DOCUMENT RESUME

ED 085 970 EC 060 958

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TITLE Factor Structures of Retarded and Nonretarded Children on Raven's Progressive Matrices. Studies in Learning Potential, Volume 3, Number 54.
INSTITUTION Research Inst. for Educational Problems, Cambridge, Mass.
SPONS AGENCY Bureau of Education for the Handicapped (DHEW/OE), Washington, D.C.; National Inst. of Education (DHEW), Washington, D.C.
PUB DATE 73
GRANT NE-G-00-3-0016; OEG-0-8-080506-4597
NOTE 21p.

EDRS PRICE MF-$0.65 HC-$3.29
DESCRIPTORS Adolescents; Average Students; Childhood; *Educable Mentally Handicapped; *Exceptional Child Research; *Factor Analysis; *Intelligence Differences; Mentally Handicapped; Testing; *Test Validity
IDENTIFIERS *Raven Progressive Matrices

ABSTRACT

Item responses of two samples each of 622 normal and 573 educable mentally retarded children, 6 to 15 years of age, on Raven's Colored Progressive Matrices were submitted to a principal components analysis and varimax rotation. The following four factors were obtained: continuity and reconstruction of simple and complex structures, discrete pattern completion, reasoning by analogy, and simple continuous pattern completion. The factors corresponded to readily identifiable problem types. Comparability of factor structures of normal and retarded subjects indicated the factorial invariance of the test with children of different IQ levels. (Author)

STUDIES IN LEARNING POTENTIAL

FACTOR STRUCTURES OF RETARDED AND NONRETARDED CHILDREN ON RAVEN'S PROGRESSIVE MATRICES

by

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Volume 3, Number 54

1973

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Abstract

Item responses of two samples of normal and educable mentally retarded (EMR) children on Raven's Coloured Progressive Matrices were submitted to a principal components analysis and varimax rotation. Four factors were obtained which corresponded to readily identifiable problem types. The factor structure for both retarded and nonretarded subjects was replicated by an independent sample. Comparability of factor structures of normal and retarded subjects indicated the factorial invariance of this test with children of different IQ levels.

FACTOR STRUCTURES OF RETARDED AND NONRETARDED CHILDREN ON RAVEN'S PROGRESSIVE MATRICES[1]

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During recent years the Raven Progressive Matrices has become widely used as a measure of mental development of children and adults. Described by Raven (1965) as a test of "observation and clear thinking," the Coloured Progressive Matrices (Sets A, AB, B) were designed to assess the chief cognitive processes of children prior to the stage in which the intellectual capacity to reason by analogy is used as a consistent mode of inferential thinking. Raven considered the ability required to solve the puzzles in the Coloured Matrices to be independent of acquired knowledge or previously developed verbal skills, and planned these sets for use with intellectually normal children under 11 years of age and children whose intellectual ability has become impaired.

The aim of this study was to determine the factorial composition of responses to items on this test by normal and educable mentally retarded (EMR) children, in order to establish the existence and the nature of the separate subskills measured by the test. Comparison of factor structures obtained with these two groups of subjects would indicate the degree of factorial invariance of the Raven test with children of different IQ levels.

Several factor analytic studies have examined the overlap between skills on the Raven and other tests of mental abilities (Banks, 1949; Burke, 1958; Vernon, 1950). These studies have most often been conducted with adult or older adolescent subjects and have furnished evidence that the 60-item Raven test (Sets A, B, C, D, E) taps perceptual and spatial abilities as well as Spearman's *g*. While these findings are relevant to the present study, the purpose of this investigation was not to determine overlap in mental abilities measured by total scores on various tests, but rather to determine the specific skills measured by items on the Raven sets appropriate for children.

Keir (1949) conducted an investigation of children's item responses by factor analyzing item scores of 300 children, aged 10 to 14, on the 60-item Raven test. Results revealed the presence of a general factor accounting for 37% of the total variance and two supplementary factors which contributed 13% of the variance. Keir believed that these two factors corresponded to the relative difficulty level and the matrix structure of different items. The method used in Keir's study cannot be considered comparable to modern factor analysis, since tetrachoric correlations among all possible pairs of items were factorized by simple summation. The table of factor loadings indicated that, as a result of this procedure, the loading of every item was highest on the first factor, thereby increasing the amount of variance accounted for by that factor. Keir's research differs from the present study in its methodology, age range of children tested, and sets of items administered.

In this investigation the Coloured Matrices were administered in two separate studies. The second study provided an independent sample of retarded and nonretarded children, in order to determine whether the factor structures obtained in the first study were replicable.

Method

Normal Subjects

The sample of 243 normal subjects in Study 1 was drawn from low income areas of a large urban community in Massachusetts. They ranged in age from seven to twelve and were in the second through sixth grade. Approximately 60% were male and about a fourth were black.

The sample in Study 2 consisted of 379 students at schools in a large urban community in upstate New York. They ranged in age from six to eleven and were evenly distributed among the first through fifth grades. Twenty-eight percent of these subjects' fathers were employed in business or professional occupations; the majority were blue collar or clerical workers. Fifty-five percent were male and 53% were black.

Retarded Subjects

The sample in Study 1 consisted of 399 EMR children who attended special classes in public schools in the New England area. They ranged in age from seven to fifteen, with a mean age of eleven years (SD = 2). Sixty percent were male, about 25% were black, and three fourths were from working class families. The mean Stanford-Binet IQ of EMR subjects for whom this information was available (N = 242) was 71.0 (SD = 7.8).

The sample in Study 2 consisted of 174 special class students in schools in a large urban community in upstate New York. Fifty-five percent were male. This sample ranged in age from 5½ to 14 years, with a mean age of 10 (SD = 2). All but 5% were from low socioeconomic backgrounds (welfare, blue collar, or clerical) as indicated by father's occupation, and 43% were black. The mean IQ of these EMR subjects on the WISC or Stanford-Binet was 68.4, with a standard deviation of 9.0.

Procedure

Study 1 was conducted from 1969 to 1971. The data from Study 2 were collected during the spring of 1973. Raven's Coloured Progressive Matrices (1956) were group administered to all subjects in both studies.

This test consists of 36 six-choice problems comprising Sets A, AB, B, with 12 problems in each set. Raven (1965) has described seven types of problems represented by the items in these sets, as follows: problems requiring simple continuous pattern completion (A1 to A8), pattern completion with change in one direction (A9 to A10), pattern completion with change in two directions (A11 to A12), discrete pattern completion (AB1 to AB3, B1 to B2), apprehension of three figures as a whole (AB4 to AB12, B3 to B5), concrete reasoning by analogy (B6 to B9), and abstract reasoning by analogy (B10 to B12).

The responses of all subjects on the 36 items were dichotomized into correct and incorrect responses (1 = incorrect, 2 = correct). These 36 item scores were then submitted to a principal components analysis with 1.00 in the diagonal, followed by a varimax rotation of the four factors obtained. This procedure was carried out separately with item scores of two samples of normal subjects and two samples of EMR subjects.

Results

Normal Subjects

With both normal samples, the four factor solution was found to best satisfy the criteria recommended by Kaiser (latent roots > 1.00) and by Cliff and Hamburger (1967) (significant breaks in latent roots). The four factors accounted for nearly identical percents of variance in

Study 1 and Study 2 (42.5 and 42.2, respectively); however, the largest amount of variance was contributed by Factor I in Study 1 (21.5%) and Factor III in Study 2 (23.0%).

Table 1 presents the items with high factor loadings (>.30) for both samples of normal subjects. Factors I, III, and IV were very similar in Studies 1 and 2. Factor II contained several items with high loadings for one sample but not the other. Most of these items, however, had high loadings on another factor, as well as on Factor II.

Insert Table 1 about here

When those items with high loadings on the same factor in both samples are considered, the factor structure for normal subjects seems to be as follows: Factor I includes items A7 to A12, AB4 to AB11, B3 to B7. Factor II includes items AB1 to AB3, B1 to B2 (A4 had high loadings on Factor II but higher loadings on Factor IV). Factor III includes items AB12, B8 to B12 (B7 had high loadings on Factor III but higher loadings on Factor I); and Factor IV includes items A1 to A6.

Labels for the factors can be provided according to the tasks required by the items: Factor I--Continuity and Reconstruction of Simple and Complex Structures, Factor II--Discrete Pattern Completion, Factor III--Reasoning by Analogy, and Factor IV--Simple Continuous Pattern Completion. Factors II, III, and IV correspond closely to item categories

TABLE 1

Normal Subjects:

Factor Loadings above .30 on Varimax Rotation

	Factor I		Factor II		Factor III		Factor IV	
	1[a]	2	1	2	1	2	1	2
Item	I[b]	III	II	II	III	I	IV	IV
A1							-.84	.64
A2							-.75	.57
A3							-.59	.59
A4			.39	-.32			-.47	.54
A5				-.37			-.64	.57
A6				-.36			-.45	.51
A7	.57	-.64						
A8	.39	-.34		-.37				
A9	.54	-.44		-.32				
A10	.51	-.57						
A11	.48	-.47						
A12	.43	-.38			.30			
AB1			.81	-.74				
AB2			.74	-.74				
AB3			.77	-.76				
AB4	.47	-.47		-.48				
AB5	.63	-.63						
AB6	.68	-.67						
AB7	.58	-.56	.31					

TABLE 1 (continued)

	Factor I		Factor II		Factor III		Factor IV	
	1	2	1	2	1	2	1	2
Item	I	III	II	II	III	I	IV	IV
AB8	.46	-.57			.35			
AB9	.47	-.52						
AB10	.42	-.45						
AB11	.37	-.32						
AB12					.59	.43		
B1			.57	-.72			-.32	
B2	.34		.54	-.65				
B3	.44	-.40		-.52				
B4	.60	-.53		-.33				
B5	.63	-.48				.35		
B6	.43	-.30				.43		
B7	.34	-.40			.33	.31		
B8					.72	.74		
B9					.82	.73		
B10					.76	.77		
B11					.80	.81		
B12					.63	.61		
% variance	21.5	5.2	11.0	9.7	5.7	23.0	4.3	4.3
Rotated Ss	5.4	4.9	3.1	4.3	3.9	3.6	3.0	2.3

TABLE 1 (continued)

	Factor I	Factor II	Factor III	Factor IV
Burt's coefficient[c]	.93	.78	.94	.89

[a]Study number.

[b]Actual factor.

[c]Burt's coefficient of congruence (Harmon, 1967) was used to provide an indication of the degree of similarity between each factor obtained with two samples. Loadings on all 36 items were used. Interpretation is similar to that of a product-moment coefficient.

postulated by Raven (1965) which were previously described.

Retarded Subjects

The four factor solution was again found to satisfy Kaiser's and Cliff and Hamburger's (1967) criteria. The four factors accounted for 48.5% of the variance in Study 1 and 40.1% in Study 2.

Table 2 presents the items with factor loadings above .30 for both EMR samples. While the order in which the four factors were extracted differed in the two studies, the overall structures were quite similar, with Factors I, II, and III very similar for the two samples. Factor IV in Study 2 contained several items which did not load highly on this factor in Study 1. Many of these items had high loadings on Factor I as well as Factor IV in Study 2 (e.g., items A7, A9, A10, AB4, AB5). Factor IV was the least consistent for the two EMR samples but accounted for a small percentage of overall variance (5.5 and 4.4% in Studies 1 and 2, respectively).

Insert Table 2 about here

When those items with high loadings on the same factor in both samples are considered, the factor structure for retarded subjects appears to be as follows: Factor I includes items A7, A9 to A11, AB4 to AB10, B3 to B6. Factor II includes items A6, AB1 to AB3, B1 to B3. Factor III consists of items AB12, B8 to B12, and Factor IV consists of items A4 and A5 (A6 loaded higher on Factor IV but more highly on Factor II).

TABLE 2

Retarded Subjects:

Factor Loadings above .30 on Varimax Rotation

	Factor I		Factor II		Factor III		Factor IV	
	1[a]	2	1	2	1	2	1	2
Item	I[b]	I	IV	II	II	III	III	IV
A1								
A2							.66	
A3				-.70			.74	
A4				-.57			.68	.34
A5				-.58			.60	.42
A6			-.45	-.56			.36	.50
A7	.55	.30						.57
A8	.32							.62
A9	.50	.37						.35
A10	.56	.34						.52
A11	.50	.37						
A12								.38
AB1			-.70	-.72				
AB2			-.54	-.68			.30	
AB3			-.63	-.69				
AB4	.59	.60						.31
AB5	.64	.64						.34
AB6	.63	.66						
AB7	.61	.62						

TABLE 2 (continued)

	Factor I		Factor II		Factor III		Factor IV	
	1	2	1	2	1	2	1	2
Item	I	I	IV	II	II	III	III	IV
AB8	.60	.62						
AB9	.56	.49						
AB10	.50	.52						
AB11	.41							.47
AB12					.53	.52		
B1			-.62	-.62				
B2			-.63	-.47				
B3	.46	.62	-.44	-.34				
B4	.59	.62						
B5	.61	.69						
B6	.32	.46						
B7	.43							.36
B8					.74	.76		
B9					.70	.64		
B10					.60	.53		
B11					.79	.60		
B12					.68	.45		
% variance	20.1	20.1	4.3	9.5	8.6	6.1	5.5	4.4
Rotated _Ss_	5.5	4.8	3.0	3.9	3.1	2.5	2.3	2.8

TABLE 2 (continued)

	Factor I	Factor II	Factor III	Factor IV
Burt's coefficient[c]	.85	.74	.92	.02

[a]Study number.

[b]Actual factor.

[c]Burt's coefficient of congruence (Harmon, 1967) was used to provide an indication of the degree of similarity between each factor obtained with two samples. Loadings on all 36 items were used. Interpretation is similar to that of a product-moment coefficient.

Comparison of the factor structures of normal and EMR subjects can be made by referring to Table 3. Factor III (Reasoning by Analogy) contains the same items for both groups of subjects. Factor I (Continuity and Reconstruction of Simple and Complex Structures) and Factor II (Discrete Pattern Completion) are very similar for EMR and normal subjects. Factor IV contains items A1 to A6 in the normal samples but only items A4 and A5 in the EMR samples. Item A6, while loading more highly on Factor II, had loadings above .30 on Factor IV for EMR subjects in both studies. Items A1 to A6 consist of linoleum block completions and are the easiest items on the test; these items were passed by a very high percentage of normal subjects in both studies.

Insert Table 3 about here

Discussion

Factor analysis of Raven's Coloured Progressive Matrices revealed a pattern of test items which could be logically as well as statistically defined. Four factors were obtained: I, Continuity and Reconstruction of Simple and Complex Structures; II, Discrete Pattern Completions; III, Reasoning by Analogy; and IV, Simple Continuous Pattern Completions. Items with high loadings on Factors II and III (and IV for normal subjects) closely resembled certain item categories postulated by Raven (1965).

TABLE 3

Items with High Loadings in Two Studies

of Normal and Retarded Subjects

Factor I Continuity and reconstruction of simple and complex structures		Factor II Discrete pattern completion		Factor III Reasoning by analogy		Factor IV Simple continuous pattern completions	
Normal	EMR	Normal	EMR	Normal	EMR	Normal	EMR
A7	A7		A6	AB12	AB12	A1	
A8		AB1	AB1	B8	B8	A2	
A9	A9	AB2	AB2	B9	B9	A3	
A10	A10	AB3	AB3	B10	B10	A4	A4
A11	A11	B1	B1	B11	B11	A5	A5
A12		B2	B2	B12	B12	A6	
AB4	AB4		B3				
AB5	AB5						
AB6	AB6						
AB7	AB7						
AB8	AB8						
AB9	AB9						
AB10	AB10						
AB11							

TABLE 3 (continued)

Factor I Continuity and reconstruction of simple and complex structures		Factor II Discrete pattern completion		Factor III Reasoning by analogy		Factor IV Simple continuous pattern completions	
Normal	EMR	Normal	EMR	Normal	EMR	Normal	EMR
B3	B3						
B4	B4						
B5	B5						
B6	B6						
B7							

The similarity of factor structures obtained with two independent samples provided evidence of the stability of the factorial composition of this test for retarded as well as nonretarded subjects. In addition, the factor structures of retarded and normal subjects were found to be highly comparable. This finding provides an indication of the factorial invariance of Raven's Coloured Progressive Matrices with children of different intellectual levels.

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Footnotes

[1]This research was supported by Grant OEG-0-8-080506-4597 from the Bureau of Education for the Handicapped, U.S. Office of Education, and Grant NE-G-00-3-0016 from the National Institute of Education, both under the Department of Health, Education, and Welfare. The authors wish to thank Dr. Robert H. Harrison for statistical consultation.