This newsletter summarizes findings from the research on secondary school mathematics published during 1972 that might be used by teachers. Covered is research on the role of materials, mathematics laboratories, computers and calculators, informing students of objectives, student computational achievement, course organization patterns, personality characteristics affecting mathematics learning, and teaching strategies. A list of 26 references is included. (DT)
Questions about what, when, or how to teach mathematical topics and ideas arise every day. Researchers as well as teachers (and remember: most researchers are teachers, too!) try to find answers to such questions. This review presents some of these investigations which were published during 1972. (For reports of such research in previous years, see Suydam, 1972a and 1972b). In selecting these studies, the primary criterion was: do the findings have meaning for the teacher—that is, are the findings applicable in the classroom? Studies whose findings would appear to be of interest primarily to other researchers are not included. Other criteria involved evaluation of the research report: how carefully the research was designed and carried out was considered.

See if any of your questions have been explored!

Materials. What Helps Whom?

In a review of research on the role of materials, Fennema concluded that research appears to indicate that the ratio of concrete to symbolic models used to convey mathematical ideas should reflect the developmental level of the learner. Thus, alternative models should be available so the learner can select the most meaningful one for him.

Researchers continue to explore such alternatives, as well as the role of materials per se. For instance, in a study involving 65 geometry students, Waters found that there were no significant achievement or attitude differences between groups using circular geoboards, the geoshet (a two-dimensional version of the geoboard), or "conventional classroom methods."

Kuhfittig looked at the role of materials in relation to "guided discovery" learning. He selected 40 seventh graders at two achievement levels, and randomly assigned them to four groups using (1) either intermediate or maximal guidance and (2) either abstract or concrete materials, for a two-day unit on ratio conversion. The intermediate guidance groups had a carefully structured sequence of questions, while the maximal guidance groups had careful explanations of individual steps. Low-ability students who used concrete materials achieved better than low-ability students who did not use materials; no difference was found between high-ability students. For intermediate-guidance groups, mean transfer-test scores for students using concrete aids were higher than scores of those not using aids; no difference was found for maximal-guidance students.

Shoecraft investigated the effects of three instructional approaches on translating selected types of algebra word problems: direct translation, high imagery with materials, and high imagery with drawings. Twelve seventh-grade mathematics classes and ten ninth-grade algebra classes spent eight days on differential treatments of number, coin, and age problems, and four days on identical treatment of work and mixture problems. It was concluded that, except for low achievers who seemed to derive particular benefit from representing problems with materials, students taught to translate directly performed comparably to those experiencing material referents and superior to those experiencing pictorial referents. Shoecraft added: "Thus the popular assumption that materials and/or pictorial representation of mathematics in and of itself enhances mathematics learning is perhaps unjustified. To expect such representation to facilitate mathematics learning is to assume that the mathematics implicit in the use of materials and drawings is descriptive of what is going on in the heads of students. The disparity between the two was evident in this study."

Mathematics Laboratories: How Effective?

Research reflects continued concern with the effectiveness of a laboratory or activity approach. Vance and Kieren reported on a ten-week investigation of the effectiveness of laboratory activities used once a week in grades 7 and 8. In the Mathematics Laboratory Group, students worked in groups of two, with written instructions and physical materials to help them discover concepts or relationships, then did practice exercises. In the Class Discovery Group, the same content was used, but the teacher demonstrated with concrete materials, leading the group to discover. The Control Group had the regular program with no laboratory work.

No significant differences were found in achievement of content covered in the regular program, even though class...
time was spent in informal exploration. Students in the two treatment groups achieved about the same on tests of content done in the laboratory, except that average- and low-ability seventh graders did better in the Class Discovery Group. Both Laboratory and Class-Discovery Groups scored higher than students in the regular program on cumulative achievement, transfer, divergent-thinking, and attitude measures.

In a study with eight tenth-grade classes using geometry content, White found that inquiry lessons used with individualized teaching-learning units significantly increased critical-thinking, achievement, and retention scores for average- and high-ability students. Laboratory lessons significantly increased achievement and retention scores for low- and average-ability students. Students in the laboratory group made the greater gain in scores for attitude toward mathematics.

Silbaugh studied 36 seventh-grade mathematics classes. Twelve classes attended multiple-activities laboratories twice a week during the school year. 12 classes were housed in the same school but did not attend the laboratories; 12 classes were in schools with no laboratories. The students who attended the laboratories appeared to achieve significantly higher on a standardized test.

In a 14-day study with eighth-graders, Whipple taught elementary concepts of metric geometry to two classes by a laboratory method emphasizing use of manipulative materials, while two classes used individualized instruction units. Students in the laboratory group scored higher on conventional written tests and showed better ability to compute areas and volumes using actual objects. No differences in spatial perception were found.

Johnson developed a nine-week curriculum for a ninth-grade class using apparatus and experiments which involve active manipulation, with game-playing, discussion, and children working in pairs or in small groups. The group taught with this curriculum scored higher on achievement and attitude measures than did a control group.

Ditmer presented responses to specific questions related to guidelines for developing a mathematics laboratory, from state supervisors and from teachers using a laboratory approach in grades 7-12.

In studies previously summarized in Suydam (1972a), Herfield and Kieren reported that use of computer programming as a problem-solving tool was especially helpful for average and above-average students in grade 7; in grade 11, it appeared best for average achievers.

Gasin assigned six classes of ninth-grade mathematics students to three treatments: a conventional algorithm set consisting of the usual textbook approach, used with or without a calculator, and an alternative algorithm set where each fractional operand was converted to a decimal, used with a calculator. Five mastery-learning units were used for an eight-week period. For low-ability or low-achieving children, the alternative algorithm with the calculator was found to be a "viable alternative" to the conventional algorithm with or without the calculator for promoting computational skill with positive rational numbers. Use of the calculator did not significantly affect performance with the conventional algorithm.

Objectives: Does Experience Help?

Loh investigated the use of behavioral objectives with two second-year algebra classes. Students who were informed of behavioral objectives did not learn or retain better than students not informed of objectives.

On the other hand, Harris found that, for four geometry and algebra classes, prescribed content with set daily goals, feedback, and systematic reinforcement increased achievement in each course.

Achievement: What is Studied and Status?

Austin and Prevost reported that computation scores for eighth graders were lower in 1967 than in 1965 or 1963; different achievement tests were used, however. In grade 10, those students who had used "modern" or "transitional" textbooks in earlier grades scored higher on some subtests than did those who had used "traditional" textbooks.

Hammons found a significant decline in computational skills in eighth grades in Louisiana schools studied during 1960-1969, but a significant change in reasoning was not found.

Organization: How is Mathematics Learning Affected?

Gaskill studied the relationship between achievement and personal adjustment in middle schools and in junior high schools. Scores from 846 eighth graders from middle schools and 381 eighth graders from junior high schools indicated that differences in mean gain in achievement significantly favored the junior high school group. No differences in personal adjustment were found between the two organizational patterns.

Buchman studied low-achievers in ninth grade from schools providing only a two-semester algebra course, a three-semester course for low achievers and "slow workers," or a four-semester course. He found no differences in achievement, but some affective aspects were better in the lengthened courses.

Personality: What Characteristics Affect Mathematics Learning?

May identified students as "sensing" or "intuitive" personality type. The 295 eighth graders' scores on achievement and attitude measures were then compared. A
significant difference in computation, concepts, and applications scores was found between sensing and intuitive types. No differences in attitudes toward mathematics were found. May concluded that teachers should consider type of personality when planning instruction.

**Teaching Strategies: What Patterns Are Used?**

*Gregory* studied 20 teachers and their seventh-grade classes. He had one of each teacher's classes audio-taped five times, and administered a reasoning test to students at the beginning and end of the semester. The teachers were ranked on the basis of analysis of the frequency of their conditional moves: that is, how often did they use "if-then" language in their teaching. Students of teachers who used such language more frequently outperformed students of teachers who made fewer such statements, on the reasoning tests. Thus the teacher, through use of logical language in a variety of situations, can help students to develop greater achievement in logic.

*Wolfe* listed eight strategies observed being used by 11 mathematics teachers in grades 9 and 10 in an investigation of the verbal activity or "justification" as it is carried out in the classroom. Criteria for identifying justification "moves" in such ventures were also noted.

Cooney and Henderson attempted to identify methods of instruction which prove effective in helping students to structure their knowledge; that is, to organize in a meaningful way the concepts, facts, and principles they learn. From videotapes of 44 instances of mathematics teaching by ten teachers in grades 7 through 12, they identified nine organizing relations: set membership, set inclusion, analysis, specifying, characterizing, explaining, implicating, generalizing, and abstracting. These are described: teachers might find it interesting to check these descriptions and compare them with their own classroom procedures.

*A complete annotated listing of studies published during 1972 is available from ERIC's SMEAC. The listing will also appear in the November 1973 issue of the Journal for Research in Mathematics Education.*

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**LIST OF SELECTED REFERENCES**


Whipple, Robert M. A Statistical Comparison of the Effectiveness of Teaching Metric Geometry by the Laboratory and Individualized Instruction Approaches. (Northwestern University, 1972.) DAI 33A: 2699-2700; Dec. 1972.


The new address of the National Council of Teachers of Mathematics is: 1906 Association Drive, Reston, Virginia 22091. NCTM moved into the new building in April; the formal dedication was early in May. It is one of the first buildings in a new educational park setting.

The Second International Congress on Mathematical Education (ICME) was held at Exeter University in England during late summer, 1972. That Congress exhibited a strong interest in discussion of aspects of mathematical education on a wide international scale. The International Commission on Mathematical Instruction (ICMI) responded by giving sponsorship to international symposia, some of which are listed here for the benefit of American mathematics educators. For information concerning particular symposia, please write directly to the person and addresses listed.

(1) Poland. Symposium at Warsaw; 1974.
Main Subject: Mathematics in Primary Schools (Children from 6 to 11 years of age.)
Professor Z. Semadeni, Institute of Mathematics, Polish Academy of Sciences, U.L. Sniadeckich 8, Warszawa 1, Poland.

Main Subject: Interactions between mathematical education and linguistics.
Dr. D. Saint-Rossey, UNESCO House, Malik Street, PO Box 30592, Nairobi, Kenya.

Main Subject: Curriculum and teachers' training.
Professor S. Iyinaga, 12-4, Otsuka 6-Chome, Bunkyo-Ku, Tokyo, Japan.

(4) India. Regional Conference; late 1974.
Main Subject: The Development of an integrated curriculum in mathematics for the underdeveloped countries.
Professor P. L. Bhatnagar, Dean of Studies, Department of Mathematics, Himachal Pradesh University, Simla 5, India.


** "DAI" refers to Dissertation Abstracts International.**

Computer searches of the ERIC document base are available from various concerns. Here is some information on several:

* Systems Development Corporation has developed a service for searching the ERIC files from a terminal in your own office. Documents may be requested by accession number, clearinghouse code, author, title, publication date, descriptors, identifiers, institution or source of origin, sponsoring agency, and/or issue. Multiple categories may be selected within a single request. If a printout of all items found in a search is desired, it can be accomplished on-line at the terminal, or off-line, which saves terminal time costs. Off-line printed items are air-mailed to your address the same day as requested. For further information, write:

  System Development Corporation
  SDC/ERIC Search Service, Room 3113
  2500 Colorado Avenue
  Santa Monica, California 90406

* The New England Research Applications Center (NERAC) at the University of Connecticut is disseminating retrospective and selective information from the ERIC files. Users of the search service will be given assistance in instructing and implementing searches. For further information, write:

  Dr. Daniel U. Wilde
  Director, New England Research Applications Center
  Mansfield Professional Park
  Storrs, Connecticut 06268

* The Resource Information Center has available a low-cost computer software package for searching the ERIC files. The first phase locates and prints a list of accession numbers. The second phase prints abstracts and other selected information found in RIE and CJE. The software package can be installed on any IBM 360 from a Model 30 upwards. For further information, write:

  Edward Kraether, or Kent Horne
  Resource Information Center
  Box 8009 University Station
  Grand Forks, North Dakota 58201

* Oregon Total Information System (OTIS) will do ERIC subject searches for institutions, on ERIC records dated 1969 or later. The logical operators “or,” “and,” “and not” may be used, with no more than 20 descriptors. Up to 130 citations can be printed. For further information, write:

  Benjamin L. Jones
  OTIS
  354 East 40th Avenue
  Eugene, Oregon 97405
The list of subscribers to *Investigations in Mathematics Education* is continuing to grow. In the journal are abstracts and analyses of recent research reports in mathematics education. For further information about subscriptions to *Investigations in Mathematics Education*, write:

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Incidentally, some printing problems delayed the publication of Volume 6, Number 2; subscribers should receive their copies soon. Volume 6, Number 3 is now at the printers.

In the last issue of this Newsletter (Vol. 5, No. 2), two errors have been found in the review of “Research on Elementary School Mathematics, 1972.” First, on page 1, Weaver (1972) reported data from 23 schools; that means 135 classes, instead of the 23 stated. And on page 2 is a similar error: in Kratzer’s (1972) study a total of 12 classes were involved. Apologies are offered for the inadvertent diminishing of the scope of these studies—and thanks go to the two authors for calling attention to the erroneous information.

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