A computer-assisted instructional (CAI) course at Pennsylvania State University uses an IBM 1500 system to introduce education majors to strategies for teaching elementary school mathematics. CAI is employed to provide individualized instruction on the theoretical aspects of the subject. The course is divided into 16 modules, each of which contains an outline of the content and a set of instructional objectives. Sequences of instructional episodes are designed for each objective; these are flow-charted, thus providing documentation not only of curricular material but also of the instructional logic. The advantages of the course as it is structured are that it offers self-paced individualized instruction and that it gives feedback for continuous modification of the program. In addition, the CAI course constitutes an instructional laboratory in which basic instructional research can be carried out. (LB)
The College of Education at Pennsylvania State University has an IBM 1500 instructional system [1] which until very recently has been dedicated to curricular and instructional research, but beginning this year a part of the system's capability has been committed to the resident instruction program. It is the intent of this paper to describe one of the two courses [2] now making use of the aforementioned CAI system--namely a course concerned with strategies for teaching elementary school mathematics.

General Nature of the Course

The course is separated conceptually into two major subdivisions, one dealing with theory, and the other with practice. The primary role of the CAI system is to provide individualized instruction on the theoretical concerns of the course, and laboratory experiences are provided to attend to matters of practice such as micro-teaching and other activities not suited for CAI. The practice component of the course also is supported by a one-day-a-week in-school internship experience in which the participants can engage in modest exercises of a tutorial nature.

Description of the CAI Component

The CAI portion of the course is divided into a total of 16 modules, as indicated in Table 1.

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<thead>
<tr>
<th>Module #</th>
<th>Title</th>
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<tbody>
<tr>
<td>1</td>
<td>Sets</td>
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<td>2</td>
<td>Addition of Whole Numbers</td>
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<tr>
<td>3</td>
<td>Subtraction of Whole Numbers</td>
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<td>4</td>
<td>Multiplication of Whole Numbers</td>
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<td>5</td>
<td>Division of Whole Numbers</td>
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<td>Relations and Functions</td>
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<td>Fractions, Rational Numbers</td>
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<td>9</td>
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<td>10</td>
<td>Ratio and Percent</td>
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<td>14</td>
<td>Measurement</td>
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<td>15</td>
<td>Real Numbers</td>
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<td>16</td>
<td>Probability</td>
</tr>
</tbody>
</table>

Each module has associated with it a carefully constructed outline of its content in addition to a correlated set of instructional objectives. A sample module outline is given in Appendix I, and several illustrative objectives growing out of it are given below. They pertain to Unit 3:

Strategy for Teaching Addition and Subtraction in Tandem.

Objective #9: The student can translate a flowchart for teaching addition into a flowchart for teaching subtraction.

Objective #10: The student can construct sample problems for each component of a flowchart for teaching subtraction.
Objective #11: Given a collection of different types of subtraction problems, the student can order them into a sequence illustrating the proper teaching arrangement.

The actual CAI instructional sequences have been organized into what we refer to as instructional episodes which have been prepared in flowchart form. Each instructional objective, then, has associated with it at least one instructional episode and great care has been taken to label episodes in such a manner as to permit their modification in the event that they do not teach (i.e., students do not achieve a specified objective). The practice of flowcharting episodes has the attribute of providing outstanding documentation not only of the content of the curricular material available on system, but also of the instructional logic which have been utilized. This practice, we feel, constitutes a significant advance in instructional programming; at the present time the writer knows of no other installation or project that is able to provide comparable documentation. For the benefit of the reader, sample instructional episodes relating to the objectives listed earlier are given below (the reader should note, however, that not all of the episodes dealing with these objectives are included). The flowcharts for teaching addition which are referred to in selected frames of the episodes may be found in Appendix II.

Episode #12

Since addition and subtraction are so closely related, it has become very popular to teach these ideas in tandem; that is, after a particular addition competence or skill has been taught, the corresponding subtraction idea is taught.

If addition and subtraction are taught in tandem, then the flowchart for teaching addition can be used as a guide for teaching subtraction.

Turn to Panel ____ of your Illustration Booklet where a Phase III Flowchart for Teaching Addition is given.

Observe that the first problem type following the Basic Addition Facts involves finding the sum of a two-digit number and a one-digit number where no regrouping is necessary.

Which one of the following is an example of this type of problem?

- $16 + 3$
- $17 + 5$

Both of these examples represent the sum of a two-digit number and a one-digit number, but $17 + 5$ is a type of problem that is classified as requiring regrouping while $16 + 3$ does not.

Go to Next Episode
If addition and subtraction are taught in tandem, which one of the following subtraction problems is an example of what would be taught after the basic subtraction facts have been learned?

1. 24 - 7
2. 33 - 9
3. 48 - 5

The only one of the above subtraction examples which does not require regrouping is 48 - 5.

Which one of the following subtraction problems is an example of what would be taught after the basic subtraction facts have been learned?

1. 53 - 5
2. 27 - 6
3. 35 - 7

Observe that in the example, 53 - 5, it was necessary to express 53 as 40 + 13. This sort of maneuver is what we refer to as regrouping.
As has been pointed out, the flowcharts for teaching addition can be used as guides for teaching subtraction.

The value of the flowcharts is that they prescribe a logical sequence of problem types to be taught.

Use the comprehensive flowchart for teaching addition as a guide for ordering the following subtraction problems according to the sequence in which they would be taught.

Which of the following problem types would be taught first?

- Problem C corresponds to a basic subtraction fact, and so this type of problem would be taught before either of the others.

Problem A involves finding the difference of a two-digit number and a one-digit number where no regrouping is necessary.

As suggested by the flowchart, this problem type occurs before either of the others.
Order the following problem types according to the sequence in which they would be taught. (You may wish to use the flowchart given in Panel ___ as an aid).

\[
\begin{array}{ccc}
A & B & C \\
67 & 16 & 59 \\
-52 & -8 & -4 \\
\end{array}
\]

Problem type B is a basic subtraction fact and so it would be taught first.

Problem type C involves finding the difference of a two-digit number and a one-digit number where no regrouping is required, and so it would be taught second. Therefore, the teaching order would be BCA.

Order the following problem types according to the sequence in which they would be taught.

\[
\begin{array}{ccc}
A & B & C \\
67 & 82 & 95 \\
-5 & -73 & -8 \\
\end{array}
\]

According to the flowchart (See Panel ___), the above problem types would be taught in the order ACB.
Discussion

The incorporation of the CAI component into our methods class has resulted in several significant advances.

1. It has permitted us to effectively individualize our instruction relative to the theoretical aspects of the course. Students can proceed through the CAI segments at their own rate of speed, and our experience to-date has shown some students completing them as early as the middle of the term.

2. The way in which the CAI modules have been constructed and documented, it is possible to continually up-date and refine them as necessary. Individual episodes are designed to enable the participants to achieve specified objectives. If a particular episode does not prove to be adequate, it must be revised. It is possible, of course, to replace or modify a given episode without affecting any others.

3. The conditions described in the foregoing item substantially enhance our opportunities for engaging in serious and disciplined instructional research. In short, the computer provides us with what may be the first true instructional laboratory--where the critical elements necessary to scientific investigation are present.

NOTES

1. A description of the nature of the system is given in schematic form in Appendix III.

2. The other course was reported at the Third Conference on Computers in the Undergraduate Curricula (1972) by G. P. Cartwright and C. A. Cartwright.
APPENDIX I
CAI MODULE #3
SUBTRACTION OF WHOLE NUMBERS

1. MEANING OF SUBTRACTION
   a. Subtraction as the Inverse of Addition (#1, #2)
   b. Converting Subtraction Problems into Addition Problems
      1) Translating Subtraction Sentences into Equivalent Addition Sentences (#3)
      2) Terminology: Missing Addends, Minuend, Subtrahend, Difference (#4, #4A)
   c. Subtraction of Whole Numbers Interpreted in Terms of the Operation of Set Subtraction (#5 - #7)

2. THE BASIC SUBTRACTION FACTS
   a. Composition of the "Basic Subtraction Facts"
   b. Relationship of the Basic Subtraction Facts to the Basic Addition Facts (#8 - #11)

3. STRATEGY FOR TEACHING ADDITION AND SUBTRACTION IN TANDEM
   a. Translation of the Phase III Flowchart for Teaching Addition into a Guide for Teaching the Corresponding Subtraction Ideas. (#12 - #18)
   b. Translation of the Comprehensive Flowchart for Teaching Addition into a Teaching Sequence for Subtraction (#19 - #22)

4. SUBTRACTION ALGORITHMS
   a. The "Decomposition" Method (#23)
   b. Other Methods (#24, #25)

5. DISCUSSION OF SELECTED STRUCTURAL PROPERTIES
   a. Subtraction of Whole Numbers not Commutative (#26)
   b. Subtraction of Whole Numbers not Associative (#27)
   c. Subtraction of Whole Numbers not Closed (#28)
APPENDIX II

Meaning of Addition of Cardinal Numbers

- Cardinal Number of a Set
  - Equivalent Sets
    - 1-1 Correspondence
      - Disjoint Sets
        - Set
          - Set Union

Phase I Flowchart for Teaching Addition

*It is not customary to teach the concept of disjoint sets prior to teaching the concept of addition since the (disjoint) condition is normally met in practical illustrations of the addition idea.

- Commutative Property of Addition
- Basic Addition Facts
- Associative Property of Addition

Fundamental Ideas
- About Place Value and the Decimal System of Numeration

Phase II Flowchart for Teaching Addition
Sum of a pair of two-digit numbers where no regrouping is necessary.

Term Rearrangement Principle

Sum of a two-digit number and a one-digit number where no regrouping is necessary.

Basic Addition Facts

Sum of multiples of ten where sum is less than 100.

Distributive Property of Multiplication over Addition

Sum of a pair of two-digit numbers where regrouping is necessary.

Sum of multiples of ten where sum is greater than or equal to 100.

Sum of a two-digit number and a one-digit number where regrouping is necessary.

Sum of a pair of two-digit numbers where no regrouping is necessary.

Phase III Flowchart for Teaching Addition

Phase IV Flowchart for Teaching Addition