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ABSTRACT

This guidebook on minimum course content was designed for students who have mastered the skills and concepts of analytic geometry. It is a short course in the basic techniques of calculus recommended for the student who has need of these skills in other courses such as beginning physics, economics or statistics. The course does not intend to teach applications of the calculus to any particular area of study nor to delve to any extent into theory. Some background work in functions and notation is provided in the first week. Overall course goals are specified; a course outline, performance objectives, and suggested teaching strategies are listed. (JP)

AUTHORIZED COURSE OF INSTRUCTION FOR THE QUINMESTER PROGRAM

TECHNIQUES OF DIFFERENTIATION AND INTEGRATION

5297.27

MATHEMATICS

DADE COUNTY PUBLIC SCHOOLS

DIVISION OF INSTRUCTION•1971

QUINMESTER MATHEMATICS

COURSE OF STUDY

FOR

TECHNIQUES OF DIFFERENTIATION AND INTEGRATION

5297.27

(Experimental)

Written by

Gary B. Forrester

for the

DIVISION OF INSTRUCTION

Dade County Public Schools
Miami, Florida 33132
1971-72

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PREFACE

The following course of study has been designed to set a minimum standard for student performance after exposure to the material described and to specify sources which can be the basis for the planning of daily activities by the teacher. There has been no attempt to prescribe teaching strategies; those strategies listed are merely suggestions which have proved successful at some time for some class.

The course sequence is suggested as a guide; an individual teacher should feel free to rearrange the sequence whenever other alternatives seem more desirable. Since the course content represents a minimum, a teacher should feel free to add to the content specified.

Any comments and/or suggestions which will help to improve the existing curriculum will be appreciated. Please direct your remarks to the Consultant for Mathematics.

All courses of study have been edited by a subcommittee of the Mathematics Advisory Committee.

CATALOGUE DESCRIPTION

A short course in the basic techniques of calculus for the student who has need of these skills in other courses such as: beginning physics, economics, statistics, etc.

Designed for the student who has mastered the skills and concepts of Analytic Geometry 2 or Mathematical Analysis 3.

TABLE OF CONTENTS

INTRODUCTION

Techniques of Differentiation and Integration is a nine-week course in the basic techniques of the calculus for the student who has need of these skills in other areas, such as beginning physics, economics, and statistics. The student should have mastered the skills and concepts of Analytic Geometry 2 or Mathematical Analysis 3.

The course does not intend to teach students applications of the calculus to any particular area of study nor to delve to any extent on theory. Most of the basic theorems and formulas needed are to be stated without proof in the interest of saving time. Some background work in functions and notation is provided in the first week. Then the basic concepts of differential calculus pertaining to curve sketching and tangents and integral calculus pertaining to areas are studied. Finally certain methods of integration, including trigonometric substitutions, partial fractions, and integration by parts are included.

OVERALL GOALS

1. Give the student an understanding of the importance of the calculus as a tool in other areas of study.
2. Prepare the student to delve further into advanced topics in other media than would normally be possible.
3. Extend the students' knowledge to a greater understanding and appreciation of the scientific world about him.
4. Help the student to attain the highest goals which he might set for himself.

OVERALL STRATEGY

Due to the complexity of the subject matter and the span of time designated for its presentation, the very nature of this course requires a complete lack of rigor in most respects. I would suggest that the instructor depend heavily on geometric intuition whenever possible. In an effort to teach techniques it would be advisable to concentrate on a problem-solving approach to as much of the material as possible. Definitions should be used as an aid to the solution of problems and not as a basis for further investigation into the abstract aspects of the calculus. A few suggestions pertaining to specific topics accompany the outline.

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I. A REVIEW OF FUNCTIONS

PERFORMANCE OBJECTIVES

The student will:

1. Describe union and/or intersection of sets using set notation.
2. Graph the absolute value function.
3. Determine the domain and range of any given function.
4. Evaluate a given function for any point in its domain.
5. Locate the intercepts of a given function.
6. Determine symmetry of a given function with respect to either axis or the origin.
7. Determine asymptotes of any given function.
8. Sketch the graph of any given function showing axes of symmetry and asymptotes.
9. Graph the sum or difference of two given functions.
10. Find the composition of two given functions.
11. Determine the domain and range of the composition of two given functions.

COURSE CONTENT

1. Review of set and interval notation.
2. Definition of the absolute value function.
3. Definition of a function as a mapping including notation, domain and range.
4. Different types of functions:

   a. Constant
   b. Polynomial
   c. Rational

d. Algebraic
e. Greatest integer
f. Transcendental
g. Natural log
h. Exponential

5. Intercepts, symmetry, horizontal and vertical asymptotes.

6. Graph of a function.

7. Sum, difference, product, quotient, and composition of two functions. Domain and range in each case.

8. Notation used for composition.

STRATEGIES

As this material is a review it should not be necessary to delve into any area in much detail. A few examples will probably be sufficient to recall the topic to the student's mind.

When determining domains, ranges, and graphs of functions it would be advisable to include some compound functions, such as:

1. $y = \left\{ \begin{array}{ll} x + 2 & \text{if } x \in [0,2) \\ -x & \text{if } x \in [2,4] \end{array} \right\}$

2. $y = \left\{ \begin{array}{ll} -2 & \text{if } x < 0 \\ x^2 - 2 & \text{if } x \in [0,3) \\ x + 4 & \text{if } x \geq 3 \end{array} \right\}$

REFERENCES

Reference Number	Chapter(s)	Section(s)
1	1	
4	1	2,3
	3	2,3,4
5	3	8
	4	4,5,6
7	1	1,2,3
	2	1,2,5
8	1	6,7,9
	2	2,3,4
	8	2,7,8,9

SAMPLE TEST QUESTIONS I

1. Sketch the set of points P (x,y) described by each of the following:

   a. $x \in (0,3]$ and $y \in [1,4)$

   b. $x \in (-4,3)$ and $y \in (-6,1)$ (1,8)*

   c. $|x + 2| \leq 3$ and $|y + 3| < 2$

2. Use absolute values and inequalities to express the following:

   a. point P is closer to 2 than to point t.

   b. point P is farther from -5 than from point t. (1,2)

   c. point P is closer to point c than to point t.

3. Solve the inequalities:

   a. $2x - 3 > 0$

   b. $x^2 - 5x - 24 \leq 0$

   c. $x^2 - 6x + 9 < 0$

   d. $(x + 3)(x - 2)(x - 4) < 0$

   e. $x^2 + 7x + 12 \leq 0$

   f. $x^2 - 1 \geq 0$ (1,3)

   g. $18x - 3x^2 > 0$

   h. $(x + 1)^2(x - 3) > 0$

4. Sketch the graph of:

   a. $y = |2x|$: $x \in [-2,3]$

   b. $f(x) = |\cos x|$: $x \in [-\pi, \pi]$

   c. $f(t) = \frac{1}{t}$

   d. $g(r) = 2^r$

   e. $y = x + \sin x$

   f. $y = \begin{cases} \frac{x}{\sqrt{|x|}} & \text{for } x \neq 0 \\ 0 & \text{for } x = 0 \end{cases}$

   g. $y = \ln x$ (2,3,4,5,9)

   h. $h(x) = e^x$

   i. $y = e^{-x}$

5. Give the natural domains of f and g and the domains of f + g, fg, $\frac{f}{g}$

   a. $f(x) = x$ $g(x) = \sqrt{x - 1}$

   b. $f(x) = \frac{1}{x - 2}$ $g(x) = \frac{1}{\sqrt{x - 1}}$ (3,4)

6. If $f = \{(0,1), (1,2), (3,4), (4,5), (5,6)\}$ and $g = \{(0,0),$ $(2,4), (4,8), (6,12)\}$ find $f \circ g$ and $g \circ f$. (10,11)

7. Find $f \circ g$ and $g \circ f$ if:

   a. $f(x) = 3x - 4$ $\quad g(x) = x^2$

   b. $f(x) = x^2$ $\quad g(x) = \sin x$ (10,11)

8. Determine symmetry about either axis and the origin, and intercepts for:

   a. $x^2 - y^2 = 1$

   b. $xy = 1$ (6)

   c. $x^2 + y^2 = 1$

9. Determine asymptotes for:

   a. $xy = 1$

   b. $y^2(x^2 - x) = x^2 + 1$ (7)

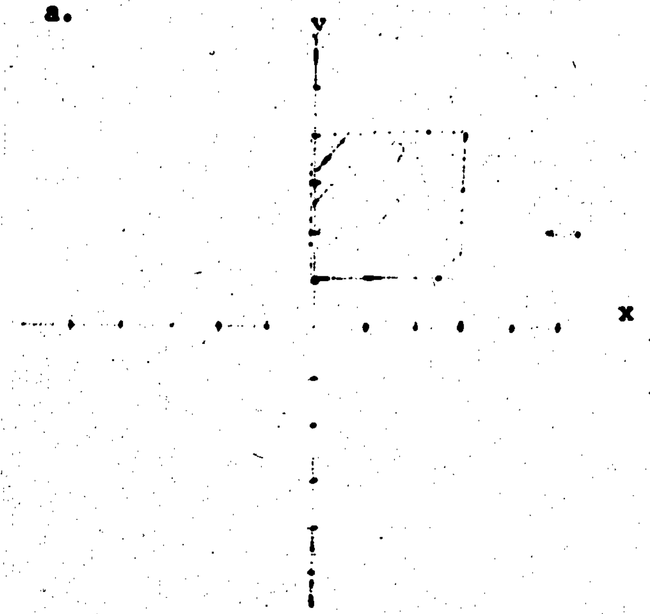
   c. $y = \dfrac{1}{x^2 - 1}$

* indicates objective which question is designed to measure.

ANSWERS 1

1.

a.



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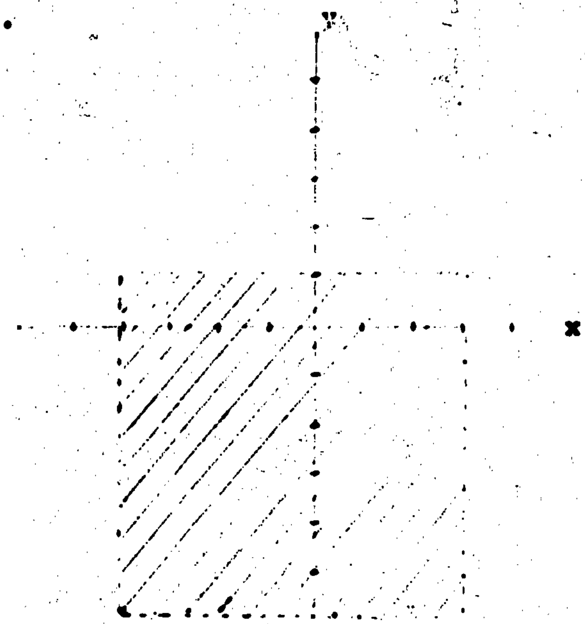

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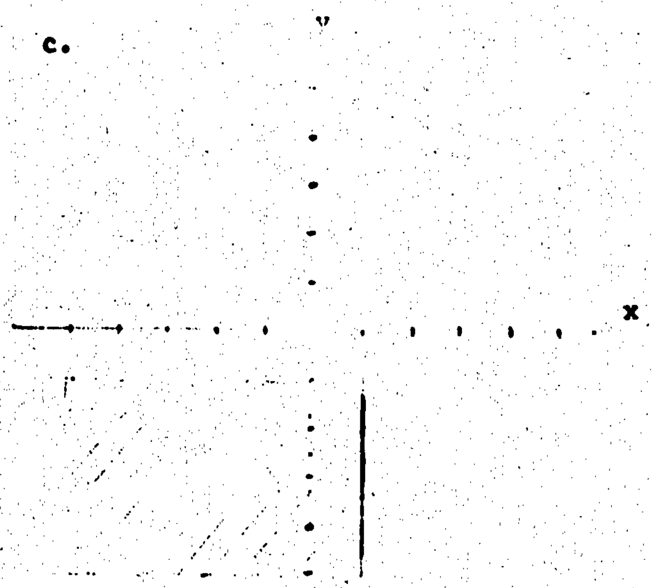


2. a. $|p - 2| < |p - t|$

   b. $|p + 5| > |p - t|$

   c. $|p - c| < |p - t|$

3. a. $x > \frac{3}{2}$

   b. $x \in [-3,8]$

   c. $\emptyset$

   d. $x \in (2,4)$ or $x < -3$

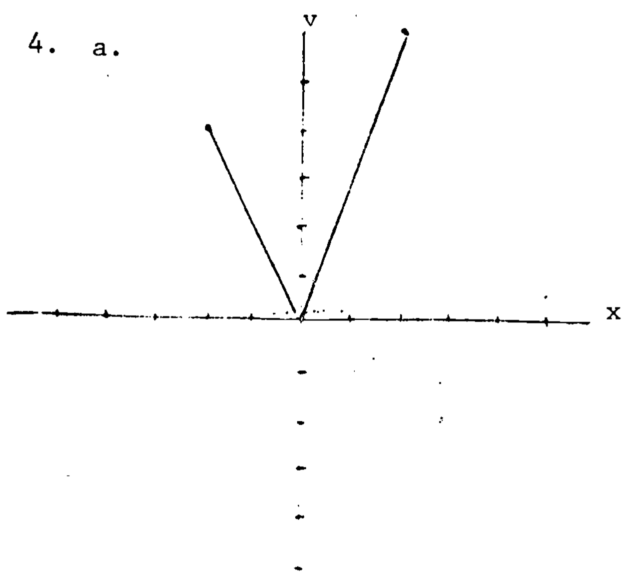
   e. $x \in [-4,-3]$

   f. $\{x \mid x \geq 1\} \cup \{x \mid x \leq -1\}$

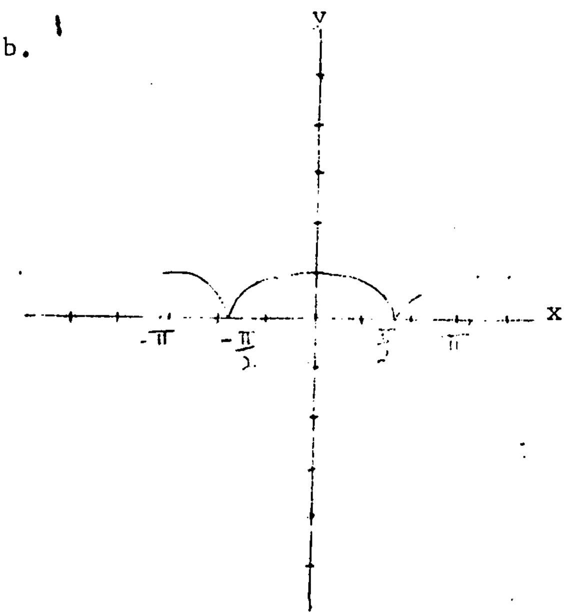
   g. $x \in (0,6)$

   h. $x > 3$

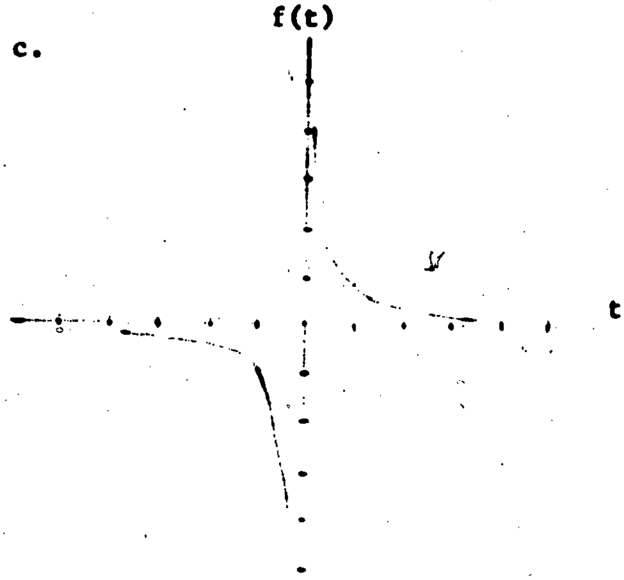
4. a.



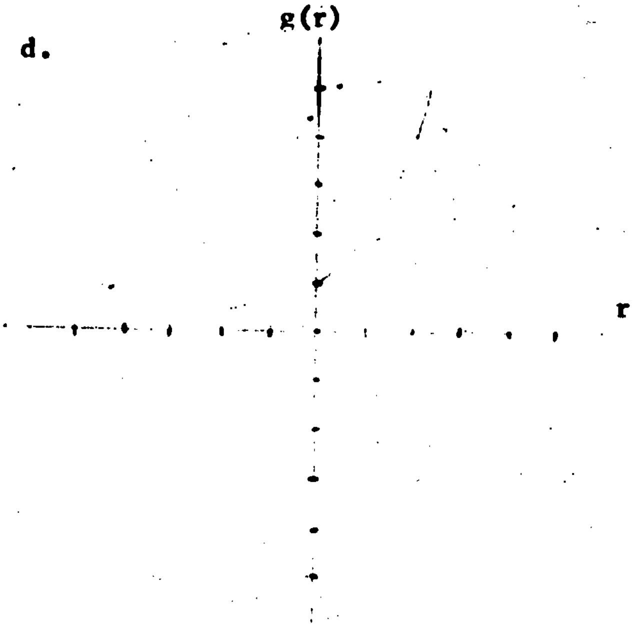
b.



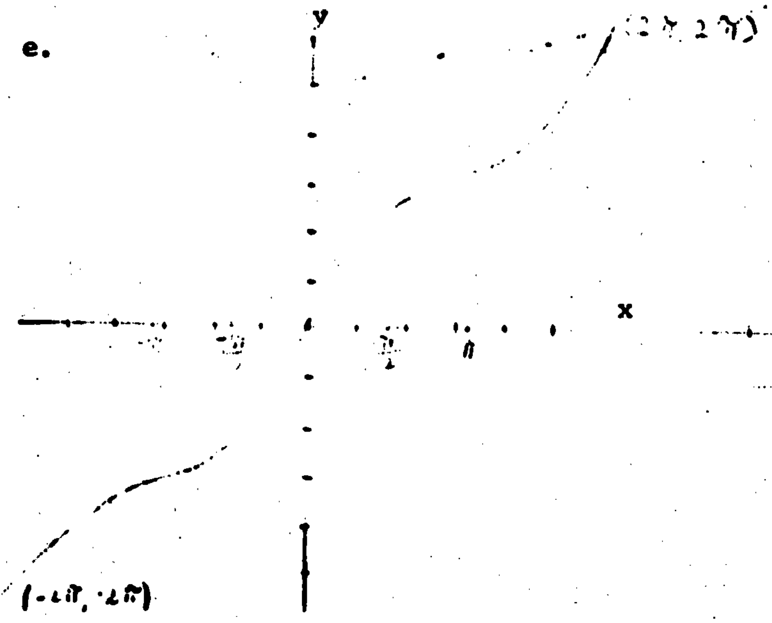
4. c.



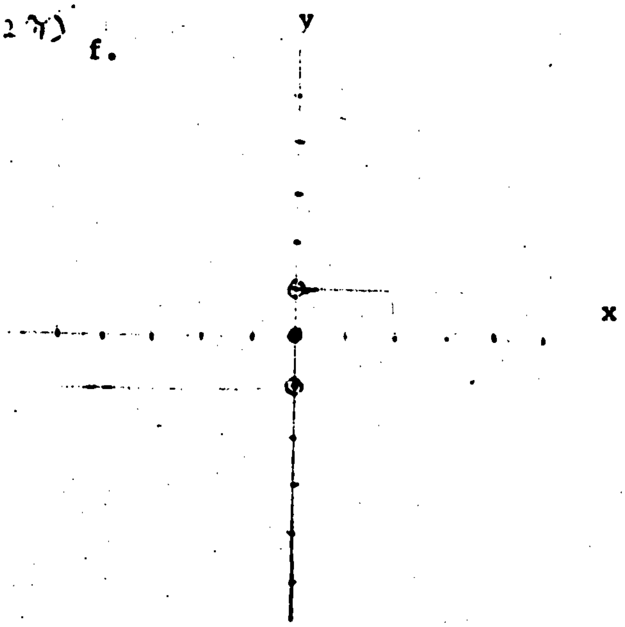
d.



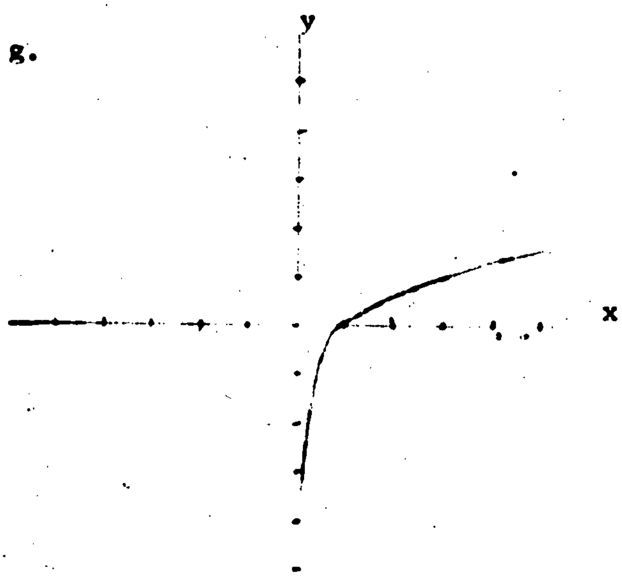
e.



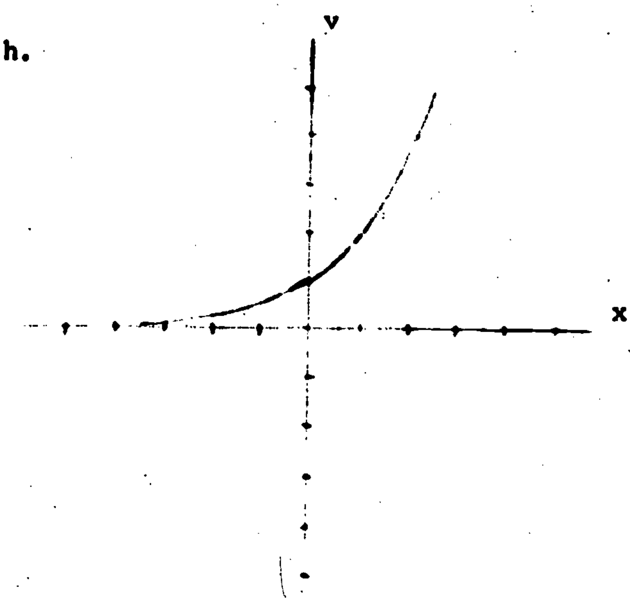
f.



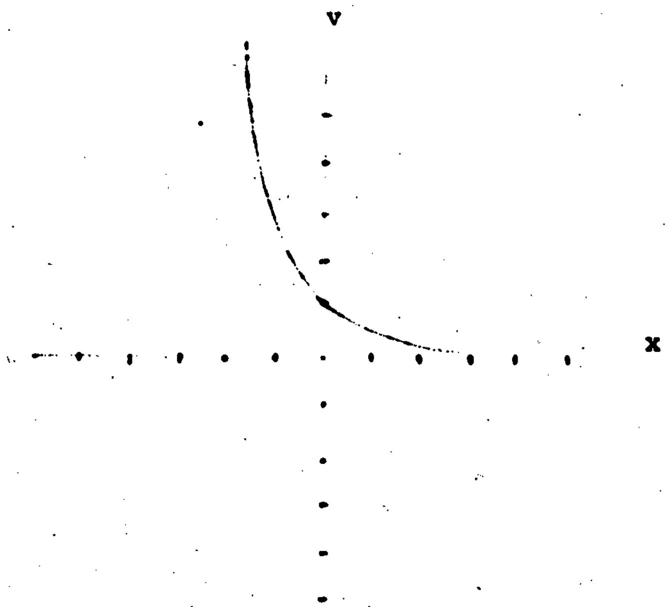
4. g.



h.



i.



5.

Problem	f	g	f + g	fg	$\frac{f}{g}$
a	reals	$x \geq 1$	$x \geq 1$	$x \geq 1$	$x > 1$
b	reals $- \{2\}$	$x > 1$	$(1,2)\cup(2,\infty)$	$(1,2)\cup(2,\infty)$	$(1,2)\cup(2,\infty)$

6. $f \circ g = \{(0,1), (2,5)\}$ $\quad g \circ f = \{(1,4), (3,8), (5,12)\}$

7. a. $f \circ g = 3x^2 - 4$ $\quad g \circ f = (3x - 4)^2$

   b. $f \circ g = \sin^2 x$ $\quad g \circ f = \sin x^2$

8. a. symmetric about x-axis, y-axis, origin; the intercepts are (1,0) and (-1,0).

   b. symmetric about origin; no intercepts.

   c. symmetric about x-axis, y-axis, origin; the intercepts are (1,0), (-1,0), (0,1) and (0,-1).

9. a. $x = 0$; $y = 0$

   b. $x = 0$; $x = 1$; $y = 0$

   c. $x = \pm 1$

II. LIMITS AND THE DERIVATIVE

PERFORMANCE OBJECTIVES

The student will:

1. Find a limit of a given polynomial or rational function and show that the answer is correct by the intuitive definition.
2. State the limit theorems.
3. Find the limit of a given function at a given point or at ∞ using theorems.
4. Find the slope function of any given differentiable function.

COURSE CONTENT

1. Definition of a limit by intuitive geometry.
2. Definition of infinite limits.
3. Limit theorems (stated only)

   a. If $f(x) = c$, then $\lim_{x \to a} f(x) = c$

   b. If $\lim_{x \to a} f(x) = L_1$ and $\lim_{x \to a} g(x) = L_2$ then

      1. $\lim_{x \to a} k\, f(x) = k \lim_{x \to a} f(x) = k\, L_1$ for k constant.
      2. $\lim_{x \to a} (f(x) \pm g(x)) = L_1 \pm L_2$
      3. $\lim_{x \to a} f(x)\, g(x) = L_1 L_2$

4. $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{L_1}{L_2}$ if $L_2 \neq 0$

5. $\lim_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{L_1}$ if L_1 is real

4. Definition of continuity of a function.

   a. at a point

   b. over an interval

5. Definition of one-sided limit:

   $\lim_{x \to a} f(x) = L$ iff $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = L$

6. Definition of the derivative of a function as an application of limits of slope functions

7. Different notations for the derivative

8. Additional limit theorems:

   a. If $f(x)$ is a polynomial function, then $\lim_{x \to a} f(x) = f(a)$

   b. If $\lim_{x \to a} f(x) = L_1$ and $\lim_{x \to a} g(x) = L_2$, then $\lim_{x \to a} [f(x) + g(x)]$

   $= L_1 + L_2$

STRATEGIES

An intuitive geometric definition of a limit might be as follows: $\lim_{x \to a} f(x) = L$ if f is defined in an open interval about a and for every pair of horizontal lines α and β with (a,L) between them there exist vertical lines h and k with (a,L) between them such that every point of f between h and k is also between α and β.

The definition of continuity at a point could then be given as follows:

1. $\lim_{x \to a} f(x) = f(a)$

2. $f(a)$ is real.

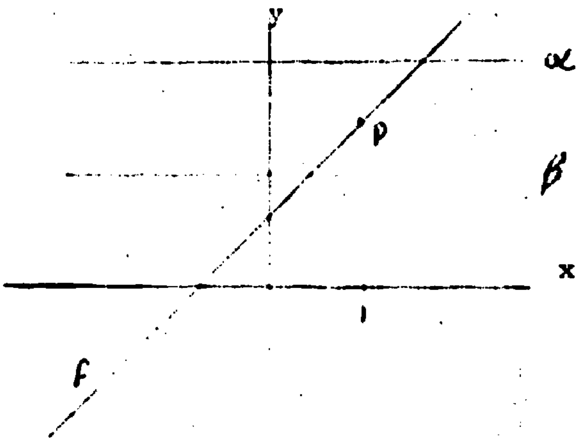
Some sample problems to illustrate continuity or discontinuity follow.

Since the slope function $\frac{f(x + h) - f(x)}{h}$ is generally easy for students to accept, the standard definition of the derivative, $\lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$, if this limit exists, should be given. If time permits, an alternate definition $\lim_{x \to x_1} \frac{f(x_1) - f(x)}{x_1 - x}$ might be derived by letting $x_1 = x + h$ in the original definition.

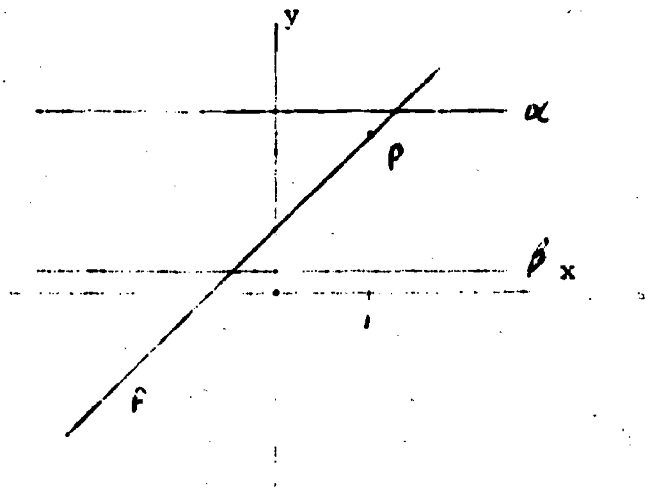
The following set of problems consists of a number of functions, and for each function f, some pairs α and β of horizontal lines with some indicated point p between them. In each case, you are to draw (if possible) vertical lines h and k with p between them so that the definition of <u>continuity at</u> p is satisfied. If this is impossible, write "not possible."

1. $f(x) = x + 1$

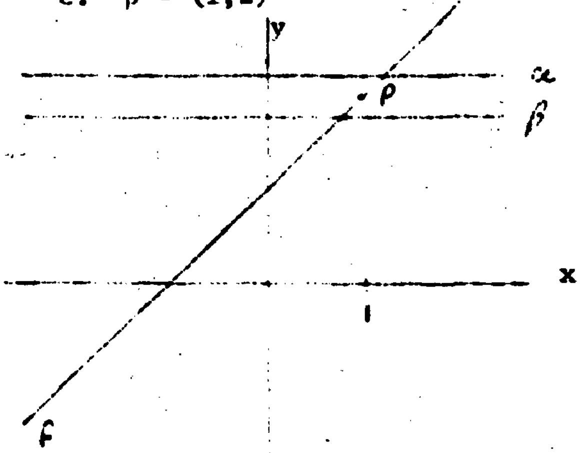
a. $p = (1,2)$



b. $p = (1,2)$

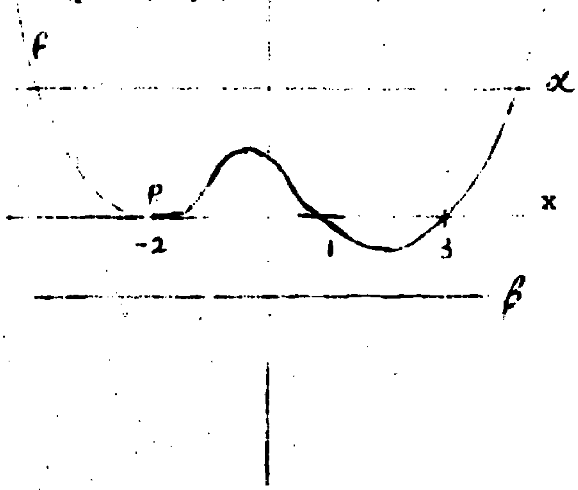


c. $p = (1,2)$

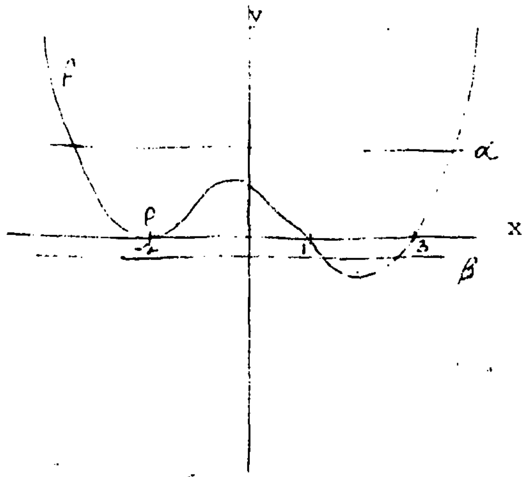


2. $f(x) = (x - 1)(x + 2)^2(x - 3)$

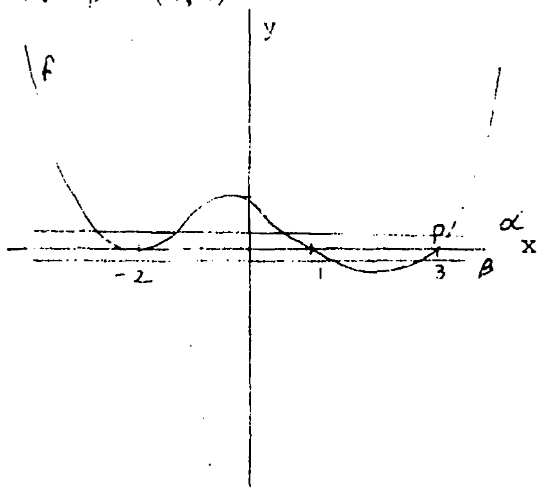
a. $p = (-2,0)$



b. p = (-2,0)

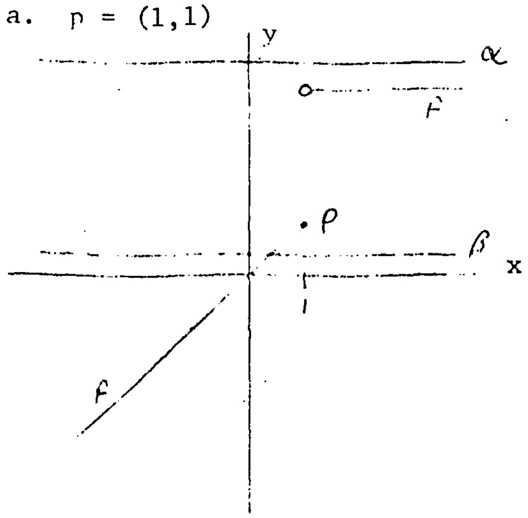


c. p = (3,0)

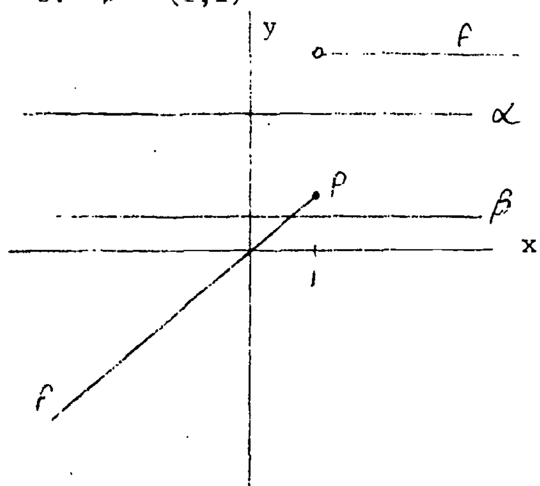


3. $f(x) = \begin{cases} 4 & \text{if } x > 1 \\ x & \text{if } x \le 1 \end{cases}$

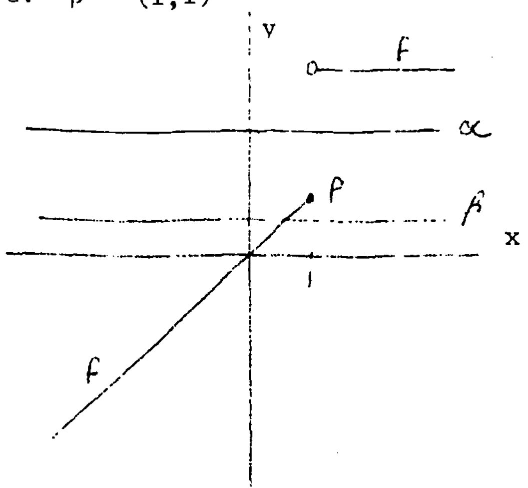
a. p = (1,1)



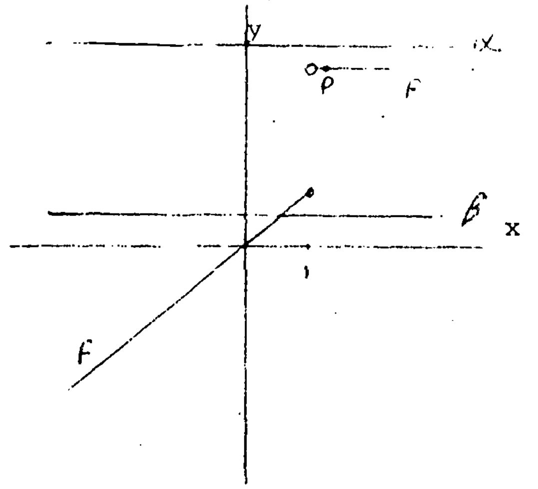
b. p = (1,1)



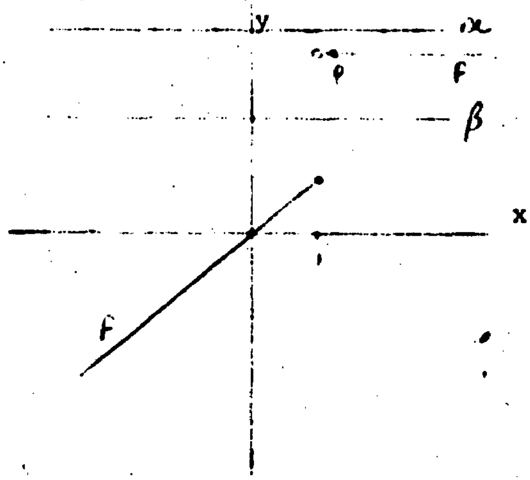
c. p = (1,1)



d. p = (1.001, 4)

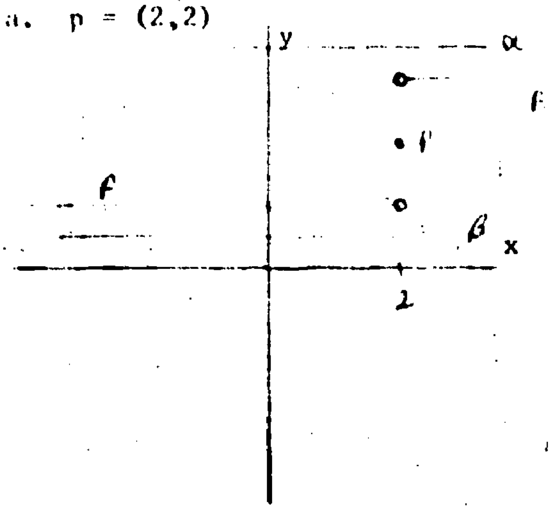


e. p = (1.001,4)

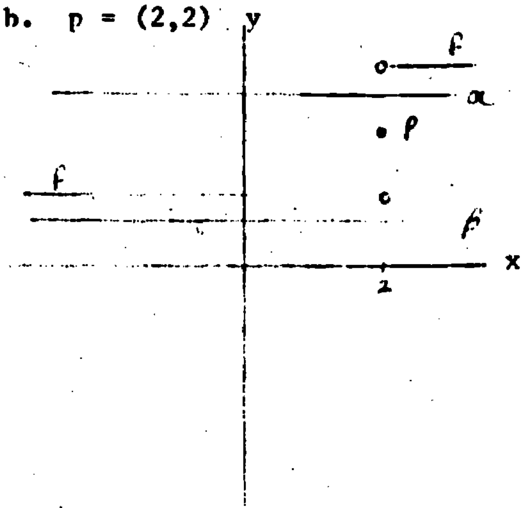


4. $f(x) = \begin{cases} 1 \text{ if } x < 2; \ 2 \text{ if } x = 2; \\ 3 \text{ if } x > 2 \end{cases}$

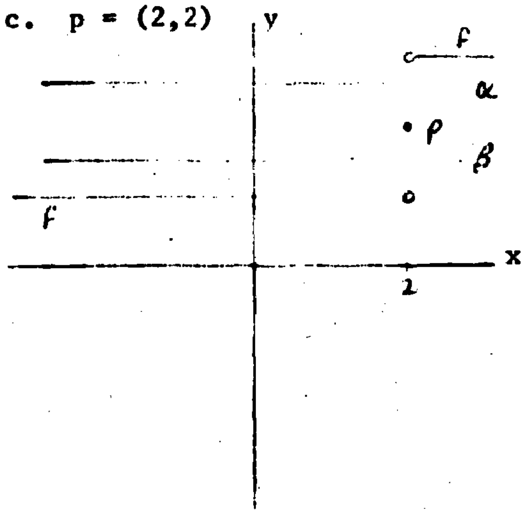
a. p = (2,2)



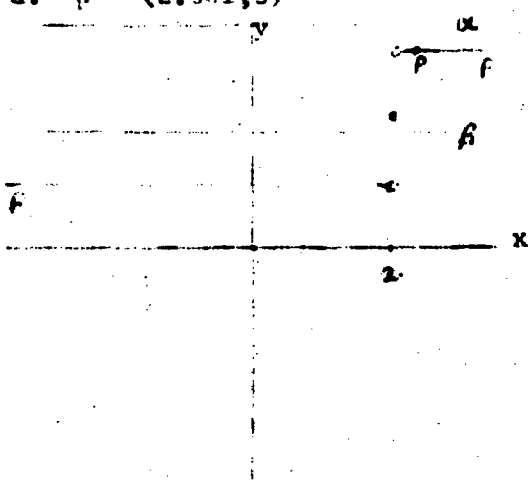
b. p = (2,2)



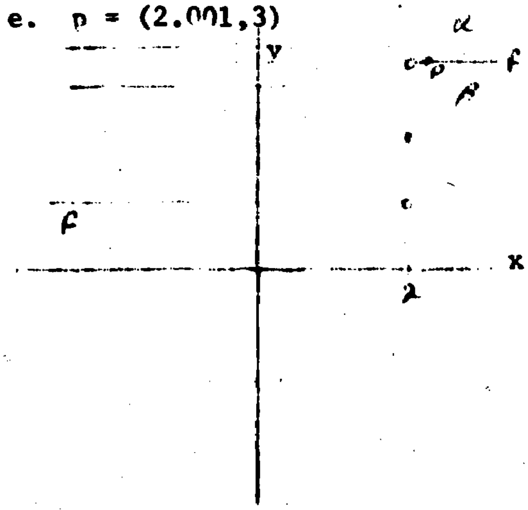
c. p = (2,2)



d. p = (2.001,3)

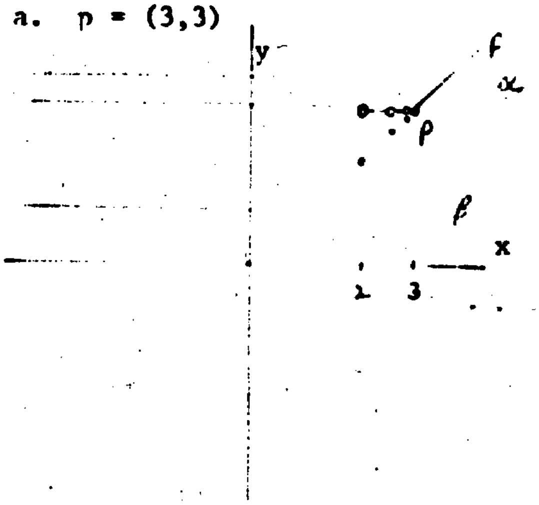


e. p = (2.001,3)

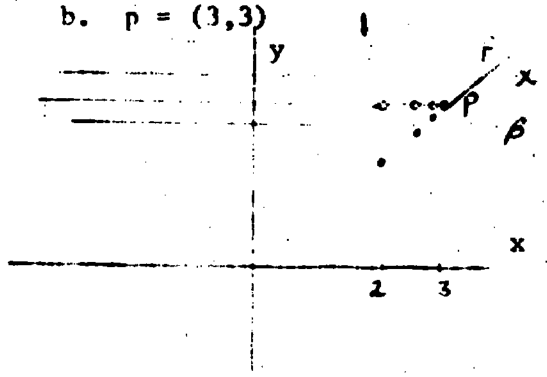


5. $f(x) = \begin{cases} x \text{ if } x > 3 \\ x \text{ if } x = 2,\ 2\frac{1}{2},\ 2\frac{3}{4},\ 2\frac{7}{8},\ \ldots \\ 3 \text{ otherwise} \end{cases}$

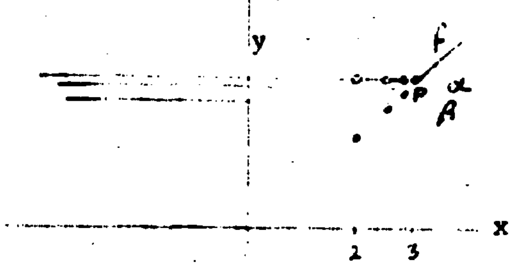
a. $p = (3,3)$



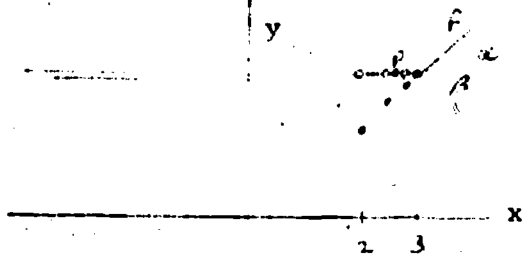
b. $p = (3,3)$



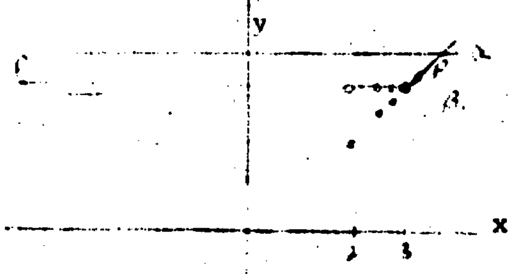
c. $p = (2\frac{3}{4},\ 2\frac{3}{4})$



d. $p = (2\frac{2}{3},\ 3)$

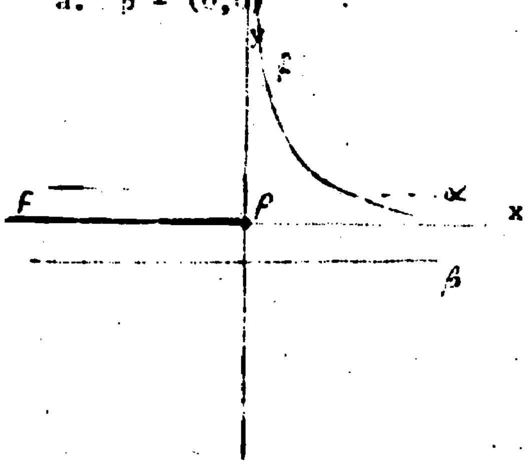


e. $p = (3.001,\ 3.001)$

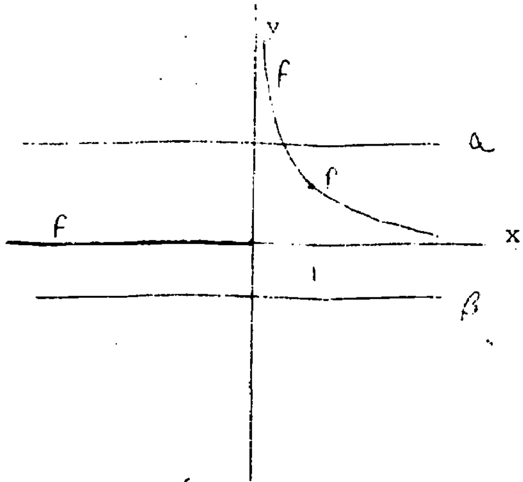


6. $f(x) = \begin{cases} 0 \text{ if } x \le 0 \\ \frac{1}{x} \text{ if } x > 0 \end{cases}$

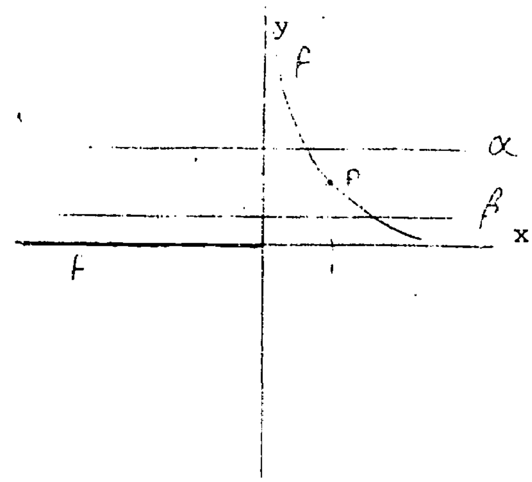
a. $p = (0,0)$



b. $p = (1,1)$

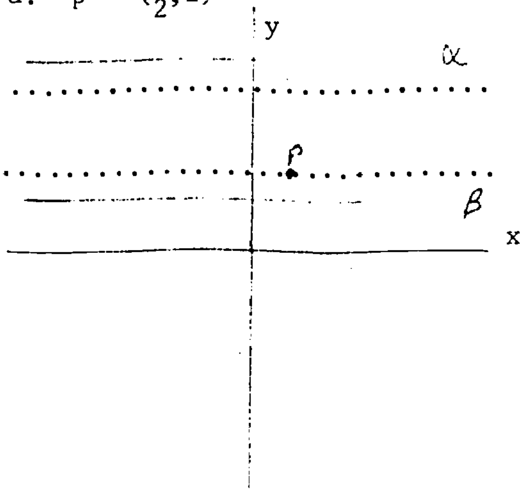


c. $p = (1,1)$

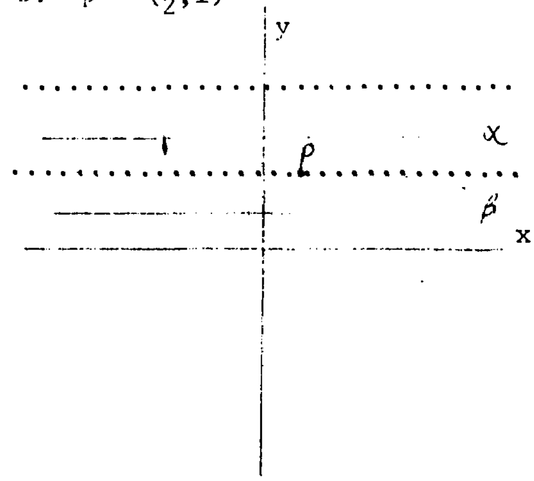


7. $f(x) = \begin{cases} 1 \text{ if } x \text{ is rational} \\ 2 \text{ if } x \text{ is irrational} \end{cases}$

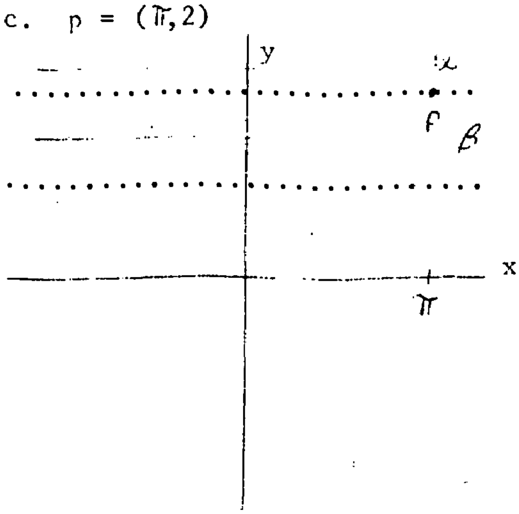
a. $p = (\frac{1}{2},1)$



b. $p = (\frac{1}{2},1)$

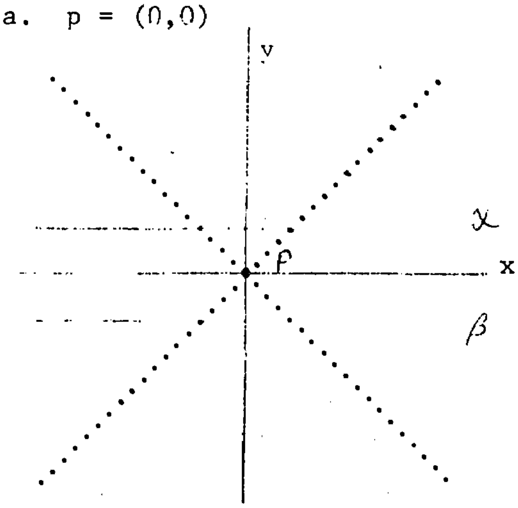


c. $p = (\pi,2)$



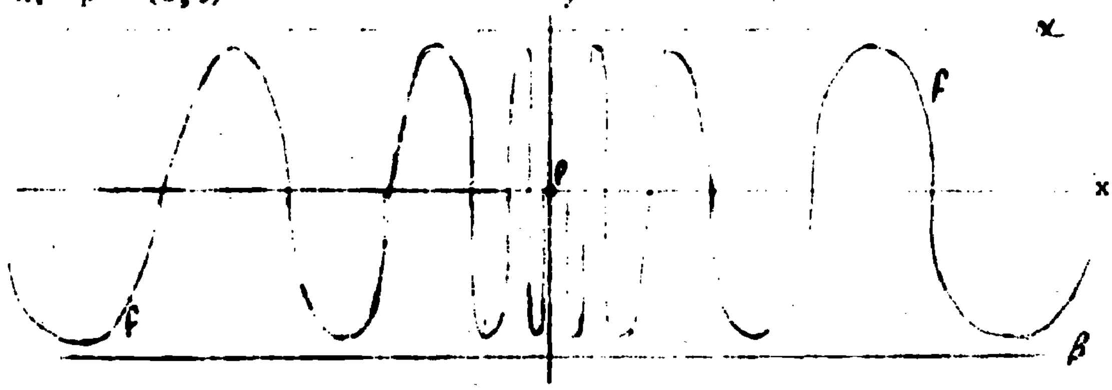
8. $f(x) = \begin{cases} x \text{ if } x \text{ is rational} \\ -x \text{ if } x \text{ is irrational} \end{cases}$

a. $p = (0,0)$

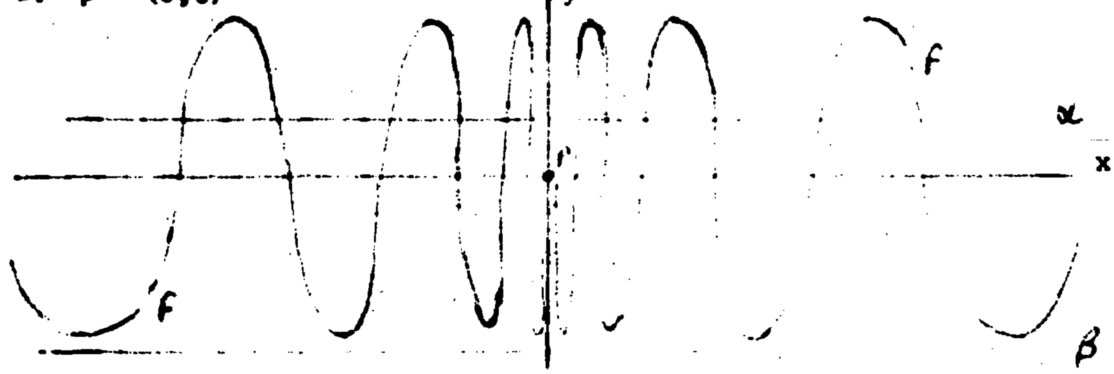


9. $f(x) = \begin{cases} \sin \frac{1}{x} \text{ if } x \neq 0 \\ 0 \text{ if } x = 0 \end{cases}$

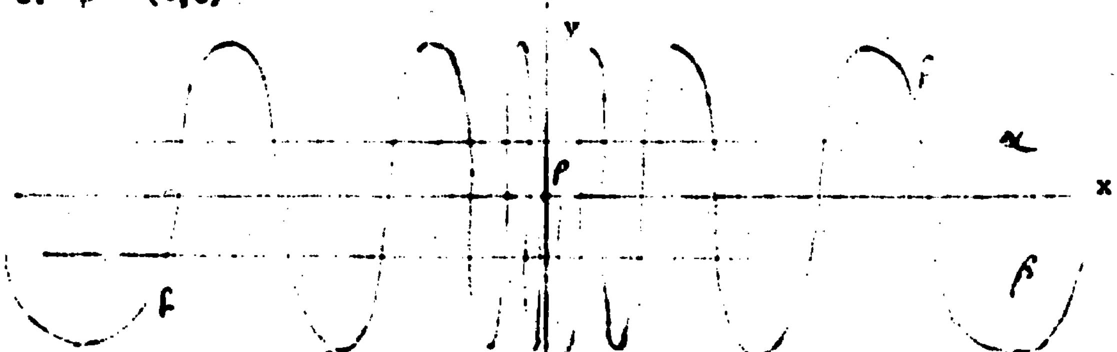
a. $p = (0,0)$



b. $p = (0,0)$

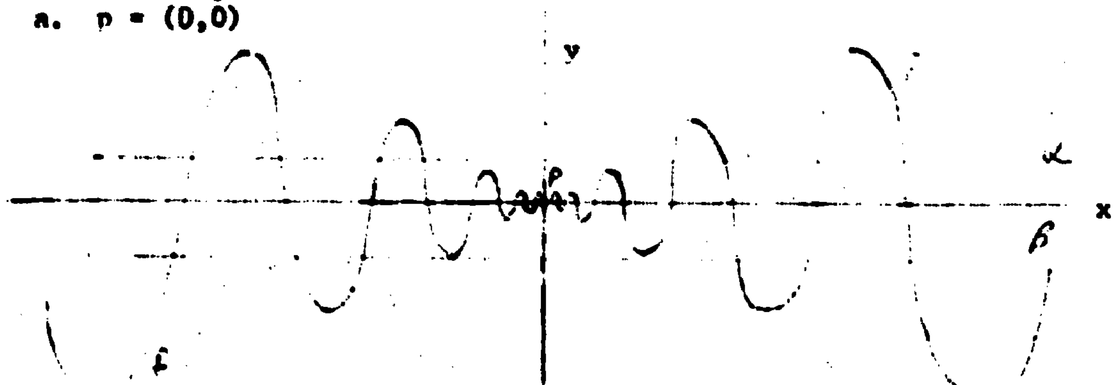


c. $p = (0,0)$



10. $f(x) = \begin{cases} x \sin \frac{1}{x} \text{ if } x \neq 0 \\ 0 \text{ if } x = 0 \end{cases}$

a. $p = (0,0)$



REFERENCES

Reference Number	Chapter(s)	Section(s)
1	2 3	
2	5	1,2,3,4,8
4	4 5	3,7 1,2
7	4 5	1,2,3,4 1,2,3,4
8	3 2	2,3,4 6,7

SAMPLE TEST QUESTIONS II

1. Find $\lim_{x \to a} f(x)$ in each of the following and illustrate that your results are correct with a graph showing appropriate α, β, h, k.

   a. $f(x) = 5x - 3;\ a = 1$

   b. $f(x) = 6;\ a = 4$

   c. $f(x) = \frac{x^2 - 4}{x - 2};\ a = 2$

   d. $f(x) = \frac{6}{x};\ a = 3$

   e. $f(x) = \frac{x^2 + x - 12}{x - 3};\ a = 3$ (1)

   f. $f(x) = \frac{12x^2 + 40x + 32}{4x + 8};$ $a = -2$

2. Determine the following limits:

   a. $\lim_{x \to 3} \frac{x^3 - 27}{x - 3}$

   b. $\lim_{x \to 2} 2x$

   c. $\lim_{x \to -\frac{1}{3}} (3x - 1)$

   d. $\lim_{x \to 0} (x^2 - 2x + 1)$

   e. $\lim_{x \to 4} \sqrt{x}$

   f. $\lim_{x \to 1} (2x - 1)x$

   g. $\lim_{x \to 2} 3(2x - 1)(x + 1)$

   h. $\lim_{x \to -7} (x + 6)^{387}$

   i. $\lim_{x \to 2} \frac{x + 3}{x + 2}$

   j. $\lim_{x \to 5} \frac{x - 5}{x^2 - 25}$ (3)

   k. $\lim_{h \to 0} \frac{(2 + h)^2 - 4}{h}$

   l. $\lim_{h \to 0} \frac{(x + h)^3 - x^3}{h}$

   m. $\lim_{x \to x_o} \sin 2x$

   n. $\lim_{x \to x_o} \cos (x + c)$

3. Find:

   a. $\lim_{x \to \infty} \frac{1}{x}$

   b. $\lim_{x \to \infty} \frac{x}{x + 1}$

   c. $\lim_{x \to \infty} \frac{2x^2 - x + 3}{3x^2 + 5}$

   d. $\lim_{x \to \infty} e^{-x}$

   e. $\lim_{x \to \infty} \ln x$ (3)

   f. $\lim_{x \to \infty} \sin 2x$

4. State at least 7 limit theorems. (2)

5. What value must be assigned to $f(1)$ if $f(x) = \frac{x^2 - 1}{x - 1}$ is to be continuous at $x_0 = 1$? (3)

6. Find the slope function, $m\,f(x)$, if:

a. $f(x) = x^2$

b. $f(x) = \frac{1}{x}$

c. $f(x) = x^2 + \frac{1}{x}$

d. $f(x) = 2x^2 - x + 5$

e. $f(x) = \frac{1}{\sqrt{x}}$ (4)

f. $f(x) = x^4$

7. Find $\frac{\Delta y}{\Delta x}$ if:

a. $y = \sqrt{x}$

b. $y = 2x + 3$

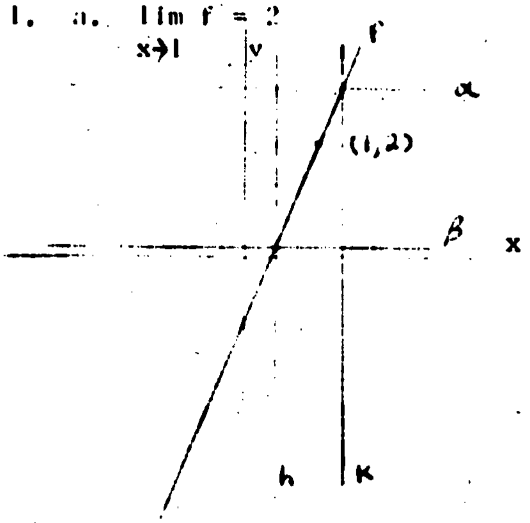
c. $y = x^3$

d. $y = \sqrt{x^2}$

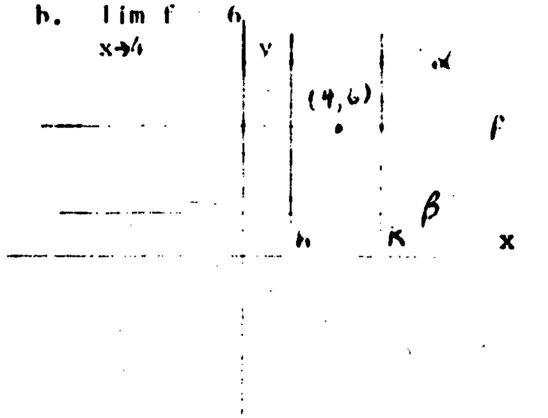
e. $y = |x|$ (4)

f. $y = ax^2 + bx + c$

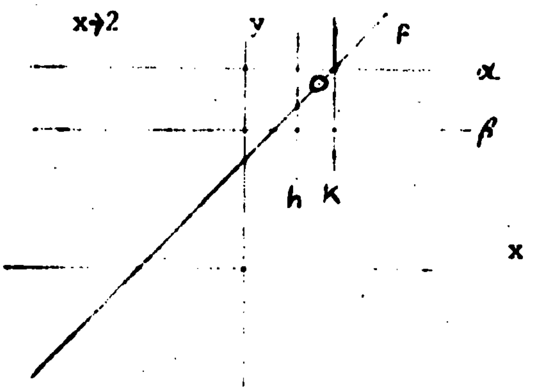
1. a. $\lim\limits_{x\to1} f = 2$



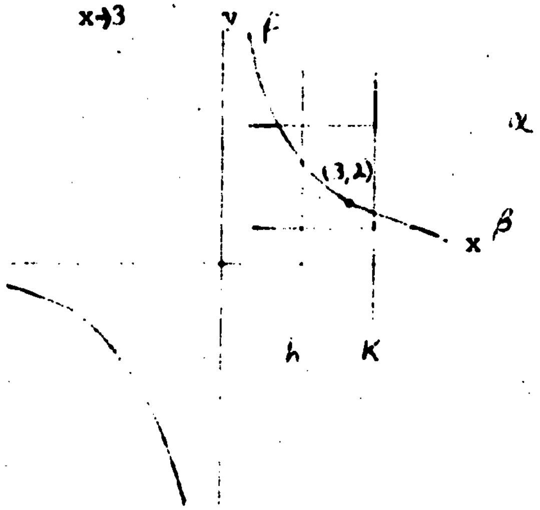
b. $\lim\limits_{x\to4} f = 6$



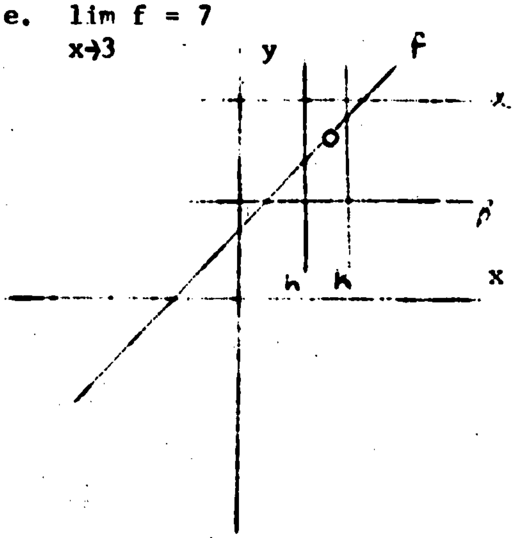
c. $\lim\limits_{x\to2} f = 4$



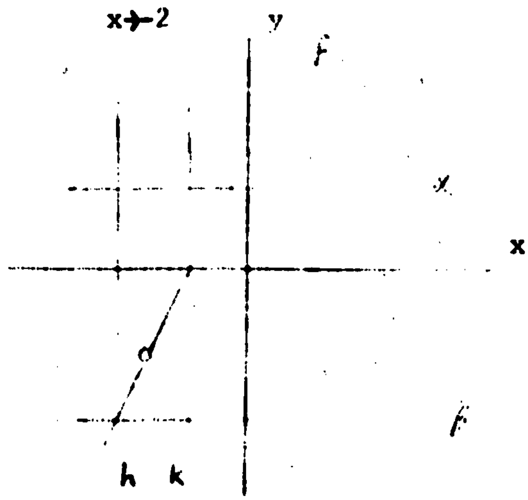
d. $\lim\limits_{x\to3} f = 2$



e. $\lim\limits_{x\to3} f = 7$



f. $\lim\limits_{x\to-2} f = -2$



2. a. 27

b. 4

c. -2

d. 1

e. 2

f. 1

g. 27

h. -1

i. $\frac{5}{4}$

j. $\frac{1}{10}$

k. 4

l. $3x^2$

m. $\sin 2x_o$

n. $\cos (x_o + c)$

3. a. 0

b. 1

c. $\frac{2}{3}$

d. 0

e. ∞

f. does not exist

4. See list in course outline

5. 2

6. a. $2x + h$

b. $\frac{-1}{x(x + h)}$

c. $2x + h - \frac{1}{x(x + h)}$

d. $4x + 2h - 1$

e. $\frac{-1}{x\sqrt{x + h} + (x + h)\sqrt{x}}$

f. $4x^3 + 6x^2h + 4xh^2 + h^3$

7. a. $\frac{1}{\sqrt{x + \Delta x} + \sqrt{x}}$

b. 2

c. $3x^2 + 3x(\Delta x) + (\Delta x)^2$

d. $\frac{2x + \Delta x}{\sqrt{(x + \Delta x)^2} + \sqrt{x^2}}$ or same as e

e. $\begin{cases} 1 \text{ if } x > 0 \\ -1 \text{ if } x < 0 \end{cases}$

f. $2ax + a(\Delta x) + b$

III. DIFFERENTIATION

PERFORMANCE OBJECTIVES

The student will:

1. Determine the derivative of any given function using the definition of derivative.

2. Differentiate any given function, using formulas which are the:

    a. sum of several functions of one variable

    b. product of two algebraic functions

    c. combinations of other functions by the fundamental operations

    d. composition of two differentiable functions

3. Apply the chain rule to find the derivative of any function given in parametric form or of a composite function.

4. Find the coordinates of any point on a given curve for which the tangent is horizontal.

5. Differentiate any function given implicitly.

6. Find the coordinates of any point on a given curve for which the tangent is parallel or perpendicular to a given line.

7. Find the equation of the tangent or normal line to a given curve at a given point.

8. Determine the missing coefficients of a function when sufficient conditions are given.

COURSE CONTENT

1. Derive by examples the formula for the derivative of a power function.

2. State the formulas to find the derivative of a sum, product or quotient of two functions.

3. Introduce the chain rule for composite functions via Leibnitz notation.

4. Method of implicit differentiation.

5. Applications of the derivative to find:

   a. slope function of a curve
   b. slope of the tangent to a curve
   c. slope of the normal to a curve

REFERENCES

Reference Number	Chapter(s)	Section(s)
1	4 5 6 7	
2	7	5,6,7,9
4	5	4,5,6,7,8,9
5	6 8	1,2 1
8	4	1,2,3,5
7	6	1,2,3,4

SAMPLE TEST QUESTIONS III

1. Find $f'(x)$ by the definition if:

   a. $f(x) = 2x^2 - x + 5$ (1)

   b. $f(x) = \frac{1}{x}$

2. Find $\frac{dy}{dx}$ by the definition if: (1)

   a. $y = \sqrt{x}$

   b. $y = ax^2 + bx + c$

3. Using formulas find the derivative of each of the following: (2,3)

   a. $y = 3x^2$

   b. $y = x^2 - 2x + 1$

   c. $g(t) = (2x + 1)^2$

   d. $h(x) = \sqrt{x^2 - 1}$

   e. $y = 2t^{\frac{1}{2}}$

   f. $f(x) = \frac{x - 1}{x + 1}$

4. Find $\frac{dy}{dx}$ if: (3)

   a. $x = 2t + 3;\ y = 4t^2 - 9$

   b. $x = 2 + \frac{1}{t};\ y = 2 - t$

   c. $x = \frac{1}{2}t;\ y = 4 - t^2$

   d. $x = t + \frac{1}{t};\ y = t + 1$

5. Find $\frac{dy}{dx}$ if: (5)

   a. $2x^2 + y^2 - 6 = 0$

   b. $x^2 + y^2 = r^2$ (r constant)

   c. $xy = 1$

   d. $\sqrt{x^2 + y^2} = 9$

6. Find the slope function of each of the following curves: (1,2,5)

   a. $y = \sqrt{x^2 + 2x + 4}$

   b. $f(x) = (x - 1)^2$

   c. $y = 3x^3 - x + 1$

   d. $x^2 + 3xy^2 - 4 = 0$

7. Find the slopes of the tangent line and the normal line to each of the following curves at the point indicated: (7)

a. $y = 3x^2$; $(-1,3)$

b. $f(x) = x^2 + 2$; $(0,2)$

c. $x^3y + xy^3 = 10$; $(1,2)$

d. $2x^2 + y^2 - 6 = 0$; $(-1,2)$

8. Find the points at which the curve has a horizontal tangent: (4)

a. $y = 4x^2 - 8x + 1$

b. $y = (x - 2)(x + 3)$

c. $y = (x^2 + 1)^5$

d. $y = \dfrac{2x + 5}{3x - 2}$

9. Find the point of tangency of the line tangent to the given curve and parallel to the chord passing through the two given points: (6)

a. $y = x^3$; $(0,0)$, $(1,1)$

b. $y = x^3 - 2x$; $(1,-1)$, $(2,4)$

c. $y = (x - 1)^2 + 2$; $(0,3)$, $(1,2)$

d. $y = x + \dfrac{1}{x}$; $(1,2)$, $(2,\dfrac{5}{2})$

10. Determine the equations of the tangent line and the normal line to the given curve at the given point: (7)

a. $y = \dfrac{x^2 - 1}{x^2 + 1}$; $(-1,0)$

b. $y = x(x - 1)(x - 2)$; $(-1,-6)$

c. $y = x + \dfrac{2}{x}$; $(2,3)$

d. $x^2 + xy + y^2 = 3$; $(-2,1)$

11. Find the values of the constants a, b, and c such that the curve $y = ax^2 + bx + c$ passes through the point $(1,2)$ and is tangent to the line $y = x$ at the origin. (8)

12. Find the constant a if $y = x^2 + a$ is to be tangent to $y = x$. (8)

ANSWERS III

1. a. $4x - 1$

   b. $-x^{-2}$

2. a. $\frac{1}{2}x^{-\frac{1}{2}}$

   b. $2ax + b$

3. a. $y' = 6x$

   b. $y' = 2x - 2$

   c. $g' = 4(2x + 1)$

   d. $h' = x(x^2 - 1)^{-\frac{1}{2}}$

   e. $y' = t^{-\frac{1}{2}}$

   f. $f' = \dfrac{2}{(x + 1)^2}$

4. a. $2(x - 3)$

   b. $(x - 2)^{-2}$

   c. $-8x^2$

   d. $\dfrac{1 - y}{x - 2y + 2}$

5. a. $\dfrac{-2x}{y}$

   b. $\dfrac{-x}{y}$

   c. $\dfrac{-y}{x}$

   d. $\dfrac{-x}{y}$

6. a. $(x^2 + 2x + 4)^{-\frac{1}{2}}(x + 1)$

   b. $2x - 2$

   c. $9x^2 - 1$

   d. $\dfrac{-3y^2 - 2x}{6xy}$

7. a. $-6, \frac{1}{6}$

   b. 0, undefined

   c. $\frac{-14}{13}, \frac{13}{14}$

   d. 1, -1

8. a. $(1, -3)$

   b. $(-\frac{1}{2}, -\frac{25}{4})$

   c. $(0,1)$

   d. None

9. a. $\left(\frac{\pm\sqrt{3}}{3}, \frac{\pm\sqrt{3}}{9}\right)$

   b. $\left(\frac{\pm\sqrt{21}}{3}, \frac{\pm\sqrt{21}}{9}\right)$

   c. $\left(\frac{1}{2}, \frac{9}{4}\right)$

   d. $\left(\pm\sqrt{2}, \pm\frac{3\sqrt{2}}{2}\right)$

10. a. $y = -x - 1$; $y = x + 1$

 b. $y = 11x + 5$; $11y + x = -67$

 c. $2y - x = 4$; $y = -2x + 7$

 d. $x = -2$; $y = 1$

11. $a = b = 1$; $c = 0$

12. $a = \frac{1}{4}$

IV. EXTREMA

PERFORMANCE OBJECTIVES

The student will:

1. Determine the difference between the average rate of change and the instantaneous rate of change over a given interval and at an end point of the interval.

2. Find the rate of change of a given variable with respect to another given variable.

3. For a given function find the point of the domain where a maximum or minimum occurs.

4. Find the maximum or minimum value of a given function.

5. Locate intervals over which a given function is concave up or down.

6. Find the points of inflection of a given function.

COURSE CONTENT

1. Average rate of change vs. instantaneous rate of change.

2. Significance of the first and second derivative to graphing.

STRATEGIES

To show that as $h \to 0$, $\frac{\sin h}{h} \to$ intuitively, use a table of values of the sine function. Note that $\lim_{h \to 0} \sin h = 0$ and $\lim_{h \to 0} \cos h = 1$ graphically. Then have students guess what $\lim_{h \to 0} \frac{\cos h - 1}{h}$ should be. With sufficient data the student should see that $h \to 0$ faster than does cos h. The proof that this limit is zero is a good exercise in algebra and the manipulation of limits. It might go as follows:

As $\lim\limits_{h \to 0} \dfrac{\cos h - 1}{h} = \dfrac{\lim\limits_{h \to 0} (\cos h - 1)}{\lim\limits_{h \to 0} h}$ gets us nowhere, try

$$\frac{\cos h - 1}{h} \cdot \frac{\cos h + 1}{\cos h + 1} \text{ where } \cos h \neq -1 \text{ as } h \to 0$$

$$= \frac{-\sin^2 h}{h(\cos h + 1)}$$

Now apply the limit theorems obtaining the desired result. At this point we can find $f'(x)$ if $f(x) = \sin x$ by the definition as follows:

$$f(x + h) = \sin(x + h) = (\sin x)(\cos h) + (\cos x)(\sin h)$$

$$\frac{f(x + h) - f(x)}{h} = \frac{\sin x (\cos h - 1) + (\cos x)(\sin h)}{h}$$

Now apply the limit operations to complete the exercise.

REFERENCES

Reference Number	Chapter(s)	Section(s)
1	8	
4	6	3,4,5
5	8	5,6
7	7	2,3,4,5
8	5	1,3,4,5

1. Determine the average rate of change of the given function over the given interval: (1)

   a. $f(x) = 2x - 1;\ x \in [-1, 3]$

   b. $g(x) = x^2 + x;\ x \in [4, 6]$

   c. $y = \sin x;\ x \in [\frac{\pi}{3}, \frac{\pi}{2}]$

   d. $y = \frac{x}{x - 3},\ x \in [0, \frac{1}{10}]$

   e. $y = |x|,\ x \in [-1, -\frac{1}{2}]$

   f. $y = x^2;\ x \in [x_o, x_o + h]$

2. Find the instantaneous rate of change of the given function at the given point: (1)

   a. $f(x) = x^2 + \frac{1}{x};\ x = -3$

   b. $g(x) = \sqrt{x};\ x = 2$

   c. $y(x) = \frac{1}{2x + 1};\ x = 0$

   d. $y = ax^2 + bx + c;\ x = x_o$

   e. $s(t) = \frac{1}{2} gt^2;\ t = t_o$

   f. $s(t) = 64t - 16t^2;\ t = -2$

3. What is the rate of change of the volume of a cube with respect to the length of an edge? (2)

4. Show that the rate of change of the volume of a sphere with respect to a radius is equal to the surface area. (2)

5. Show that the rate of change of the area of a circle with respect to a radius is equal to the circumference. (2)

6. Discuss the following functions for relative maxima and minima and determine those intervals over which the function is increasing or decreasing: (3,5)

   a. $y = x^2 + 4x + 2$

   b. $y = -2x^2 + 5x - 6$

   c. $y = 3x^{\frac{1}{2}} - x^{\frac{3}{2}}$

   d. $y = \frac{2x}{x^2 + 1}$

   e. $f(x) = x^{\frac{5}{3}} + 5x^{\frac{2}{3}}$

   f. $y = x + \frac{1}{x}$

7. Determine the point in the domain of each function where a relative maxima or minima occurs.

   a. $y = \frac{x}{x + 1}$

   b. $y = -x^3 + 2x^2 - x + 1$

c. $y = \frac{x - 1}{x + 1}$

d. $y = |x|$ (4)

8. State the absolute maxima and minima for each of the following:

a. $f(x) = 1 + 12x - 3x^2;\ x \in [-1,4]$

b. $y = |x - 2|;\ x \in [0,3]$

c. $y = \frac{1}{x^2 - 1};\ x \in [1,3]$ (3,4)

d. $y = x^2 - 3x + 4;\ x \in [-2,5]$

9. Determine the interval over which the given functions are concave up or down and determine points of inflection:

a. $y = x^3$

b. $y = 9x - x^2$

c. $y = \frac{x - 1}{x + 1}$

d. $f(x) = x^4 - 4x^3 + x - 7$

e. $f(x) = x^3 - 12x + 1$

f. $f(x) = ax^2 + bx + c$

(5,6)

10. Graph each of the following:

a. $y = x + \frac{1}{x}$

b. $y = 5 - x^{\frac{2}{3}}$

c. $y = x^2 + 4x^{-1}$

d. $y = x^4 - 2x^2$

(3,4,5,6)

ANSWERS IV

1. a. 2

   b. 11

   c. $\frac{-3(\sqrt{3} - 2)}{\pi}$

   d. $\frac{-30}{29}$

   e. -1

   f. $2x_o + h$

2. a. $\frac{-55}{9}$

   b. $\frac{\sqrt{2}}{4}$

   c. -2

   d. $2ax_o + b$

   e. gt_o

3. $3e^2$

4. $\frac{dv}{dr} = 4\pi r^2 = S$

5. $\frac{dA}{dr} = 2\pi r = C$

6.

FN	Max	Min	Increasing	Decreasing
a	None	$x = -2$	$x > -2$	$x < -2$
b	$x = \frac{5}{4}$	None	$x < \frac{5}{4}$	$x > \frac{5}{4}$
c	$x = 1$	None	$x < 1$	$x > 1$
d	$x = 1$	$x = -1$	$x \in (-1,1)$	$x > 1$ or $x < -1$
e	$x = -2$	None	$x < -2$	$x > -2$
f	$x = -1$	$x = 1$	$x > 1$ or $x < -1$	$x \in (-1,0)$ or $x \in (0,1)$

7. a. None

   b. Max. $x = 1$; Min. $x = \frac{1}{3}$

   c. None

   d. Min. $x = 0$

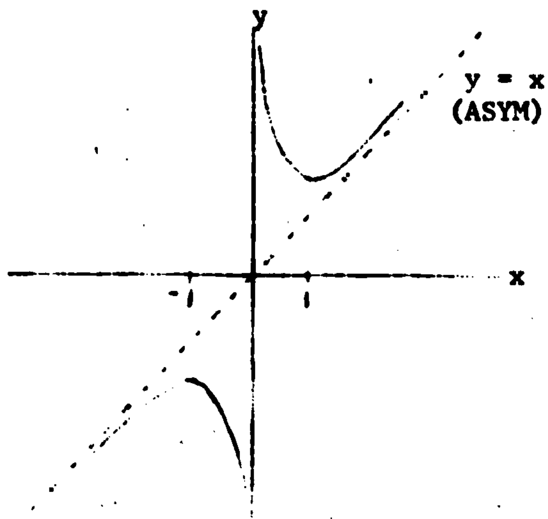
8.

Prob.	Absolute Max.	Absolute Min.
a	13	None
b	2	0
c	None	$\frac{1}{8}$
d	None	$1\frac{3}{4}$

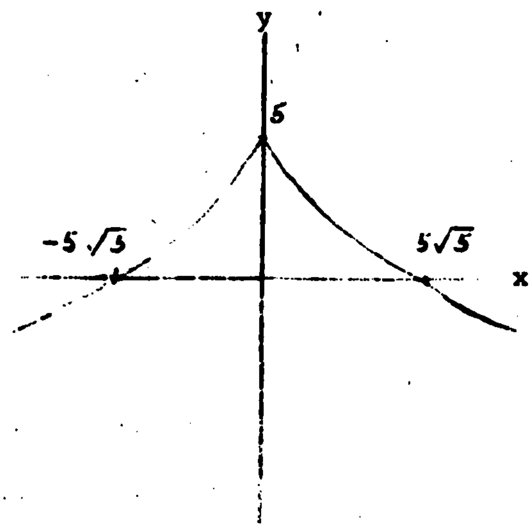
9.

Prob.	Concave up	Concave down	Point of inflection
a	$x > 0$	$x < 0$	(0,0)
b	Never	Reals	None
c	$x < -1$	$x > -1$	$x = -1$
d	$x < 0$ or $x > 2$	$x \in (0,2)$	$x = 0,2$
e	$x > 0$	$x < 0$	$x = 0$
f	$a > 0$	$a < 0$	None

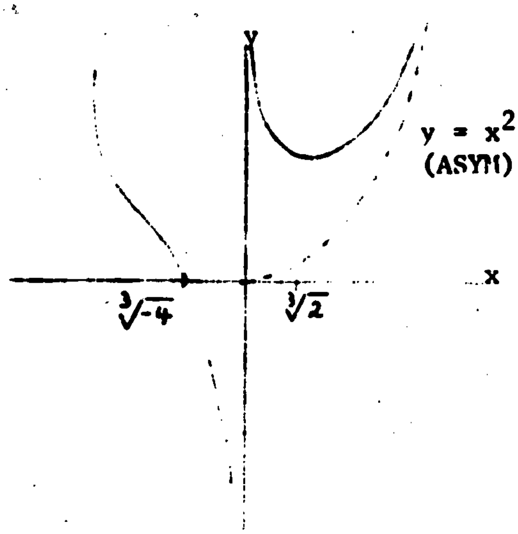
10. a.



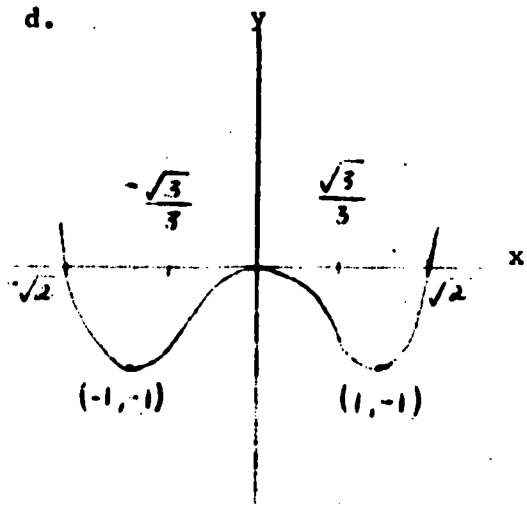
b.



c.



d.



V. INTEGRATION

PERFORMANCE OBJECTIVES

The student will:

1. Find the families of curves whose slope functions are given.
2. Solve selected differential equations given boundary conditions.
3. Determine the area under a given curve.
4. Determine the area between two curves.

COURSE CONTENT

1. Antiderivative of functions.
2. Define definite integral by using area under a curve.
3. Area between two curves.
4. Formulas for integration.

STRATEGIES

Formulas for integration might include:

1. $\int u^n \, du = \dfrac{u^{n+1}}{n+1} + C$, if $n \neq -1$

2. $\int (f(x) \pm g(x)) \, dx = \int f(x) \, dx \pm \int g(x) \, dx$

   which can be extended to a finite sum or difference of functions

3. $\int c\, f(x) \, dx = c \int f(x) \, dx$

REFERENCES

Reference Number	Chapter(s)	Section(s)
1	3 4	
2	9	5,7
4	6 7	8 6,8,10
5	5	1,2,3,4,5
7	8 9	1,3,4,8 2
8	6 7	2,8,9 2

SAMPLE TEST QUESTIONS V

1. Find an antiderivative, $F(x)$, of each of the following: (1)

   a. $f(x) = -32x$

   b. $f(x) = 2x^2 + 3x + 7$

   c. $f(x) = (x + 1)(2x + 6)$

   d. $f(x) = \frac{(x^2 + 6x - 2)^2}{9}$

2. Find the family of curves whose slope function, $m(f)$ is given: (1)

   a. $m(f) = \sqrt{x} + \frac{1}{\sqrt{x}}$

   b. $m(f) = 5 - 7t^2$

   c. $m(f) = \sqrt[3]{x^2}$

   d. $m(f) = (2t + 1)^2$

3. Determine the function y from the following information: (2)

   a. $\frac{dy}{dx} = 2x + 3$; $v = 3$ when $x = 1$

   b. $y' = \frac{3t^2}{2} - 3t + 1$; $y = 4$ when $t = 2$

   c. $\frac{dy}{dt} = 6 - 2t - 3t^2$; $y = 0$ when $t = 0$

   d. $y'' = -32$; $y' = 96$ when $t = 0$ and $y = 0$ when $t = 0$

   e. $\frac{d^2y}{dx^2} = -20$; $y' = 8$ and $y = 0$ when $t = 0$

4. Solve the following differential equations: (2)

   a. $\frac{dv}{dx} = \frac{x - 1}{y}$; $v = 5$ when $x = 2$

   b. $\frac{dv}{dx} = \frac{1}{x}$; $y = 2$ when $x = 1$

   c. $\frac{dv}{dx} = \frac{x}{y}$; $y = -5$ when $x = 4$

   d. $\frac{dv}{dx} = 2y^2$; $y = 1$ when $x = 2$

   e. $\frac{d^2v}{dx^2} = 6x - 2$; $y = 3$ and $\frac{dv}{dx} = -2$ when $x = -1$

5. Evaluate the following definite integrals:

a. $\int_{1}^{3} \frac{x\,dx}{2}$

b. $\int_{1}^{4} \frac{dx}{1 + x}$

c. $\int_{-3}^{-1} \frac{dx}{1 - x}$

d. $\int_{-1}^{2} x^2\,dx$

e. $\int_{2}^{5} \sqrt{1 + x}\,dx$

f. $\int_{-1}^{1} x^3\,dx$

(3)

6. Find the area between the given curve and the x-axis over the given interval:

(3)

a. $y = x^2 - 3x + 2;\ x \in [1,3]$

b. $y = \sqrt{x} + \frac{1}{\sqrt{x}};\ x \in [1,4]$

c. $f(t) = (2t + 1)^2;\ t \in [1,2]$

d. $y = t^2;\ t \in [0,x]$

e. $f(t) = (t + 1)(2t + 6);\ t \in [0,2]$

f. $y = 4x^3 - 3x^2 + 2;\ x \in [-3,-1]$

7. Find the area of the region bounded by:

a. $x = 1,\ x = 2,\ y = 3x,\ y = x^2$

b. $y = x,\ y = x^3$

c. $y^2 = x - 1,\ y = x - 3$

d. $y^2 = x,\ x = 4$

e. $y = 1,\ x = y,\ xy^2 = 4$ (4)

f. $y = x^2,\ x = y^2$

g. $y = x^2,\ y = x^4$

ANSWERS V

1\. a. $-16x^2 + C$

b. $\frac{2x^3}{3} + \frac{3x^2}{2} + 7x + C$

c. $\frac{2x^3}{3} + 4x^2 + 6x + C$

d. $\frac{x^5}{45} + \frac{x^4}{3} + \frac{32x^3}{27} - \frac{4x^2}{3} + \frac{4x}{9} + C$

2\. a. $\frac{2}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + C$

b. $5t - \frac{7}{3}t^3 + C$

c. $\frac{3}{5}x^{\frac{5}{3}} + C$

d. $\frac{4}{3}t^3 + 2t^2 + t + C$

3\. a. $y = x^2 + 3x - 1$

b. $y = \frac{1}{2}t^3 - \frac{3}{2}t^2 + t + 4$

c. $y = 6t - t^2 - t^3$

d. $y = -16t^2 + 96t$

e. $y = -10t^2 + 8t$

4\. a. $y^2 = (x - 1)^2 + 24$

b. $y = \ln |x| + 2$

c. $y^2 - x^2 = 9$

d. $x = -\frac{1}{2}y^{-1} + \frac{5}{2}$

e. $y = x^3 - x^2 - 7x - 2$

5\. a. 2

b. $\ln \frac{5}{2}$

c. $\ln \frac{1}{2}$

d. 3

e. $2\sqrt{3}\,(2\sqrt{2} - 1)$

f. 0

6\. a. 1

b. $6\frac{2}{3}$

c. $16\frac{1}{3}$

d. $\frac{1}{3}x^3$

e. $33\frac{1}{3}$

f. 102

7\. a. $2\frac{1}{6}$

b. $\frac{1}{2}$

c. $4\frac{1}{2}$

d. $10\frac{2}{3}$

e. $\frac{9}{2} - 3\sqrt[3]{2}$

f. $\frac{1}{3}$

g. $\frac{4}{15}$

VI. TRANSCENDENTAL FUNCTIONS

PERFORMANCE OBJECTIVES

The student will:

1. Be able to differentiate equations involving trigonometric functions.
2. Be able to integrate the following, for u a function of x:

a.	sin u du	i.	$\sin^2 u$ du
b.	cos u du	j.	$\cos^2 u$ du
c.	tan u du	k.	$\sec^2 u$ du
d.	cot u du	l.	$\csc^2 u$ du
e.	sec u du	m.	$\sin^3 u$ du
f.	csc u du	n.	$\cos^3 u$ du
g.	sec u tan u du	o.	$\frac{du}{u}$
h.	csc u cot u du	p.	e^u du

COURSE CONTENT

1. Differentiation and integration of trigonometric functions.
   a. trig identities needed:
      1. Pythagorean
      2. $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$
      3. $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$
      4. reciprocal identities
      5. inverse trig functions, as $x = a \sin y$ iff $y = \sin^{-1} \frac{x}{a}$, etc.

h. theorems needed:

1. $\lim_{h \to 0} \frac{\sin h}{h} = 1$

2. $\lim_{h \to 0} \frac{\cos h - 1}{h} = 0$

Both of the above are needed to find $f'(x)$ if $f(x) = \sin x$.

2. Differentiation and integration of $y = e^u$

3. Define $\ln x = \int_1^x \frac{dx}{x}$ for $x > 0$

4. $y = \ln u$ differentiate

5. integrate $dy = \frac{du}{u}$

REFERENCES

Reference Number	Chapter(s)	Section(s)
1	12 14	
2	11	1,2,4,5,9,10,11
4	9 10	1,2,3,4,6,7 1,2,3
5	6 7	5,6,7 2,3,4,5
7	9 11	5,6 1,2,3,7,8,9,10
8	8	1,4,5,6,7,8

SAMPLE TEST QUESTIONS VI

1. Differentiate each of the following trigonometric functions: (1)

   a. $f(x) = \tan 3x$
   b. $y = \sec t^2$
   c. $y = \sin(\cos x)$
   d. $g(x) = 2 \sin x \cos x$
   e. $y = \ln(x^2 + 2x)$
   f. $y = \ln(\cos x)$
   g. $y = x \ln x - x$
   h. $y = (\ln x)^3$
   i. $y = \sin^2 3x$
   j. $f(x) = \dfrac{\sin 2x}{1 + \cos 2x}$
   k. $y = \csc^3 \dfrac{x}{3}$
   l. $y = (1 + \sin^2 2x)^{\frac{1}{2}}$
   m. $y = \ln(\ln x)$
   n. $y = x^2 e^x$
   o. $y = e^{\frac{1}{x}}$
   p. $y = \frac{1}{2}(e^x - e^{-x})$

2. Integrate each of the following: (2)

   a. $\int \sin 4x \, dx$
   b. $\int \tan^2 x \, dx$
   c. $\int \sin^3 x \, dx$
   d. $\int \sec(3 - x) \tan(3 - x) dx$
   e. $\int \cot 2x \, dx$
   f. $\int \dfrac{\cos x}{\sin x} dx$
   g. $\int x^3 \sec^2(4x^2) \, dx$
   h. $\int \cos 8t \, dt$
   i. $\int \sec^2 \left(\dfrac{x - 5}{2}\right) dx$
   j. $\int \dfrac{\cos 2x \, dx}{\sin^2 2x}$
   k. $\int \cos^2 \dfrac{x}{2} \, dx$
   l. $\int \dfrac{4dx}{e^{3x}}$
   m. $\int \cot^2(2x - 6) \, dx$
   n. $\int x \sin(x^2) \, dx$
   o. $\int \dfrac{\sin x \, dx}{\cos^5 x}$
   p. $\int \dfrac{dx}{2x + 3}$
   q. $\int \dfrac{x \, dx}{4x^2 + 1}$
   r. $\int \dfrac{x \, dx}{x + 1}$
   s. $\int \dfrac{\sin x \, dx}{2 - \cos x}$
   t. $\int e^{2x} \, dx$
   r. $\int xe^{x^2} \, dx$
   v. $\int \dfrac{e^x \, dx}{1 + 2e^x}$

3. Evaluate the following definite integrals:

a. $\int_{-\frac{\pi}{3}}^{\frac{\pi}{6}} \sin^2 x \cos x \, dx$ (2)

b. $\int_0^{\frac{\pi}{2}} \sqrt{2 + \sin 3t} \cos 3t \, dt$

c. $\int_0^1 e^x \, dx$

d. $\int_1^0 \frac{(2x + 4) \, dx}{x^2 + 4x - 5}$

e. $\int_e^{e^2} \frac{dx}{x \ln x}$

f. $\int_0^2 \frac{1}{x^2 + 2} \, dx$

g $\int_2^e \frac{du}{u}$

h. $\int_0^1 e^{2t} \, dt$

ANSWERS VI

1\.

a. $3 \sec^2 3x$

b. $2t \sec t^2 \tan t^2$

c. $-\sin x \cos(\cos x)$

d. $2 \cos 2x$

e. $\dfrac{2x + 2}{x^2 + 2x}$

f. $-\tan x$

g. $\ln x$

h. $\dfrac{3 (\ln x)^2}{x}$

i. $6 \sin 3x \cos 3x = 3 \sin 6x$

j. $\dfrac{2}{\cos 2x + 1}$

k. $-\csc^3 \dfrac{x}{3} \cot \dfrac{x}{3}$

l. $\dfrac{\sin 4x}{(1 + \sin^2 2x)^{\frac{1}{2}}}$

m. $(x \ln x)^{-1}$

n. $xe^x (x + 2)$

o. $-x^{-2} e^{\frac{1}{x}}$

p. $\dfrac{1}{2}(e^x + e^{-x})$

2\.

a. $-\dfrac{1}{4} \cos 4x + C$

b. $\tan x - x + C$

c. $\dfrac{1}{3} \cos^3 x - \cos x + C$

d. $-\sec(3 - x) + C$

e. $\dfrac{1}{2} \ln |\sin x| + C$

f. $\ln |\sin x| + C$

g. $\dfrac{x^2}{8} \tan(4x^2)$

$+ \dfrac{1}{32} \ln |\cos(4x^2)| + C$

h. $\dfrac{1}{8} \sin 8t$

i. $2 \tan \dfrac{(x - 5)}{2} + C$

j. $-\dfrac{1}{2} \csc(2x) + C$

k. $\dfrac{1}{2} x + \dfrac{1}{2} \sin x + C$

l. $-\dfrac{4}{3} e^{-3x} + C$

m. $-\dfrac{1}{2} \cot(2x - 6) - x + C$

n. $-\dfrac{1}{2} \cos(x^2) + C$

o. $\dfrac{1}{4} \sec^4 x + C$

p. $\dfrac{1}{2} \ln |2x + 3| + C$

q. $\dfrac{1}{8} \ln(4x^2 + 1) + C$

r. $x - \ln |x + 1| + C$

s. $\ln(2 - \cos x) + C$

t. $\frac{1}{2} e^{2x} + C$

u. $\frac{1}{2} e^{x^2} + C$

v. $\frac{1}{2} \ln(1 + 2e^x) + C$

3. a. $\frac{1}{24}(1 - 3\sqrt{3})$

b. $\frac{2}{9}(1 - 2\sqrt{2})$

c. $e - 1$

d. $\ln \frac{5}{8}$

e. $\ln 2$

f. $\frac{\sqrt{2}}{2} \tan^{-1} \sqrt{2}$

g. $1 - \ln 2$

h. $\frac{1}{2}(e^2 - 1)$

VII. TECHNIQUES OF INTEGRATION

PERFORMANCE OBJECTIVES

The student will:

1. Integrate any given function requiring the use of:
    a. trigonometric substitutions
    b. partial fractions
    c. integration by parts

COURSE CONTENT

1. Techniques of integration:
    a. Trigonometric substitutions including changes in limits of integration
    b. Partial fractions
    c. Integration by parts

REFERENCES

Reference Number	Chapter(s)	Section(s)
1	26 28 29	
2	13	3,8
4	11	1,2,3,4
7	15	4,6,7
8	9	1,2,4,6,7

SAMPLE TEST QUESTIONS VII

1. Integrate using partial fractions: (1b)

   a. $\int \frac{(x + 5)\,dx}{(x - 1)^2(x + 2)}$

   b. $\int \frac{3x - 2}{(x + 2)(x + 1)(x - 1)}\,dx$

   c. $\int \frac{x^2 + 2x + 3}{x^3 - x}\,dx$

   d. $\int \frac{x^3 + 1}{(x^2 - 1)^2}\,dx$

2. Integrate using trig substitutions: (1a)

   a. $\int \frac{dx}{x^2 + 4}$

   b. $\int_{-1}^{\sqrt{3}} \sqrt{4 - x^2}\,dx$

   c. $\int_3^6 \frac{\sqrt{x^2 - 9}}{x}\,dx$

   d. $\int_0^1 \frac{x^2\,dx}{\sqrt{4 - x^2}}$

   e. $\int \frac{2dx}{x\sqrt{x^2 - 5}}$

   f. $\int \frac{dx}{(a^2 - x^2)^{\frac{3}{2}}}$

3. Integrate by parts: (1c)

   a. $\int xe^x\,dx$

   b. $\int x^2e^x\,dx$

   c. $\int \ln x\,dx$

   d. $\int e^x \sin x\,dx$

   e. $\int_1^3 x^3 \ln x\,dx$

   f. $\int 2x^3e^{x^2}\,dx$

4. Integrate using any method: (1 abc)

   a. $\int \frac{3x^2 + x - 2}{(x - 1)(x^2 + 1)}\,dx$

   b. $\int \frac{x^2 + 3x + 4}{x - 2}\,dx$

   c. $\int \frac{(x^2 - 2)\,dx}{(x + 1)(x - 1)^2}$

   d. $\int \frac{dx}{(x^2 + a^2)^2}$

   e. $\int_{-6}^{-2\sqrt{3}} \frac{dx}{\sqrt{4 - x^2}}$

   f. $\int (\ln x)^2\,dx$

   g. $\int \frac{\sqrt{x^2 - a^2}}{x}\,dx$

   h. $\int x \ln x\,dx$

ANSWERS VII

1. a. $\frac{1}{3} \ln \left| \frac{x+2}{x-1} \right| - \frac{2}{x-1} + C$

   b. $\frac{1}{6} \ln \left| \frac{(x+1)^{15}(x-1)}{(x+2)^{16}} \right| + C$

   c. $3 \ln \left| \frac{(x-1)(x+1)^{\frac{1}{3}}}{x} \right| + C$

   d. $\frac{1}{4} \ln \left| (x+1)^3 (x-1) \right| - \frac{1}{2(x-1)} + C$

2. a. $\frac{1}{2} \tan^{-1} \frac{x}{2} + C$

   b. $\pi + \ 3$

   c. $3 \ 3 - \frac{\pi}{3}$

   d. $\pi - \frac{\sqrt{3}}{2}$

   e. $\frac{2\sqrt{5}}{5} \sec^{-1} \frac{x}{\sqrt{5}} + C$

   f. $\frac{}{a^2 \sqrt{a^2 - x}} + C$

3. a. $e^x (x - 1) + C$

   b. $x^2 e^x - 2xe^x + 2e^x + C$

   c. $x \ln x - x + C$

   d. $\frac{e^x}{2} (\sin x - \cos x)$

   e. $\frac{81}{4} \ln 3 - 5$

   f. $e^{x^2} (x^2 - 1) + C$

4. a. $\ln \left| (x-1)(x^2+1) \right| + 3 \tan^{-1} x + C$

   b. $\frac{x^2}{2} + 5x + 14 \ln |x - 2| + C$

   c. $\frac{1}{4} \ln \left| \frac{(x-1)^5}{x+1} \right| + \frac{1}{2} (x-1)^{-1} + C$

   d. $\frac{1}{a^3 \sqrt{x^2 + a^2}} \left(x - \frac{x^3}{3(x^2 + a^2)} \right) + C$

   e. not defined

   f. $x (\ln x)^2 - 2(x \ln x - x) + C$

   g. $\sqrt{x^2 - a^2} - a \sec^{-1} \frac{x}{a} + C$

   h. $\frac{1}{2} x^2 \ln x - \frac{1}{4} x^2 + C$

VIII. LENGTHS AND LOGS

PERFORMANCE OBJECTIVES

The student will:

1. Find the length of a given curve or arc.

2. Find the distance a point travels in a given time interval when the coordinates of the point are given parametrically in terms of t.

3. Find the derivative of a given function by the method of logarithmic differentiation.

COURSE CONTENT

1. Formulas for the length of a curve (including parametric).

2. Logarithmic differentiation.

REFERENCES

Reference Number	Chapter(s)	Section(s)
1	14 16 19 41	
2	11	4
4	8 9 15	6 7 10
7	11 12	9 1,2,3
8	4 7 8 14	5 6 9 1,2

SAMPLE TEST QUESTIONS VIII

1. Find the length of arc in each case:

   a. $y = x^{\frac{3}{2}}$ from A (1,1) to B $(2,2\sqrt{2})$ (1)

   b. $x = t^3 + 1$, $y = 2t^{\frac{9}{2}} - 4$ from $t = 1$ to $t = 3$

   c. $x = r \cos t$, $y = r \sin t$ for $t \in [0,2]$

   d. $y = \sqrt{36 - x^2}$, $x \in [0,3]$

   e. $x = e^{-t} \cos t$, $y = e^{-t} \sin t$, $t \in [0,\frac{\pi}{2}]$

   f. $x = a \cos t + at \sin t$
      $y = a \sin t - at \cos t$, $t \quad [0, \frac{\pi}{2}]$, $a > 0$

   g. $y = \frac{1}{3}(x^2 + 2)^{\frac{3}{2}}$, $x \in [0,3]$

   h. $y = \frac{x^3}{3} + \frac{1}{4x}$, $x \in [1,3]$

2. The position of a particle P (x,y) at time t is given by $x = \frac{1}{3}(2t + 3)^{\frac{3}{2}}$, $y = \frac{t^2}{2} + t$. Find the distance it travels between $t = 0$ and $t = 3$. (2)

3. Use logarithmic differentiation to find $\frac{dy}{dx}$ if: (3)

   a. $y = x^x$

   b. $y = x^n$ (n constant)

   c. $y = e^{2x}$

   d. $y = x^{\sin x}$

   e. $y = 4^{-2x}$

   f. $y = x^4e^{-3x}$

   g. $y = e^{2x}\ln x$

   h. $y = e^{x^2 - 3x + 7}$

   i. $y = (\ln x)^x$

   j. $y = x^{\ln x}$

ANSWERS VIII

1. a. $\frac{1}{27}(22\sqrt{22} - 13\sqrt{13})$

   b. $\frac{2}{27}(488\sqrt{61} - 10\sqrt{10})$

   c. $2r$

   d. π

   e. $\sqrt{2}(1 - e^{-\frac{\pi}{2}})$

   f. $\frac{1}{8} a \pi^2$

   g. 12

   h. $\frac{53}{6}$

2. $\frac{21}{2}$

3. a. $x^x(1 + \ln x)$

   b. $n\, x^{n-1}$

   c. $2\, e^{2x}$

   d. $x^{\sin x}\left(\frac{\sin x}{x} + \cos x \ln x\right)$

   e. $-\ln 16\ (4^{-2x})$

   f. $x^3 e^{-3x}(-3x + 4)$

   g. $e^{2x}\left(\frac{1}{x} + 2 \ln x\right)$

   h. $e^{x^2 - 3x + 7}(2x - 3)$

   i. $(\ln x)^x\, [(\ln x)^{-1} + \ln(\ln x)]$

   j. $\frac{2}{x} \ln x\ (x^{\ln x})$

IX. APPROXIMATIONS

PERFORMANCE OBJECTIVES

The student will:

1. Use the trapezoidal rule to approximate a given definite integral.

2. Use Simpson's rule to approximate a given definite integral.

COURSE CONTENT

1. Approximating definite integrals using:

   a. trapezoidal rule

   b. Simpson's rule

REFERENCES

Reference Number	Chapter(s)	Section(s)
1	55	
2	13	5,6
4	8	7,8
7	16	14
8	6 9	10 11

SAMPLE TEST QUESTIONS IX

1. Use the trapezoidal rule to approximate the following definite integrals. Use n subintervals as given. When possible compare your result with the exact value of the integral. (1)

   a. $\int_1^2 x^2\,dx$, n = 4

   b. $\int_0^2 x\,dx$, n = 4

   c. $\int_0^2 \sqrt{x}\,dx$, n = 4

   d. $\int_1^2 \frac{dx}{x^2}$, n = 2

   e. $\int_0^2 x^3\,dx$, n = 4

   f. ln 2, n = 2

2. Use Simpson's Rule to approximate the following definite integrals with the given number, n, of subintervals. Where possible, compare your answers with the exact value. (2)

   a. $\int_0^2 x^2\,dx$, n = 4

   b. $\int_0^{\pi} \sqrt{\sin x}\,dx$, n = 4

   c. $\int_0^{\pi} \frac{\sin x}{x}\,dx$, n = 2 and n = 4

   d. $\int_0^1 \frac{dx}{x^2+1}$, n = 2

   e. $\int_0^{\pi} \cos x\,dx$, n = 4

   f. $\int_0^{\pi} \frac{dx}{2+\cos x}$, n = 2

ANSWERS IX

1.

Prob.	Trapezoidal Approx.	Exact Value	Approx. Error
a	$\frac{75}{32}$	$\frac{7}{3}$	.4%
b	2	2	None
c	$\frac{1}{2} + \sqrt{2}(\frac{1}{2} + \frac{\sqrt{3}}{4})$	$\frac{4\sqrt{2}}{3}$	3.45%
d	$\frac{77}{144}$	$\frac{1}{2}$	7%
e	$\frac{17}{4}$	4	6.25%
f	$\frac{17}{24}$	$\ln 2 = \int_1^2 \frac{dx}{x} \doteq .6931$	2.2%

2.

Prob.	Simpson Approx.	Exact Value	Approx. Error
a	$\frac{8}{3}$	$\frac{8}{3}$	None
b	$\doteq 2.28$		
c	$\doteq 1.857$; $\doteq 1.852$		
d	$\doteq .78$	$\frac{\pi}{4} \doteq .785$	.6%
e	0	0	None
f	$\frac{145\pi}{252} \doteq 1.8$	$\frac{\pi\sqrt{3}}{3} \doteq 1.81$	1.1%