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ABSTRACT

This is the second of a two quin series which introduces the student to all the theorems usually included in high school geometry; emphasis is on understanding and use of these theorems without proof. The course develops definitions and properties of the plane and solid figures and formulates methods for finding their linear measure, lateral and total area measure, and volume measures. New material and enrichment activities include the following topics: application of metric measure, right triangle trigonometry, coordinate geometry, tanagrams, tesselations, flexagons, projections, polyhedral models, topology, non-Euclidean geometry, and architectural design applications. Overall course goals are specified; a course outline, performance objectives, and suggested teaching strategies are listed. (JP)

AUTHORIZED COURSE OF INSTRUCTION FOR THE QUINMESTER PROGRAM

Mathematics

Geometry 3
5228.32

DADE COUNTY PUBLIC SCHOOLS

DIVISION OF INSTRUCTION•1971

Dr. E. L. Whigham, Superintendent of Schools
Dade County Public Schools
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QUINMESTER MATHEMATICS

COURSE OF STUDY

FOR

GEOMETRY 3

5228.32

(EXPERIMENTAL)

Written by

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for the

DIVISION OF INSTRUCTION
Dade County Public Schools
Miami, Florida 33132
1971-72

PREFACE

The following course of study has been designed to set a superior standard for student performance after exposure to the material described, and to specify additional enrichment sources which can be the basis for the planning of daily activities by the teacher. There has been no attempt to prescribe teaching strategies; those strategies listed are merely suggestions which have proved successful at some time for some class.

The course sequence is suggested for a guide; an individual teacher should feel free to rearrange the sequence whenever other alternatives seem more desirable. Since the course content represents a maximum nine week unit, a teacher should feel free to omit some of the enrichment topics.

Any comments and/or suggestions which will help to improve the existing curriculum will be appreciated. Please direct your remarks to the Consultant for Mathematics.

All courses of study have been edited by a subcommittee of the Mathematics Advisory Committee.

CATALOGUE DESCRIPTION

The second of a two quin sequence which introduces the student to all of the theorems usually included in high school geometry ; emphasis is on understanding and use of these theorems without proof. Develops definitions and properties of plane and solid figures. Formulates methods for finding their linear measures, lateral and total area measures, and volume measures.

Develops an extensive view of the Pythagorean Theorem and special right triangle relationships.

This quin includes detailed development of the following new material and enrichment activities not covered in 5218.22: Application of Metric Measure, Right Triangle Trigonometry, Coordinate Geometry, Tanagrams, Tesselations, Flexagons, Projections, Polyhedral Models, Topology, Non-Euclidean Geometry, Suggested Challenge Problem, and Architectural Design Applications.

TABLE OF CONTENTS

OVERALL GOALS

The student will:

1. Review linear measure and extend his measurement knowledge to include area and volume.

2. Improve his ability to visualize, sketch, and construct models of two and three-dimensional figures.

3. Build an intuitive understanding of the properties of plane and solid figures, including triangles, quadrilaterals, other polygons, circles, prisms, pyramids, cylinders, cones, spheres, and compound figures.

4. Develop and apply the many formulas related to the forementioned plane and solid figures.

5. Correlate the appropriate scientific principles with their mathematical applications in this quin. (Cavalierri's principle, Archimedes' principle etc.)

6. Enrich his appreciation of modern geometric construction through the comparative study of methods used throughout history.

7. Strengthen his preparation for future science courses through stress on the measurement formulas introduced in this quin; in particular, the special right triangle and volume formulas.

8. Improve his ability to reason informally deductively and inductively and add to his foundation for future development of formal proof.

9. Increase his speed and accuracy in computation.

10. Extend his use of mathematical and scientific symbols, notations, vocabulary, and formulas necessary to the material introduced in this quin.

11. Add to his special reading techniques for mathematics and science.

12. Be equally proficient in the use of English and Metric systems of measurement.

13. Become aware of some of the topological properties of geometry and their applications, such as the Moebius Strip.

14. Extend his visual awareness by being able to describe the orthogonal projections of both plane and solid figures.

OVERALL GOALS (cont.)

15. Be able to select and apply the fundamental right triangle ratios to the solution of appropriate problems.

16. Become aware of the existence of systems of Geometry other than Euclid's, and the main basis for their differences.

17. Review fundamental vocabulary skills and principles used in graphing in the coordinate plane, and apply these to the developing of the geometric properties of triangles and quadrilaterals.

KEY TO STATE ADOPTED REFERENCES

M- Moise, Edwin E. and Downs, Floyd L., Jr. Geometry. Reading, Massachusetts: Addision-Wesley Publishing Co., Inc., 1967.

L- Lewis, Harry. Geometry, A Contemporary Course. Princeton, New Jersey: D. Van Nostrand Company, Inc., 1968.

JD- Jurgensen, Donnelly, Dolciani. Modern School Mathematics Geometry. Boston: Houghton Mifflin Company, 1966.

A- Anderson, Garon, Gremillion. School Mathematics Geometry. Boston: Houghton Mifflin Company, 1966.

I. TRIANGLES

A. KINDS AND RELATED PARTS

PERFORMANCE OBJECTIVES

The student will:

1. Define and illustrate items selected from the vocabulary list, labeling them correctly on appropriate diagrams.

2. Develop and apply items selected from the list of associated properties given under Course Content.

3. Compute the measure of the third angle of a triangle, given the measures of two angles.

4. Find the measure of an exterior angle of a triangle, given appropriate information.

5. Find the measure of a remote interior angle of a triangle given appropriate information.

6. Construct a triangle with a straight edge and compass given:

   a. SSS
   b. ASA
   c. SAS
   d. SAA
   e. HL of rt △
   f. HA of rt △

STATE ADOPTED REFERENCES

	M	L	JD	A
CHAPTER	4, 5, 7, 9	5, 10, 15	5, 6	5

Vocabulary

polygon
triangle
  side
  angle
  vertex
  adjacent sides
  scalene
  isosceles
  equilateral

equiangluar
exterior angle
included angles
exterior
interior
right
obtuse
acute
legs

hypotenuse
vertex angle
remote interior angle
base
base angles
included angles

I. TRIANGLES

A. KINDS AND RELATED PARTS (continued)

Associated Properties

A triangle is a three-sided polygon.

If two sides of a triangle are congruent, the angles opposite those two sides are congruent; and, conversely, if two angles of a triangle are congruent, the sides opposite those angles are congruent.

Every equilateral triangle is equiangular and vice versa.

For every triangle, the sum of the measures of the angles is 180.

The acute angles of a right triangle are complimentary.

If two pairs of angles of two triangles are congruent, then the third pair of angles i congruent.

For any triangle, the measure of an exterior angle is the sum of the measures of its two remote interior angles.

Constructions

Construct a triangle with a straight edge and compass given:

a. SSS
b. ASA
c. SAS
d. SAA
e. HL of rt $\triangle$
f. HA of rt $\triangle$

SUGGESTED LEARNING ACTIVITIES

1. Cut out and fold paper models of triangles to demonstrate measure relationships of the angles of a triangle.

2. Repeat the above constructions using protractor and ruler and compare the constructions done with compass and straightedge.

I. TRIANGLES

B. CONGRUENT TRIANGLES

PERFORMANCE OBJECTIVES

The student will:

1. Define and illustrate items selected from the vocabulary list.

2. Label corresponding parts of given marked figures.

3. Name congruent triangles using notation which shows corresponding vertices of a given marked figure.

4. State the postulate which justifies the congruence of given pairs of triangles.

5. Identify all corresponding parts of given congruent triangles, where the correspondence is shown:

   a. pictorially
   b. by letters without a figure

6. With only a compass and straightedge copy a given triangle using each of the following methods:

a.	SSS	d.	SAA
b.	SAS	e.	HL of rt $\triangle$
c.	ASA	f.	HA of rt $\triangle$

STATE ADOPTED REFERENCES

	M	L	JD	A
CHAPTER	4, 5	5	6	6, 7

COURSE CONTENT

Vocabulary

one-to-one correspondence	rotate
corresponding sides	flip
corresponding angles	slide
congruent sides	rigid
congruent angles	identity congruence
congruent triangles	self congruence

I. TRIANGLES

B. CONGRUENT TRIANGLES (continued)

Associated Properties

Two polygons are congruent if, for some pairing of their vertices, each side and each angle of one polygon is congruent to the corresponding part of the other polygon.

The correspondence ABC$\leftrightarrow$ABC is called the identity congruence.

If AB $\cong$ AC, then the correspondence ABC$\leftrightarrow$ACB is called a self congruence, which is not an identity congruence.

Congruence postulates and theorems: SSS, SAS, ASA, SAA, HL, HA

Constructions

Copy a triangle if given: SSS, ASA, SAS, SAA, HL of rt Δ, HA of rt Δ

SUGGESTED LEARNING ACTIVITIES

1. Cut and fold paper models to demonstrate correct correspondence in figures. (Especially helpful to illustrate self-congruence in polygons with 2 or more congruent sides.)
2. Use filmstrip #A542-4 on Congruence by S. V. E.
3. Use filmstrip #1122 Congruent Triangles by F. O. M. and #1157 Using Congruence in Construction by F. O. M. (both are listed in annotated bibliography in back of quin.)
4. Use selected items from Overhead Visuals for Geometry by Jurgensen, Donnelly, Dolciani, Volumes III & V.
5. Overhead Transparency Masters for Geometry - Moise & Downs #9.

I. TRIANGLES
C. INEQUALITIES

PERFORMANCE OBJECTIVES

The student will:

1. Use the order properties (transitivity, addition, multiplication by a positive number) in the solution of simple inequalities.

2. Find the missing angle measures of a triangle if given measures of an exterior angle and one remote interior angle.

3. Use exterior angle properties of a triangle and transitivity to establish relative sizes of angles in a given figure.

4. Determine the largest angle of a triangle if given measures of the sides.

5. Determine the longest side of a triangle if given the measures of the angles.

6. Determine the largest angle of a figure positioned into triangles if given appropriate information.

7. Determine the longest side of a figure partitioned into triangles if given appropriate information.

8. Solve numerical problems involving the definition of distance from a point to a line or plane.

9. Determine whether three given measures could be the lengths of the sides of a triangle.

10. Use the "Hinge Theorem" and its converse to compare lengths of segments and measures of angles in 2 triangles.

STATE ADOPTED REFERENCES

	M	L	JD	A
CHAPTERS	7	16	7	8

I. TRIANGLES
C. INEQUALITIES (continued)

COURSE CONTENT

Vocabulary

Order of inequality

Associated Properties

Properties of order in inequalities:

trichotomy	addition
transitivity	multiplication by a positive number

An exterior angle of a triangle is larger than each of its remote interior angles.

The measure of an exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles.

If a triangle has one right angle, then its other angles are acute.

If two sides of a triangle are not congruent, then the angles opposite those sides are not congruent and the larger angle is opposite the longer side; and, conversely.

The shortest segment joining a point and a line is the perpendicular segment from the point to the line.

Triangle Inequality: The sume of the lengths of any two sides of a triangle is greater than the length of the third side.

Hinge Theorem: If two sides of one triangle are congruent to two sides of a second triangle and the included angle of the first triangle is larger than the included angle of the second triangle, then the third side of the first is longer than the third side of the second triangle; and, conversely.

The distance from a point to a line or from a point to a plane is the length of the perpendicular segment.

I. TRIANGLES

C. INEQUALITIES (continued)

1. Use 2 blackboard compasses with different openings against the blackboard to demonstrate the hinge theorem and its converse. Draw the 3rd side with chalk, noting the length of the 3rd side in comparison with the size of the opposite angle.

2. Have selected students construct wooden models to demonstrate:
   a. the shortest segment joining a point to a line
   b. the Hinge Theorem
   c. the Triangle Inequality theorem

3. Draw a large $\triangle$ with all sides extended. Number all angles. Use a pointer to indicate an angle which the class must decide is or is not, an exterior angle of the $\triangle$. If they decide it is, then they must call out the numbers of its remote interior angles.

I. TRIANGLES

D. PERIMETER AND AREA

PERFORMANCE OBJECTIVES

The student will:

1. Find the perimeter of a triangle given the measures of the three sides.

2. Find the length of a side of a triangle given the perimeter and the measures of two sides.

3. Find the smallest number of triangular regions into which a given polygonal region can be divided.

4. Find the area of a triangle given the necessary measures.

5. Find a missing part of a triangle given the area and necessary measures.

6. Express the area of a triangle in terms of its perimeter and measures of its sides. (Hero's Formula: $A=\sqrt{s(s-a)\ (s-b)\ (s-c)}$
$s = 1/2\ (a+b+c)$)

7. Compare the areas of two triangles having a) equal bases and altitudes b) equal bases c) equal altitudes.

STATE ADOPTED REFERENCES

	M	L	JD	A
CHAPTERS	9(254-256) 2(23-24) 12(352, Ex. 14)	11	9	13

COURSE CONTENT

Vocabulary

triangular region
polygonal region
altitude
perimeter
area
ratio
triangulate

I. TRIANGLES
D. PERIMETER AND AREA (continued)

Associated Properties

If two triangles are congruent, then the triangular regions determined by them have the same area.

To every polygonal region there corresponds a unique positive number.

Given that the region R is the union of two regions R_1 and R_2, and R_1 and R_2 intersect in at most a finite number of points and segments, than $aR = aR_1 + aR_2$.

The area of a square region is the square of the length of its edge. (Emphasize as unit of measure).

The area of a rectangle is the product of the lengths of its base and its altitude.

The area of a right triangle is half the product of the lengths of its legs.

The area of a triangle is half the product of the lengths of any base and the corresponding altitude.

The perimeter of a triangle is equal to the sum of the measures of its sides.

The area of a triangle $=\sqrt{s(s-a)\ (s-b)\ (s-c)}$, where $s=1/2\ (a+b+c)$.

If two triangles have the same base and equal altitudes, then they have the same area.

If two triangles have the same altitude, then the ratio of their areas is equal to the ratio of their bases.

SUGGESTED LEARNING ACTIVITIES

1. Have students bring in scale drawings of their rooms and compute the cost of:
   a. wallpaper or painting
   b. carpeting

2. Have students make scale drawings of playing regions for popular sports (tennis, football, baseball, etc.) Devise an appropriate list of questions based on these drawings.

I. TRIANGLES

D. PERIMETER AND AREA (continued)

3. Make a model for demonstrating triangular area using a rigid base segment and a track parallel to the base segment a convenient distance above it. Use a nail in track and elastic attached to endpoints of base. Slide nail along track to demonstrate triangles with different shapes but same area.

4. Overhead transparency #17 - Moise-Downs: Geometry.

I. TRIANGLES

D. PERIMETER AND AREA - METRIC SYSTEM-ENRICHMENT ACTIVITY

(This enrichment activity is not optional, and should be applied to all calculations throughout the quin.)

The metric system is actually much easier to use than the system which is now in common use, and was adopted by the U.S. in 1866 but never enforced.

The metric system is called the decimal system of numbers, where place value depends on powers of ten. The prefixes in the metric systems also depend on the powers of ten.

Refer to this table to help you complete the following questions.

Table

Prefixes of the Metric System	Powers of Ten	Decimal Number System
milli--	.001	thousandths
centi--	.01	________
deci--	.1	tenths
unit	1.	units
deka--	10.	________
hecto--	100.	________
kilo--	1000.	thousands

1. 1/10 or .1m = 1 decimeter (dm)
2. 1/100 or .01 m = 1 centimeter (cm) = .1 dm
3. 1/1000 or .001 m = 1 millimeter (mm) = .1 cm = .01 dm
4. 1 m = ________ dm
5. 1 m = ________ cm
6. 1 m = ________ mm
7. 1dm = ________ cm

I. TRIANGLES

D. PERIMETER AND AREA - METRIC SYSTEM - ENRICHMENT ACTIVITY (continued)

8. 1 dm = ________mm

9. 1 cm = ________mm

10. 1 cm = ________dm

11. ________m = 1 kilometer (km)

12. ________m = 1 hectometer (hm)

13. ________m = 1 dekameter (dkm)

Now let's try converting from the English System to the Metric System and vice versa. Use the following facts to help you:

1 in. = 2.54 cm.

1 mile = 5280 feet

1. 7 inches = ______mm = ______cm = ______m

2. 50 miles = ______km

3. 30 ft. = ______cm = _______m

4. 40.64 cm = _______in. = ______ft. = ______yd.

5. 300 km = _________miles

The teacher may desire to use supplementary materials here.

I. TRIANGLES

E. PYTHAGOREAN THEOREM AND SPECIAL RIGHT TRIANGLE RELATIONSHIPS AND APPLICATIONS

PERFORMANCE OBJECTIVES

The student will:

1. Apply the Pythagorean Theorem to find the missing measure of the sides of a given right triangle.
2. Strengthen his ability to simplify radicals.
3. Use the converse of the Pythagorean Theorem to determine if a triangle is a right triangle.
4. Apply the appropriate formulas to solve for the missing parts is special right triangles. (30-60-90 and 45-45-90)
5. Demonstrate a minimum of five developments of the Pythagorean Theorem. *
6. Develop and apply the formula for the area of an equilateral triangle.
7. Determine the lengths of the sides and the altitude of an equilateral triangle given the area of the triangle.
8. Memorize the three most used Pythagorean triplets: 3, 4, 5; 5, 12, 13; and 8, 15, 17.
9. Search for at least 2 more Pythagorean triplets.
10. Use metric units of measure in the above objectives where appropriate.

STATE ADOPTED REFERENCES

	M	L	JD	A
CHAPTERS	9 (254-256) 11 2 (pp. 23-24) 12 (pg. 352 ex. 14)	11	9	13

* Additional reference for Developments of the Pythagorean Theorem. (See The Pythagorean Proposition, Elisha Scoot Loomis, NCTM, Washington, D.C.)

I. TRIANGLES
E. PYTHAGOREAN THEOREM AND SPECIAL RIGHT TRIANGLE RELATIONSHIPS AND APPLICATIONS (continued)

COURSE CONTENT

Vocabulary

square	radicand
square root	right angle
radical	isosceles right triangle
index	

Associated Properties

Pythagorean Theorem and its converse.

Speical right triangle relationships and converses. (30-60-90 and 45-45-90).

The altitude to the hypotenuse of a right triangle in terms of its legs is equal to: $\dfrac{l_1 l_2}{\sqrt{l_1^2 + l_2^2}}$

The area of an equilateral triangle equals $\dfrac{S^2}{4}\sqrt{3}$

SUGGESTED LEARNING ACTIVITIES

1. Have students construct various right triangles with squares upon the three sides. Have them measure and compute areas of the squares to develop the Pythagorean relationship.

2. Have students present reports on historical background of the Pythagorean Theorem.

3. Use filmstrip by FOM #1168 Special Right Triangles

4. Overhead transparency #19, Moise-Downs, Geometry.

I. TRIANGLES

F. SIMILAR TRIANGLES

PERFORMANCE OBJECTIVES

The student will:

1. Find the missing term of a proportion when given the other three terms.

2. Find the geometric and arithmetic mean of two terms given in numerical or literal form.

3. Determine whether figures in a given marked diagram are similar; and if so, identify the property which justifies the conclusion.

4. List all proportions determined by a line intersecting two sides of a triangle and parallel to the third side.

5. Determine whether a segment is parallel to one side of a triangle if given certain proportional parts.

6. Identify the proportions determined by a bisector of an angle of a triangle.

7. Solve for the missing measures of a right triangle if given the measure of the altitude to the hypotenuse and one other appropriate appropriate measure.

8. If given the ratio of corresponding parts of a similar figures and other appropriate information, apply the proper formulas to find:

   a. perimeter
   b. area
   c. volume

9. Apply the definition of similarity of triangles to write a proportion and solve for an unknown length of a side, given the appropriate information.

10. Use similar triangles to solve problems involving indirect measurement.

I. TRIANGLES

F. SIMILAR TRIANGLES

STATE ADOPTED REFERENCES

	M	L	JD	A
CHAPTERS	11, 12	11	8, 9	13, 14

COURSE CONTENT

Vocabulary

means
extremes
geometric mean
arithmetic mean
projection
similar polygons
similar triangles
proportional segments

Associated Properties

Two polygons are similar if, for some pairing of their vertices, corresponding angles are congruent and corresponding sides are in proportion.

Similarity theorems: AAA, AA, SAS, SSS

If a line is parallel to one side of a triangle and intersects the other two sides, then a) it divides the sides into proportional segments; b) it cuts off segments proportional to these sides, and conversely.

If a ray bisects one angle of a triangle, it divides the opposite side into segments which are proprotional to the given sides.

Transitivity of similarity with respect to triangles.

The altitude to the hypotenuse of a right triangle forms two triangles which are similar to the given triangle and similar to each other.

The altitude to the hypotenuse of a right triangle is the mean proportional (geometric mean) between the segments into which it divides the hypotenuse.

The altitude to the hypotenuse in a 30-60-90 rt $\triangle$, divides the hypotenuse in the ratio of 1:3.

Associated Properties (continued)

If two triangles are similar, then the ratio of their perimeters is equal to the ratio of any two corresponding sides, altitudes, or medians.

SUGGESTED LEARNING ACTIVITIES

1. Film catalog #1-01506, Similar Triangles

2. FOM Filmstrips
   #1161 Mean Proportion and Right Triangles
   #1116 Similar Triangles

3. Overhead transparencies by Moise-Downs #21, 22

4. Have students bring in examples of similarity relationships such as: model planes, cars, photography enlargements, doll clothes and furniture, blue prints, and maps.

5. Cut out paper models of figures and slide them over each other to demonstrate similarity and the constant ratio of corresponding sides.

I. TRIANGLES
G. CONSTRUCTIONS

PERFORMANCE OBJECTIVES

The student will:

1. Using only a compass and straightedge, construct:

   a. Isosceles triangles
   b. Equilateral triangles
   c. Right triangles
   d. Angle bisectors of triangles
   e. Perpendicular bisectors of the sides of a triangle
   f. Medians of a triangle
   g. Altitudes of a triangle
   h. Inscribed circle of a triangle
   i. Circumscribed circle of a triangle

2. Demonstrate the difference in the terms "sketch" and "construct".

STATE ADOPTED REFERENCES

	M	L	JD	A
CHAPTERS	15	11, 15	6, 11	13, 17

COURSE CONTENT

Vocabulary

Sets of points related to a triangle:

angle bisector
perpendicular bisector of a side
median
circumcenter
incenter
orthocenter
centroid
circumscribed circle
inscribed circle

Associated Properties

The medians of every triangle are concurrent at a point two-thirds of the distance from one vertex to the mid-point of the opposite side.

I. TRIANGLES
G. CONSTRUCTIONS (continued)

Associated Properties (continued)

The segment joining the midpoints of two sides of a triangle is parallel to the third side and one-half as long.

The median to the hypotenuse of a right triangle is half as long as the hypotenuse.

In a triangle, if a median is half as long as the side which it bisects, then the triangle is a right triangle and the side is its hypotenuse.

SUGGESTED LEARNING ACTIVITIES

1. Have students report on development of construction implements from collapsible compass to modern day drafting tools.

2. Have students report on trisection on an angle.

3. Have students do constructions by two methods:

   compass and straightedge
   ruler and protractor

I. TRIANGLES

H. RIGHT TRIANGLE TRIGONOMETRY: ENRICHMENT ACTIVITY

PERFORMANCE OBJECTIVES

The student will:

1. Develop informally the sine, cosine, and tangent ratios.

2. Apply the appropriate trigonometric ratios to solve for any missing length, of a side of a right triangle, given the lengths of two sides.

3. Use Metric System of Measure in problem solving.

4. Use appropriate associated properties to solve identity problems.

5. Master the solution of verbal problems through drawing of a diagram, labeling it, selecting the correct ratio and accurately completing the computation.

STATE ADOPTED REFERENCES

	M	L	JD	A
CHAPTERS	12	19	9	19

Vocabulary

trigonometric ratios
  sine
  cosine
  tangent
ratio
trigonometry
indirect measurement
identity

angle of depression
angle of elevation
hypotenuse
leg
radian measure
complement
degree
minute
second

Associated Properties

The sine of an acute angle is equal to the cosine of its complement.

I. TRIANGLES

H. RIGHT TRIANGLE TRIGONOMETRY: ENRICHMENT ACTIVITY (continued)

Associated Properties (continued)

The product of the tangent of an acute angle and the tangent of its complement is 1.

Trigonometric Ratios: $\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$ $\tan A = \frac{\text{opposite}}{\text{adjacent}}$

$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$

$(\sin A)^2 + (\cos A)^2 = 1$

SUGGESTED LEARNING ACTIVITIES

1. Use sextant and transit to measure objects outdoors.

2. Apply the trigonometric ratios and/or the Pythagorean Theorem to solve problems involving the inclined plane, the wedge, and the screw. An excellent source for both teacher and student is: O'Brien, Robert, Machines, Life Science Library, New York: New York: Time Inc., 1964.

II. QUADRILATERALS

A. KINDS AND RELATED PARTS

PERFORMANCE OBJECTIVES

The student will:

1. Define and illustrate items from vocabulary list.

2. Identify and sketch items from vocabulary list, if given their definitions.

3. Label and identify parts of figures correctly.

4. Draw a Venn diagram to illustrate the relationships of quadrilaterals and their subsets.

5. Develop and apply the associated properties listed under "Course Content" to numerical and non-numerical exercises.

6. Tell if given information is sufficient to classify a quadrilateral as a trapezoid or an isosceles trapezoid.

STATE ADOPTED REFERENCES

	M	L	JD	A
CHAPTERS	5, 9	8, 17	5, 7, 15	10, 13

COURSE CONTENT

Vocabulary

quadrilateral
opposite angles
consecutive or successive angles
opposite sides
adjacent sides
consecutive or successive sides
diagonal
convex quadrilateral
vertices
clockwise
counterclockwise
supplementary angles
parallel sides
rectangle
square
trapezium
trapezoid
rhombus
isosceles trapezoid
median of a trapezoid
base angles of trapezoid
kite
trisect

II. QUADRILATERALS

A. KINDS AND RELATED PARTS (continued)

Associated Properties

Every quadrilateral is a four-sided polygon.

The sum of the measures of the angles of a quadrilateral is 360°.

The median of a trapezoid is parallel to the bases and equal in measure to one-half the sum of the bases.

The median of a trapezoid bisects both diagonals.

If two consecutive angles of a trapezoid are congruent but not supplementary, the trapezoid is isosceles.

The bisectors of the opposite angles of a nonrhombic parallelogram are parallel.

(Optional) If a trapezoid has two nonparallel sides both congruent to one of the parallel sides, then the diagonals bisect the angles at the other parallel side.

SUGGESTED LEARNING ACTIVITIES

1. Chart of quadrilaterals and their properties may be begun here and continued throughout II - B.

2. Cut out, fold and measure paper models etc.

3. Use Venn Diagrams to help student learn properties of quadrilaterals.

4. Have students construct models using hinged sides, and elastic to demonstrate properties of special quadrilaterals.

II. QUADRILATERALS

B. PARALLELOGRAMS

PERFORMANCE OBJECTIVES

The student will:

1. Define and illustrate items from vocabulary list.
2. State properties of all parallelograms.
3. State special properties of rectangle, rhombus, and square.
4. Given a property, name all quadrilaterals which have that property.
5. Complete a chart of all quadrilaterals and their properties.
6. Tell if given information is sufficient to classify a quadrilateral as a parallelogram, rectangle, rhombus, square, trapezoid, isosceles trapezoid and/or kite.
7. State all sets of conditions which would guarantee that a quadrilateral is a parallelogram, rectangle, rhombus, square trapezoid, isosceles trapezoid and/or kite.
8. Determine the measures of three angles of a parallelogram, given the measure of one angle.
9. Determine the missing measures of angles in compound plane figures composed of triangles and quadrilaterals.
10. Determine the missing lengths of sides and/or diagonals given necessary and sufficient information.
11. Name the figure formed by joining the consecutive mid-points of the sides of;

    a. any quadrilateral
    b. a parallelogram
    c. a rectangle
    d. a rhombus
    e. a square

12. Determine if sufficient, insufficient or too much information is given to find the solution of a problem.

II. QUADRILATERALS
B. PARALLELOGRAMS (continued)

Associated Properties

Each diagonal of a parallelogram separates a parallelogram into two congruent triangles.

Any two opposite sides of a parallelogram are congruent.

Any two opposite angles of a parallelogram are congruent.

Any two consecutive angles of a parallelogram are supplementary.

The diagonals of a parallelogram bisect each other.

If the bisectors of two consecutive angles of a parallelogram intersect, they are perpendicular.

Given a quadrilateral in which both pairs of opposite sides are congruent, hen the quadrilateral is a parallelogram.

If two sides of a quadrilateral are parallel and congruent, then the quadrilateral is a parallelogram.

If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

The segments joining the mid-points of opposite sides of any quadrilateral bisect each other.

If a parallelogram has one right angle, then it has four right angles, and the parallelogram is a rectangle.

In a rhombus, the diagonals are perpendicular to one another.

If the diagonals of a quadrilateral bisect each other and are perpendicular, then the quadrilateral is a rhombus.

The diagonals of a rectangle are congruent.

The diagonals of a rhombus bisect the angles of the rhombus.

If three parallel lines intercept congruent segments on one transversal T then the segments they intercept on every transversal T, which is parallel to T, are congruent to each other and to the segments of T.

II. QUADRILATERALS
B. PARALLELOGRAMS (continued)

Associated Properties (continued)

If three or more parallel lines intercept congruent segments on one transversal, then they intercept congruent segments on any other transversal.

A kite has two pairs of congruent sides, but its opposite sides are not congruent.

The figure formed by the segments joining the mid-points of consecutive sides of any:

(a) quadrilateral is a quadrilateral
(b) parallelogram is a parallelogram
(c) rectangle is a rhombus
(d) rhombus is a rectangle
(e) square is a square

If two lines are parallel, then all points of each line are equidistant from the other line.

Every parallelogram is an isosceles trapezoid.

SUGGESTED LEARNING ACTIVITIES

1. Continue activities #1-4, from II A

2. Filmstrip by FOM #1113 (Parallelograms and Their Properties)

II. QUADRILATERALS
C. PERIMETER AND AREA

PERIMETER OBJECTIVES

The student will:

1. Develop and apply the standard formulas for finding the perimeter of any quadrilateral, any parallelogram, special parallelograms, trapezoids, kites and compound figures.

2. Develop and apply the standard formulas for finding the area of any quadrilateral, any parallelogram, special parallelograms, trapezoids, kites and compound figures.

3. Find missing lengths of parts needed in order to use standard perimeter and area formulas, given appropriate angle measures and lengths of segments.

4. Find measures of diagonals, bases and/or altitudes, given other appropriate measures.

5. Find ratios of perimeters, areas, sides, and/or altitudes of figures.

6. Find the perimeter of a figure, given the perimeter of another figure and the ratio of the two perimeters or other parts.

7. Find the area of a figure, given the area of another figure and the ratio of the two areas.

8. Apply metric units of measure to the above objectives where appropriate.

COURSE CONTENT

Vocabulary

base
parallel
square unit
linear unit
compound figures

II. QUADRILATERALS

C. PERIMETER AND AREA (continued)

Associated Properties

1. The perimeter of a square, rectangle, rhombus, parallelogram, trapezoid or any quadrilateral is equal to the sum of the lengths of its sides.

2. The area of a square equals the square of the length of a side.

3. The area of a rectangle equals its base times its height (altitude).

4. The area of a parallelogram equals its base times its altitude.

5. The area of a trapezoid equals $\frac{1}{2}h(b_1 + b_2)$.
h = height
b_1 and b_2 = lengths of bases

6. If the diagonals of a rhombus, square or kite are d_1 and d_2, then the area of the enclosed region is $\frac{1}{2}d_1\ d_2$.

7. If the diagonals of a convex quadrilateral are perpendicular to each other, then the area of the quadrilateral equals one-half the product of the lengths of the diagonals.

SUGGESTED LEARNING ACTIVITIES

1. Use cut out figures to illustrate area formulas and relate the area of a parallelogram to that of a rectangle by replacing the right triangle, etc.

2. Overhead transparency Moise-Downs Geometry #18

3. Distribute cardboard cutouts of various quadrilaterals and have students measure required parts and compute perimeter and area.

4. Discussion Question: Does perimeter determine area? Take 4 strips of cardboard 1/2 inch by 4 inches and fasten them together so that they make a square. Adjust the frame to different positions and trace the inside outline on graph paper. Estimate the area of each figure.

II. QUADRILATERALS
D. CONSTRUCTIONS

PERFORMANCE OBJECTIVES

Using only a compass and straightedge, the student will construct:

1. A parallelogram starting with (a) its diagonals (b) a pair of consecutive sides (c) a pair of parallel segments (d) a given side, acute angle, and the longer diagonal (e) 2 segments and 1 angle given, representing 2 sides and the included angle.

2. A rhombus given (a) two line segments representing its diagonals (b) a segment and an angle representing its sides and one angle.

3. A square

4. A rectangle

5. A trapezoid

6. A kite

7. An altitude from any given vertex of a quadrilateral

8. A median of a trapezoid

9. A rhombus using a given line segment of lengths as one side and a given angle A as one of the angles.

COURSE CONTENT

Associated Properties

Four sets of conditions which determine a parallelogram are:

1. The diagonals of a quadrilateral bisect each other.
2. Both pairs of opposite sides of a quadrilateral are parallel.
3. Both pairs of opposite sides of a quadrilateral are congruent.
4. One pair of opposite sides of a quadrilateral are congruent and parallel.

SUGGESTED LEARNING ACTIVITIES

1. Repeat selected constructions using modern construction implements
   a) T square
   b) ruler
   c) protractor

2. As a review or a test, distribute envelopes to each student containing construction instructions for different quadrilaterals, and have him construct the required quadrilateral using straightedge and compass.

3. Throughout this quin, the teacher should include more challenging construction problems from the State-adopted texts and from the sources listed in the annotated bibliography at the end of this quin.

II. QUADRILATERALS
E. COORDINATE GEOMETRY: ENRICHMENT ACTIVITY

PERFORMANCE OBJECTIVES

The student will:

1. Develop selected properties of triangles and quadrilaterals through the use of coordinate geometry.
2. Compute the measures of sides, areas, perimeters, diagonals, altitudes, etc. of triangles and quadrilaterals, given the coordinates of appropriate points.
3. Determine the relationship (parallel, perpendicular, intersecting between two lines through given points, by applying the slope and/or Pythagorean formulas.
4. Write the equation of a line through a given point with a given slope in point-slope and slope-intercept form.
5. Convert from a point-slope equation to a slope-intercept form, and vice-versa.
6. Given a general equation of a line, determine its slope and the coordinates of 2 points on the line.
7. Sketch the graph of a given linear equation.
8. Given a linear graph, produce its equation in point-slope or slope-intercept form.

STATE ADOPTED REFERENCES

	M	L	JD	A
CHAPTERS	13	12, 13	12, 13	15

COURSE CONTENT

Vocabulary

ordered pair	x-axis	midpoint formula
coordinate axes	y-axis	slope
origin	coordinate system	vertical
quadrant	distance formula	non-vertical

II. QUADRILATERALS

E.. COORDINATE GEOMETRY: ENRICHMENT ACTIVITY (cont.)

Vocabulary (continued)

horizontal
3-D coordinate system
abcissa
x-coordinate
y-coordinate
reciprocal
octant
z-axis
z-coordinate
graph of a condition

Associated Properties

On a non-vertical line, all segments have the same slope.

2 non-vertical lines are parallel, if and only if, they have the same slope.

The slopes of perpendicular lines are the negative reciprocals of each other

The distance between the points (x_1, y_1) and (x_2, y_2) is

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Given $P = (x_1, y_1)$ and $P_2 = (x_2, y_2)$, the coordinates of the midpoints of $P_1 P_2$ are $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

If P is between P_1 and P_2, and $r = \frac{P_1P}{P_2P}$ then the coordinates of P are $\left(\frac{x_1 + rx_2}{1 + r}, \frac{y_1 + ry_2}{1 + r}\right)$.

If $P_1 = (x_1, y_1)$ and $P = (x, y)$, then the slope (m) of P_1P is $\frac{y-y_1}{x-x_1}$

Point-Slope Formula $(y-y_1) = m(x-x_1)$

Slope-Intercept Formula is $y = mx + b$ where b is the y intercept $(x=0)$

II. QUADRILATERALS

E. COORDINATE GEOMETRY: ENRICHMENT ACTIVITY

SUGGESTED LEARNING ACTIVITIES

1. Have a student construct a model of a 3-D coordinate system, and locate points by supporting them in space with wire.

2. Dittos can be made of sets of equations and inequalities whose graphs together form pictures of familiar objects such as a boat, house, etc.

3. Students may create their own sets of equations and inequalities and submit them to the class for graphing.

II. QUADRILATERALS

F. TANAGRAMS - ENRICHMENT ACTIVITY

Objective: To gain a better understanding of some basic geometric shapes and their relationships through puzzle solving.

Special Materials: scissors, tanagrams, tanagram cards.

Commentary: Throughout the ages, man has been fascinated by geometric shapes and their relationships. Manipulating shapes both mentally and physically provides interesting problems and challenging thinking situations.

Tanagrams (Tangram) is an ancient Oriental Puzzle composed of seven geometric shapes and a set of silhouettes. The object of the game is to duplicate the silhouette using all seven pieces. Sam Loyd, one of the world's greatest puzzle experts, fabricated a very delicate history of Tanagrams. According to Loyd, the puzzle originated about four thousand years ago with a Chinese nobleman by the name of Tan, who had a square ceramic tile which he valued highly. One day he accidentally dropped the tile and broke it into seven pieces. He then spent the rest of his life trying to put it back together. In the process, he wrote The Seven Books of Tan using gold leaf figures on parchment, showing the designs he had made with the pieces. Loyd went on to compile a list of famous personages (many non-existent) who were disciples of Tan. One of the most accepted tall tales was that Napolean spent his declining years in exile playing with the Tanagrams. Loyd wrote <u>The Eighth Book of Tan</u>

containing this imaginative history and giving many new configurations. It was several decades before anyone dared to question Loyd's history.

Activities: (The cut-out and diagram sheets for this activity are found at the end of this Guide.)

I. Cut out the seven pieces of the Basic Tanagram puzzle. (Figure 1)

1. Give a geometric name to each of the seven pieces.
2. Give the properties of each of the geometric figures.
3. Arrange the seven pieces to form:
   a) a square
   b) a rectangle
   c) a parallellogram
   d) a trapezoid
   e) a triangle

Sketch the position of the pieces in each solution.

4. Using the patterns given in figure 2 arrange the puzzle pieces to form each of the shapes given. Sketch the position of the pieces in each solution. Remember, you are to use all seven puzzle pieces each time!

II. Variations on the Tanagram Puzzle

1. Cut out the puzzle pieces in figure 3. Arrange them to form the following:

II. (continued)

a) a rectangle

b) a parallelogram

c) a cross

2. Cut out the pieces in figure 4 and arrange them to form:

a) a square

b) a pentagon

3. The 5 x 5 square in figure 5 is divided into triangles and trapezoids. What is the area of this square? ________.

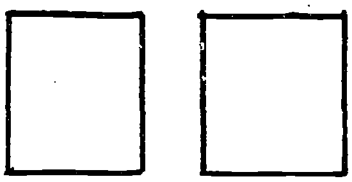
4. Cut out the pieces and rearrange them into a rectangle.

What is the area of the rectangle? ____________________

Can you explain this?

Challenge Question:

In some cases it is possible to make two congruent (identical) figures using just the seven basic pieces. Before you attempt solutions, decide which sets are possible and which are not. What makes you think so?



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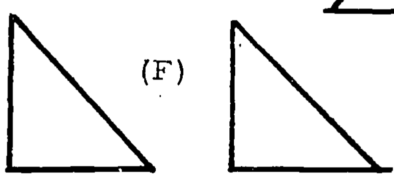
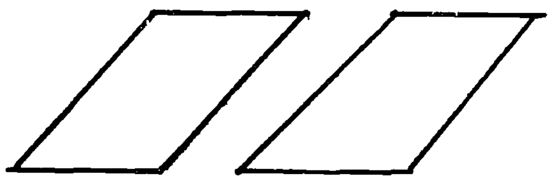


(F)



(E)

Figure 1 The Basic Tanagram

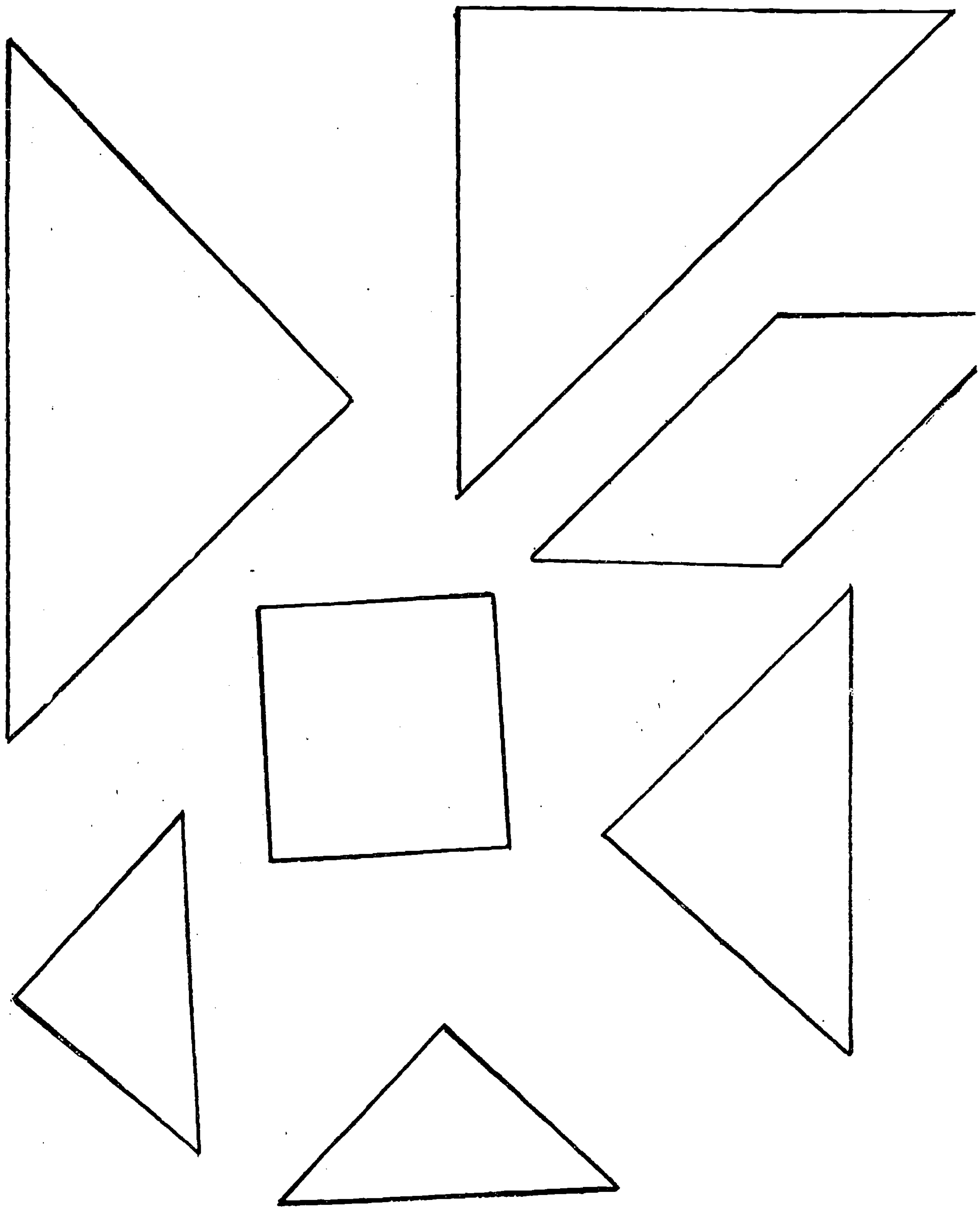


Figure 2 Patterns for use with the Basic Tanagram

After constructing each figure, sketch in the position of the pieces of your solution in the proper drawing below. REMEMBER YOU ARE TO USE ALL SEVEN TANAGRAM PIECES EACH TIME.

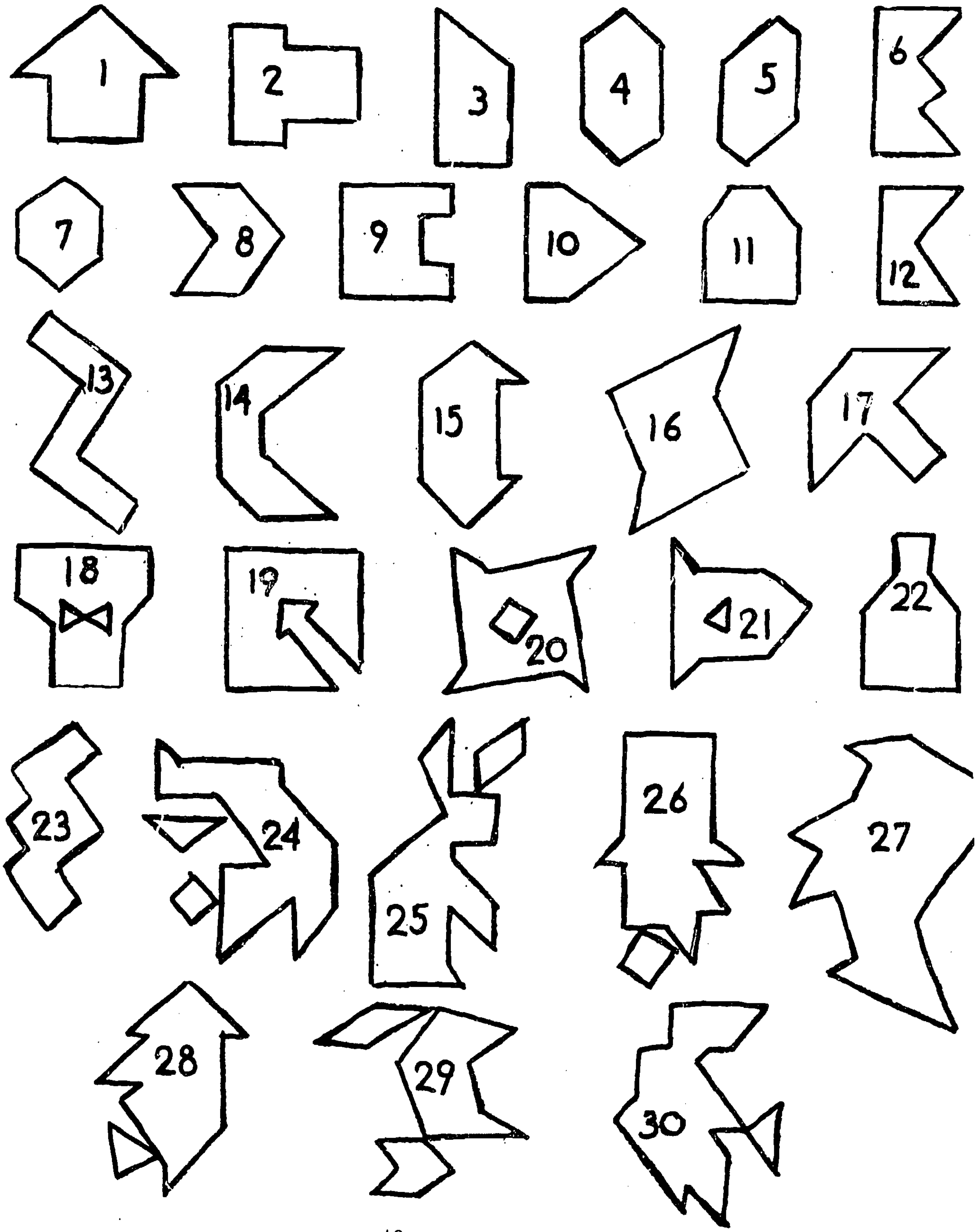


Figure 3 Variation A on the Basic Tanagram

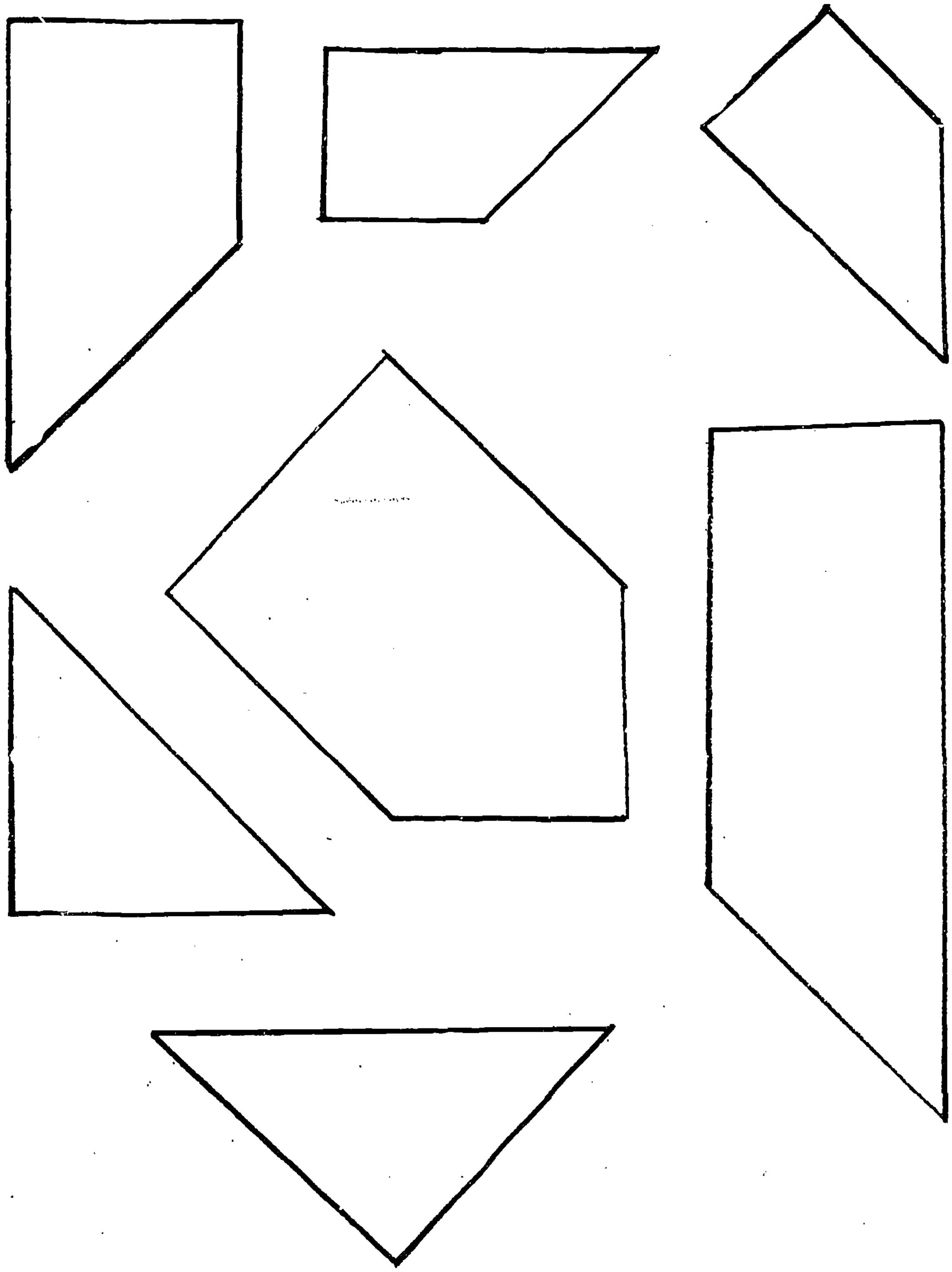


Figure 4 Variation B on the Basic Tanagram

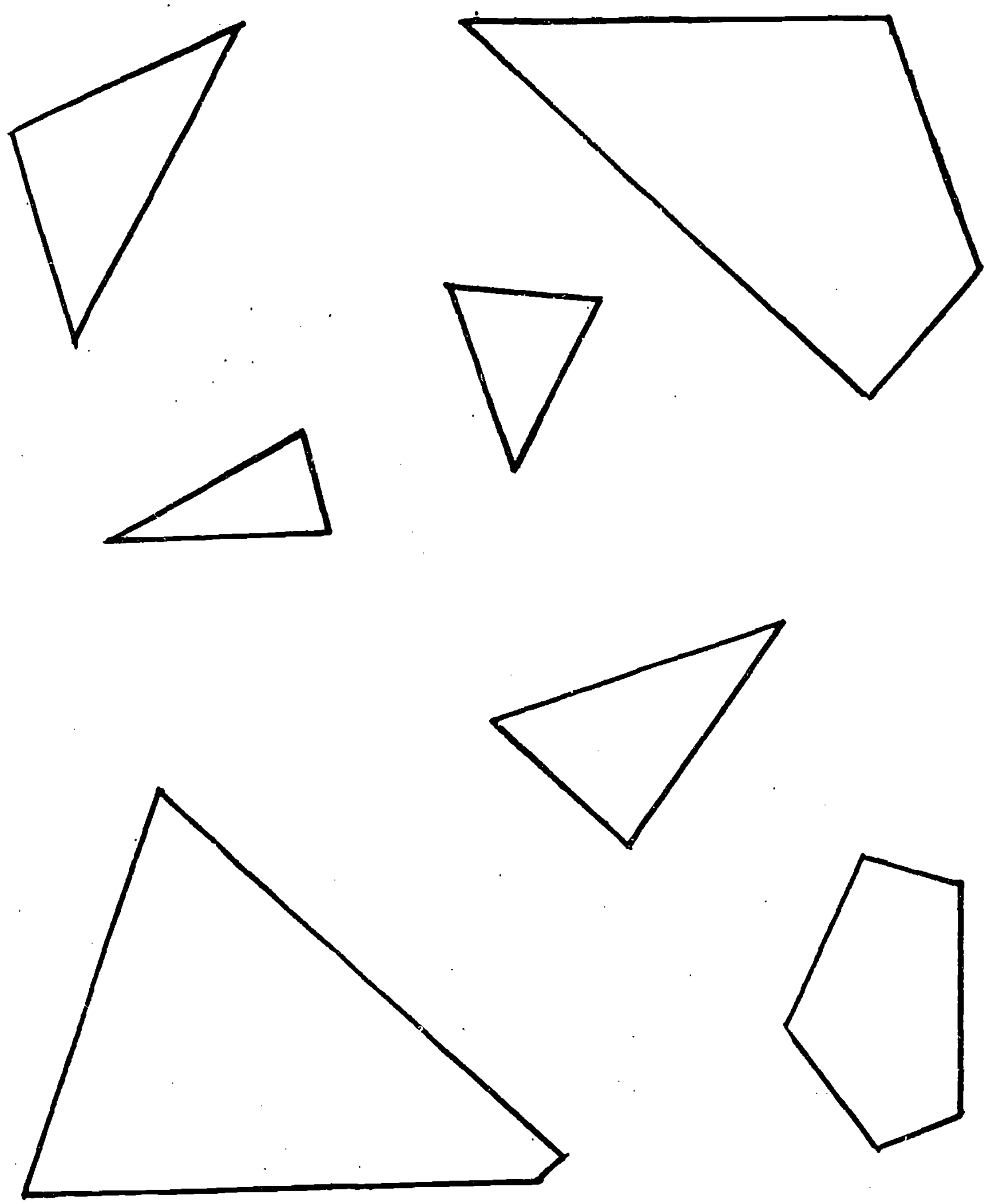
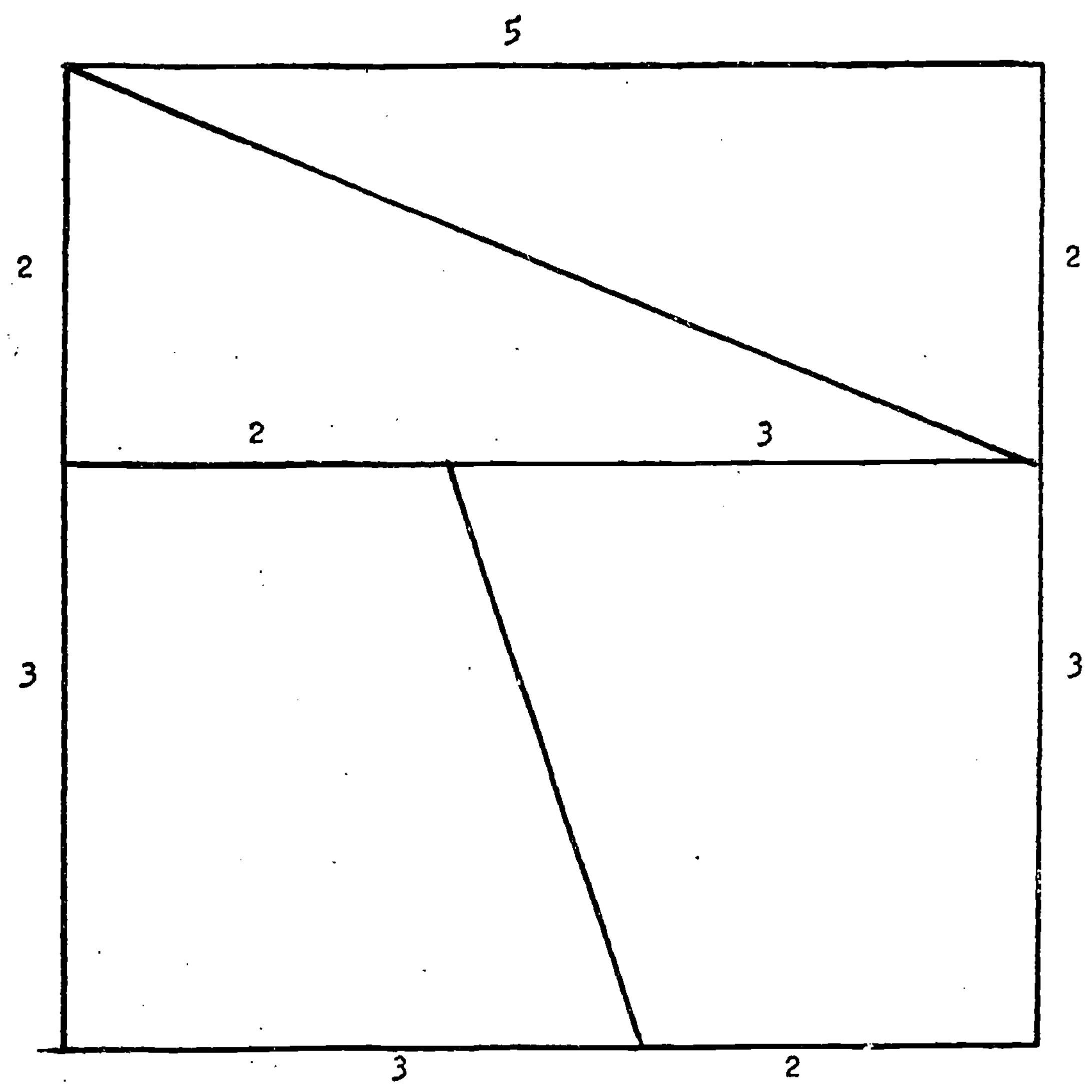


Figure 5 Variation C on the Basic Tanagram



III. CIRCLES

A. RELATED PARTS

PERFORMANCE OBJECTIVES

The student will:

1. Identify and label the related parts of a circle in a given diagram.
2. Draw a diagram of a circle and its related parts and label the figure, given a list of the required parts.
3. Name the related parts of a circle, given their definitions.
4. Define items from the vocabulary list.
5. Solve numerical and non-numerical problems based on items in the vocabulary list and the associated properties list.
6. Develop items selected from the list of associated properties intuitively.

STATED ADOPTED REFERENCES

(to be used for Circles: A - F)

	M	L	JD	A
CHAPTERS	14, 15, 16	14, 15, 17	10, 15	16, 17, 18

COURSE CONTENT

Vocabulary

circle
center
sphere
radius
compass
straightedge
diameter
chord
tangent (line, segment, ray)
secant (line, segment, ray)
point of tangency

circumference
concentric circles
congruent circles
segment
sector
semi-circle
arc
minor arc
major arc
intercepted arc
inscribed polygon

circumscribed polygon
great circle

III. CIRCLES

A. RELATED PARTS (continued)

Vocabulary (continued)

internally tangent
externally tangent
common internal tangent
common external tangent

pi (π)
irrational
arc in which angle is inscribed

Associated Properties

Pi is the constant ratio of the circumference of a circle to its diameter.

The diameter equals twice the radius.

The interception of a sphere with a plane through its center is a circle with the same center and the same radius as the sphere.

A line perpendicular to a radius at its outer end is tangent to the circle.

Every tangent to a circle is perpendicular to the radius drawn to the point of contact.

The perpendicular from the center of a circle to a chord bisects the chord.

The segment from the center of a circle to the mid-point of a chord is perpendicular to the chord.

In the plane of a circle, the perpendicular bisector of a chord passes through the center.

If two circles are tangent, their centers are collinear with their point of tangency.

No circle contains three collinear points.

In the same circle or in congruent circles, chords equidistant from the center are congruent.

In the same circle or in congruent circles, any two congruent chords are equidistant from the center.

III. CIRCLES

A. RELATED PARTS (continued)

Associated Properties (continued)

If a line, intersects the interior of a circle, then it intersects the circle in two and only two points.

In any circle, the mid-points of all chords congruent to a given chord form a circle concentric with the given circle and with a radius equal to the distance of any one of the chords from the center.

In a circle, if two chords which have a common end points determine congruent angles with a diameter from the same point, then the chords are congruent.

If two congruent chords of a circle intersect on a diameter, they determine congruent angles with the diameter.

Any three noncollinear points lie on a circle.

A plane perpendicular to a radius at its outer end is tangent to the sphere.

Every tangent plane to a sphere is perpendicular to the radius drawn to the point of contact.

If a plane intersects the interior of a sphere, then the intersection of the plane and the sphere is a circle. The center of this circle is the foot of the perpendicular from the center of the sphere to the plane.

The perpendicular from the center of a sphere to a chord bisects the chord.

The segment from the center of a sphere to the mid-point of a chord is perpendicular to the chord.

If two diameters of a sphere are perpendicular, the figure formed by the segments joining their end points in succession is a square.

Any two great circles of a sphere intersect at the end points of a diameter of the sphere.

If two planes intersect a sphere and their distances from the center are equal, then the intersections are either two points or two congruent circles.

III. CIRCLES

A. RELATED PARTS (continued)

Associated Properties (continued)

Given two circles of radius a and b, with c as the distance between their centers. If each of the numbers a, b, and c is less than the sum of the other two, then the circles intersect in two points, on opposite sides of the line through the centers.

SUGGESTED LEARNING ACTIVITIES

1. Overhead transparency - Moise & Downs #27 Circles.

2. Teacher could mimeograph a matching type question using a diagram of a large circle with related parts indicated by a, b, c, etc., to be matched with a given vocabulary list.

3. Filmstrip by FOM - #1151

III. CIRCLES

B. SPECIAL ANGLE FORMULAS

PERFORMANCE OBJECTIVES

The student will:

1. Define selected items from vocabulary list and/or match items from vocabulary list with their given definitions.

2. Identify the kinds of angles related to a circle named in the vocabulary list, given a sketch.

3. Sketch the angles related to a circle, given their names.

4. Develop the angle-measurement formulas listed under "Associated Properties".

5. Apply the angle-measurement formulas to find measures or angles and/or arcs given appropriate information, with or without a diagram.

6. Solve numerical exercises involving lengths of segments, given appropriate information with or without a diagram.

7. Develop and apply selected associated properties involving chords, radii, secants, tangents, inscribed and circumscribed circles and polygons.

COURSE CONTENT

Vocabulary

central angle
inscribed angle
angle formed by intersection of two secants intersecting at a point in the interior of the circle
angle with its vertex on a circle, formed by a secant ray and a tangent ray

III. CIRCLES

B. SPECIAL ANGLE FORMULAS

Vocabulary (continued)

angle formed by two secants or a secant and a tangent or two tangents of a circle intersecting at a point in the exterior of the circle
degree
degree measure
second
minutes
angle inscribed in an arc

Associated Properties

If B is a point of $\overset{\frown}{AC}$, then $m\overset{\frown}{ABC} = m\overset{\frown}{AB} + m\overset{\frown}{BC}$.

The bisector of a central angle of a circle bisects the corresponding minor arc.

The measure of an inscribed angle is half the measure of its intercepted arc.

Any angle inscribed in a semicircle is a right angle.

All angles inscribed in the same arc are congruent.

If two circles are tangent internally such that the smaller circle contains the center of the larger circle, then any chord of the larger circle having one end point at the point of tangency is bisected by the smaller circle.

In any circle, parallel chords intercept arcs having equal measures.

In a circle, a diameter perpendicular to a chord bisects each arc determined by the end points of the chord.

If an angle inscribed in a circular arc is a right angle, the arc is a semicircle.

The opposite angles of an inscribed quadrilateral are supplementary.

In the same circle or in congruent circles, if two chords are congruent, then so are the corresponding minor arcs.

Associated Properties (continued)

In the same circle or in congruent circles, if two arcs are congruent, then so are the corresponding chords.

Given an angle with its vertex on a circle, formed by a secant ray and a tangent ray, The measure of the angle is half the measure of the intercepted arc.

If two tangents to a circle intersect each other, they form an isosceles triangle with the chord joining the points of tangency.

If two arcs are congruent, then any angle inscribed in one of the arcs is congruent to any angle inscribed in the other arc.

The mid-point of the arc intercepted by an angle formed by a secant ray and a tangent ray with its vertex on a circle, is equidistant from the sides of the angle.

The measure of an angle formed by two secants of a circle intersecting at a point in the interior of the circle is one-half the sum of the measures of the arcs intercepted by the angle and its vertical angle.

The measure of an angle formed by two secants, or a secant and a tangent, or two tangents of a circle intersecting at a point in the exterior of the circle is one-half the difference of the intercepted arcs.

The two tangent segments to a circle from a point of the exterior are congruent and determine congruent angles with the segment from the exterior point to the center of the circle.

SUGGESTED LEARNING ACTIVITIES

1. Teacher should develop angle formulas in relationship to the position of the vertex.
   - center of circle
   - circle's interior
   - on the circle
   - circle's exterior

2. To review or test, use a ditto question of the type at the bottom of p. 386 of Modern School Mathematics Geometry by Jurgensen, Donnelly, Dolciani.

3. Overhead transparency - Moise-Downs #28

III. CIRCLES

C. PROPORTIONS INVOLVING CHORDS, SECANTS, and TANGENTS (POWER OF A POINT WITH RESPECT TO A CIRCLE)

PERFORMANCE OBJECTIVES

The student will:

1. Develop properties intuitively and apply them to both numerical and non-numerical exercises.

2. Determine the lengths of tangent segments, secant segments, and segments of chords, by substituting in the appropriate "power of a point" formulas.

3. Determine the measures of angles in a given figure by applying selected associated properties.

COURSE CONTENT

Vocabulary

ratio
proportion
"power of a point"
secant segment
tangent segment
common internal tangent
common external tangent

Associated Properties

Given a circle C, and a point Q of its exterior. Let L_1 be a secant line through Q, intersecting C in points R and S; and let L_2 be another secant line through Q, intersecting C in points U and T. Then

$$QR \times QS = QU \times QT$$

Given a tangent segment $\overline{QT}$ to a circle, and a secant line through Q, intersecting the circle in points R and S. Then

$$QR \times QS = QT^2$$

Let $\overline{RS}$ and $\overline{TU}$ be chords of the same circle, intersecting at Q. Then $QR \times QS = QU \times QT$

III. CIRCLES

C. PROPORTIONS INVOLVING CHORDS, SECANTS, and TANGENTS (POWER OF A POINT WITH RESPECT TO A CIRCLE) (continued)

Associated Properties (continued)

If the measure of the angle determined by two tangent segments to a circle from a point of the exterior is 60, then the tangent segments form an equilateral triangle with the chord joining the points of tangency.

The common external tangent segments of two circles are congruent.

If two circles and a line intersect in the same point, or points, then the line bisects each common external tangent segment of the circles.

The common internal tangent segments of two nonintersecting circles are congruent.

SUGGESTED LEARNING ACTIVITIES

1. In developing the common tangent relationship in circles, use ex. 23 page 459 of Moise-Downs Geometry.

2. Develop the Power of a Point Theorem through proportional sides of similar triangles. Then use "product of the means equals product of the extremes." Convey the idea that the word "power" is a unique number representing the answer to a multiplication problem.

III. CIRCLES
D. CONSTRUCTIONS

PERFORMANCE OBJECTIVES

The student will, using only compass and straightedge, construct:

1. A circle containing three given noncollinear points.
2. A circle circumscribed about a triangle.
3. A circle inscribed in a triangle.
4. The center of a given arc.
5. A tangent to a circle from a given external point.
6. A tangent to a circle at a given point on the circle.
7. Three congruent circles, each tangent to the other two, given the radius.
8. Two circles internally tangent, given the radius of each circle.
9. An equilateral triangle, given the radius of the inscribed circle.

The student will, using only compass and straightedge, try to construct a square which has the same area as a given circle.
(Impossible construction ---Moise, p. 506)

COURSE CONTENT

Vocabulary

external point	isosceles
noncollinear	equilateral

Associated Properties

Concurrence theorems (angle bisectors, perpendicular bisectors of sides, medians, altitudes of a triangle)

III. CIRCLES

D. CONSTRUCTIONS (continued)

Associated Properties (continued)

The Perpendicular Bisector Theorem:
Two points, each equidistant from the endpoints of a given segment, determine the perpendicular bisector of the segment.

A line perpendicular to a radius at its outer end is tangent to the circle.

Every tangent to a circle is perpendicular to the radius drawn to the point of contact.

The set of all points in a plane equidistant from the sides of an angle is the angle bisector.

SUGGESTED LEARNING ACTIVITIES

1. Filmstrip by Educational Projections Inc. #372, Locus.

2. Filmstrip by FOM - #1144, Locus problems

3. Do selected constructions using modern drafting implements.

4. Selected students will report onthe Impossible Constructions of Antiquity.

5. Throughout this quin, the teacher should inlcude more challenging construction problems from the many sources available in both the State adopted texts and the annotated bibliography at the end of this quin.

III. CIRCLES
E. CIRCUMFERENCE AND AREA

PERFORMANCE OBJECTIVES

The student will:

1. Find the circumference and/or area of a circle, given the radius or diameter or other necessary information.

2. Find the radius or diameter of a circle, given the circumference and/or area or other necessary information.

3. Find the ratio of the circumferences, areas, or radii of two circles given the measures of their circumferences, areas or radii.

4. Apply the associated properties from III A, B, and C to solve circumference and area problems, including those involving compound figures.

5. Use metric measure in above performance objectives.

COURSE CONTENT

Vocabulary

apothem annulus

Associated Properties

The circumference of a circle equals pi times the diameter, or two times pi times the radius. (πd or $2\pi r$)

The area of a circle equals pi times the square of the radius. (πr^2)

SUGGESTED LEARNING ACTIVITIES

1. Compare the areas available when serving dinner from a square, rectangular, and circular table, given a fixed perimeter.

2. Given a constant area, which shaped swimming pool would require the longest concrete walkway? (circular, rectangular, or square)

3. Include an annulus problem (flower bed, etc.)

III. CIRCLES

F. SPECIAL MEASUREMENT FORMULAS

PERFORMANCE OBJECTIVES

The student will:

1. Find the lengths of arcs, given necessary, information.
2. Find the areas of segments of a circle and sectors, given necessary information.
3. Find missing measures, given lengths of arcs and other necessary information.
4. Find missing measures, given area of segment of a circle or sector of a circle and other necessary information.

COURSE CONTENT

Vocabulary

length of arc
proportional
segment of a circle

Associated Properties

If two arcs have equal radii, then their lengths are proportional to their measures.

If an arc has measure q and radius r, then its length, L, is:

$$L = \frac{q}{180} \cdot r$$

The area of a sector is half the product of its radius and the length of its arc.

If a sector has radius r and its arc has measure q, then its area is:

$$A = \frac{q}{180} \cdot r^2$$

III. CIRCLES

F. SPECIAL MEASUREMENT FORMULAS

SUGGESTED LEARNING ACTIVITIES

1. Review algebra skills needed to solve formulas for any one of the variables.

2. Refine students' ability to select the appropriate formula by using a matching type question with a list of formulas to be matched with the proper question.

3. Have students create original designs with and without a spirograph.

4. Have students report on architectural applications of the circle, such as archways, etc.

IV. POLYGONS

A. KINDS AND RELATED PARTS

PERFORMANCE OBJECTIVES

The student will:

1. Identify convex polygons from assorted convex and non-convex diagrams.

2. Sketch a convex and non-convex polygon of a given number of sides.

3. Name the polygons having 3, 4, 5, 6, 7, 8, 9, 10, 12 and n sides.

4. Determine the number of diagonals of any given convex polygon.

5. Find the sum of the measures of the interior angles of any convex polygon having a given number of sides.

6. Given the sum of the measures of the interior angles, find the number of sides of any convex polygon.

7. Calculate the measures of the interior angles, central angle, and exterior angles of any regular polygon having a given number of sides.

8. Given the measure of an interior angle, central angle, or exterior angle of a regular polygon, find the number of sides.

9. Develop and apply the formula for the sum of the measures of the exterior angles of a convex polygon.

10. Distinguish between regular and non-regular polygons, given appropriate information in a diagram or verbal description.

STATE ADOPTED REFERENCES

	M	L	JD	A
CHAPTERS	11, 12, 16	10, 17	8, 15	13, 18

IV. POLYGONS

A. KINDS AND RELATED PARTS (continued)

Vocabulary

polygon
- triangle
- quadrilateral
- pentagon
- hexagon
- octagon
- decagon
- duodecagon
- heptagon
- n-gon
- nonagon
- vertex, vertices
- adjacent sides
- consecutive or successive sides and angles

polygonal region
interior
convex polygon
non-convex polygon
diagonal
interior angle
exterior angle
regular polygon
- central angle
- center
- radius
- apothem

Associated Properties

In any polygon the number of diagonals which can be drawn from any one vertex is (n-3).

In any given polygon the number of triangular regions formed by drawing all diagonals from one vertex is (n-2).

The number of diagonals of any n-gon is $\frac{n}{2}(n-3)$.

The sum of the measures of the interior angles of a convex polygon of n sides is $(n-2) \cdot 180$.

The sum of the measures of the exterior angles of a convex polygon of n sides is 360.

The sum of the measures of the central angles of a convex polygon is 360.

The measure of an interior angle of a regular n-gon is $180 - \frac{360}{n}$ or $\frac{(n-2)\,180}{n}$.

Associated Properties (continued)

The measure of a central angle of a regular n-gon is $\frac{360}{n}$.

The measure of an exterior angle of a regular n-gon is $\frac{360}{n}$.

The number of sides of a regular polygon is $\frac{360}{\text{measure of exterior angle}}$

$\frac{360}{\text{measure of central angle}}$.

The only three regular polygons which may be repeated to fill out a plane are an equilateral triangle, a square, and a regular hexagon.

If any given polygon, the number of sides, interior angles, exterior angles and central angles (in reg. polygon only) is the same.

SUGGESTED LEARNING ACTIVITIES

1. Filmstrip by FOM - #1140 Angle sums for Polygons

2. Overhead transparency, Moise-Downs #29

3. Selected students can report on "space filling figures." Refer to pages 19-21 of Irving Adler's "A New Look at Geometry."

4. Have students create original designs using regular polygons.

IV. POLYGONS

B. CONGRUENT AND SIMILAR POLYGONS

PERFORMANCE OBJECTIVES

The student will:

1. State and apply the definitions of similar polygons and congruent polygons.

2. Enlarge or reduce the size of a given polygonal drawing according to a give scale.

3. Construct a polygon congruent to a given polygon.

4. Given similar polygons and the measures of some of their sides, find the lengths of the other corresponding sides.

5. Determine if polygons are similar under a given set of conditions.

COURSE CONTENT

Vocabulary

irrational	reflexive
scale drawing	symetric
similar polygons	transitive
congruent polygons	equivalence

Associated Properties

Similarity of convex polygons is reflexive, symmetric, and transitive, and is, therefore, an equivalence relation. (See Moise, pg. 118)

All congruent figures are similar, but not all similar figures are congruent.

SUGGESTED LEARNING ACTIVITIES

1. Cut out cardboard models to match and compare shapes and sizes.

2. Students should research Home magazine for "odd" shaped floor plans.

3. Have students develop original plans for a room, house, table, etc.

4. Students can develop original tile designs for a floor and research Turkish & Indian tile designs using familiar polygons.

IV. POLYGONS

C. PERIMETER AND AREA

PERFORMANCE OBJECTIVES

The student will:

1. Determine the perimeter of a polygon given the length of its sides.

2. Determine the perimeter of a regular polygon given the length of one side.

3. Determine the area of a regular polygon given sufficient information.

4. Determine the perimeter and area of compound figures, given the necessary measures.

5. Given the area of a regular polygon and other necessary measures of the polygon, find the missing measures of other parts.

6. Determine the area of a regular octagon, given the radius of the octagon.

7. Determine the area of a regular hexagon, given the radius of the hexagon.

8. Develop and apply the special formula for the area of a regular hexagon $\frac{a\ p}{2}$.

9. Determine the radius, circumference, and area of the inscribed and circumscribed circles, given appropriate measures of a regular polygon.

10. Determine the ratio of the perimeters and areas given the ratio of the apothems or sides of two similar polygons, and vice versa.

11. Determine the area of a regular hexagon given the perimeter, and vice versa.

12. Develop and apply the formula for the area of a regular polygon.

13. Apply metric measure, where appropriate, to above objectives.

IV. POLYGONS

C. PERIMETER AND AREA

COURSE CONTENT

Vocabulary

limit
compound figures
regular

Associated Properties

The area of a regular hexagon is $3\sqrt{\frac{3}{2}}\ S^2$

The ratio of the areas of similar polygons equals the square of ratio of their corresponding apothems, sides, radii, perimeters.

The perimeter of a regular polygon equals the product of the length of a side times the number of sides.

The area of a regular polygon equals the product of the length of a side times the number of sides.

The area of a regular polygon equals 1/2 the product of the length of the apothem and perimeter.

As the number of sides of a regular polygon inscribed in a circle increases, its perimeter approaches the circumference of the circumscribed circle; its apothem approaches the radius of the circumscribed circle; the length of its sides approach zero; the measure of an interior angle approaches 180; the area of the inscribed polygon approaches the area of the circumscribed circle.

SUGGESTED LEARNING ACTIVITIES

1. Use selected designs & floor plans from IV B, assign appropriate measures, then have students compute perimeters & areas.

2. Refer to Tanagrams & games such as "Pythagoras" and "Euclid"

IV. POLYGONS

C. PERIMETER AND AREA (continued)

Suggested Learning Activities (continued)

3. Students can create their own tanagrams starting with a piece of graph paper and a square 20 boxes by 20 boxes. They are to cut 5 or 6 different shapes from that square, compute each area, and add them up again to match the area of the original square.

4. Use geoboard to create and compute measurements of various polygonal shapes.

IV. POLYGONS

D. CONSTRUCTIONS

PERFORMANCE OBJECTIVES

Using only a compass and straightedge, the student will construct:

1. a circle inscribed in a regular polygon.
2. a circle circumscribed about a regular polygon.
3. a regular hexagon
4. an equilateral triangle inscribed in a circle.
5. a square inscribed in a circle.
6. a regular octagon inscribed in a circle.
7. a polygon congruent to a given polygon.
8. a smaller or larger polygon similar to a given polygon using a given ratio of the lengths of the sides.

COURSE CONTENT

Vocabulary

inscribed polygon
circumscribed polygon

Associated Properties

A circle can be inscribed in and circumscribed about, any regular polygon.

In a circle, the length of a side of an inscribed regular hexagon equals twice the apothem of an inscribed equilateral triangle.

The radius of a regular hexagon is equal to the length of its side.

SUGGESTED LEARNING ACTIVITIES

1. Any mechanical drawing or graphics book will provide excellent additional problems for this unit.

2. Use "Kaleidoscope Geometry" to construct various polygons, as explained in Readings in Geometry an NCTM publications mentioned in the annotated bibliography.

3. Throughout this quin, the teacher should include more challenging construction problems from the State-adopted texts and the sources listed in the annotated bibliography.

IV. POLYGONS

E. TESSELLATIONS - ENRICHMENT ACTIVITY

PERFORMANCE OBJECTIVES

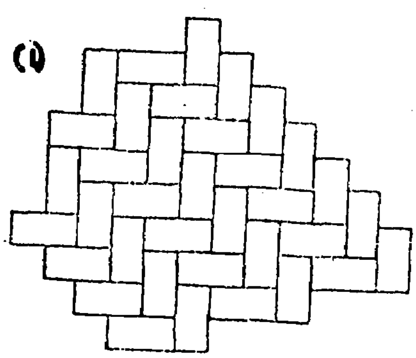
The student will:

1. Identify tessellation as using a repeated pattern to fill a plane.
2. Name the three regular tessellations.
3. Name correctly a mathematical, semi-regular tessellation.
4. Draw a tessellation.

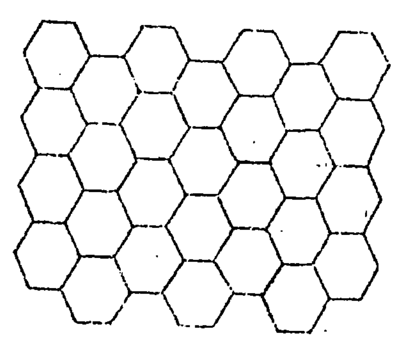
Activities: Read the following and do the problems.

I. The ancient Pythagoreans began the study of space-filling figures. The object of their study was to find patterns using polygonal regions which could be repeated in order to completely fill a plane. These patterns became known as tessellations. Here are three examples of tessellations:

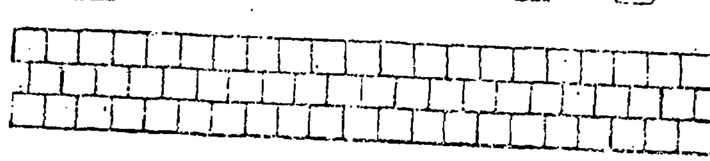
(i)



(ii)



(iii)

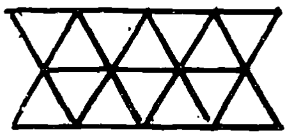


The unit of tessellation in (i) is a rectangular region; in (ii) is a hexagonal region; in (iii) is a square region. We say,

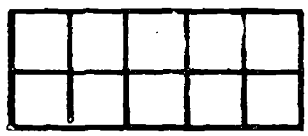
in each case, that the unit <u>tessellates</u> because the whole plane can be completely filled by repeating the smaller congruent regions. (Note: henceforth in this unit, terms such as triangle, square, etc. will be used to mean the region bounded by that figure.)

There are three, <u>regular</u> tessellations (see figures below). That is, tessellations formed by repeating a single regular polygon. These are formed by using equilateral triangles, squares, or regular hexagons.

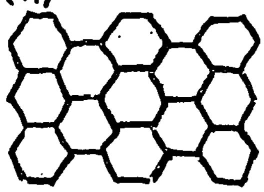
(iv)



(v)



(vi)



Problems: 1. Find another arrangement of squares which will still form a regular tessellation. Are there other possible arrangements using the triangle? the hexagon?

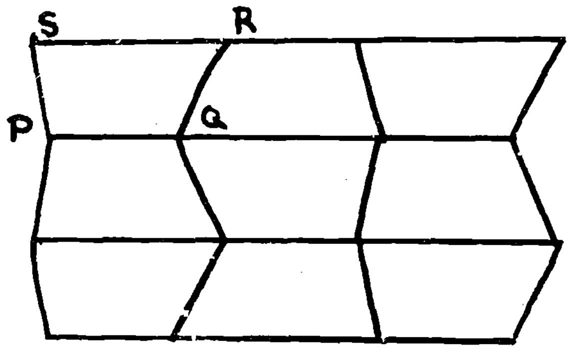
2. Try to make a tessellation by using a regular pentagon. What happens? Try using any other regular polygon. What happens?

IV. POLYGONS

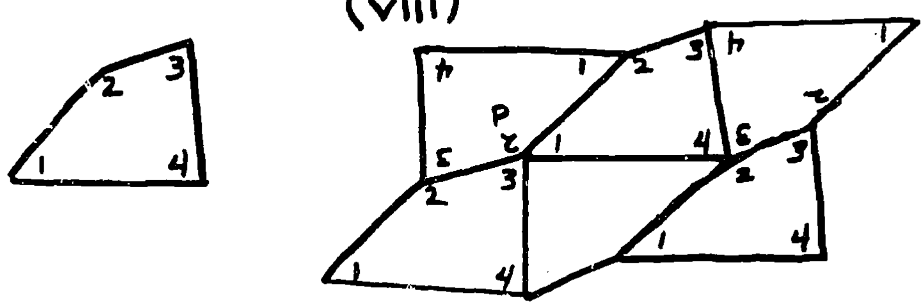
E. TESSELLATIONS - ENRICHMENT ACTIVITY (continued)

Other tessellations can be made by repeating a single polygon, but only if the polygon is non-regular. Look at the following examples:

(vii)



(viii)



In example (vii) the trapezoid PQRS is repeated and in example (vii) the trapezium at the left is repeated. In fact, any quadrilateral will tessellate a plane, as will any triangle.

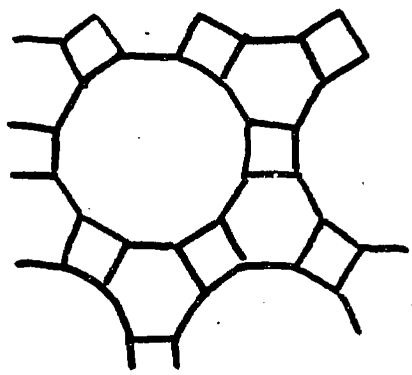
3. Draw a tessellation using a non-regular polygon.

II. So far the only tessellations you have seen have been formed by repeating a single polygon. Can tessellations be found that use two or more polygons? Yes.

The first group of combination tessellations we shall look at are called the semi-regular tessellations. These, as you

might guess from the name, are formed by using combinations of regular polygons. There are only eight semi-regular tessellations.

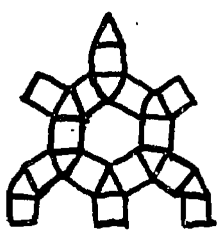
Look at the following example. It is semi-regular and it is also "mathematical".



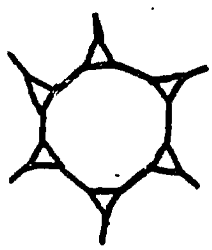
To be a "mathematical" tessellation, the polygons surrounding <u>any</u> vertex must always be the same. In the above, the polygons are a squre (4 sides), a regular hexagon (6 sides) and a regular dodecagon (12 sides). This is called a 4 6 12 tessellation. Can you guess why ?

Here are three more of the semi-regular tessellations and their names.

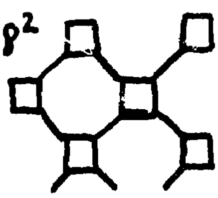
$3 \cdot 4^2 \cdot 6$



$3 \cdot 12^2$



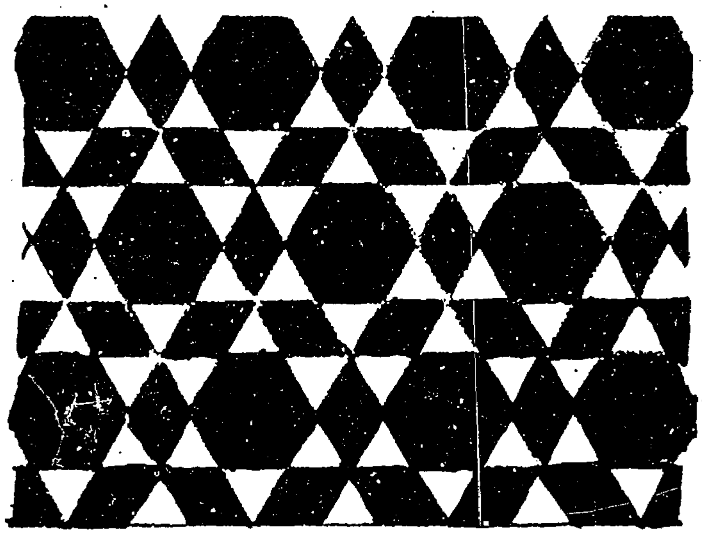
$4 \cdot 8^2$



Problems: 1. Find the other four semi-regular tessellations and name them. (Hint: You will use combinations of squares, triangles, and/or hexagons.)

Lastly, we have those tessellations which are formed by using combinations containing non-regular polygons. There are many of these, some are mathematical, some are not.

2. Draw another tessellation using a regular and a non-regular polygon. Here is one example which is not mathematical:



3. Make a list of all the present day uses you can find of tessellations (example: chess board, bathroom floor tile, etc.). See who in your class can come up with the longest list.

IV. POLYGONS

F. FLEXAGONS - ENRICHMENT ACTIVITY

PERFORMANCE OBJECTIVES

The student will:

1. Define, construct, and flex a trihexaflexagon.

Special Materials: Scissors, paste.

Activity:

Follow directions carefully!

I. Constructing the special hexagon.

1. Cut out a strip on the second page after this.

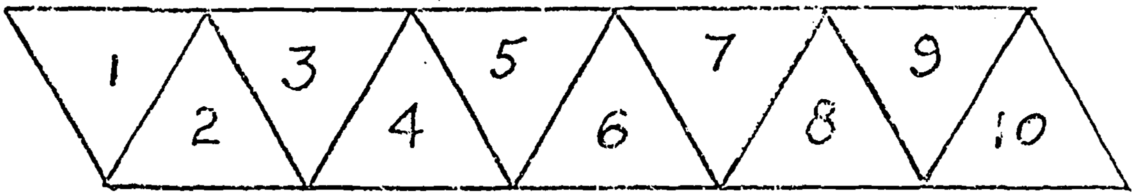


figure 1

2. Fold the strip backward along segment AB as shown in figure 2. Note: Solid segments represe. edges of the paper and dotted segments represent sides of triangles. Always keep 1, 2, 3, in the position as shown.

3. Fold the top of the strip backward again along segment CD, slipping triangle 9 on top of triangle 1 as shown in figure 3

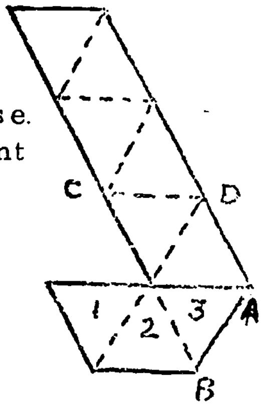


figure 2

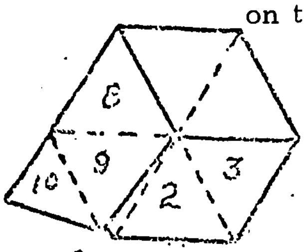


figure 3

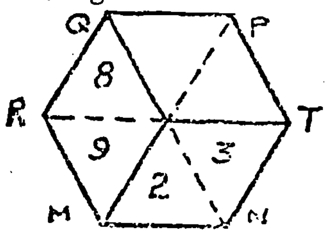


figure 4

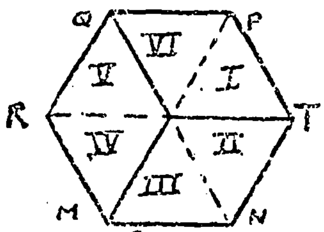


figure 5

4. Triangle 10 is now folded back under triangle 1 and glued in place.

5. You should have a hexagon as shown in figure 4.

IV. POLYGONS

F. FLEXAGONS - ENRICHMENT ACTIVITY (continued)

6. Shade one side of the hexagon with your pencil and the other side with a pen.

7. You now have a trihexaflexagon.

8. Now follow the directions carefully and discover the hidden face which will then give your model three different faces which can be brought into view. That's why we call it a trihexaflexagon.

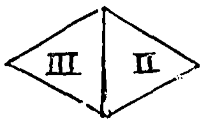
II. Flexing the trihexaflexagon

1. To make it easier to flex the model you have made, fold the hexagon in half along segment RT. Crease it carefully. Unfold it. Now fold in half along MP. Unfold. Now fold in half along QN. Unfold.

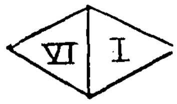
2. Now hold the trihexaglexagon horizontally in front of you.

3. Pinch two adjacent triangles (I and II of figure 5) and fold at T.

4. Fold the opposite two triangles under (IV and V) and in so that you have a figure like this:



facing you



underside

5. Unfold this figure from the top. (If it won't unfold, try again with different triangle combinations).

6. You should now have a hexagon. Is it the same? How can you tell? Remember, you colored one face of the hexagon with pen and one face with pencil in step 7 when you were making the model.

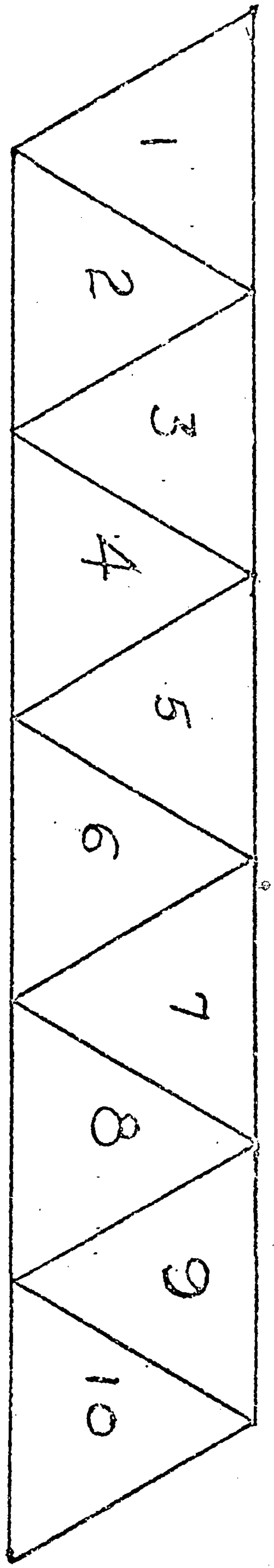
7. Try steps 3 - 6 again.

Conclusion:

Although we made our flexagon primarily for mathematical amusement, many distinguished mathematicians have tried to work out the mathematical theory of flexagation. The theory of how to construct a flexagon of any desired size or species has been a challenge to many learned men. Perhaps you would like to do more work with flexagation.

Ask your teacher to help you make a hexahexaflexagon. (Refer to the Scientific American Book of Mathematical Puzzles and Diversions by Martin Gardner, Simon and Schuster, New York).

MODEL FOR TRIHEXAFLEXAGON



III. The Hexa-tetraflexagon

Certain paper polygons folded from strips of paper which change their faces when flexed are called flexagons. They were "discovered" in 1939 by a young English graduate student at Princeton as he fiddled with inch-wide strips he had torn from his American notebook sheets so that they would fit his English binder. Flexagons were soon the rage of the Princeton graduate school. In fact, a committee was promptly formed to investigate their mysteries.

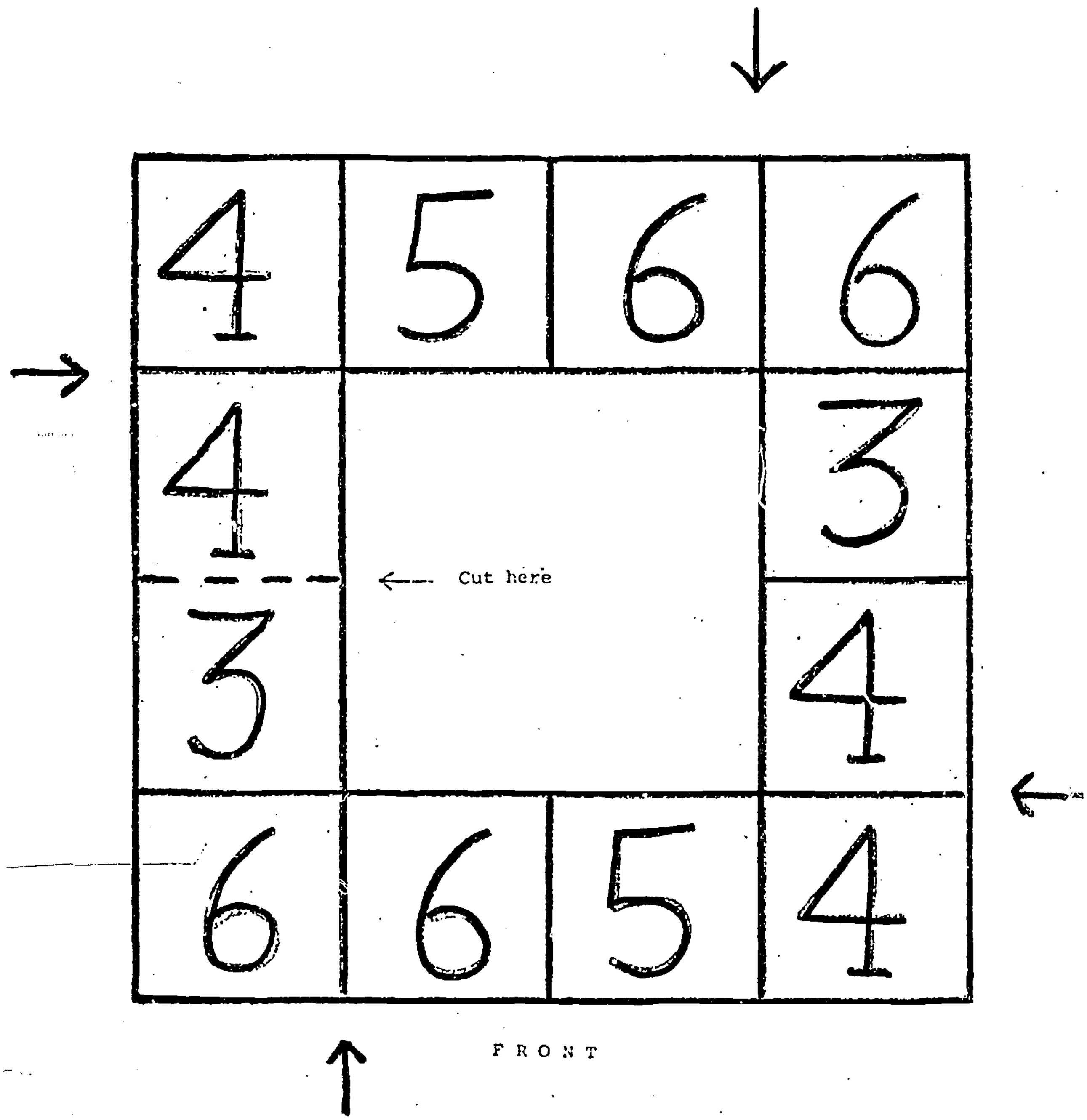
You will find that your students--high achievers or low, elementary school students or high schoolers--will be just as fascinated with flexagons as were the Princeton graduate students 30 years ago. The great challenge to elementary school students will be folding and flexing the flexagons. Older, more mathematically mature students will want to count the number of differant faces possible, find the easiest way to bring out all faces of a flexagon, and learn how to make flexagons of any size and species.

In this issue you will find directions and a pattern for one of the simplest types of flexagons called a hexa-tetra flexagon. (Ask your students to explain the reason for the name given the flexagon.) You may find it easiest to ditto the pattern off on construction paper for your students.

Beware! You too may become a flexagon addict!

IV. POLYGONS

F. FLEXAGONS - ENRICHMENT ACTIVITY (continued)



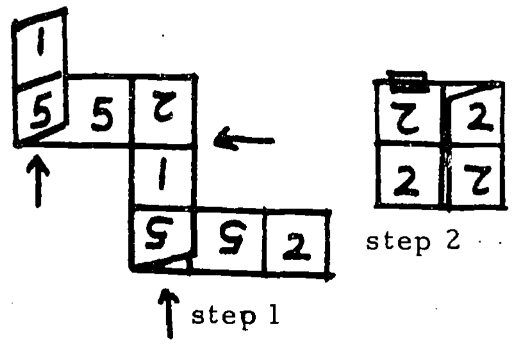
DIRECTIONS FOR HEXA-TETRAFLEXAGON

TO FOLD:

1. Crease up along all lines indicated by arrow in pattern.

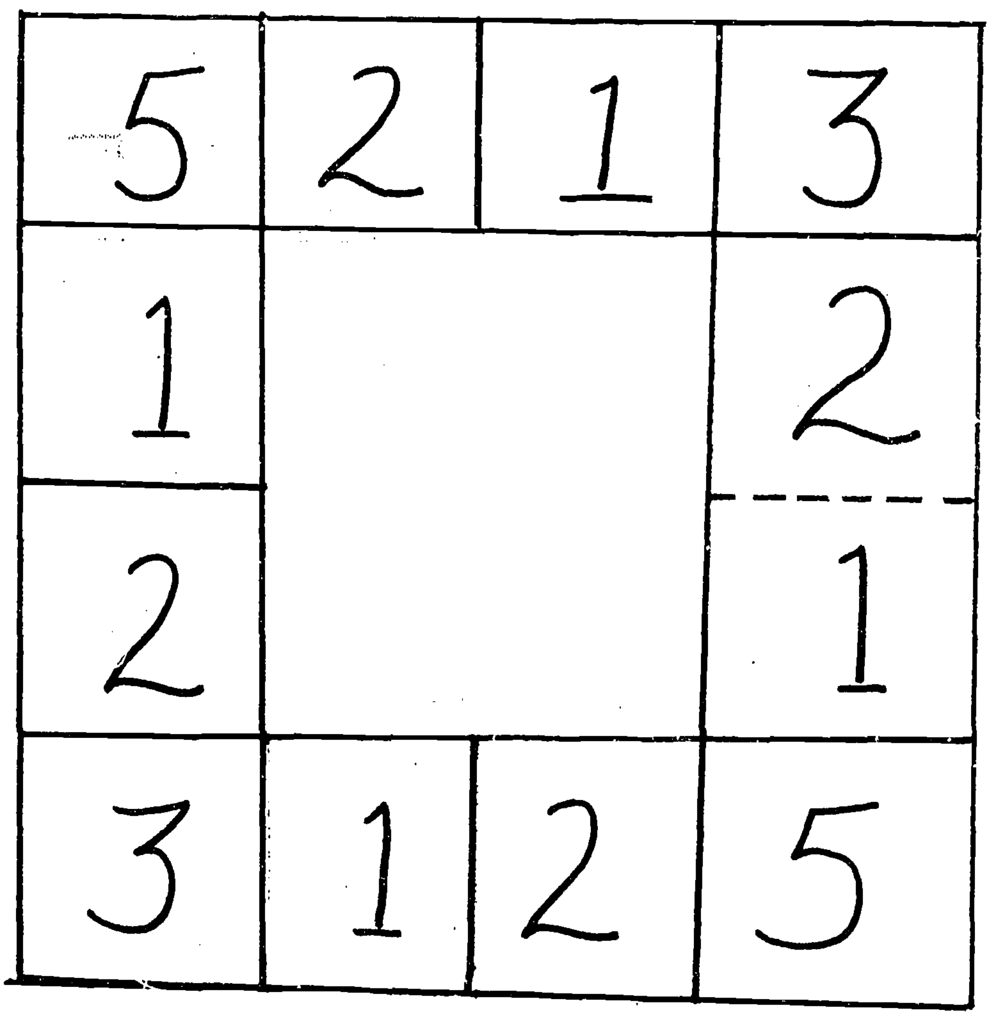
2. Crease all lines indicated by arrows in diagram for Step 1. Overlap so that all 2's show.

3. Tape with transparent tape as shown in diagram for Step 2. Overlap tape so that it hits 1 square on other side.



TO FLEX

4. Fold on both horizontal and vertical axes. How many faces can you expose?



BACK

V. THREE-DIMENSIONAL FIGURES (SOLIDS)
A. KINDS AND RELATED PARTS

PERFORMANCE OBJECTIVES

The student will:

1. Describe the method of generation and the required parts for a given solid.

2. Sketch the solids named in the vocabulary list and identify their parts.

3. Match items in a verbal list of solids and related parts with items given pictorially.

STATE ADOPTED REFERENCES

	M	L	JD	A
CHAPTERS	17	18	16	20

ADDITIONAL REFERENCES

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COURSE CONTENT

Vocabulary

solid
prism
pyramid
cylinder
cone
right prism
lateral surface
total surface
volume
interior
exterior
regular pyramid
axis
base
face
edge
cross sectionplane section
polyhedral angle
triangular prism
tetrahedron
regular solid
Platonic solid
frustum
slant height
lateral face
lateral edge
regular octahedron
circular cylinder
right circular cylinder
right circular cone
cylinder of revolution
tangent plane
parallelepiped
rectangular parallelepiped
cube
similar solids
congruent solids
dodecahedron
icosahedron
platonic solids

Associated Properties

All cross sections of a prism are congruent to the base.

The lateral faces of a prism are parallelogram regions.

The lateral faces of a right prism are rectangular regions.

Dual Solids: The centers of the six faces of a cube are the vertices of an octahedron. The centers of the eight faces of an octahedron are the vertices of a cube. The dodecahedron and the icosahedron are also dual solids.

The five regular or platonic solids are:

cube - six faces (squares)
four vertices
twelve edges

V. THREE-DIMENSIONAL FIGURES (SOLIDS)
A. KINDS AND RELATED PARTS (continued)

Associated Properties (continued)

tetrahedron - four faces (equilateral triangles)
four vertices
six edges

octahedron - eight faces (equilateral triangles)
six vertices
twelve edges

dodecahedron - twelve faces (regular pentagons)
twenty vertices
thirty edges

icosahedron - twenty faces (equilateral triangles)
twelve vertices
thirty edges

SUGGESTED LEARNING ACTIVITIES

1. Students can make models of the various polyhedrons as discussed in a book by Wenninger, Magnus J. Polyhedron Models for the Classroom. Washington, D.C., 1970

V. THREE-DIMENSIONAL FIGURES (SOLIDS)
B. AREA (LATERAL, TOTAL, CROSS-SECTIONAL)

PERFORMANCE OBJECTIVES

The student will:

1. Compute the lateral, total or cross-sectional area of any prism, pyramid, cone, cylinder, or sphere, given the appropriate measures.

2. Compute the lateral and total area of a compound figure given the necessary measures.

3. Compute the length of an edge, the slant height, the altitude, etc., given the lateral, total, or cross-section area of a certain solid and other necessary measures.

4. Compute the area of a cross-section of a solid given a distance from the base of the solid, given the altitude and base of the solid.

5. Compare cross-sectional areas of the same solid given the necessary measurements.

6. Compare lateral edges, altitudes and bases of two similar solids, given the ratio of the areas of two corresponding cross-sections.

7. Apply metric measure where appropriate to above objectives.

COURSE CONTENT

Vocabulary

area	radius (r)
lateral area (LA)	length of base edge (e)
total area (TA)	altitude (h)
cross sectional area	area of Base (B)
slant height (l)	perimeter of Base (p)

Associated Properties

Right Prism	Regular Pyramid
LA = ph	LA = 1/2 pl
TA = ph + 2B	TA = 1/2 pl + B

Associated Properties (continued)

Right circular cylinder

$LA = 2\pi rh$

$TA = 2\pi(rh + r^2)$

Right circular cone

$LA = \pi rl$

$TA = \pi rl + \pi r^2$

Sphere

$SA = 4\pi r^2$

Frustum of a regular pyramid

$LA = (1/2)l\ (p_1 + p_2)$

Frustum of a right circular cone

$LA = (1/2)l\ (2\pi r_1 + 2\pi r_2) = l\ (R_1 + R_2)$

The lateral areas and total surface areas of 2 similar solids have the same ratio as the squares of the lengths of any pair of corresponding altitudes, slant heights, radii, edges, etc.

Every cross section of a triangular pyramid, between the base and the vertex, is a triangular region similar to the base. If h is the altitude, and k is the distance from the vertex to the cross section, then the area of the cross section is equal to k^2/h^2 times the area of the base.

In any pyramid, the ratio of the area of a cross section to the area of the base is k^2/h^2, where h is the altitude of the p pyramid and k is the distance from the vertex to the plane of the cross section.

If two pyramids have the same base area and the same altitude, then cross sections equidistant from the vertices have the same area.

Every cross section of a circular cylinder has the same area as the base.

Given a cone of altitude h, and a cross section made by a plane at a distance k from the vertex. The area of the cross section is equal to k^2/h^2 times the area of the base.

V. THREE-DIMENSIONAL FIGURES (SOLID)

C. VOLUMES

PERFORMANCE OBJECTIVES

The student will:

1. Compute the volume of a certain solid given the appropriate measurements.
2. Compute the length of a selected segment or area, given the volume of a certain solid and other needed measurements.
3. Find the ratio of the volumes of similar solids given the ratio of their corresponding segments and vice versa.
4. Apply Cavalieri's Principle.
5. Find the volume of the second of two solids, given the volume of the first solid and an appropriate ratio of a pair of corresponding parts of the two solids.
6. Apply the metric measure where appropriate to above objectives.

COURSE CONTENT

Vocabulary

volume (v)
altitude (h)
area of base (B)
edge (e)
radius (r)

Associated Properties and Formulas

Frustum = $\frac{1}{3}h(B_1 + B_2 + \sqrt{B_1B_2})$

Right Prism V = (Bh)

V. THREE-DIMENSIONAL FIGURES (SOLID)
C. VOLUMES (continued)

Associated Properties (continued)

Regular Pyramid $V = \frac{1}{3} Bh$

Right Circular Cylinder $V = \pi r^2 h$

Right Circular Cone $V = \frac{1}{3} \pi r^2 h$

Sphere $V = \frac{4}{3} \pi r^3$

The volumes of similar solids have the same ratio as the cubes of the lengths of any pair of corresponding altitudes, slant heights, edges, radii, etc.

Cavalieri's Principle: Given two solids and a plane. Suppose that every plane parallel to the given plane, intersecting one of the two solids, also intersects the other, and gives cross sections with the same area, then the two solids have the same volume.

V. THREE-DIMENSIONAL FIGURES (SOLIDS)
D. POLYHEDRON MODELS - ENRICHMENT ACTIVITY

PERFORMANCE OBJECTIVES

The student will:

1. Be able to identify cubes, rectangular prisms, triangular prisms, pentagonal prisms, square pyramids, triangular pyramids, oblique prisms, cylinders and cones.

2. Tell the number of faces and edges and vertices in each of these figures.

Special Materials: Patterns distributed by the teacher, scissors, paste or tape.

Activities: I. Working in groups of 4 or 5, each group will assemble one set of solid models and study the shapes of each model. II Each student will then complete the following table.

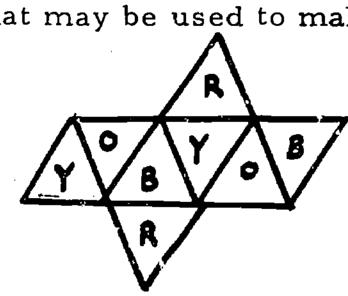
1.

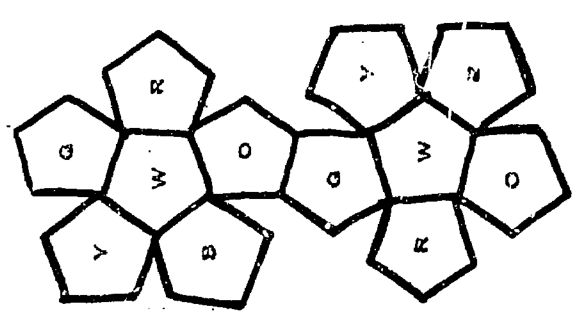
solid	cube	rectangular prisms	triangular prism	pentagonal prism	square pyramid	triangular pyramid	cylinder	cone
faces								
edges								
vertices								

2. Compute f - e + v for each solid. What conclusion do you reach?

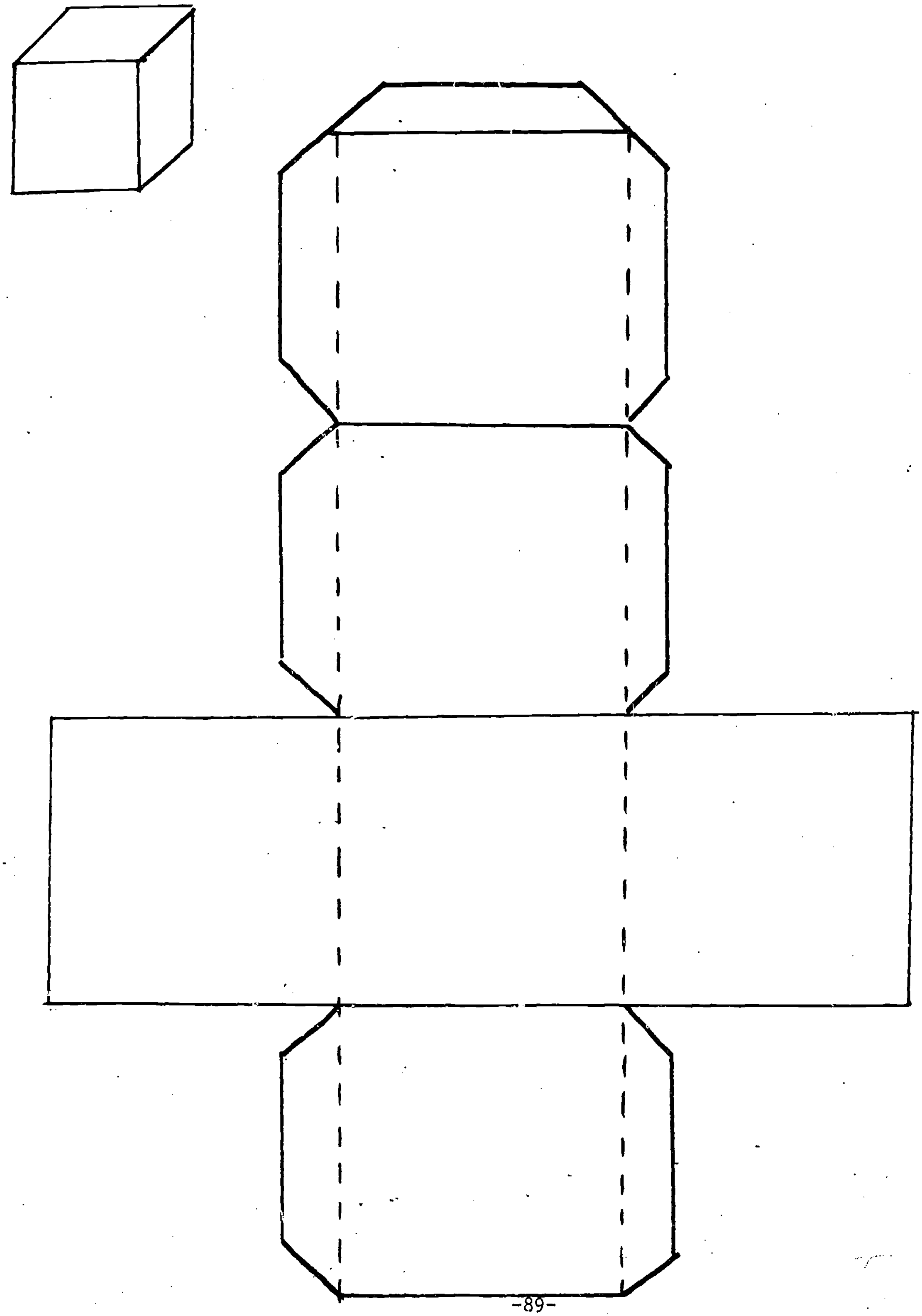
3. Try to make a pattern for a hexagonal prism.

III. Bonus: Use the following patterns as guides and make other models. Try to find out the name of these models. The letters suggest colors that may be used to make the models more interesting.

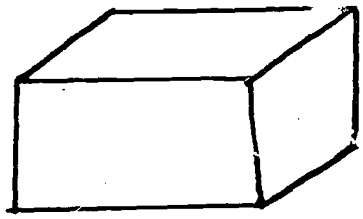


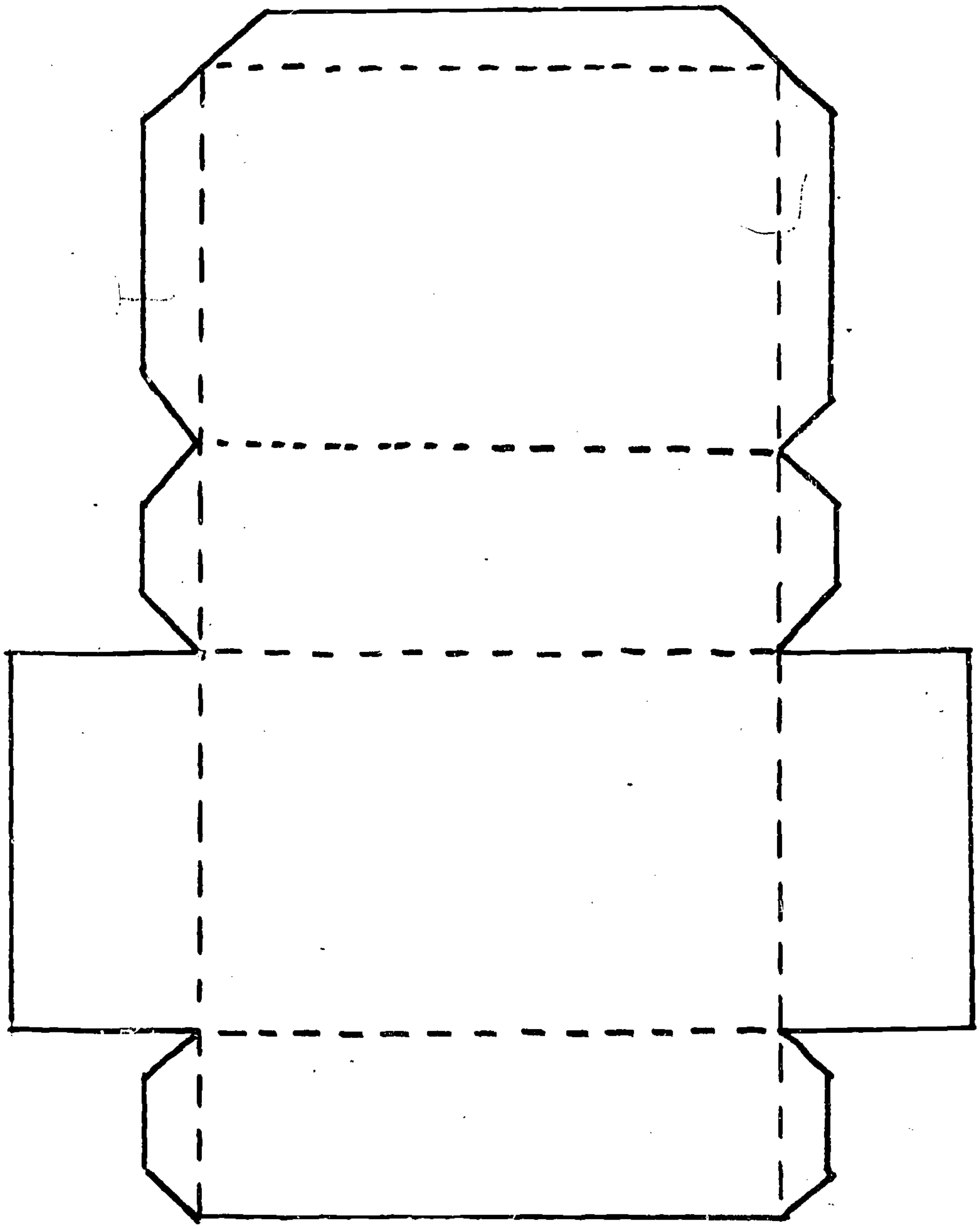


1. Pattern for a cube. When the model is finished it should look like a closed box.

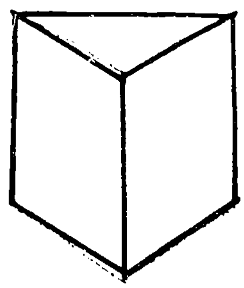


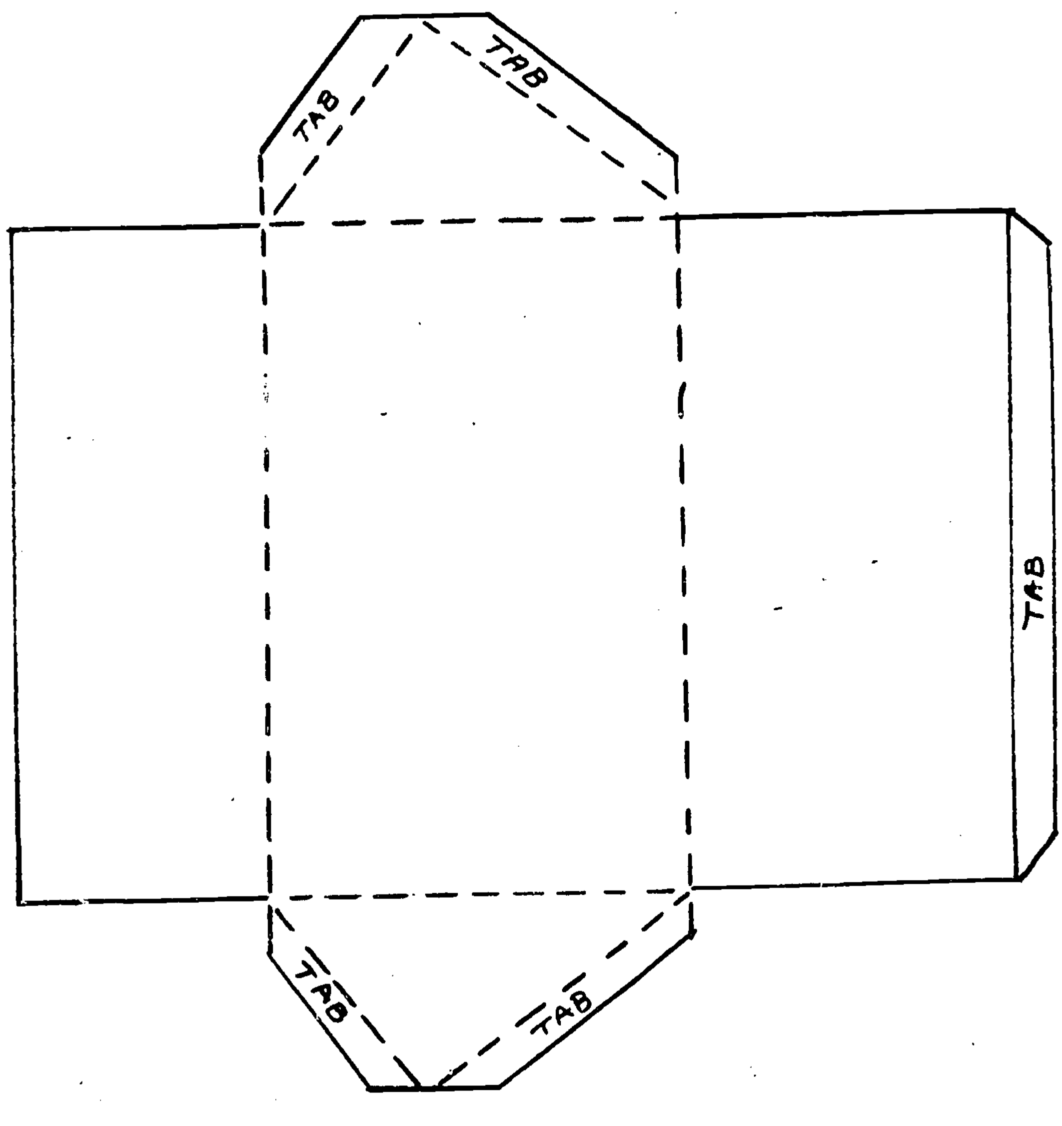
2. Pattern for a rectangular prism. The finished model should look like this:



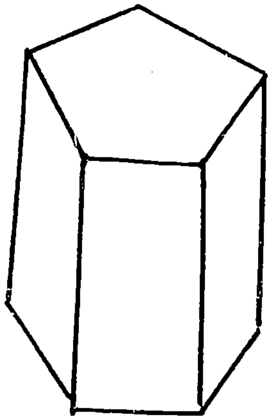


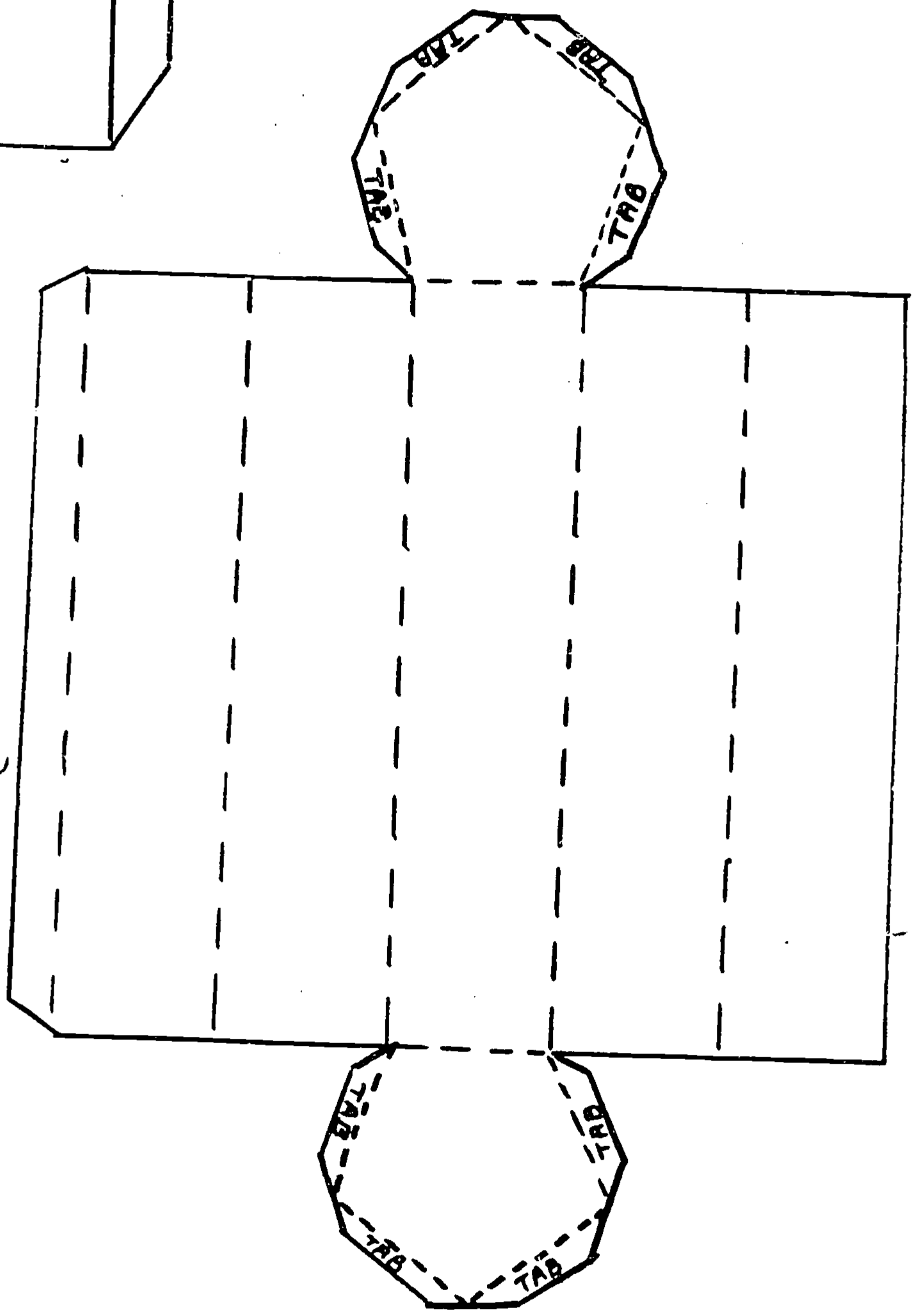
3. Pattern for a triangular prism. The finished model should look like this:



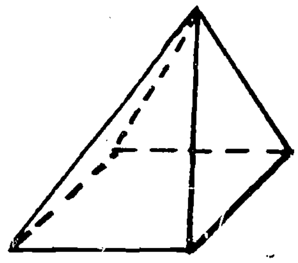


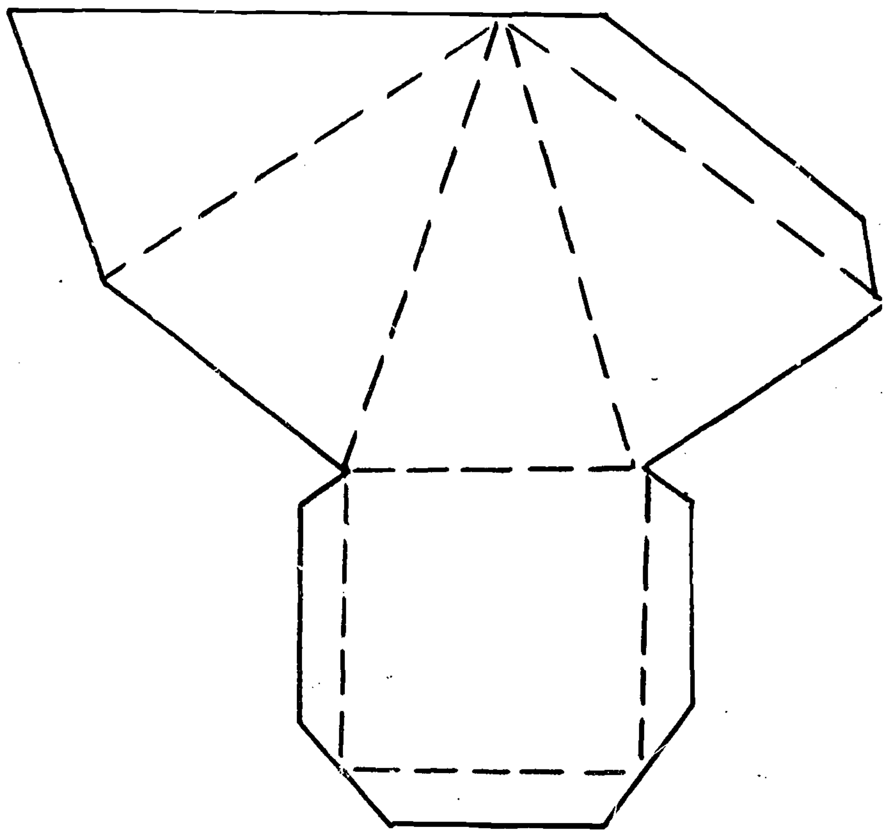
4. Pattern for Pentagonal Prism. The finished model should look like this:



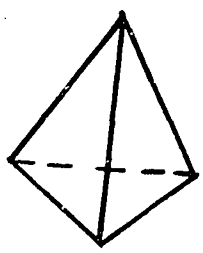


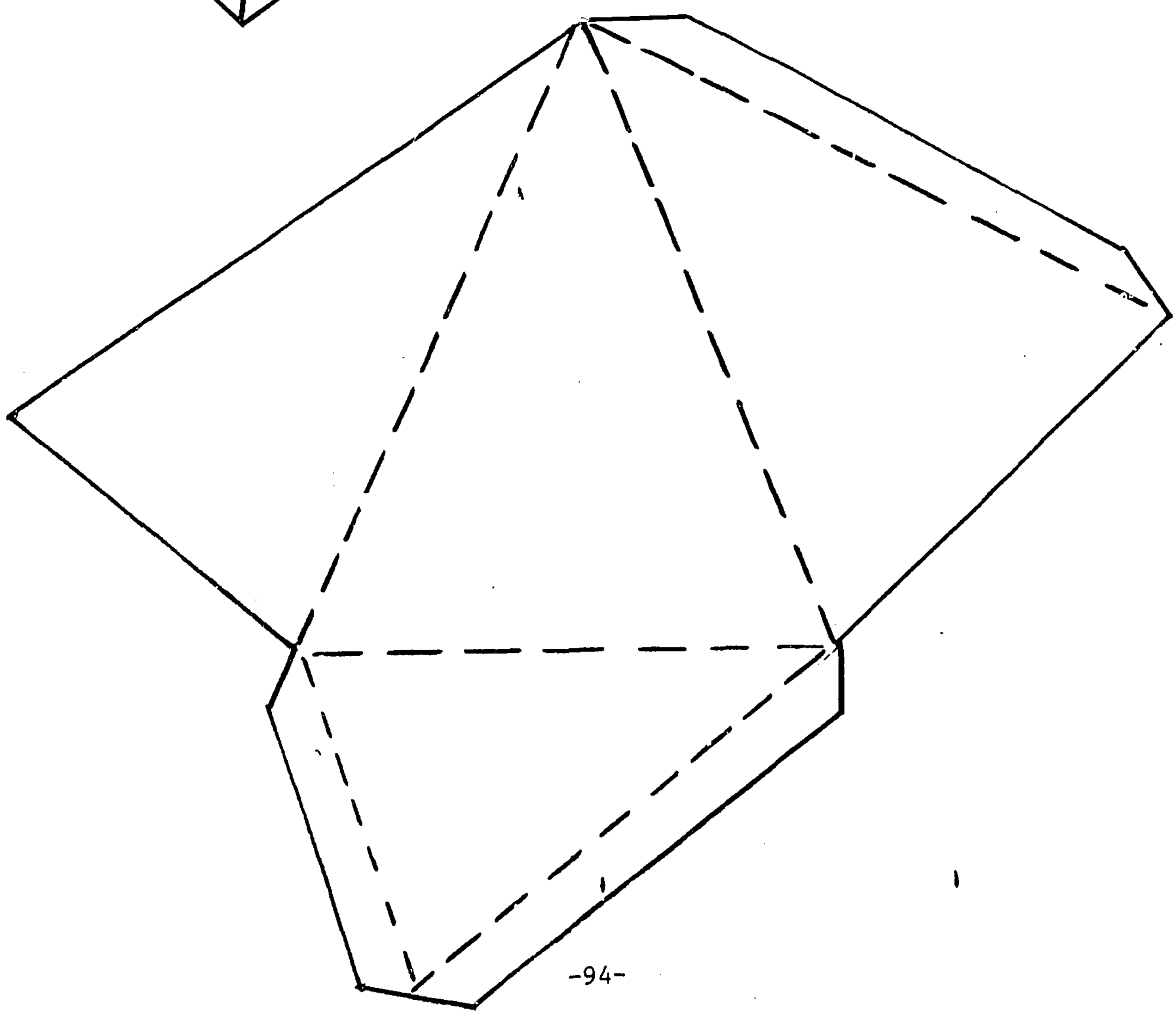
5. Pattern for a regular square pyramid. The finished model should look like this:



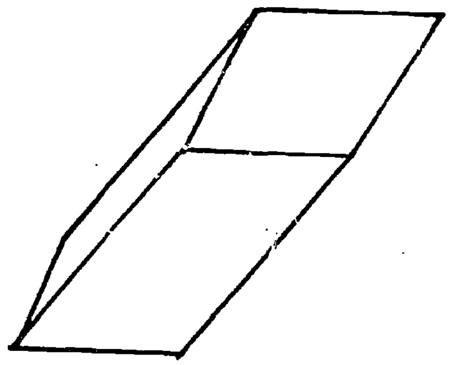


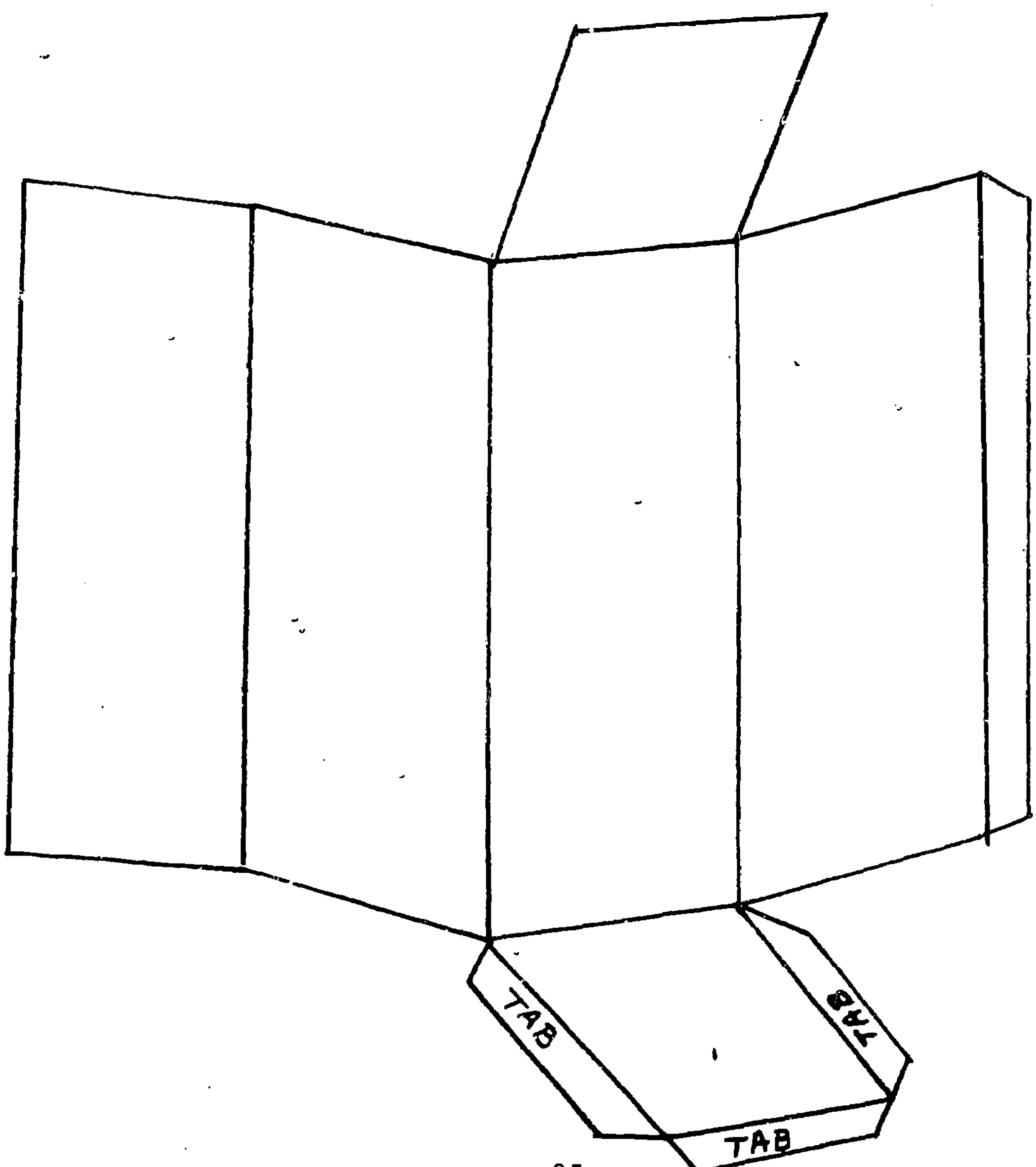
6. Pattern for a triangular pyramid (tetrahedron). The finished model should look like this:



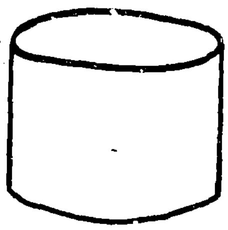


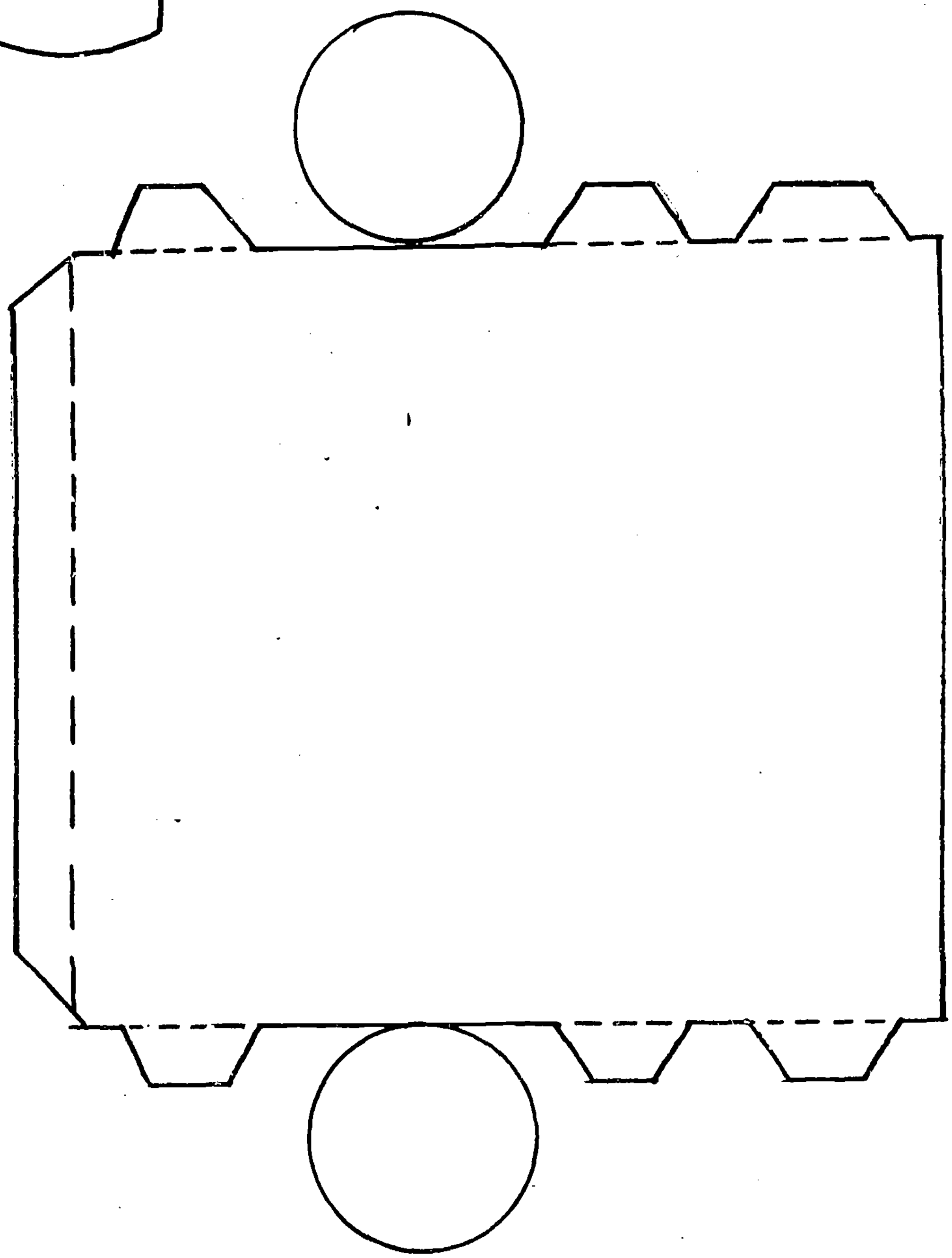
7. Pattern for a model of an oblique prism. The model should look like this:



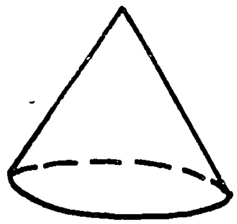


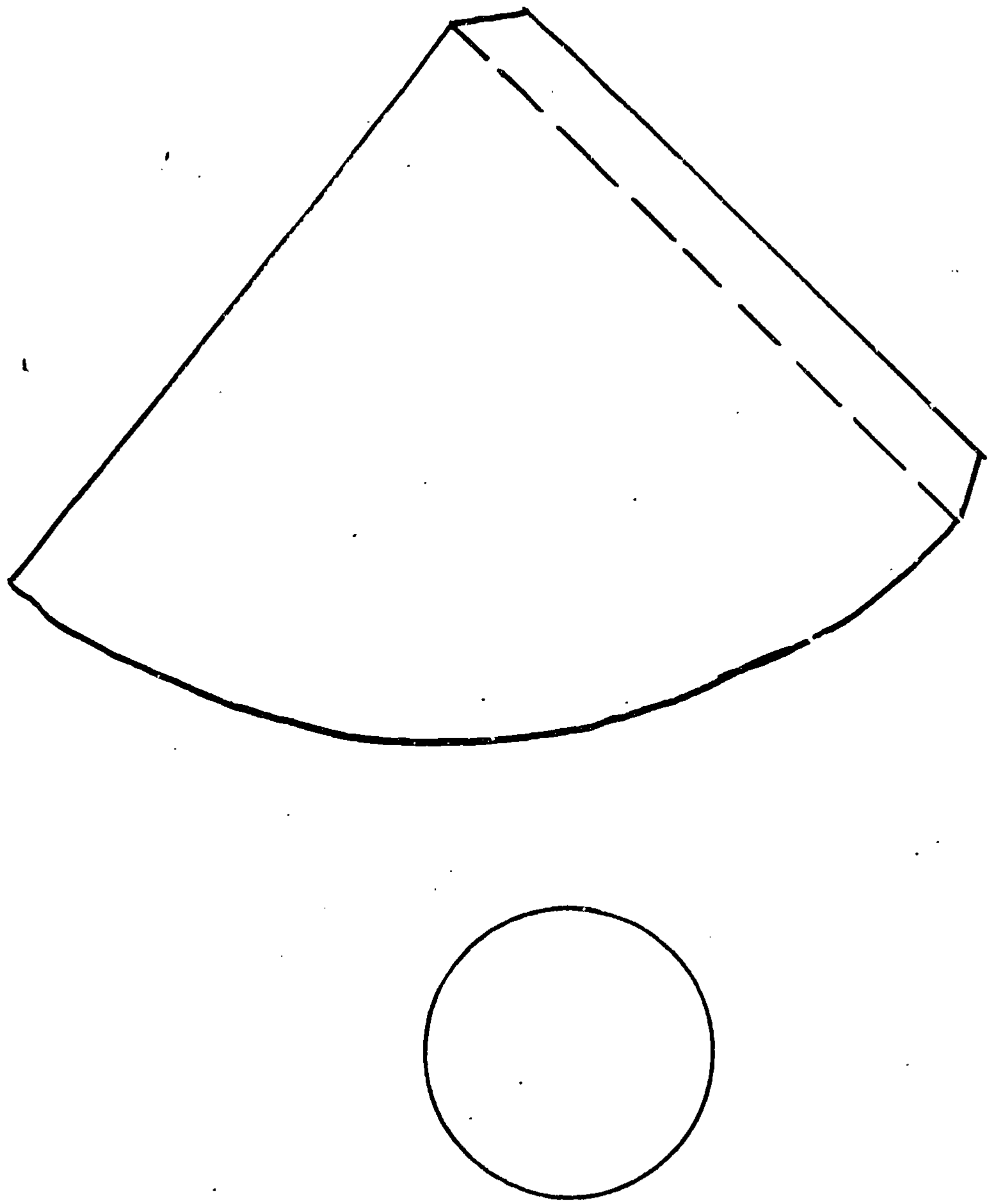
8. Pattern for a right circular cylinder. Your model should look like this:





9. Pattern for a right circular cone. The finished model should look like this:





V. THREE-DIMENSIONAL FIGURES (SOLIDS)
E. PROJECTIONS - ENRICHMENT ACTIVITY

PERFORMANCE OBJECTIVES

The student will:

1. Learn to visualize the different shadows that may be cast by solid models when they are held in various positions.

2. Be able to identify the shapes which could cast a given shadow.

Special Materials: A strong light such as an overhead projector, slide projector, flashlight, etc. Several sets of polyhedron models including: cylinder, cone, sphere, cube, rectangular prism, triangular prism, square pyramid, triangular pyramid and octahedron. Any other available models may also be used.

Activities: Sit in small groups, 4 or 5 students in a group. Each group should have a set of polyhedron models. (You may use the ones you made in class earlier.)

I. 1. Your teacher is holding one of these models in her hand to produce the shadow you see.

a. What model do you think she's holding ______________________

b. What model was she holding? ______________________

c. Will any other model you have produce a shadow with the same shape? ________ If so which one(s)? ________________

2. Now study the models you have in front of you and see how many different shadows you think you could produce using each of these models. Make a list of the shadows that you think each model will make.

Cylinder	Cone	Sphere
______________	______________	______________
______________	______________	______________
______________	______________	______________
Cube	Rectangular Prism	Triangular Prism
______________	______________	______________
______________	______________	______________
______________	______________	______________
Square Pyramid	Triangular Pyramid	Octahedron
______________	______________	______________
______________	______________	______________
______________	______________	______________

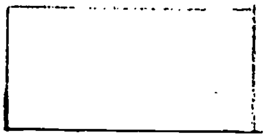
3. Your teacher will have students come up and demonstrate the various shadows which can be produced. If you see one that you did not have, be sure that you add it to your list.

II. Now without looking at the lists you made above see how well you can remember what you have learned by completing the following quiz.

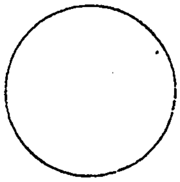
Below are shadows of geometric models you have studied. Below each shadow write the name or names of the models that might make such a shadow.

1.

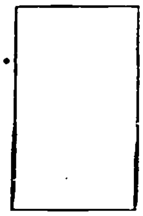
2.



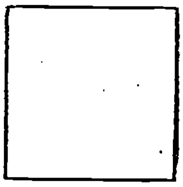
3.



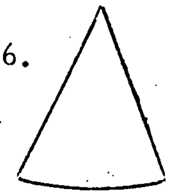
4.



5.

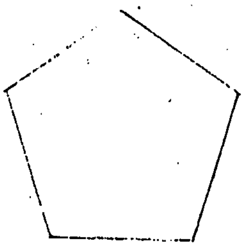


6.

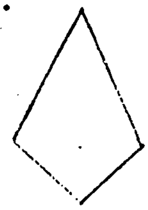


7.

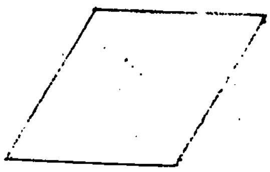
8.



9.



10.



VI. INTRODUCTION TO TOPOLOGY - ENRICHMENT ACTIVITY

PERFORMANCE OBJECTIVES

The student will:

1. Make a Moebius strip and list at least two properties of it.

Special Materials: Notebook paper; scissors; tape, glue, or staples.

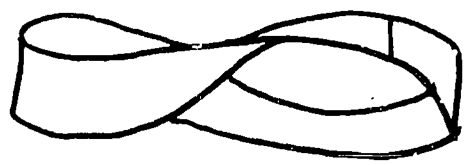
Activity: A German mathematician by the name of Augustus Ferdinand Moebius (1790-1868) made a discovery which was very important to geometry and fun to play with. He created a one-sided piece of paper!

Look at this piece of paper. If you draw a line down the center of this page, it appears on one side only. You have to lift your pencil and turn the page over in order to draw another line on the back.

Now try this:

1. Cut a strip of paper about 1 inch wide from the long side of a sheet of notebook paper.

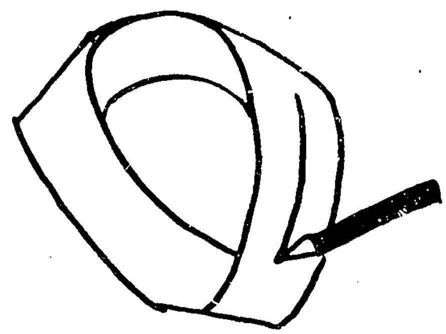
2. Give this strip a half-twist.



3. Connect the ends by using tape, glue, or staples. Your completed look should look like this:
   This loop became known by its discoverer's name: a Moebius Strip

Now try the following experiments:

1. Justify for yourself that this loop has only one side by drawing



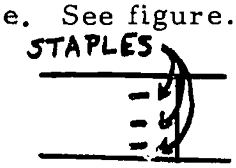
a line down its center. Did you have to life your pencil in order to do "both sides"? No! How many sides does the Moebius Strip have?

2. Cut a second strip of paper, also about an inch wide. Make a simple circular loop and connect the ends. Cut this loop down the center. How many loops do you get? ___________

Suppose you cut your Moebius Strip the same way, how many loops do you think you will get? ______________ Write your guess and then actually cut the loop. Were you right? _________

A mathematician confided
That a Moebius band is one-sided,
And you'll get quite a laugh
If you cut one in half,
For it stays in one piece when divided.

3. Make another Moebius Strip about $1\frac{1}{2}$ inches wide. (If you use staples to connect the ends you must use three. See figure.)



Cut this loop into thirds. If you do this for a simple circular loop, you have to make two cuts to get three circular loops. How many

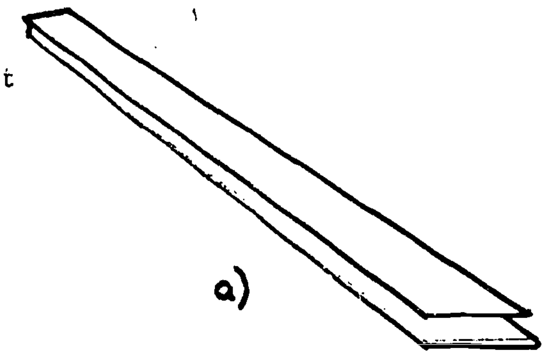
cuts were necessary to cut the Moebius Strip into thirds?

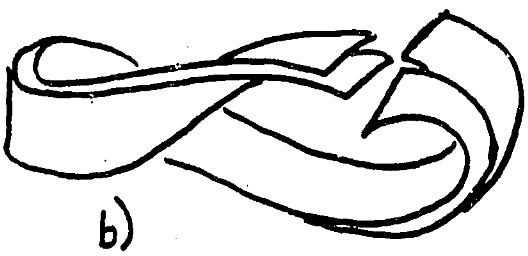
________________ Did you get three Moebius Strips when

you did this? ____________

4. Now lets make a puzzle that you can give to your family and friends. Cut two strips of paper each about one inch wide and twelve inches long. (It's more fun if they are different colors, but they don't have to be.)

a. Holding them together, twist into a Moebius Strip. (look back at the instructions if you need them.)





b. Separate the ends on one side so that the other two ends go between them.

c. Connect the two ends on the top and the two ends on the bottom. Refer to diagram b. (Be careful! Don't let the two loops fall apart!) Your completed doubles loop should look like this:

Study the resulting double loop carefully. Separate the strips (you should now have only one loop.) And shake it so

it's all mixed up. NOW--put it back together again!

When you have mastered this, try your "topological puzzle" on your family and friends.

CONCLUSION: The Moebius Strip is only one example of the many interesting activities which could be used to demonstrate properties of topology, "rubber-sheet" geometry. Refer to Life Science Library Mathematics by David Bergamini and the editors of Life, pages 176 through 191, to find other examples such as the transformation of a doughnut into a coffee cup, the removal of a man's vest without removing his coat, a bottle with no inside, the four-color map puzzle, non-circular circles and mazes with no insides, and a topological redhead.

VII. NON-EUCLIDEAN GEOMETRIES - ENRICHMENT ACTIVITY

PERFORMANCE OBJECTIVES

The student will:

1. Identify the main differences between Euclidean and non-Euclidean geomtries

2. You will name three different types of geometries and one mathematician associated with each.

Reference: Moise; p. 289
Lewis; p. 245-47
p. 313-319
Ransom, William, Three Famous Geometries- see annotated bibliography

Activity: Read the following and do the included experiment. Earlier you learned a little about Euclid, the Greek mathematician who collected and organized the known work in geometry in the fourth century B.C. in a book called The Elements. Work in geometry at that time was concerned with earlier three-dimensional space or two-dimension flat surfaces (or planes). The importance of this fact will be explained shortly.

Euclid began the section of this book on geometry by stating five basic assumptions, or postulates, which were simply to be accepted as true. From these he proved (that is, he showed by logical reasoning from these "known facts") that other properties must also be true. One of these properties was that the sum of the measures of the interior angles of a triangle is 180°. But remember--this was based on the assumption that the first five basic assumptions were true!

Euclids' five basic assumptions were that:

1) A segment can be drawn from any point to any other point.

2) Any segment can be extended into a line

3) A circle can be drawn using any point as the center and any segment as the radius.

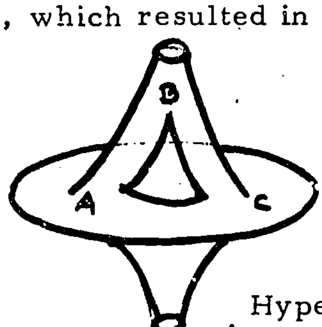
4) All right angles are equal in measure.

5) Given a line and a point not on the line, there is one and only one line through the point parallel to the given line.

Through the years mathematicians had been able to justify all of Euclid's basic assumptions except <u>the last one</u> (known as the Parallel Postulate).

During the nineteenth century, mathematicians began to question why this property could not be justified and, consequently, if Euclid was correct in assuming that it was true. One of the first men to do this was Carl Fredrick Gauss (1777-1855), a German, who as a young man had worked on this problem, but had never published his thoughts on the subject.

In 1832, he received a letter from an old friend in Hungary, Farkas Bolyai, telling Gauss of the work of his son, Janos. The young Bolyai had devised a geometry on a trumpet shaped surface such as the one below, which resulted in a triangle on this inwardly curving surface the sum of the measures of the interior angles is less than 180°. This type of geometry became known as hyperbolic geometry.

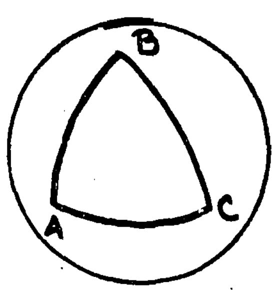


Hyperbolic Space
$M\angle A + M\angle B + M\angle C < 180^{\circ}$

Gauss wrote his friend that he thought the idea was great, but he couldn't help but boast that he had thought of this years ago. Shortly after this paper was published by a Russian, Nicolai Ivanovitsch Lobachevsky (1793-1856), on this same subject. After this double blow, Bolyai retired and did no more work in mathematics.

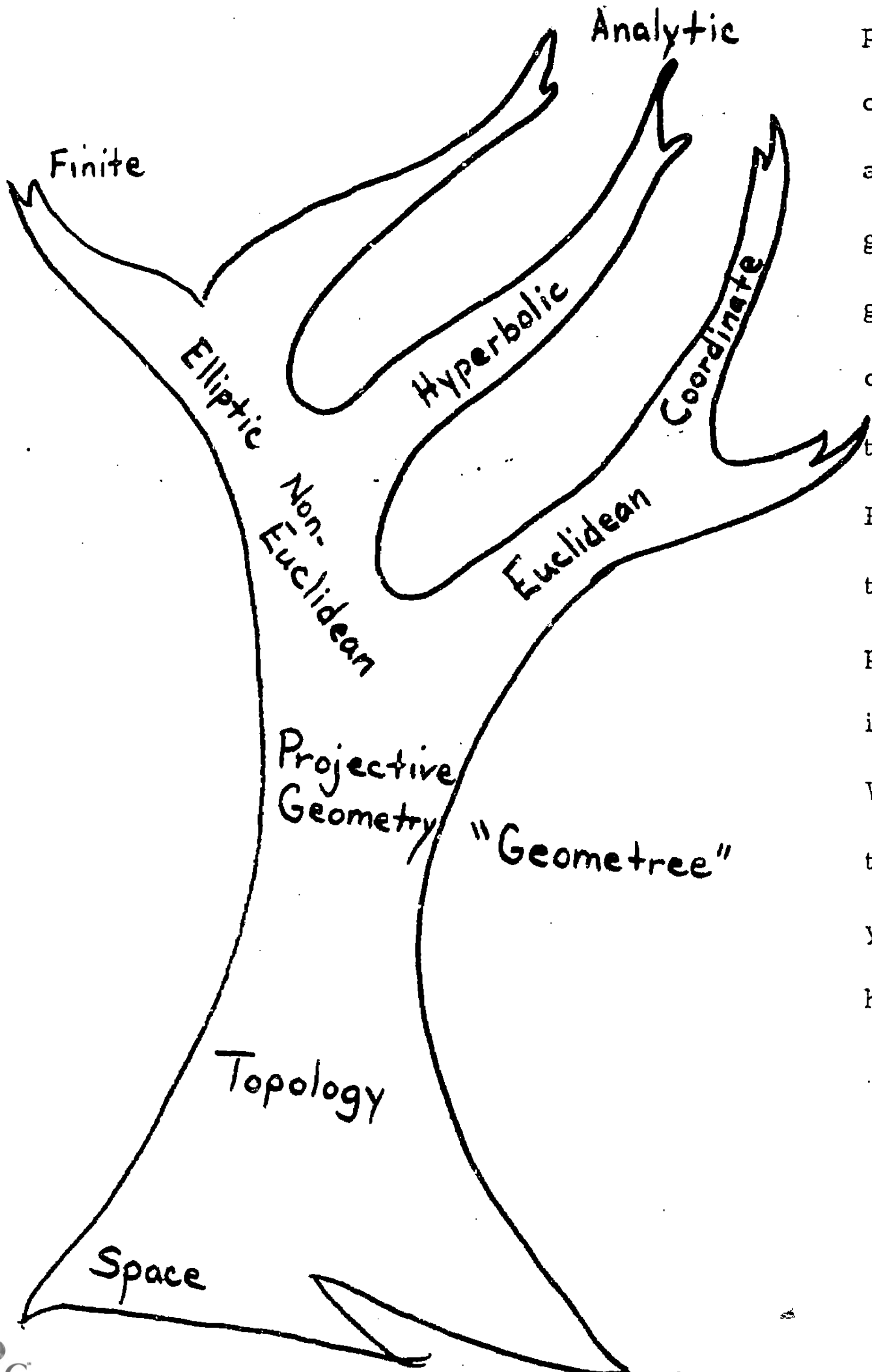
In 1854, a former pupil of Gauss, Georg Fredrich Barnhard Riemann (1826-1866), also a German, published a paper on a third type of geometry which became known as elliptic geometry. Whereas Euclidean geometry (or parabolic geometry, as it is now called) used flat surfaces and Bolyai's hyperbolic geometry used an inward curving surface, Riemann used an outward curving surface such as the shell of an egg. In this geometry there are no parallel lines (Think of longitudinal lines on Earth. They all intersect at the North or South Poles.) and the sum of the measures of the interior angles of a triangle can be anywhere between 180° and 540°!

Riemann was eventually able to extend his ideas to three; four; and n-dimensional geometry. So, even though we can't "see" beyond three-dimensions, Riemann showed that mathematically speaking other dimensions do exist. His work also opened up a whole new area in mathematics which is known as topology, the study of surfaces. You saw a little of this subject when you worked with the Moebius strip.



Elliptic Space
$M\angle A + M\angle B + M\angle C \neq 180^{\circ}$

Several years later another German mathematician, Felix Klein (1849-1925) showed that the three geometries described above were really just variations on what became known as real projective geometry. As man studied the properties of space other geometries were also developed such as finite geometries (where only a limited number of points may be used), coordinate geometry (where algebra is used along with geometry), and analytic geometry (which requires calculus). We might diagram the topics in geometry thus:



From this you should be able to see that only a very small part of the study of geometry is included in your course. Who knows how large this tree may grow by the time your grandchildren are in high school!

EXPERIMENT:

Now - if you had difficulty visualizing the properties of the triangles mentioned above, do the following experiment.

1. Cut a small triangle (about 1/2' inches for the longest side) from a piece of paper.
2. Make a fist with your hand.
3. Place the small paper triangle on the back of your hand. It should lie reasonably flat. This represents a parabolic (Euclidean) space, a flat surface. The sum of the measures of the interior angles of the triangle is 180^{o}.
4. Place the triangle on top of one of your knuckles. Does it lie flat now? In order for it to fit the surface it would have to be stretched (and the angles of the triangle become larger). This represents an elliptic space, a curve that turns only outward.
5. Now, try to fit the triangle between two knuckles, in the depression. It doesn't lie flat here either, does it? In order to fit, the triangle would have to be "squashed" (and the angles become smaller). This represents hyperbolic space, a curve that turns only inward.

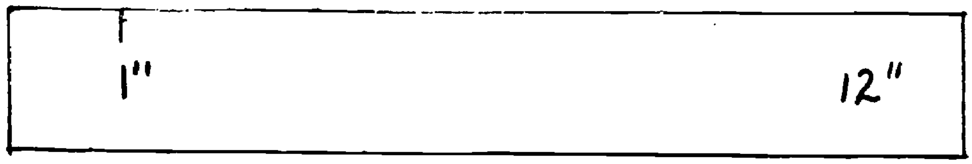
CONCLUSION:

KIND OF GEOMETRY	NUMBER OF PARALLELS	SUM OF MEASURES OF ANGLES OF TRIANGLE
Euclidean (Parabolic)		
Lobachevskian (Hyperbolic)		
Riemannian (Elliptic)		

ACKNOWLEDGMENT

The following "Challenge" problems are "Think" problems from the books School Mathematics I and II by Robert E. Eicholz, et al, copyright in 1967. Permission obtained from Addison-Wesley Publishing Company, Inc.

VIII. Suggested Challenge Problems - Enrichment Activity

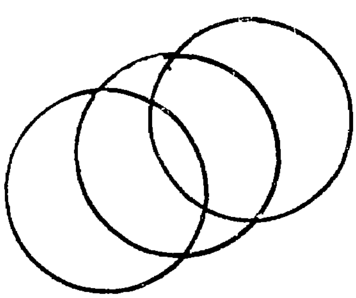


1. If you had a 12" ruler with only the above marking, could you make only three more marks on the ruler and still be able to measure all the whole number lengths between 1 and 12? There is more than one way. Indicate your way by marking the drawing above.

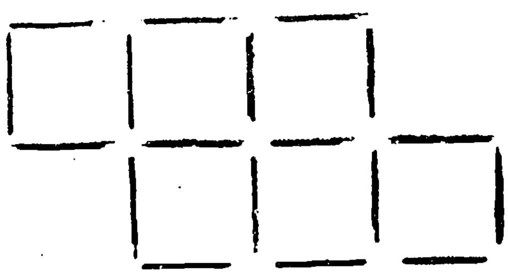
2. Can you arrange nine dots in:

   1. Eight rows of three each?
   2. Nine rows of three each?
   3. Ten rows of three each?

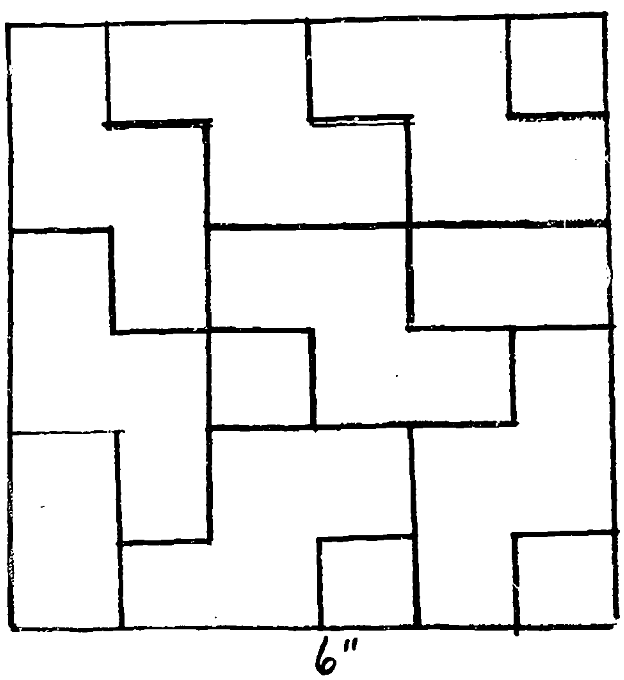
   Rows must be collinear.

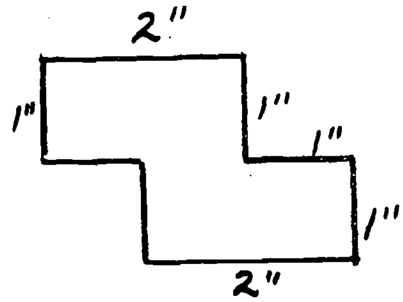


3. Start at any point and draw the figure without retracing any "lines" or crossing over any "lines."

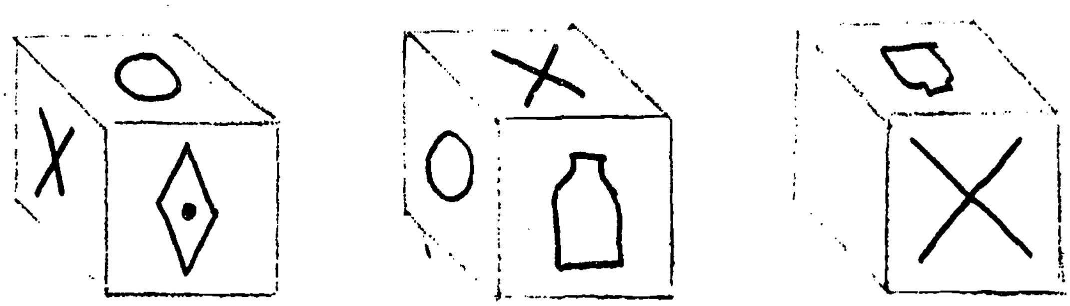


1. The sticks form six small squares. Can you remove only two sticks and form four squares of the same size?

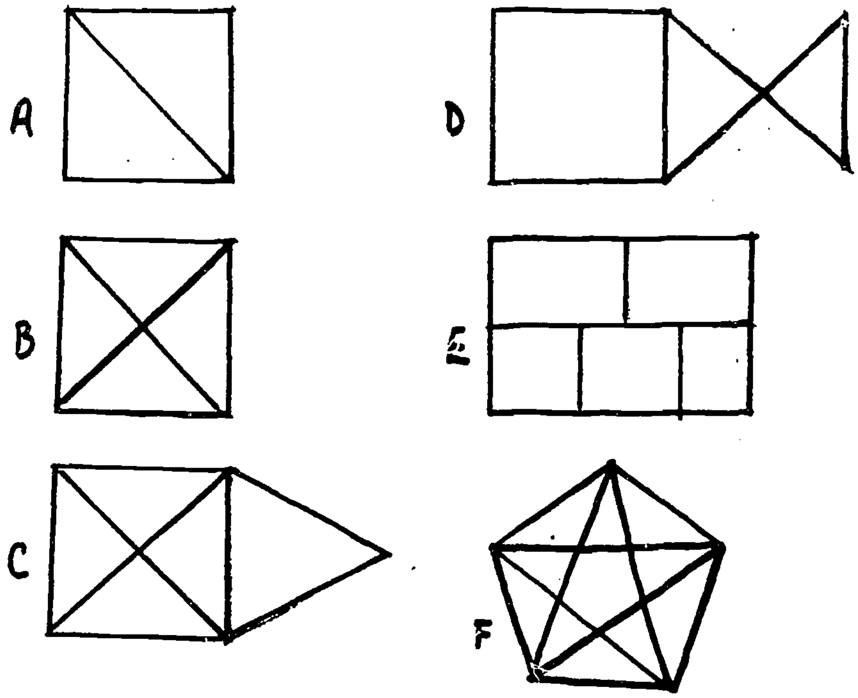




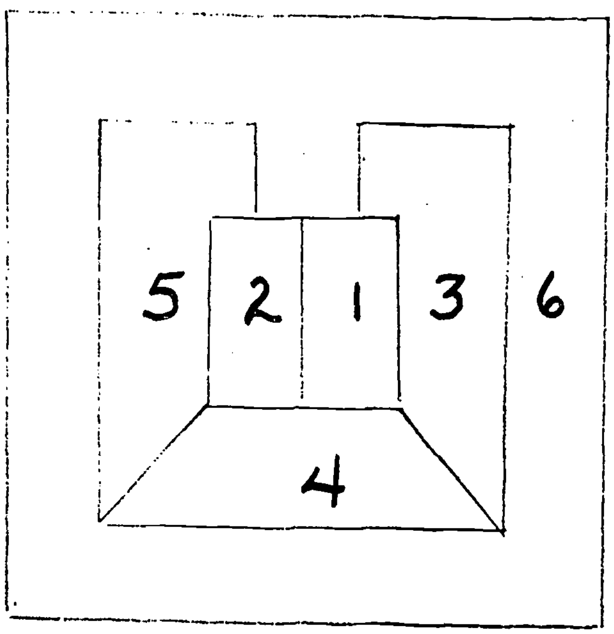
2. The figure shows how seven of the shapes can be fitted in a 6" square. Can you show how to fit eight of the shapes in a 6" square?



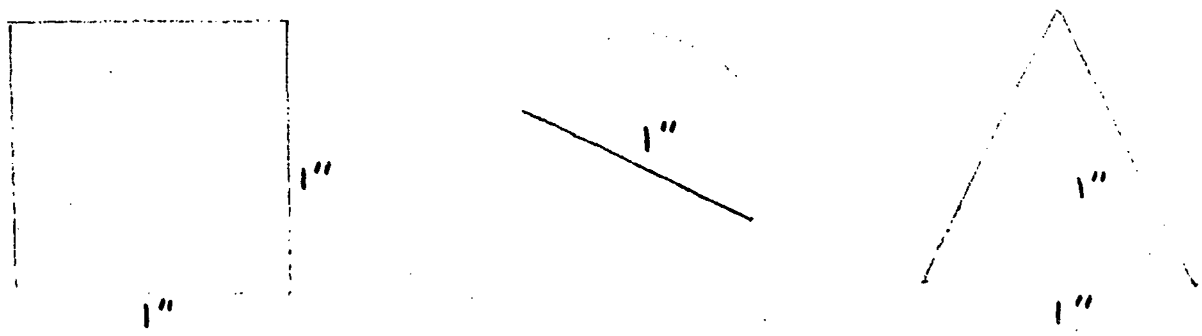
3. Three different views of the same cube are shown. In view C, one of the faces previously shown has been painted. What design was painted over?



4. Which of the figures can be traced without lifting your pencil from the paper or retracing? ________
Is there a method for determining this without tracing?

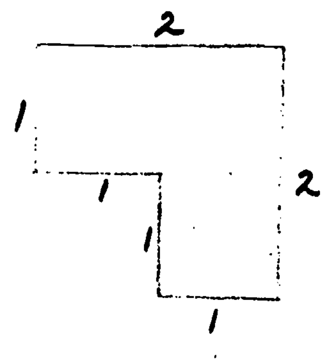


5. What is the least number of colors needed so you can color each region and not have any bordering regions the same color?

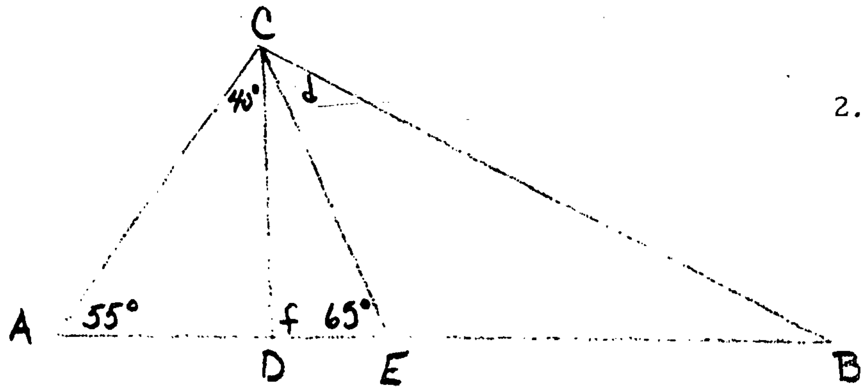


4. There is a certain solid that can be passed through each of the above holes. For each hole, at some stage of passing the solid through the hole, the solid will completely fill the hole. Can you make such a solid?

Unit VI

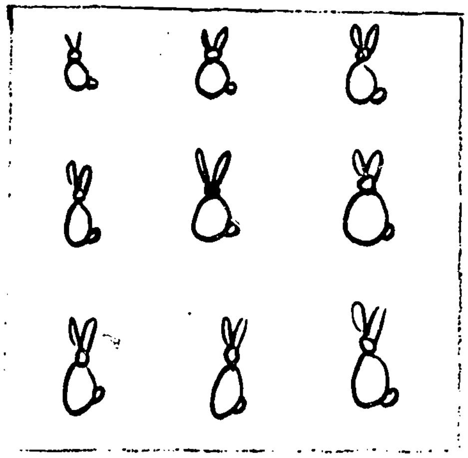


1. a. Divide the figure into two congruent figures.
   b. Divide the figure into three congruent figures.
   c. Divide the figure into four congruent figures.

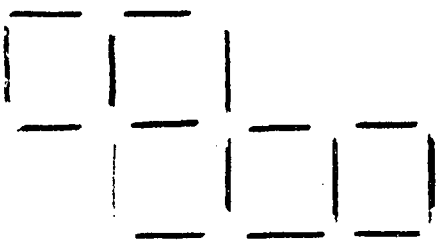


2. Triangle ABC is a right triangle. m ACB $=90^{\circ}$
Find the measures of $\angle e$, $\angle f$, $\angle d$.
m $\angle$ e = ______
m $\angle$ f = ______
m $\angle$ d = ______

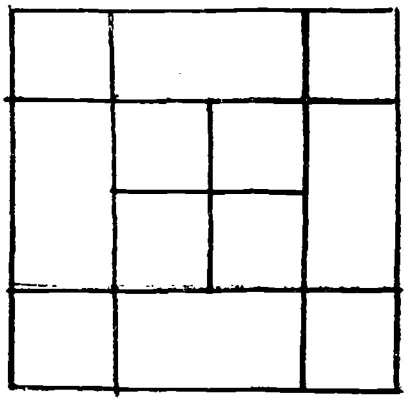
3. Using six sticks of the same length, place them in such a way that you have exactly four congruent triangles. The end of each stick must be at a vertex.



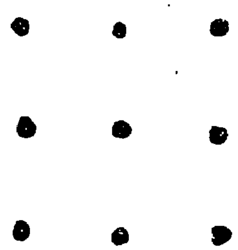
1. The farmer has nine rabbits in one square pen. He can keep each rabbit in a separate pen by building two more square pens within the original. Can you sketch how he was able to do this?



2. Sixteen pencils are arranged as shown to form five squares. Can you move only two of the pencils and form four squares?

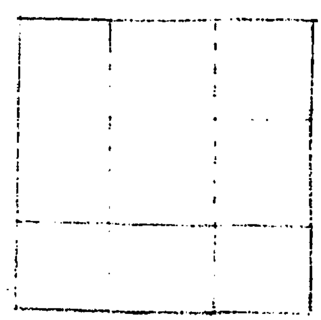


3. How many squares are in this figure?

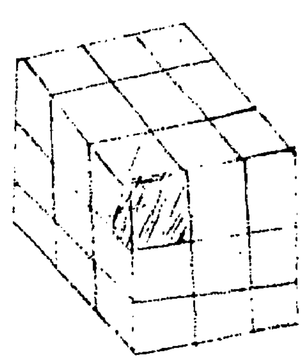


4. Start at any dot. Without lifting your pencil or retracing, draw four segments so that each dot lies on one of the segments

Unit X



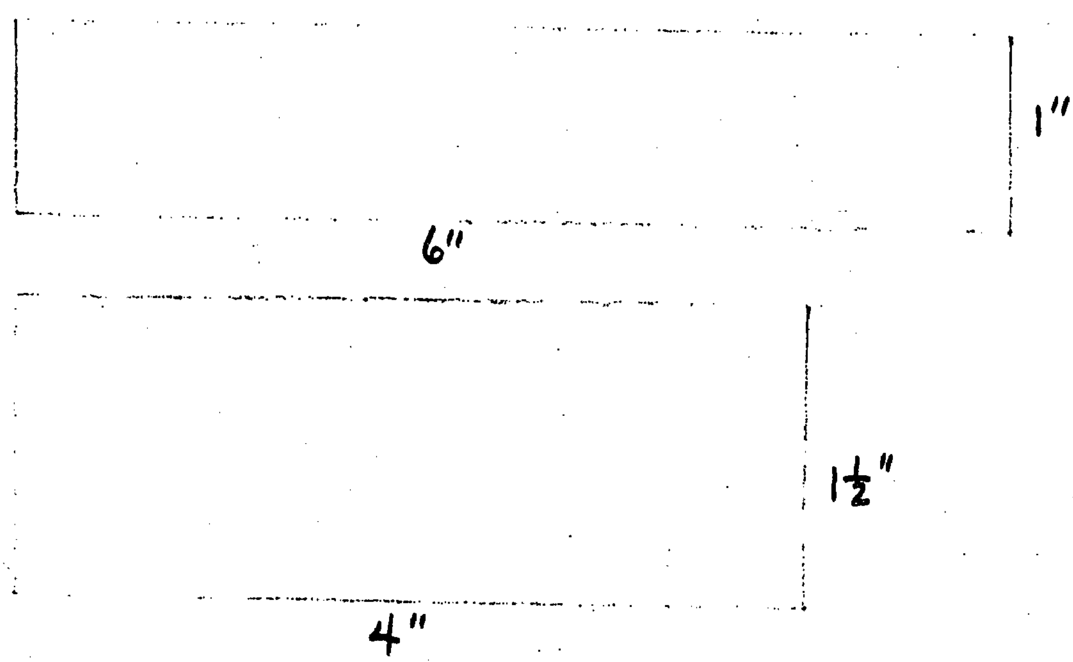
1. Draw a 3" square as shown and divide it into nine 1" squares as shown. Shade or color it. After cutting out the large square, cut along the dotted lines. Now fold the figure to make a cube that has all faces shaded.



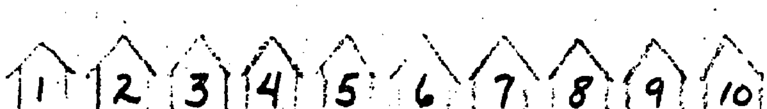
2. a. Find the surface area of the large cube______.
   b. Find the volume of the cube________.
   c. If the shaded cube is removed, what will the surface area be? _______
   d. By removing one block at a time, and never changing the surface area, what is the smallest volume you can get? ________
   e. By starting with the large cube and removing one block at a time, what is the largest surface area you can get? ______

3. If you have one 4 quart and one 9 quart pail with no markings, how can you measure 6 quarts of water? You may use only the two pails that you start with.

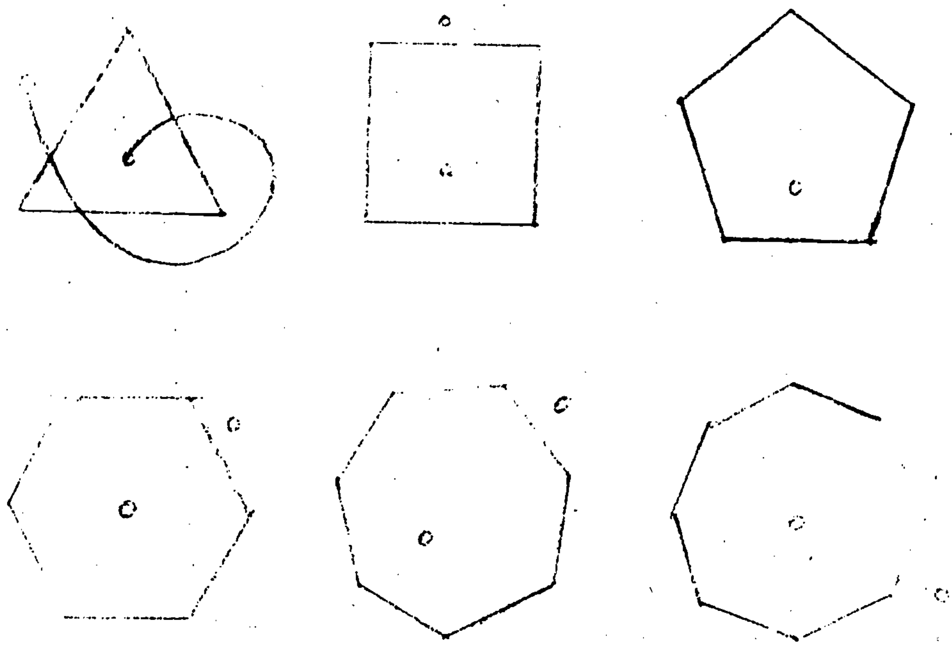
GENERAL



1. Can you cut the 4" x 1 1/2" rectangle into two congruent shapes that will exactly cover the 6" x 1" rectangle?



2. The electric meter reader must stop at each of the ten houses located in a row and built on equal size lots. He wants to waste time because he is paid by the hour. If he starts at any one of the houses, what is the longest route he can take?



3. In each of the six figures above, there is an interior and exterior point. By drawing a continuous curve to connect the interior and exterior point, can you cross each side of the figure exactly once? The first figure is completed as an example.

SAMPLE POSTTEST ITEMS

I. TRIANGLES

TRUE-FALSE. Correct the underlined portion of each of the false statements.

1. The sum of the measures of the interior angles of a triangle is half the sum of the measures of its three exterior angles formed by extending the sides of a triangle in one direction.

2. In any triangle, the three medians intersect in a common point called the centroid.

3. The segment joining the midpoints of two sides of a triangle is twice the length of the third side.

4. Two triangles can be proved congruent if three angles of one are respectively congruent to three angles of the other.

5. The bisector of any angle of a scalene triangle divides the triangle into two congruent triangles.

6. The shortest side of a 30-60-90 triangle is one-third the length of the hypotenuse.

7. Two angles of a triangle are 40 and 60. The number of degrees in the obtuse angle formed by the bisectors of these two angles is 130.

8. Two angles that are both congruent and supplementary must be right angles.

9. The bisectors of the angles of a triangle meet at a point which is equidistant from the three vertices of the triangle.

10. Two of the exterior angles of a right triangle must be acute.

II. QUADRILATERALS

1. An equilateral quadrilateral that is not equiangular is called a(n) ________________.

2. If the diagonals of a parallelogram are not congruent and are perpendicular to each other, the figure is a ______________.

3. If the diagonals of a quadrilateral are congruent and bisect each other, the figure is a ________________.

II. QUADRILATERALS (continued)

4. If the ratio of the measures of two angles of an isosceles trapezoid is 4:1, the measures of the angles are ______ and ______ .

5. If the bases of a trapezoid measure 12 in. and 18 in., find the length of the segment joining the midpoints on the non-parallel sides.

6. Name the figure formed by the segments joining the midpoints of the consecutive sides of any quadrilateral.

TRUE - FALSE. Correct the underlined portion of each of the false statements.

7. If the diagonals of a rectangle are perpendicular to each other, the figure must be a <u>square</u>.

8. If the diagonals of a parallelogram are congruent, the figure must be a <u>square</u>.

9. A parallelogram must be a <u>rhombus</u> if it is equilateral.

10. If the midpoints of two adjacent sides of a rhombus are joined, the triangle formed must be <u>equilateral</u>.

III. CIRCLES

For each of the following statements indicate whether the information given in the hypothesis is (a) too little (b) just enough or (c) too much to justify the conclusion.

1. If from one end of a diameter two congruent chords are drawn, they will be equidistant from the center.

2. The measure of the angle formed by the intersection of two tangents is equal to one-half the difference of the measures of the intercepted arcs.

3. Two equilateral triangles inscribed in a circle are congruent.

4. If two chords of a circle are congruent and parallel, then the chord passing through their midpoints is a diameter.

5. In the same or congruent circles, congruent chords are equidistant from the center(s) of the circle(s).

III. CIRCLES (continued)

6. The sum of the measures of the opposite arcs intercepted by two chords which meet inside a circle to form vertical angles of 50 degrees is 100 degrees.

TRUE-FALSE

7. Tangent segments PA and PB are drawn from an external point P to points A and B of circle O. They form an angle of 70 degrees. If radii OA and OB are drawn, the measure of AOB is 110 degrees.

8. If a secant and a tangent to a circle are parallel, the diameter drawn to the point of tangency is perpendicular to the secant.

9. If two chords of a circle are perpendicular to each other, one is a diameter.

10. If two tangents are drawn to a circle from an external point and the major arc intercepted by the tangents is twice the minor arc, then the measure of the angle formed by the tangents is 60.

IV. POLYGONS

1. Using a given segment as a side, construct a regular hexagon.

2. Using a given segment as the radius of the circumscribed circle, construct a regular octagon.

Complete each of the following statements:

3. Two regular pentagons are always __________.

4. If a polygon has ten sides, then it is called a ______________.

5. Regular polygons of the same number of sides are __________.

TRUE-FALSE

6. If a regular hexagon is inscribed in a circle whose radius is 10 inches, the difference between the area of the circle and the area of the hexagon (Correct to the nearest square inch) is 55 square inches. (pi = 3.14 and $\sqrt{3}$ = 1.73)

7. The number pi is a constant which represents the ratio between the circumference and the diameter of the circle.

8. The regular polygon whose apothem is one-half its side is a square.

9. The number of degrees in each interior angle of a regular hexagon is 120.

10. The number of degrees in each exterior angle of a regular octagon is 135.

V. SOLIDS

1. A cylindral gasoline tank is 3.5 feet long and 16 inches in diameter. What is the volume of the tank.

2. Find the volume of a water tank 20 inches deep if its bottom is in the form of a rectangle with a semicircle at each end, and the rectangle is 18 inches wide and 80 inches long.

3. Find the length of a bar that can be made from a cubic foot of iron, if the bar has a rectangular cross section 1 inch by 1 1/4 inches.

4. Find the volume of a cube whose area is 600 square inches.

5. Two corresponding edges of two similar tetrahedrons measure 3 inches and 5 inches respectively. Compare their volumes.

6. The volume of one polyhedron is 216 cubic inches and its total area is 108 square inches. If the total area of a similar polyhedron is 294 square inches, what is its volume?

7. If a polyhedron has 8 faces and two more than twice as many edges, how many vertices does it have?

8. Find the volume of a regular octahedron if its total area is $72\sqrt{3}$ square inches.

9. The bases of the frustum of a pyramid are squares having sides of four inches and nine inches respectively. Find the volume of the frustum if its altitude is 7/9 inches.

10. Show that the volume of a solid formed by revolving an equilateral triangle with a side "s" about an altitude is $\frac{1}{24}\pi S^3\sqrt{3}$.

KEY TO SAMPLE POSTTEST ITEMS

I. TRIANGLES

1. True
2. True
3. False - Half
4. False - Similar
5. False - Equilateral
6. False - One-Half
7. True
8. True
9. False - Sides
10. False - Obtuse

II. QUADRILATERALS

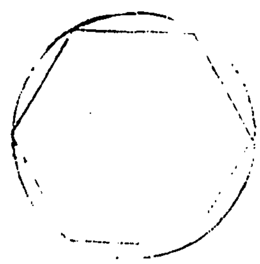
1. Rhombus
2. Rhombus
3. Rectangle
4. 36° and 144°
5. 15 inches
6. Parallelogram
7. True
8. False - Rectangle
9. True
10. False - Isosceles

III. CIRCLES

1. c
2. a
3. b
4. b
5. b
6. b
7. True
8. True
9. False
10. True

IV. POLYGONS

1. Construct an equilateral △ using given segment as length of a side. Continue the construction of equilateral △'s until six have been completed.

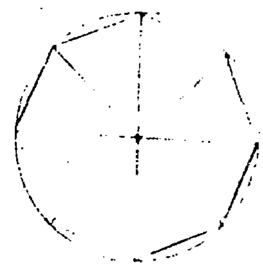


<u>or</u> Construct a ci cle having given segment as radius. Use compass to mark off 6 chords around circle equal in length to the radius of the circle.

IV. (continued)

2. ____________________

Given segment



Construct circle having radius equal to length of given segment.
Draw diameter. Construct $\perp$ bisector of diameter. Bisect 2 adjacent right angles and extend the bisectors to intersect circle in two points. Connect points of intersection on circle. (A, B, C, D, E, F, G, H)

3. Similar
4. Decagon
5. Similar
6. True
7. True
8. True
9. True
10. False

V. SOLIDS

1. $\frac{56\pi}{9}$ cubic feet

2. $(28,000 + 1620\pi)$ cubic inches

3. $115\frac{1}{\ }$ feet

4. 1,000cubic inches

5. 27 : 125

6. $686\sqrt{2}$ cubic inches

7. 12

8. $72\sqrt{2}$ cubic inches

9. $267\frac{1}{27}$ cubic inches $(V = \frac{1}{3}h(B + b + \sqrt{Bb}))$

10. Solid will be a cone. Area of base = $\pi\left(\frac{s}{2}\right)^2$

$$V = \frac{1}{3} \cdot \frac{\pi s^2}{4} \cdot \frac{s\sqrt{3}}{2} = \frac{1}{24}\pi s^3 \sqrt{3}$$

ANNOTATED BIBLIOGRAPHY

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2. Adler, Irving, Mathematics - The Story of Numbers, Symbols and Space. New York: Golden Press, 1961.
   A very basic, elementary but interesting treatment of many of the items covered in this quin. This is an excellent source book for students.

3. Dade County Experimental Project for Geometry Level I.
   Excellent for activities and procedures developing intuitive conclusions.

4. Dodes, Irving A. Geometry. New York: Harcourt, Brace and World, Inc., 1965.
   Good for practical and commercial applications of geometry. Especially chapt. 4, 6-9, 11-14.

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   Chpts. 3, 6, 7, 9-15, 17-19 excellent for test questions and quiz material. Very good intuitive approach in the Discovery Exercises.

6. Gardner, Martin. Mathematical Puzzles and Diversions. New York: Simon and Schuster, 1959.
   Marvelous source for the curious and resourceful student looking for something different.

7. Goodwin, Wilson, A. and Vanaata, Glen D. Geometry. Columbus, Ohio: Charles E. Merrill Publishing Co., 1970.
   Chpts. 6, 8-13, 16-19 all very helpful in this quin. Excellent 3-D drawings and explanation.

8. Keedy, Marvin, L. and Jameson, Richard, E. Exploring Geometry. New York: Holt, Rinehart, and Winston, Inc., 1967.
   Excellent diagrams and explanations of three-dimensional figures. Teachers commentary gives helpful additional explanations. Very good source book for entire quin. Excellent "applied" problems in each chapter.

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   Another interesting and readable book for the students as well as the teacher.

10. Munro, Thomas and Wilson, Catherine M. Tenth Year Mathematics. New York: Oxford Book Co., 1960.
Good basic review material. Excellent for additional questions for homework or tests. Explanations very brief and basic.

11. N. C. T. M. Geometry, Unit Four of Experiences in Mathematical Discovery (Washington D. C.: National Council of Teachers of Mathematics, 1966.
Concise but excellent references on many points covered in this quin from the intuitive and discovery angle.

12. N. C. T. M. Readings in Geometry from the Arithmetic Teacher. Washington, D. C.: National Council of Teachers of Mathematics, 1971.
Excellent understandable explanation of fun topics covered in this quin. For both teacher and student, a delightful introduction to unusual elementary approaches.

13. Ransom, William. Can and Can't in Geometry. Portland, Maine: J. Weston Walch, 1960.
Unusual explanations of quin topics useful for teacher research.

14. Smith, Rolland R. and Ulrich, James F. Geometry, A Modern Course. New York: Harcourt, Brace and World, Inc., 1964.
Excellent chapter on constructions and three-dimensional geometry. (Chapter 3). Chpts. 10, 11, 15, 17, 19 20 all very helpful in this quin.

15. Ulrich, James, F., and Payne, Joseph N. Geometry. New York: Harcourt, Brace and World, Inc., 1969.
Excellent for definitions and explanations of work covered in this quin. Chapter 11 covers the Pythagorean Theorem in depth.

16. Wilcox, Marie S. Geometry a Modern Approach. Reading, Mass: Addison-Wesley Publishing Co., 1968.
Good for quiz and text materials as well as general spacial concepts. Chpts. 5, 8, 11-14 apply particularly to this quin.

Annotated Bibliography (continued)

AUDIO-VISUAL MATERIAL

Films - By request only through the school library

Cat. Number

1-01506 Similar Triangles (12 min.)

1-01348 Triangles: Types and Uses (11 min.)

Curriculum Full-Color Filmstrips by Educational Projections, Inc.

366 Triangles. Types of triangles, median, angle bisection, altitude hypotenuse, exterior angles.

372 Locus. Very good explanation and examples.

Filmstrips by S. V. E.

A542-4 Congruence.

Filmstrips by McGraw-Hill Book Co.

9 Indirect measurement. (Excellent) Use after study of right triangles

Set 1; 7 & 8 Constructions with compass

Set 1; Geometric Figures

Filmstrips by PSP (Pop. Sci. Pub. Co. NYC)

1181 Right Angles in 3-D Figures

1185 Applied Geometry

Filmstrips by FOM (Pop. Sci. Pub. Co., NYC)

1168 Special right triangles

1113 Parallelograms and their properties

1122 Congruent triangles

1140 Angle sums for polygons

1151 Diameter perpendicular to a chord

1161 Mean proportion and right triangles