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ABSTRACT

Mastery learning is an approach to learning whereby students are expected to demonstrate competence of one level, or unit, of learning objectives before advancing to the next level. Most junior college instruction is group-paced, with the instructor determining the rate with which units are presented. One compromise between existing instruction and ideal mastery learning techniques is to provide specific supplementary instruction for those students who fail to master a given unit, while maintaining the group pace. The research reported focused on one form of that compromise: the effect of teaching one additional lesson per unit to those students who did not achieve mastery of that unit. Another part of the research examined the effect of providing students with detailed behavioral objectives. The research was conducted in selected English and algebra courses at five community colleges in Southern California. Dependent variables were scores on semester exam and mastery rates, as defined by the proportion of the number of students who received A's, B's, and C's to the total number enrolled. The results of the research are reported separately for the English and algebra classes. In the algebra classes, the students received testing and remediation for mastery, and they achieved significantly higher final exam scores than control students. There was no significant difference between mastery rates of experimental and control students. In the English classes, the students who received detailed behavioral objectives scored significantly higher on their final exam than did control subjects who did not receive the objectives. (Author/DB)

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BEHAVIORAL OBJECTIVES AND MASTERY LEARNING APPLIED TO TWO AREAS OF JUNIOR COLLEGE INSTRUCTION

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SUMMARY

Mastery learning is an approach to learning whereby students are expected o demonstrate competence of one level, or unit, of learning objectives before dvancing to the next level. This approach has two basic featureo: first, be-avioral objectives, or learning units, are clearly defined and hierarchically rdered; second, the amount of time students spend on learning a unit will vary rom student to student, thereby necessitating some form of individually-paced nstruction. Most junior college instruction is group-paced, with the instruc-or determining the rate that units are presented. One compromise between ex-sting instruction and ideal mastery learning techniques is to provide specific upplementary instruction for those students who fail to master a given unit, hile maintaining the group pace. This research focused on one form of that ompromise: the effect of teaching one additional lesson per unit to those tudents who did not achieve mastery of that unit. Another part of the research xamined the effect of providing students with detailed behavioral objectives.

The research was conducted in selected English and Algebra courses at five ommunity colleges in Southern California. Dependent variables were scores on semester exam and mastery rates, as defined by the proportion of the number f students who received A's, B's, and C's to the total number enrolled.

The results of the research are reported separately for the English and lgebra classes. In the Algebra classes the students received testing and re-ediation for mastery, and they achieved significantly higher final exam scores han control students ($\alpha = .05$). There was no significant difference between astery rates of experimental and control students. In the English classes, he students who received detailed behavioral objectives scored significantly igher on their final examination than control students who did not receive the bjectives.

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I. THE PROBLEM

Introduction

Education in America has long proceeded on the assumption that, since students are normally distributed with respect to ability to learn, they will also be normally distributed with respect to achievement of educational objectives. With the exception of special classes for the very low or very high ability, most classes involve equal time and equal amounts of instruction for all students. The resulting achievement usually approximates a normal distribution of grades from A to F. The "good" students are those with grades of B and A, while those assigned C, D, and F grades are considered to have learned some, but not enough to be considered for direct entrance to four-year college or to many occupations. "We proceed in our teaching as though only the minority of students should be able to learn what we have to teach (Bloom, 1968, p. 2)."

Mastery Learning

In recent years, educational researchers have been developing and testing instructional strategies which enable a majority of students to learn well. One set of these strategies was originally discussed by Carroll (1963) in "A Model of School Learning," then elaborated by Bloom (1968) in "Learning for Mastery." This model for mastery learning suggests that almost all students can master a subject or achieve at a desired level of competition in a subject. As summarized by Block:

> [Mastery learning] suggests procedures whereby each student's instruction and learning can be so managed, <u>within the context of ordinary group-based classroom instruction</u>, as to promote his fullest development. Mastery learning enables 75 to 90 per cent of the students to achieve to the same high level as the top 25 per cent learning under typical group-based instructional methods. It also makes student learning more efficient than conventional approaches. Students learn more material in less time. Finally, mastery learning produces markedly greater student interest toward the subject learned than usual classroom methods (1971, p. 3).

The essential components of mastery learning are:

1. Mastery is defined in terms of particular educational objectives each student is expected to achieve. These are commonly described as behavioral objectives. Identification of the objectives is the first step in designing a mastery learning program.
2. Instruction is organized into well-defined units. Each unit consists of a systematic collection of specific behavioral objectives.
3. Complete mastery of each unit is required before proceeding to the next. This is important because the units are hierarchically sequenced so that successful learning of each unit depends upon prior learning (Gagné, 1961, 1962).
4. An ungraded, diagnostic-progress test is administered after each unit to certify mastery of unit objectives. The mastery level, or criterion level, is pre-determined by the instructor or course designer, and is usually between 75 and 95 per cent of the unit objectives.

5. Supplemental instruction is provided for those students who do not ıeet the mastery level on the diagnostic test. A highly individualized program ·ill provide corrective instruction for the specific errors of each student. here resources permit, this supplemental instruct'on is individually prescribed or the errors of each student. This ideal is se'dom attainable, and supple- ıental instruction is in practice an alternate form of the original instruction.

6. Time is used as a variable in individualizing instruction. Students ace themselves, taking as long as they individually need to achieve mastery.

Mastery learning has been studied on several levels, including junior high Kim, 1969; Collins, 1969), college undergraduate (Keller, 1968; Biehler, 1970) nd college graduate (Airasian, 1967). Each of these studies describes a dif- erent form of class organization and instructional techniques, while maintain- ng the basic format of mastery learning. In each case the researcher maintained high degree of administrative control over the instruction. The results of hese reported studies have been generally supportive of the virtues of master- ng learning described by Block.

Problem with Junior College Instruction

One function of comprehensive junior colleges throughout the country is to rovide the first two years of college work acceptable for transfer to a four- ear institution. The stated philosophy of the junior college is "open door," .e., any high school graduate or eighteen-year old citizen will be accepted as student.

The difficulty of relating the diverse goals of "open door" admissions and ompetence in university-level transfer courses is associated with instructional trategies. One such strategy is to give entry or placement tests in basic aca- emic skills, then provide remedial courses for those students who lack the kills prerequisite for university-level-courses. However, many colleges which ared to evaluate their remedial programs found that very few students who are neligible on the criteria of testing programs and grade point average for col- ege English and math ever successfully pass college level work (Benjamin Gold, 965; Richard Bossone, September 1966).

In summary, a frequent pattern for entering junior college students follows testing-failure-dropout sequence. Students enter the junior college because hey are not eligible for a senior institution and do not have the prerequisite kills to successfully accomplish college level work. They are given a test attery and placed in one or more remedial courses. These courses are group- aced, at a rate determined by the instructor or the department. The success ate, as measured by students who master most of the skills needed for college evel work, is at most 40 per cent, and usually much less. The remedial stu- ents suffer discouragement, not only from being placed in remedial courses, ut from having failed to master the content of the remedial courses. Academic robation and eventual dropout are frequent occurrences.

The problem, then, is to significantly increase the number of students who aster the skills taught in remedial English and math. courses in junior college. ncreases in mastery rates from 40 per cent to 60 per cent and higher have al- eady been achieved at the junior high and college levels through application f mastery learning techniques (Kim, 1969; Keller, 1968). Particular improve- ent has been noted with lower I.Q. students (Kim, 1969). Junior college stu- ents may also, because of some academic deficiencies, find particular improve- ent.

Research on the Problem

This report documents research on mastery learning in five Southern California community (junior) colleges. The subjects chosen were elementary Algebra, and remedial English, comparable to a 10th or 11th grade grammar and composition course. Mastery of these subjects is important for success in most college level math or English courses, so the experiment was not an artificial test of mastery procedures.

Research Objective and Questions

The objective of the research was to examine the effect of analyzing course material into discreet sequential learning blocks and testing for mastery after each learning block was completed. The research questions are:

----Will junior college students who are permitted to proceed with instruction in sequential blocks after mastery of previous blocks, or after a second attempt at mastery: (1) learn more than a comparable group of students who are not so instructed?; (2) have a mastery rate which is higher than a comparable group who are not so instructed?

Educational Significance

Though attrition rates at junior colleges vary greatly, almost all comprehensive junior colleges have a very large proportion of dropouts. One important reason is the "falling behind" of many students. When one does not master the first learning block in a sequence, one does not have the prerequisite skills to learn and understand the next. The result is a continually falling further and further behind in classroom work. This frequently leads to a self-perception of inadequacy by the learner. This research is an attempt to examine the effect of assuring the prerequisite skills in two important subjects of junior college work. It provides information to two-year colleges across the country about their entrance testing and an alternative approach to remediation programs.

Definition of Terms

1. Achievement test - The final semester examination given to both control and experimental classes.
2. Behavioral Objective - A statement of what the learner will be doing when he is demonstrating that he has reached the objective (Mager, 1962, p. 52).
3. Community College - A junior college.
4. Control Group - An intact junior-college class of either English or Algebra which was taught without interference by the experiment.
5. Elementary Algebra - A one-semester junior-college course which covers approximately the content of one year of high school algebra.
6. Experimental Group - An intact junior-colege class of either English or Algebra which was taught in the same manner as the control group, except:
   a. There was a different instructor,
   b. If instruction was in Algebra, the class was tested for mastery at the end of each unit of instruction, and those students who failed to master a unit were provided self-instructional materials and were retested.
   c. If instruction was in English, the students were provided with behavioral objectives for the course.

The instruction in both English and Algebra was group-paced as in the control group.

7. Mastery of an Objective - A student will be considered to have mastered an objective if he can consistently provide the correct response to questions designed to elicit the behavior specified by the objective, making allowance only for clerical errors.

8. Mastery of a Unit - A student will be considered to have mastered a unit when he has demonstrated mastery of a pre-determined percent of the objectives of the unit.

9. Mastery Rate - The ratio of the number of students who complete a course with a grade of A, B or C to the total number who were enrolled and in attendance the first week.

10. Remedial English - The junior-college course which immediately precedes college English.

11. Remediation - Providing self-instructional materials to students who failed to master the first test of a unit.

12. SCAT-Q and SCAT-V - School and College Aptitude Test, Quantitative and Verbal portions. This test is given as part of the placement testing series by most California community colleges.

13. Self-Instructional Materials - For this experiment, audio cassette tapes, programmed instruction, or other self-paced materials, selected and/or prepared by the participating instructors. The same materials were used at each unit in all the participating colleges.

14. Unit of Instruction - A series of instructional objectives taught and tested as a group.

<u>Limitations</u>

The research was limited to students in elementary Algebra and remedial English courses at the following junior colleges in southern California:

Cerritos College
Los Angeles City College
Rio Hondo College
San Diego City College
San Diego Mesa College

Some compromises with mastery techniques were made in this project. In order to ensure maximum cooperation by the diverse instructors in the experiment, some of the changes prescribed by mastery techniques were either dropped or modified. The most significant change was that of time as a variable. Extra time was planned for those students who failed to master a given unit, but the instruction was still group-paced, rather than individually-paced. That is, instruction proceeded from unit to unit for the group as a whole, with non-mastery students learning the new unit, while also coming in on their own time for supplemental instruction on the previous unit.

Additional changes were in supplemental instruction and in mastery requirements. Ideally, supplemental instruction should be prescribed for each student according to the problems he missed on the unit test. After such instruction he would be tested again for mastery and, if he failed to achieve the mastery level, new supplemental instruction would be prescribed. The process would continue until mastery was achieved. Changes in these mastery ideals were necessitated by the amount of time and effort the participating instructors had available for testing-retesting and for the design of supplemental instruction. It was decided to use one set of additional instruction programmed for individual student use, but not individually prescribed. One retest was to be given, but no additional instruction was to be offered.

In addition to the planned compromises with mastery learning noted above, one Algebra instructor found that the constraints of the project were too much for his courses and changes were made. This instructor at Los Angeles City College was unable to complete instruction for all of the objectives, so he gave a different final exam from that which was used by the others for reporting

achievement scores. The results of the Los Angeles City experimental group are therefore reported separately.

All ten of the Algebra classes (five experimental, five control) used the same textbook (Drooyan, 1969).

Because of the close correspondence of the stated behavioral objectives and the common textbook, the Algebra part of this research was able to develop and use common unit tests for mastery and common self-paced instructional materials for each unit.

The English group was more diverse than the Algebra group, with the result that several changes were made with the planned research. There was no agreement in the English group on textbook or on teaching method, so that no unit tests or common instructional materials were prepared. The agreement on a common set of detailed behavioral objectives and a common objective final examination was a significant accomplishment in itself, for each college's remedial English course has traditionally taken a different view of how to prepare students for a college English course. Consequently, the final examination scores for the English part of this research project are a test of the behavioral objectives part of the mastery learning technique, and do not reflect that part of mastery instruction based on test-retest of each unit of instruction. Additionally, the non-objective part of the English test, a written essay, was different for each instructor. As a consequence they are incomparable and cannot be used for criterion data.

II. DESIGN AND METHODOLOGY

Experimental Population and Sample Selection

Five public community colleges participated in this research. They were listed in the Limitations section of Chapter 1. These colleges are representative of the population of the 92 community colleges in California and, to some extent, of the nearly one thousand in the country. The student subjects for this research are representative of every type of student which public comprehensive community colleges serve. They come from every social class, every ethnic background, and every population density that our country offers.

Where possible, two instructors from each college were identified to participate in the experiment. They were asked to participate by the president of the college or by the department chairman after the college had agreed to participate. Instructions were given that those selected should be representative teachers. There was one faculty member teaching a section of remedial English and one teaching Elementary Algebra. The subjects for the experimental groups were those students in the appropriate English and Algebra classes of the selected instructors during the Spring semester of the academic year 1971-1972. The control groups were a sample of comparable sections of the same course. Control group instructors were selected by the same means as the experimental instructors. There was no systematic error in the assignment of students to control or experimental sections, and many sections of each class were taught during a semester. In order to eliminate as much sampling error as possible, the control sections were selected which were taught at approximately the same times as the experimental classes.

Variations in the planned number of instructors were necessitated by the inability to get volunteer instructors for the English classes at Cerritos and San Diego City College. Consequently, two experimental English classes were selected at San Diego Mesa. Thus, there were a total of four experimental English instructors; one each at Los Angeles City and Rio Hondo, and two at San Diego Mesa. There was one experimental Algebra instructor at each of the five colleges.

Treatment Variables

For each subject-matter area, English and Math, two dependent variables were used. One variable was the set of scores on the semester achievement test given during final exam week. The second dependent variable was the mastery rate for each of the experimental and control classes.

To help understand sources of variance on the achievement scores, data was also collected on sex (male/female) and measures of verbal and quantitative ability (SCAT-V and SCAT-Q).

Developmental Procedures

In the Spring of 1971, instructors were identified for the experimental and control classes and group leaders were chosen for the math and English groups. In math, the leader was one of the experimental instructors, while in English the leader was the English department chairman at a non-participating junior college. The instructors and group leaders are listed in the Appendix, p.

Meetings were then called by the group leaders to: (a) orient the experimental instructors to the concept of mastery learning; (b) form teams responsible

for development of behavioral objectives, unit tests, final test and self-instructional materials. The development of each of the experimental subject areas proceeded in different ways.

Development of the Algebra Course

In the Fall of 1971, seven meetings were held by all of the experimental instructors to develop objectives and materials. In the Spring of 1972, the experimental semester, one meeting was held to discuss problems and adjust instructional strategies as needed. In addition to these meetings there was frequent communication by teams of instructors responsible for various elements of the experimental program.

Responsibility for development of material was distributed as follows: Two instructors developed behavioral objectives and the final examination based on those objectives; one instructor developed the 17-unit tests for mastery; two instructors developed alternate learning materials. As each of these materials was completed, a meeting was held and the materials were edited by the entire group so that the same material could be used by each instructor.

The behavioral objectives were elicited primarily from problems to be learned in the textbook. From the detailed objectives a representative sample of problems was chosen for the final exam. The unit objectives were chosen according to their apparent place in a hierarchical ordering of the course objectives. The hierarchy was determined by discussion among all of the instructors as to which objectives were prerequisite to others toward mastery of the terminal objectives.

The level of mastery on each of the unit exams was an important issue among the instructors. Although some recent research indicates that optimum learning is effected by mastery of 80 to 85 per cent of the objectives (Block, 1970), most of the instructors felt that a somewhat lower level was more realistic in their classes. These instructors felt that the motivation of their students would be adversely affected by a level of 80 per cent. They agreed to set their own levels, with one instructor choosing 70 per cent, three choosing 75 per cent and one choosing 80 per cent.

The alternate learning materials were linear programmed and self-instructional. They each consisted of a cassette tape and a ditto-reproduced set of exercises. There was one unit of material designed for students who failed to master each of the 17 units of the course. The material was specific to the objectives of each unit, but was not designed for diagnosis and remediation of individual errors on the mastery test. The instructors considered such individual diagnosis, but the development of suitable branching programs was deemed beyond the resources of this project.

Development of the English Course

In the Fall of 1971, meetings were held by all of the experimental instructors. After initial discussion, it was decided to concentrate on the development of a common set of behavioral objectives and a final objective examination based on those objectives. Since each of the instructors had a different idea about how to teach to the terminal objectives, there was no agreement as to unit objectives in an instructional hierarchy, and there was consequently no development of unit tests for mastery or self-instructional materials for alternate learning on each unit. The instructors also felt that the development of a complete set of objectives would, by itself, command all of the time and resources

which they had for the project. At the first meeting they found that their differences on objectives for a course in remedial English were quite diverse, and they were aware that reaching agreement on a single set of objectives would be a long and arduous experience. They required nine meetings in all, including two, two-day meetings.

The actual method of developing objectives was worked out by the group. At one meeting the instructors brought samples of writing from their classes, graded according to levels of acceptability for a regular college English course. The grading measures were: would succeed in a college English course, might succeed, and would not succeed. The teachers then graded each other's papers and found broad disagreement. After extensive sharing of views on what constitutes good writing, they regraded each other's papers and found significant agreement. The elements of good writing which had emerged in discussion were recorded and used as the basic outline for development of objectives. Each of the instructors took one of the elements of the outline home for refinement into sub-objectives, then returned it for editing by the group. In this way, a single and very extensive set of objectives was developed out of what had been five very diverse approaches to teaching remedial English.

The final examination was developed from the objectives, with a multiple-choice exam to test the sub-objectives of writing style, and a written exam to test the ability of students to integrate the elements of style.

Experimental Procedures

The instructors for the Algebra component of the experiment followed the mastery learning procedures of providing students with clear behavioral objectives, testing for mastery at the end of each one-week unit, and providing remedial self-instructional materials for those students who failed to master a unit. After one try at remediation on a unit, non-mastery students were retested, but they were allowed to continue with the class even if they did not achieve mastery on the second try. Ideal mastery learning presumes self-paced instruction and continued remedial work until mastery is achieved on each unit, but the participating instructors were not prepared to individualize their classes to that extent. They did not feel that there was sufficient time or resources to design a completely individualized course.

The experimental Algebra classes used two distinct procedures of mastery learning: providing students with detailed behavioral objectives, and testing and remediation for mastery on unit tests. If statistical results were to indicate a significant difference between experimental and control groups, there would be no way to determine which effect, behavioral objectives or testing/remediation, was responsible for the difference. To preclude such confusion of effects, it was decided to provide the control classes with copies of the objectives. Therefore, significant differences could be attributed to the testing/remediation effect.

The experimental semester for the English classes was different than for the Algebra classes because all of the developmental effort had been toward creating the behavioral objectives. Without the second effect of test/remediate for mastery on each unit of instruction, the English experiment was a test for the effect of behavioral objectives. Consequently, the control English classes did not receive the objectives, although the control instructors received copies of the final examination. The actual instruction was as varied between experimental classes as between experimental and control classes. The

actual instruction was as varied between experimental classes as between experimental and control classes. The experimental instructors attempted to teach to the behavioral objectives while retaining the style and manner of their former instruction.

At the end of the experimental semester, both Algebra and English classes gave final examinations. The scores on these exams were used for judging experimental effects. The Algebra exam which was initially created by the experimental instructors, was rewritten by a professional expert from outside the experiment in order to minimize the tendency to teach to the test. The rewritten exam tested the same objectives as the original, and was distributed to both experimental and control instructors shortly before the final.

Included in the Appendix (p. 35) are copies of the behavioral objectives for both English and Algebra as well as a sample of a unit test and its remediation materials for Algebra.

Research Questions and Null Hypotheses

After development of experimental materials, the original research questions applied only to the Algebra group. These questions are:

----Will junior college students in an elementary Algebra class, who are permitted to proceed with instruction in sequential blocks after mastery of previous blocks, or after a second attempt at mastery: (1) learn more than a comparable group of students who are not so instructed?; (2) have a mastery rate which is higher than a comparable group who are not so instructed?

An additional, or revised, question was posed by the experimental procedures of the English group:

(3) Will junior college students in a remedial English class who are provided with a clear and detailed outline of behavioral objectives, learn more than a comparable group of students who are not provided with behavioral objectives?

Null Hypotheses

1. There is no difference, at the .05 level of confidence, between the final examination scores of Algebra students who receive the testing and remediation for mastery treatment, and the exam scores of a comparable group of students who do not receive such treatment.

2. There is no difference, at the .05 level of confidence, between the mastery rates of Algebra students who receive the testing and remediation for mastery treatment, and the mastery rates of a comparable group of students who do not receive such treatment.

3. There is no difference, at the .05 level of confidence, between the final examination scores of English students who receive behavioral objectives and the exam scores of a comparable group of students who do not receive behavioral objectives.

Statistical Treatment

The method of analysis of the achievement variable was a two-way analysis of variance, using achievement scores on the final examination treated as the dependent variable, and sex (male-female) and treatment (experimental-control)

treated as the two independent variables. Separate analyses were computed for the Algebra and English components of the experiment.

The experimental unit was the individual student, rather than the classroom. This method precludes a quantitative description of college/method interaction, but allows for a sample size of greater than 30, rather than an N of 8 which would result if classroom means were compared (noted in Glass and Stanley, pp. 501-508, "The Experimental Unit and the Unit of Statistical Analysis: Comparative Experiments with Intact Groups."). Replications of the experiment over four different colleges and two subject fields will permit qualitative description of the college/method interaction.

To analyze the possible effect on post-test scores of initial differences between experimental and control groups, SCAT-Q and SCAT-V scores were collected and computed in two-way analyses of variance. The SCAT scores, computed in separate ANOVAs for the quantitative and verbal parts, were considered the dependent variable, while sex and group were considered as independent variables. A non-significant difference between groups on the SCAT scores would satisfy the conditions for the experimental design described by Campbell and Stanley as Design 10: The nonequivalent Control Group Design (1963, 47-50). This design does not assume random selection of subjects; the lack of systematic recruitment of students to either experimental or control groups will satisfy the requirement of a Nonequivalent control group and allow for analysis by Kirk's "completely randomized analysis of variance, CR-2" (1969, 102-104). As noted by Campbell and Stanley (p. 40 and 48-50), this design controls for the following sources of invalidity: history, maturation, testing, instrumentation, selection and mortality.

In general use, this design leaves open some question of validity threats by regression and several interaction effects. However, regression is a hazard only if one of the comparison groups has been selected for its extreme scores on the SCAT-Q test, and this is not the case. There will be no systematic selection of students, by entry scores or any other measure. The hypothesis of an interaction between selection and maturation should not be tenable because both groups should have the same rate of maturity or autonomous change. The hypothesis of an interaction of selection and the experimental treatment should not be tenable because there will be four replications of the experiment, in junior colleges as varied in student populations as students throughout the nation. The additional error induced by reaction effect of this design should be minimal (Campbell and Stanley, p. 50).

Data on mastery rates is expressed as proportions and is tested for the significance of the difference between experimental and control groups by a z-test for proportions (Guilford, 1965).

III. RESULTS

In this chapter the null hypothesis for each research question is repeated, followed by the results of the experiment testing that question. The numeric input data for Questions 1 and 3 of this experiment included values for: student number, college, treatment group (control or experimental), final examination score, SCAT-V and SCAT-Q scores, and sex (male or female). Note that the number of students varies depending on the analysis, since SCAT scores and sex data were unavailable for some students.

Null Hypothesis 1: There is no difference, at the .05 level of confidence, between the final examination scores of Algebra students who receive the testing and remediation for mastery treatment, and the exam scores of a comparable group of students who do not receive such treatment.

Data for Question 1. The data for this question are presented in two parts. The first part, presented in Tables 1, 2, 3 and 4, is a summary of the two-way analysis of variance which tests for significant differences between entry test scores, as measured by SCAT-V and SCAT-Q, of the experimental and control groups. The second part of the data presented in Tables 5 and 6, is a summary of the two-way analysis of variance which tests for significant differences between final examination scores, the measure of the main experimental effect.

Discussion of Question 1. Algebra Achievement

PART 1: SCAT scores

Tables 3 and 4 reflect the summary data for differences between experimental and control groups, as measured by their entry test scores of SCAT-V and SCAT-Q. The F-Test values for the treatment group comparisons are low: (SCAT-V, $F = 2.09$; SCAT-Q, $F = 2.67$). There is no significant difference at the .05 level of confidence, between the entry scores of the treatment groups. This result will be discussed in conjunction with results of the ANOVA for achievement scores.

Other results in these tables reflect minor effects to the experiment. There is no significant difference at the .05 level of confidence between male and female SCAT scores (SCAT-V, $F = 2.37$; SCAT-Q, $F = 1.69$). There is no significant difference for the SCAT-V group X sex interaction ($F = 0.02$), but there is a significant difference ($\alpha = .05$) for the SCAT-Q group X sex interaction ($F = 4.14$). This interaction does not affect the experimental results, but probably reflects the relatively low mean score (24.42) of the female control group.

PART 2: Achievement Scores

Table 6 summarizes the analysis of variance for testing differences between final examination scores of experimental and control groups, and of male and female students. The F-Test score for the treatment group is significant ($F = 12.31$): the null hypothesis for Question 1 is rejected at the .05 level of confidence. This result is probably the most important of the entire experiment, for coupled with the lack of difference on the entry data (SCAT scores), we may assume that the testing and remediation treatment contributed to significant differences between mean final examination scores of the treatment groups. The difference between means was 7.79 out of a total score of 68.

PART 1: ANOVA for Algebra SCAT scores

TABLE 1

Two-Way Statistics for SCAT-V

Group	Sex: Female	Sex: Male	Row Marginals
Experimental			
Mean	38.91	33.74	36.33
SD	16.34	14.76	
N	22	58	
Control			
Mean	34.09	27.85	31.9C
SD	19.63	12.92	
N	11	13	
Column Marginals:			
Mean	36.50	30.79	33.65

SD = Standard Deviation
N = Number of Students

TABLE 2

Two-Way Statistics for SCAT-Q

Group	Sex: Female	Male	Row Marginals
Experimental			
Mean	38.00	35.27	36.64
SD	14.92	16.80	
N	24	58	
Control			
Mean	24.42	36.77	30.59
SD	20.61	14.37	
N	12	17	
Column Marginals:			
Mean	31.21	36.02	33.61

SD = Standard Deviation
N = Number of Students

TABLE 3

Analysis of Variance for SCAT-V

(Total N = 104)

Source	Sum of Squares	DF	Mean Square	F-Test
Treatment Group	497.89	1	497.89	2.09
Sex	564.98	1	564.98	2.37
Group X Sex	5.03	1	5.03	0.02
Error	23,881.55	100	238.82	

NOTE: The sums of squares are calculated assuming all cell counts equal 17.35 (the harmonic mean of cell N's). (Applies to Tables 3, 4, 8 and 9).

TABLE 4

Analysis of Variance for SCAT-Q

(Total N = 111)

Source	Sum of Squares	DF	Mean Square	F-Test
Treatment Group	727.51	1	727.51	2.67
Sex	460.64	1	460.64	1.69
Group X Sex	1,129.84	1	1,129.84	4.14*
Error	29,179.61	107	272.71	

PART 2: ANOVA for Algebra Achievement Scores

TABLE 5

Two-Way Statistics for Achievement

Group	Sex: Female	Sex: Male	Row Marginals
Experimental			
Mean	45.80	41.16	43.48
SD	12.81	13.69	
N	49	114	
Control			
Mean	38.95	32.44	35.69
SD	14.48	14.44	
N	20	39	
Column Marginals:			
Mean	42.37	36.80	39.59

SD = Standard Deviation
N = Number of Students

TABLE 6

Analysis of Variance for Achievement

(Total N = 222)

Source	Sum of Squares	DF	Mean Square	*F*-Test
Treatment Group	2,312.13	1	2,312.13	12.31*
Sex	1,186.50	1	1,186.50	6.32*
Group X Sex	33.73	1	33.58	0.18
Error	40,951.73	218	187.85	

A significant difference $\alpha = .05$, is also noted for the sex variable. Female students had a mean score 5.57 higher than the male student mean score. Sex difference was not part of the main research question, but the result is reported here.

Null Hypothesis 2: There is no difference, at the .05 level of confidence, between the mastery rates of Algebra students who receive the testing and remediation for mastery treatment, and the mastery rates of a comparable group of students who do not receive such treatment.

Data for Question 2. Data for Question 2 is found in Table 7. The mastery rate is defined in this report as the number of students who finish the course with an A, B, or C, divided by the number of students who were enrolled and in attendance during the first week. Mastery rates were calculated for the experimental and control groups at each of the four colleges which submitted final examination data for Algebra. Mastery rates were not calculated for the English groups because there was insufficient data.

A z-test for significance of differences between uncorrelated proportions (Guilford, 186) was made for the treatment groups at each college. There were, then, four replications of the experiment.

Discussion of Question 2. Mastery Rates for Algebra

The differences between control group and experimental group mastery rates are tested for significance four times, once for each of the colleges reporting final examination data for Algebra. Table 7 shows the z-test scores for significance between proportions. These scores are -.441, .856, .409, and .786. Since the z-score for rejection of the null hypothesis at the .05 level of confidence is 1.96 or more, we may not reject the null. There is no significant difference in mastery rates for any of the four groups.

Null Hypothesis 3: There is no difference, at the .05 level of confidence, between the final examination scores of English students who receive behavioral objectives and the exam scores of a comparable group of students who do not receive behavioral objectives.

Data for Question 3. As reported in Question 1, the data for Question 3 will be reported in two parts. The first part (Tables 8, 9, 10 and 11) relate SCAT entry scores, while the second part (Tables 12 and 13) relate the achievement test scores.

Discussion of Question 3. English Achievement Scores.

PART 1: SCAT Scores

Tables 10 and 11 summarize the analyses of variance for SCAT-V and SCAT-Q. The F-tests for the treatment groups yielded values of 0.50 and 0.01, both well below the criterion value for a significant difference, $\alpha = .05$. Likewise, the F-test values for six differences yielded low values of 1.88 and 1.07, well below the criterion for the .05 level of significance. There is no significant difference between SCAT scores for the treatment groups, or for male and female students.

DATA FOR QUESTION 2

TABLE 7

Mastery Rates for Algebra

(X = Experimental Group; C = Control Group)

College	Number Enrolled (N)	Number Mastered (m)	Mastery Rate p = m/N	z-score
Cerritos:				
X	87	34	.391	-.441
C	25	11	.440	
Rio Hondo:				
X	38	20	.526	.856
C	33	14	.424	
San Diego City:				
X	78	30	.385	.409
C	71	25	.352	
San Diego Mesa				
X	105	63	.600	.786
C	47	25	.532	

Cont'd next page

TABLE 7
(Continued)

Calculation of z-scores ($\bar{z}$ in Guilford, p. 186):

For: N_1 = number enrolled in experimental group (X).

N_2 = number enrolled in control group (C).

m_1 = number mastering X course.

m_2 = number mastering C course.

p_1 = mastery rate, or proportion, of X.

p_2 = mastery rate of C.

$\bar{p}e$ = estimated population proportion

$$= \frac{m_1 + m_2}{N_1 + N_2}$$

$$\bar{q}_e = 1 - \bar{p}_e$$

Then:

$$\bar{z} = \frac{p_1 - p_2}{\sqrt{\bar{p}_e \, \bar{q}_e \, \frac{N_1 + N_2}{N_1 \, N_2}}}$$

TABLE 8

Two-Way Statistics for SCAT-V

(N = 30)

Group	Sex: Female	Sex: Male	Row Marginals
Experimental			
Mean	13.33	21.11	17.22
SD	10.56	15.35	
N	6	9	
Control			
Mean	10.00	16.91	13.46
SD	6.48	15.01	
N	4.00	11.00	
Column Marginals:			
Mean	11.67	19.01	15.33

SD = Standard Deviation
N = Number of Students

TABLE 9

Two-Way Statistics for SCAT-Q

(N = 30)

Group	Sex: Female	Sex: Male	Row Marginals
Experimental			
Mean	6.67	21.44	14.06
SD	5.47	22.52	
N	6 9		
Control			
Mean	16.00	13.09	14.55
SD	12.70	9.09	
N	4.00	11.00	
Column Marginals:			
Mean	11.33	17.27	14.30

SD = Standard Deviation
N = Number of Students

TABLE 10

Analysis of Variance for SCAT-V

Source	Sum of Squares	DF	Mean Square	F-Test
Treatment Group	91.78	1	91.78	0.50
Sex	348.65	1	348.65	1.88
Group X Sex	1.22	1	1.22	0.01
Error	4,821.13	26	185.43	

TABLE 11

Analysis of Variance for SCAT-Q

Source	Sum of Squares	DF	Mean Square	F-Test
Treatment Group	1.55	1	1.55	0.01
Sex	227.69	1	227.69	1.07
Group X Sex	505.63	1	505.63	2.38
Error	5,516.46	26	212.17	

NOTE: The sums of squares are calculated assuming all cell counts equal 6.47 (the harmonic mean of cell N's.

PART 2: Achievement Scores

TABLE 12

Two-Way Statistics for Achievement Scores

(N = 120)

Group	Sex: Female	Sex: Male	Row Marginals
Experimental			
Mean	59.32	58.30	58.81
SD	14.36	15.25	
N	28	54	
Control			
Mean	44.50	49.92	47.21
SD	8.77	7.68	
N	12	26	
Column Marginals:			
Mean	51.91	54.11	53.01

SD = Standard Deviation
N = Number of Students

TABLE 13

Analysis of Variance for Achievement Scores

Source	Sum of Squares	DF	Mean Squares	F-Test
Treatment Group	3,056.29	1	3,056.29	17.54*
Sex	109.88	1	109.88	0.63
Group X Sex	236.21	1	236.21	1.36
Error	20,212.29	116	174.24	

PART 2: Achievement Scores

Table 13 summarizes the analysis of variance for testing differences between final examination scores of the elementary English students. The F-test score for the treatment group is significant ($F = 17.54$): the null hypothesis for Question 3 is rejected at the .05 level of confidence. Since there was no significant difference on the entry data (SCAT scores), we may assume that the behavioral objectives treatment contributed to significant differences between mean final examination scores of the treatment groups. The difference between the treatment means was 11.60 out of a total score of 100.

No significant difference ($\alpha = .05$) was found between achievement means of female and male students.

IV. CONCLUSIONS AND RECOMMENDATIONS

Conclusion 1 - It is possible, by means of testing for mastery and providing remediation after each unit of an instructional sequence, to achieve a significant increase in final examination scores by a junior college elementary Algebra class.

Conclusion 2 - The use of the techniques of testing for mastery and providing remediation after each unit of an instructional sequence does not result in a significant increase in the mastery rate of a junior college elementary Algebra class.

Comment: The mastery rate was based on the subjective measure of teacher assigned marks (A, B, C, D, F or Withdraw). The experimental result of no significant difference between control and experimental mastery rates may be due to the tendency of instructors to assign a normal distribution of grades, in spite of the fact that their class may have scored in a non-normal manner on an objective criterion measure.

Conclusion 3 - It is possible, by means of providing detailed and hierarchically ordered behavioral objectives, to achieve a significant increase in final examination scores by a junior college remedial English class.

Recommendations

The statistical results of this study, combined with anecdotal information from the experimental instructors, indicate that courses based on clearly specified and quite detailed behavioral objectives were superior to control courses. The objectives in both subject areas served to direct instruction and learning toward the foundation skills which were prerequisite to mastery of the semester objectives.

The Algebra instructors were very enthusiastic about the use of testing and remediation for mastery, although they felt pressed for time in giving unit tests so frequently. Further individualization of the course was recommended by the Algebra instructors, especially in the area of providing time and instructional methods for self-pacing by students. The English instructors, who did not have the time or resources to carry out the mastery component of the research, expressed interest in mastery procedures for their classes. They stressed the need, however, for some consensus about optimum sequencing of objectives and provision for alternate means of instruction.

The research did not control for selection of instructors, a problem inherent in a project which depends on participation of motivated volunteers. Since no accurate measures of teacher effectiveness currently exist, the degree of treatment effect due to excellence of instruction cannot be ascertained.

In further research of this scope, it is recommended that sufficient time be budgeted for development of each of the components of ideal mastery learning, including individual pacing (Block, 1971, pp. 64-76). Where a course in Algebra may be fully developed in two semesters, a course in English, which is traditionally taught by instructors of diverse instructional methods, may take three or four semesters to achieve a common set of objectives and to develop a sufficiently rich set of alternative instructional strategies.

It is recommended that further research be done on the relationship between mastery rates and scores on the final examination. The results of this research suggest that students who received the experimental treatment scored significantly higher on their semester exam than control students, yet the distribution of A's, B's and C's was about the same for both groups. The grading system itself, based on individual teacher criteria, deserves close scrutiny.

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APPENDIX

Contents:

PARTICIPATING INSTRUCTORS AND GROUP LEADERS

College	Math	English
Cerritos	John D. Baley	(none)
Los Angeles City	Hyman Z. Solomon	Virginia H. Fick
Rio Hondo	Gerald E. Bruce	Mahlon A. Woirhaye, Jr.
San Diego City	Harry L. Baldwin, Jr.	Sidney H. Forman (did not finish the experiment)
San Diego Mesa	John M. Carman	Beverly Erickson Emil Hurtik (replaced Sidney Forman)

GROUP LEADERS:

College	Math	English
Golden West		Edith A. Freligh (led, but did not have a participating class)
Cerritos	John D. Baley (participant and math leader)	

ELEMENTARY ALGEBRA

Behavioral Objectives for a course based on

Elementary Algebra, Structure and Skills

by Drooyan, Hadel, and Fleming

(Following each objective is a sample of a problem that might be used to test the student's ability to meet the objective.)

UNIT NUMBER 1 -- SETS

At the conclusion of this unit, the student will be able to:

1. Describe a set in at least two ways: (1) by listing the elements of the set (called roster method) and (2) by stating a rule which defines the set.
   ++(a) Rewrite the set P = {1,2,3,4,5} by stating a rule which identifies the set.
   (b) Rewrite the set M = the set of all counting numbers which are multiples of two by listing its elements.

2. Translate English sentences concerning sets into mathematical notation.
   ++Translate the following expressions into mathematical notation:
   (a) The intersection of sets A and B is the empty set.
   (b) Set M is a subset of set N.
   (c) Set R is the complement of set S.
   (d) "a" is an element of set Q,
   (e) "d" is not an element of set Q.

3. When given a universal set U and three sets A,B, and C, find such sets as A', $A \cap B$, $A \cup B$, $A \cap C$, $A \cup C$, $B \cap C$, $B \cup C$, $(A \cap B)'$, $(A \cup B) \cup C$, etc.
   ++Given a universal set U = {a,b,c,d,e,f,h,i,j} , A = {a,b,c}, B = {d,e,f}, and C = {b,d,f,h}, find the following:

(a) $A \cap B$ = ________	(e) $(A \cap B)'$ = ________
(b) $A \cup B$ = ________	(f) $(A \cup B)'$ = ________
(c) A' = ________	(g) $B \cap C$ = ________
(d) B' = ________	(h) $(A \cap B) \cap C$= ________

4. When given a set, tell whether it is finite or infinite.
   ++State whether the following sets are finite or infinite:
   (a) A = the set of all people on the earth.
   (b) B = the set of counting numbers greater than one million.
   (c) C = the set of all even counting numbers.
   (d) D = the set of all multiples of three less than one million.

5. When given a list of sets, identify the pairs of equal sets and unequal sets.
   ++Given that A = {1,2,3}, B = {2,3,4}, and C = {3,4,2}, indicate "=" or "≠" in the appropriate blanks: (a) A__B, (b) A__C, (c) B__C.

6. List the subsets of a given set with no more than four elements.
   ++(a) List the subsets of the set A = {1,2,3}.
   (b) List the subsets of the set B = {a,b,c,d}.

UNIT NUMBER 2 -- WHOLE NUMBERS

At the conclusion of this unit, the student will be able to:

1. Identify whether a given number is used in a counting sense or in an order sense, given a statement in which the number is used.

   ++In each of the following sentences identify each number as to its use (counting or order) and write the correct answer in the blank after each number:

   (a) Sam is the fifth _____ bug in the line.

   (b) Jim ranked number three _____ in a class of 50 _____.

2. Identify the correct order relation between numbers and use the proper symbol to so indicate.

   ++Make each statement true by inserting either $>$ or $<$ or $=$.

   (a) $3 + 5$ __ 7 (b) $9 \div 3$ __ $9 - 3$.

3. Use set builder notation to express a well defined set:

   ++Write the following sets in set builder notation.

   (a) $\{2,3,4,5\}$

   (b) $\{0,1\}$

   (c) $\{6,7,8\ldots\}$

4. Graph various subsets of the whole numbers.

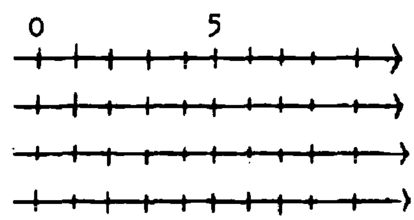
   ++On the given number lines, graph,

   (a) $\{2,3,6,7\}$

   (b) {all whole numbers between 1 and 8}

   (c) $\{x \mid x \leq 4 \text{ and } x \; \Sigma \; W\}$

   (d) $\{y \mid y > 6 \text{ and } y \; \Sigma \; N\}$



5. Identify which order axiom (reflexive of equality, symmetric of equality, transitive of equality, substitution of equality, trichotomy, or transitive of inequality) is the correct justification for various statements.

   ++Name the order axiom that justifies each of the following statements:

   (a) If $3 = 2 + 1$, then $2 + 1 = 3$. ____________________

   (b) If $6 > 4$ and $4 > 2$, then $6 > 2$. ____________________

6. Identify which operation axiom (closure of add. or mult., commutativity of add. or mult., associativity of add. or mult., identity of add. or mult., or distributive law) is the correct justification for vaious statements.

   ++Name the operation axiom that justifies each of the following statements:

   (a) $5 \cdot 7 = 7 \cdot 5$ ____________________

   (b) $(3 + 7) \cdot a = 3 \cdot a + 7 \cdot a$ ____________________

   (c) $(4 + 7) + 3 = 4 + (7 + 3)$ ____________________

   (d) $1 \cdot (a + b) = a + b$ ____________________

7. Given an operation axiom rewrite an algebraic expression using only the specified axiom.

++Rewrite $(a + b)2$ using only the distributive law

Rewrite $(a + b)2$ using only the commutative law for addition.

Rewrite $(a + b)2$ using only the commutative law for multiplication.

8. Use the distributive law to rewrite various expressions without parentheses

++Use the distributive law to rewrite the following expressions without parentheses:

(a) $a(b + c) =$ ________________

(b) $(x + 5)7 =$ ________________

(c) $(a + 3)(b + 6) =$ ________________

(d) $8(r + s + t) =$ ________________

9. Identify which theorem (equality of addition, equality of multiplication, or zero factor) is the correct justification for various statements.

++Name the theorem that justifies each of the following statements:

(a) If $2 + 5 = 7$, then $(2 + 5) + 4 = 7 + 4$. ________________

(b) $5 \cdot 7 \cdot 0 = 0$ ________________

10. Factor into primes composite numbers whose prime factors do not exceed 13 and where no factor is used more than three times.

++Express the following composite numbers as products of primes:

(a) 54 = ________________

(b) 462 = ________________

(c) 180 = ________________

(d) 3500 = ________________

UNIT NUMBER 3 -- INTEGERS

At the conclusion of this unit, the student will be able to:

1. Demonstrate knowledge of the Additive Inverse Law by giving an illustrative example like $5 + (-5) = 0$.

++Write an example of the Inverse law of addition.

2. Express the additive inverse of any integer and thus demonstrate that if a is positive, then -a is negative or if a is negative then -a is positive.

++Write the additive inverse of each of the following:

(a) $3 \rightarrow$ ____ (b) $-5 \rightarrow$ ____ (c) $y \rightarrow$ ____ (d) $-z \rightarrow$ ____.

3. Find the sum of any series of integers.

++Evaluate: (a) $(7) + (-9) =$ ____ (b) $(-3) + (5) + (6) + (-13) =$ _____

4. Use the definition of subtraction to rewrite expressions of the form $a + -b$ in the form $a - b$, or $a - b$ in the form $a + -b$.

++Use the definition of subtraction to rewrite,

a. $x + -5 =$

b. $y - 4 =$

5. Subtract integers by the process of adding the additive inverse of the integer being subtracted.

++Evaluate: (a) $(-6) - (5) =$ ____ (b) $7 - 9 =$ ____ (c) $5 - (-3) =$ __ .

6. Multiply any number of integers.

++Evaluate: (a) $(-3)(7) =$ ____ (b) $(2)(-3)(-1) =$ ____

7. Recognize whether the quotient of two integers is an integer and if so, what integer.

++Evaluate the following quotients. If no such integer exists, so state.

(a) $\frac{6}{-2} =$ ____ (b) $\frac{-32}{4} =$ ____ (c) $\frac{-10}{-5} =$ ____ (d) $\frac{17}{3} =$ ____ (e) $\frac{5}{0} =$ ____

(f) $\frac{0}{5} =$ ____

UNIT NUMBER 4 -- RATIONALS

At the conclusion of this unit, the student will be able to:

1. Demonstrate knowledge of the Multiplicative Inverse Law by giving an illustrative example like $4 \cdot \frac{1}{4} = 1$.

++Write an example of the inverse law of multiplication.

2. Express the reciprocal of any non-zero rational number.

++Write the reciprocal of each of the following:

(a) $3 \rightarrow$ ____ (b) $-7 \rightarrow$ ____ (c) $\frac{3}{8} \rightarrow$ ____

3. Determine the equality or inequality of any pair of rational numbers.

++Is $\frac{4}{7}$ equal or unequal to $\frac{20}{35}$? Is $\frac{3x}{7}$ equal or unequal to $\frac{6x}{9}$?

4. Use the fundamental principle of fractions to express rational numbers as basic fractions.

++Express the following rational numbers as basic fractions (i.e. simplest form):

(a) $\frac{12}{30} =$ ____. (b) $\frac{30xy}{38yz} =$ ____.

5. Express any rational number in the required "higher terms."

++Fill in the missing numerators: (a) $\frac{3}{7} = \frac{\quad}{42}$ (b) $\frac{2x}{3y} = \frac{\quad}{12y}$

6. Use the definition of division to rewrite expressions of the form $a \cdot \frac{1}{b}$ in the form $\frac{a}{b}$, or $\frac{a}{b}$ in the form $a \cdot \frac{1}{b}$.

++Use the definition of division to rewrite the following:

a. $3 \cdot \frac{1}{5} =$

b. $\frac{7}{8} =$

7. Find the Lowest Common Denominator of a series of fractions to be added or subtracted.

++Find the L.C.D. of the following groups of fractions:

(a) $\frac{1}{12}, \frac{1}{15} \longrightarrow$ (b) $\frac{1}{8}, \frac{1}{18}, \frac{1}{24} \longrightarrow$

8. Given a basic fraction, or the additive inverse of a basic fraction, express as a basic fraction in standard form:

a) $\frac{2}{-5}$ b) $-\frac{3}{-7}$ c) $-\frac{-1}{-2}$

9. Add or subtract groups of rational numbers and express the answers as basic fractions.

++ a. $\frac{2}{5} + \frac{3}{4} =$ b. $\frac{3}{8} - \frac{1}{4} =$ c. $\frac{5}{6} - \frac{3}{4} + \frac{1}{5} =$

10. Multiply or divide groups of rational numbers and express the answers as basic fractions.

++Perform the indicated operations and simplify each answer:

(a) $\frac{3}{7} \cdot \frac{4}{9} =$ ____ (b) $\frac{4}{3x} \div \frac{-2}{3y} =$ ____ (c) $\dfrac{\frac{3}{8}}{\frac{1}{4}} =$ ______.

11. Distinguish between "a square root of 4" and $\sqrt{4}$.

++(a) What is the square root of 4? ______ (b) What is $\sqrt{4}$? ______

12. Given a simple radical expression state whether or not it is an element of the set of real numbers.

++Place a yes or no on the blank after each expression to indicate whether of not it is an element of the set of real numbers:

a. $\frac{-4}{3}$ ____ b. $-\sqrt{4}$ ____ c. $\sqrt{-4} =$ ____ d. $\frac{0}{6}$ ____

e. $\frac{6}{0}$ ____

13. Evaluate simple radical expressions.

++Evaluate: (a) $\sqrt{9}$ = _____ (b) $-\sqrt{25}$ = _____ (c) $\sqrt{-4}$ = _____

14. Recognize the relationships that exist among the natural numbers N, the whole numbers W, the integers J, the rational numbers Q, the irrational numbers H, and the real numbers R.

++True or False: (a) $H \subseteq Q$ → ___ (b) If $x \in N$, then $x \in R$. → ___

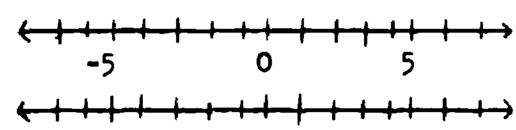
(c) $Q \cup H = R$. → ___

15. Graph subsets of the real numbers.

++Graph the following sets on the number lines provided:

(a) $\{x \mid -3 < x \leq 2 \text{ and } x \in J\}$

(b) $\{y \mid -1 \leq y < 5 \text{ and } y \in R\}$



UNIT NUMBER 5 -- POLYNOMIALS

At the conclusion of this unit, the student will be able to:

1. Express simple expressions without exponents in exponential form.

++Express in exponential form: (a) $3 \cdot 3rrrt$ = _____ (b) 6xyyzz = _____

2. Express simple expressions in exponential form without exponents.

++Express without exponents: (a) $2ab^2$ = _________ (b) x^2y^3 = _________

3. Use the vocabulary of polynomials: such as term, coefficient, degree, monomial, binomial, and trinomial to answer questions about polynomials.

++Given: $2x^2$: $x^2 - 3x + 4$: $2x + 1$

(a) What degree is the monomial above?

(b) What is the coefficient of the first degree term in the trinomial above?

(c) What is the constant term in the binomial above?

4. Use the rule of order to simplify algebraic expressions.

++Simplify completely: (a) $2 + 3 \cdot 6$ = _____ (b) $\frac{3^3+3}{5(2)} + \frac{2+2^3}{5(2)} - \frac{8^2-7^2}{3(5)}$ = _______

5. Multiply or divide groups of monomials with positive integer exponents.

++Perform the indicated operations and express the answers as basic fractions in standard form:

(a) $(4x)(3x)(2x)$ = _______ (b) $\frac{10x^4y}{-5x^2}$ = _______

6. Simplify expressions with zero or negative integer exponents by expressing them with only positive integer exponents.

++Express the following using only positive integer exponents:

(a) 5^0 = ______ (b) $2x^{-1}$ = ______ (c) $\left(\frac{1}{3}\right)^{-2}$ = ______

7. Rewrite rational expressions containing integer positive, zero, or negative exponents using only positive exponents.

++Express the following in simplest form:

(a) $\frac{4^{-1}\,2^0}{2^2}$ = ______ (b) $\left(\frac{3x^{-1}}{2y^2}\right)^2$ = ______

8. Add and subtract polynomials by removing grouping symbols and combining terms.

(a) $(3x^2 + 4x - 6) + (2x^2 - 7x + 5)$ = ______

(b) $(3x - 4y + 2) - (-5y + 4y + 5)$ = ______

9. Multiply polynomials by using the distributive law to remove grouping symbols and then combining terms.

++Perform the indicated operations and simplify:

(a) $3x^2(4x^2-5x+2)$ = ______ (b) $(2y-1)^2$ = ______

(c) $4- [3x-2(x-5)+x]$ = ______

10. Express polynomials in one variable with powers in descending order with all intermediate powers expressed. For example. $x^3+3x^4-7 = 3x^4+x^3+0x^2+0x-7$

++Rewrite the following polynomials with powers in descending order with all intermediate powers expressed: (a) x^3-1 = ______

(b) $3x-x^4+5$ = ______

11. Perform long divisions of polynomials by first degree polynomials and express the answer as the quotient plus the remainder over the divisor.

++Evaluate: $x+2\overline{)x^2+8x+12}$ or perform $\frac{x^3+x^2+4}{x-2}$ by long division

UNIT NUMBER 6 -- FACTORING

At the conclusion of this unit, the student will be able to:

1. Recognize common factors in the terms of a polynomial and use the distributive law to factor the polynomial.

++Factor completely: (a) $8p - 16$ = ______ (b) $3x^2 + 6xy - 9x$ = ______

(c) x^2y+x = ______

2. Recognize when a binomial that cannot be factored by the distributive law is a difference of squares and then factor it.

++Factor completely: (a) a^2-b^2 = ______ (b) $4x^2-1$ = ______

3. Recognize when a trinomial cannot be factored by the distributive law and use the factoring methods for trinomials to factor it, if it is factorable.

++Factor completely: (a) $x^2 - 5x + 6$ = ______ (b) $x^2 + x + 4$ = ______

(c) $3x^2 - 5x - 2$ = ______

4. Recognize and factor all factorable expressions using all methods (1. distributive law, 2. count the terms left in the parentheses, a. if two terms left, use the difference of squares, or b. if three terms left, use other methods).

++Factor completely: (a) $3x^3 - 12x$ = ______ (b) $4x^3-10x^2y-6xy^2$ = ______.

(c) $x^4 - 16$ = ______

UNIT NUMBER 7 -- RATIONAL EXPRESSIONS

At the conclusion of this unit, the student will be able to:

1. Express rational expressions in simplest form by factoring and then using the fundamental principle of fractions.

++Simplify completely: (a) $\frac{5x+10}{5}$ = ______ (b) $\frac{2x^2-x-10}{x+2}$ = ______

(c) $\frac{x^2+5x+6}{x^2-4}$ = ______

2. Find the least Common Denominator of a sequence of rational expressions.

++Find the L.C.D. of the following groups of fractions:

(a) $\frac{1}{6xy}, \frac{1}{8x^2}, \frac{1}{3xy^2} \longrightarrow$ (b) $\frac{1}{x^2-x-2}, \frac{1}{x^2-4x+4} \longrightarrow$

3. Add or subtract groups of rational expressions and give the answers in simplest form.

++Perform the indicated operations and simplify:

(a) $\frac{2}{x^2} + \frac{3}{xy}$ = ______ (b) $\frac{7}{x-2} - \frac{4}{x+2}$ = ______ (c) $\frac{1}{y^2-16} + \frac{2}{y+4}$ = ______

4. Multiply rational expressions where the numerators and denominators of all the fractions are monomials.

(a) $\frac{-21r^2s}{8} \cdot \frac{-14t^2}{3rs}$ =

5. Divide rational expressions where the numerators and denominators of all the fractions are monomials.

++Perform the indicated operations and simplify:

(a) $\dfrac{-x^2y}{u^2v^2} \div \dfrac{xy^2}{u^2v} =$ ____________

6. Express all numerators and denominators in completely factored form when multiplying or dividing rational expressions where the numerators and denominators are <u>not</u> monomials.

++Express the numerators and denominators in completely factored form:

$$\frac{4y^2-1}{2y^2-9y+4} \cdot \frac{y^2-4y}{x}$$

7. Multiply rational expressions where the numerators and denominators of all the fractions are <u>not</u> monomials. (a) $\dfrac{t^2-4}{t^2-1} \cdot \dfrac{1-t}{2t^2+4t} =$

8. Divide rational expressions where the numerators and denominators of all the fractions are <u>not</u> monomials.

++Perform the indicated operations and simplify:

(a) $\dfrac{x^2-3x}{x^2-3x-4} \div \dfrac{x^2+2x-3}{x^2-5x+4} =$ ______________

9. Simplify complex fractions.

++Simplify completely:

(a) $\dfrac{3+\frac{1}{5}}{2-\frac{3}{5}} =$ __________ (b) $\dfrac{1+\frac{1}{2y}}{1-\frac{1}{4y^2}} =$ ____________

UNIT NUMBER 8 -- FIRST-DEGREE EQUATIONS IN ONE VARIABLE

At the conclusion of this unit, the student will be able to:

1. Find the solution set of a first-degree equation of the type $ax = b$, where a and b are integers.

++Find the solution set of: (a) $3x = 6$ (b) $-2y = 8$ (c) $7x = -12$

2. Write an equivalent equation of the form $ax = b$ when given a multi-term equation with no grouping symbols and with integer coefficients.

++Write $4x + 2 = 3 - 5x$ in the form $ax = b$.

3. Write an equivalent equation with no grouping symbols and with integer coefficients when given a multi-term equation with no grouping symbols and with rational coefficients.

++Write $\frac{1}{2}x + 2 = 3x - \frac{1}{4}$ without fractions.

4. Write an equivalent equation with no grouping symbols and with integer coefficients when given an equation with grouping symbols and with rational coefficients.

++Write $\frac{1}{2}(x - 2) = 3\frac{1}{2}x - 5)$ without grouping symbols or fractions.

5. Write an equivalent equation without the variable in any denominator when given an equation with the variable in one or more denominators.

++Write $\frac{4}{x + 1} = \frac{3}{x}$ without fractions.

6. Perform steps 1 through 5 above, in reverse order, to find the solution set of an equation that reduces to the form $ax = b$.

++Find the solution sets for: (a) $3x - 5 = -7$ (b) $3(2x - 4) = x + 13$ (c) $5(x - 4) = \frac{5}{9}(x + 1)$ (d) $3x - 7x + \frac{1}{4}x = 8$ (e) $\frac{7}{-2x} + \frac{1}{x} = 5$

7. Remove fractions, remove grouping symbols, apply the addition law, factor and apply the multiplication law, in order to find an equation of the form $x = A$, where A is an algebraic expression not containing x. The equation $x = A$ is to be equivalent to an equation which may contain fractions, grouping symbols, and other variables, but which can be reduced to $x = A$.

++Solve for x: (a) $ax = cx + 5$ (b) $4(a - x) = 3x + b$ (c) $\frac{2x}{a} + 5 = cx$

UNIT NUMBER 9 -- WORD PROBLEMS

At the conclusion of this unit, the student will be able to:

1. Express in mathematical language certain sentences and expressions given in the English language. These expressions will involve only the symbols presented in earlier sections. Also be able to translate into math language such expressions as:

<u>a</u> <u>fraction</u> <u>of</u> a quantity	(like "two-thirds of ...")
<u>a</u> <u>percentage</u> <u>of</u> a quantity	(like "forty percent of ...")
<u>more</u> <u>than</u> a quantity	(like "seven more than ...")
<u>less</u> <u>than</u> a quantity	(like "four less than ...")
<u>the</u> <u>difference</u> <u>between</u> two quantities	

a multiple of a quantity

the square of or the cube of a quantity

the next consecutive integer (or the next integer)

the next consecutive even or odd integer (the next even integer)

++(a) If x is an even integer, express in terms of x:

1. the next even integer.
2. four more than twice the integer.
3. the sum of the integer and twice its opposite.
4. one less than the square of the integer.
5. the difference between the square of the integer and the cube of the next integer.

(b) If x is a number, write in mathematical language:

1. the number is four less than its square.
2. twice the number is greater than half the number.
3. eight less than twice a third of the number is the number.
4. a third of the number is the same as half of four more than the number.
5. the square of the number is less than the number.

(c) If x is the number of gallons of gas I put into my car last week, then represent in terms of x:

1. two-thirds of the amount of gas I put into my car last week.
2. the amount of gas I put into my car this week, if I put in four gallons less than twice the amount I put in last week.
3. the amount of oil I put into my car last week, if I use three times as much oil as gas.
4. the amount of gas Sam put into his car this week, if that amount is three times what he put in last week, and last week he only put in three-fourths as much as I did.

2. On a given problem, (a) select an appropriate variable for a quantity of interest in the problem, (b) translate all other quantities in the problem into expressions in terms of the original variable, (c) use these quantities to form an equation concerning the problem, (d) solve the equation, and (e) interpret the solution in terms of the original word problem.

++Solve the following word problems:

(a) The sum of an integer and 3 more than twice that integer is 18. Find the integer.

(b) If a collection of 23 dimes and nickels is worth $1.70, how many nickels are there?

(c) Jim is three years older than his sister Liz. In two years the sum of their ages will be 33. How old is Liz?

UNIT NUMBER 10 -- FUNCTIONS AND GRAPHS

At the conclusion of this unit, the student will be able to:

1. Examine a given set of ordered pairs (roster notation or set builder notation) and determine whether that relation is or is not a function. Identify the domain and range of the set. List the elements in the domain and range of each set.

   ++Are the following functions or not: (a) $\{(1,2),(1,3),(2,2),(2,3)\}$
   (b) $\{(x,y) \mid x \in \{1,4,9\} \text{ and } y = \sqrt{x}\}$

2. Evaluate the corresponding elements in the range of a function when given elements from the domain and the rule of the function in functional notation.

   ++If $f(x) = 2x^2 - 5$, find: (a) $f(2)$ (b) $f(-1)$ (c) $f(x+1)$

3. Express a variable in an equation explicitly when given an equation that expresses the variable implicitly.

   ++Solve $\frac{2y-4}{5} = -2x$ explicitly for y in terms of x.

4. Find ordered pairs that satisfy a first-degree equation in two variables.

   ++Find 3 ordered pairs that satisfy the equation $2x - 3y = 6$.

5. Graph ordered pairs of rationals with denominators less than 5.

   ++On a given coordinate system, plot the points (a) $(1,-3)$ (b) $(0,5)$
   (c) $(1/2,6)$

6. Find the slope of the line passing through two given points.

   ++Find the slope of the line determined by: (a) $(1,2)$ and $(-3,5)$
   (b) $(7,4)$ and $(0,-1)$

7. Write in standard form the equation of a line if the line is given in non-standard form with rational coefficients.

   ++Write in standard form: (a) $y = \frac{3}{2}x - 5$ (b) $2(x + 1) = 3(y - 2)$

8. Write in slope-intercept form the equation of a line.

++Write in slope-intercept form: (a) $3x - 2y = 12$ (b) $\frac{1}{2}(x - 5) = 2(y + 1)$

9. Graph the equation of a line either by plotting points on the line (obj. 3) or by applying the slope and y-intercept (obj. 7).

++Graph the following equations: (a) $y = \frac{-2}{3}x + 5$ (b) $6x = 2y = 9$

UNIT NUMBER 11 -- SYSTEMS OF EQUATIONS

At the conclusion of this unit, the student will be able to:

1. Graph a system of two linear (first-degree) equations in two variables.

++Graph by the slope-intercept method: $\begin{cases} 4x - 6y = 12 \\ 3x + 2y = 6 \end{cases}$

2. Identify, from graphs, the geometric possibilities that may occur (intersecting, parallel, or coincident lines) and pair these possibilities with independent, inconsistent, and dependent systems.

++Graph and state as independent, inconsistent, or dependent:

(a) $\begin{cases} 4x - 7y = 28 \\ 3x + 2y = 6 \end{cases}$ (b) $\begin{cases} 10x + 7y = 35 \\ 5x + \frac{7}{2}y = 14 \end{cases}$ (c) $\begin{cases} 4x + 5y = 20 \\ 8x + 10y = 40 \end{cases}$

3. Show that the point of intersection of independent systems satisfies both equations in the system by substituting the measured coordinates of the point into the equations.

++Graph the following system. Measure the coordinates of the point of intersection and show that this ordered pair satisfies the given system: $\begin{cases} 5x - 3y = -9 \\ 3x + 4y = 12 \end{cases}$

4. Show analytically that inconsistent and dependent systems can be identified by determining the slopes and y-intercepts of the two lines. When the two slopes are equal but the intercepts are not, the system is inconsistent. And when the slopes are equal and the intercepts are also equal, the system is dependent.

++Determine the slopes and y-intercepts of the lines in the following systems and thus judge as to whether each system is inconsistent or dependent:

(a) $\begin{cases} 10x + 7y = 35 \\ 5x + \frac{7}{2}y = 14 \end{cases}$ (b) $\begin{cases} 4x + 5y = 20 \\ 8x + 10y = 40 \end{cases}$

5. "Solve" analytically (either by substitution or addition) a system of equations.

++Solve the following systems analytically:

(a) $\begin{cases} 4x - 7y = 28 \\ 3x + 2y = 6 \end{cases}$ (b) $\begin{cases} 10x + 7y = 35 \\ 5x - 2y = 10 \end{cases}$

6. Starting with any system of 2 linear equations, determine whether the system falls into independency, inconsistency, or dependency, and if independent, find the solution of the system.

++Determine whether the following systems are independent, inconsistent, or dependent. If independent, find the solution point:

(a) $\begin{cases} 3x + 5y - 9 = 0 \\ 5y - \frac{7}{2}x + 4 = 0 \end{cases}$ (b) $\begin{cases} \frac{2x-1}{2} - \frac{4y-1}{2} = \frac{1}{2} \\ \frac{3x+1}{3} + \frac{1}{2} = \frac{4y+1}{2} \end{cases}$

UNIT NUMBER 12 -- ROOTS AND RADICALS

At the conclusion of this unit, the student will be able to:

1. Simplify a term containing radicals until there are no perfect squares as factors under the radical. This requires the use of the statement: $\sqrt{a}\sqrt{b} = \sqrt{ab}$,

++Simplify the following: (a) $\sqrt{75x^3}$ (b) $3\sqrt{32y}$ (c) $\sqrt{36x^9}$

2. Simplify a term containing radicals until there are no fractions under the radical. This requires the use of the statement: $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

++Simplify: (a) $\sqrt{\frac{3}{4}}$ (b) $\sqrt{\frac{2x}{9}}$

3. Simplify a term containing radicals until there are no radicals in the denominator. This requires the use of the Fundamental Principle of Fractions.

++Simplify: (a) $\frac{1}{\sqrt{13}}$ (b) $\frac{2}{\sqrt{2x}}$

4. Combine objectives 1,2,3 to simplify any term containing radicals.

++Simplify: (a) $\sqrt{\frac{27x}{y^3}}$ (b) $\frac{1}{3}\sqrt{\frac{3x}{y}}$

5. Use the distributive law to add or subtract terms containing radicals.

++Combine: $\sqrt{7} + 3\sqrt{7}$, $\sqrt{8} - \sqrt{2}$

6. Multiply many termed expressions containing radicals.

++Multiply: (a) $\sqrt{3}(3\sqrt{3} - \sqrt{2})$ (b) $5\sqrt{2}(3 - 2\sqrt{6})$

(c) $(\sqrt{3} - \sqrt{2})(\sqrt{3} + \sqrt{2})$ (d) $(\sqrt{x} + 3)^2$

7. Rationalize expressions with 2 terms containing radicals by expressing the problem as a fraction and multiplying numerator and denominator by the conjugate of the denominator.
++Divide (a) $(3) \div (\sqrt{2} - 1)$ (b) $\frac{3\sqrt{x} - 1}{\sqrt{x} + 1}$

UNIT NUMBER 13 -- QUADRATIC EQUATIONS

At the conclusion of this unit, the student will be able to:

1. Rewrite quadratic equations in standard form.
++Write in standard form: (a) $5x(x - 2) + 1 = 1 = 3x$ (b) $31x = 6x^2 + 35$

2. Use factoring as a method of solving quadratic equations.
++Solve: (a) $x^2 - 5x + 6 = 0$ (b) $11x = 2 + 15x^2$

3. Solve simple quadratic equations by "extracting roots."
++Solve: (a) $x^2 = 4$ (b) $x^2 - 16 = 0$ (c) $(x - 5)^2 = 7$

4. Solve quadratic equations by completing the square.
++Solve by completing the square: (a) $x^2 - 6x - 2 = 0$ (b) $3x^2 - 12x + 5 = 0$

5. Write the quadratic formula from memory.

6. Identify the coefficients a, b and c when given a quadratic equation.

7. Use the quadratic equation to solve quadratic equations.
++Solve by use of the quadratic formula:
(a) $2x^2 - 3x - 5 = 0$ (b) $x(3x + 2) = 5(3x + 4)$

GOAL 1: Using periods, commas, semicolons, and connecting words, the student will construct and appropriately punctuate a variety of sentences in simple, compound, and complex patterns. He will also edit out of his own writing such problems as run-ons, comma splice, and sentence fragments.

OBJECTIVES

PERIODS

1.1 Given a sentence which includes periods, commas, semicolons, and apostrophes, the student will be able to identify each punctuation symbol with 100% accuracy.

1.2 Given a list of 10 word groups including simple sentences or phrases but with periods omitted, the student will show that he recognizes sentences by the fact that he places a period at the end of any word group which includes both subject and verb.

1.3 Given a list of 10 simple sentences with either the subject or the verb missing, the student will recognize the omission and insert an appropriate subject or verb.

1.4 Given a paragraph of at least 5 simple sentences . . . the student will be able to recognize omissions and/or inappropriately placed periods with 100% accuracy.

1.5 Given a list of 5 simple sentences which begin with the first-person singular pronoun, or with proper nouns, and which require but omit only periods, the student will be able to insert periods at appropriate places with 100% accuracy.

1.6 Given a paragraph of at least 5 simple sentences . . . the student will be able to insert periods at appropriate places with 100% accuracy.

1.7 The student will be able to generate a list of 5 simple sentences, appropriately punctuated with periods, with 100% accuracy.

1.8 Given a paragraph of 10 simple sentences which begin with I or with proper nouns, and some of which are run together without punctuation, the student will identify run-on sentences with 100% accuracy.

OPTIONAL OR BRANCHING

1.9 The student will be able to generate a paragraph of at least 5 simple sentences, all of which are appropriately punctuated with periods, with 100% accuracy.

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COMMAS IN SERIES

1.10 Given a list of 5 simple sentences which include single words in a series,
the student will be able to recognize omissions and/or inappropriately placed commas, including the final comma before the word <u>and</u>, with 100% accuracy.

1.11 Given an otherwise correctly punctuated paragraph of at least 5 simple sentences which include single words in a series,
the student will be able to recognize omissions and/or inappropriately placed commas, including the final comma before the word <u>and</u>, with 100% accuracy.

1.12 Given a list of 5 simple sentences which include single words in a series, and which require but omit only commas,
the student will be able to insert commas at appropriate places, including the final comma before the word <u>and</u>, with 100% accuracy.

1.13 Given a paragraph of at least 5 simple sentences . . .
the student will be able to insert commas at appropriate places, including the final comma before the word <u>and</u>, with 100% accuracy.

1.14 The student will be able to generate a list of 5 simple sentences which include words in a series and which are appropriately punctuated with commas with at least 90% accuracy.

OPTIONAL OR BRANCHING

1.15 The student will be able to generate a paragraph of at least 5 simple sentences which include words in a series and which are appropriately punctuated with commas with at least 80% accuracy.

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COMMAS - INTRO. WORDS, PHRASES

1.16 Given a list of 5 simple sentences which include introductory words or phrases, but which require no other internal punctuation,
the student will be able to recognize omissions and/or inappropriately placed commas with 100% accuracy.

1.17 Given an otherwise correctly punctuated paragraph of at least 5 simple sentences with introductory words or phrases,
the student will be able to recognize omissions and/or inappropriately placed commas with 100% accuracy.

1.18 Given a list of 5 simple sentences with introductory words or phrases which require but omit commas,
the student will be able to insert commas at appropriate places with 100% accuracy.

GOAL 1: (Cont'd.)

1.19 Given a paragraph of at least 5 simple sentences . . .
the student will be able to insert commas at appropriate places with 100% accuracy.

1.20 The student will be able to generate a list of 5 simple sentences which include introductory words or phrases, and which are appropriately punctuated with commas, with at least 90% accuracy.

OPTIONAL OR BRANCHING

1.21 The student will be able to generate a paragraph . . . with at least 80% accuracy.

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COMMAS DATES, ETC.

1.22 Given a list of 10 simple sentences, each of which includes at least one example of dates, places, or appositives which require comma punctuation,
the student will be able to recognize omissions and/or inappropriately placed commas with 100% accuracy.

1.23 Given a list of 10 simple sentences each of which includes . . . but with commas ommitted,
the student will be able to insert commas at appropriate places with 100% accuracy.

1.24 Given a paragraph of at least 5 simple sentences several of which include . . .
the student will be able to insert commas at appropriate places with 100% accuracy.

1.25 The student will be able to generate at least 5 correctly punctuated simple sentences which include dates, places, and appositives with 100% accuracy.

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COORD. COMPOUND SENTENCES

1.26 Given a list of 25 coordinating conjunctions, subordinating conjunctions, and transitional words (i.e., alternately called transitional or conjunctive adverbs, or "phony" conjunctions, or "shifty old bridges" - SOB's):
the student will identify the 7 coordinating conjunctions with 100% accuracy.

1.27 The student will be able to write the 7 coordinating conjunctions with 100% accuracy.

1.28 Given a list of 10 simple and compound sentences which include coordinating conjunctions in compound construction,
the student will be able to insert commas between coordinate clauses with 100% accuracy.

SEMICOLONS

1.29 Given a list of 10 compound sentences with the comma and the coordinating conjunction omitted,
the student will be able to insert both commas and coordinating conjunctions with 100% accuracy.

1.30 Given a list of 10 simple and compound sentences with no punctuation or coordinating conjunction between compound clauses,
the student will insert semicolons between compound clauses with 100% accuracy.

1.31 Given a list of 10 compound sentences with compound clauses run together without punctuation or coordinating conjunctions,
the student will insert semicolons between the compound clauses with 100% accuracy.

1.32 Given a list of 15 compound sentences with the comma and coordinating conjunction omitted,
the student will be able to insert either a comma and coordinating conjunction, or a semicolon, or a period, with 100% accuracy.

1.33 Given a list of 10 simple sentences,
the student will be able to expand each of them into compound sentences which are appropriately connected with commas and coordinating conjunctions, or with semicolons, with 100% accuracy.

1.34 The student will be able to generate a list of 10 appropriately punctuated or connected compound sentences with 100% accuracy.

OPTIONAL OR BRANCHING

1.35 The student will be able to generate a paragraph of at least 5 sentences, some of which are appropriately punctuated/connected compound constructions, with 100% accuracy.

COMPOUND SENTENCES - TRANS. WORDS

1.36 Given 10 sentences,
the student will recognize comma splice or run-on errors with at least 70% accuracy.

1.37 Given a list of 20 appropriately punctuated compound sentences which include underlined coordinating and "phony" conjunctions,
the student will be able to identify the coordinating conjunctions and transitional words ("phony" conjunctions, SOB's, etc.) with 100% accuracy.

1.38 Given a list of 20 compound sentences connected with either coordinating conjunctions or "phony" conjunctions, but with punctuation omitted,
the student will be able to insert commas or semicolons in appropriate places with 100% accuracy.

GOAL 1: (Cont'd.)

1.39 Given a list of 10 compound sentences connected with semicolons, but with pairs of commas indicating where transitional words have been omitted and should be inserted in parenthetical positions <u>within the 2nd clause instead of between clauses</u> (i.e., illustrating that "shifty bridges" need not be placed between sentences),
the student will be able to insert appropriate transitional words (SOB's) between the commas with 100% accuracy.

1.40 Given a list of 15 compound sentences which include SOB's in the conjunctive or parenthetical position, but with punctuation omitted,
the student will be able to insert commas and semicolons in appropriate places with 100% accuracy.

1.41 Given a list of 20 compound sentences with all punctuation, coordinating conjunctions, and transitional words (both conjunctive and parenthetical) omitted, and a matching list of coordinating conjunctions mixed with SOB expressions,
the student will be able to insert, in appropriate places, coordinating conjunctions, "phony" conjunctions, parenthetical transitional words, and commas and semicolons with 100% accuracy.

1.42 The student will be able to generate at least 5 compound sentences using coordinating conjunctions, and "phony" conjunctions in both conjunctive and parenthetical positions with 100% accuracy.

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SUBORD.

1.43 Given a list of 25 coordinating conjunctions, transitional words, and "pure" subordinating conjunctions (i.e., excluding relative-pronoun),
the student will identify the subordinating conjunctions with 100% accuracy.

1.44 Given a list of 20 correctly punctuated complex sentences, which include underlined coordinating conjunctions, "phony" conjunctions (i.e., transitional words), and subordinating conjunctions,
the student will identify the subordinating conjunctions with 100% accuracy.

1.45 Given a list of 10 pairs of independent clauses,
the student will insert a subordinating conjunction between each 2 clauses to make the second one a dependent adverbial clause, 100% accuracy.

1.46 Given a list of 10 pairs of independent clauses,
the student will supply a subordinating conjunction before the first clause to make it a dependent adverbial clause, and he will insert a comma after the introductory adverbial clause with 100% accuracy.

1.47 Given a list of 10 pairs of independent clauses,
the student will supply a subordinating conjunction either before the first clause or between the clauses to create one dependent adverbial clause, and he will punctuate with a comma when appropriate. 100% accuracy.

1.48 Given a list of 10 complex sentences with introductory and ending adverbial clauses, requiring no other punctuation than commas,
the student will recognize omissions and/or inappropriately placed commas with 100% accuracy.

1.49 Given an otherwise correctly punctuated paragraph of at least 5 complex sentences with introductory and ending adverbial clauses, requiring no other punctuation than commas,
the student will recognize omissions and/or inappropriately placed commas with 100% accuracy.

1.50 Given an otherwise correctly punctuated paragraph of at least 5 complex sentences with introductory and ending adverbial clauses with commas omitted, but requiring no other punctuation,
the student will insert commas at appropriate places with 100% accuracy.

1.51 Given a list of 10 simple sentences,
the student will expand each of them into an appropriately punctuated complex sentence with 100% accuracy.

1.52 The student will generate a list of 5 complex sentences with introductory and ending adverbial clauses, and with commas appropriately placed, with at least 90% accuracy.

1.53 The student will generate at least 5 sentences which include at least one simple, one compound, and one complex sentence.

1.54 The student will generate a correctly punctuated paragraph of at least 5 sentences . . . with at least 80% accuracy.

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GOAL 1: (Cont'd.)

OPTIONAL 1.55 Given a list of 10 complex sentences in which the subordinating conjunction is a relative pronoun or an adverb,
the student will identify the subordinating conjunction with 100% accuracy.

OPTIONAL 1.56 Given a list of 5 complex sentences each of which includes a restrictive or non-restrictive clause,
the student will recognize omissions and/or inappropriately placed commas with 80% accuracy.

OPTIONAL 1.57 Given a list of 10 sentences each of which includes . . .
the student will insert commas at appropriate places with 80% accuracy.

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1.58 Given 10 clauses,
the student will discriminate between a simple sentence and a subordinate clause with 70% accuracy.

FRAGS.

1.59 Given 10 word groups which include phrases and subordinate clauses,
the student will distinguish between the sentences and the fragments with at least 80% accuracy.

1.60 Given 5 fragments which include both phrases and clauses,
the student will create simple sentences with at least 80% accuracy.

1.61 Given 5 fragments which include both phrases and clauses,
the student will create simple sentences and/or add independent clauses to make complex sentences.

OPTIONAL OR BRANCHING 1.62 The student will generate a paragraph of at least 6 sentences which include both phrases and clauses, none of which are fragments.

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OPTIONAL 1.63 Given 10 complex sentences,
the student will recognize adjective, noun, and adverb clauses with 70% accuracy.

OPTIONAL 1.64 The student will generate at least 5 sentences which include at least one noun clause, one adverb clause, and one adjective clause.

OPTIONAL 1.65 Given a paragraph of at least 5 simple sentences,
the student will convert them to logically coordinated and subordinated sentences.

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OPTIONAL 1.66 Given 10 simple sentences having either action or linking verbs, the student will discriminate between the action verb and the linking verb with 80% accuracy.

OPTIONAL 1.67 Given 10 simple sentences in the five basic patterns (S-V, S-V-DO, S-V-IO-DO, S-V-SC, and There-V-S), the student will recognize each pattern with 70% accuracy.

OPTIONAL 1.68 The student will be able to generate several sentences illustrating the linking and action verbs.

OPTIONAL 1.69 The student will generate at least 5 sentences illustrating each of the basic sentence patterns.

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PRON. REF. - AGREEMENT

__GOAL 2: The student will distinguish between acceptable and unacceptable (i.e., standard) usage and diction.__

2.1 Given a list of 15 words, including personal pronouns,
the student will recognize the personal pronouns with 100% accuracy.

2.2 Given a list of 10 sentences which include personal pronouns,
the student will recognize the personal pronouns with 100% accuracy.

2.3 Given a list of 10 sentences which include singular and plural personal pronouns,
the student will recognize the personal pronouns and indicate whether they are singular or plural with 100% accuracy.

2.4 Given a list of 10 nouns (which are plural if they end with _s_),
the student will indicate which are singular and which are plural with 100% accuracy.

2.5 Given a list of 10 simple sentences containing a personal pronoun and a noun or pronoun antecedent,
the student will recognize the personal pronoun and its antecedent with 100% accuracy.

2.6 Given a list of 10 sentences containing a choice between a singular or a plural personal pronoun and a noun or pronoun antecedent,
the student will recognize the antecedent, determine whether it is singular or plural, and select the appropriate pronoun with 100% accuracy.

2.7 Given a list of 10 simple sentences with the pronoun omitted and a noun or pronoun antecedent,
the student will insert an appropriate personal pronoun of his choice with 100% accuracy.

2.8 Given a paragraph of 10 simple and complex sentences with the personal pronouns omitted,
the student will insert an appropriate personal pronoun of his choice with 100% accuracy.

2.9 Given a paragraph as above, with personal pronouns and antecedents omitted,
the student will insert appropriate personal pronouns and antecedents of his choice with 100% accuracy.

2.10 The student will compose a list of 10 sentences which contain pronouns and their antecedents with 90% accuracy.

OPTIONAL OR BRANCHING

2.11 The student will compose a one-half-page paragraph which uses personal pronouns and their antecedents with 90% accuracy.

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2.12 Given a list of 20 words including non-personal pronouns,
the student will recognize the pronouns with 100% accuracy.

2.13 Given a list of 10 sentences which include non-personal pronouns,
the student will recognize the pronouns with 100% accuracy.

PRON. REF.- FAULTY

2.14 Given a list of 10 sentences which include singular and plural non-personal pronouns, and antecedents when appropriate,
the student will recognize the pronouns and whether they are singular or plural with 100% accuracy.

2.15 Given a list of 10 complex sentences which include personal and non-personal pronouns with vague or ambiguous reference,
the student will identify which pronouns are used in vague or ambiguous (unclear) ways.

2.16 Given a list of 10 complex sentences which include . . .
the student will correct the faulty reference by substituting nouns and/or inserting additional words.

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PRON. CASE

2.17 Given a list of 10 sentences using pronouns in the subjective and objective case (excluding possessive case),
the student will recognize subjective and objective case forms with 100% accuracy.

2.18 Given a list of 10 sentences which use compound subjects and objects, including objects of prepositions, and which have some errors in case form,
the student will recognize errors in the use of subjective or objective case with 100% accuracy.

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GOAL 2: (Cont'd.)

2.19 Given a list of 25 words which include plurals of words as words, symbols, numbers and letters and possessives of compound nouns,
the student will be able to insert the apostrophes in the appropriate position with 100% accuracy.

2.20 Given a list of 20 contractions with apostrophes omitted,
the student will be able to insert the apostrophe between the appropriate letters with 100% accuracy.

2.21 Given a list of 20 word-groups frequently contracted,
the student will be able to form the contraction correctly with 100% accuracy.

2.22 Given a paragraph of 5 or 6 sentences having errors in apostrophe usage for contractions,
the student will be able to identify the errors with 100% accuracy.

2.23 Given a list of 15 possessive pronouns and <u>whose</u> correctly and incorrectly formed,
the student will be able to recognize those which are incorrect with 100% accuracy.

2.24 Given a list of 20 personal pronouns and <u>who</u> in the nominative case,
the student will be able to form the possessive forms with 100% accuracy.

2.25 Given a paragraph of 5 or 6 sentences having errors in apostrophe usage for possessive pronouns and <u>whose</u>,
the student will be able to identify the errors with 100% accuracy.

2.26 Given a list of 20 possessives,
the student will be able to recognize the singulars and plurals with 100% accuracy.

2.27 Given 10 sentences with two possessive forms (boy's, boys')
the student will be able to choose the correct form with 80% accuracy.

2.28 The student will be able to generate a sentence illustrating the use of the apostrophe for singular possessive of a noun.

2.29 The student will be able to generate a sentence illustrating the use of the apostrophe for possessive form of a plural noun ending in <u>s</u>.

2.30 The student will be able to generate a sentence illustrating the use of the apostrophe for possessive form of a plural noun not ending in <u>s</u>.

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VERBS - RECOGNITION

2.31 Given a list of 10 sentences with both verbs and non-verbs underlined,
the student will recognize the verbs with 100% accuracy.

2.32 Given a list of 10 sentences, which incorporate the use of helping verbs,
the student will recognize the helping verbs with 100% accuracy.

2.33 Given a list of 10 sentences which incorporate the use of helping verbs.
the student will recognize both the helping and the main verbs with 100% accuracy.

2.34 Given a list of 10 two- to four-word declarative sentences in the present tense,
the student will recognize standard and non-standard usages in agreement with 100% accuracy.

2.35 Given a paragraph of at least 10 sentences with some helping and some main verbs omitted,
the student will insert verbs of his choice in the appropriate form with 90% accuracy.

VERBS - TENSE, S-V AGREEMENT

2.36 Given 10 regular and irregular verbs, including the verbs <u>do</u>, <u>see</u>, <u>be</u>, and <u>have</u> with a conjugation chart labeling all 6 tenses, all 3 persons under each tense, and singular and plural number,
the student will fill in the correct person, number, and tense using personal pronouns and the appropriate form of each of the given 10 verbs.

2.37 Given a list of 20 simple sentences with the verbs underlined, and including examples of all 6 tenses, singular and plural, and all 3 persons,
the student will identify the person, number, and tense of each verb with 90% accuracy.

2.38 Given a list of 20 sentences containing regular and irregular verbs in the infinitive form,
the student will change the infinitive to past tense form with 100% accuracy.

2.39 Given a list of 20 sentences containing regular and irregular verbs with an appropriate helping verb for a past participle form, but with the main verb identified by the infinitive form,
the student will change the main verb to the appropriate past participle form with 90% accuracy.

GOAL 2: (Cont'd.)

S-V AGREEMENT

2.40 Given a list of 20 sentences containing regular and irregular verbs in the infinitive form, and specifying present, past, or past participle form,
the student will change the infinitive to the specified present, past, or past participle form with 90% accuracy.

2.41 Given a list of 20 simple, compound, and complex sentences with a verb omitted,
the student will insert an appropriate verb in acceptable form with 90% accuracy.

2.42 Given a paragraph of at least 20 sentences containing 10 main regular or irregular verbs misused,
the student will recognize the misused verbs and insert the appropriate usage with 90% accuracy.

2.43 The student will generate a list of 10 sentences using specified person and tense, singular and plural, with at least 90% accuracy.

OPTIONAL OR BRANCHING

2.44 Given a paragraph which uses inconsistent tense,
the student will revise for consistent verb tense.

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2.45 Given a list of 10 nouns which use <u>s</u> or <u>es</u> for plural, including collective nouns,
the student will be able to designate which are singular with 100% accuracy.

2.46 Given a list of 10 sentences which include underlined nouns which use <u>s</u> or <u>es</u> for plural, including collective nouns,
the student will be able to designate which nouns are singular with 100% accuracy.

2.47 Given a list of 10 sentences in the present tense (verb in the infinitive form) with prepositional phrases following the single-noun or indefinite-pronoun subject,
the student will recognize the subject and change the infinitive to appropriate present-tense form with 100% accuracy.

2.48 Given a list of 10 sentences in the present tense (verb in the infinitive form) with compound subjects joined by <u>and</u>,
the student will recognize the subject and appropriate present-tense form with 100% accuracy.

2.49 Given a list of 10 sentences in the present tense (verb in the infinitive form) with compound subjects joined by <u>or</u> or <u>nor</u>,
the student will recognize the nearer subject and change the infinitive to the present-tense form with 100% accuracy.

2.50 Given a list of unusual nouns or pronouns (e.g., tweezers, measles, couple, a lot, athletics)
the student will indicate the subjects that take a singular verb with 90% accuracy.

2.51 Given a list of 10 sentences in the present tense with a variety of singular and plural subjects with the verb omitted,
the student will select an appropriate present-tense verb with 90% accuracy.

OPTIONAL OR BRANCHING 2.52 The student will compose a paragraph of at least 5 sentences in which subjects and verbs are appropriately matched.

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VOICE

OPTIONAL 2.53 Given a list of 10 sentences which include verbs in both active and passive voice,
the student will distinguish between active and passive voice with at least 70% accuracy.

OPTIONAL 2.54 The student will be able to generate at least 5 sentences in the active voice.

OPTIONAL 2.55 The student will generate at least 5 sentences in the passive voice.

OPTIONAL 2.56 Given a list of 5 sentences written in the passive voice,
the student will change the voice to active with 80% accuracy.

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USAGE

2.57 Given examples of writing illustrative of different levels of usage,
the student will recognize in context the levels of usage as described by any of the collegiate dictionaries.

2.58 Given a list of words,
the student will find in a collegiate dictionary the level of usage as indicated by a specific dictionary.

2.59 The student will take a specimen passage of prose and analyze all the words in it for their standard or non-standard usage.

2.60 Given a variety of "formal" and "informal" situations, and a parallel variety of specimen passages,
the student will match passages and situations appropriately.

GOAL 2: (Cont'd.)

2.61 The student will write of an event or experience within both formal and informal contexts (e.g., he will distinguish in his own writing the difference between a "gripe" and a "complaint", etc.)

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2.62 Given a list of 10 sentences which include misplaced modifying words and phrases, the student will recognize which sentences include mis-placed modifiers with 100% accuracy.

2.63 Given a list of 10 sentences which include misplaced modifying words and phrases, the student will identify which words or phrases are misplaced with 100% accuracy.

2.64 Given the same list of sentences as above, the student will move the misplaced words or phrases to appropriate positions which clarify sentence meaning. 90% accuracy.

2.65 Given a list of 10 words and phrases which are likely to be used as adverbs or adjectives within the context of a sentence, the student will construc 10 sentences which incorporate the given words and phrases and which so place them as to avoid misplaced modifiers. 90% accuracy.

2.66 Given a list of 20 sentences which include both dangling and correctly used adjective phrases, the student will identify which sentences use dangling modifiers. 100% accuracy.

2.67 Given a list of 10 sentences which include dangling modifiers, the student will re-write the sentences to correct the errors. 90% accuracy.

2.68 Given a list of 20 verbal phrases, the student will construct sentences to include them in such a way as to avoid both misplaced and dangling modifiers. 90% accuracy.

OPTIONAL 2.69 Given two lists, one of words or short phrases and the second of pretentious words or longer phrases,
the student will match the short word/concise phrase with the pretentious word/wordy phrase with 100% accuracy

OPTIONAL 2.70 Given a list of 20 paired sentences, one wordy and one concise,
the student will select the concise sentence with 100% accuracy.

OPTIONAL 2.71 Given a list of 10 sentences containing wordy phrases or pretentious words and a matching list of concise phrases or unpretentious words,
the student will recognize the wordiness/pretentiousness and substitute the concise/unpretentious alternate.

OPTIONAL 2.72 Given a list of 10 sentences containing unnecessary or redundant words,
the student will recognize (cross out) the unnecessary or redundant words with 90% accuracy.

OPTIONAL 2.73 Given a list of 10 concise sentences containing simple words,
the student will load up the sentence with wordy phrases, redundancies, and pretentious words. (Double the number of words per sentence or letters per word with 100% accuracy.)

OPTIONAL 2.74 Given a paragraph of at least 15 sentences containing redundancies, wordiness, and pretentious words,
the student will revise the paragraph. (Eliminate half the words and simplify the pretentious words with 90% accuracy.)

OPTIONAL 2.75 Given 10 pair of simple sentences which contain wordiness, redundancies, and pretentious words,
the student will eliminate the wordiness/redundous/pretentious and combine the paired sentences into a single sentence.

OPTIONAL 2.76 The student will write a 20 sentence paragraph and then revise it so that it is free from wordiness and pretentious words.

GOAL 3: The student will write, within a 50-minute period, a paragraph of at least 5 sentences (or at least 150 words) including each of the following qualities: an appropriately narrowed (limited) topic, a clear topic sentence, no irrelevant or extraneous detail (i.e., all sentences relate to and support the topic sentence), coherence, appropriate diction, and an acceptable standard level of mechanics (i.e., "edited english"). The style will include authentic detail which clearly reflects the maturity, personality, and experience of the writer in a formal situation.

3.1 Given a picture or cartoon which incorporates at least 3-4 separate situations (e.g., a tourist family in a heavily loaded car which has a flat tire, is stranded in deserted area, stopped beside a phone booth in which a bear is using the phone, etc.),
the student will generate a single sentence incorporating all of the elements of the pictured situation.

3.2 Given a list of 10 sentences containing rather ordinary verbs, adverb-verb combinations, and verbals (all underlined),
the student will supply an average of 10 vivid alternatives for each.

3.3 Given a paragraph of at least 10 sentences,
the student will suggest vivid substitute verbals and verbs in appropriate places.

3.4 Given a list of 20 nouns or adjective-noun combinations,
the student will supply an average of 7 specific noun synonyms for each.

3.5 Given a list of 10 sentences containing rather abstract nouns and adjective-noun combinations,
the student will recognize them and supply more specific noun synonyms where appropriate.

3.6 Given a series of 10 sentences with colorless nouns and/or noun-adjective combinations,
the student will be able to substitute 5-6 more colorful alternatives for each, including re-writing the sentences.

3.7 Given 5-10 general (abstract) sentences,
the student will transform each into a sentence using concrete detail.

3.8 The student will write and revise 10 sentences which use concrete detail, including explicitly vivid verbs, verbals, and nouns.

3.9 The student will write and revise a paragraph of at least 5 sentences which contain vivid nouns, verbs, and verbals.

3.10 Given 3 social situations illustrative of 3 levels of usage (e.g., leaving a note for the milkman, writing a congressman, writing a friend, etc.),
the student will generate 3 sentences appropriate to the 3 situations and levels of usage.

3.11 Given a paragraph of 5-10 sentences arranged in scrambled sequence and including both a topic sentence and irrelevant sentences,
the student will arrange most of the sentences into a paragraph which omits irrelevant sentences, and coordinate or subordinate those sentences which logically belong together.

3.12 Given 5-10 sentences arranged in scrambled sequence and including irrelevant sentences,
the student will develop a topic sentence, arrange most of the sentences into a paragraph which omits irrelevant sentences, and coordinate or subordinate those sentences which logically belong together.

3.13 Given an abstract topic,
the student will narrow the topic and create a topic sentence with an argumentative edge.

3.14 The student will write a paragraph of at least 5 sentences, on a topic of personal concern or interest, in which he narrows the topic, creates a topic sentence with an argumentative edge, arranges his sentences in logical sequence, and uses concrete/vivid detail and diction.

3.15 The student will write, outside of class, a paragraph of at least 5 sentences (or at least 150 words) including each of the following qualities: an appropriately narrowed (limited) topic, a clear topic sentence, no irrelevant or extraneous detail (i.e., all sentences relate to and support the topic sentence), coherence, appropriate diction, and an acceptable standard level of mechanics (i.e., "edited english"). The style will include authentic detail which clearly reflects the maturity, personality, and experience of the writer in a formal situation.

This is a Unit Test for Mastery.

ELEMENTARY ALGEBRA
TEST 12 FORM A

NAME______________________
DATE______________ HR________

10 points per problem

Express each phrase symbolically using "x" as a variable.

1. a) The sum of twice an integer and 3. ________
   b) Three times an integer subtracted from its square. ________

2. a) The sum of an even integer and the next consecutive odd integer. ________
   b) The sum of an odd integer and the preceding odd integer. ________

The sum of two real numbers is 37. If x represents the larger number, represent in terms of x:

3. a) The small number. ________
   b) Three times the smaller number. ________

4. a) Two times the larger number. ________
   b) The sum of the larger number and the smaller number. ________

I have 4 more dimes in my bank as quarters. If I have "x" dimes, represent in terms of x:

5. a) The number of quarters in the bank. ________
   b) The value of the quarters in the bank. ________

6. a) The value of the dimes in the bank. ________
   b) The total value of the dimes and quarters in the bank. ________

7. Find three consecutive integers whose sum is 360. ________

8. A 27 foot rope is cut into 2 pieces, so that one piece is 7 feet longer than the other.
   a) If x represents the length of the shorter piece, represent in terms of x the length of the longer piece. ________
   b) Find the length of the shorter piece. ________

9. A grocer mixes 3 lbs. of 29¢ tea with x lbs. of 48¢ tea to obtain a new mixture of 6 lbs. of tea selling for 37¢ per pound.
   a) Represent in terms of x the number of pounds of 48¢ tea. ________
   b) Represent the cost of 3 lbs. at 29¢ per pound. ________

10. One truck has a capacity of 9 tons more than another. Together in 6 trips, they haul 126 tons. How many tons does the larger truck carry in one trip?

____

This is a linear program which follows verbal instructions given on cassette tape.

ELEMENTARY ALGEBRA

LOOP #12

1.

Algebra	English
+	more than, increased by, greater than
=	is, equal
x	times, of
-	less than

---

2. If n represents a number, we can represent

twice the number as ____2n____

three more than the number as ____n + 3____

a. four more than the number as ________

b. two less than the number as ________

c. the sum of the number and six as ________

one-half of the number as ____$\frac{1}{2}n$ or $\frac{n}{2}$____

d. one-third of the number as ________

e. two-fifths of the number as ________

3.

consecutive integers

2, 3, 4	2, 2 + 1, 2 + 2	n, n + 1, n + 2

consecutive even integers

6, 8, 10	6, 6 + 2, 6 + 4	n, n + 2, n + 4

consecutive odd integers

7, 9, 11	7, 7 + 2, 7 + 4	n, n + 2, n + 4

---

4. The sum of two consecutive odd integers is 16.

Find the integer.

Let n = first integer

n + 2 = second integer

first integer + second integer = 16

$$n + n + 2 = 16$$

$$2n + 2 = 16$$

$$2n = 14$$

$$n = 7$$

$$n + 2 = 9$$

---

5. The sum of an even integer and three times the next consecutive even integer is 46. Find the integers.

Let x = first integer

a. ________ = second integer

b. ________ = 3 times the second integer

c. Write an equation to indicate

first even integer + 3 times second even integer = 46

________________ + ________________ = 46

d. Solve the equation to find the integers.

---

6. One integer is four times another. The sum of the smaller integer and six more than the larger is 26. Find the integers.

7. How many pounds of nuts selling for 90 cents a pound must a grocer mix with nuts selling for 60 cents a pound to get 18 pounds of a mixture selling for 80 cents a pound?

Let x = the number of pounds of 90 cent nuts

a. The number of pounds of 60 cent nuts is represented by ____________________

b. The price or value of all the 90 cent nuts is ________________

c. The price of all the 60 cent nuts is ________________

d. The price of 18 pounds of 80 cent nuts is _____________

value of 90 cent nuts	+	value of 60 cent nuts	=	value of 80 cent nuts
e. ________	+	_______	=	_______

---

8. $90x + 60(18 - x) = 80(18)$

$90x + 60 \cdot 18 - x = 1440$

$90x - 60x + 1080 = 1440$

$30x + 1080 = 1440$

$30x = 1440 - 1080$

$30x = 360$

$x = 12$

---

9. Practice Problems

1. The sums of two numbers is 76. One of the numbers is 22 more than the other number Find the numbers.
2. The sum of two numbers is 438. One of the numbers is 26 less than the other number. Find the numbers.
3. The sum of two numbers is 118. One of the numbers is 8 less than the other number. Find the numbers.
4. The sum of two numbers is 147. One of the numbers is twice the other number. What are the numbers?
5. Find three consecutive integers whose sum is 174.
6. Find three consecutive even integers whose sum is 90.
7. How many pounds of peanuts that sell for 60 cents per pound must mix with cashew nuts that sell for 90 cents per pound to make a mixture of 10 pounds that will sell for 70 cents per pound?
8. A grocer mixes two grades of tea that sell for 60 cents and 70 cents per pound, respectively. How many pounds of each must he use to make a mixture of 40 pounds that will sell for 67 cents per pound?

Answers - Loop 12

Frame 2

a. $n + 4$

b. $n - 2$

c. $n + 6$

d. $\frac{1}{3}n$ or $\frac{n}{3}$

e. $\frac{2}{5}n$ or $\frac{2n}{5}$

Frame 5

a. $x + 2$

b. $3(x + 2)$

c. $x + 3(x + 2) = 46$

$x + 3x + 6 = 46$

$4x = 40$

$x = 10, \ x + 2 = 12$

$10 + 3(12) = 46$

Frame 6

x = first integer

$4x$ = second integer

$x + (4x + 6) = 26$

$5x + 6 = 26$

$5x = 20$

$x = 4, \ 4x = 16$

Frame 8

the length of the other piece is ___40 - x___

x = length of first piece, $3x$ = length of second piece

$x + 3x = 40$

$4x = 40$

$x = 10, \ 3x = 30 = 40 - x$

Frame 11

a. $18 - x$ b. $90x$ c. $60(18 - x)$ d. $18(80)$

e. $90x + 60(18 - x) = 18(80)$

Frame 13

1. x = first number

$x + 22$ = second number

$$x + (x + 22) = 76$$
$$2x + 22 = 76$$
$$2x = 54$$
$$x = 27, \quad x + 22 = 49$$

2. x = first number

$x - 26$ = second number

$$x + (x - 26) = 438$$
$$2x - 26 = 438$$
$$2x = 464$$
$$x = 232, \quad x - 26 = 206$$

3. x = first number

$x = 8$ = second number

$$x + (x - 8) = 118$$
$$2x - 8 = 118$$
$$2x = 126$$
$$x = 63, \quad x - 8 = 55$$

4. x = first number

$2x$ = second number

$$x + 2x = 147$$
$$3x = 147$$
$$x = 49, \quad 2x = 98$$

5. x = first number

$x + 1$ = second number

$x + 2$ = third number

$$x + (x + 1) + (x + 2) = 174$$
$$3x + 3 = 174$$
$$3x = 171$$
$$x = 57, \quad x + 1 = 58, \quad x + 2 = 59$$

6. x = first number

$x + 2$ = second number

$x + 4$ = third number

$$x + (x + 2) + (x + 4) = 90$$
$$3x + 6 = 90$$
$$3x = 84$$
$$x = 28,\ x + 2 = 30,\ x + 4 = 32$$

7. x = the number of pounds of peanuts

$60x$ = value of the 60¢ peanuts

$10 - x$ = number of pounds of cashews

$90(10 - x)$ = value of 90¢ cashews

$10(70)$ = value of mixture nuts at 70¢ per pound

$$60x + 90(10 - x) = 10(70)$$
$$60x + 900 - 90x = 700$$
$$-30x + 900 = 700$$
$$900 - 700 = 30x$$
$$200 = 30x$$
$$6\frac{2}{3} = \frac{20}{3} = x,\ 3\frac{1}{3} = 10 - x$$

$6\frac{2}{3}$ pounds of peanuts, $3\frac{1}{3}$ pounds of cashews

8. x = number of pounds of 60¢ tea

$60x$ = value of 60¢ tea

$40 - x$ = number of pounds of 70¢ tea

$70(40 - x)$ = value of 70¢ tea

$40(67)$ = value of mixture of tea at 67¢ per pound

$$60x + 70(40 - x) = 40(67)$$
$$60x + 2800 - 70x = 2680$$
$$10x = 120$$
$$x = 12,\ 40 - x = 28$$

12 pounds of 60¢ tea: 28 pounds of 70¢ tea

Sample Remediation Material: This is a script of a programmed dialogue which is on cassette tape. The tape follows a written linear program.

ELEMENTARY ALGEBRA

LOOP #12

Frame 1

Word problems are a part of algebra that students find difficult. This may be because word problems require translating the language of English into the language of algebra. Also, the student is usually very familiar with the language of English, but learning algebra is like learning a new language. The language of English is not as precise as the language we use in algebra. Therefore, it is often difficult to understand exactly what is meant by the problem, as stated in English, and to express this problem in the language of algebra. However, there are some rules of translation that can make this easier. Like, a plus sign in the language of algebra is usually read as, more than, increased by, or greater than, in the language of English. An equal sign in the language of algebra can almost always be translated to, is equal or the same as in English. A multiplication sign usually means "times" or "of", like half of ten in English can be written one-half times 10 in algebra. A subtraction sign in algebra means less than, but watch out, because in English when we say four less than 10, you write 10 subtraction sign 4. Notice that the 10 came first, not the four.

Frame 2

This frame is some practice representing simple quantities in letters. If "n" represents a number, we can represent twice that number as 2 times n or 2n. We can represent three more than the number as n plus 3. Question A: How can we represent four more than the number? Write this quantity on the blank. We can represent it as n plus four. Now work problems B, C, D and E. Check the answers in the back of this loop before you turn on the tape recorder for Frame 3.

Frame 3 - Ways to represent consecutive integers

Consecutive integers are integers that follow right after each other, like 2, 3, 4, 5, 6, and so forth. Now we could represent 2, 3, and 4 as 2, 2 plus 1, and 2 plus 2, which seems a little strange. But suppose we didn't know what the first integer was. We could just call it "n" and you could represent the next

integer as n plus 1, and the integer after that n plus 2, and the integer after that n plus 3, and so on. Consecutive even integers are integers that differ by two, like 6, 8, and 10. So you could represent 6, 8, and 10 as 6, 6 plus 2, 6 plus 4. Or if we didn't know what the first integer was, we could say that the first integer was "n", the integer after that was n plus 2, the integer after that was n plus 4. Consecutive odd integers also differ by 2. For instance, 7, 9, and 11 could be written 7, 7 plus 2, and 7 plus 4. We can represent consecutive odd integers n, n plus 2, n plus 4. But you may look at the representations for consecutive even and consecutive odd integers and say to yourself, "Wait, it's the same representation", and you're right. The secret is what we let "n" equal. If we let "n" equal an odd integer then n plus 2 is the next odd integer. If "n" stands for an even integer, then n plus 2 is the next even integer.

Frame 4

Here's an example of a basic type of a stated problem: The sum of two consecutive odd integers is 16. Find the integers. First, we let n equal the first integer. Then, as we did in Frame 3 we can say n plus 2 is equal to the second integer. Now the problem tells us that the sum of the two consecutive odd integers is 16. That translates to say that the first integer plus the second integer is equal to 16. And if n represents the first integer then n plus 2 equals the second integer. So we can write an equation that describes this situation. n plus, n plus 2 equals 16. Combining like terms, n and n are 2n plus 2 equals 16. Subtracting a 2 from both sides, 2n equals 14. Multiplying both sides by one-half, n equals 7. That's the first integer. The second integer is n plus 2 or, 7 plus 2 which is 9. Then we check to be sure that this is the right answer. Is it true that the sum of 7 and 9 is 16? Yes, then we have found the two integers.

Frame 5

Here is a problem for you to work with integers. I have given you some hints. If you answer each question in order, you should have enough information to be able to solve the problem. When you think that you have worked out the problem check the answers at the back of this package before you proceed to the next frame. Shut off the tape while you work.

Frame 6

Here is another problem with integers. Try to work it on your own. Check your answer at the end of this package before going onto the next frame.

Frame 7

Suppose that you have a board that is 15 feet long and you cut two feet off the board. How many feet are left on the board? Well, the answer is 13. How did you get that? You took two feet from 15 feet and said, if I cut two feet off the board, then 15 minus 2 feet is how much board is left. Now let's generalize this problem. Suppose that you cut 3 feet off the board, you would have 15 minus 3 feet left. Now suppose that you cut 5 feet off the board, well, there's 15 minus 5 feet left for 10 feet. But suppose that you cut x feet off the board. How many feet are left? The answer is 15 minus x feet. This trick is often used in algebra. When we want to divide a quantity into two parts, and we do not know exactly the size of each part, we let x represent one part and the original quantity minus x then represents the other part.

Frame 8

A forty-foot rope is cut into two pieces. If we let x represent the length of one piece, how do you represent the length of the other piece? Shut off the tape recorder if you need to think before writing the answer in the blank. 40 minus x is the way that you represent the length of the other piece and is what you should have written in the blank. Now if one piece is 3 times the length of the other piece, what are the lengths of the two pieces? To solve this problem, write an algebraic equation that utilizes your knowledge that one piece can be represented as x and the other piece as 40 minus x. And have this equation express the second part of the statement, that if one piece is three times the length of the other piece. Remember that "is" is an equal sign. Solve this problem in the space provided. The complete solution to this problem is written at the end of this loop, if you want to check.

Frame 9

How much money must you pay for 6 pounds of oranges at 10 cents a pound. You'll probably answer instantly 60 cents. Stop and notice that you got that by multiplying 6 times 10 cents a pound. How much for 7 pounds of oranges? 70 cents or 7 times 10 cents a pound. O.K. Then how much money must you pay for x pounds of oranges at 10 cents a pound? The answer is 10x or the price of one pound of oranges times x, the number of pounds of oranges that you want to buy.

Frame 10

How much money must you pay for 3 pounds of apples at 15 cents a pound and 5 pounds of pears at 20 cents a pound? There's no need to use variables here.

Just fill in the blanks and turn the tape recorder back on when you are ready to hear the answer. There are 3 pounds of apples at 15 cents a pound so the price of the apples is 3 times 15 or 45, and there are 5 pounds of pears at 20 cents a pound so the price of the pears is 5 times 20 or 100 cents. Therefore the total price for the apples and the pears is 45 plus 100 or 145 cents. Now notice that I kept this all in terms of cents rather than saying a dollar forty-five. You could have said a dollar forty-five, but it makes it much easier to solve the equation if you translate all dollar and cent figures into cents. That way we do not have to work with decimals.

Frame 11

How many pounds of nuts selling for 90 cents a pound must the grocer mix with nuts selling for 60 cents a pound to get 18 pounds of a mixture selling for 80 cents a pound? This is an example of a dreaded mixture problem. Don't panic, they can be solved. I learned how to do them, you can learn how to do them. The thing to do is to go back and use the trick that we learned in Frame 7 about cutting a board into two pieces. Notice that the problem said that we are to get 18 total pounds of nuts when we are finished. Where are the 18 pounds of nuts to come from? Well, they are to come from the number of pounds of 90 cent a pound nuts and the number of pounds of 60 cent a pound nuts. If we let x equal the number of pounds of 90 cent nuts then the number of pounds of 60 cent nuts can be represented as the total number of pounds minus x, or 18 minus x. Question B is what is the price or value of all the 90 cent nuts. If there were five pounds of the 90 cent nuts it would be 5 times 90 or $4.50. But there is x pounds of 90 cent nuts so the value of all the 90 cent nuts must be 90 times x. Well, how about the price of the 60 cent nuts. Well, if there were 4 pounds of 60 cent nuts the price would be 4 times 60. We don't know how many pounds, but we have represented the number of pounds of the 60 cent nuts as 18 minus x. So the price of the 60 cent nuts must be 60 times (18 minus x). Part D, what would be the price of 18 pounds of the 80 cents a pound nuts? Remember that 18 pounds is the final result, and we are going to sell them for 80 cents per pound. The answer is 80 times 18. Now we can write an equation that describes the situation. The equation is based on the principle that the value or cost of both kinds of nuts that go into the mixture added together must equal the cost of the total mixture at the end. Shut off the tape recorder and write in the blank in E, an equation that describes this situation. Turn on the tape recorder when you are ready to hear the answer.

Frame 12

The equation that you should have written in Frame 11 was 90x to represent the value or the price of the 90 cent nuts, namely, 90 cents a pound times x, the pounds of 90 cent nuts, plus 60 times 18 - x. That is 60 cents a pound, the price per pound of the nuts, times 18 minus x, the number of pounds of the 60 cent nuts, equal to 80 times 18, because 80 cents per pound is the price of the 80 cent a pound nuts and 18 represents the number of pounds of 80 cent nuts. Now we can solve the equation and find x, which is the number of pounds of 90 cent nuts we should mix. Applying the distributive property: 90x plus (60 times 18) minus (60 times x) equals 80 times 18 or 1440. Collecting like terms, 90x minus 60x is 30x plus 1080 equals 1440. Subtracting 1080 from both sides we get 30x equals 1440 minus 1080 or 30x equals 360, and multiplying both sides by 1/30th or by dividing both sides by 30, we get x equals 12. This tells us that our grocer should mix in 12 pounds of 90 cent a pound nuts and he should mix in 18 minus 12 or 6 pounds of 60 cent a pound nuts to get himself 18 pounds total of 80 cent a pound nuts.

Frame 13

Here's some practice problems for you to try. The answers are written out at the end of the loop. I hope this helps you to learn word problems. Go back now to your book and start with the easiest work problems in the book. Do not skip any of the easy problems because it's practice on these problems that makes it possible for you to solve harder problems in the later section. This is the end of loop 13.