The use of Kruskal's nonmetric multidimensional scaling model for analysis of classroom interaction data is discussed. Four distance models are proposed which lead to multidimensional representation of single sequences, sets of sequences, and behavior categories using symmetric and conditional proximity options of the model. Results of application of the four models to real data revealed that single sequence and sets of sequences were adequately represented in spaces of two or three dimensions. The dimensions were interpretable as classroom climate variables and/or affective or cognitive content of verbal behavior. Relative advantages of symmetric and conditional proximity models are discussed. (Author)
MULTIDIMENSIONAL SCALING OF CLASSROOM INTERACTION DATA*

Robert E. Rumery
Illinois State University
Barbara M. Hartnett
Lincoln College

Sequential categorization of classroom behavior has been an important source of data in analysis of the dynamics of classroom instruction for some years. A number of category systems have appeared since Flanders' introduction of the technique, but models for quantitative analysis of data so produced have been conspicuously absent. To be sure, indexes such as direct-indirect influence ratios are well known, but these are essentially ad hoc indexes based on frequency counts of behavior in sets of categories clustered on what appear to be essentially intuitive grounds. A notable exception to the absence of coherent models is the suggestion by Darwin (1959) that classroom interaction sequences can be interpreted as realizations of one-dependent (Markov) probability chains. He derived a series of four likelihood ratio tests to evaluate the hypothesis that two or more interaction sequences are realizations of the same Markov chain.

Pena (1972) reported an empirical study of the applicability of the Darwin tests to real data and concluded that long chains were not, in fact, one-dependent; therefore the Darwin procedures were inappropriate. Unfortunately there were certain logical and methodological flaws in her study which compromised this conclusion. Some of the flaws in her study are discussed in another paper presented at this convention (Hartnett & Rumery, 1973), and evidence is presented which suggests that chains of lengths typically encountered in classroom situations are, indeed, one-dependent. Applicability of the Darwin

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tests provides a basis for determining whether two or more interaction sequences differ and whether difference is due to differences in the time spent in any one category before behavior in another category is observed (steady-state probabilities), differences in transition probabilities independent of steady-state probabilities, or differences in total time devoted to any category of behavior (occupation probabilities).

When Markov chains are used as models representing psychological processes such as memory, concept attainment, or the like, transitions are ordinarily between hypothetical states inferred from behavior—not between discrete classes of observable behavior. It seems plausible to suggest that a variety of different interaction sequences could be manifestations of a reduced set of state to state transitions. Additionally, the conclusion that two observed chains are different, based on application of the tests proposed by Darvin, leaves unanswered the question of how these chains differ. In principle, the question of whether two or more distinct transitions are manifestations of a single latent or hypothetical state transition, as well as the question of what latent characteristics underly manifest differences in interaction sequences is amenable to multidimensional scaling.

Torgerson (1958) has treated multidimensional scaling as a two-stage process. In the first stage, empirical observations are transformed into interpoint distances representing dissimilarity between observed events or objects. In the second stage, points are represented in a coordinate space (usually, but not always Euclidean) preserving certain properties of the interpoint distances. This paper is concerned with application of one nonmetric multidimensional scaling model (Kruskal, 1964) in which points are represented in a metric space which optimizes goodness of fit to the ordinal properties of
of the interpoint distances. First, I will discuss two models for transforming data in interaction matrices to interpoint distances. Then, I will describe some specimen results obtained from analysis of interpoint distances using the Kruskal method. Finally, I will discuss some persisting methodological questions and some implications of the method for research on teaching.

The complement of interpoint distance is, of course, interpoint proximity. Coombs (1964) has identified two distinct forms of proximity data: symmetric proximity data and conditional proximity data. Symmetric proximity data almost invariably involve relations between elements of a homogeneous set: subject-by-subject; stimulus-by-stimulus; observed variable-by-observed variable, as in factor analysis; or, in the circumstances we are considering here, category-by-category or classroom-by-classroom. Conditional proximity data involve relations between elements of heterogeneous sets: subject-by-stimulus or classroom-by-category. Symmetric proximity data lead to multidimensional representation of elements of a single set in an object- or subject-space. Conditional proximity data lead to multidimensional representation of elements of two distinct sets in a joint space.

The probability of occurrence in a classroom $I$ of behavior in category $J$ (in Markov chains, identified as occupation probabilities) can be directly interpreted as conditional proximity of classroom $I$ to category $J$. In a sense, the order of occupation probabilities can be said to represent the order of "preference" of the classroom for behavior in various observed categories. Transition frequencies or probabilities can be transformed to symmetric distances following a procedure proposed by Restle (1959). Following Restle's procedure, the total number of events in a category (row or column sum) is considered to be the number of elements in a set. The total number
of occurrences of binary sequences in which behavior in category $i$ is followed by behavior in category $j$ and the number of occurrences of behavior in category $i$ followed by behavior in category $j$ are considered as the number of elements common to sets $I$ and $J$. The distance between sets is determined by the total number of non-common elements; that is, the distance between sets $I$ and $J$ is the difference between the total number of elements in the two sets and the number of elements common to both sets.

Data were obtained in connection with a project intended to develop a program to train future teachers in techniques for fostering divergent problem solving. Classroom interactions were recorded in 39 classroom situations involving 20 teacher trainees. Of these, 10 were students who had received training and 10 were controls. In each group, two participants were assigned to each of five grade levels: Kindergarten, Grade 3, Grade 7, Grade 10, and Grade 12. Classroom interaction sessions were videotaped in the Laboratory Schools at Illinois State University and were coded by a single Observer using an expanded interaction category system (Amidon, Amidon, and Rosenshine, 1969). Of the thirty categories in the system, nine categories proved to be codable. These are listed in Table 1.

Insert Table 1 about here.

Estimated occupation probabilities for the nine categories of behavior in the 39 interaction sequences were analyzed as conditional proximities using control parameters appropriate to multidimensional unfolding (Coombs, 1964). The configuration producing the lowest minimum stress ($S = 1.69$) represented the 39 teaching sequences and nine categories in three dimensions. The
configuration appears in Fig. 1. In this configuration, the points represented by open circles (a - i) correspond to the nine interaction categories. The dark circles represent classroom sessions. The first dimension is defined by categories Factual Questions and Evaluative Student Talk at the positive extreme and Praise, Acceptance of Student Ideas at the negative extreme (along with the "wastebasket" category Silence, Confusion, Begin, End). The second dimension is defined by Factual Questions and Factual Student Talk at the positive extreme and Lectures, Divergent Questions, and Divergent Student Talk at the negative extreme. The third dimension is defined by Evaluative Questions and Divergent Questions at the positive extreme and Divergent Student Talk at the other.

It can be seen that three fairly distinct clusters of classroom sessions emerged. The first cluster of 10 sessions located in the first octant of the space consisted entirely of controls: three Kindergarten, four Grade 3, one Grade 7, two Grade 10, and one Grade 12. A second cluster, near the origin of the configuration, included five sessions: one Grade 3, two Grade 7, and two Grade 12; all controls. The third, near the surface of the third quadrant of the (I, II) plane, consisted of four sessions involving trained subjects and three sessions involving controls. The controls included one Kindergarten session, one Grade 10, and one Grade 12. The sessions involving trained participants included one each of Kindergarten, Grade 3, Grade 7, and Grade 12. The (I, III) plane separated sessions involving trained subjects from those involving untrained subjects ($\chi^2 = 10, df = 2, p < .005$). The untrained subjects
conducted sessions characterized by Evaluative Questions. Factual Student Talk and Evaluative or Student-initiated Student Talk. Trained subjects conducted sessions which were much more variable in behavioral emphasis. In four sessions the dominant behavior was Divergent Student Talk. Others ranged from nearest to Divergent Questions and Lectures to approximately equidistant from Divergent questions, Evaluative Questions and Evaluative or Student-initiated Student Talk.

Symmetric distances were analyzed for 20 sessions in which behavior was coded in eight categories. Of the 20 sessions, 12 were representable in one dimension; usually involving clustering of six or seven behavior categories at one extreme and one or two at the other. In six of the twelve sessions represented in one dimension, the isolated category was Divergent Student Talk. Similar conditions were observed in two- and three-dimensional configurations except that fewer behavior categories were clustered two and three categories were isolated respectively in two- and three-dimensional configurations. Categories most frequently isolated were student talk categories. The implication of this general tendency is that behavior in various student talk categories tended to be somewhat independent of teacher talk categories.

In this study, it appears that multidimensional scaling was not instrumental in identifying possible latent states or latent characteristics of individual interaction sessions. This may have been due to any of several factors. First, all of the participants, whether trained or untrained, were inexperienced. The absence of coherent organization of interactions seems plausible under these conditions. Second, most of the observed interaction sequences were rather short, and were of somewhat different intent than might be expected in "real" classroom interactions. Third, certain questions could be raised about the general method-
ology of multidimensional scaling and about specific aspects of this analysis.

Perhaps the most important general methodological question is whether necessary empirical conditions are satisfied which may lead to representation of points in a space with the properties of a power metric. The nature of these conditions is outlined in Beals, Krantz and Tversky (1968). They point out that failure to satisfy (or even test) the necessary ordinal assumptions compromises the interpretability of multidimensional or takes advantage of random variation in achieving one. Given that the necessary assumptions hold, and theoretically justifiable distance functions exist, it is not at all clear that a Euclidean representation is the appropriate one. A city-block representation, for example, might be more appropriate.

In conclusion, it must be admitted that the questions with which we began remain, in the main, unanswered. Nevertheless, it should not be concluded that the methods are thereby not useful. The separation of trained and untrained subjects in the conditional proximity analysis suggests at least empirical utility. What seems to be called for as a crucial component is testing of foundational assumptions from a coherent theoretical analysis of the phenomenon of classroom interaction. Lacking such analysis and testing, multidimensional scaling methods are unlikely to rise above the status of elegant and interesting procedures for summarizing and displaying data.
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Figure 1. Joint configuration of interaction sessions and behavior categories.
Table 1.

Selected Categories from Expanded Classroom Interaction Category System.

1. Teacher Talk

   a. Teacher praises student or accepts student ideas.
   b. Teacher asks factual questions.
   c. Teacher asks divergent questions.
   d. Teacher asks evaluative questions.
   e. Teacher lectures.

2. Student Talk

   f. Factual student talk.
   g. Divergent student talk.
   h. Evaluative student talk and student-initiated talk.
   i. Silence, confusion, begin sequence, end sequence.