The European literature on algorithms for learning and instruction is reviewed in this document. The word "algorithm" is defined, the relationship between the European literature and current trends in research on learning and instruction in the United States is described, the important practical uses of algorithms are discussed, and potential high-yield research activities related to the use of algorithms are suggested. (DT)
ALGORITHMS IN LEARNING AND INSTRUCTION
A CRITICAL REVIEW

Vernon S. Gerlach
and
Fritz H. Brecke

Arizona State University
Tempe, Arizona

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Instructional Resources Laboratory
Arizona State University
Tempe, Arizona
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The term algorithm is no longer restricted to the domains of mathematics and computer science. In education and psychology, the algorithm has become a significant means for precisely representing the structure of mental and instructional processes.

In education, the algorithm has been used to represent aspects of programmed instruction, computer-assisted instruction, task analysis, and systems application. Psychologists have employed it to model cognitive processes. In a broader sense, the recent emphasis on artificial intelligence is another illustration of the application of algorithms.

Outside the United States, the emphasis on algorithms is seen in the growing body of literature in the area of educational cybernetics which has emerged during the past decade. This literature emphasizes learning, instruction, and psychology-related phenomena. Most of it is in German or Russian; very little has been translated into English. Most of this literature is
theoretical. Much of it has important implications for the increasing work with algorithms in the United States.

The purposes of this paper are (1) to review the European literature on algorithms for learning and instruction, (2) to describe relationships between the European literature and current trends in research on learning and instruction in this country, (3) to discuss important practical uses of algorithms, and (4) to suggest potentially high-yield research activities related to their use.

**Definitions**

The word algorithm is defined with varying degrees of precision. The most precise definition is used by mathematicians. Behavioral scientists use the term in a somewhat more general sense. In this paper the term is used to designate a set of rules which specify a sequence of discriminations and operations. This set of rules yields the solution to any problem of a class of problems.

For example, the system of rules to be followed for finding the greatest common divisor of two natural numbers is the classical Euclidean Algorithm. When this algorithm is applied to any two natural numbers (see Figure 1), the solution is the greatest

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**Insert Figure 1 about here**

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common divisor.
The Euclidean Algorithm

Domain: Any set of two natural numbers
Range: The greatest common divisor for any set in the domain
Entry skill: Can factor natural numbers

A: Convert both numbers into products of prime factors including 1

B: Find the smallest factor of the first product
   a: Is that same factor among the factors of the second product?

C: Mark it down
D: Strike this factor from both products

E: Strike this factor from the first product
   b: Is there a factor left in the first product?

F: The product of all factors you have marked down is the greatest common divisor.
An algorithm consists of operators and discriminators joined by connective elements. Operators are the elementary operations which must be performed by the information processing system which is using the algorithm. (They are printed in capital letters in Figure 1.) The discriminators (lower case letters in Figure 1) are always dichotomous, i.e., they instruct the user to check for the presence or absence of a condition. The connective elements consist of such expressions such as "then", "go to", and "If - then", "If not - then". (In Figure 1 they appear as arrows or as + and - signs.)

The set of input elements to which an algorithm is applied is the domain of the algorithm. The set of solutions or output elements is the range of the algorithm.

In order to use a given algorithm, an information processing system must satisfy certain antecedent conditions. For example, a learner must possess certain entry skills, or a machine must be able to accept input information as specified and/or execute the operations and discriminations listed.

Algorithms vary in the degree to which they are deterministic. A set of inputs may produce unique or multiple solutions, depending on such factors as the size or complexity of elementary operations, the degree to which operations and discriminations can be formalized, and the nature of the problem at hand. Obviously,
then, problems vary in the degree to which they are amenable to solution by algorithms or algorithmic procedures.

Algorithms can be classified in a number of ways, but any attempt at a complete, comprehensive taxonomy appears premature at this time. Three types of algorithms are discussed in this paper:

1. **Teaching algorithms** specify sequences of operations which the teaching system (such as a live teacher or a computer) is to execute. These operations may be contingent upon the presence or absence of certain conditions (branching or adaptive algorithms) or they may be strictly linear.

2. **Learning algorithms** are those algorithms which students are supposed to learn and those algorithms which a learner applies in order to learn something.

3. An important subset of the latter is the **search algorithm**, which represents a generic problem solving strategy or strategies.

**Literature**

The Russian psychologist L. N. Landa (1966) was one of the first to describe use of algorithms in instruction. Landa's teaching algorithms are characterized by (1) a precise specification of the objective and the contents of instruction and (2) a precise specification of the operations which teacher and learner must perform in order to attain a given objective and
which the teacher must perform in response to any given response from the universe of possible learner responses. When these operations are performed, they are indicative of the fact that instruction according to a specific algorithm is completed.

When a given thought process is successfully represented by an algorithm, its components and structure become explicit and thus amenable to systematic improvement. Consequently, Landa contends that the main emphasis of instruction should lie in the teaching of efficient generic algorithms (i.e., search algorithms). A teaching algorithm is the complete set of instructions for a specific program of teacher behaviors which are, in turn, contingent upon specified learner responses or behaviors. The actions or behaviors of the teacher are the operators; the responses or behaviors of the learner(s) are the logical conditions. One uses a teaching algorithm to design programmed instruction, but it is not necessary (indeed, it is rarely, if ever, done!) to teach this algorithm to the pupils.

A learner algorithm is one which is "taught" to a pupil; he uses it to guide himself to the attainment of a specific objective, since it specifies the responses he is to make and it specifies the conditions under which the responses are to occur.

The essence of Landa's work is his effort to define the potential and the limitations of describing teaching behavior by means of algorithms. He assumes that it is not possible to create
a universal teaching algorithm simply because no one can specify all the psychological and educational learner parameters with sufficient precision. In order to generate an algorithm for any given task, Landa insists that seven steps must be followed. If it is impossible to follow his steps, no algorithm can be produced.

Landa has demonstrated that certain teaching and learning processes are amenable to and in need of algorithmic descriptions. He emphasizes that the prime goal of education is the development of systematic and efficient methods of thinking in the learner and that algorithms are models of such methods.

Bung (1967, 1971) has introduced a concept which he calls a "quasi-algorithm". Algorithms are procedures which can be carried out by automata as well as by humans, while quasi-algorithms are procedures which can be carried out by humans only. All algorithms are also quasi-algorithms, but not all quasi-algorithms are algorithms. The term algorithm, according to Bung, is restricted to that subset which is strictly deterministic or completely formalizable. He places particularly strong faith in the notion of a kind of heuristic process by means of which a teacher can develop quasi-algorithms in which those operations are specified which will enable the learner to arrive at a correct solution to problems such as "Which is the correct form of the third person possessive adjective?"
While the extrapolations which Bung makes are generally extremely theoretical, his work does reveal some interesting empirical aspects which should be investigated in a rigorous research context.

Helmar Frank (1969), a German theoretician who has written extensively in the field of educational cybernetics, has applied Landa's concepts in the context of Gluschkow's (1963) theory of abstract automata. He has developed a sophisticated mathematical instrumentation as a part of his theoretical treatment of teaching algorithms. His didactic algorithms are generic computer programs which generate programmed instruction. These computer-generated programmed texts vary with respect to their adaptability to learner responses. While empirical data on the effectiveness of such computer-generated programs are scarce, this work does represent an important initial step towards automated instructional design.

Milos Lansky (1969) has used algorithms to facilitate teaching. He has also developed an algorithm for the optimal distribution of new concepts in a teaching program. Lansky extends the algorithmic programming strategies developed by Herbert Anschütz (1965) by making them responsive to certain learner characteristics.

The work of Frank and Lansky is quite similar in three respects. Each uses programmable strategies designed for generating programmed instruction as a model for the automated generation of
algorithms for teaching. Each makes use of explicit mathematical models of the learner in order to construct algorithms. Each is strongly oriented toward use of highly developed theories, such as information theory and the theory of automata, for developing the model describing the information processing system of the learning system as it responds to the teaching algorithm. It is quite possible that this approach has much to offer instructional designers in the United States.

B. A. Trakhtenbrot (1963), a Russian mathematician, has suggested a number of applications of algorithms to the teaching and learning processes. His research has concentrated on distinguishing problems which can be solved by means of algorithms from those which cannot and on describing the characteristics of each. The logical extension of his research is the identification of those teaching and learning problems which are amenable to algorithmic solution and those which are not.

Empirical Research and Practical Applications

While the theoretical literature on algorithms is quite substantial in both quality and quantity, there are very few reports of empirical research and even fewer which approach the generally accepted standards of educational research in the United States. Landa (1966) used an experimental treatment involving sixth and seventh graders in an in-class development of algorithms for learning Russian syntax. His experimental Ss dramatically out-
performed Ss in a control group taught without algorithms. Bussmann (1971) reports a study on the of which he concludes that algorithms inhibit "productive thinking" of a problem solving type; however, his methods and procedures are so questionable that his conclusions are highly suspect.

The two reports cited are not merely representative empirical studies; they are virtually the only ones.

Future research should attempt to determine the effect of such independent variables as type of teaching algorithms. For example, what effects do Landa's "discovery-type" algorithms or Bussmann's "directed-type" algorithms have on learning? The effect of form of presentation (verbal or flowchart) on pupil performance is another variable which needs to be studied. Research endeavors in which algorithms are studied in relationship to problem solving and rule learning are particularly needed, since such activity has the potential of establishing quickly what relationships, if any, exist between the European approaches discussed in this paper and areas which are of considerable interest to American educators and psychologists.
References


