operators" in the logical sense of being a statement or proposition the truth of which can be ascertained or assumed. It is not intended to reflect the "grammatical" sense of sentence. With this in mind, the conditional sentence is considered to be composed of an antecedent (logical sentence) and a consequent (a second logical sentence). A logical connective between components of a conditional is "If . . . then . . ."

The biconditional connective between two sentences is a linkage that suggests a relationship of mutual implication between the two sentence components. Biconditional relationships may be true by defintion or may be assigned to conditionals whose converses also happen to be true. In a biconditional a conditional and its converse are true;

hence, a proper connective between components of the biconditional is " . . . if, and only if . . ."

A conjunction unites two sentences called conjuncts. Each of the two conjuncts may be treated independently of the other; in fact, two separate assertions may be conjoined at one's convenience. Since in a true conjunction both conjuncts are true, a proper connective between them is " . . . and . . ."

The negajunction is the denial of a conjunction. Each component of a negajunction is called a negajunct. In a true negajunction at least one of the negajuncts is false. A logical connective for negajunction is "Not both . . . and . . ."

Disjunctions (alternations) unite two component sentences at least one of which is true. The common form of logical connective between two component disjuncts (alternants) is ". . . or . . ." which may be either inclusive, weak, or strong depending on context. Inclusive alternation is a special application to conditionals such that the antecedent consists of the alternation, at least one alternant being understood to be true and such that the second alternant may be true. Affirmation of one weak alternant does not imply a thing about the other whereas affirmation of either alternant in strong alternation implies denial of the other alternant. Moreover, in both strong and weak alternation, denial of one alternant implies affirmation of the other. Weak alternation may be translatable into a conditional (if not p, then q); strong alternation into a biconditional (not p, if and only if q).

DETERMINING TRUTH VALUE FOR STATEMENTS HAVING LOGICAL CONNECTIVES

If one evaluates arguments of any kind, in education or in any other field, he needs to be able to assign a truth value to each logical statement used. This assignment may be made from assumption, from empirical considerations, from logical considerations, or from a combination of logical and empirical considerations.

In the following analysis generality is maintained by introducing the concept of a well-formed-formula, WFF. A basic WFF is an alphabetic character, a "sentence letter," representing a statement which has a truth value. Well formed formulas may be derived from this basic WFF and other derived WFF's in the following ways:

1. The denial of a WFF is a WFF.
2. Conjoining (with "and") two well formed formulas generates a WFF.
3. Disjoining (with "or") two well formed formulas generates a WFF.
4. Connecting two well formed formulas with a conditional (If . . . then . . .) produces a well formed formula.
5. Connecting two well formed formulas with a biconditional (. . . if, and only if . . .) produces a well formed formula.

A statement may be symbolized as a basic well formed formula only if it is a proposition, assumption, or sentence with inherrent truth value or if it may be derived by denial, conjunction, disjunction, conditional linkage, or biconditional linkages from basic propositions, assumptions, or sentences, or from other statements which already qualify for symbolization as well formed formulas.

Valuation of truth status of statements (propositions) containing logical connectives is an activity which occurs in sequence after assignment of truth values to all component sentences. One assigns, either from evidence or by assumption, truth values to each sentence component of an argument, he valuates statements produced from basic sentence or WFF components. Five rules for valuation following assignment of truth values to each sentence may be listed as follows:

1. A denial is true if, and only if the result of deleting the denying qualifier is false.

2. A conjunction is true if, and only if, every component is true.

3. A disjunction is true if, and only if, at least one component is true.

4. A conditional is true if, and only if, either the antecedent component is false, or the consequent component is true, or both antecedent is false and consequent is true.

5. A biconditional is true if, and only if, both of its components have the same truth value.

RELATIONS

The theory supporting an investigation represents the researcher's capacity for rational explanation of his problematic situation, his problem, and his potential solution(s) for his problem. Eventually, this theory should be constructed of statements of relationships. The investigation, thereby, enlarges from considerations within theory, enlarges particularly with respect to empirical data gathering activities that culminate in performing statistical tests on the theory's relational statements (theorems).

Cattell[1] synthesized a conceptual framework around which relational statements can be made. (His writings were fundamental for development of the analyses of relations that follows in this section.) Educational and other behavioral- or social-science investigations may be designed from a study of relationship structures accounted for with Cattell's theory. On this basis, the investigator would select the best relationship(s) in his empirical settings for testing his theoretical structure. Relations selected should point to statistical analysis procedures (tests) that could be applied to data collected for the entities related. Inferential testing of the empirical evidence would suggest whether or not tentatively to accept hypotheses in the theory; that is, whether or not tentatively to believe the theory to be acceptable as an explanation for phenomena investigated.

Data collected under stimulation of educational theory will usually represent cases for which theoretical classifications can be employed. Cattell's relational systems specify that this range of theoretical classifications may be extended ". . . to academic and research areas usually considered beyond psychology."[2] These classifications (concepts-by-postulation, concepts-by-inspection, constructs, "class words") include proto-types in their mean or standard condition

---

[1]Raymond B. Cattell, "The Data Box: Its Ordering of Total Resources in Terms of Possible Relational Systems," Handbook of Multivariate Experimental Psychology, edited by Raymond B. Cattell (Chicago: Rand McNally and Company, 1966), pp. 67-128.

[2]Ibid., p. 94.

and patterns of deviation or distributions for the proto-types. The proto-types are:

1. The persons, organisms, or other units representing the cases to be measured

2. The focal stimuli that yield measurements complete with instructions for responding

3. The non-focal environmental background stimuli at the time and in the place of observation

4. The responses that could be made to the focal stimuli

5. The observers that supervise, record, and/or otherwise manage all data-gathering operations

Consider an example problematic situation such that some theory led to the logically consistent conclusion (theorem) that "chemistry students in Lee County, Alabama will perform better on the American Chemical Society standardized test (when intelligence is statistically controlled) if taught the CBA program than if taught the CHEM study or traditional programs." All students on whom data are collected serve together as the persons set; the ACS standardized test along with the instrument providing for intelligence measures constitute the focal stimuli set; the ranges of answers possible from both instruments represent the responses set; chemistry laboratories and/or chemistry classrooms and/or other places of testing are elements of the environmental backgrounds set; and the observers' set contains all teachers, administrators, guidance counsellors, and so forth, who administered, coded, and analyzed the instruments yielding data. Merits of the three types of experience given (CBA, CHEM, traditional) can be compared after analyses accommodating the above classificatory dimensions have been completed.

Data classifications in the above list are for standard or mean conditions only for each person, stimulus, et cetera. To have a truly adequate, full system of classification, provisions would need to be made for the various states, conditions, or dispersions for each dimension in the proto-type list above. Doing this would provide five additional classifications for a total of ten. Classifications that should be added to the former list are:

6. States (conditions, roles) that persons can assume around the state for each person during time of observation.

7. Variants possible for the stimuli distributed on either side of the stimuli (questions) used.

8. Phases of environmental backgrounds ranging between their possible extremes.

9. Styles (qualities) of responses arranged about the responses actually made.

10. Conditions possible for the observers with respect to the condition of each during the time he was observing.

If the CBA-CHEM-traditional example were expanded to provide for these five additional dimensions, it might develop as follows. Additional information on each person would need to be collected such that each person could be located with respect to states studied; that is, his attitude state, his vocational interest state, or his motivational state; his role-playing state; or the state he assumes as a consequence of directions given to him while he is being observed. Variants for the stimuli might include forms of the ACS test actually used, conceptual areas surveyed, type of questions asked, mathematical nature of questions, difficulty level for the questions, and so forth. Environmental backgrounds might be conceived to range along a comfort distribution for temperature, light, humidity conditions; to range across seasons or times of year; background sounds (music); interruptions while test is in progress; or other variations of the setting in which the test is administered. Styles of responses may range across possible quality states; intensities or strengths of convictions; degree of insightfulness or complexity; trial-and-error or logical answers; grammatical correctness; scholarly presentation format; and other styles. Observer conditions might enlarge upon the observer's educational background, his experiences in science or in teaching, his ability as a test administrator and/or teacher and/or counsellor, his past performance as a recorder of data, his emotional or psychological state, et cetera.

The designed experiment that is truly capitalizing on relational possibilities would provide for sampling of all variants and/or states that the entities had assumed in the past or were likely to assume in the future. The researcher would, therefore, need to have an intimate knowledge of his problematic situation from which he could synthesize a viable set of theoretical relationships. Some of the relationships should be predictions amenable to observations leading to their verification in terms of empirical truth or to their rejection due to empirical falsity.

Each of the ten classificatory <u>sets</u> that have been noted may be considered as a <u>dimension</u> for describing an empirical event. Within each dimension are particular <u>cases</u>, <u>elements</u>, <u>ids</u>, or <u>entities</u> which may be assigned nominal numeric values. An assignment of numbers uniquely to individuals observed, another independent numerical assignment to states of individuals, still another independent assignment to stimuli, and so on through all dimensions would generate a ten-dimensional coördinate space representing an empirical component of the complete investigation, say the CBA component in the example cited. Note that although the cells are not quantitative they can nevertheless be used to designate empirical events through specifying the id from each set in a vector descriptive of the cell in which the vector terminates. The cell that is the event is at the intersection of one id from each dimension. (A cell may be specified by its unique combination of coordinates from all dimensions of the events matrix.) A cell also may be called a node, or a pattern.

In practice, data would be collected for each cell of the events matrix. The events matrix gives direction for data collecting as the researcher will attempt to provide a datum to each cell rather than to haphazardly collect data duplicating many cells and leaving others empty. The data collected should either represent measures (modes) from instruments that reflect theoretical considerations associated with the problem or the data should be relational. Such instruments may be commercially available or may be constructed specifically for the investigation at hand, the latter course being pursued when existing instruments are too remotely connected with the theoretical considerations to be useful to the investigation. Also, instruments may be (subjectively) sets of non-written criteria giving direction to the making of assessments or judgements needed but this is not recommended practice. Relational data initially collected may be dichotomous or reflect "moreness," "equality," or "ordinality," or be scaled to equal intervals. Both relational and instrumental data differentiate ids such that the ids can be ordered in some way.

Data thus obtained would be entered into appropriate cells of the events matrix. This first data, as collected, is called raw data which in an events matrix may be referred to as a raw data relations matrix. "Relations" appears as a qualifier since the data matrix either reflects relations or is beneficial for relational analyses. Each datum is also called an entry.

Data from instruments will usually be scalar (not vector) values as outcomes of measurements scaled to some continua specific to the instrument used. They may reflect a difference relation between ids in a set or between some initial and final measure representing, perhaps, before- and after-instruction. The data may be analyzable with parametric or non-parametric approaches as appropriate.

Entries into a data matrix may also be transformations of the raw data. While there seems to be no limit to the number of transformations possible, each one used should represent something more than a "fishing expedition." Often transformations have a basis in theory, perhaps as simple a basis as that greater regularity exists in some transformed state than is reflected in the raw state. A common transformation generates standardized scores whose mean is zero and whose standard deviation is one.

Entries may be classified as attribute data or as relational data. Attribute data may be thought of as measurements that are adjectival or predicate in nature; that is, that describe. An entry may reflect a description of ids in any coordinate set of the events matrix. Attribute data entries in education could be scores (raw data) representing response correctness, response speed, personal fear, personal motivation, stimulus step-size, stimulus intensity, environmental stress, environmental affluence, observer age, observer bias and many other theoretically qualified attributes from each of the ten dimensions of the events matrix. Relational data entries, on the other hand, generally reflect qualities between at least two coordinate sets of the events matrix. The "galvanic skin response," for an example, might represent a relation among a person, his stimulus, the environment, the observer, et cetera. The relation entered might be a frequency of encounter between persons and stimuli, or it could be the difference in stimuli when changing from one environment to another. Relations may also be entered as a result of operations performed between data from two or more events matrices, perhaps subtraction of one conforming matrix from another. (There are an unlimited number of singular mathematical operations that could be used in producing relations.) Also, relations may be generated from complex functions of two or more operations on one to several variables. In addition, relations may be derived relations from initial attribute data particularly the very important derived relations known as covariance, correlation, partial correlation, or regression, discriminant function, and "is distributed as chi square (students t, F, etc.)," to specify but a few.

The range of entity dimensions included in an events matrix will rarely exhaust the possibilities inherent in the ten dimensions that could be used. Thus, one decision that should be made is which group of the ten sets need to be specified. This decision relates to theoretical considerations and to their particular data requirements. Unless theory holds, for example, that condition of observer is a factor in explaining one's problem, its solution, or phenomena related to both problem and solution, then there is little reason to collect observer condition information. In the final analysis, however, the researcher should include in his matrix all dimensions that will account for significant variations in the data collected (experimentation being the control of variance such that effects of various treatments may be assessed), and that are supported with theory.

A basic premise of this argument for relational analysis is that total variance could be partitioned as variance due to persons, stimuli, environmental backgrounds, responses, observers, the variants or states of each, and one-way through nine-way interactions of these sources. In addition there is variance due to--

1. the attribute or relational measures used (the distributions of entries),
2. mismatching of ideal entities with observed entities (in imperfect epistemic correlations),
3. distribution of ids in their sets,
4. sampling error from one experiment to another, and
5. experimental error by the amount of variance associated with the uncontrolled, unstated off-sets.

Omitting some of the ids from sets may change the original distributions in the sets as well as affect the discrimination properties of the ids remaining. (Suppose one does an analysis of variance on some attribute between classes called "high achievers" and "low achievers." If there is little difference between the two classes because of sampling bias or other reasons, then the classifications may not be discriminating.)

Because of the foregoing considerations, ids composing each set should be representative with respect to the reported general conclusions associated with similar problematic situations and representative with respect to all theoretical considerations associated with the theory on which the study is based. (One should bear in mind that each set of ids composing a coordinate axis of an events matrix may have its own unique collection of sampling assumptions. He should also note that variance is typically different for different coordinate sets.)

THE DECISION PROCESS OF THEORY BUILDING

Before reaching the stage of deductively formulated theory, the researcher probably will have established insufficiency of traditional theories for guiding his investigation to the discovery of evidence needed to solve his problem. A new or modified set of hypotheses (a new theory) will thereby be indicated as being essential for satisfactorily completing the study.

Having a problematic situation which calls for a new or modified theory the researcher needs to decide if natural history facts alone are capable of suggesting a fruitful theory or if unobservable entities and relations need to be proposed. In the latter case one proposes (hypothesizes, assumes, or otherwise designates) what exists with the most unambiguous designations possible from which theorems or consequences are deduced with formal logic. Among the consequences which should follow if the theory holds, one searches for theorems (theoretical conclusions) which define experiments that can be performed. Entities and relations hypothesized are provisionally confirmed if all experiments generate the results (epistemic correlations) predicted by the theorems.

Figure 5 diagrams an hypothesized decision process accompanying theory building. In practice, however, a researcher, particularly the student researcher, closes the investigation with the first testing of his theory whatever the outcome may be. In these cases the cycle in the lower right corner of the diagram would not hold.

SYNTHESIZING THEORY FOR A PROBLEMATIC SITUATION

Is the problem resolvable with facts that have been determined, described, and classified in the natural history stage?

No

Create concepts by perception, concepts by imagination, and/or concepts by intellection.

Proceed to fruitful and relevant hypothesis (theory) structure as hypothetically inferred, deductively-formulated theory of postulates and theorems.

Yes

Formalize logical concepts by intuition.

Proceed to fruitful and relevant hypotheses (theory) structure as abstractive, deductively-formulated theory.

Apply (test) the theory. Does it hold?

Yes

Problem is provisionally solved.

No

Modify the theory structure of the instrument used for testing.

Test the modified version.

Figure 5. A diagram relating decision processes accompanying theory building.

III. FUNCTIONS OF THEORETICAL SYSTEMS

The primary functions of theory are to predict in an organized, logically consistent, explanatory fashion what kinds of data are needed to solve a problem and to suggest whether the data obtained are reasonable, that is, whether any wires were crossed in data collection or processing. In order to accomplish these functions theories are often reduced to mathematical or diagrammatic symbolism, or are reflected in a mechanical model of some kind. The theoretician may explain by analogy, with photographs, sketches, equations, geometric figures or other means. Any such procedure may be called a theoretical "model."

A danger in modeling is sometimes associated with over application of imported meanings from the model being imposed on the theory--meanings that contain a surplus of interpretations some of which do not have status in the new theoretical system itself. This danger seems to be associated with the remoteness of the model from the theory it serves and with the complexity of the model.

Whether related to a model or not, the theoretical rationale (framework) for a problem should result from the researcher's focus on real or introspective phenomena, on the conceptual literature, and on logical interactions between phenomena and literature as they are studied or reflected upon. Theoretical hypotheses should be advanced from background assumptions and "primitives." These hypotheses should coherently interconnect in a theoretical network that joins the problem, with its variable bases, to some possible solution(s). All activities associated with this theoretical development require reasoning, an ability which the scientist ordinarily, perhaps unconsciously, accepts as a generally reliable tool of research when conformable to related rules of logic.

Following are some considerations concerning the postulate of the "reliability of reasoning" as reasoning is usually applied to educational research.

Gephart and Ingle claim that research results are "worthwhile only to the extent of the strength of the logical argument that demonstrated their probable truth or falsities."[1] Unless the logical argument is valid and unless the argument fits into some larger theoretical frame of reference, there probably is no advance in educational knowledge by having done research even though this research may have generated much data.

Precision in the development of logical arguments is also important since language at its best (particularly English) is usually defective with respect to the possible meanings that may be associated with particular words, concepts or sentences. In English, different "tokens" often express the same proposition since it is possible to express the same idea with a number of equivalent sentence forms and combinations of words

---

[1] William J. Gephart and Robert B. Ingle, eds., Educational Research: Selected Readings, Merrill's International Series in Education (Columbus, Ohio: Charles E. Merrill Publishing Co., 1969), p. 115.

from background assumptions and "primitives." These hypotheses should coherently interconnect in a theoretical network that joins the problem, with its variable bases, to some possible solution(s). All activities associated with this theoretical development require reasoning, an ability which the scientist ordinarily, perhaps unconsciously, accepts as a generally reliable tool of research when conformable to related rules of logic.

Following are some considerations concerning the postulate of the "reliability of reasoning" as reasoning is usually applied to educational research.

Gephart and Ingle claim that research results are "worthwhile only to the extent of the strength of the logical argument that demonstrated their probable truth or falsities."[1] Unless the logical argument is valid and unless the argument fits into some larger theoretical frame of reference, there probably is no advance in educational knowledge by having done research even though this research may have generated much data.

Precision in the development of logical arguments is also important since language at its best (particularly English) is usually defective with respect to the possible meanings that may be associated with particular words, concepts or sentences. In English, different "tokens" often express the same proposition since it is possible to express the same idea with a number of equivalent sentence forms and combinations of words

[1]William J. Gephart and Robert B. Ingle, eds., Educational Research: Selected Readings, Merrill's International Series in Education (Columbus, Ohio: Charles E. Merrill Publishing Co., 1969), p. 115.

within each sentence form. Also, the English language may be considered defective in the sense that different sentence types express the same proposition, for a proposition may be developed in a paragraph of simple sentences only or that proposition may be resolved with a single highly complex sentence. A third possible defect in English appears to be in its use of the same tokens for expressing different propositions; that is, the same word may have a great number of interpretations depending upon the context in which it is used or the way the word is pronounced. For examples, consider "to," "too," and "two" in written English or "receipt" in spoken English.

In the development of sound documents associated with educational theory it seems essential that the yardstick of deductive validity be employed. If an argument, which is an ordered finite set of sentences composing propositional premises which lead to a conclusion is deductively valid, then it is impossible that the premises are true and at the same time that the conclusion is false. Any weakness of this generalization is probably in the word "impossible." Impossibility here does not refer to physical impossibility but to logical impossibility in which a contradiction ensues if the speaker or writer has said or written a series of statements that speak against or argue against themselves. In a contradictory argument there would be statements such that one is the explicit denial of another or that one implies the denial or the negation of another.

There is a connection between the validity of an argument and the validity of its form. Thus, it is possible to judge an argument by reducing it to sentence tokens separated from each other by logical connectives and sets of parentheses thereby developing an expanded well-formed formula, WFF, for each argument--a formula capable of being resolved through mechanical application of logical rules. In this case a logical event would by any finite sequence or ordered set of letters representing sentences connected with the logical operators "and," "not both-and," "or," "if-then," or "if and only if." Also, sentence letters may be prefixed with "not" which implies the denial of the sentence represented by the letter. The puntuation marks of logical formula development are the opening parenthesis and the closing parenthesis.

Logical connectives or operators and prefixes may be referred to with a variation of names each having varying symbolic representations. Throughout this discussion--

1. the denial is represented by a dash before a sentence letter or well-formed formula,

2. the conjunction is represented with a caret, $\wedge$, between adjacent well-formed formulas or sentence letters in a definite sequence,

3. the disjunction is represented in logical shorthand by an upside down caret, $\vee$, connecting sentence letters or well-formed formulas,

4. the conditional is a result of writing an arrow pointing to the right between sentence letters or well-formed formulas, and

5. the biconditional is connotated with a double headed arrow between sentence letters or well-formed formulas.

The negajunction is a denied conjunction.

Associativity fails for the conditional and for the biconditional since formulas generated by performing logical operations across conditionals or biconditionals are not the same as will be obtained with the same operations after first reducing the conditional or biconditional to a truth value before removing parentheses.

If the educational researcher knows the truth value of each sentence token (proposition) associated with an argument, then he may mechanically apply rules to decide upon the validity of the total argument. The rules of assignment for mechanical decision making follow:

1. A denial is true if and only if the result of deleting the denial is false.

2. A conjunction is true if and only if both conjuncts are true.

3. A disjunction is true if and only if at least one disjunct is true.

4. A conditional is true if and only if either the antecedent is false or the consequence is true or both obtain.

5. A biconditional is true if and only if both components have the same truth value.

Criteria given above for mechanical assignment of the truth-value of "true" to a conditional assumes the conditional to be what Ennis refers to as the "material conditional."[1] It is for material conditionals that the assignment rule holds since, in these cases,

> "If p then q" is interpreted as having semantical equivalency with the negajunction "Not both p and not q."

Statements are semantically equivalent if the statements have the same truth tables.

Assumptions supportive of arguments used in the theoretical rationale for a problem should be made explicit for critical review of

---

[1]Ennis, Logic in Teaching, pp. 60-62.

others interested in the investigation. From clear statements of assumptions the theoretical framework for the complete investigation should have evolved obviously by a comprehensive, logical chain of reasoning.

Hypotheses should be used if there are logical bases for predictions, otherwise questions should be proposed (leading to exploratory or survey studies), the exploration of which may lead to expansion into a multivariate context amenable to theoretical analyses by construction of an hypothesis system.

Practical arguments are more than guesses in that they have postulates or they imply consequences that can be observed and known as either confirmed or not-confirmed pure fact. Speculative guesses in an argument may not be convertible to fact nor may hypotheses that are stated in vague, ambiguous, obscure language or language with other communication defects.

Theories could arise or be modified as inadequacies in existing theory are determined within investigations for which there are unexplainable (with present theory) facts or relationships. These new theories should evolve largely from the Inductive-Hypothetico-Deductive activities of the scientist as he observes regularities in the literature or in his preparatory empirical observations of problematic situations. From these theories hypothetical constructs could develop within a system from a set of chosen postulates by relationship with analogous situations in remote fields or as a product of deduction.

Also, theories may grow, as did Einstein's $E = mc^2$, as creative generalizations to a higher order from empirical constructs maturing almost simultaneously in several subdisciplines. Whatever their origins, new theories should be adjusted to account for the broadest possible base of empirical facts and relationships and to extrapolate to new situations for which evidence has not been collected. By extending a theory to several new situations, one may quickly find the point at which the theory becomes inadequate raising new theoretical problems for recursive cycles of knowledge development.

Tentative verification of a new or existing educational theory is usually accomplished by statistically testing the conclusions (hypotheses) reached through reasoning within that theory. Relationships or propositional statements stated from the hypotheses, not facts or variables as such, are tested statistically. Some relationships or propositions, for testing might be $=$, $\neq$, $>$, $<$, "is described by a normal distribution," "is correlated with," "will be," and so forth.

Sometimes the researcher may have an array of known facts at his disposal but finds that they are inadequate for arriving at a solution to his problem. However, these same facts, when theoretically synthesized with some other considerations which may exist as speculations only (concepts-by-postulation) may advance plausible solutions which may be testable. If these solutions are verified, then the speculative elements of the underlying theory are supported but not in an unqualified sense.

Qualifications are needed when verifications for an argument are based on finding that its conclusion holds for there may be other theoretical structures leading to that conclusion. When more than one argument supports a conclusion, the simpliest one, with fewest assumptions, will usually be the one provisionally advocated.

It was just noted that there may be more than one argument supporting a particular conclusion (solution) for a problem. It also happens that several arguments, leading to several solutions, may follow from analysis of the facts collected from a problematic situation in the natural history stage of observations and literature reviewing. Each of the possibilities that the researcher uncovers should be given a due place in the investigation and each should be impartially tested. Perhaps several explanations for a problem, when viewed in collective perspective, will be more illuminating than any one in isolation in that each may be more discriminative of particular facets. Moreover, intellectual and scholarly honesty would seem to dictate a need for the researcher to give attention to all explained solutions. To the extent he leaves some of them open, thus unknown, he leaves his study open to negative criticism. If he cannot close his investigation on all possible consequences of his problematic situation because of time, money or other constraints, he may be well advised to redefine in an effort to effect a singularly qualified conclusion (warranted by a single set of premises) to the portion of the overall problem that does fit into his constraints.

Unification of generalizations from the field of education should be a goal of theoreticians. This can be done by creating theories having more and more comprehensiveness--theories that account for increasing numbers of observable facts. Each such fact should be obtainable with purposeful probing stimulated by objective theoretical considerations. If facts are at variance with predictions from theory, then the theory should be retailored to fit the facts.

In addition to functions of theory that have been previously discussed, one could add the two following functions: (1) its classificatory function and (2) its function as a guide for future developments that are designed to produce theory where knowledge is known to be deficient. An example of a theory which serves educators through its classificatory function is the "Taxonomy of Educational Objectives" synthesized by Bloom and others. An example of the guiding function in an area needing further attention might be "step-size" in programmed learning; that is, specifying differences between "large" and "small" steps.

IV. FORMAL LOGICAL METHODS OF THE THEORETICAL SCIENTIST

METHOD OF DEFINITION

The method of definition makes its first impact on theory development as _defined concepts_ are derived from the theory's _primitive concepts_. One also uses the method of definition in stating new concepts in terms of existing defined concepts.

METHOD OF HYPOTHESES

Formal logical methods of the theoretical scientist are required to establish links between the problematic situation that initiated inquiry and consequences for empirical examination. Each confirmation of a consequence would provisionally support one or more components of the theoretical system giving rise to that consequence. Perhaps the problem is such that the complete theory behind it consists of observable entities or relations only, interconnected in a structure whose consequences may be satisfactorily confirmed only by application of logical methods. In this case each premise may be given a truth value (true or false) and if all are true and if valid reasoning has been used, then the conclusion (consequence) of the argument would be affirmed from the basic consideration of logic that in valid arguments it is logically impossible for a conclusion to be false and each of its premises to be true. Although the foregoing holds, it is worthwhile to verify the conclusion with direct observation when observation is possible.

If unobservable entities and relations are needed to link a problematic situation to one or more tentative, provisional solutions, then the <u>method of hypothesis</u> will be required for the construction of a hypothetically inferred, deductively formulated system. In this case experience, creativity, intuition, familiarity with conceptual structures in the area of the problem, a propensity for good fortune, and perhaps other supportive characteristics of the theoretician will be applied to the coordinative adjustment of the present state of knowledge with new, unique, hypothetical entities and relations that may permit synthesis of a theoretical system sufficient to suggest potential solutions to the problem and to thereby suggest some possible consequences for empirical testing.

In this procedure, logical considerations applied to one's hypothesized entities and relations are believed adequate to permit deduction of consequences (theorems, conclusions, solutions) for arguments based on his hypotheses. (Hypothesized entities and relations are termed <u>concepts-by-postulation</u>.) Some consequences should link with the observable world, that is, should have meaning in intervening variables for which raw data (the aesthetic component for epistemic correlations) may be collected.

Conclusions from these observations relating to the hypothetically inferred, deductively formulated, theoretical structures producing their associated observational variables, however, should be advanced with caution. Qualifications are needed that reflect the investigator's awareness that--

1. there may be more than one theory accounting for the observations,

2. there may be some observational variables more closely connected to the hypothesis (theory) structure than the ones used (these were missed as fertile epistemic correlates), and

3. assignment of a truth value of true to an antecedent in a conditional because of affirmation of its consequent is logically questionable.

The student researcher engaged in synthesizing a hypothetically inferred, deductively formulated theory should use the method of hypothesis coordinatively with several activities each of which may have a reciprocative influence upon all of the others. Those activities are--

1. to formalize theoretical requirements from the aesthetic continuum by specifying logical-concepts-by-induction;

2. to hypothesize what is proposed to exist with basic assumptions and concepts-by-perception, concepts-by-imagination, concepts-by-intellection, or a combination of these;

3. to apply formal logic to deduce theorems or consequences (conclusions, solutions);

4. to search the consequences for definitions (implications) of experiments (epistemic correlations) that can be performed;

5. to perform the experiment(s) both logically and empirically, (logic allows the researcher to hypothesize predictions for collected data, such hypothesized predictions being conclusions for arguments);

6. to confirm (tentatively) the hypotheses if all experimental instances produce results called for by the theory; and

7. to alter the hypotheses if an experimental result is negative.

METHOD OF EPISTEMIC CORRELATIONS

The scientific method used to relate components of hypothetically inferred, deductively formulated theory to data available from the aesthetic continuum may be referred to as the method of epistemic correlations.

According to Northrop, any kind of theory whatever, even metaphysical and religious theories, can be tested with respect to its truth or falsity.[1] In every case, testing can result only from relating concepts-by-imagination and/or concepts-by-intellection to concepts-by-perception or concepts-by-intuition, the latter two types of concept being verifiable with directly inspectable, empirical data. The relation between concepts is not the relation of identity but epistemic correlation. It is the relation which reflects two ways of knowing--(1) knowing from logical methods which lead from postulates to theories (speculations or predictions) concerning what relations should be observable in the apprehendable world, and (2) knowing from empirical data what relations are in fact observed in the apprehendable world. If relations observed are what should have been observed, then there is an epistemic correlation giving indirect evidence for provisionally or tentatively accepting the truth of the theory that was epistemically correlated with experience. Should relations which were observed be different from those which should be observed in terms of predictions from theory then the researcher considers the theory to be false without qualification. In this latter case, there

---

[1]Northrop, Logic of the Sciences and the Humanities, p. 114.

must be some other theory that does account for the observations. In the former case of positive epistemic correlation, however, there may be some other theory which could account for what was observed. Thus a theory having concepts-by-intellection or -imagination is either provisionally accepted pending development of new knowledge of unconditionally rejected from evidence at hand.

It should be noted that abstractive, deductively formulated theory built only upon logical-concepts-by-intuition has theoretical concepts-by-postualtion for which there exist an identity relation with concepts used for the apprehendable continuum. Theorems of such theories may be logically verified after empiricism confirms the postulated entities and relations that enter into all of their theoretical arguments. Logical-concepts-by-intuition epistemically correlate with the undifferentiated aesthetic continuum more readily than any other theoretic component. Of all theoretic components only the logical-concepts-by-intuition have functional status in both universes or knowledge. They denote aesthetic qualities and have status in deductively formulated theory; thus, they are simultaneously sensual and abstract and their manifestations in both continua are known and verifiable, therefore, epistemically correlatable.

Epistemic correlations are associated with scientific objectivity. Concepts-by-intuition have ineffable meanings which probably vary from person to person; there is no assurance, for example, that observation of the same patch of blue sky has the same effect on each observer. The theoretically known, postulationally designated sky, on the other hand, is invariant, by comparison, from observer to observer. Theoretically designated entities and relations are the scientifically more objective ones although each verification through an epistemic correlation requires observations which, being functions of observer sensations and/or manipulations, are subjective. The color temperature meter used as an index of sky color may be held at varying angles to the vertical or may be held for varying degrees of parallax, or the observers' eyes themselves may have defects which subjectively affect the readings obtained.

Epistemic correlations relate two ways of demonstrating truth in knowledge. Logical methods usually serve as one way for establishing truth for some theory reflecting a way of knowing. The second approach is with empirical methods which show what conditions in the apprehendable environment are those that were predicted by theoretical methods. If there is a form of knowing in which there are nonobservable objects or relations, then epistemic correlations also are necessary to tie those unobservables to the apprehendable continuum. It is the <u>task of the deductive scientist</u> to find directly inspectable data which epistemically correlate with his postulated objects and relations.

Some epistemic correlates for which deductive scientific procedures establish the apprehendable component for theoretic entities and relations are given in Table 1.

An important <u>relation</u> (epistemic correlation) predicted with deductively formulated theory and observable in empirical data may be called the "is of identity." Consider epistemic correlates for the "moon" as it is theoretically known and directly apprehendable. From theories concerning its formation, the moon can be predicted to <u>be</u> a

TABLE 1

EPISTEMIC CORRELATIONS
BY DEDUCTION

Aesthetic Component	Linking Concept	Theoretic Component
Concept-by-intuition	*Epistemic correlation*	Logical-concept-by-intuition
Concept-by-intuition Concept-by-perception	*Epistemic correlation*	Concept-by-imagination Concept-by-intellection
Ineffable, sensuous		Scientifically postulated
White, sweet	*Sugar*	Sucrose
White, salty	*Salt*	Sodium chloride
Illness, painful	*Appendicitis*	Appendicitis
Musical sounds	*Music*	Numeric relationships Spatial qualities Memory Perception
Conversational qualities Creativity	*Intelligence*	Reasoning Word fluency Memory
Two demensional colored part of a photograph	*Desk*	Three dimensional, 4 legs, drawers
Fuzzy, angular, whitish	*One's nose*	One portion of the surface of a three dimensional anatomical object
Yellow, circular disk in dark night sky	*Moon*	Three dimensional, spherical, orbiting body
A color	*Blue*	An electromagnetic wave of specified wavelength
Wilson cloud chamber experiment with directly inspectable flashes of light	*Electron*	An ionization caused by collision of negatively charged particle(s) with a molecule of gas

TABLE 1--Continued

Aesthetic Component	Linking Concept	Theoretic Component
Educational employees fixing responsibility for educational activities, content, functions, programs, etc.	*Accountability*	A relationship between educators and the community
The chicken pecks a red circular disk; students assume study postures when a bell sounds	*Operant-conditioning*	A method for fixing particular responses to specific stimuli
What one can directly inspect		Postulated, unobservable scientific objects and relations
Ten potentials in a physical field	*General theory of Relativity*	Einstein's tensor equation of ten variables for four basic entities
	4 terms for dimensions 3 terms for two-way interactions 2 terms for three-way interactions 1 term for four-way interaction	
Potentials	*10 terms*	Variables

spherical body which was confirmed by manned and unmanned orbiting satellites. Theories of perspective coupled with the theory that the moon _is_ spherical predict that the moon should be observed with the naked eye from earth as a circular disk. Observations confirm that the moon _is_ so observed. The relation, "is," epistemically correlates theoretical considerations (predictions) with the immediately apprehendable. The epistemic correlation has joined a concept-by-postulation--"moon" to its corresponding concept-by-inspection--"moon." The apprehendable "moon" probably _is_ the theoretical "moon." These concepts for epistemic correlation are schematically illustrated in Figure 6.

Observed relation	← Epistemic correlation →	Theoretically deduced relation (prediction)
.		.
.		.
.		.
Concept by inspection	← Epistemic correlation →	Concept by postulation

---

A concept; e.g. "blue," "intelligence"

There may be an operational theory for the concept with purely empirically-given properties or operationally-defined meanings	The concept in deductively formulated theory has denotatively-given properties or meanings prescribed by the method of definition

epistemic correlation of quite different meanings

An operation; e.g. "performance based education" "open education"

An immediately apprehended sequence of events	Denotatively-given, theoretically conceived properties

epistemic correlations of apprehendable with theoretical criteria

---

Figure 6. Illustrations of applications of the "epistemic correlation" concept.

Epistemic correlations are extensions of a theory built upon concepts-by-postulation. The apprehendable versions of concepts, operations, or relations may be addition to theory, though not a substitute for some or all of the designated concepts, operations, or relations of the theory. This addition of apprehendable, empirically based components functions both for provision of epistemic correlates and for clarification of pragmatic applications of the theory.

Notice that "relation" has three senses of meaning as presented in this document. There is the relation between two ways of knowing--the epistemic correlation, there is the relation observed in immediately apprehendable data, and there is the relation between entities which is known only from the theoretical context that denotes the relation. The latter two relations may be mathematical, logical, or descriptive.

Some mathematical relations are "are equal to," "are not equal to," "are greater than," "are less than," "are correlated with," "are completed by," "are linearly determined by," and "are normalized." Some logical relations are "are implied by," "are equivalent to," or "are negated by." Finally, some descriptive relations are "are distributed as," "are characterized by," "are associated with," "are controlled by," "are preceeded by," "are on a hierarchical level with," "are reducible to," "are classified as," "are partitioned into," "are confounded with," "are suppressed by," "are predicted by," "are influenced by," and "has expected value of."

Without theory there are no criteria for determining when data collected from the immediately apprehendable are valid for the problematic situation being investigated. Theory indicates possibilities in terms of dispersions for the data, expected values, et cetera. This gives the researcher a feeling for when wires are crossed (data being miscoded, computational procedures being erratic, and so forth) and provides an approach for diagnostically teasing out the difficulties.

Epistemic correlations are also useful to the inductive scientist as he reasons with natural history data. The inductive scientist tries to postulate entities and relations which theoretically account for his natural history data, that is, which are epistemic correlates of his directly apprehendable data; he tries to develop a logically uniform explanation that accounts for his sensory experiences. Usually, inductive scientific procedures lead to epistemic correlates only by trial-and-error; therefore, the researcher attempts to establish, by trial-and-error, the correct epistemic correlate from the theoretically possible ones. Northrop uses an example which might be diagrammatically represented as in Figure 7.

Directly inspected data	Theoretic explanations possible
Possible epistemic correlations	
Scenery for a play impinges upon the visual portion of the apprehendable continuum	Two dimensional images of books are painted on a backdrop
(Some books seem to be on a shelf. Are they?)	Three dimensional books are sitting on a shelf
A character in the play pulls out a book	
Epistemic correlation by trial-and-error	

Figure 7. An example of establishing an epistemic correlation by trial and error. After Northrop.

An early phase of development for deductively formulated theory is the giving of postulational meanings to all concepts-by-intuition that were used for classifications of natural history data. That is, all concepts for apprehendable data from the problematic situation should also have status as concepts-by-postulation such that each concept-by-postulation in epistemic correlation with a concept-by-intuition has a postulationally prescribed meaning given in terms of primitives by means of the method of definition. Moreover, the portion of deductively formulated theory that developed to account for natural history data may constitute the entire theory giving abstractive, deductively formulated theory; or the portion may be an addition to a theoretical structure having a more complete set of concepts-by-postulation some of which are not epistemically correlated with the immediately apprehendable. The more complete theory is called hypothetically inferred, deductively formulated theory.

If a theory is to have credibility, it should be epistemically correlatable with some elements of the immediately apprehendable. Such credibility can result only if each epistemic correlation has an operational definition component in which meanings from observations are interwoven with theoretical meanings. Not every theoretical concept, however, needs to have an operational translation. Educational researchers may, in fact, have been too much concerned with operationalism and too little concerned with theory. Northrop states it as follows:

> As the century has moved on, it has become evident in the field of pragmatic philosophy, realistic pragmatic law, education and the other social sciences that this [over] emphasis upon the practical, upon instrumentalism and upon operations has tended to solve very few problems and to introduce more and more rhetoric and less and less science into the subject matter.[1]

---

[1]Northrop, Logic of the Sciences and the Humanities, p. 125.

Care should be taken to insure that operations are defined by theory rather than being used to define theory. Operations denoted within the immediately apprehendable should be organically united with concepts-by-intuition and should be theoretically specified. Furthermore, the concepts-by-postulation, while denoted by deductively formulated theory, may be connotatively enlarged by that theory to have operational components specified by concepts-by-perception. Such theoretically-conceived, empirical operations would be verifications of scientific entities and relations postulated in the theory.

As has been noted, it is the relation of epistemic correlation that connects operational meanings to theoretical concepts. These operational meanings should be theoretically proposed within deductively formulated theory. Each such operational meaning may be considered as an operational theory for a concept-by-postulation. By this approach, the operation is defined by the theory, not the theory by the operation; however, operational theory in this sense has both an antecedent theoretical component and a consequent operational component thereby, epistemically correlating evidence from the antecedent theoretic continuum with evidence from the consequences which are apprehendable.

There may also be operational concepts that arise from observing, describing, and classifying data in the natural history stage of an investigation. These operational concepts would be denotatively created from specific observations; therefore, they would be concepts-by-intuitior, not concepts-by-postulation. Concepts-by-postulations, on the other hand, are always denotatively specified by their deductively formulated theory, not by antecedent operations.

The procedure for epistemically correlating immediately apprehendable data with the theoretic component of knowing is probably the most difficult transition for a beginning researcher in the coordination of his problematic situation with verifications of possible theoretical solutions. There is no formal deductive manipulation for deriving concepts-by-inspection from concepts-by-postulation. Theorems of deductive logic never contain more than is implicit in their premises; therefore, concepts-by-inspection that have potential in epistemic correlations should be formalized with status as primitives or as logical-concepts-by-intuition. The theoretician cannot deduce theorems referring to more observable factors than are contained in his primitives and concepts-by-postulation combined. Directly observable data should be classified, therefore, with concepts that are both concepts-by-inspection and concepts-by-postulation. This classification links the two distinct worlds of discourse in such a way that concepts in the theorems of the deduced theory can guide the search for empirical data that provisionally verify that which is implicit in its theoretical premises. The only concepts in deductively formulated theory are either primitive concepts of the postulates or concepts which are derived from the primitives by the method of definition.

METHODS FOR VERIFYING THEORIES

Abstractive, Deductively Formulated Theory. A reseacher may confirm empirically theories whose entities and relations are abstractions from the continuum of sense awareness such as is typical in the field of economics. Each postulate of these theories may have a truth value assigned to it by observation. The truth value of the theorems derived from these postulates may be assigned by logical considerations as each theorem will be the logical consequence of the empirically known postulates.

In any discipline similar theories whose arguments are based only on logical-concepts-by-intuition will be empirically confirmed through their postulates. In these cases concepts-by-intuition related to the aesthetic continuum are identical with concepts-by-postulation with status in the theoretic continuum. (The relation of identity holds between concepts-by-intuition and logical-concepts-by intuition.)

Hypothetically Inferred, Deductively Formulated Theory. Unlike abstractive, deductively formulated theory which is verified from empirical confirmation of its postulates, hypothetically inferred, deductively formulated theory is indirectly and provisionally verified by empirically verifying (epistemically correlating) its theorems which are propositions that are logical consequences of the postulates and, being propositions, have truth value. Theories having mathematical consequences typically are indirectly verified by testing those consequences, but only if the consequences are logically consistent with their supporting postulates.

Concepts-by-postulation in hypothetically inferred, deductively formulated theories are indirectly verified from empirical verification of theorems based on concepts-by-imagination and concepts-by-intellection. Concepts-by-perception link deductively formulated theory to the external world of common sense objects such as tables, chairs, one's persisting self, other people, and so forth. Confirmations of concepts-by-perception typically occur automatically (without conscious effort); thus, errors are made easily. Concepts-by-perception in educational research activities are usually unquestioned. In legal matters, on the other hand, there are cross-examination efforts to assure that errors of inference have not occurred in the linkage of postulations, to perceptions, to confirmations of witnesses. (These linkages, for most observers, are automatically conditioned reactions in patterns of habits extending to early childhood.)

Concepts-by-imagination and concepts-by-intellection, on the other hand, are not as readily confirmed. They do not enjoy intuitively self-evident linkages with the aesthetic continuum but are linked with increasing theoretical effort as their levels of abstraction increase or their degree of familiarity in the field being researched decreases. However, investigators may often become so familiar with some of their abstract concepts-by-postulation that they treat them as though they were concepts-by-perception, thereby creating considerable confusion for other researchers who are less familiar with both the conceptually designated denotations and the directly unobservable, postulationally designated scientific entities and relations that are connotated by them.

If there is to be publicly valid, objective, plausible, though qualified, verification of hypothetically inferred, deductively formulated theory, then its concepts-by-imagination and concepts-by-intellection need to account for phenomena connotated by the basic scientific concepts known as logical-concepts-by-intuition and concepts-by perception both of which may be derived by the method of definition. Concepts-by-perception defined from postulated scientific entities and relations thereby give rise to common-sense objects such as clocks, Geiger counters, tests, galvanic skin response instruments, computers, and so forth. It should be emphasized that empirically verifiable concepts-by-perception are <u>defined in terms of the more basic</u> scientific concepts-by-imagination and concepts-by-intellection.

Thus, the method for empirically testing deductively formulated theories seems to involve five major manipulative activities:

1. Specification of primitive concepts and postulates.
2. Definition of the defined concepts-by-postulation.
3. Deduction of theorems from the concepts-by-postulation utilizing the logical methods of formal implication.
4. Union of concepts-by-postulation in deduced theorems to concepts-by-inspection with the method of epistemic correlations.
5. Verification of entities and relations specified in the epistemic correlations by methods of observations yielding empirical, apprehendable data.

This process of developing deductively formulated theory and verifying it might be diagrammatically represented as in Figure 8.

Each researcher should be aware of the logical problem raised with acceptance of a theory on the strength of empirically confirmed logical consequences (theorems). Unqualifiedly accepting theories on this basis produces a logical error known as "the fallacy of the hypothetical syllogism," or "the fallacy of affirming the consequent."

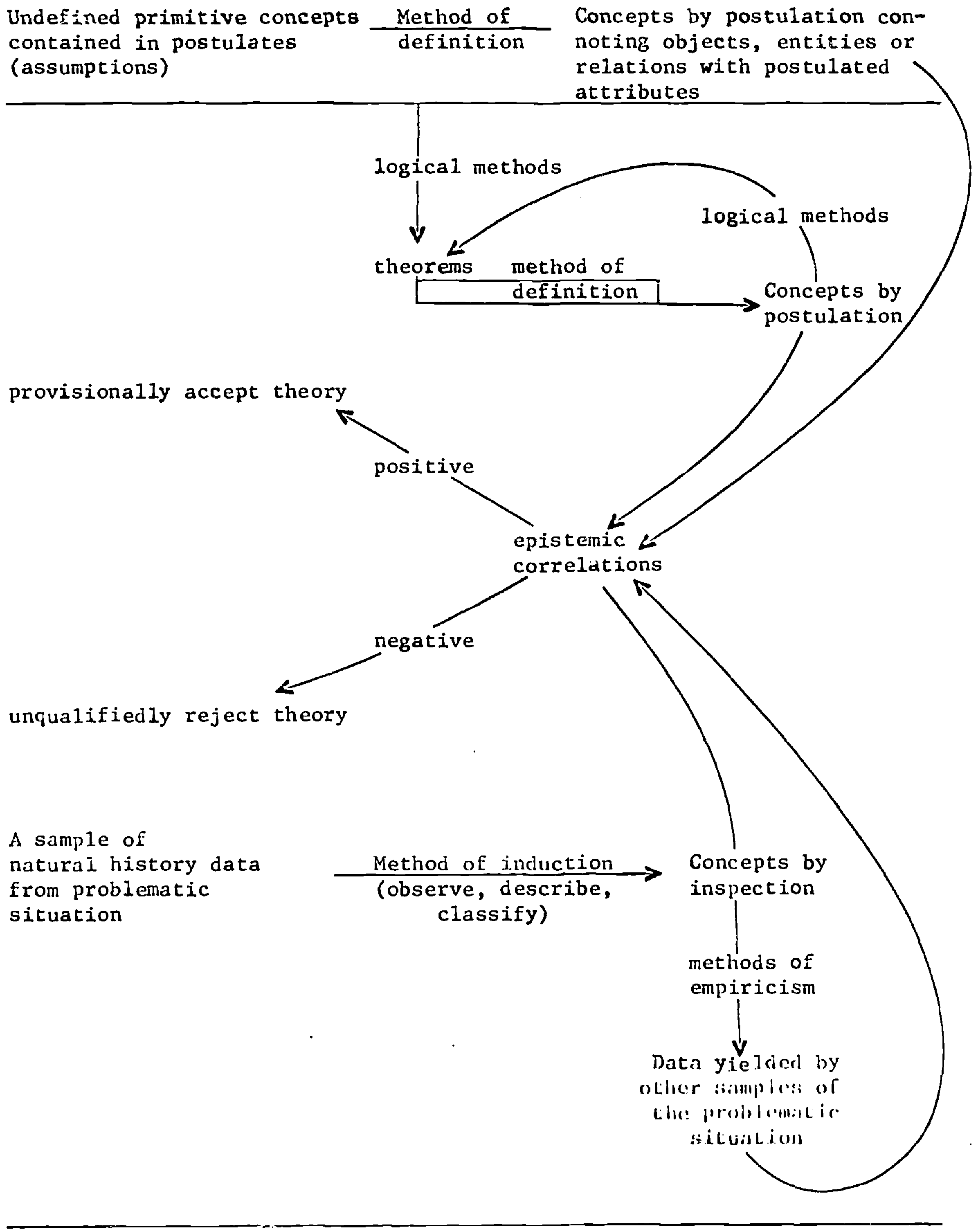


Figure 8. A schematic diagram illustrating development and verification of deductively formulated theory.

Suppose one derives a set of theorems (X, Y, Z) from a set of postulates (A, B, C). Confirmation of X, Y, and Z by empirical methods merely provides a possibility for the truth of A, B, and C. A, B, and C could be guaranteed true by confirmation of X, Y, and Z only if they are the only postulates from which X, Y, and Z could be deduced.

When a theory is not confirmed, the researcher should reexamine all components of his investigation particularly the deductively formulated theory under which the data were collected. The question to ask is whether the relation tested is inaccurate or the entities it unites are incorrectly specified or nonexistent. Answers to this question can be advanced only from intimate knowledge of the problem, the disciplines associated with it, and the full range of conditions associated with the problematic situation.

Theoretical investigation should develop every tenable hypothesis (theory) structure that rationally explains the observable phenomena of a problematic situation. Each such theory should be impartially pursued in a manner assuring thoroughness. This elevates each tenable theory to a status of potential acceptance. From such an approach either a more comprehensive theory may arise coordinating two or more explanations or the problem itself may be fractured into components leading to the development of new, more highly discriminative theories.

Theoretical investigations should be further concerned with establishing the uniqueness of a particular theory for accounting for a particular set of empirically confirmed consequences. The objective is either to demonstrate weakness in other theories that might lead to those conclusions, or to show that no other theoretical approach to those conclusions is possible. This requires an investigation of every theoretical possibility.

> The introduction of crucial experiments in order to mitigate the danger of the fallacy of affirming the consequent in the mere experimental confirmation of a scientific theory without the consideration of other theoretical possibilities, means that there can be no trustworthy science, even with experimental confirmation, in the mature stage of development of an empirical science unless as much attention is given by scientists to the consideration of theory and of rival theoretical possibilities and to deductive logic as is given to induction, factual data and experimentation. The point is that the mere experimental confirmation of a scientific theory through its deductive consequences is not generally regarded by competent scientists as a sufficient criterion of the scientific validity of that theory. One must go further and show as far as is possible that the theory in question is the only one which is capable, through its deductive consequences, of taking care of the natural history data.
>
> . . . .
>
> Only by formulating rival theories, learning to think deductively with respect to them by pressing them to their deductive consequences--in short, only by emphasis upon theory and deductive logic as well as upon facts and experimentation can the uniqueness of an experimentally confirmed scientific theory be established.[1]

The process for establishing the validity of a theory to account for or explain apprehendable data is one of converging evidence. The theory is increasingly validated as predictions from it are more and more verified. It is also increasingly validated if there is an absence of rival theories accounting for the predicted, observable data.

METHOD OF PROOF (SENTENCE REASONING)

The most capable researchers seem to use deductive reasoning with theoretical propositions in their field or in related fields. With

---

[1]Northrop, Logic of the Sciences and the Humanities, pp. 149-50.

deductive reasoning initial theoretical propositions develop into problem hypotheses (solutions) that are linked to the immediately apprehendable in epistemic correlations. Data are collected from the immediately apprehendable component of the investigation and the data are analyzed by appropriate statistical techniques, the results of which lead the researcher to provisionally accept or reject his statistical hypotheses. Unless the first moves are properly performed the later moves leading to statistical hypotheses testing may be of little value. Research results are "worthwhile only to the extent of the strength of the logical argument that demonstrated their probable truth or falsity."[1]

As reasoning from initial premises through establishment of epistemic correlations is deductive, it may be verified with deductive methods, particularly the method of proof. One may employ this method if his goal is to show that an argument is deductively valid; that is, that the argument leads to true conclusions if the premises are true. There is no showing by this method, however, that the premises used are the <u>only</u> premises generating the particular true conclusion obtained for the argument whose validity is being tested.

The method is called the "method of proof" as one "proves" arguments valid. This should be distinguished from showing statements to be <u>true</u> and from performing a statistical <u>test</u>. An argument is <u>proven</u>

---

[1]William J. Gephart and Robert B. Ingle, eds., <u>Educational Research: Selected Readings</u>, Merrill's International Series in Education (Columbus, Ohio: Charles E. Merrill Publishing Co., 19^9), p. 115.

when it is shown that its conclusion <u>follows</u> necessarily from its premises. To "follow necessarily" is to deny that a case can be found for which all premises are true and the conclusion is false. For an argument that has been proven to be deductively valid, it is a contradiction (logical impossibility) that the premises are true and the conclusion false.

Deduction is used both in the development of arguments and in their *a posteriori* proofs. When deduction is used to directly prove a conclusion, steps are--

1. the premises are either assumed true or shown to be true,
2. the argument is shown to be valid (that the conclusion follows necessarily); therefore,
3. the justification is established that the conclusion must be true.

Deduction may be used to indirectly prove at least one premise is false by--

1. showing the argument to have a conclusion that follows necessarily from its premises (proving valid),
2. showing the conclusion to be empirically false (testing); therefore,
3. concluding at least one premise is false.

It is also possible to use deduction to establish that an argument is technically invalid. One would first show that its premises are true then he would demonstrate its proposed conclusion to be empirically false. He would finally conclude that the argument must be technically invalid for a technically valid argument cannot have true premises and a false conclusion. The researcher, in this case, should reexamine his argument as initially posed to determine if the conclusion drawn is a contradiction of the validly drawn conclusion.

The same criteria for showing arguments to be valid establish conditions which ordinarily generate true conclusions from true premises. In the discussion which follows logical sentences are symbolically represented with alphabetic characters: premises are above a horizontal line and conclusions below. A double line separates arguments. Valid conclusions may serve as premises in more complex arguments.

PROVING ARGUMENTS HAVING A CONDITIONAL

An argument based on a conditional premise can be proven (shown to be valid) in one of three ways. If a second premise in the argument either affirms the antecedent or denies the consequent, then the conclusion that results is valid if the conclusion is a statement affirming the consequent in the first case or denying the antecedent in the second case. Both affirmation of the antecedent and denial of the consequent in a subsequent premise may be used in a proof though both are not required. Symbolically, <u>valid arguments</u> having a conditional as premise may be represented as follows:

If A, then C	conditional	(premise)
A	affirmation of antecedent	(premise)
C	affirmation of consequent	(conclusion)

If A, then C	conditional	(premise)
not C	denial of consequent	(premise)
not A	denial of antecedent	(conclusion)

Note that the premises in the above arguments might be inverted without affecting the conclusions drawn or the validity of the arguments as a whole.

Invalid arguments containing sentences connected with the conditional may be symbolically represented as follows:

If A, then C	conditional	(premise)
not A	denial of antecedent	(premise)
not C	denial of consequent	(invalid conclusion)

If A, then C	conditional	(premise)
C	affirmation of consequent	(premise)
A	affirmation of antecedent	(invalid conclusion)

In the first case for the invalid arguments there may be another set of premises which would establish the consequent so to claim that its denial is certain on the basis of denial of its antecedent would be invalid as this claim does not follow necessarily from the evidence at hand.

The second case also represents a conclusion that does not follow necessarily from the premises given. Only if no other set of premises accounted for the consequent, could its affirmation result in affirmation of its antecedent.

In practical research, however, this second type of invalid argument is commonly used. The consequent of the conditional being empirically verified in practical investigations gives credibility to the argument containing that affirmed consequent pending the discovery of other arguments leading to that consequent. The practical researcher should, therefore, examine all possible avenues to the consequent in attempting to show that the antecedent conditions in the theory containing the particular conditional empirically tested are the only antecedent conditions that are tenable. Even then these antecedent conditions are only provisionally or tentatively accepted.

Two adjustments to conditionals often appear in arguments: the converse and the contrapositive. The converse of a conditional is not necessarily true, but its contrapositive does follow. A converse is the interchange of antecedent with consequent statements; a contrapositive is the interchange of the denial of antecedent with the denial of consequent. A valid argument concluding the contrapositive of a conditional is

If A, then C	conditional	(premise)
If not C, then not A	contrapositive of conditional	(conclusion)

An invalid argument concluding the converse of a conditional is

If A, then C	conditional	(premise)
If C, then A	converse of conditional	(invalid conclusion)

The logical connective known as the conditional has several equivalent practical forms of usage. The following statements are representative of the conditional:

1. If A, then C
2. If not C, then not A
3. C, if A
4. Only if C, A
5. C, provided that A
6. C, unless not A

7. A is adequate (sufficient) for C
8. C is essential (necessary) for A

The standard form of the conditional used in most texts is "If A, then C."

Frequently, conditionals are linked to one another in a series of premises. If two premise conditionals are such that the antecedent of one conditional is in common with (the same as) the consequent of the second conditional, then the argument can be formed in terms of a conditional chain. The conclusion for a conditional chain is a conditional containing the non-common antecedent and consequent of its premise conditionals, respectively, as concluding antecedent and consequent.

Invalidly chained arguments (1) conclude a conditional from premise conditionals whose common components are both antecedents or are both consequents, or (2) conclude a conditional from a pair of conditional premises having no components in common.

PROVING ARGUMENTS HAVING
A BICONDITIONAL

The biconditional may also serve as a premise in a logical argument. The only form of it that will be employed here will be ". . . if, and only if, . . ." Arguments based on the biconditional can be proven (found <u>valid</u>) in one of the following ways:

A if, and only if, C	biconditional	(premise)
C if, and only if, A	converse of biconditional	(conclusion)
If C, then A	converse of conditional	(conclusion)
If A, then C	conditional	(conclusion)

A if, and only if, C	biconditional	(premise)
Not C if, and only if, not A	contrapositive of biconditional	(conclusion)

A if, and only if, C	biconditional	(premise)
A	affirmation of component	(premise)
C	affirmation of other component	(conclusion)

A if, and only if, C	biconditional	(premise)
Not A	denial of either component	(premise)
Not C	denial of other component	(conclusion)

The following are invalid arguments based on the biconditional:

A if, and only if, C	biconditional	(premise)
A	affirmation of either component	(premise)

Not C	denial of other component	(invalid conclusion)

A if, and only if, C	biconditional	(premise)
Not A	denial of either component	(premise)
C	affirmation of other component	(invalid conclusion)

PROVING ARGUMENTS HAVING
A CONJUNCTION

When the conjunction serves as a premise in a logical argument, two sentences called "conjuncts" are connected with a conjunction, either "and" or translatable to "and." The following represent valid arguments based on a conjunctional premise:

P and R	conjunction	(premise)
P	affirmation of one conjunct	(premise)
R	affirmation of second conjunct	(conclusion)

P and R	conjunction	(premise)
Not P	denial of one conjunct	(premise)
Not R	denial of second conjunct	(conclusion)

P	affirmation of one statement	(premise)
R	affirmation of second statement	(premise)
P and R	conjunction	(conclusion)

P and R	conjunction	(premise)
R and P	same conjunction with conjuncts reversed	(conclusion)

The following are <u>invalid</u> arguments based on the conjunction:

P and R	conjunction	(premise)
Not P	denial of one conjunct	(premise)
R	affirmation of second conjunct	(invalid conclusion)

P and R	conjunction	(premise)
P	affirmation of one conjunct	(premise)
Not R	denial of second conjunct	(invalid conclusion)

P	affirmation of one statement	(premise)
R	affirmation of second statement	(premise)
Not (P and R)	denial of conjunction	(invalid conclusion)

PROVING ARGUMENTS HAVING A NEGAJUNCTION

A negajunction, denial of a conjunction, may serve as a premise in an argument. In such cases valid arguments assume one of the following forms:

Not (P and R)	negajunction	(premise)
P	affirmation of one negajunct	(premise)
Not R	denial of second negajunct	(conclusion)

Not (P and R)	negajunction	(premise)
Not (R and P)	negajunction with premise negajuncts reversed	(conclusion)

The following are invalid arguments based on the negajunction:

Not (P and R)	negajunction	(premise)
Not P	denial of one negajunct	(premise)
R	affirmation of second negajunct	(invalid conclusion)

Not (P and R)	negajunction	(premise)
P	affirmation of one negajunct	(premise)
R	affirmation of second negajunct	(invalid conclusion)

The material conditional is an "If A, then C" relationship in which the if-then relationship is called material implication. The material conditional is equivalent to the negajunction if "If A, then C" is understood to mean the same as "not both (A and not C)." Both the material conditional and the given negajunction are made true by the falsity of A or the truth of C. Mechanical decision procedures are permitted by this definition--procedures such as those associated with logical decision (truth) trees for determining assignments of truth values to well-formed formulas and their derivatives.

PROVING ARGUMENTS HAVING A DISJUNCTION

Disjunctions (alternations) are statements in which two sentences are connected with or. The component sentences are called disjuncts (alternants). There are three types of alternation: strong-, weak-, and inclusive-alternation. Weak-alternation employs "or" in such a way that at least one alternant being true implies nothing about the truth value of the second alternant. Strong-alternation uses "or" in such a way that affirmation of one alternant requires denial of the other and denial of one alternant requires affirmation of the other--this is the "either P or R is true but not both are true" relationship. Inclusive-alternation occurs in conditionals as antecedents employing "or" in such a way that at least one of the alternants is true if the consequent is true. Weak-, strong-, and inclusive-alternation also differ in that both weak and strong imply a connection between alternants whereas no connection is implied with inclusive.

Some valid arguments utilizing an alternation as premise are:

P or R	Weak-alternation: (If not P, then R) or (If not R, then P) or (P and R)	(premise)
Not P	denial of either alternant	(premise)
R	Affirmation of other alternant	(conclusion)

P or R	Strong-alternation: (Not R if, and only if, P) and (Not P if, and only if, R) and [not (P and R)]	(premise)
P	affirmation of one alternant	(premise)
Not R	Denial of other alternant	(conclusion)

P or R	Strong-alternation: (Not R if, and only if, P) and (Not P if, and only if, R) and [not (P and R)]	(premise)
Not P	denial of either alternant	(premise)
R	Affirmation of other strong-alternant	(conclusion)

P	Affirmation of a sentence	(premise)
P or R	Weak-alternation or inclusive-alternation	(conclusion)

Not R	Denial of a sentence	(premise)
P	affirmation of a sentence	(premise)

P or R	Strong-alternation: (Not R if, and only if, P) and (Not P if, and only if, R) and [not (P and R)]	(conclusion)

Not R	denial of a condition	(premise)
P	affirmation of a condition	(premise)
P or R	Weak- or inclusive-alternation in the sense that (If not R, then P) or (If not P, then R) or (P and R)	(conclusion)

P or R	Inclusive-alternation	(premise)
not P	denial of an alternant	(premise)
R	affirmation of other alternant	(conclusion)

Some invalid arguments using an alternation as premise are:

P or R	Weak-alternation in the sense (If not P, then R) or (If not R, then P) or (P and R)	(premise)
P	affirmation of either weak alternant	(premise)
not R	denial of other weak alternant	(invalid conclusion)

P or R	Strong-alternation in the sense (Not R if, and only if, P) and (Not P if, and only if, R) and [not (P and R)]	(premise)

not P	denial of either alternant	(premise)
not R	denial of other alternant	(invalid conclusion)

P or R	inclusive-alternation	(premise)
P	affirmation of either alternant	(premise)
R	affirmation of other alternant	(invalid conclusion)

P or R	inclusive-alternation	(premise)
not P	denial of either alternant	(premise)
not R	denial of other alternant	(invalid conclusion)

P or R	inclusive-alternation	(premise)
P	affirmation of either alternant	(premise)
not R	denial of other alternant	(invalid conclusion)

METHOD OF PROOF (INDIRECT PROOF)

Indirect proof may be used as a strategy for determining the validity of arguments. In these cases the denial of a proposed conclusion is assumed to be true. If from this assumption there is even one assignment function for the premises that does not produce a contradiction, then the argument being "proven" is shown invalid. The denial of a validly drawn conclusion from a valid deductive argument should contradict the actual assertion of the premises, and contradictions must be denied.

One approach to indirect proof utilizes truth trees in which the denial of the conclusion and the assertions of the premises serve as initial considerations. This model requires shifting of the argument into idealized, but reliable, trunk and branching pattern symbols (sentence letters) according to established rules for truth conditions for each type of logical connective and for the denials of statements having and not having logical connectives. Shifting into idealized patterns requires elimination of such implicit and explicit qualifiers as "probably," "likely," "generally," "for the most part," "under normal conditions,' "other things being equal," "by and large," "roughly speaking," "by this principle," "approximately"; and so forth. Idealizing an argument also requires culling from context and representing as sentence letters the essence of the argument, omitting irrelevant details and statements; that is, isolating structural premises and conclusions only. To the resulting skeletal structure for the argument explicit rules for truth conditions can be applied, each rule yielding either a trunk fragment, a branch, or a trunking condition appended to each arm of a branch, all arms of each branch being entered simultaneously. Entries as a trunk imply that their parent statements require each trunk entry to hold for truth to hold in the parent statement; entries as branches imply that the parent statement will be true if either branch holds; and entries that are trunks appended to simultaneously entered branches imply that either trunk condition so entered being true can establish truth in the parent statement. Figure 9 represents an attempt to clarify these rules.

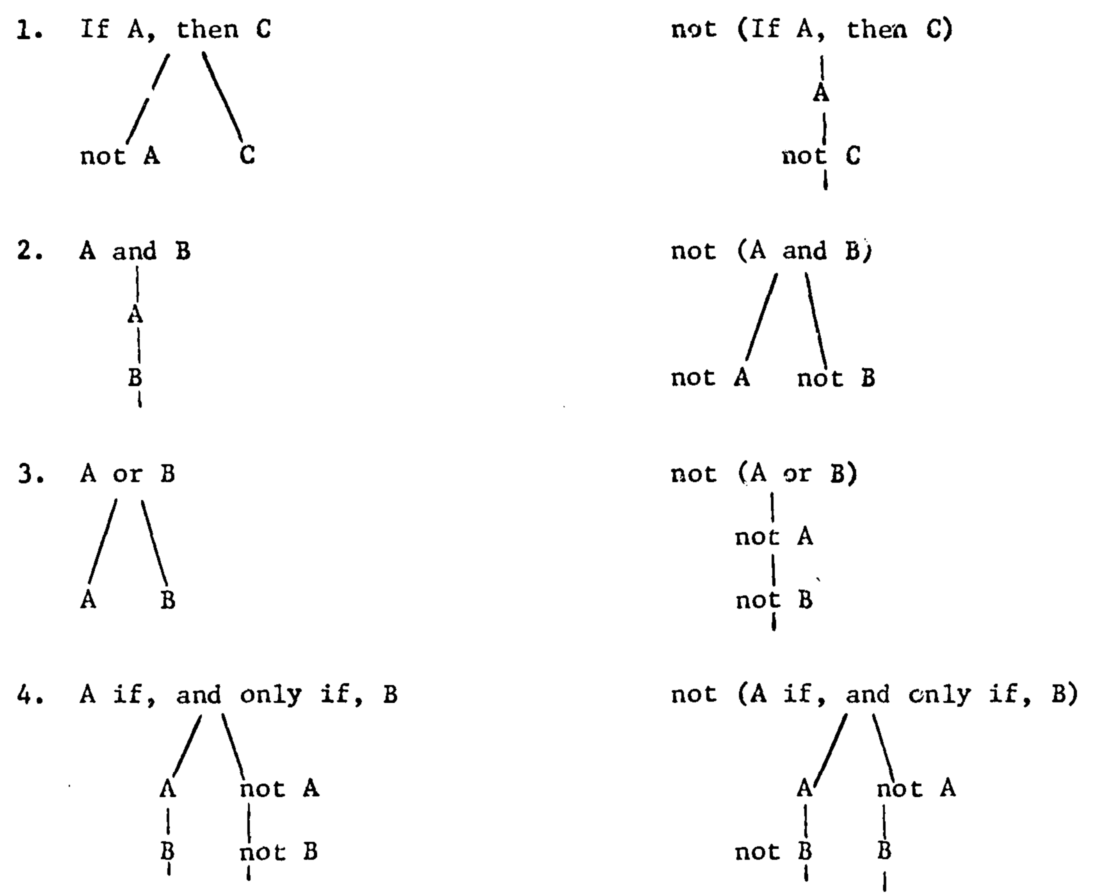


Figure 9. Rules for Valuation of Logical Connectives

A practical mechanical procedure for judging arguments may be outlined as follows:

1. Write premises, one under the other, as sentence letters.

2. Write denial of conclusion under the last premise, with sentence letters.

3. Apply relevant rule for valuation under every open branch. Use rules that involve no branching first.

4. Close each path that develops into a contradiction. In each contradiction there is specified a well formed formula followed by its denial at the terminus of the branch that should be closed to further activity.

5. Conclude the argument's validity:

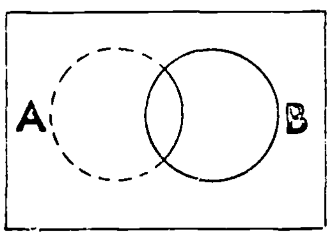
   A. If an open branch is left after applying every relevant rule for valuation, then the argument is invalid since there is a possible case in which the premises are true and the conclusion is false.

   B. If all branches are closed, then the argument is valid.

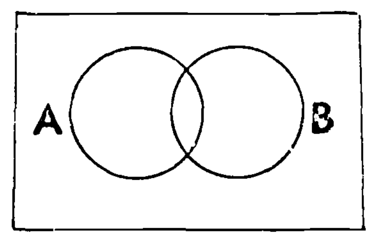
It should be noted that the previous procedure is completed only if either all branches are closed or all well formed formulas of the premises have been reduced by rules of valuation and rewritten according to the mechanical rules as branches or trunks composed of basic sentence letters or denied sentence letters.

METHOD OF PROOF (CLASS REASONING)

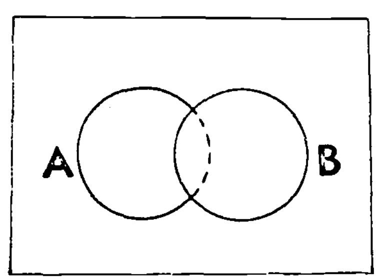
In class reasoning, sentences are transformed to fit the class relationship pattern of a subject class contained in a predicate class as reflected in linkages of subject to predicted through forms of the infinitive "to be." Relationships of inclusion or class membership may be demonstrated with Euler Circle Systems such than an encompassing boundary, circle or rectangle, representing the universe of discourse contains circles for the subject and predicate classes and crosses, "X"'s, for individuals. The universe of discourse is an encompassing class including members and non-members of all classes associated with an argument. If no members of a given class are members of a second class, then their circles are drawn without overlap. If it is known that at least some members and perhaps all members of one class are contained in a second class, circles are drawn to overlap with indeterminancy shown by breaks in the arc representing the possibly excluded members of the first class. If some, but not all, members of the subject class belong to the predicate class,the two overlapping circles are each solid. If at least some of the subject class, and perhaps all of it, are not in the predicate class, indeterminancy within the predicate class is shown by breaks in the arc of the subject class that is inside the predicate class. These concepts are illustrated in the diagrams below.



At least some, and perhaps all A's are B's.



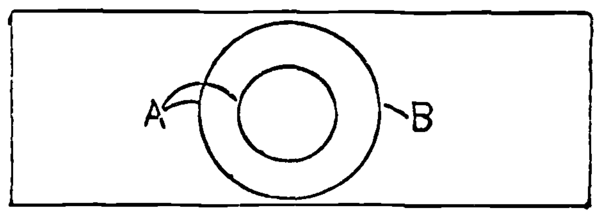
Some, but not all, A's are B's and Some, but not all, A's are not B's.



At least some, and perhaps all, A's are not B's.

All that are A's are not B's.

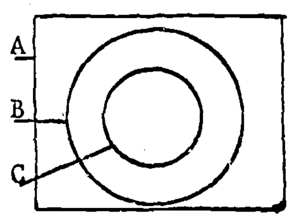
The extent of the predicate class may not be explicitly known from its description. If it is stated that "All A's are B's" there are two possibilities open: (1) there may be B's which are not A's and (2) there may be no B's which are not A's. One usually does not have the right to assume that the extent of the predicate class is larger than the subject class although this may be a common tendency. The Euler Circle diagram for uncertainty in extent of the predicate class is as follows:

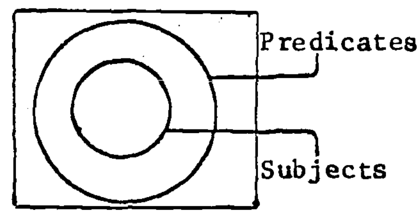


All A's are B's

Each premise in class reasoning relates, with respect to such concepts as class membership and class inclusion, a subject class to a predicate class. Premises are combined into complete arguments by equating common classes between the premises. (The subject class of one premise is equated to the predicate class of another, et cetera.) If, in working against the conclusion, the conclusion is inescapable in the representation of all premises combined in an Euler Circle System, then the argument is valid. To work against a conclusion is to try to find inconsistent possibilities left open by the premises--inconsistent possibilities with respect to the conclusion drawn.

Sentences may need to be transformed to fit the class relationship pattern of "'subject class' 'to be' 'predicate class.'" A predicate class may need to be created or made explicit, particularly the predicate class defining the universe of discourse. With the universe of discourse explicitly determined, transformations among negatives and positives may be effected. The following diagrams represent some possible transformations:





All C's are B's

All not B's are not C's

All B complements are C complements

All not A's are not B's

Subjects are predicates

Non-predicates are non subjects.

Each valid argument using class reasoning is of such content that the predicate class of one premise is unambiguously contained in the subject class of a second premise, the predicate class of the second premise being unambiguously contained in the subject class of the third, et cetera, leading inescapably to the conclusion which is a subject class (from one premise) inclusion or membership association with a predicate class (of another premise). If the argument is evaluated using Euler Circle Systems, then the evaluator tries not to diagram the proposed conclusion by drawing circles or crosses to include every possibility left open by the premises. If, in so doing, the drawing cannot be made to be inconsistent with the conclusion given, then the completed argument is valid.

Some invalid arguments derived from class reasoning include conclusions generated by interpreting from "all A's are B's" either that

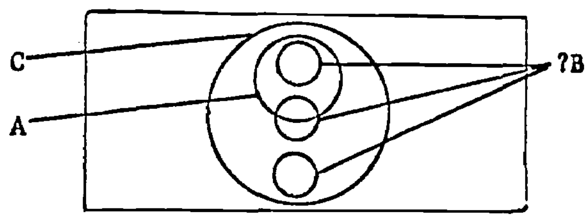
(1) all B's are A's or (2) at least some B's are not A's. Unambiguous arguments require explicit designation in premises of extents of subject classes (with respect to predicate classes) involved. Anytime an argument leads to a conclusion that omits some possibilities consistent with the premises, the argument may be judged invalid. Some common errors associated with invalid arguments are stated in the following list:

1. If two subject classes of two premises have the same predicate class, it is invalid to conclude that one subject class is wholly contained in the other subject class.

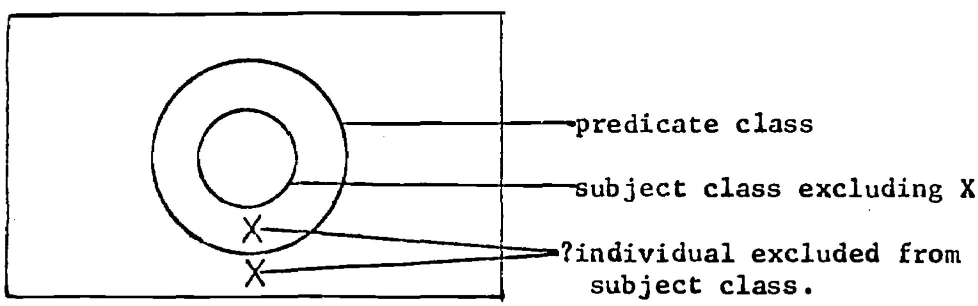
   All A's are C's

   <u>All B's are C's</u>

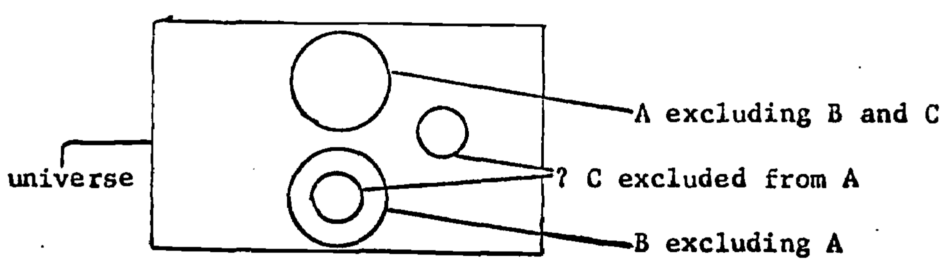
   All A's are B's



2. Given a sufficiently general universe of discourse containing a premise having a subject class within a predicate class, if an individual or object is known to be excluded from the subject class of the given premise, it is invalid to conclude either that the individual <u>is</u> or <u>is</u> <u>not</u> contained in the predicate class.

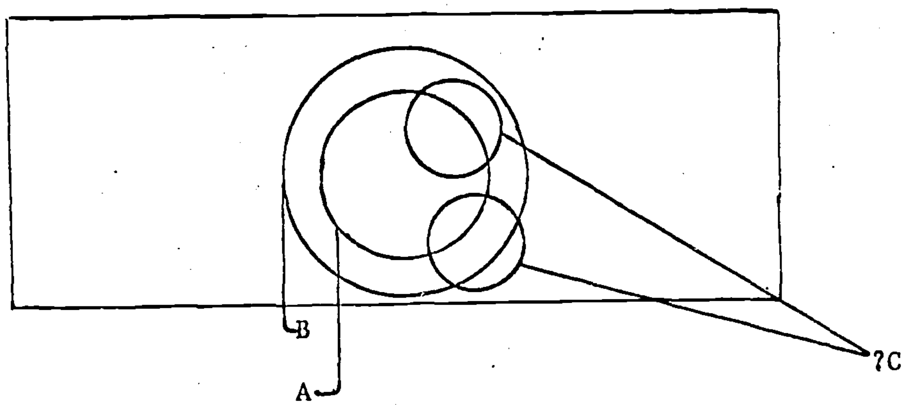


3. Given two mutually exclusive classes in a universe of discourse and given that a third class (individual or object) is excluded from one of the mutually exclusive classes. It is invalid to conclude that the third class is included in the second of the mutually exclusive classes.



4. Given an argument of the following form:

All A's are B's	(premise)
Some, but not all, C's are A's	(premise)
Some, but not all, C's are B's	(invalid conclusion)



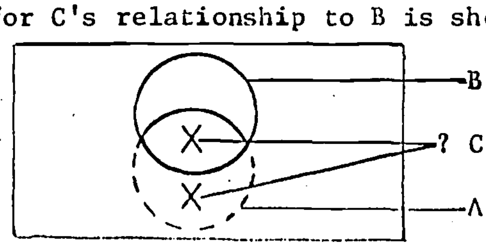
The premises provide a possibility that "some, but not all, C's are B's" but the premises also provide the possibility that "all the C's are B's." A valid conclusion would be "at least some of the C's are B's."

When it is not clear whether "some" should be interpreted "at least some" or "some, but not all" then both possibilities should be considered in judging arguments.

5. Given an argument of the following form:

Some, but not all A's are B's	(premise)
C is an A	(premise)
C is a B	(invalid conclusion)

Ambiguity for C's relationship to B is shown as follows:



Class and sentence reasoning techniques may be combined for proving arguments. Such proofs, while based on extensions of considerations given above for the two types of reasoning will not be developed in this analysis. Many standard texts in deductive reasoning include examples that clarify the techniques used.

V. SUMMARY

The theoretical framework for the researcher's investigation should evolve from his integration of the major references from his related literature with his creative analysis of facts observed. Postulates based on these references may serve as some of the premises for a series of deductions culminating in theoretical solutions (conclusions) for the problem.

Theories explaining solutions for problems are synthesized in conjunction with developments within each activity of the investigation. As the theory matures into an adequate explanatory system uniting a problem to a solution, it may reciprocatively influence other research activities. The schematic of Figure 10 places the hypothesis (theory) development stage in context with other investigative activities and their products. Note the reciprocative influencing effect associated with the various cyclic paths shown by directed lines between activities.

DISCRIMINATING CONCEPTS-BY-INTUITION FROM CONCEPTS-BY-POSTULATION

Concepts-by-intuition are products of natural history investigations while concepts-by-postulation are denotations from deductive theory formulations. Each type of concept has several attributes that overlay only in the case of logical-concepts-by-intuition which link apprehendable phenomena to postulates and theorems.

A comprehensive contrast of concepts-by-intuition with concepts-by-postulation is attempted in Table 2. It is a collection of statements

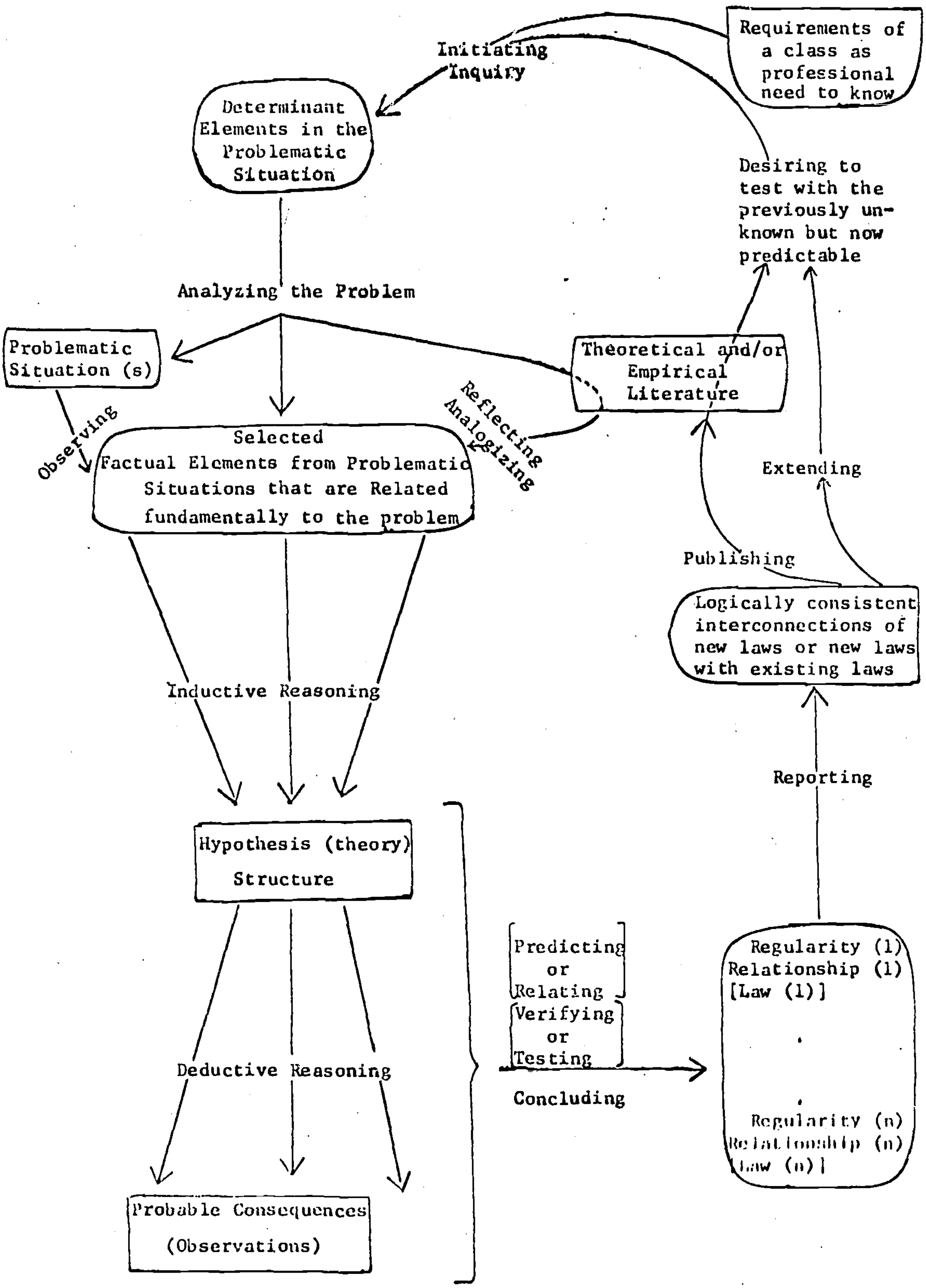


Figure 10. The hypothesis (theory) structure stage in schematic context with other investigative activities and their products. All operations are not enclosed. All focal elements or objectives are enclosed.

representing meanings that have been attached to the two types of concepts. The meanings are presented numerically under an arrangement for levels of content similarity in order to effect an ease of comparison.

TABLE 2

PROPERTIES OF SCIENTIFIC CONCEPTS
AND SOME COMPARISONS BETWEEN THEM

CONCEPTS-BY-INTUITION vs	CONCEPTS-BY-POSTULATION
1. Connotes facts which can be immediately apprehendable.	1. Connotes meanings which are designated within the context of a specific, deductively-formulated theory.
	2. Definitions require designation of methods for knowing.
	3. Postulates give meanings to entities and relations denoted by concepts by postulation.
4. Nothing whatever can be deduced from immediately apprehendable fact classified as concepts-by-intuition.	4. Meanings for concepts by postulation cannot be found by observing anything.
	5. A concept-by-postulation is a meaningless mark if considered apart from some specific deductively formulated theory.
6. Relations between different species of apprehendable facts are external and contingent upon inner and outer sensations.	6. Relations between concepts-by-postulation are internal and dependent upon context.
	7. Common sense objects and inferences induced by them denote concepts used to label those objects.
8. Concepts-by-intuition may become logical-concepts-by-intuition (a type of concept by postulation) by postulating a logical status for their intuitively given meaning.	8. A "common-sense," "concept-by-intuition," could be used to denote a concept-by-postulation.

TABLE 2--Continued

CONCEPTS-BY-INTUITION	vs	CONCEPTS-BY-POSTULATION
Concept by intuition A. intellect is passive B. world of sensation C. transitory D. temporal E. meaning is ineffable	RED	Concept by postulation A. intellect is active B. real world C. permanent D. immortal E. meaning is a function of theoretical context
Meaning is immediately apprehendable		A band of defined wavelengths in the spectrum of electromagnetic relations
	SPACE	
Immediately apprehendable		Absolute space of Newton's deductively formulated theories
	TIME	
Immediately apprehendable		Defined periods of oscillation for a standard atom; changes in position of hands on a clock or shadows cast by a sundial
9. Concepts by intuition are not--		9. Concepts by postulation may frequently be--
	a. speculative hunches b. wild guesses c. axioms assumed to be self-evident	
10. Since concepts by intuition always connote fragments of the immediately apprehendable abstracted from the totality of immediately apprehended fact, valid interpretations of natural history data will always require cognizance of--		10. Meanings for concepts by postulation, being dependent only on postulates of specific deductive theories, do not require consideration of--
	empirical context: a. social environment b. historical circumstances c. political structures d. economic frames of reference e. psychological or emotional sets of observer	

TABLE 2--Continued

CONCEPTS-BY-INTUITION vs	CONCEPTS-BY-POSTULATION
11. Concepts by intuition may be classified according to the ways they are known to the observer: A. Concepts by sensation (outer senses) B. Concepts by introspection (inner feelings)	11. Concepts by postulation may be classified according to their remoteness from the immediately apprehendable: A. Logical-concepts-by-induction B. Concepts-by-perception C. Concepts-by-imagination D. Concepts-by-intellection
	12. "...any concept in deductively formulated scientific theory, whether it appears in the postulates or in the theorems of that theory, is a concept by postulation." (Northrop, p. 204)
13. After apprehension comes the search for descriptions with concepts-by-intuition. Fact described by these concepts always exists in the undifferentiated aesthetic continuum. Full meanings for these apprehended facts are ineffable and probably unique for all observers. There is no public world.	13. After the concept by postulation exists comes the search for the directly inspectable empirical facts needed to verify it. The latter may not exist or they may be elusive if they do exist sometimes requiring new technology and sense extending instruments such as electron microscopes, galvanic skin response meters, et cetera for their detection and observation.

One approach to educational research is as follows. From observations of a problematic situation in education one may classify apprehendable data in terms of concepts-by-inspection. The educational researcher's task then is to find a deductively formulated theory having an axiomatic sub-structure exhibited in or leading to his natural history data. The empirical data will thereby tentatively verify his deductively formulated theory. A second approach to educational research begins with concepts, relations, and entities in the <u>postulates</u> of deductively formulated theory. The task of the researcher is to find relations and entities connoted with concepts-by-inspection from immediately apprehended problematic situations. Positive epistemic correlations in this case verify theorems logically implied by the postulated system prescribing them.

SOURCES CONSULTED

BOOKS

Dewey, John J. How We Think, Boston: D. C. Heath, 1933.

_______. Logic: The Theory of Inquiry. New York: Henry Holt and Company, 1938.

_______. Essays in Experimental Logic. New York: Dover Publications, 1953.

Englehart, Max D.; Furst, Edward J.; Hill, Walker H.; and Krathwohl, David. Taxomony of Educational Objectives: The Classification of Educational Goals. Handbook 1: Cognitive Domain. Edited by Benjamin S. Bloom. New York: David McKay Company, Inc., 1956.

Ennis, Robert H., Logic in Teaching. Englewood Cliffs, N. J.: Prentice-Hall, Inc., 1969.

Gephart, William J., and Ingle, Robert B., eds. Educational Research: Selected Readings. Merrill's International Series in Education. Columbus, Ohio: Charles E. Merrill Publishing Co., 1969.

Glaser, Edward. "An Experiment in the Development of Critical Thinking." Teachers College Contribution to Education, No. 843, 1941.

Helmstadter, G. C. Research Concepts in Human Behavior: Education, Psychology, Sociology. New York: Appleton-Century-Crofts, 1970.

Inhelder, B., and Piaget, J. The Growth of Logical Thinking from Childhood to Adolescence. New York: Basic Books, 1958.

Kerlinger, Fred N. Foundations of Behavior Research: Educational and Psychological Inquiry. New York: Holt, Rinehart and Winston, Inc., 1965.

Kochen, Manfred, ed. The Growth of Knowledge: Readings on Organization and Retrieval of Information. New York: John Wiley and Sons, Inc., 1967.

Northrop, F. S. C. The Logic of the Sciences and Humanities. Cleveland: The World Publishing Co., 1947.

Smith, B. Othanel, and Meux, Milton O. In collaboration with Jerrold Combs, Daniel Eierdam, and Ronald Szoke. A Study of the Logic of Teaching. Urbana: University of Chicago Press for the College of Education, 1970.

Van Dalen, Deobold B. and Meyer, William J. Understanding Educational Research: An Introduction. Rev. ed. McGraw Hill Series in Education. New York: McGraw Hill Book Co., 1966.

COMPONENT PART BY ONE AUTHOR IN WORK EDITED BY ANOTHER

Cattell, Raymond B. "Psychological Theory and Scientific Method." Handbook of Multivariate Experimental Psychology. Edited by Raymond B. Cattell. Chicago: Rand McNally & Co., 1966.

__________. "The Principles of Experimental Design and Analysis in Relation to Theory Building." Handbook of Multivariate Experimental Psychology. Edited by Raymond B. Cattell. Chicago: Rand McNally & Co., 1966.

__________. "The Data Box: Its Ordering of Total Resources in Terms of Possible Relational Systems." Handbook of Multivariate Experimental Psychology. Edited by Raymond B. Cattell. Chicago: Rand McNally & Co., 1966.

Chamberlin, T. C. "The Method of Multiple Working Hypotheses." Educational Research: Selected Readings. Edited by William J. Gephart and Robert B. Ingle. Merrill's International Series in Education. Columbus, Ohio: Charles E. Merrill Publishing Co., 1969.

Cornell, Francis G. "Jumping to Conclusions." Educational Research: Selected Readings. Edited by William J. Gephart and Robert B. Ingle. Merrill's International Series in Education. Columbus, Ohio: Charles E. Merrill Publishing Co., 1969.

Good, Carter V. "Criteria For Selection of the Research Problem." Educational Research: Selected Readings. Edited by William J. Gephart and Robert B. Ingle. Merrill International Series in Education. Columbus, Ohio: Charles E. Merrill Publishing Co., 1969.

Lamke, T. A. "A Primer in Research . . . Lesson 1: Defining the Problem." Educational Research: Selected Readings. Edited by William J. Gephart and Robert B. Ingle. Merrill International Series in Education. Columbus, Ohio: Charles E. Merrill Publishing Co., 1969.

Lindvall, C. M. "The Review of Related Research." Educational Research: Selected Readings. Edited by William J. Gephart and Robert B. Ingle. Merrill's International Series in Education. Columbus, Ohio: Charles E. Merrill Publishing Co., 1969.

Masson, Henry James. "An Outline of Certain Processes of Reasoning in Research," Educational Research: Selected Readings. Edited by William J. Gephart and Robert B. Ingle. Merrill's International Series in Education. Columbus, Ohio: Charles E. Merrill Publishing Co., 1969.

Rose, Arnold M. "Generalizations in the Social Sciences." Educational Research: Selected Readings. Edited by William J. Gephart and Robert B. Ingle. Merrill's International Series in Education. Columbus, Ohio: Charles E. Merrill Publishing Co., 1969.

Van Dalen, D. B. "The Role of Hypotheses in Educational Research." Educational Research: Selected Readings. Edited by William J. Gephart and Robert B. Ingle. Merrill International Series in Education. Columbus, Ohio: Charles E. Merrill Publishing Co., 1969.